Montogue

Quiz AS106
Aircraft Performance

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Problems

Problem 1

An airplane weighing 225,000 kg is flying at standard sea level with a velocity of 700 km/h. At this velocity, the L/D ratio is a maximum. The wing area and aspect ratio are 450 m^2 and 8.0, respectively. The Oswald efficiency factor is 0.93. Calculate the total drag on the airplane.

A) D = 19.4 kN

B) D = 28.5 kN

C) D = 39.7 kN

D) D = 49.8 kN

Problem 2

An aircraft with a mass of 650 kg and a wing area of 8.4 m^2 is flying steadily at sea level with a constant speed of 60 m/s. The aircraft engine generates 750 N of thrust. Assuming a zero angle of attack, determine the lift-to-drag ratio.

A) E = 6.35

B) E = 8.49

C) E = 10.6

D) E = 13.0

Problem 3

An aircraft with a mass of 3200 kg, a wing area of 30 m^2, and a drag coefficient of 0.05 is cruising at a speed of 200 knots at constant altitude. If the pilot increases the engine thrust by 40%, calculate the acceleration and the final velocity. You may assume that the drag coefficient is constant throughout the flight and that the aircraft is flying at sea level.

A) a = 1.22 m/s^2 and V_2 = 237 knots

B) a = 1.22 m/s^2 and V_2 = 265 knots

C) a = 2.44 m/s^2 and V_2 = 237 knots

D) a = 2.44 m/s^2 and V_2 = 265 knots

Problem 4.1

Calculate and plot the thrust-to-weight ratio, T/W, versus wing loading, W/S, for a generic airliner with the following specifications, flying in steady, level, unaccelerated flight at an airspeed of 900 km/h. Assume that the aircraft weight varies from the heavier weight to the lighter weight given in the table.

Parameter	Specification
Weight, light	1,900,000 N
Weight, heavy	3,000,000 N
Wing span	63.3 m
Wing area	535 m^2
Zero-lift drag, $C_{D,0}$	0.041
Span efficiency factor, e	0.71
Stall speed, V_s	200 km/h

Problem 4.2

Reconsidering the aircraft of Problem 4.1, compute the velocity for minimum thrust required and the lift coefficient for minimum thrust required.

A) $V_{F_{R,\min}}$ = 378 km/h and $C_{L,F_{R,\min}}$ = 0.444

B) $V_{F_{R,\min}}$ = 378 km/h and $C_{L,F_{R,\min}}$ = 0.828

C) $V_{F_{R,\min}}$ = 550 km/h and $C_{L,F_{R,\min}}$ = 0.444

D) $V_{F_{R,\min}}$ = 550 km/h and $C_{L,F_{R,\min}}$ = 0.828

Problem 4.3

Calculate and plot the thrust available and thrust required from the stall speed to a speed of 900 km/h, using the specifications given in Problem 4.1, then determine the maximum velocity. Assume the aircraft is at the heavy weight in the table. The aircraft is powered by four Pratt & Whitney turbofan engines, each of which produces a static sea level thrust of 200 kN. The sea level thrust of each engine, $F_{\text{eng,SL}}$, varies according to the equation

$$F_{\text{eng,SL}} = 200 + 2.37\times10^{-4} V_\infty^{\,2} - 0.244 V_\infty$$

where V_∞ has units of m/s.

Problem 4.4

Estimate the range and endurance of the generic airliner specified in Problem 4.1 for an airspeed of 300 km/h at sea level. Assume an initial weight of 2,800,000 N, a final weight of 1,900,000 N, and a thrust specific fuel consumption, $TSFC$, of 1.68×10^{-4} N/N·s.

A) R = 1540 km and E = 2.91 h

B) R = 1540 km and E = 5.83 h

C) R = 2980 km and E = 2.91 h

D) R = 2980 km and E = 5.83 h

Problem 5.1

Consider a generic twin-jet attack aircraft. The airplane has the following characteristics and is equipped with two jet engines with 40,298 N of static thrust each at sea level. Using power required and power available curves, estimate the maximum velocity.

Parameter	Specification
Weight	103,047 N
Wing area	47 m²
Aspect ratio	6.5
Oswald efficiency fator	0.87
Zero-lift drag coefficient	0.032

A) $V_{\max}$ = 450 km/h

B) $V_{\max}$ = 630 km/h

C) $V_{\max}$ = 800 km/h

D) $V_{\max}$ = 1060 km/h

Problem 5.2

Reconsidering the twin-jet airplane introduced in the previous problem, it is known that the thrust specific fuel consumption is 1.0 N of fuel per newton of thrust per hour, the fuel capacity is 1900 gal, and the maximum gross weight is 136,960 N. Calculate the range and endurance at a standard altitude of 8 km. Let the weight of fuel = 30 N/gal.

A) R = 2770 km and E = 3.82 h

B) R = 2770 km and E = 6.33 h

C) R = 4250 km and E = 3.82 h

D) R = 4250 km and E = 6.33 h

Problem 5.3

It was verified that the maximum rate of climb of the craft introduced in the previous problem was 50 m/s at sea level and 25 m/s at an altitude of 5 km. Assuming that the maximum ROC varies linearly with altitude, estimate the absolute ceiling of the airplane.

A) h = 2.5 km

B) h = 5 km

C) h = 7.5 km

D) h = 10 km

Problem 6

Consider a single-engine light plane powered by a single piston engine of 345 hp maximum at sea level. The aspect ratio is 6.2, the wing area is 181 ft^2, and the Oswald efficiency factor is 0.91. The zero-lift drag coefficient is 0.027. The craft has specific fuel consumption of 0.42 lb of fuel per horsepower per hour, fuel capacity of 44 gal, and maximum gross weight of 3400 lb, The two-blade propeller has an efficiency of 0.83. Calculate the range and endurance at standard sea level. Let the weight of fuel = 5.64 lb/gal.

A) R = 365 mi and E = 3.73 h

B) R = 365 mi and E = 7.47 h

C) R = 725 mi and E = 3.73 h

D) R = 725 mi and E = 7.47 h

Problem 7.1

Consider a generic turbojet aircraft whose wing loading is 3600 N/m^2, TSFC is 0.76 h^{-1}, and the cruise-fuel weight fraction is 0.3. The aspect ratio of the craft's wing is 6.85 and the Oswald efficiency factor is 0.83. The zero-lift drag coefficient is 0.015. The aircraft started cruising flight at an altitude of 9 km with an airspeed of 800 km/h. True or false?

1.( ) The range of constant altitude- lift coefficient flight is greater than 5500 km.
2.( ) The endurance of constant altitude- lift coefficient flight is greater than 7.5 h.
3.( ) The range of constant airspeed-lift coefficient flight is greater than 5500 km.
4.( ) The endurance of constant airspeed-lift coefficient flight is greater than 7.0 h.
5.( ) The range of constant altitude-airspeed flight is greater than 5000 km.
6.( ) The endurance of constant altitude-airspeed flight is greater than 7.0 h.

Problem 7.2

Find the maximum ranges of each flight program. True or false?

1.( ) The maximum-range airspeed for constant h-C_L flight is greater than 800 km/h.
2.( ) The maximum range for constant h-C_L flight is greater than 5500 km.
3.( ) The maximum-range airspeed for constant V-C_L flight is greater than 850 km/h.
4.( ) The maximum range for constant V-C_L flight is greater than 5500 km.
5.( ) The maximum-range airspeed for constant h-V flight is greater than 800 km/h.
6.( ) The maximum range for constant h-V flight is greater than 5000 km.

Problem 7.3

Investigate the flight parameters for fastest turn and tightest turn at an altitude of 6 km. The maximum thrust-to-weight ratio of the craft is 0.33. True or false?

1.( ) The turning rate for fastest turn is greater than 9 degrees per second.
2.( ) The turning radius for fastest turn is greater than 1000 m.
3.( ) The turning rate for tightest turn is greater than 7 degrees per second.
4.( ) The turning radius for tightest turn is greater than 700 m.

Problem 8

A jet transport aircraft is cruising at 30,000 ft altitude with a transonic speed of Mach 0.82. The plane has a mass of 50,000 kg at the beginning of cruise flight and will land when 8000 kg of its fuel is depleted. The turbofan engines that drive the craft have a TSFC of 0.76 kg/h/kg. The zero-lift drag coefficient is 0.02, the Oswald efficiency factor is 0.7, the wing span is 28 m, and the wing area is 100 m^2. True or false?

1.( ) The craft is flying in a constant altitude-airspeed flight program. Accordingly, its range is greater than 2400 km.

2.( ) The maximum range for the constant altitude-airspeed flight program of the aircraft is greater than 2800 km.

3.( ) The thrust required to sustain the (non-optimized) flight of the craft is greater than 42 kN.

4.( ) The fuel consumption rate with the thrust obtained in the calculations of statement 3 is greater than 2800 kg/h.

5.( ) The duration of the flight with the fuel consumption rate obtained in statement 4 is greater than 3 hours.
6.( ) Suppose the craft were flying in a cruise-climb (constant airspeed-lift coefficient) flight program. In this case, the maximum range of the aircraft would have been greater than 2500 km.

Problem **9**

A sluggish transport aircraft with a takeoff mass of 125,000 kg has a wing area of 240 m^2. The aircraft cruising speed at 26,300 ft is 500 knots. The induced drag correction factor is 0.052, the zero-lift drag coefficient at takeoff is 0.048, the stall speed V_S = 53.8 knots and the ground roll speed V_R = 1.3V_S. The thrust developed during the ground roll is assumed constant at 60 kN. Determine the takeoff ground roll if the friction coefficient between the plane and the runway surface is 0.01. Assume there is no flap deflection during takeoff.

A) s_G = 840 m
B) s_G = 1100 m
C) s_G = 1540 m
D) s_G = 2010 m

Problem **10.1**

A business jet aircraft has a mass of 16,500 kg and two turbofan engines each generating a thrust of 24.2 kN with an airspeed of 120 m/s. The aircraft drag is 15,000 N. Determine the rate of climb and the climb angle. Ignore variations in aircraft weight, drag, and thrust during climb.

A) ROC = 12.4 m/s and α = 11.9°
B) ROC = 12.4 m/s and α = 22.1°
C) ROC = 24.8 m/s and α = 11.9°
D) ROC = 24.8 m/s and α = 22.1°

Problem **10.2**

How long does it take for the aircraft specified in the previous problem to ascend to an altitude of 20,000 ft?

A) Δt = 2.3 min
B) Δt = 4.1 min
C) Δt = 6.3 min
D) Δt = 8.0 min

Problem **11.1**

A generic fighter jet with two turbofan engines is flying with a mass of 18,000 kg and has a wing span of 13 m, a wing area of 25 m², and an Oswald efficiency factor of 0.9. Each turbofan supplies 20.5 kN of thrust. The maximum lift coefficient and the zero-lift drag coefficient are 2.0 and 0.015, respectively. Determine the airspeed for maximum rate of climb and the angle for maximum rate of climb. Assume mass, thrust, and other relevant parameters to be constant during the climb phase.

A) $V_{ROC_{\max}}$ = 124 m/s and $\alpha_{ROC_{\max}}$ = 8.73°
B) $V_{ROC_{\max}}$ = 124 m/s and $\alpha_{ROC_{\max}}$ = 16.4°
C) $V_{ROC_{\max}}$ = 249 m/s and $\alpha_{ROC_{\max}}$ = 8.73°
D) $V_{ROC_{\max}}$ = 249 m/s and $\alpha_{ROC_{\max}}$ = 16.4°

Problem **11.2**

Reconsidering the aircraft introduced in the previous problem, determine the rate of climb that corresponds to the maximum climb angle.

A) $ROC_{\alpha_{\max}}$ = 10.4 m/s
B) $ROC_{\alpha_{\max}}$ = 15.2 m/s
C) $ROC_{\alpha_{\max}}$ = 20.1 m/s
D) $ROC_{\alpha_{\max}}$ = 25.8 m/s

Problem 12

A small jet aircraft is equipped with a turbojet engine and is utilizing 82% of its maximum engine thrust in a cruising flight at 30,000 ft. The mass of the aircraft is assumed constant at 52,000 kg, the zero-lift drag coefficient is 0.025, the induced drag correction factor is 0.051, and the wing area is 135 m^2. The maximum thrust at sea level is 120 kN. Determine the cruising speed for this aircraft at 30,000 ft altitude.

A) V_C = 727 km/h

B) V_C = 889 km/h

C) V_C = 933 km/h

D) V_C = 1050 km/h

Problem 13

A jet transport aircraft has a mass of 55,000 kg and a wing area of 98 m^2. The craft has zero-lift drag coefficient of 0.019, an induced drag correction factor of 0.048, and is equipped with two Rolls-Royce engines that produce a sea-level thrust of 60 kN each. Estimate the cruise altitude and the cruising speed in terms of Mach number. Ignore other flight phases (including climb) and assume that the cruise altitude is to maximize the range.

A) h_C = 20,500 ft and M_C = 0.611

B) h_C = 20,500 ft and M_C = 0.839

C) h_C = 29,700 ft and M_C = 0.611

D) h_C = 29.700 ft and M_C = 0.839

Problem 14

An aircraft enters a level turn at a constant airspeed of 400 km/h. It completes a full 360° circle in 30 sec, maintaining the entry speed constant throughout the turn. Calculate the turn radius and the bank angle.

A) R = 531 m and ϕ = 44.2°

B) R = 531 m and ϕ = 67.1°

C) R = 760 m and ϕ = 44.2°

D) R = 760 m and ϕ = 67.1°

Additional Information

Table 1 Standard atmosphere data

Alt. (m)	Alt. (ft)	Density Ratio, σ	Density (kg/m³)	Speed of Sound (m/s)
0	0	1	1.225	340.3
1000	3281	0.9075	1.112	336.4
2000	6562	0.8217	1.007	330.8
3000	9843	0.7422	0.909	328.6
4000	13123	0.6689	0.819	324.6
5000	16404	0.6012	0.736	320.5
6000	19685	0.5389	0.660	316.4
7000	22966	0.4816	0.590	312.3
8000	26247	0.4292	0.526	308.1
9000	29528	0.3813	0.467	303.9
10,000	32808	0.3376	0.414	299.5
11,000	36089	0.2978	0.365	295.1
12,000	39370	0.2546	0.312	295.1
13,000	42651	0.2176	0.267	295.1
14,000	45932	0.186	0.228	295.1
15,000	49213	0.159	0.195	295.1

→ **Equations**

Equation 1: Thrust-to-weight ratio as a function of wing loading

$$\frac{F}{W} = q_\infty C_{D,0}\left(\frac{1}{W/S}\right) + \frac{1}{q_\infty \pi e AR}\left(\frac{W}{S}\right)$$

where F is thrust, W is weight, q_∞ is dynamic pressure, $C_{D,0}$ is the zero-lift drag coefficient, S is wing area, e is the span efficiency factor, and AR is the wing aspect ratio.

Equation 2: Velocity for minimum thrust required

$$V_{F_R \min} = \left(\frac{4W^2}{\rho^2 S^2 C_{D,0} \pi e AR}\right)^{1/4}$$

where W is weight, ρ is the air density, S is wing area, $C_{D,0}$ is the zero-lift drag coefficient, e is the span efficiency factor, and AR is the wing aspect ratio.

Equation 3: Lift coefficient for minimum thrust required

$$C_{L,F_R \min} = \sqrt{C_{D,0} \pi e AR}$$

where $C_{D,0}$ is the zero-lift drag coefficient, e is the span efficiency factor, and AR is the wing aspect ratio.

Equation 4: Maximum aerodynamic efficiency (i.e., lift-to-drag ratio)

$$E_m = \left(\frac{L}{D}\right)_{\max} = \frac{\sqrt{\pi e (AR) C_{D,0}}}{2C_{D,0}}$$

where e is the span efficiency factor, AR is the wing aspect ratio, and $C_{D,0}$ is the zero-lift drag coefficient. Another way to state this equation, using the induced drag correction factor $K = 1/\pi ARe$, is

$$E_m = \left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{C_{D,0} K}}$$

Equation 5: Range of jet-powered aircraft

$$R = \frac{2}{TSFC}\left(\frac{\sqrt{C_L}}{C_D}\right)\sqrt{\frac{2}{\rho S}}\left(\sqrt{W_0} - \sqrt{W_1}\right)$$

where $TSFC$ is the thrust specific fuel consumption, C_L is the lift coefficient, C_D is the drag coefficient, ρ is the air density, S is the wing area, W_0 is the initial weight, and W_1 is the final weight.

Equation 6: Endurance of jet-powered aircraft

$$E = \frac{1}{TSFC}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_0}{W_1}\right)$$

where $TSFC$ is the thrust specific fuel consumption, C_L/C_D is the lift-to-drag ratio, W_0 is the initial weight, and W_1 is the final weight.

Equation 7: Range of propeller-driven aircraft

$$R = \frac{\eta_p}{c}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_0}{W_1}\right)$$

where η_p is the propeller efficiency, c is the specific fuel consumption, C_L/C_D is the lift-to-drag ratio, W_0 is the initial weight, and W_1 is the final weight.

Equation 8: Endurance of propeller-driven aircraft

$$E = \frac{\eta_p}{c}\left(\frac{C_L^{3/2}}{C_D}\right)\sqrt{2\rho S}\left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}}\right)$$

where η_p is the propeller efficiency, c is the specific fuel consumption, C_L is the lift coefficient, C_D is the drag coefficient, ρ is the air density, S is the wing area, W_0 is the initial weight, and W_1 is the final weight.

Equation 9: Range of a constant altitude-constant lift coefficient flight (similar to equation 5)

$$x_{h-C_L} = \frac{2}{TSFC}\sqrt{\frac{2W_0}{\rho S C_L}}\frac{C_L}{C_D}\left(1-\sqrt{\frac{W_1}{W_0}}\right)$$

where $TSFC$ is the thrust specific fuel consumption, ρ is the air density, S is the wing area, C_L is the lift coefficient, C_D is the drag coefficient, W_0 is the initial weight, and W_1 is the final weight. A condensed form of this equation is

$$x_{h-C_L} = \frac{2EV}{TSFC}\left(1-\sqrt{1-\zeta}\right)$$

where E is the aerodynamic efficiency (i.e., the lift-to-drag ratio), V is velocity, and ζ is the cruise-fuel weight fraction.

Equation 10: Endurance of a constant lift coefficient flight (similar to equation 5)

$$t_{C_L} = \frac{E}{TSFC}\ln\left(\frac{1}{1-\zeta}\right)$$

where E is the aerodynamic efficiency (i.e., the lift-to-drag ratio) and ζ is the cruise-fuel weight fraction.

Equation 11: Range of a constant airspeed-constant lift coefficient flight

$$x_{V-C_L} = \frac{V}{TSFC}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_0}{W_1}\right)$$

where V is airspeed, $TSFC$ is the thrust specific fuel consumption, C_L is the lift coefficient, C_D is the drag coefficient, W_0 is the initial weight, and W_1 is the final weight. A condensed form of this equation is

$$x_{V-C_L} = \frac{EV}{TSFC}\ln\left(\frac{1}{1-\zeta}\right)$$

where E is the aerodynamic efficiency (i.e., the lift-to-drag ratio) and ζ is the cruise-fuel weight fraction.

Equation 12: Range of a constant altitude-constant airspeed flight

$$x_{h-V} = \frac{V}{TSFC\sqrt{KC_{D,0}}}\arctan\left[\frac{q_\infty S\sqrt{KC_{D,0}}\left(W_0-W_1\right)}{q^2S^2C_{D,0}+KW_0W_1}\right]$$

where V is velocity, $TSFC$ is thrust specific fuel consumption, $K = 1/\pi ARe$ is the induced drag correction factor, $C_{D,0}$ is the zero-lift drag coefficient, q_∞ is dynamic pressure, S is wing area, W_0 is initial weight, and W_1 is final weight. A condensed form of this equation is

$$x_{h-V} = \frac{2VE_m}{TSFC}\arctan\left[\frac{E\zeta}{2E_m\left(1-KEC_L\zeta\right)}\right]$$

where $E_m = 1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency, E is the aerodynamic efficiency (i.e., the lift-to-drag ratio), ζ is the cruise-fuel weight fraction, $K = 1/\pi ARe$ is the induced drag correction factor, and C_L is the lift coefficient.

Equation 13: Maximum-range airspeed for constant altitude-constant lift coefficient flight (similar to equation 2)

$$V_{MR,h-C_L} = \left[\frac{2\left(W/S\right)}{\rho_{SSL}\sigma}\right]^{1/2}\left(\frac{3K}{C_{D,0}}\right)^{1/4}$$

where W is weight, S is wing area, $\rho_{SSL} \approx 1.225$ kg/m^3 is the air density at sea level, σ is the air density ratio, $K = 1/\pi ARe$ is the induced drag correction factor, and $C_{D,0}$ is the zero-lift drag coefficient.

Equation 14: Aerodynamic efficiency for maximum range in constant altitude-constant lift coefficient flight

$$E_{MR,h-C_L} = \frac{\sqrt{3}E_m}{2}$$

where $E_m = 1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency.

Equation 15: Maximum-range airspeed for constant altitude-constant airspeed flight

$$V_{MR,h-V} = V_{MR,h-C_L}\left(1-\zeta^*\right)^{1/4}$$

where $V_{MR,h-C_L}$ is the maximum-range airspeed for constant altitude-constant lift coefficient flight (equation 13) and ζ^* is the cruise-fuel weight fraction.

Equation 16: Lift coefficient for maximum-range constant altitude-constant airspeed flight

$$C_{L,MR;h-V} = \left[\frac{C_{D,0}}{3K\left(1-\zeta^*\right)}\right]^{1/2}$$

where $C_{D,0}$ is the zero-lift drag coefficient, $K = 1/\pi ARe$ is the induced drag correction factor, and ζ^* is the cruise-fuel weight fraction.

Equation 17: Aerodynamic efficiency for maximum-range constant altitude-constant airspeed flight

$$E_{MR,h-V} = \frac{2E_m\sqrt{3\left(1-\zeta^*\right)}}{1+3\left(1-\zeta^*\right)}$$

where $E_m = 1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency and ζ^* is the cruise-fuel weight fraction.

Equation 18: Load factor

$$n = \sqrt{\left(\frac{\dot{\chi}V_\infty}{g}\right)^2 + 1}$$

where $\dot{\chi}$ is the turning rate, V_∞ is the airspeed, and g is the acceleration due to gravity.

Equation 19: Turn radius

$$R = \frac{V_\infty^2}{g\sqrt{n^2-1}}$$

where V_∞ is airspeed, g is the acceleration due to gravity, and n is the load factor.

Equation 20: Bank angle

$$\phi = \arccos\left(1/n\right)$$

where n is the load factor.

Equation 21: Airspeed for fastest turn (similar to equation 2)

$$V_{FT} = \left[\frac{2\left(W/S\right)}{\rho_{SSL}\sigma}\right]^{1/2}\left(\frac{K}{C_{D,0}}\right)^{1/4}$$

where W is weight, S is wing area, ρ_{SSL} is the air density at sea level, σ is the altitude air density ratio, $K = 1/\pi ARe$ is the induced drag correction factor, and $C_{D,0}$ is the zero-lift drag coefficient.

Equation 22: Load factor for fastest turn

$$n_{FT} = \sqrt{2\left(F/W\right)E_m - 1}$$

where F/W is the thrust-to-weight ratio and $E_m = 1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency.

Equation 23: Fastest turn bank angle

$$\phi_{FT} = \arccos\left(1/n_{FT}\right)$$

where n_{FT} is the load factor for fastest turn (equation 22).

Equation 24: Lift coefficient for fastest turn

$$C_{L,FT} = \left\{ \frac{\left[2\left(F/W\right)E_m - 1\right]C_{D,0}}{K} \right\}^{1/2}$$

where F/W is the thrust-to-weight ratio, E_m = $1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency, $C_{D,0}$ is the zero-lift drag coefficient, and $K = 1/\pi ARe$ is the induced drag correction factor.

Equation 25: Aerodynamic efficiency for fastest turn

$$E_{FT} = \frac{1}{F/W}\sqrt{2\left(F/W\right)E_m - 1}$$

where F/W is the thrust-to-weight ratio and E_m = $1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency.

Equation 26: Turning rate for fastest turn

$$\dot{\chi}_{FT} = \frac{g}{V_{FT}}\sqrt{n_{FT}^2 - 1}$$

where g is the acceleration due to gravity, V_{FT} is the airspeed of fastest turn (equation 21), and n_{FT} is the fastest turn load factor (equation 22).

Equation 27: Turning radius for fastest turn

$$r_{FT} = \frac{V_{FT}^2}{g\sqrt{n_{FT}^2 - 1}}$$

where g is the acceleration due to gravity, V_{FT} is the airspeed of fastest turn (equation 21), and n_{FT} is the fastest turn load factor (equation 22).

Equation 28: Airspeed for tightest turn

$$V_{TT} = 2\left[\frac{K\left(W/S\right)}{\rho_{SSL}\sigma\left(F/W\right)}\right]^{1/2}$$

where $K = 1/\pi ARe$ is the induced drag correction factor, W is weight, S is wing area, $\rho_{SSL} \approx 1.225$ kg/m³ is the air density at sea level, σ is the altitude air density ratio, and F/W is the thrust-to-weight ratio.

Equation 29: Load factor for tightest turn

$$n_{TT} = \left[2 - \frac{1}{E_m^2\left(F/W\right)^2}\right]^{1/2}$$

where E_m = $1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency and F/W is the thrust-to-weight ratio.

Equation 30: Tightest turn bank angle

$$\phi_{TT} = \arccos\left(1/n_{TT}\right)$$

where n_{TT} is the load factor for tightest turn (equation 29).

Equation 31: Lift coefficient for tightest turn

$$C_{L,TT} = \frac{1}{2KE_m}\left[2\left(F/W\right)^2 E_m^2 - 1\right]^{1/2}$$

where F/W is the thrust-to-weight ratio, E_m = $1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency, and $K = 1/\pi ARe$ is the induced drag correction factor.

Equation 32: Aerodynamic efficiency for tightest turn

$$E_{TT} = \frac{1}{E_m (F/W)^2}\left[2(F/W)^2 E_m^2 - 1\right]^{1/2}$$

where F/W is the thrust-to-weight ratio, and $E_m = 1/2(KC_{D,0})^{1/2}$ is the maximum aerodynamic efficiency.

Equation 33: Turning rate for tightest turn

$$\dot{\chi}_{TT} = \frac{g}{V_{TT}}\sqrt{n_{TT}^2 - 1}$$

where g is the acceleration due to gravity, V_{TT} is the airspeed of tightest turn (equation 28), and n_{TT} is the tightest turn bank angle (equation 29).

Equation 34: Turning radius for tightest turn

$$r_{TT} = \frac{V_{TT}^2}{g\sqrt{n_{TT}^2 - 1}}$$

where g is the acceleration due to gravity, V_{TT} is the airspeed of tightest turn (equation 28), and n_{TT} is the tightest turn bank angle (equation 29).

Equation 35: Ground roll distance

$$s_G = -\frac{1}{2B}\ln\left(\frac{A}{A + BV_R^2}\right)$$

where V_R is the ground roll velocity (commonly 1.1 to 1.3 times the stall speed); A is a parameter given by

$$A = \frac{F}{m} - \mu g$$

where F is thrust, m is takeoff mass, μ is the friction coefficient between the aircraft wheels and the airfield surface, and g is the acceleration due to gravity; in turn, B is a parameter given by

$$B = -\frac{\rho_{SSL} S}{2m}\left(C_{D_{TO}} - \mu C_{L_{TO}}\right)$$

where $\rho_{SSL} \approx 1.225$ kg/m³ is the sea-level air density, S is the wing area, $C_{D_{TO}}$ is the takeoff drag coefficient, and $C_{L_{TO}}$ is the takeoff lift coefficient.

Equation 36: Rate of climb

$$ROC = \frac{(F - D)V}{W}$$

where F is thrust, D is drag, V is flight velocity, and W is weight.

Equation 37: Airspeed for maximum rate of climb

$$V_{ROC_{\max}} = \sqrt{\frac{F}{3\rho_{SSL} C_{D,0} S}\left\{1 + \sqrt{1 + \frac{3}{\left[\left(\frac{L}{D}\right)_{\max}\frac{F}{W}\right]^2}}\right\}}$$

where F is thrust, $\rho_{SSL} \approx 1.225$ kg/m³ is the air density at sea level, $C_{D,0}$ is the zero-lift drag coefficient, S is the wing area, $(L/D)_{\max}$ is the maximum lift-to-drag ratio (equation 4), and W is weight.

Equation 38: Angle corresponding to maximum rate of climb

$$\alpha_{ROC_{\max}} = -\arcsin\left\{\frac{1}{4KW}\left[\sqrt{\rho_{SSL}^2 S^2 V_{ROC_{\max}}^4 \left(1 + 4KC_{D,0}\right) - 8K\rho_{SSL} FSV_{ROC_{\max}}^2} - \rho_{SSL} SV_{ROC_{\max}}^2\right]\right\}$$

where $K = 1/\pi ARe$ is the induced drag correction factor, W is weight, $\rho_{SSL} = 1.225$ kg/m³ is the air density at sea level, S is wing area, $V_{ROC_{\max}}$ is the maximum-rate-of-climb airspeed (equation 37), $C_{D,0}$ is the zero-lift drag coefficient, and F is thrust.

Equation 39: Steepest climb angle

$$\alpha_{\max} = \arcsin\left[\frac{1}{W}\left(F_{\max} - 2W\sqrt{KC_{D,0}}\right)\right]$$

where W is weight, $F_{\max}$ is maximum thrust, $K = 1/\pi ARe$ is the induced drag correction factor, and $C_{D,0}$ is the zero-lift drag coefficient.

Equation 40: Flight velocity

$$AV^2 + \frac{B}{V^2} - nCF_{\max} = 0$$

where V is flight velocity, $F_{\max}$ is the thrust at the appropriate altitude, and n is the percentage of engine thrust being used, which often ranges from 0.65 to 0.9 for cruise flight conditions. Coefficient A is given by

$$A = \frac{1}{2}\rho SC_{D,0}$$

where ρ is air density, S is wing area, and $C_{D,0}$ is the zero-lift drag coefficient. Coefficient B is given by

$$B = \frac{2KW^2}{\rho S}$$

where $K = 1/\pi ARe$ is the induced drag correction factor and W is weight. Coefficient C depends on the engine type and the flight altitude, as follows.

Turbojet flying In the troposphere	Turbojet flying In the stratosphere	Turbofan engine
$C = \left(\frac{\rho}{\rho_{SSL}}\right)^{0.9}$	$C = \left(\frac{\rho_{11,000}}{\rho_{SSL}}\right)^{0.9}\left(\frac{\rho}{\rho_{11,000}}\right)$	$C = \left(\frac{\rho}{\rho_{SSL}}\right)^{1.2}$

where ρ is air density, $\rho_{SSL} \approx 1.225$ kg/m³ is the sea-level air density, and $\rho_{11,000}$ is the air density at an altitude of 11 km.

Equation 41: Air density at cruise altitude

$$\rho_C = \left[\frac{1.155W\rho_{SSL}^{1.2}}{(L/D)_{\max} F_{SSL}}\right]^{1/1.2}$$

where ρ_C is the air density at cruise altitude, W is weight, $\rho_{SSL} \approx 1.225$ kg/m³ is the sea-level air density, $(L/D)_{\max}$ is the maximum lift-to-drag ratio (equation 4), and F_{SSL} is the thrust at sea level.

Solutions

P.1 ■ Solution

The expression for total drag is

$$D = q_\infty S\left(C_{D,0} + C_{D,i}\right)$$

Here, q_∞ is the dynamic pressure, S is the wing area, $C_{D,0}$ is the zero-lift drag coefficient, and $C_{D,i}$ is the coefficient of drag due to lift. Since the airplane is flying at maximum lift-to-drag ratio conditions, we have $C_{D,0} = C_{D,i}$ and the equation above becomes

$$D = q_\infty S\left(C_{D,0} + C_{D,i}\right) = 2q_\infty SC_{D,i}$$

The dynamic pressure is calculated as

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\times 1.225\times(700/3.6)^2 = 23,200\text{ Pa}$$

In view of steady flight, the lift coefficient can be determined as

$$C_L = \frac{W}{q_\infty S} = \frac{225,000\times 9.81}{23,200\times 450} = 0.211$$

The coefficient of drag due to lift is determined next,

$$C_{D,i} = \frac{C_L^2}{\pi eAR} = \frac{0.211^2}{\pi \times 0.93 \times 8.0} = 0.0019$$

It remains to compute the total drag,

$$D = 2q_\infty SC_{D,i} = 2 \times 23{,}200 \times 450 \times 0.0019 = \boxed{39.7 \text{ kN}}$$

★ The correct answer is **C**.

P.2 ■ Solution

For steady flight, lift equals weight, or

$$W = L = mg$$

$$\therefore L = 650 \times 9.81 = 6380 \text{ N}$$

The lift coefficient is then

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{6380}{\frac{1}{2} \times 1.225 \times 60^2 \times 8.4} = 0.344$$

In addition, thrust equals drag, so that $F = D = 750$ N. The drag coefficient follows as

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 S} = \frac{750}{\frac{1}{2} \times 1.225 \times 60^2 \times 8.4} = 0.0405$$

It remains to compute the lift-to-drag ratio,

$$E = \frac{C_L}{C_D} = \frac{0.344}{0.0405} = \boxed{8.49}$$

★ The correct answer is **B**.

P.3 ■ Solution

Before the engine is accelerated, thrust equals drag. Accordingly,

$$F_1 = D_1 = \frac{1}{2}\rho V_1^2 SC_D = \frac{1}{2} \times 1.225 \times (200 \times 0.5144)^2 \times 30 \times 0.05 = 9720 \text{ N}$$

Once the engine thrust is increased, the acceleration a can be obtained from Newton's second law,

$$T_2 - T_1 = ma \rightarrow a = \frac{T_2 - T_1}{m}$$

$$\therefore a = \frac{1.4 \times 9720 - 9720}{3200} = \boxed{1.22 \text{ m/s}^2}$$

The acceleration will increase the velocity of the craft until the drag equates the thrust again. The final velocity will then be

$$T_2 = D \rightarrow 1.4T_1 = \frac{1}{2}\rho V_2^2 S$$

$$\therefore V_2 = \sqrt{\frac{1.4T_1}{\frac{1}{2}\rho SC_D}} = \sqrt{\frac{1.4 \times 9720}{\frac{1}{2} \times 1.225 \times 30 \times 0.05}} = 122 \text{ m/s} = \boxed{237 \text{ knots}}$$

That is, increasing the thrust by 40% will increase the velocity by about 18.5%.

★ The correct answer is **A**.

P.4 ■ Solution

Part 1: The wing aspect ratio is determined first,

$$AR = \frac{b^2}{S} = \frac{63.3^2}{535} = 7.49$$

The flight velocity expressed in m/s is V_∞ = 900/3.6 = 250 m/s, and the dynamic pressure corresponding to a velocity of 900 km/h at sea level is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\times 1.225\times 250^2 = 38,300 \text{ Pa}$$

The wing loading for a weight of 3,000,000 N follows as

$$\frac{W}{S} = \frac{3,000,000}{535} = 5610 \text{ N/m}^2$$

The wing loading is generally expressed in units of kgf/m^2; that is,

$$\frac{W}{S} = 5610\,\frac{\text{N}}{\text{m}^2}\times\frac{1}{9.81}\,\frac{\text{kgf}}{\text{N}} = 572 \text{ kgf/m}^2$$

The thrust-to-weight ratio is given by equation 1, namely,

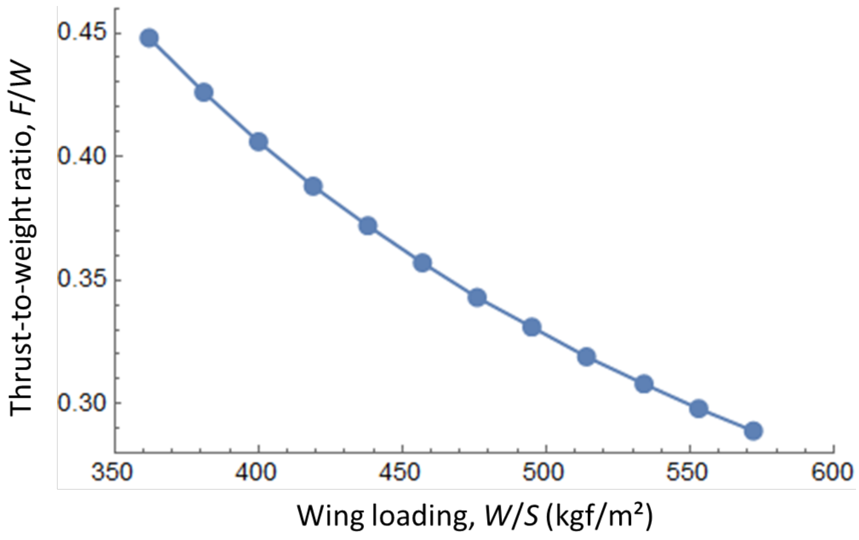
$$\frac{F}{W} = q_\infty C_{D,0}\left(\frac{1}{W/S}\right) + \frac{1}{q_\infty \pi e AR}\left(\frac{W}{S}\right)$$

$$\therefore \frac{F}{W} = 38,300\times 0.041\times\frac{1}{5610} + \frac{1}{38,300\times 3.14\times 0.71\times 7.49}\times 5610 = 0.289$$

The remaining calculations are tabulated below.

Weight (N)	W/S (N/m²)	W/S (kgf/m²)	F/W
3.00E+06	5607	572	0.289
2.90E+06	5421	553	0.298
2.80E+06	5234	534	0.308
2.70E+06	5047	514	0.319
2.60E+06	4860	495	0.331
2.50E+06	4673	476	0.343
2.40E+06	4486	457	0.357
2.30E+06	4299	438	0.372
2.20E+06	4112	419	0.388
2.10E+06	3925	400	0.406
2.00E+06	3738	381	0.426
1.90E+06	3551	362	0.448

The graph we are looking for is one of thrust-to-weight ratio (the red column) versus wing loading (the blue column).



Part 2: Applying equation 2, the velocity for minimum thrust is determined to be

$$V_{F_R\,\min} = \left(\frac{4W^2}{\rho_\infty^2 S^2 C_{D,0}\pi e AR}\right)^{1/4} = \left[\frac{4\times\left(3.0\times 10^6\right)^2}{1.225^2\times 535^2\times 0.041\times\pi\times 0.71\times 7.49}\right]^{1/4} = 105 \text{ m/s}$$

or 378 km/h. The lift coefficient for minimum thrust, in turn, is calculated with equation 3,

$$C_{L,F_R\,\min} = \sqrt{C_{D,0}\pi e AR} = \sqrt{0.041\times\pi\times 0.71\times 7.49} = \boxed{0.828}$$

★ The correct answer is **B**.

Part 3: As an example, we shall perform the calculations for an airspeed of 300 km/h, or, equivalently, 83.3 m/s. Appealing to the definition of lift coefficient, we have

$$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{3{,}000{,}000}{\frac{1}{2}\times 1.225\times 83.3^2\times 535} = 1.32$$

Recall that the aspect ratio of the craft is AR = 63.3²/535 = 7.49. The total aircraft drag coefficient is then

$$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR} = 0.041 + \frac{1.32^2}{\pi\times 0.71\times 7.49} = 0.145$$

The lift-to-drag ratio follows as

$$C_L/C_D = 1.32/0.145 = 9.10$$

The thrust required is calculated as

$$F_R = \frac{W}{C_L/C_D} = \frac{3{,}000{,}000}{9.10} = 330\text{ kN}$$

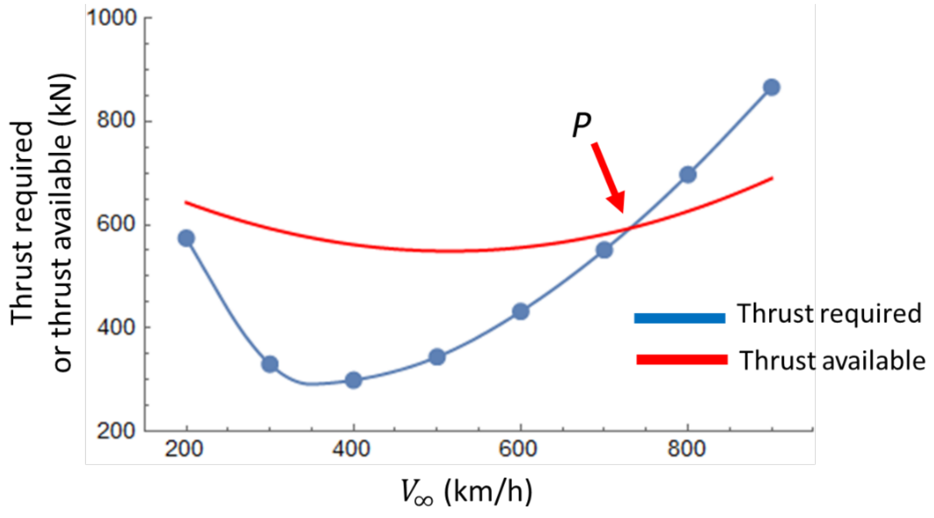
The total thrust available from the four engines is given by

$$F_A = 4\times\left[200 + \left(2.37\times 10^{-4}\right)\times 83.3^2 - 0.244\times 83.3\right] = 725\text{ kN}$$

The calculations for other values of velocity are summarized below.

V_∞ (km/h)	V_∞ (m/s)	C_L	C_D	L/D	F_R (kN)	F_A (kN)
200	55.6	2.97	0.568	5.22	574	749
300	83.3	1.32	0.145	9.09	330	725
400	111.1	0.74	0.074	10.03	299	703
500	138.9	0.47	0.054	8.71	344	683
600	166.7	0.33	0.048	6.94	432	664
700	194.4	0.24	0.045	5.44	551	646
800	222.2	0.19	0.043	4.31	697	630
900	250.0	0.15	0.042	3.46	866	615

The plots we are looking for are thrust required versus velocity (i.e., blue column versus yellow column) and thrust available versus velocity (i.e., red column versus yellow column), as follows.



The graphs intersect at point P, where $V_\infty \approx$ 730 km/h. Accordingly, the maximum velocity is about 730 kilometers per hour.

Part 4: The wing area is S = 535 m² and, from Problem 4.3, the lift and drag coefficients for an airspeed of 300 km/h are 1.32 and 0.145, respectively. The range is determined by dint of equation 5,

$$R = \frac{2}{TSFC}\left(\frac{\sqrt{C_L}}{C_D}\right)\sqrt{\frac{2}{\rho_\infty S}}\left(\sqrt{W_0}-\sqrt{W_1}\right)$$

$$\therefore R = \frac{2}{1.68\times 10^{-4}}\times\left(\frac{\sqrt{1.32}}{0.145}\right)\times\sqrt{\frac{2}{1.225\times 535}}\times\left(\sqrt{2.8\times 10^6}-\sqrt{1.9\times 10^6}\right) = 1.54\times 10^6\text{ m}$$

$$\therefore \boxed{R = 1540\text{ km}}$$

The endurance, in turn, is determined with equation 6,

$$E = \frac{1}{TSFC}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_0}{W_1}\right) = \frac{1}{1.68\times10^{-4}}\times\left(\frac{1.32}{0.145}\right)\ln\left(\frac{2.8\times10^6}{1.9\times10^6}\right) = 21{,}000 \text{ s}$$

$$\therefore \boxed{E = 5.83 \text{ h}}$$

★ The correct answer is **B**.

P.5 ■ Solution

Part 1: Assume the airspeed to be 50 m/s. The lift coefficient is

$$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{103{,}047}{\frac{1}{2}\times1.225\times50^2\times47} = 1.43$$

The coefficient of drag due to lift follows as

$$C_D = C_{D,0} + \frac{C_L^2}{\pi eAR} = 0.032 + \frac{1.43^2}{\pi\times0.87\times6.5} = 0.147$$

and the required thrust is

$$F_R = \frac{W}{C_L/C_D} = \frac{103{,}047}{1.43/0.147} = 10.6 \text{ kN}$$

The required power is then

$$P_R = F_R V = 10.6\times50 = 531 \text{ kW}$$

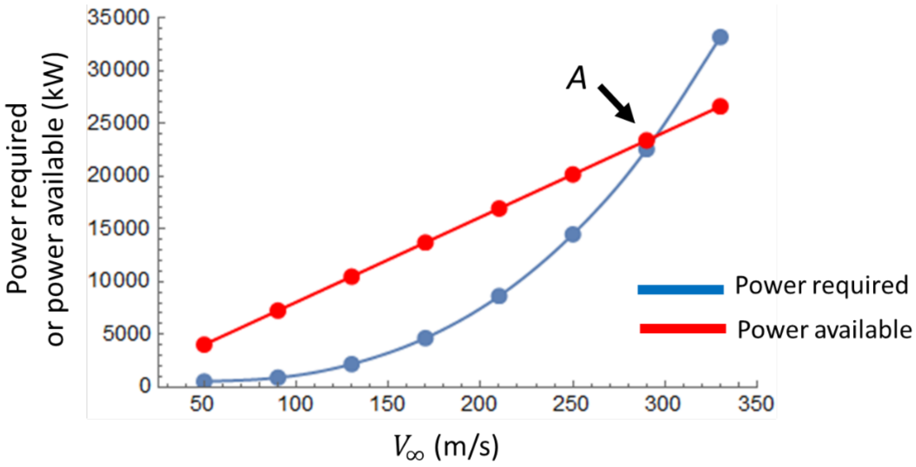
The power available for an airspeed of 50 m/s is

$$P_A = 2F_A V_\infty = 2\times40{,}298\times50 = 4030 \text{ kW}$$

Calculations for other airspeeds are summarized below.

V_∞ (m/s)	C_L	C_D	C_L/C_D	F_R (kN)	P_R (kW)	P_A (kW)
50	1.432	0.1475	9.71	10.6	531	4030
90	0.442	0.0430	10.28	10.0	902	7254
130	0.212	0.0345	6.13	16.8	2184	10477
170	0.124	0.0329	3.77	27.3	4648	13701
210	0.081	0.0324	2.51	41.1	8630	16925
250	0.057	0.0322	1.78	57.9	14477	20149
290	0.043	0.0321	1.33	77.7	22539	23373
330	0.033	0.0321	1.03	100.5	33168	26597

The graphs we require are one of power required (the blue column) versus airspeed (the yellow column) and another of power available (the red column) versus airspeed, as follows.



The maximum velocity occurs at point A, where the two curves intersect and the power required curve surpasses the power available curve. The maximum velocity is read as V_{max} = 295 m/s, or 1060 km/h.

★ The correct answer is **D**.

Part 2: The air density at 8-km altitude is ρ_∞ = 0.526 kg/m³ (Table 1). The weight of fuel at takeoff is W_{fuel} = 1900 × 30 = 57,000 N and the weight of the craft after depletion of all fuel is W_1 = 136,960 − 57,000 ≈ 80,000 N. Expressed in N/N·s, the thrust specific fuel consumption is

$$TSFC = 1.0\,\frac{\text{N}}{\text{N}\cdot\cancel{\text{hr}}}\times\frac{1}{3600}\frac{\cancel{\text{hr}}}{\text{s}} = 2.78\times10^{-4}\ \text{N/N}\cdot\text{s}$$

We also require the ratio

$$\left(\frac{C_L^{1/2}}{C_D}\right)_{\max} = \frac{\left[\frac{1}{3}C_{D,0}\pi e(AR)\right]^{1/4}}{\frac{4}{3}C_{D,0}} = \frac{\left[\frac{1}{3}\times0.032\times\pi\times0.87\times6.5\right]^{1/4}}{\frac{4}{3}\times0.032} = 15.5$$

Then, the range is given by equation 5,

$$R = \frac{2}{TSFC}\left(\frac{\sqrt{C_L}}{C_D}\right)\sqrt{\frac{2}{\rho_\infty S}}\left(\sqrt{W_0}-\sqrt{W_1}\right)$$

$$\therefore R = \frac{2}{2.78\times10^{-4}}\times15.5\times\sqrt{\frac{2}{0.526\times47}}\left(\sqrt{136{,}960}-\sqrt{80{,}000}\right) = 2.77\times10^6\ \text{m}$$

$$\therefore \boxed{R = 2770\ \text{km}}$$

In order to compute the endurance, we must compute the maximum lift-to-drag ratio (equation 4),

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{\sqrt{\pi e(AR)C_{D,0}}}{2C_{D,0}} = \frac{\sqrt{\pi\times0.87\times6.5\times0.032}}{2\times0.032} = 11.8$$

The endurance follows from equation 6,

$$E = \frac{1}{TSFC}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_0}{W_1}\right) = \frac{1}{2.78\times10^{-4}}\times11.8\times\ln\left(\frac{136{,}960}{80{,}000}\right) = 22{,}800\ \text{s}$$

$$\therefore \boxed{E = 6.33\ \text{h}}$$

★ The correct answer is **B**.

Part 3: The rate of climb can be estimated by the linear relation

$$ROC_{\max} = \alpha h + \beta$$

where h is the altitude and α and β are coefficients. These can be obtained by substituting the two data points we were given. We know that the ROC is 50 m/s at sea level, that is, when h = 0. Thus,

$$ROC_{\max} = \alpha h + \beta \rightarrow 50 = \alpha\times0+\beta$$

$$\therefore \beta = 50$$

In addition, we know that the ROC = 25 m/s when h = 5000 m. Thus,

$$ROC_{\max} = \alpha h + \beta \rightarrow 25 = \alpha\times5000+50$$

$$\therefore \alpha = -25/5000 = -0.005$$

so that

$$ROC_{\max} = -0.005h + 50$$

Putting ROC = 0, we can establish the absolute ceiling of the craft,

$$-0.005h + 50 = 0 \rightarrow h = \frac{50}{0.005} = 10{,}000$$

$$\therefore \boxed{h = 10\ \text{km}}$$

★ The correct answer is **D**.

P.6 ■ Solution

The weight of fuel is W_f = 44 × 5.64 = 248 lb and the mass of the aircraft after all fuel has been depleted is W_1 = 3400 − 248 ≈ 3150 lb. Expressed in ft^{-1}, the fuel consumption is

$$c = 0.42\,\text{lb} \times \frac{1}{550} \frac{1}{\frac{\text{lb ft}}{\text{s}}} \times \frac{1}{3600\ \text{s}} = 2.12 \times 10^{-7}\ \text{ft}^{-1}$$

The maximum lift-to-drag ratio follows from equation 4,

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{\sqrt{\pi e(AR) C_{D,0}}}{2C_{D,0}} = \frac{\sqrt{\pi \times 0.91 \times 6.2 \times 0.027}}{2 \times 0.027} = 12.8$$

To calculate the range, we appeal to equation 7,

$$R = \frac{\eta_p}{c}\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_0}{W_1}\right) = \frac{0.83}{2.12 \times 10^{-7}} \times 12.8 \times \ln\left(\frac{3400}{3150}\right) = 3.83 \times 10^6\ \text{ft}$$

$$\therefore \boxed{R = 725\ \text{mi}}$$

The endurance is calculated next. We first compute the ratio

$$\frac{C_L^{3/2}}{C_D} = \frac{\left[3C_{D,0}\pi e(AR)\right]^{\frac{1}{2}}}{4C_{D,0}} = \frac{(3 \times 0.027 \times \pi \times 0.91 \times 6.2)^{1/2}}{4 \times 0.027} = 11.1$$

Then, we substitute in equation 8, giving

$$E = \frac{\eta_p}{c}\left(\frac{C_L^{3/2}}{C_D}\right)\sqrt{2\rho_\infty S}\left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}}\right)$$

$$\therefore E = \frac{0.83}{2.12 \times 10^{-7}} \times 11.1 \times \sqrt{2 \times 0.00238 \times 181} \times \left(\frac{1}{\sqrt{3150}} - \frac{1}{\sqrt{3400}}\right) = 26{,}900\ \text{s}$$

$$\therefore \boxed{E = 7.47\ \text{h}}$$

★ The correct answer is **D**.

P.7 ■ Solution

Part 1: At an altitude of 9 km, the density ratio σ = 0.3813 (Table 1). The induced drag correction factor is given by

$$K = \frac{1}{\pi ARe} = \frac{1}{\pi \times 6.85 \times 0.83} = 0.0560$$

The TSFC is 0.76/3600 = 2.11×10^{-4} s^{-1} and the airspeed is V = 800/3.6 = 222 m/s. The lift coefficient is given by

$$C_L = \frac{2(W/S)}{\rho_{SSL}\sigma V^2} = \frac{2 \times 3600}{1.225 \times 0.3813 \times 222^2} = 0.313$$

while the drag coefficient follows as

$$C_D = C_{D,0} + KC_L^2 = 0.015 + 0.056 \times 0.313^2 = 0.0205$$

The aerodynamic efficiency (i.e., the lift-to-drag ratio) is $E = C_L/C_D$ = 0.313/0.0205 = 15.3. The thrust-to-weight ratio is $F/W = 1/E$ = 1/15.3 = 0.0654. Appealing to equation 9, the range for constant h-C_L flight is determined to be

$$x_{h-C_L} = \frac{2EV}{TSFC}\left(1 - \sqrt{1-\zeta}\right) = \frac{2 \times 15.3 \times 222}{2.11 \times 10^{-4}} \times \left(1 - \sqrt{1-0.3}\right) = 5.26 \times 10^6\ \text{m}$$

$$\therefore x_{h-C_L} = 5260\ \text{km}$$

The corresponding endurance is given by equation 10,

$$t_{C_L}=\frac{E}{TSFC}\ln\left(\frac{1}{1-\zeta}\right)=\frac{15.3}{2.11\times10^{-4}}\ln\left(\frac{1}{1-0.3}\right)=25{,}900\ \text{s}$$

$$\therefore t_{C_L}=7.19\ \text{h}$$

The range for constant V-C_L flight follows from equation 11,

$$x_{V-C_L}=\frac{EV}{TSFC}\ln\left(\frac{1}{1-\zeta}\right)=\frac{15.3\times222}{2.11\times10^{-4}}\ln\left(\frac{1}{1-0.3}\right)=5.74\times10^{6}\ \text{m}$$

$$\therefore x_{V-C_L}=5740\ \text{km}$$

The endurance for contant V-C_L flight is the same as that for constant h-C_L flight; that is,

$$t_{V-C_L}=t_{h-C_L}=t_{C_L}=7.19\ \text{h}$$

Before proceeding, we require the maximum aerodynamic efficiency E_m, namely

$$E_m=\frac{1}{2\sqrt{KC_{D,0}}}=\frac{1}{2\times\sqrt{0.0560\times0.015}}=17.3$$

Then, the range for constant h-V flight is obtained with equation 12.

$$x_{h-V}=\frac{2E_mV}{TSFC}\arctan\left[\frac{E\zeta}{2E_m\left(1-KEC_L\zeta\right)}\right]$$

$$\therefore x_{h-V}=\frac{2\times17.3\times222}{2.11\times10^{-4}}\times\arctan\left[\frac{15.3\times0.3}{2\times17.3\times\left(1-0.0560\times15.3\times0.313\times0.3\right)}\right]=5.22\times10^{6}\ \text{m}$$

$$\therefore x_{h-V}=5220\ \text{km}$$

The endurance of a constant h-V flight is obtained simply by dividing the range obtained above by the velocity V = 800 km/h. Accordingly,

$$t_{h-V}=\frac{5220}{800}=6.53\ \text{h}$$

The flight figures calculated in this problem are summarized below. Clearly, constant airspeed-lift coefficient flight provides the best range, about 9% greater than those of the other two flight types; the endurance is the same for constant altitude-lift coefficient and constant airspeed-lift coefficient flight. The asterisk (*) for constant altitude-airspeed flight parameters indicates conditions that apply at the start of cruise.

Flight program	C_L	C_D	E	F/W	Range (km)	Endurance (h)
Constant altitude-lift coefficient	0.313	0.0205	15.3	0.0654	5260	7.19
Constant airspeed-lift coefficient	0.313	0.0205	15.3	0.0654	5740	7.19
Constant altitude-airspeed	0.313*	0.0205*	15.3*	0.0654*	5220	6.53

★ Statements **3**, **4** and **5** are true, while statements **1**, **2** and **6** are false.

Part 2: The first step is to compute the maximum-range airspeed at the start of cruise using equation 13,

$$V_{MR,h-C_L}=\left[\frac{2(W/S)}{\rho_{SSL}\sigma}\right]^{1/2}\left(\frac{3K}{C_{D,0}}\right)^{1/4}$$

$$\therefore V_{MR,h-C_L}=\left(\frac{2\times3600}{1.225\times0.3813}\right)^{1/2}\times\left(\frac{3\times0.0560}{0.015}\right)^{1/4}=227\ \text{m/s}=817\ \text{km/h}$$

In addition, the aerodynamic efficiency for maximum range in constant altitude-constant lift coefficient flight is given by equation 14,

$$E_{MR,h-C_L}=0.866E_m=0.866\times17.3=15.0$$

Inserting these results into equation 9 gives the maximum range for the flight type in question,

$$x_{MR,h-C_L} = \frac{E_{MR,h-C_L} V_{MR,h-C_L}}{TSFC}\left(1-\sqrt{1-\zeta}\right)$$

$$\therefore x_{MR,h-C_L} = \frac{15.0\times 227}{2.11\times 10^{-4}}\times\left(1-\sqrt{1-0.3}\right) = 5.27\times 10^{6}\text{ m}$$

$$\therefore x_{MR,h-C_L} = 5270\text{ km}$$

The maximum-range airspeed for constant airspeed-lift coefficient flight is the same as that for constant altitude-lift coefficient flight; that is,

$$V_{MR,V-C_L} = V_{MR,h-C_L} = 227\text{ m/s} = 817\text{ km/h}$$

The aerodynamic efficiency is also such that $E_{MR,V-C_L} = E_{MR,h-C_L} = 15.0$. Then, inserting these results into equation 11 yields

$$x_{MR,V-C_L} = \frac{E_{MR,V-C_L} V_{MR,V-C_L}}{TSFC}\ln\left(\frac{1}{1-\zeta}\right)$$

$$\therefore x_{MR,V-C_L} = \frac{15.0\times 227}{2.11\times 10^{-4}}\times\ln\left(\frac{1}{1-0.3}\right) = 5.76\times 10^{6}\text{ m}$$

$$\therefore x_{MR,V-C_L} = 5760\text{ km}$$

The maximum-range airspeed for constant altitude-constant airspeed flight is given by equation 15,

$$V_{MR,h-V} = V_{MR,h-C_L}\left(1-\zeta\right)^{1/4}$$

$$\therefore V_{MR,h-V} = 227\times\left(1-0.3\right)^{1/4} = 208\text{ m/s} = 749\text{ km/h}$$

The lift coefficient for maximum-range constant h-V flight follows from equation 16,

$$C_{L,MR;h-V} = \left[\frac{C_{D,0}}{3K\left(1-\zeta^{*}\right)}\right]^{1/2} = \left[\frac{0.015}{3\times 0.0560\times\left(1-0.3\right)}\right]^{1/2} = 0.357$$

The aerodynamic efficiency for maximum-range constant h-V flight follows from equation 17,

$$E_{MR,h-V} = \frac{2E_m\sqrt{3\left(1-\zeta^{*}\right)}}{1+3\left(1-\zeta^{*}\right)} = \frac{2\times 17.3\times\sqrt{3\times\left(1-0.3\right)}}{1+3\left(1-0.3\right)} = 16.2$$

It remains to insert these results into range equation 12, giving

$$x_{MR,h-V} = \frac{2E_m V_{MR;h-V}}{TSFC}\arctan\left[\frac{E_{MR;h-V}\zeta}{2E_m\left(1-KE_{MR;h-V}C_{L,MR;h-V}\zeta\right)}\right]$$

$$\therefore x_{MR,h-V} = \frac{2\times 17.3\times 208}{2.11\times 10^{-4}}\arctan\left[\frac{16.2\times 0.3}{2\times 17.3\times\left(1-0.0560\times 16.2\times 0.357\times 0.3\right)}\right] = 5.26\times 10^{6}\text{ m}$$

$$\therefore x_{MR,h-V} = 5260\text{ km}$$

The results for different maximum range flight programs are summarized below.

Maximum range flight program	V (km/h)	Range (km)
Constant altitude-lift coefficient	817	5270
Constant airspeed-lift coefficient	817	5760
Constant altitude-airspeed	749	5260

★ Statements **1**, **4** and **6** are true, while statements **2**, **3** and **5** are false.

Part 3: For an altitude of 6 km, the density ratio σ = 0.5389 (Table 1). The airspeed for fastest turn is given by equation 21,

$$V_{FT} == \left[\frac{2\left(W/S\right)}{\rho_{SSL}\sigma}\right]^{1/2}\left(\frac{K}{C_{D,0}}\right)^{1/4}$$

$$\therefore V_{FT} = \left[\frac{2\times 3600}{1.225\times 0.5389}\right]^{1/2} \times \left(\frac{0.0560}{0.015}\right)^{1/4} = 145 \text{ m/s} = 523 \text{ km/h}$$

It is assumed here that the aircraft has first climbed to an altitude of 6 km under constant throttle setting with a negligible amount of fuel consumption. This brings to $F/W = (F_{SSL}/W)\sigma$ = 0.33×0.5389 = 0.178. The load factor for fastest turn follows from equation 22,

$$n_{FT} = \sqrt{2(F/W)E_m - 1} = \sqrt{2\times 0.178\times 17.3 - 1} = 2.27$$

while the fastest turn bank angle is obtained with equation 23,

$$\phi_{FT} = \arccos(1/n_{FT}) = \arccos(0.441)$$

$$\therefore \phi_{FT} = 63.8^{\circ}$$

The lift coefficient for fastest turn is calculated according to equation 24,

$$C_{L,FT} = \left\{\frac{\left[2(F/W)E_m - 1\right]C_{D,0}}{K}\right\}^{1/2}$$

$$\therefore C_{L,FT} = \left[\frac{(2\times 0.178\times 17.3 - 1)\times 0.015}{0.0560}\right]^{1/2} = 1.18$$

while the aerodynamic efficiency is computed with equation 25,

$$E_{FT} = \frac{1}{F/W}\sqrt{2(F/W)E_m - 1} = \frac{1}{0.178}\times\sqrt{2\times 0.178\times 17.3 - 1} = 12.8$$

The turning rate for fastest turn is given by equation 26,

$$\dot{\chi}_{FT} = \frac{g}{V_{FT}}\sqrt{n_{FT}^2 - 1} = \frac{9.81}{145}\times\sqrt{2.27^2 - 1} = 0.138 \text{ rad/s} = 7.91 \text{ deg/s}$$

and the turning radius is obtained with equation 27 or, alternatively, simply by dividing the turning velocity by the turning rate,

$$V_{FT} = \dot{\chi}_{FT} r_{FT} \rightarrow r_{FT} = \frac{V_{FT}}{\dot{\chi}_{FT}}$$

$$\therefore r_{FT} = \frac{145}{0.138} = 1050 \text{ m}$$

The airspeed for tightest turn is given by equation 28,

$$V_{TT} = 2\left[\frac{K(W/S)}{\rho_{SSL}\sigma(F/W)}\right]^{1/2}$$

$$\therefore V_{TT} = 2\left[\frac{0.0560\times 3600}{1.225\times 0.5389\times 0.178}\right]^{1/2} = 82.8 \text{ m/s} = 298 \text{ km/h}$$

The load factor for tightest turn follows from equation 29,

$$n_{TT} = \left[2 - \frac{1}{E_m^2(F/W)^2}\right]^{1/2} = \left(2 - \frac{1}{17.3^2\times 0.178^2}\right)^{1/2} = 1.38$$

while the tightest turn bank angle is obtained with equation 30,

$$\phi_{TT} = \arccos(1/n_{TT}) = \arccos(0.725)$$

$$\therefore \phi_{TT} = 43.5^{\circ}$$

The lift coefficient for tightest turn is calculated with equation 31,

$$C_{L,TT} = \frac{1}{2KE_m}\left[2(F/W)^2 E_m^2 - 1\right]^{1/2}$$

$$\therefore C_{L,TT} = \frac{1}{2\times 0.0560\times 17.3}\times\left(2\times 0.178^2\times 17.3^2 - 1\right)^{1/2} = 2.19$$

while the aerodynamic efficiency is computed with equation 32,

$$E_{TT} = \frac{1}{E_m (F/W)^2}\left[2(F/W)^2 E_m^2 - 1\right]^{1/2} = \frac{1}{17.3\times 0.178^2}\times\left[2\times 0.178^2\times 17.3^2 - 1\right]^{1/2} = 7.73$$

The turning rate for tightest turn is given by equation 33,

$$\dot{\chi}_{TT} = \frac{g}{V_{TT}}\sqrt{n_{TT}^2 - 1} = \frac{9.81}{82.8}\times\sqrt{1.38^2 - 1} = 0.113 \text{ rad/s} = 6.46 \text{ deg/s}$$

Lastly, the turning radius is obtained with equation 34 or, alternatively, simply by dividing the turning velocity by the turning rate,

$$V_{TT} = \dot{\chi}_{TT} r_{TT} \rightarrow r_{TT} = \frac{V_{TT}}{\dot{\chi}_{TT}}$$

$$\therefore r_{FT} = \frac{82.8}{0.113} = 733 \text{ m}$$

The parameters obtained in this analysis are summarized below.

	Fastest turn	Tightest turn
Airspeed	523 km/h	298 km/h
Load factor	2.27	1.38
Bank angle	63.8$^{\text{o}}$	43.5$^{\text{o}}$
Lift coefficient	1.18	2.19
AD Efficiency	12.8	7.73
Turning rate	7.91 deg/s	6.46 deg/s
Turning radius	1050 m	733 m

★ Statements **2**, and **4** are true, while statements **1** and **3** are false.

P.8 ■ Solution

The aspect ratio is

$$AR = \frac{b^2}{S} = \frac{28^2}{100} = 7.84$$

The induced drag correction factor is

$$K = \frac{1}{\pi AR e} = \frac{1}{\pi\times 7.84\times 0.7} = 0.0580$$

The maximum aerodynamic efficiency is

$$E_m = \left(\frac{C_L}{C_D}\right)_{\text{max}} = \frac{1}{2\sqrt{KC_{D,0}}} = \frac{1}{2\times\sqrt{0.0580\times 0.02}} = 14.7$$

The cruise-fuel weight fraction is

$$\zeta = \frac{m_f}{m_i} = \frac{8000}{50{,}000} = 0.16$$

At 30,000 ft, the speed of sound is 303 m/s (Table 1), and the velocity of the craft is V = 0.82×303 = 248 m/s. The lift coefficient is determined next,

$$C_L = \frac{W}{\frac{1}{2}\rho_{SSL}\sigma V^2 S} = \frac{50{,}000\times 9.81}{\frac{1}{2}\times 1.225\times 0.374\times 248^2\times 100} = 0.348$$

The drag coefficient, in turn, is

$$C_D = C_{D,0} + KC_L^2 = 0.02 + 0.0580\times 0.348^2 = 0.0270$$

The aerodynamic efficiency follows as

$$E = \frac{C_L}{C_D} = \frac{0.348}{0.0270} = 12.9$$

Lastly, the range for a constant altitude-constant airspeed flight such as the present one is given by equation 12, namely

$$x_{h-V}=\frac{2E_mV}{TSFC}\arctan\left[\frac{E\zeta}{2E_m(1-KEC_L\zeta)}\right]$$

$$\therefore x_{h-V}=\frac{2\times14.7\times248}{(0.76/3.6)}\times\arctan\left[\frac{12.9\times0.16}{2\times14.7\times(1-0.0580\times12.9\times0.348\times0.16)}\right]=2530\text{ km}$$

The maximum-range airspeed for cruise-climb flight is obtained by dint of equation 13,

$$V_{MR}=\left[\frac{2(W/S)}{\rho_{SSL}\sigma}\right]^{1/2}\left(\frac{3K}{C_{D,0}}\right)^{1/4}$$

$$\therefore V_{MR;h-C_L}=\left[\frac{2\times(50,000\times9.81)/100}{1.225\times0.374}\right]^{1/2}\times\left(\frac{3\times0.0580}{0.02}\right)^{1/4}=251\text{ m/s}$$

The maximum-range airspeed for constant altitude-constant lift coefficient follows from equation 15,

$$V_{MR;h-V}=V_{MR;h-C_L}\left(1-\zeta^*\right)^{1/4}=251\times(1-0.16)^{1/4}=240\text{ m/s}$$

The lift coefficient for maximum-range constant h-C_L flight is given by equation 16,

$$C_{L,MR;h-V}=\left[\frac{C_{D,0}}{2K\left(1-\zeta^*\right)}\right]^{1/2}=\left[\frac{0.02}{2\times0.0580\times(1-0.16)}\right]^{1/2}=0.453$$

The aerodynamic efficiency for maximum-range constant h-C_L flight is given by equation 17,

$$E_{MR;h-V}=\frac{2E_m\sqrt{3\left(1-\zeta^*\right)}}{1+3\left(1-\zeta^*\right)}=\frac{2\times14.7\times\sqrt{3(1-0.16)}}{1+3\times(1-0.16)}=13.3$$

Finally, the maximum range for constant h-C_L flight is given by applying the pertaining results to equation 12,

$$x_{MR;h-V}=\frac{2E_mV_{MR;h-V}}{TSFC}\arctan\left[\frac{E_{MR;h-V}\zeta}{2E_m\left(1-KE_{MR;h-V}C_{L,MR;h-V}\zeta\right)}\right]$$

$$\therefore x_{MR;h-V}=\frac{2\times14.7\times240}{(0.76/3.6)}\arctan\left[\frac{13.3\times0.16}{2\times14.7\times(1-0.0580\times13.3\times0.453\times0.16)}\right]=2560\text{ km}$$

At steady conditions, thrust equals drag. Thus,

$$F=D=\frac{1}{2}\rho_{\text{SSL}}\sigma V^2SC_D$$

$$\therefore F=\frac{1}{2}\times1.225\times0.374\times248^2\times100\times0.0270=38\text{ kN}$$

Knowing the thrust F = 38,000 N and the TSFC = 0.76 kg/h/kg, the fuel consumption rate is determined as

$$Q_f=\frac{F\times TSFC}{g}=\frac{38,000\times0.76}{9.81}=2940\text{ kg/h}$$

The flight duration can be estimated by dividing the mass of fuel lost by the fuel consumption rate,

$$t=\frac{m_f}{Q_f}=\frac{8000}{2940}=2.72\text{ h}$$

Finally, suppose the craft in question is flying in an optimized cruise-climb flight. The maximum-range aerodynamic efficiency is given by equation 14,

$$E_{MR,V-C_L}=0.866E_m=0.866\times14.7=12.7$$

Inserting the pertaining data into equation 11 gives

$$x_{MR,V-C_L} = \frac{E_{MR,V-C_L} V_{MR,V-C_L}}{TSFC} \ln\left(\frac{1}{1-\zeta}\right) = \frac{12.7\times 251}{(0.76/3.6)} \ln\left(\frac{1}{1-0.16}\right) = 2630 \text{ km}$$

★ Statements **1**, **4**, and **6** are true, while statements **2**, **3** and **5** are false.

P.9 ■ Solution

At an altitude of 26,300 ft, the air density is about 0.526 kg/m³ (Table 1). The corresponding lift coefficient is

$$C_{L_C} = \frac{2W}{\rho_{SSL}\sigma V_C^2 S} = \frac{2\times(125,000\times 9.81)}{0.526\times(500\times 0.5144)^2\times 240} = 0.294$$

The takeoff lift coefficient is, with no allowance for high lift devices,

$$C_{L_{TO}} = C_{L_C} + \Delta C_L = 0.294 + 0 = 0.294$$

The takeoff drag coefficient follows as

$$C_{D_{TO}} = C_{D,0_{TO}} + KC_{L_{TO}}^2 = 0.048 + 0.052\times 0.294 = 0.0633$$

. Parameter A in equation 35 is given by

$$A = \frac{F}{m} - \mu g = \frac{60,000}{125,000} - 0.01\times 9.81 = 0.382$$

while parameter B is given by

$$B = -\frac{\rho_{SSL} S}{2m}\left(C_{D_{TO}} - \mu C_{L_{TO}}\right) = \frac{1.225\times 240}{2\times 125,000}\times(0.0633 - 0.04\times 0.294) = 6.06\times 10^{-5}$$

The roll velocity is V_R = 1.3×(53.8×0.5144) = 36.0 m/s. Substituting the pertaining variables in equation 35 yields

$$s_G = -\frac{1}{2B}\ln\left(\frac{A}{A+BV_R^2}\right)$$

$$\therefore s_G = -\frac{1}{2\times\left(6.06\times 10^{-5}\right)}\ln\left(\frac{0.382}{0.382+6.06\times 10^{-5}\times 36^2}\right) = \boxed{1540 \text{ m}}$$

★ The correct answer is **C**.

P.10 ■ Solution

Part 1: The ROC is given by equation 36,

$$ROC = \frac{(F-D)V}{W} = \frac{(2\times 24,200 - 15,000)\times 120}{16,500\times 9.81} = \boxed{24.8 \text{ m/s}}$$

The climb angle is determined next,

$$ROC = V\sin\alpha \rightarrow \alpha = \arcsin\left(\frac{ROC}{V}\right)$$

$$\therefore \alpha = \arcsin\left(\frac{24.8}{120}\right) = \boxed{11.9^{\circ}}$$

★ The correct answer is **C**.

Part 2: The altitude in question is 20,000×0.3048 = 6100 m. At a ROC of 24.8 m/s, the time required to ascend to this level is

$$\Delta t = \frac{6100}{24.8} = 246 \text{ s} = \boxed{4.1 \text{ min}}$$

★ The correct answer is **B**.

P.11 ■ Solution

Part 1: The wing aspect ratio is

$$AR = \frac{b^2}{S} = \frac{13^2}{25} = 6.76$$

The induced drag correction factor follows as

$$K = \frac{1}{\pi ARe} = \frac{1}{\pi \times 6.76 \times 0.9} = 0.0523$$

The maximum lift-to-drag ratio is determined next,

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{1}{2\sqrt{KC_{D,0}}} = \frac{1}{2\times\sqrt{0.0523\times0.015}} = 17.9$$

The airspeed for maximum rate of climb is established with equation 37,

$$V_{ROC_{\max}} = \sqrt{\frac{F}{3\rho_{SSL}C_{D,0}S}\left\{1+\sqrt{1+\frac{3}{\left[\left(\frac{L}{D}\right)_{\max}\frac{F}{W}\right]^2}}\right\}}$$

$$\therefore V_{ROC_{\max}} = \sqrt{\frac{2\times20,500}{3\times1.225\times0.015\times25}\left\{1+\sqrt{1+\frac{3}{\left[17.9\times\left(\frac{2\times20,500}{18,000\times9.81}\right)\right]^2}}\right\}} = \boxed{249\text{ m/s}}$$

This result should be compared with the stall speed at the current weight; that is,

$$V_S = \sqrt{\frac{2(W/S)}{\rho_{SSL}C_{L,\max}}} = \sqrt{\frac{2\times(18,000\times9.81/25)}{1.225\times2.0}} = 75.9\text{ m/s}$$

Since $V_{ROC_{\max}} > V_S$, the rate of climb obtained is acceptable. The angle corresponding to maximum rate of climb follows from equation 38,

$$\alpha_{ROC_{\max}} = -\arcsin\left\{\frac{1}{4KW}\left[\sqrt{\rho_{SSL}^2S^2V_{ROC_{\max}}^4\left(1+4KC_{D,0}\right)-8K\rho_{SSL}FSV_{ROC_{\max}}^2}-\rho_{SSL}SV_{ROC_{\max}}^2\right]\right\}$$

$$\therefore \alpha_{ROC_{\max}} = -\arcsin\left\{\frac{1}{4\times0.0523\times(18,000\times9.81)}\left[\sqrt{\begin{array}{l}1.225^2\times25^2\times249^4\left(1+4\times0.0523\times0.015\right)\\-8\times0.0523\times1.225\times(2\times20,500)\times25\times249^2\end{array}}-1.225\times25\times249^2\right]\right\}$$

$$\therefore \boxed{\alpha_{ROC_{\max}} = 8.73^\circ}$$

Equipped with $V_{ROC_{\max}}$ and $\alpha_{ROC_{\max}}$, we could easily estimate the maximum rate of climb,

$$ROC_{\max} = V_{ROC_{\max}}\sin\alpha_{ROC_{\max}} = 249\times\sin8.73^\circ = 37.8\text{ m/s}$$

★ The correct answer is **C**.

Part 2: The first step is to compute the airspeed for steepest climb angle, which can be shown to be equivalent to the velocity for minimum thrust required (equation 2). Thus,

$$V_{\alpha_{\max}} = \left[\frac{2(W/S)}{\rho_{SSL}\sigma}\right]^{1/2}\left(\frac{K}{C_{D,0}}\right)^{1/4}$$

$$\therefore V_{\alpha_{\max}} = \left[\frac{2(18,000\times9.81/25)}{1.225}\right]^{1/2}\left(\frac{0.0523}{0.015}\right)^{1/4} = 147\text{ m/s}$$

The steepest climb angle is given by equation 39,

$$\alpha_{\max} = \arcsin\left[\frac{1}{W}\left(F_{\max} - 2W\sqrt{KC_{D,0}}\right)\right]$$

$$\therefore \alpha_{\max} = \arcsin\left\{\frac{1}{18,000\times 9.81}\times\left[(2\times 20,500) - 2\times(18,000\times 9.81)\times\sqrt{0.0523\times 0.015}\right]\right\}$$

$$\therefore \alpha_{\max} = 10.1^{\circ}$$

Lastly, the ROC that corresponds to the steepest climb angle is

$$ROC_{\alpha_{\max}} = V_{\alpha_{\max}} \sin\alpha_{\max} = 147\times\sin 10.1^{\circ} = \boxed{25.8\text{ m/s}}$$

★ The correct answer is **D**.

P.12 ■ Solution

The density correction for an altitude of 30,000 ft is σ = 0.347. The cruising speed V_C can be estimated with equation 40,

$$AV_C^2 + \frac{B}{V_C^2} - 0.82CF_{\max,SSL} = 0$$

Here, coefficient A is given by

$$A = \frac{1}{2}\rho SC_{D,0} = \frac{1}{2}\times(0.347\times 1.225)\times 135\times 0.025 = 0.717$$

Coefficient B is given by

$$B = \frac{2KW^2}{\rho S} = \frac{2\times 0.051\times(52,000\times 9.81)^2}{(0.347\times 1.225)\times 135} = 4.63\times 10^8$$

Lastly, for a turbojet flying in the troposphere, coefficient C is expressed as

$$C = \left(\frac{\rho}{\rho_0}\right)^{0.9} = 0.374^{0.9} = 0.413$$

Backsubstituting in the first equation yields

$$0.717V_C^2 + \frac{4.63\times 10^8}{V_C^2} - 0.82\times 0.413\times 120,000 = 0$$

$$\therefore 0.717V_C^2 + \frac{4.63\times 10^8}{V_C^2} - 40,600 = 0$$

Solving this equation yields two negative solutions and two positive solutions. The latter are $V_C \approx$ 126 m/s and $V_C \approx$ 202 m/s. The ceiling speed is the greater result, V_C = 202 m/s = 727 km/h.

★ The correct answer is **A**.

P.13 ■ Solution

The solution is started by computing the maximum lift-to-drag ratio,

$$\left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{KC_{D,0}}} = \frac{1}{2\times\sqrt{0.048\times 0.019}} = 16.6$$

The cruise altitude air density is given by equation 41,

$$\rho_C = \left[\frac{1.155W\rho_{SSL}^{1.2}}{(L/D)_{\max}F_{SSL}}\right]^{1/1.2}$$

$$\therefore \rho_C = \left[\frac{1.155\times(55,000\times 9.81)\times 1.225^{1.2}}{16.6\times(2\times 60,000)}\right]^{1/1.2} = 0.465\text{ kg/m}^3$$

The altitude that corresponds to this air density is about 9040 m, or about 29,700 ft. This is the cruise altitude of the craft in question. Next, the cruise speed, assuming maximum-range conditions, is given by equation 13,

$$V_C = V_{MR;h-C_L} = \left[\frac{2(W/S)}{\rho_{SSL}\sigma}\right]^{1/2}\left(\frac{3K}{C_{D,0}}\right)^{1/4}$$

$$\therefore V_C = \left[\frac{2\times(55{,}000\times 9.81/98)}{0.465}\right]^{1/2}\times\left(\frac{3\times 0.048}{0.019}\right)^{1/4} = 255\text{ m/s}$$

The speed of sound at an altitude of 9040 m is about 304 m/s. Accordingly, the cruise Mach number is

$$M_C = \frac{255}{304} = \boxed{0.839}$$

★ The correct answer is **D**.

P.14 ■ Solution

The airspeed is V_∞ = 400/3.6 = 111 m/s. The turn rate is found as

$$\dot{\chi} = \frac{2\pi}{30} = 0.209\text{ rad/s}$$

The load factor is obtained with equation 18,

$$n = \sqrt{\left(\frac{\dot{\chi}V_\infty}{g}\right)^2 + 1} = \sqrt{\left(\frac{0.209\times 111}{9.81}\right)^2 + 1} = 2.57$$

Equation 19 is used to solve for the turn radius,

$$R = \frac{V_\infty^2}{g\sqrt{n^2-1}} = \frac{111^2}{9.81\times\sqrt{2.57^2-1}} = \boxed{531\text{ m}}$$

Equation 20, in turn, is used to solve for the bank angle,

$$\phi = \arccos\left(\frac{1}{n}\right) = \arccos\left(\frac{1}{2.57}\right)$$

$$\therefore \boxed{\phi = 67.1^\circ}$$

★ The correct answer is **B**.

Answer Summary

Problem 1		C
Problem 2		B
Problem 3		A
Problem 4	4.1	Open-ended pb.
	4.2	B
	4.3	Open-ended pb.
	4.4	B
Problem 5	5.1	D
	5.2	B
	5.3	D
Problem 6		D
Problem 7	7.1	T/F
	7.2	T/F
	7.3	T/F
Problem 8		T/F
Problem 9		C
Problem 10	10.1	C
	10.2	B
Problem 11	11.1	C
	11.2	D
Problem 12		A
Problem 13		D
Problem 14		B

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