Montogue

Quiz AS105
Airfoils and Wings

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Problems

Problem 1

Consider a thin symmetric airfoil at 6° angle of attack. From the results of thin airfoil theory, calculate the lift coefficient and the moment coefficient about the leading edge.

A) c_l = 0.332 and $c_{m,LE}$ = − 0.114
B) c_l = 0.332 and $c_{m,LE}$ = − 0.165
C) c_l = 0.660 and $c_{m,LE}$ = − 0.114
D) c_l = 0.660 and $c_{m,LE}$ = − 0.165

Problem 2.1 (Anderson, 2016, w/ permission)

The infinite-wing lift slope for the NACA 23012 airfoil is a_0 = 0.108 degree^{-1}, and the zero-lift angle is $\alpha_{L=0} = -1.3^{\circ}$. Consider a finite wing using this airfoil, with AR = 8 and taper ratio of 0.8. Calculate the lift and induced drag coefficients for this wing at a geometric angle of attack of 7°.

A) C_L = 0.357 and $C_{D,i}$ = 0.0212
B) C_L = 0.357 and $C_{D,i}$ = 0.0424
C) C_L = 0.713 and $C_{D,i}$ = 0.0212
D) C_L = 0.713 and $C_{D,i}$ = 0.0424

Problem 2.2

The infinite-wing lift slope for the NACA 23012 airfoil is a_0 = 0.142 degree^{-1}, and the zero lift angle is $\alpha_{L=0} = -0.81^{\circ}$. Consider a finite wing using this airfoil, with AR = 10 and taper ratio of 0.6. Calculate the lift and induced drag coefficients for this wing at a geometric angle of attack of 3°.

A) C_L = 0.427 and $C_{D,i}$ = 0.00601
B) C_L = 0.771 and $C_{D,i}$ = 0.0104
C) C_L = 0.427 and $C_{D,i}$ = 0.00601
D) C_L = 0.771 and $C_{D,i}$ = 0.0104

Problem 3.1 (Anderson, 2016, w/ permission)

A light general-purpose aircraft has a wing area of 17 m² and a wing span of 9.75 m. Its maximum gross weight is 1100 kg. The wing uses an NACA 65-415 airfoil, which has an infinite-wing lift slope a_0 = 0.105 degree^{-1} and a zero-lift angle $\alpha_{L=0} = -2.6^{\circ}$. Assume the correction parameter used in the lift slope formula to be τ = 0.12. If the airplane is cruising at 200 km/h at standard sea level at its maximum gross weight and is in straight-and-level flight, calculate the geometric angle of attack of the wing.

A) α = 0.771°
B) α = 1.83°
C) α = 2.92°
D) α = 3.41

Problem 3.2

A fast general-purpose aircraft has a wing area of 181 ft² and a wing span of 33.5 ft. Its maximum gross weight is 4400 lb. The wing uses an NACA 63(2)-615 airfoil, which has an infinite-wing lift slope a_0 = 0.121 degree^{-1} and a zero-lift angle $\alpha_{L=0}$ = −1.2°. Assume the correction parameter used in the lift slope formula to be τ = 0.10. If the airplane is cruising at 250 mi/h at standard sea level at its maximum gross weight and is in straight-and-level flight, calculate the geometric angle of attack of the wing.

A) α = 0.538°

B) α = 1.71°

C) α = 2.61°

D) α = 3.81

Problem 3.3

The span efficiency factor for a finite wing in an aircraft is generally much less than that for a finite wing alone. Reconsider Problem 3.2, assuming that e = 0.58. Calculate the induced drag.

A) D_i = 36.4 lb

B) D_i = 58.9 lb

C) D_i = 79,8 lb

D) D_i = 90.6 lb

Problem 4 (Anderson, 2016, w/ permission)

The NACA 4412 airfoil has a mean camber line given by

$$\frac{z}{c}=\begin{cases}0.25\left[0.8\dfrac{x}{c}-\left(\dfrac{x}{c}\right)^2\right] & ;\ 0\le\dfrac{x}{c}\le 0.4\\ 0.111\left[0.2+0.8\dfrac{x}{c}-\left(\dfrac{x}{c}\right)^2\right] & ;\ 0.4\le\dfrac{x}{c}\le 1\end{cases}$$

True or false?

1.( ) The absolute value of the zero-lift angle is greater than 3.5°.

2.( ) The per-unit-span lift coefficient for α = 3° is greater than 0.82.

3.( ) The per-unit-span moment coefficient about the quarter chord is, when expressed in absolute value, greater than 0.08.

4.( ) The relative location x_{cp}/c of the center of pressure is greater than 0.42.

Problem 5 (Rathakrishnan, 2013, w/ permission)

A sail plane of wing span 16 m, aspect ratio 15 and taper ratio 0.32 is in level flight at an altitude where the relative density is 0.7. The true airspeed measured by an error-free airspeed indicator is 118 km/h. The lift and drag acting on the wing are 3800 N and 250 N, respectively. The pitching moment coefficient about the quarter-chord point is −0.025. True or false?

1.( ) The mean chord of the wing is greater than 0.8 m.

2.( ) The lift coefficient is greater than 0.3.

3.( ) The drag coefficient is greater than 0.025.

4.( ) The absolute value of the pitching moment about the leading edge is greater than 800 N·m.

Problem 6 (Rathakrishnan, 2013, w/ permission)

A wing with elliptical loading distribution has a span of 18 m, a planform area of 46 m^2, is in level flight at 800 km/h, at an altitude where the air density is 0.65 kg/m^3. The induced drag acting on the wing is 3000 N. Determine the lift coefficient. True or false?

1.( ) The lift coefficient is greater than 0.35.

2.( ) The downwash velocity is greater than 2.5 m/s.

3.( ) The wing loading is greater than 4500 N/m^2.

Problem 7 (Anderson, 2012, w/ permission)

Consider a finite wing with an aspect ratio of 7; the airfoil section of the wing is a symmetric airfoil with an infinite-wing lift slope of 0.11 degree^{-1}. The lift-to-drag ratio for this wing is 29 when the lift coefficient is equal to 0.35. If the angle of attack remains the same and the aspect ratio is simply increased to 10 by adding extensions to the span of the wing, what is the new value of the lift-to-drag ratio? Assume that the span efficiency factor e = 0.9 for both cases.

A) E' = 30.1

B) E' = 34.4

C) E' = 38.5

D) E' = 40.1

Problem 8

Consider a finite wing at an angle of attack of 6°. The normal and axial force coefficients are 0.8 and 0.06, respectively. Calculate the corresponding lift and drag coefficients.

A) c_l = 0.577 and c_d = 0.0912

B) c_l = 0.577 and c_d = 0.143

C) c_l = 0.789 and c_d = 0.0912

D) c_l = 0.789 and c_d = 0.143

Problem 9

Using the Prandtl-Glauert rule, calculate the lift coefficient for an NACA 2412 airfoil at 10° angle of attack in a Mach 0.65 freestream.

A) c_j = 1.52

B) c_j = 1.71

C) c_j = 1.89

D) c_j = 2.04

Problem 10

In low-speed incompressible flow, the peak pressure coefficient (at the minimum pressure point) on an NACA 0012 airfoil is −0.41. Estimate the critical Mach number for this airfoil using the Prandtl-Glauert rule.

A) M_{cr} = 0.505

B) M_{cr} = 0.609

C) M_{cr} = 0.743

D) M_{cr} = 0.880

Problem 11

Consider an airfoil in a Mach 0.5 freestream. At a given point on the airfoil, the local Mach number is 0.86. Using the compressible flow information in Table 2, calculate the pressure coefficient at that point.

A) C_p = −1.22

B) C_p = −1.53

C) C_p = −1.87

D) C_p = −2.12

Problem 12

Under low-speed incompressible flow conditions, the pressure coefficient at a given point on an airfoil was calculated to be −0.54. Calculate the pressure coefficient at this point when the freestream Mach number is 0.6, using the Prandtl-Glauert rule, the Karman-Tsien rule, and the Laitone rule. Rank your results in **decreasing order of absolute value**. Use γ= 1.4.

A) L > KT > PG

B) KT > L > PG

C) PG > KT > L

D) PG > L > KT

Problem 13.1 (Anderson, 2012, w/ permission)

Consider an airfoil mounted in a low-speed subsonic wind tunnel. The flow velocity in the test section is 150 ft/s, and the conditions are standard sea level. If the pressure at a point on the airfoil is 2080 lb/ft^2, what is the pressure coefficient?

A) $C_p = -1.34$

B) $C_p = -1.51$

C) $C_p = -1.60$

D) $C_p = -1.66$

Problem 13.2

In the previous problem, if the flow velocity is increased so that the free-stream Mach number is 0.6, what is the pressure coefficient at the same point on the airfoil?

A) $C_p = -1.68$

B) $C_p = -1.90$

C) $C_p = -2.11$

D) $C_p = -2.29$

Problem 14 (Anderson, 2012, w/ permission)

An airplane is flying at a velocity of 160 m/s at a standard altitude of 3 km. The pressure coefficient at a point on the fuselage is −2.5. What is the pressure at this point?

A) p = 10,800 N/m²

B) p = 21,100 N/m²

C) p = 30,900 N/m²

D) p = 41,100 N/m²

Problem 15 (Anderson, 2012, w/ permission)

Consider two different points on the surface of an airplane wing flying at 75 m/s. The pressure coefficient and flow velocity at point 1 are −1.4 and 120 m/s, respectively. The pressure coefficient at point 2 is −0.7. Assuming incompressible flow, calculate the flow velocity at point 2.

A) V_2 = 45.5 m/s

B) V_2 = 69.4 m/s

C) V_2 = 88.2 m/s

D) V_2 = 102 m/s

Additional Information

Figure 1 Induced drag factor δ as a function of taper ratio.

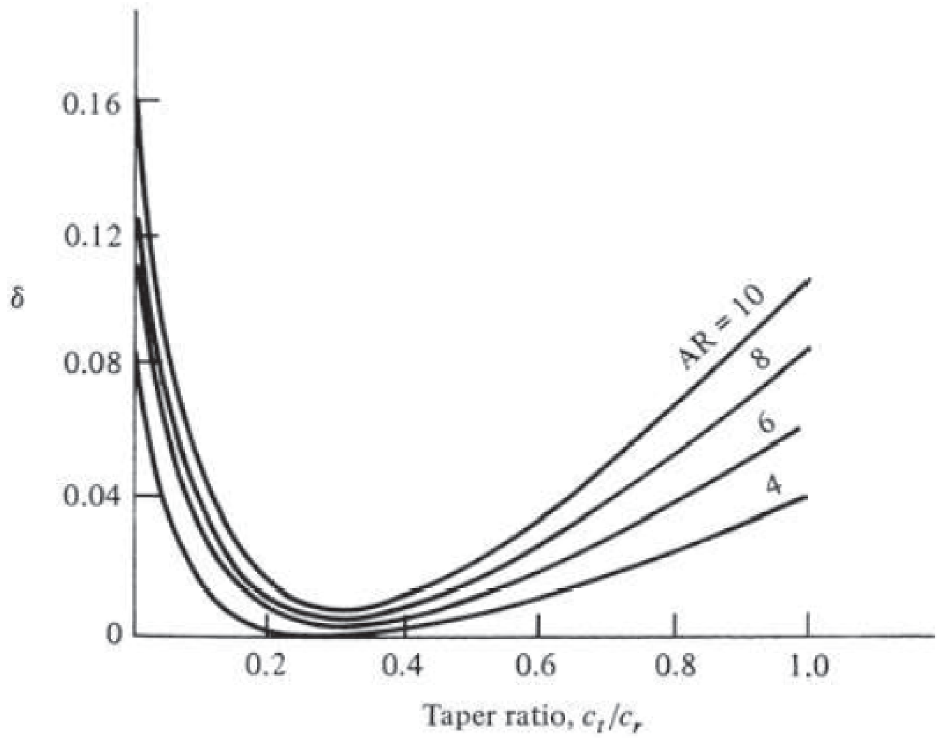


Figure 2 Variation of lift and moment coefficient for the NACA 2412 airfoil.

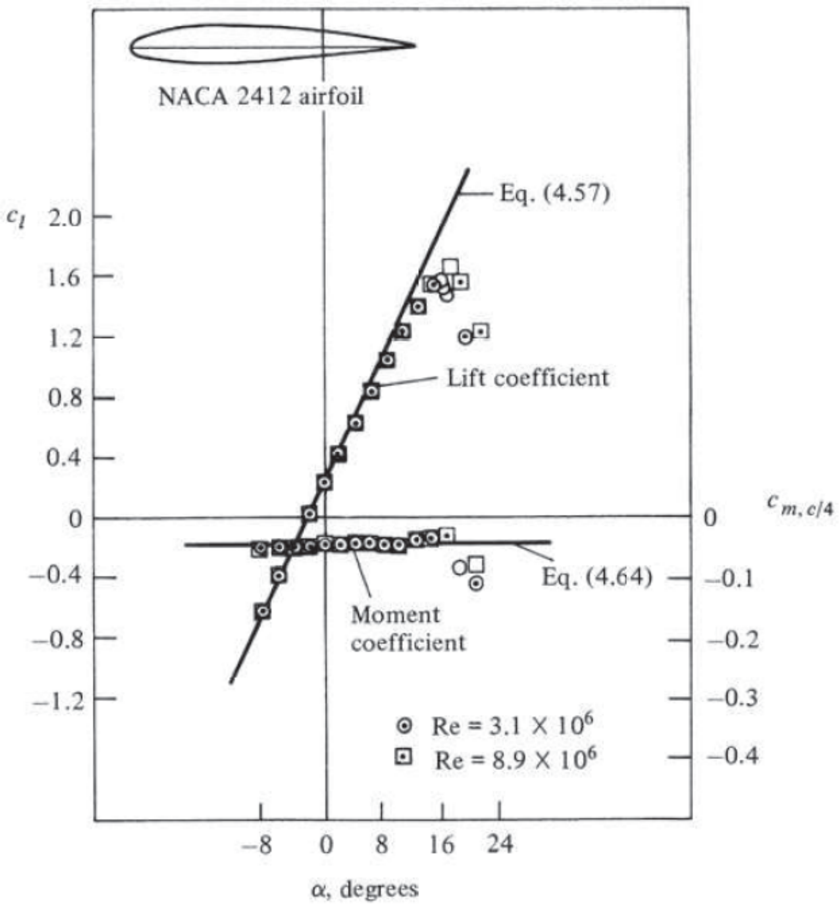


Table 1 Standard atmosphere – SI units

Altitude (m)	p (N/m²)	ρ (kg/m³)
1000	8.99×10^{4}	1.11
2000	7.95×10^{4}	1.01
3000	7.01×10^{4}	0.909
4000	6.17×10^{4}	0.819

Table 2 Isentropic flow properties

Mach number	p/p_0	T/T_0
0.2	1.1028	1.1008
0.3	1.1064	1.1018
0.4	1.1117	1.1032
0.5	1.1186	1.1050
0.6	1.1276	1.1072

Solutions

P.1 ■ Solution

The angle of attack is converted as α = 6 × π/180 = 0.105 rad. The lift coefficient is then

$$c_l = 2\pi\alpha = 2\pi \times 0.105 = \boxed{0.660}$$

The moment coefficient about the leading edge easily follows,

$$c_{m,LE} = -\frac{c_l}{4} = -\frac{0.660}{4} = \boxed{-0.165}$$

★ The correct answer is **D**.

P.2 ■ Solution

Part 1: The measured lift curve slope is converted as a_0 = 0.108 ×180/π = 6.19 rad^{-1}. Referring to Figure 1, the induced drag factor for an aspect ratio of 8 and a taper ratio of 0.8 is read as 0.05. We shall assume that the τ parameter used in the calculation of the lift slope is equal to the induced drag correction factor δ; that is, τ = δ = 0.05. The lift curve slope a is then

$$a=\frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)}=\frac{6.19}{1+\frac{6.19}{\pi\times 8}\times(1+0.05)}=4.92\text{ rad}^{-1}$$

or, equivalently, a = 4.92 × π/180 = 0.0859 deg^{-1}. The lift coefficient is established next,

$$C_L=a(\alpha-\alpha_{L=0})=0.0859\times\left[7^\circ-(-1.3^\circ)\right]=\boxed{0.713}$$

The induced drag coefficient, in turn, is given by

$$C_{D,i}=\frac{C_L^2}{\pi AR}(1+\delta)=\frac{0.713^2}{\pi\times 8}\times(1+0.05)=\boxed{0.0212}$$

★ The correct answer is **C**.

Part 2: The infinite-wing lift slope is converted as a_0 = 0.142 × 180/π = 8.14 rad^{-1}. Appealing to Figure 1, the induced drag factor for an aspect ratio of 10 and a taper ratio of 0.6 is established as 0.035. In the same manner as Problem 2.1, we postulate that τ = δ = 0.035. The lift curve slope a is then

$$a=\frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)}=\frac{8.14}{1+\frac{8.14}{\pi\times 10}\times(1+0.035)}=6.42\text{ rad}^{-1}$$

or, equivalently, a = 6.42 × π/180 = 0.112 deg^{-1}. We proceed to determine the lift coefficient,

$$C_L=a(\alpha-\alpha_{L=0})=0.112\times\left[3^\circ-(-0.81^\circ)\right]=\boxed{0.427}$$

The induced drag coefficient follows as

$$C_{D,i}=\frac{C_L^2}{\pi AR}(1+\delta)=\frac{0.427^2}{\pi\times 10}\times(1+0.035)=\boxed{0.00601}$$

★ The correct answer is **A**.

P.3 ■ Solution

Part 1: The aspect ratio of the wing is AR = 9.75^2/17 = 5.59. The flight speed is converted as 200/3.6 = 55.6 m/s. Since lift equals weight in level flight, we write L = W = 1100 × 9.81 = 10,800 N. The lift coefficient is determined next,

$$C_L=\frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 S}=\frac{10,800}{\frac{1}{2}\times 1.225\times 55.6^2\times 17}=0.336$$

The infinite-wing lift slope is converted as a_0 = 0.105 × 180/π = 6.02 rad^{-1}. The lift curve slope is then

$$a=\frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)}=\frac{6.02}{1+\frac{6.02}{\pi\times 5.59}\times(1+0.12)}=4.35\text{ rad}^{-1}$$

or, equivalently, a = 0.0759 deg^{-1}. It remains to compute the geometric angle of attack,

$$C_L=a(\alpha-\alpha_{L=0})\rightarrow\alpha=\frac{C_L}{a}+\alpha_{L=0}$$

$$\therefore\alpha=\frac{0.336}{0.0759}-2.6^\circ=\boxed{1.83^\circ}$$

★ The correct answer is **B**.

Part 2: The aspect ratio of the wing is AR = 33.5^2/181 = 6.20. The flight speed is converted as 250 × 1.47 = 368 ft/s. Since lift equals weight in steady flight, we write L = W = 3000 lb. The lift coefficient is then

$$C_L=\frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 S}=\frac{3300}{\frac{1}{2}\times 0.00238\times 368^2\times 181}=0.151$$

The infinite-wing lift slope is converted as a_0 = 0.121 × 180/π = 6.93 rad^{-1}. The value of a is calculated to be

$$a=\frac{a_0}{1+\frac{a_0}{\pi AR}(1+\tau)}=\frac{6.93}{1+\frac{6.93}{\pi\times 6.20}\times(1+0.10)}=4.98\text{ rad}^{-1}$$

which is equivalent to 0.0869 deg^{-1}. We now have the information necessary to compute the angle of attack,

$$C_L=a\left(\alpha-\alpha_{L=0}\right)\to\alpha=\frac{C_L}{a}+\alpha_{L=0}$$

$$\therefore\alpha=\frac{0.151}{0.0869}-1.2^{\circ}=\boxed{0.538^{\circ}}$$

★ The correct answer is **A**.

Part 3: We first determine the induced drag coefficient,

$$C_{D,i}=\frac{C_L^2}{\pi eAR}=\frac{0.151^2}{\pi\times 0.58\times 6.20}=2.02\times 10^{-3}$$

The induced drag follows as

$$D_i=\frac{1}{2}\rho_\infty V_\infty^2 SC_{D,i}=\frac{1}{2}\times 0.00238\times 368^2\times 181\times\left(2.02\times 10^{-3}\right)=\boxed{58.9\text{ lb}}$$

★ The correct answer is **B**.

P.4 ■ Solution

1. True. Deriving the equations that describe the airfoil with respect to x, we obtain

$$\left(\frac{dz}{dx}\right)_1=0.2-0.5\frac{x}{c}\;;\;\left(0\le\frac{x}{c}\le 0.4\right)$$

and

$$\left(\frac{dz}{dx}\right)_2=0.0888-0.222\frac{x}{c}\;;\;\left(0.4\le\frac{x}{c}\le 1.0\right)$$

We introduce the variable modification

$$x=\frac{c}{2}(1-\cos\theta)$$

which, inserting in the two previous equations, yields

$$\left(\frac{dz}{dx}\right)_1=-0.05+0.25\cos\theta\;;\;\left(0\le\theta\le 1.37\right)$$

and

$$\left(\frac{dz}{dx}\right)_2=-0.0223+0.111\cos\theta\;;\;\left(1.37\le\theta\le\pi\right)$$

The zero-lift angle follows from the relation

$$\alpha_{L=0}=-\frac{1}{\pi}\int_0^\pi\frac{dz}{dx}(\cos\theta-1)\,d\theta$$

$$\therefore\alpha_{L=0}=-\frac{1}{\pi}\int_0^{1.37}\left(-0.05+0.25\cos\theta\right)\left(\cos\theta-1\right)d\theta\;(\ldots)$$

$$(\ldots)-\frac{1}{\pi}\int_{1.37}^{\pi}\left(-0.0223+0.111\cos\theta\right)\left(\cos\theta-1\right)d\theta$$

$$\therefore\alpha_{L=0}=-\frac{1}{\pi}\times(-0.0298)-\frac{1}{\pi}\times(0.258)$$

$$\therefore\alpha_{L=0}=-0.0726=\boxed{-4.16^{\circ}}$$

2. False. The per-unit-span lift coefficient is given by

$$c_l = 2\pi\left(\alpha - \alpha_{L=0}\right) = 2\pi\left[3\times\frac{\pi}{180} - (-4.16)\times\frac{\pi}{180}\right] = \boxed{0.785}$$

3. True. The moment coefficient about the quarter chord is given by

$$c_{m,c/4} = \frac{\pi}{4}\left(A_2 - A_1\right)$$

Coefficient A_1 is evaluated as

$$A_1 = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos\theta d\theta$$

$$\therefore A_1 = \frac{2}{\pi}\int_0^{1.37}\left(-0.05 + 0.25\cos\theta\right)\cos\theta d\theta + \frac{2}{\pi}\int_{1.37}^{\pi}\left(-0.0223 + 0.111\cos\theta\right)\cos\theta d\theta$$

$$\therefore A_1 = \frac{2}{\pi}\times 0.147 + \frac{2}{\pi}\times 0.109$$

$$\therefore A_1 = 0.163$$

Coefficient A_2, in turn, is evaluated as

$$A_2 = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos 2\theta d\theta$$

$$\therefore A_2 = \frac{2}{\pi}\int_0^{1.37}\left(-0.05 + 0.25\cos\theta\right)\cos 2\theta d\theta + \frac{2}{\pi}\int_{1.37}^{\pi}\left(-0.0223 + 0.111\cos\theta\right)\cos 2\theta d\theta$$

$$\therefore A_2 = \frac{2}{\pi}\times 0.0784 + \frac{2}{\pi}\times\left(-0.0348\right)$$

$$\therefore A_2 = 0.0278$$

Backsubstituting into $c_{m,c/4}$ gives

$$c_{m,c/4} = \frac{\pi}{4}\left(A_2 - A_1\right) = \frac{\pi}{4}\times\left(0.0278 - 0.163\right) = \boxed{-0.106}$$

4. False. Ratio x_{cp}/c is expressed as

$$\frac{x_{\text{cp}}}{c} = \frac{1}{4}\left[1 + \frac{\pi}{c_l}\left(A_1 - A_2\right)\right] = \frac{1}{4}\times\left[1 + \frac{\pi}{0.785}\times\left(0.163 - 0.0278\right)\right] = \boxed{0.385}$$

P.5 ■ Solution

1. True. With the relative density σ = 0.7, the air density at the altitude of interest is ρ = 1.225×0.7 = 0.858 kg/m³. The equivalent airspeed is $V_\infty = V_1/\sqrt{\sigma}$ = 118/0.7$^{1/2}$ = 141 km/h = 39.2 m/s. Given the span 2b = 16 m and the aspect ratio AR = 15, the mean chord is calculated as

$$\bar{c} = \frac{2b}{AR} = \frac{16}{15} = 1.07 \text{ m}$$

2. True. The wing area is

$$S = 2b\times\bar{c} = 16\times 1.07 = 17.1 \text{ m}^2$$

The lift coefficient is determined next,

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{3800}{\frac{1}{2}\times 0.858\times 39.2^2\times 17.1} = 0.337$$

3. False. Likewise, the drag coefficient is

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 S} = \frac{250}{\frac{1}{2}\times 0.858\times 39.2^2\times 17.1} = 0.0222$$

4. False. The pitching moment coefficient about the leading edge is

$$C_{M_0} = -\frac{C_L}{4}\left(1+\frac{4C_{M,c/4}}{C_L}\right) = -\frac{0.337}{4}\times\left[1+\frac{4\times(-0.025)}{0.337}\right] = -0.0593$$

so that

$$M_o = \frac{1}{2}\rho V^2 S \bar{c}\, C_{M_0}$$

$$\therefore M_o = \frac{1}{2}\times 0.858\times 39.2^2\times 17.1\times 1.07\times(-0.0593) = -715\ \text{N}\cdot\text{m}$$

The negative sign indicates a nose-down moment.

P.6 ■ Solution

1. False. The airspeed is converted as V_∞ = 800/3.6 = 222 m/s. The aspect ratio of the wing is AR = 18²/46 = 7.04. The induced drag coefficient is

$$C_{D,i} = \frac{D_i}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{3000}{\frac{1}{2}\times 0.65\times 222^2\times 46} = 0.00407$$

The lift coefficient can be obtained by adjusting the relation

$$C_{D,i} = \frac{C_L^2}{\pi e AR} \rightarrow C_L = \sqrt{C_{D,i}\pi e AR}$$

$$\therefore C_L = \sqrt{0.00407\times\pi\times 1.0\times 7.04} = 0.30$$

2. True. The downwash velocity is given by

$$w = \frac{k_0}{4b}$$

However, the circulation component k_0 is stated as

$$k_0 = \frac{C_L VS}{\pi b}$$

Substituting in the equation for w and manipulating, we get

$$w = \frac{k_0}{4b} \rightarrow w = \frac{\frac{C_L VS}{\pi b}}{4b}$$

$$\therefore w = \frac{C_L V\times(2b\times c)}{\pi b\times 4b}$$

$$\therefore w = \frac{C_L V}{\pi(2b/c)}$$

$$\therefore w = \frac{C_L V}{\pi AR}$$

Substituting the available data gives

$$w = \frac{0.30\times 222}{\pi\times 7.04} = \boxed{3.01\ \text{m/s}}$$

3. True. In level flight, lift equals weight and, accordingly,

$$W = L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L = 0.5\times 0.66\times 222^2\times 46\times 0.30 = 224,000\ \text{N}$$

The wing loading is then

$$\frac{W}{S} = \frac{224,000}{46} = 4870\ \text{N/m}^2$$

P.7 ■ Solution

The infinite-wing lift slope is converted as 0.11 × 180/π = 6.30. We first compute the geometric angle of attack,

$$C_L = \frac{a_0\alpha}{1+\dfrac{a_0}{\pi eAR}} \rightarrow \alpha = \frac{C_L}{a_0}\left(1+\frac{a_0}{\pi eAR}\right)$$

$$\therefore \alpha = \frac{0.35}{0.11}\times\left[1+\frac{0.11\times(180/\pi)}{\pi\times 0.9\times 7}\right] = 4.2^{\circ}$$

Given the aerodynamic efficiency E = 29, the drag coefficient is determined as

$$E = \frac{C_L}{C_D} \rightarrow C_D = \frac{C_L}{E}$$

$$\therefore C_D = \frac{0.35}{29} = 0.0121$$

so that

$$C_D = c_d + \frac{C_L^2}{\pi eAR} \rightarrow c_d = C_D - \frac{C_L^2}{\pi eAR}$$

$$\therefore c_d = 0.0121 - \frac{0.35^2}{\pi\times 0.9\times 7} = 0.00591$$

Let the aspect ratio be equal to 10. The updated lift coefficient is

$$C_L' = \frac{a_0\alpha}{1+\dfrac{a_0}{\pi eAR'}} = \frac{0.11\times 4.2}{1+\dfrac{0.11\times(180/\pi)}{\pi\times 0.9\times 10}} = 0.378$$

The drag coefficient then becomes

$$C_D' = c_d + \frac{C_L^2}{\pi eAR} = 0.00591 + \frac{0.378^2}{\pi\times 0.9\times 10} = 0.0110$$

The updated aerodynamic efficiency is

$$E' = \frac{C_L'}{C_D'} = \frac{0.378}{0.0110} = \boxed{34.4}$$

As can be seen, increasing the wing aspect ratio increases the lift coefficient and the aerodynamic efficiency somewhat.

★ The correct answer is **B**.

P.8 ■ Solution

The lift and drag coefficients are related to the normal and axial force coefficients by the expressions

$$c_l = c_N\cos\alpha - c_A\sin\alpha$$

$$c_d = c_N\sin\alpha + c_A\cos\alpha$$

which, substituting the available data, yield

$$c_l = 0.8\times\cos 6^{\circ} - 0.06\times\sin 6^{\circ} = \boxed{0.789}$$

$$c_d = 0.8\times\sin 6^{\circ} + 0.06\times\cos 6^{\circ} = \boxed{0.143}$$

Observe that, since the angle of attack is quite small, the lift force coefficient and the normal force coefficient are quite close to each other.

★ The correct answer is **D**.

P.9 ■ Solution

Referring to Figure 2, we see that the per-unit-span lift coefficient for an angle of attack of 10° is close to 1.3. Appealing to the Prandtl-Glauert rule, we obtain

$$c_l = \frac{c_{l,0}}{\sqrt{1-M_\infty^2}} = \frac{1.3}{\sqrt{1-0.65^2}} = \boxed{1.71}$$

★ The correct answer is **B**.

P.10 ■ Solution

From the Prandtl-Glauert rule, we write

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}} = \frac{-0.41}{\sqrt{1-M_\infty^2}}$$

Some values of pressure coefficient C_p are computed with the Prandtl-Glauert rule and tabulated below.

M_∞	0.3	0.4	0.5	0.6	0.7	0.8
C_p	-0.43	-0.447	-0.473	-0.512	-0.574	-0.683

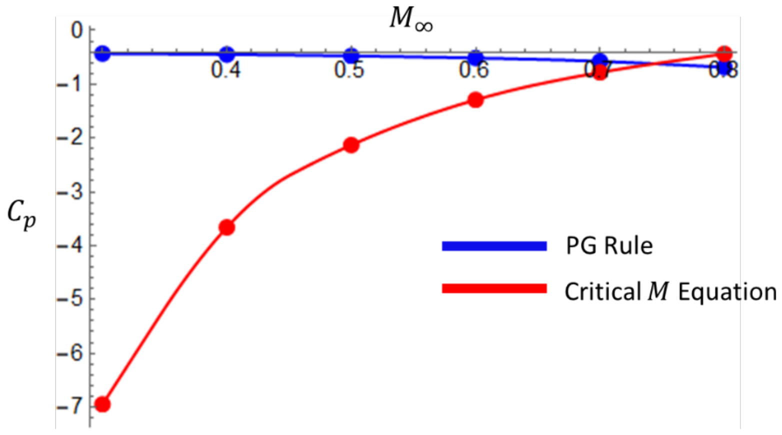
However, we know that the coefficient of pressure is related to the critical Mach number by an equation of the form

$$C_{p,\mathrm{cr}} = \frac{2}{\gamma M_{\mathrm{cr}}^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_{\mathrm{cr}}^2}{1+\frac{\gamma-1}{2}}\right)^{\gamma/(\gamma-1)} - 1\right]$$

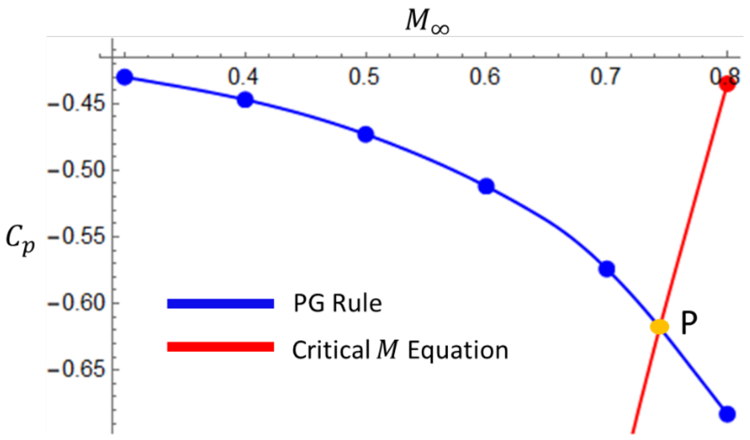
Accordingly, we tabulate some values of C_p with this relation as well.

M_∞	0.3	0.4	0.5	0.6	0.7	0.8
C_p	-6.95	-3.66	-2.13	-1.29	-0.78	-0.435

The two sets of data are plotted on the same plane.



Clearly, the two curves intersect at a point, but the plot above makes it difficult to distinguish where this occurs. The following plot shows the point of intersection more clearly. Reading the coordinates of point P below, we conclude that the critical Mach number of this airfoil is about 0.743.



★ The correct answer is **C**.

P.11 ■ Solution

For a freestream Mach number M_∞ = 0.5, we have, from Table 2, p_0/p_∞ = 1.186. For a Mach number of 0.86, we read p_0/p = 1.621. In view of these results, we write

$$p/p_\infty = \frac{p_o/p_\infty}{p_o/p} = \frac{1.186}{1.621} = 0.732$$

The pressure coefficient follows as

$$C_p = \frac{p-p_\infty}{q_\infty} = \frac{p-p_\infty}{\frac{1}{2}\gamma p_\infty M_\infty^2} = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty}-1\right)$$

$$\therefore C_p = \frac{2}{1.4\times 0.5^2}\times(0.732-1) = \boxed{-1.53}$$

A second, more direct way to obtain this result is to use the formula

$$C_p = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma}{\gamma-1}}-1\right]$$

$$\therefore C_p = \frac{2}{1.4\times 0.5^2}\left[\left(\frac{1+\frac{1.4-1}{2}\times 0.5^2}{1+\frac{1.4-1}{2}\times 0.86^2}\right)^{\frac{1.4}{1.4-1}}-1\right] = -1.53$$

As expected, the result is the same.

★ The correct answer is **B**.

P.12 ■ Solution

Applying the Prantl-Glauert rule yields

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}} = \frac{-0.54}{\sqrt{1-0.6^2}} = -0.675$$

With recourse to the Karman-Tsien rule, we obtain

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}+\frac{M_\infty^2}{1+\sqrt{1-M_\infty^2}}\frac{C_{p,0}}{2}} = \frac{-0.54}{\sqrt{1-0.6^2}+\frac{0.6^2}{1+\sqrt{1-0.6^2}}\times\frac{(-0.54)}{2}} = -0.724$$

Applying the Laitone rule, we have

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}+\frac{M_\infty^2\left[1+\frac{(\gamma-1)}{2}M_\infty^2\right]}{2\sqrt{1-M_\infty^2}}C_{p,0}}$$

$$\therefore C_p = \frac{-0.54}{\sqrt{1-0.6^2}+\frac{0.6^2\times\left[1+\frac{(1.4-1)}{2}\times 0.6^2\right]}{2\sqrt{1-0.6^2}}\times(-0.54)} = -0.806$$

Observe that there is substantial disagreement between the rules, as the lowest result in terms of absolute value, obtained from the Prandtl-Glauert rule, differs from the highest result, obtained from the Laitone rule, by nearly 20%. Of the three results, experience has shown the Karman-Tsien rule to be the most accurate.

★ The correct answer is **A**.

P.13 ■ Solution

Part 1: The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\times 0.00238\times 150^2 = 26.8 \text{ lb/ft}^2$$

The atmospheric pressure at sea level is p_∞ = 2116 lb/ft^2. The pressure coefficient is then

$$C_p = \frac{p-p_\infty}{q_\infty} = \frac{2080-2116}{26.8} = \boxed{-1.34}$$

★ The correct answer is **A**.

Part 2: First, we note that, at standard sea level, T_s = 518.7°R and the speed of sound is

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{1.4\times 1716\times 518.7} = 1120 \text{ ft/s}$$

It follows that, in the previous situation, the Mach number was M_∞ = 150/1120 = 0.134, a very low value. Hence the flow in the previous problem is essentially incompressible, and the pressure coefficient is a low speed value; that is, $C_{p,0} = -1.34$. If the flow Mach number is increased to 0.6, the pressure coefficient can be established with the Prandtl-Glauert rule,

$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}} = \frac{-1.34}{\sqrt{1-0.6^2}} = \boxed{-1.68}$$

★ The correct answer is **A.**.

P.14 ■ Solution

At a standard altitude of 3 km, the atmospheric pressure p_∞ = 7.01×10^4 N/m^2 and the air density ρ_∞ = 0.909 kg/m^3 (Table 1). The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\times 0.909\times 160^2 = 11,600 \text{ N/m}^2$$

Appealing to the definition of pressure coefficient, we can establish the pressure at the point in question,

$$C_p = \frac{p-p_\infty}{q_\infty} \rightarrow p = C_p q_\infty + p_\infty$$

$$\therefore p = -2.5\times 11,600 + 70,100 = \boxed{41,100 \text{ N/m}^2}$$

★ The correct answer is **D.**.

P.15 ■ Solution

To begin, we manipulate the equation for pressure coefficient at point 1,

$$C_{p,1} = \frac{p_1-p_\infty}{q_\infty} \rightarrow p_1 - p_\infty = C_{p,1}q_\infty$$

Likewise for point 2,

$$C_{p,2} = \frac{p_2-p_\infty}{q_\infty} \rightarrow p_2 - p_\infty = C_{p,2}q_\infty$$

Subtracting one equation from the other yields

$$p_1 - p_\infty - (p_2 - p_\infty) = q_\infty C_{p,1} - q_\infty C_{p,2}$$

$$\therefore p_1 - p_2 = q_\infty (C_{p,1} - C_{p,2})$$

However, from Bernoulli's equation, we have

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$\therefore p_1 - p_2 = \frac{1}{2}\rho (V_2^2 - V_1^2)$$

Equating the two previous results, we find that

$$q_\infty\left(C_{p,1}-C_{p,2}\right)=\frac{1}{2}\rho_\infty\left(V_2^2-V_1^2\right)$$

$$\therefore \frac{1}{2}\rho_\infty V_\infty^2\left(C_{p,1}-C_{p,2}\right)=\frac{1}{2}\rho_\infty\left(V_2^2-V_1^2\right)$$

$$\therefore V_\infty^2\left(\frac{p_1}{q_\infty}-\frac{p_2}{q_\infty}\right)=V_2^2-V_1^2$$

$$\therefore \frac{p_1-p_2}{q_\infty}=\left(\frac{V_2}{V_\infty}\right)^2-\left(\frac{V_1}{V_\infty}\right)^2$$

which can be adjusted a bit further to yield

$$\frac{q_\infty\left(C_{p,1}-C_{p,2}\right)}{q_\infty}=\left(\frac{V_2}{V_\infty}\right)^2-\left(\frac{V_1}{V_\infty}\right)^2$$

$$\therefore C_{p,1}-C_{p,2}=\left(\frac{V_2}{V_\infty}\right)^2-\left(\frac{V_1}{V_\infty}\right)^2$$

Lastly, we substitute our data and solve for V_2,

$$C_{p,1}-C_{p,2}=\left(\frac{V_2}{V_\infty}\right)^2-\left(\frac{V_1}{V_\infty}\right)^2\rightarrow -1.4-(-0.7)=\left(\frac{V_2}{75}\right)^2-\left(\frac{120}{75}\right)^2$$

$$\therefore -0.7=\left(\frac{V_2}{75}\right)^2-2.56$$

$$\therefore V_2=\sqrt{1.86}\times 75=\boxed{102\text{ m/s}}$$

As noted by Anderson, the solution did not require explicit knowledge of density. This is because we dealt with pressure difference in terms of the difference in pressure *coefficient*, which, in turn, is related to the difference of the squares of the nondimensional velocity through Bernoulli's equation.

★ The correct answer is **D.**.

Answer Summary

Problem 1		D
Problem 2	2.1	C
	2.2	A
Problem 3	3.1	B
	3.2	A
	3.3	B
Problem 4		T/F
Problem 5		T/F
Problem 6		T/F
Problem 7		B
Problem 8		D
Problem 9		B
Problem 10		C
Problem 11		B
Problem 12		A
Problem 13	13.1	A
	13.2	A
Problem 14		D
Problem 15		D

References

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