Quiz EL401
Amplitude and Phase Modulation

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PROBLEMS

Problem 1

A carrier, amplitude-modulated to a depth of 50% by a sinusoid, produces side frequencies of 4.004 MHz and 3.996 MHz. The amplitude of each side frequency is 36 V. Find the frequency and amplitude of the carrier signal.

Problem 2

A sinusoidal carrier signal of 5 V peak amplitude and 100 kHz frequency is amplitude-modulated by a 5-kHz signal of peak amplitude equal to 3 V. What is the modulation index?

A) 0.5
B) 0.6
C) 0.7
D) 0.8

Problem 3

A message signal given by

$$m(t)=\frac{1}{3}\cos(\omega_1 t)-\frac{1}{2}\cos(\omega_2 t)$$

is amplitude-modulated with a carrier of frequency ω_c to generate

$$s(t)=\left[1+m(t)\right]\cos(\omega_c t)$$

The power efficiency achieved by this AM scheme is, most nearly:

A) 8%
B) 12%
C) 16%
D) 20%

Problem 4

A 1-MHz sinusoidal carrier is amplitude-modulated by a symmetric square wave of period equal to 40 μs. Which of the following frequencies will be present in the modulated signal?

A) 1010 kHz
B) 1020 kHz
C) 1030 kHz
D) 1040 kHz

Problem 5 (Lathi, 1998)

You are given the baseband signals **(i)** $m(t) = \cos(1000\pi t)$, **(ii)** $m(t) = 2\cos(1000\pi t) + \sin(2000\pi t)$, **(iii)** $m(t) = \cos(1000\pi t)\cos(3000\pi t)$, and **(iv)** $m(t) = \exp(-10|t|)$. For each one, do the following:

1. Sketch the spectrum of $m(t)$.
2. Sketch the spectrum of the DSB-SC signal $m(t)\cos(10{,}000\pi t)$.
3. Identify the upper sideband (USB) and the lower sideband (LSB) spectra.

Problem 6 (Haykin, 2001)

Using the message signal

$$m(t)=\frac{1}{1+t^2}$$

determine and sketch the modulated waves for the following methods of modulation:

Problem 6.1: Amplitude modulation with 50 percent modulation.
Problem 6.2: Double sideband-suppressed carrier modulation.
Problem 6.3: Single sideband modulation with only the upper sideband transmitted.
Problem 6.4: Single sideband modulation with only the lower sideband transmitted.

Problem 7 (Haykin, 2001)

Problem 7.1: Show that any scheme that can be used to generate double-sideband suppressed carrier transmission (DSB-SC) can also generate AM. Is the converse true? Explain.
Problem 7.2: Show that any scheme that can be used to demodulate DSB-SC can also demodulate AM. Is the converse true? Explain.

Problem 8

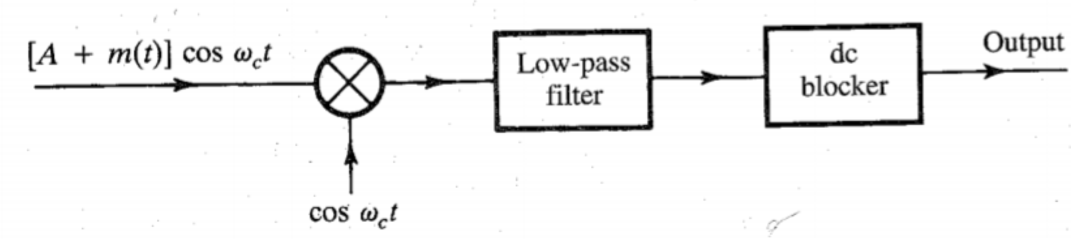
An amplitude modulator has output

$$s(t)=A\cos(\pi 400t)+B\cos(\pi 360t)+B\cos(\pi 440t)$$

The carrier power is 100 W and the power efficiency (ratio of sideband power to total power) is 40%. Compute A, B, and the modulation factor μ.

Problem 9 (Haykin, 2001)

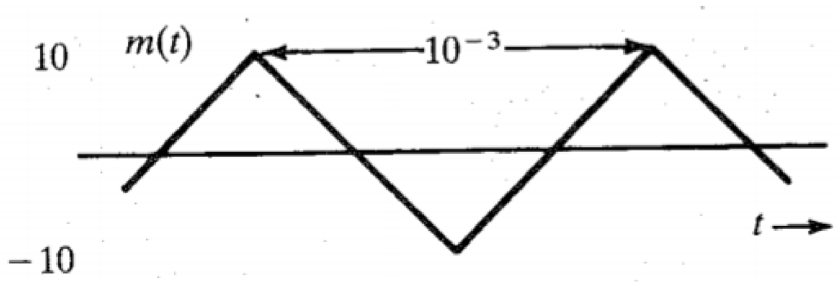
The following figure shows a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the AM signal $[A + m(t)]\cos(\omega_c t)$ regardless of the value of A.



Problem 10 (Lathi, 1998)

Sketch the AM signal $[A + m(t)]\cos(\omega_c t)$ for the periodic triangle signal $m(t)$ shown below corresponding to modulation index

1. $\mu = 0.5$
2. $\mu = 1$
3. $\mu = 2$
4. $\mu = \infty$



Problem 11 (Lathi, 1998)

For the AM signal of the previous problem with a modulation index μ = 0.8:

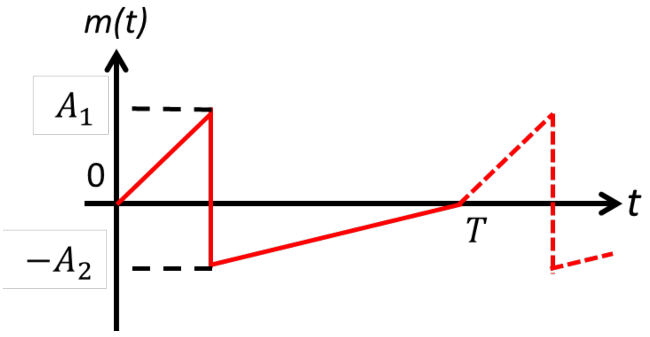
Problem 11.1: Find the amplitude and power of the carrier.
Problem 11.2: Find the sideband power and the power efficiency η.

Problem 12 (Kasturi, 2021)

For the message signal illustrated below, compute the power efficiency (the ratio of sideband power to total power) in terms of the amplitude sensitivity k_a, A_1, A_2, and T. The message is periodic with period T, has zero mean, and is AM modulated according to

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

Assume that $T \gg 1/f_c$.



Problem 13 (Kasturi, 2021)

Consider a message signal $m(t)$ whose spectrum extends from 300 Hz to 3.4 kHz, as illustrated below. This message is SSB-modulated to give

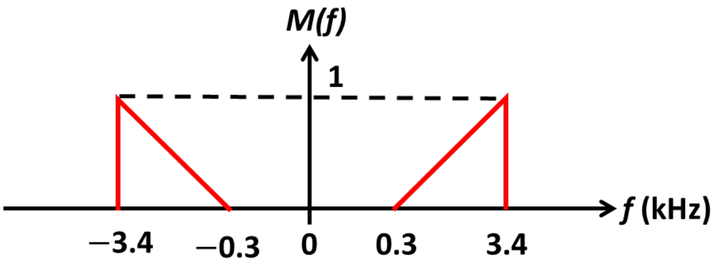
$$s(t) = A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t)$$

At the receiver, $s(t)$ is demodulated using a carrier of the form $\cos[2\pi(f_c + \Delta f)\,t]$. Plot the spectrum of the demodulated signal at the output of the lowpass filter when

1. $\Delta f = 10$ Hz

2. $\Delta f = -10$ Hz

Assume that the lowpass filter is ideal with unity gain and extends over [−4, 4] kHz, and $f_c = 20$ kHz. Indicate all the important points on the XY-axes.



Problem 14

A frequency-modulated signal is given by

$$x_c(t) = 8\cos\left[2\pi \times 10^8 t + 6\sin(600\pi t)\right]$$

Determine:

1. The carrier frequency.

2. The modulating signal frequency.

3. The modulation index.

4. The peak frequency deviation.

Problem 15

An FM signal with single tone modulation has a frequency deviation of 12 kHz and a bandwidth of 35 kHz. Find the frequency of the modulating signal.

Problem 16 (Kasturi, 2021)

Consider a wideband PM signal produced by a sinusoidal modulating wave $m(t) = A_m\cos(2\pi f_m t)$, using a modulator with phase sensitivity k_p.

Problem 16.1: Show that if the phase deviation of the PM signal is large compared to one radian, the bandwidth of the PM signal varies linearly with the modulation frequency f_m.

Problem 16.2: Compare this bandwidth of the wideband PM signal with that of a wideband FM signal produced by $m(t)$ and frequency sensitivity k_f.

Problem 17 (Lathi, 1998)

A signal $x(t) = 7\cos(24\pi \times 10^3 t)$ angle modulates a carrier signal $A_c \cos(\omega_c t)$. Determine the modulation index and the bandwidth of the modulated signal for:

Problem 17.1: an FM system with frequency deviation constant k_f = 14 kHz/V.

Problem 17.2: a PM system with phase deviation constant k_p = 1.2 rad/V.

Problem 18

An FM radio link has a frequency deviation of 36 kHz. The modulating frequency is 3.2 kHz. Calculate the bandwidth needed for the link. What will be the bandwidth if the deviation is reduced to 20 kHz?

Problem 19

Problem 19.1: An angle-modulated signal has the form

$$v(t) = 120\cos\left[2\pi f_c t + 3.6\sin(2100\pi t)\right]$$

where f_c = 8 MHz. Determine:

1. The average transmitted power.

2. The peak phase deviation.

3. The peak frequency deviation.

Problem 19.2: Is this a FM or a PM signal? Explain.

Problem 20 (Haykin, 2001)

Given the message signal $m(t) = \sin(2000\pi t)$, the modulation sensitivity $k_f = 200{,}000\pi$, and the phase sensitivity $k_p = 10$,

Problem 20.1: Estimate the bandwidths of the FM signal $\varphi_{FM}(t)$ and the PM signal $\varphi_{PM}(t)$.

Problem 20.2: Repeat part 1 if the message signal amplitude is doubled.

Problem 20.3: Repeat part 1 if the message signal frequency is doubled.

Problem 20.4: Comment on the sensitivity of FM and PM bandwidths to the spectrum of $m(t)$.

Problem 21 (Haykin, 2001)

A carrier wave is frequency modulated using a sinusoidal signal of frequency f_m and amplitude A_m.

Problem 21.1: Determine the values of the modulation index β for which the carrier component of the FM signal is reduced to zero.

Problem 21.2: In a certain experiment conducted with f_m = 1 kHz and increasing A_m starting from zero volts, it is found that the FM signal is reduced to zero for the first time when A_m = 2 V. What is the frequency sensitivity of the modulator? What is the value of A_m for which the carrier component is reduced to zero for the second time?

Problem 22

Given $m(t) = e^{-t^2}$, carrier frequency $f_c = 10^4$ Hz, and modulation sensitivities $k_f = 6000\pi$ and $k_p = 8000\pi$:

Problem 22.1: Find Δf, the frequency deviation for FM and PM.

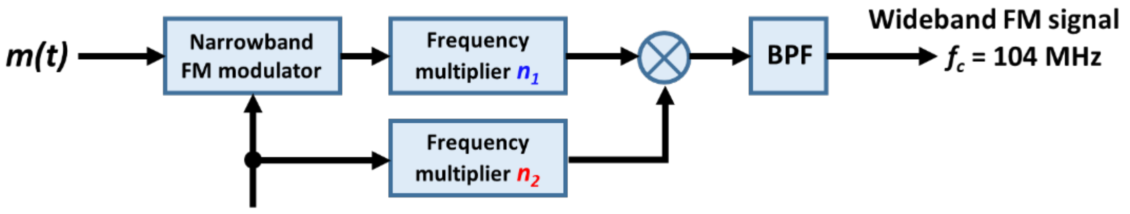
Problem 22.2: Estimate the bandwidths of the FM and PM waves. *Hint:* Find $M(\omega)$ and observe the rapid decay of this spectrum. Its 3-dB bandwidth is even smaller than 1 Hz ($B \ll \Delta f$).

Problem 23 (Haykin, 2001)

To generate wideband FM, we can first generate a narrowband FM signal, and then use frequency multiplication to spread the signal bandwidth. This is illustrated below, which is called the Armstrong-type FM modulator. The narrowband FM has a frequency deviation of 1 kHz.

Problem 23.1: If the frequency of the first oscillator is 1 MHz, determine n_1 and n_2 that are necessary to generate an FM signal at a carrier frequency of 104 MHz and a maximum frequency deviation of 75 kHz. The bandpass filter allows only the difference frequency component.

Problem 23.2: If the error in the carrier frequency f_c for the wideband FM signal is to be within $\pm$200 Hz, determine the maximum allowable error in the 1-MHz oscillator.



Problem 24

Consider the frequency-modulated signal

$$s(t) = 10\cos\left[2\pi \times 2 \times 10^5 t + +10\cos\left(2\pi \times 1,500t\right) + 15\sin\left(2\pi \times 2500\pi t\right)\right]$$

The modulation index of *s(t)* is

A) 10 rad
B) 15 rad
C) 25 rad
D) 30 rad

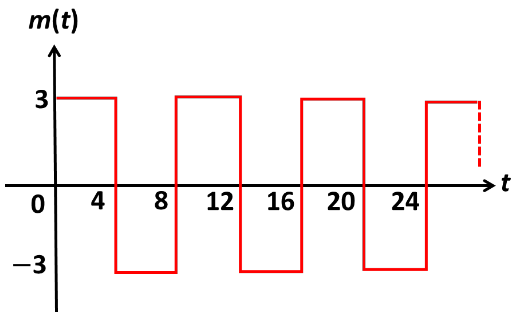
Problem 25

Compute the bandwidth of the following FM signal.

$$\varphi_{FM}(t) = 5\cos\left[\omega_c t + 20\sin\left(1000\pi t\right) + 10\sin\left(2000\pi t\right)\right]$$

Problem 26

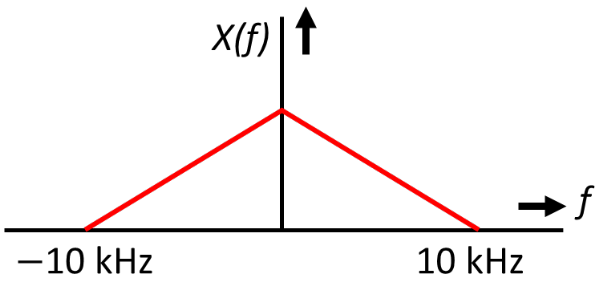
The signal *m(t)* with the waveform shown is applied to a phase modulator with k_p as phase constant and to a frequency modulator with k_f as frequency constant. Both modulators have the same carrier frequency. The ratio k_p/k_f for the same maximum phase deviation in both modulation schemes is:



A) 2π
B) 4π
C) 8π
D) 16π

Problem 27

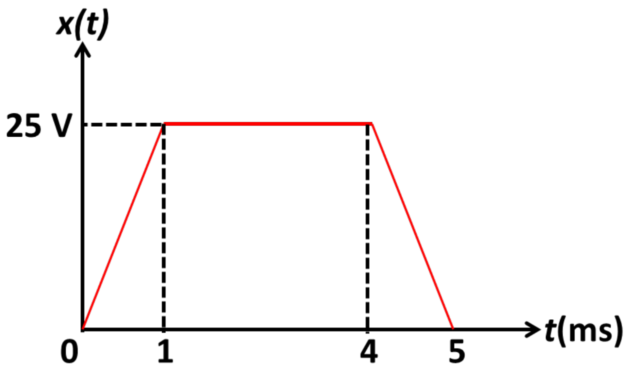
A signal *x(t)*, whose Fourier transform *X(f)* is shown below, is normalized so that $|x(t)| \leq 1$. This signal is to be transmitted using FM with a frequency deviation constant k_f = 75 kHz per volt. What will be the bandwidth required for transmission?



Problem 28

A modulating signal *x(t)* with a trapezoidal form as shown is used for
Problem 28.1: frequency modulating a carrier signal of 5 MHz frequency with a frequency deviation constant, k_f, equal to 6 kHz/V.
Problem 28.2: phase modulating a carrier with a phase deviation constant, k_p, of 6 rad/V.

In each of these cases, find the maximum instantaneous frequency of the modulated signal.



Problem 29

In a FM system, a carrier of 10 MHz is modulated by a sinusoidal signal of 2 kHz. Per Carson's rule, the bandwidth required is estimated to be 0.1 MHz. If signal *y(t)* = (*Modulated waveform*)^3, then the bandwidth of *y(t)* around 30 MHz is:

A) 0.1 MHz
B) 0.2 MHz
C) 0.3 MHz
D) 0.5 MHz

Problem 30 (Kasturi, 2021)

Consider the message signal illustrated below, which is used to frequency modulate a carrier. Assume a frequency sensitivity of k_f Hz/V and that the FM signal is given by

$$s(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$$

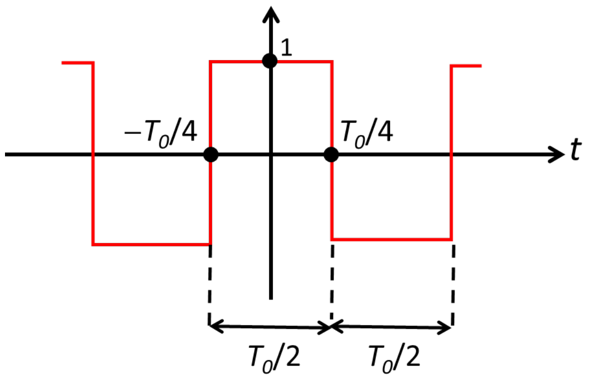
Problem 30.1: Sketch the waveform corresponding to the total instantaneous frequency *f(t)* of the FM signal for $-T_0/4 \le t \le 3T_0/4$. Label all the important points on the axes.
Problem 30.2: Sketch $\phi(t)$ for $-T_0/4 \le t \le 3T_0/4$. Label all the important points on the axes. Assume that $\phi(t)$ has zero mean.
Problem 30.3: Write down an expression for the complex envelope of *s(t)* in terms of $\phi(t)$.
Problem 30.4: The FM signal *s(t)* can be written as

$$s(t) = \sum_{n=-\infty}^{\infty} c_n \cos\left(2\pi f_c t + 2n\pi t/T_0\right)$$

where

$$c_n = \left[a_n \operatorname{sinc}\left(\frac{\beta - n}{2}\right) + b_n \operatorname{sinc}\left(\frac{\beta + n}{2}\right)\right]$$

in which $\beta = k_f T_0$. Compute a_n and b_n. Assume the limits of integration (for computing c_n) to be from $-T_0/4$ to $3T_0/4$.



Problem 31 (Kasturi, 2021)

A tone-modulated FM signal with the form

$$s(t) = A_c \sin\left[2\pi f_c t + \beta_f \cos\left(2\pi f_m t\right)\right]$$

is passed through an ideal unity-gain bandpass filter with center frequency equal to the carrier frequency and bandwidth equal to $3f_m$ ($\pm 1.5 f_m$ on either side of the carrier), yielding the signal *z(t)*.
Problem 31.1: Derive an expression for *z(t)*.
Problem 31.2: Assuming that *z(t)* has the form

$$z(t) = a(t)\cos\left[2\pi f_c t + \theta(t)\right]$$

compute *a(t)* and $\theta(t)$.

Hint: *Use the relations*

$$J_n(\beta) = \frac{1}{2\pi}\int_{x=-\pi}^{x=\pi} \exp\left[j\left[\beta \sin(x) - nx\right]\right] dx$$

and

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

Problem 32

A carrier *c(t)* and a message *m(t)* are used to generate an FM signal. If the peak frequency deviation of the generated FM signal is five times the maximum frequency f_M = 2 kHz, then the coefficient of the term $\cos[2\pi(1006\times10^3)t]$ in the FM signal equals which of the following? *J* denotes a Bessel function; take f_c = 1 MHz as the carrier frequency.

A) $J_5(3)$
B) $J_2(5)$
C) $J_3(5)$
D) $J_4(5)$

SOLUTIONS

P.1 ➡ Solution

The upper side-frequency is f_c+f_m = 4.004 MHz, while the lower side-frequency is f_c-f_m = 3.996 MHz. Adding the two parameters and solving for carrier frequency, we get

$$(f_c+f_m)+(f_c-f_m)=4.004+3.996=8\,\text{MHz}$$
$$\therefore 2f_c=8.0\,\text{MHz}$$
$$\therefore \boxed{f_c=4.0\,\text{MHz}}$$

If the carrier is $A_c\cos(\omega_c t)$, μ is the modulation index, and f_m is the modulating signal frequency, we may write

$$s(t)=A_c\left[1+\mu\cos(\omega_m t)\right]\cos(\omega_c t)=A_c\cos(\omega_c t)+\frac{\mu A_c}{2}\cos\left[(\omega_c+\omega_m)t\right]+\frac{\mu A_c}{2}\cos\left[(\omega_c-\omega_m)t\right]$$

Since the amplitude of each side frequency is 36 V and the modulation index equals 0.5, we can solve for the carrier amplitude A_c,

$$\frac{\mu A_c}{2}=36\rightarrow A_c=\frac{72}{\mu}$$
$$\therefore A_c=\frac{72}{0.5}=\boxed{144\,\text{V}}$$

The carrier has a frequency of 4 MHz and an amplitude of 144 V.

P.2 ➡ Solution

The carrier signal is given by

$$s(t)=A_c\left[1+\mu\cos(\omega_m t)\right]\cos(\omega_c t)=5\left[1+\mu\cos(\omega_m t)\right]\cos(\omega_c t)$$
$$\therefore s(t)=5\cos(\omega_c t)+5\mu x(t)\cos(\omega_c t)$$

Since $|x(t)|\le 1$, the factor shown in red must equal 3 V. The modulation index is then

$$5\mu=3\rightarrow\boxed{\mu=0.6}$$

▶ The correct answer is **B**.

P.3 ➡ Solution

Substituting *m(t)*, expanding, and using the appropriate trigonometric identities, we obtain

$$s(t)=\left[1+m(t)\right]\cos(\omega_c t)=\left[1+\frac{1}{3}\cos(\omega_1 t)-\frac{1}{2}\cos(\omega_2 t)\right]\cos(\omega_c t)$$
$$\therefore s(t)=\cos(\omega_c t)+\frac{1}{3}\cos(\omega_1 t)\cos(\omega_c t)-\frac{1}{2}\cos(\omega_2 t)\cos(\omega_c t)$$
$$\therefore s(t)=\cos(w_c t)+\frac{1}{6}\cos\left[(\omega_c+w_1)t\right]+\frac{1}{6}\cos\left[(\omega_c-\omega_1)t\right]-\frac{1}{4}\cos\left[(\omega_c+\omega_2)t\right]-\frac{1}{4}\cos\left[(\omega_c-\omega_2)t\right]$$

Of the five terms above, the first one is the carrier component and the remaining four are side band components. Noting that power is proportional to squared amplitude, the power efficiency in the AM scheme at hand is given by

$$\text{Power efficiency of AM} = \frac{\text{Side band power}}{\text{Total power}} = \frac{\frac{1}{6^2}+\frac{1}{4^2}}{1+\frac{1}{6^2}+\frac{1}{4^2}} = 0.0828$$

$$\therefore \text{Power efficiency of AM} \approx \boxed{8.3\%}$$

▶ The correct answer is **A**.

P.4 ➡ *Solution*

If the symmetric square wave has a period of 20 μs, its fundamental frequency is $f_0 = 1/(20\times10^{-6}) = 50$ kHz. Now, the square wave can be represented by the Fourier series

$$m(t) = \frac{4A}{\pi}\left[\cos(2\pi f_0 t) + \frac{\cos(6\pi f_0 t)}{3} + \frac{\cos(10\pi f_0 t)}{5} + \ldots\right]$$

which means that $m(t)$ has frequencies f_0, $3f_0$, $5f_0$, and so forth. It follows that if we amplitude-modulate a 1-MHz carrier using this signal, the modulated signal will have frequencies

$$10^6 \pm 20\times10^3,\ 10^6 \pm 60\times10^3,\ 10^6 \pm 100\times10^3 \text{ Hz}, \ldots$$

$$\therefore 1020\,\text{kHz},\ 1060\,\text{kHz},\ 1100\,\text{kHz}, \ldots$$

▶ The correct answer is **B**.

P.5 ➡ *Solution*

The spectra can be easily obtained if we consider the following Fourier transform pairs,

$$\cos(2\pi f_0 t) \Leftrightarrow \frac{1}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right] \quad (1)$$

$$\sin(2\pi f_0 t) \Leftrightarrow \frac{1}{2j}\left[\delta(f-f_0)-\delta(f+f_0)\right] \quad (2)$$

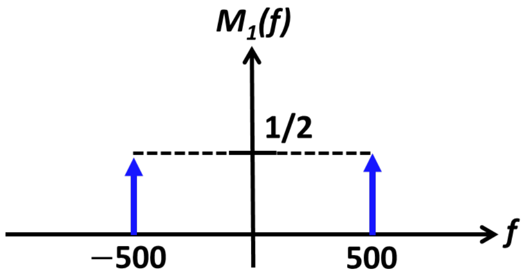
$$e^{-a|t|} \Leftrightarrow \frac{2a}{a^2+(2\pi f)^2} \quad (3)$$

$$x(t)y(t) \Leftrightarrow X(f)*Y(f) \quad (4)$$

Baseband signal (i): In this case, $m_1(t) = \cos(1000\pi t) = \cos(2\pi \times 500t)$. With recourse to identity (1) above, we have

$$M_1(f) = \frac{1}{2}\left[\delta(f-500)+\delta(f+500)\right]$$

The corresponding magnitude spectrum $|M_1(f)|$ is shown below.



The DSB-SC signal is

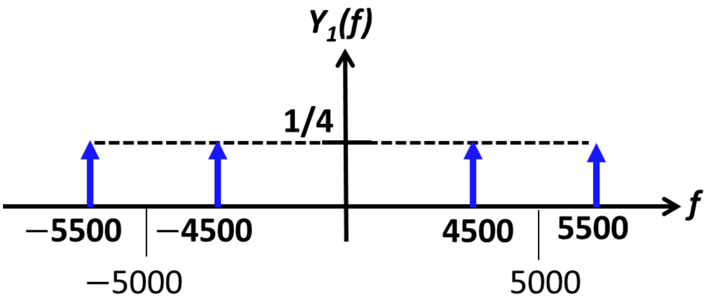
$$y_1(t) = m_1(t)\cos(10{,}000\pi t)$$

Applying Fourier transforms,

$$Y_1(t) = M_1(f) * \frac{1}{2}\left[\delta(f-5000)+\delta(f+5000)\right]$$

$$\therefore Y_1(t) = \frac{1}{2}\left[M_1(f-5000)+M_1(f+5000)\right]$$

The spectrum $Y_1(f)$ should have two copies of the spectrum $M_1(f)$ centered at ±5000 Hz; also, the amplitudes should be multiplied by 1/2.



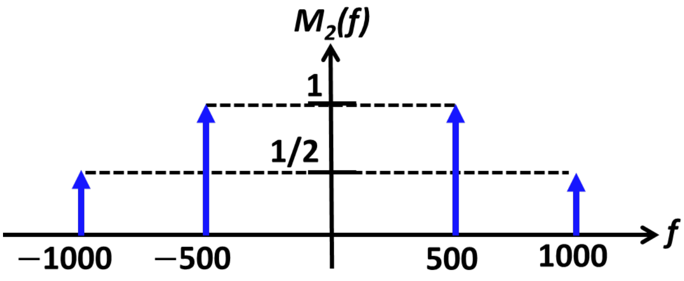
Finally, we surmise that the USB spectrum is made up of Dirac deltas at frequencies 5500 and −5500 Hz, while the DSB spectrum is made up of unit impulses at 4500 and −4500 Hz.

Baseband signal (ii): In this case, $m_2(t) = 2cos(1000\pi t) + sin(2000\pi t)$. Transforming to the frequency domain brings to

$$M_2(f) = 2\left(\frac{1}{2}\right)\left[\delta(f-500)+\delta(f+500)\right]+\frac{1}{2j}\left[\delta(f-1000)-\delta(f+1000)\right]$$

$$\therefore M_2(f) = \left[\delta(f-500)+\delta(f+500)\right]+\frac{1}{2j}\left[\delta(f-1000)-\delta(f+1000)\right]$$

The corresponding magnitude spectrum $|M_2(f)|$ is shown below.



The DSB-SC signal is

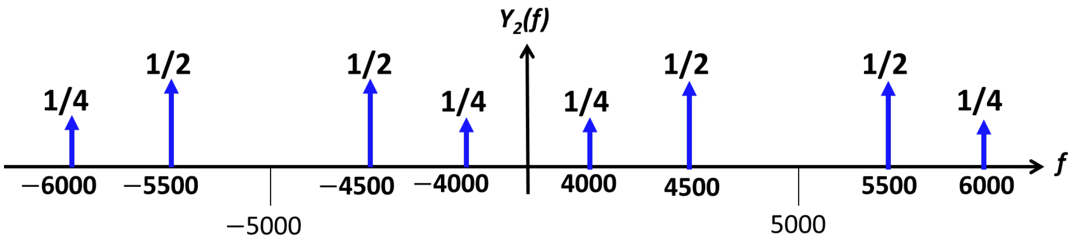
$$y_2(t) = m_2(t)\cos(10{,}000\pi t)$$

Applying Fourier transforms,

$$Y_2(t) = M_2(f) * \frac{1}{2}\left[\delta(f-5000)+\delta(f+5000)\right]$$

$$\therefore Y_2(t) = \frac{1}{2}\left[M_2(f-5000)+M_2(f+5000)\right]$$

The spectrum $Y_2(f)$ should have two copies of the spectrum $M_2(f)$ centered at ±5000 Hz; also, the amplitudes should be multiplied by 1/2.



The USB spectrum is made up of Dirac deltas at 5500, 6000, −5500, and −6000 Hz. The DSB spectrum is made up of unit impulses at 4000, 4500, −4000, and −4500 Hz.

Baseband signal (iii): In this case, $m_3(t) = cos(1000\pi t)cos(3000\pi t)$. Using trigonometry,

$$m_3(t) = \cos(1000\pi t)\cos(3000\pi t) = \frac{1}{2}\left[2\cos(3000\pi t)\cos(1000\pi t)\right]$$

$$\therefore m_3(t) = \frac{1}{2}\left[\cos(3000\pi t+1000\pi t)+\cos(3000\pi t-1000\pi t)\right]$$

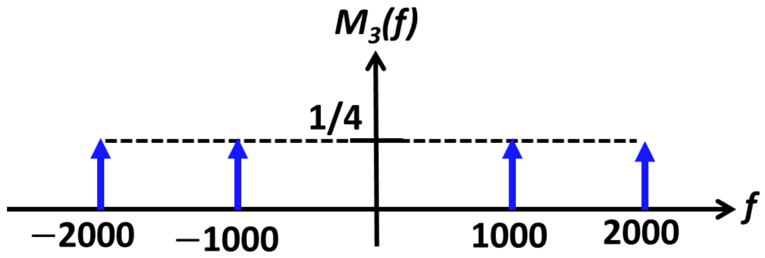
$$\therefore m_3(t) = \frac{1}{2}\left[\cos(4000\pi t)+\cos(2000\pi t)\right]$$

$$\therefore m_3(t) = \frac{1}{2}\left[\cos(2\pi 2000t)+\cos(2\pi 1000t)\right]$$

Transforming to the frequency domain yields

$$M_3(f)=\frac{1}{4}\left[\delta(f+1000)+\delta(f-1000)+\delta(f+2000)+\delta(f-2000)\right]$$

The magnitude spectrum $|M_3(f)|$ is shown below.



The DSB-SC signal is

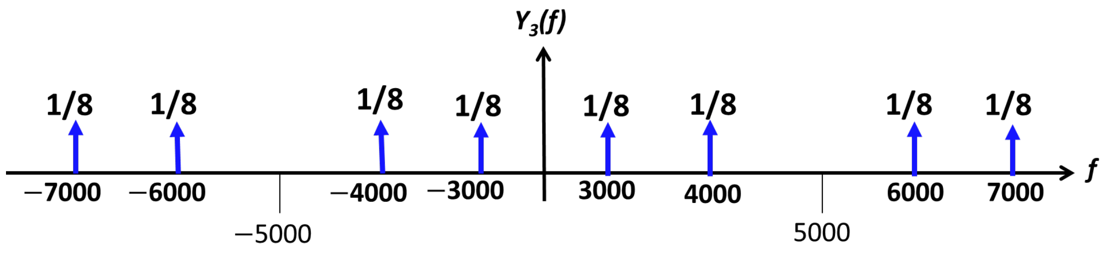
$$y_3(t)=m_3(t)\cos(10{,}000\pi t)$$

Applying Fourier transforms,

$$Y_3(t)=M_3(f)*\frac{1}{2}\left[\delta(f-5000)+\delta(f+5000)\right]$$

$$\therefore Y_3(t)=\frac{1}{2}\left[M_3(f-5000)+M_3(f+5000)\right]$$

The spectrum $Y_3(f)$ should have two copies of the spectrum $M_3(f)$ centered at ±5000 Hz; also, the amplitudes should be multiplied by 1/2.



The USB spectrum is made up of Dirac deltas at 6000, 7000, −6000, and −7000 Hz. The DSB spectrum is made up of unit impulses at 3000, 4000, −3000, and −4000 Hz.

Baseband signal (iv): In this case, $m_4(t) = -exp(-10|t|)$. With recourse to identity (3) above,

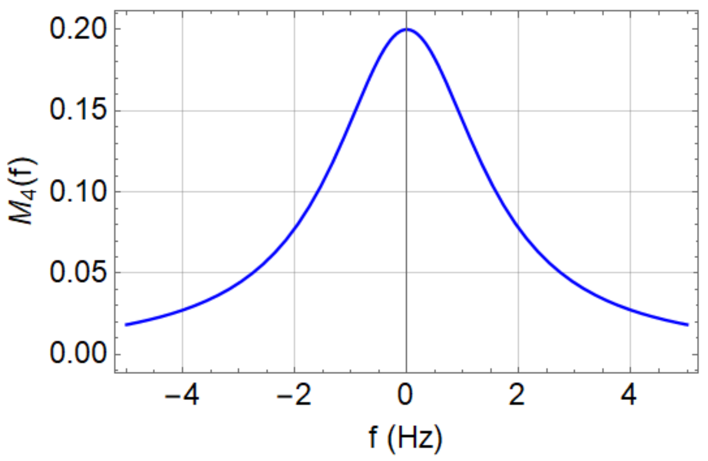
$$M_4(f)=\frac{2a}{a^2+(2\pi f)^2}=\frac{2\times 10}{10^2+(2\pi f)^2}$$

$$\therefore M_4(f)=\frac{0.2}{1+\left(\frac{\pi f}{5}\right)^2}$$

This is a continuous spectrum and can be plotted with the Mathematica code

```
In[94]:= Plot[0.2/(1 + (Pi*f/5)^2), {f, -5, 5}, Frame → True, GridLines → Automatic,
  PlotStyle → Blue]
```

The plot is shown below.



The DSB-SC signal is

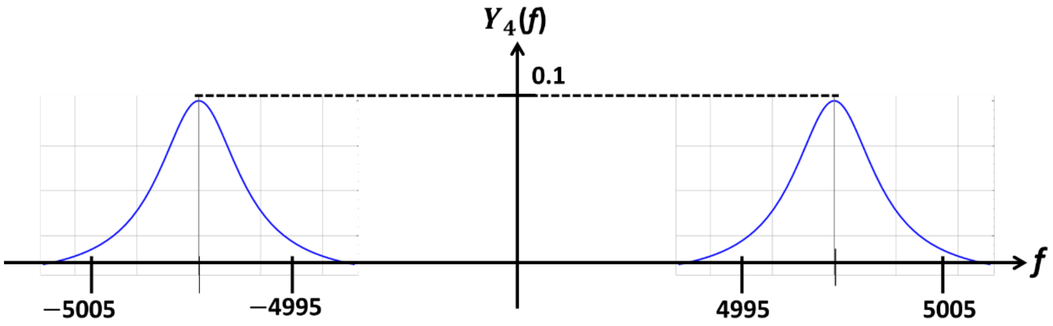
$$y_4(t) = m_4(t)\cos(10{,}000\pi t)$$

Applying Fourier transforms,

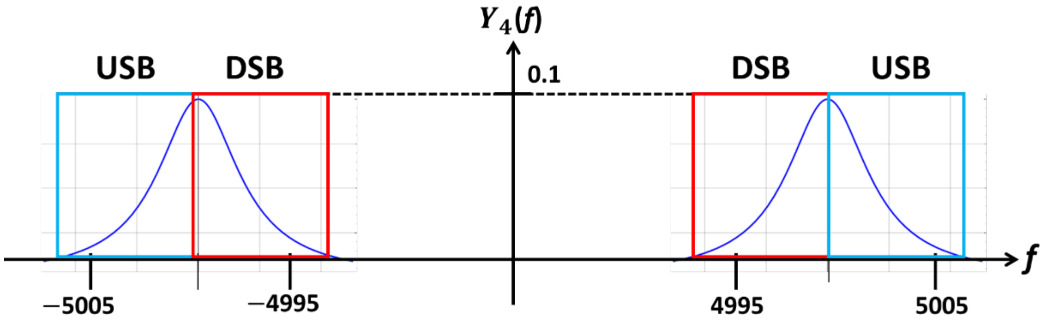
$$Y_4(t) = M_4(f) * \frac{1}{2}\left[\delta(f-5000)+\delta(f+5000)\right]$$

$$\therefore Y_4(t) = \frac{1}{2}\left[M_4(f-5000)+M_4(f+5000)\right]$$

The spectrum $Y_4(f)$ should have two copies of the spectrum $M_4(f)$ centered at ±5000 Hz; also, the amplitudes should be multiplied by 1/2.



The DSB and USB spectra are highlighted below.



P.6 ➡ *Solution*

Problem 6.1: The modulated wave can be expressed as

$$s(t) = A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)$$

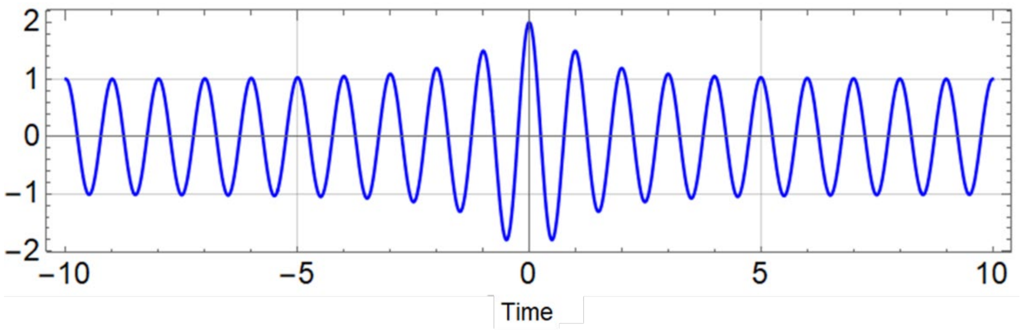
which, substituting *m(t)*, becomes

$$s(t) = A_c\left(1+\frac{k_a}{1+t^2}\right)\cos(2\pi f_c t)$$

For 50 percent modulation to be achieved, the amplitude sensitivity k_a must equal 1, in which case we obtain

$$s(t) = \boxed{A_c\left(1+\frac{1}{1+t^2}\right)\cos(2\pi f_c t)} \quad \text{(I)}$$

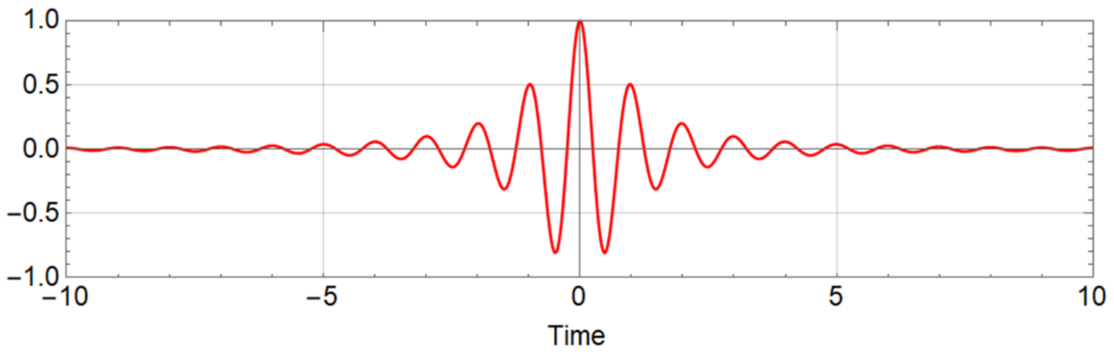
Equation (I) is plotted below.



Problem 6.2: For a double sideband-suppressed carrier modulation, the modulated wave is given by

$$s(t) = A_c m(t)\cos(2\pi f_c t) = \boxed{\frac{A_c}{1+t^2}\cos(2\pi f_c t)} \quad \text{(II)}$$

Equation (II) is plotted below.

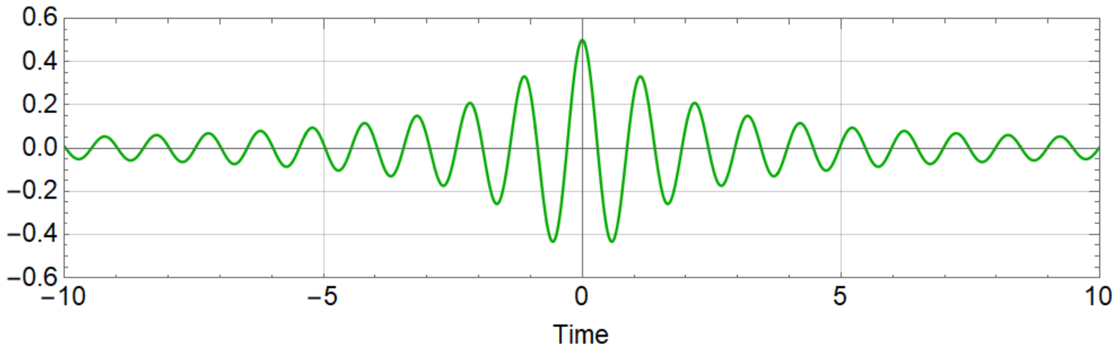


Problem 6.3: In this case, *s(t)* is given by

$$s(t)=\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t)-\hat{m}(t)\sin(2\pi f_c t)\right]$$

$$\therefore s(t)=\boxed{\frac{A_c}{2}\left[\frac{1}{1+t^2}\cos(2\pi f_c t)-\frac{t}{1+t^2}\sin(2\pi f_c t)\right]}\quad \text{(III)}$$

Equation (III) is plotted below.

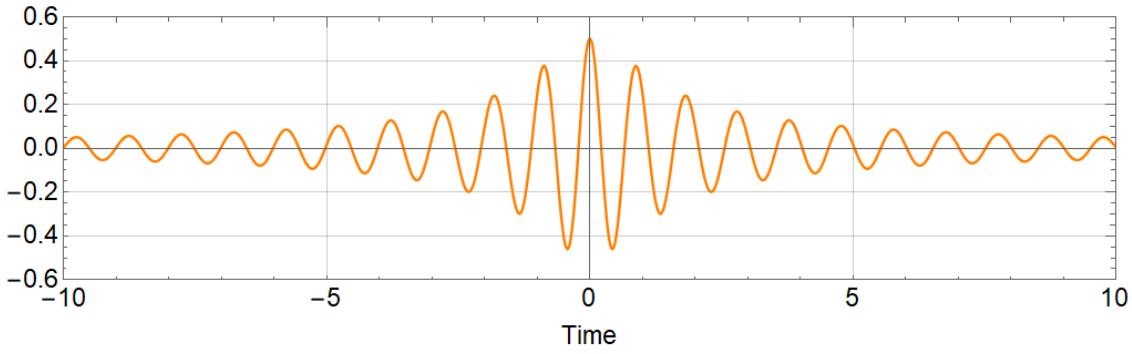


Problem 6.4: In this case, *s(t)* is given by

$$s(t)=\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t)+\hat{m}(t)\sin(2\pi f_c t)\right]$$

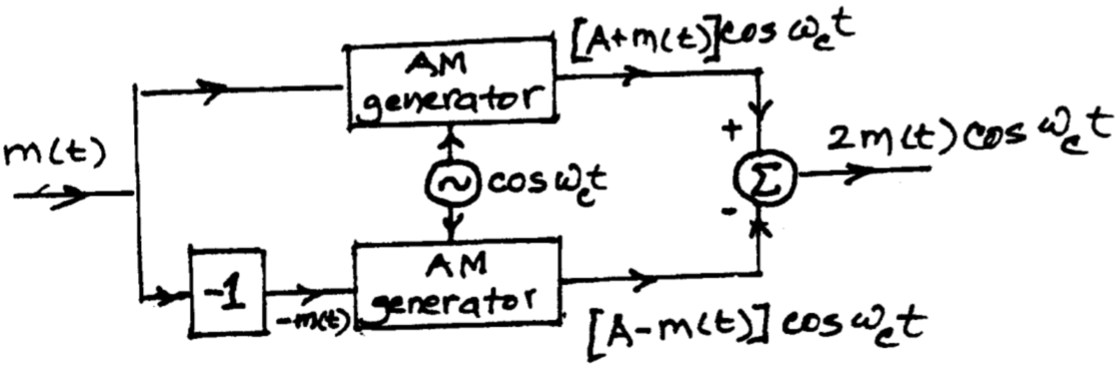
$$\therefore s(t)=\boxed{\frac{A_c}{2}\left[\frac{1}{1+t^2}\cos(2\pi f_c t)+\frac{t}{1+t^2}\sin(2\pi f_c t)\right]}\quad \text{(IV)}$$

Equation (IV) is plotted below.



P.7 ➡ *Solution*

Problem 7.1: When an input to a DSB-SC generator is *m(t)*, the corresponding output is $m(t)\cos(\omega_c t)$. Clearly, if the input is $A + m(t)$, the corresponding output will be $[A + m(t)]\cos(\omega_c t)$. This is precisely the AM signal. Thus, by adding a dc of value *A* to the baseband signal *m(t)*, we can generate an AM signal using a DSB-SC generator. However, we can generate DSB-SC using AM generators if we use two identical AM generators in the balanced scheme illustrated below to cancel out the carrier component.



Problem 7.2: When an input to a DSB-SC demodulator is $m(t)\cos(\omega_c t)$, the corresponding output is $m(t)$. It follows that if the input is $[A + m(t)]\cos(\omega_c t)$, the corresponding output will be $A + m(t)$. By blocking the dc component A from this output, we can demodulate the AM signal using a DSB-SC demodulator.

The converse, unfortunately, is not true. This is because when an input to an AM demodulator is $m(t)\cos(\omega_c t)$, the corresponding output is $|m(t)|$ (the envelope of $m(t)$). Hence, unless $m(t) \geq 0$ for all t, it is not possible to demodulate a DSB-SC signal using an AM demodulator.

P.8 ➡ Solution

As the reader surely knows by now, the general expression for a tone-modulated AM signal is

$$s(t) = A_c\left[1+\mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

and can be expanded to yield

$$s(t) = A_c\cos(2\pi f_c t) + \frac{\mu A_c}{2}\cos\left[2\pi(f_c - f_m)t\right] + \frac{\mu A_c}{2}\cos\left[2\pi(f_c + f_m)t\right]$$

Comparing this with the amplitude modulator at hand, it is apparent that

$$A = A_c$$

$$B = \frac{\mu A}{2}$$

$$2f_c = 400 \rightarrow f_c = 200\,\text{Hz}$$

$$2(f_c - f_m) = 360 \rightarrow f_c - f_m = 180\,\text{Hz}$$

$$2(f_c + f_m) = 440 \rightarrow f_c + f_m = 220\,\text{Hz}$$

Since $A_c = A$ and the carrier power is 100 W, we may write

$$P_c = \frac{A_c^2}{2} = \frac{A^2}{2} = 100 \rightarrow A = \sqrt{200}$$

$$\therefore \boxed{A = 14.1}$$

Since the given AM wave is tone-modulated, the modulation factor is given by

$$\frac{\mu^2}{2+\mu^2} = 0.4 \rightarrow \mu^2 = 0.4\times(2+\mu^2)$$

$$\therefore \mu^2 = 0.8 + 0.4\mu^2$$

$$\therefore 0.6\mu^2 = 0.8$$

$$\therefore \mu = \sqrt{\frac{0.8}{0.6}} = \boxed{1.15}$$

Finally, B is calculated to be

$$B = \frac{\mu A}{2} = \frac{1.15\times 14.1}{2} = \boxed{8.11}$$

P.9 ➡ Solution

Initially, $g_a(t) = [A + m(t)]\cos(\omega_c t)$. The signal that enters the low-pass filter is

$$g_b(t) = \left[A + m(t)\right]\cos^2(\omega_c t) = \frac{1}{2}\left[A + m(t)\right] + \frac{1}{2}\left[A + m(t)\right]\cos(2\omega_c t)$$

The first term is a lowpass signal because its spectrum is centered at $\omega = 0$. The lowpass filter allows this term to pass, but suppresses the second term, whose spectrum is centered at $\pm 2\omega_c$. Hence the output of the lowpass filter is

$$y(t) = A + m(t)$$

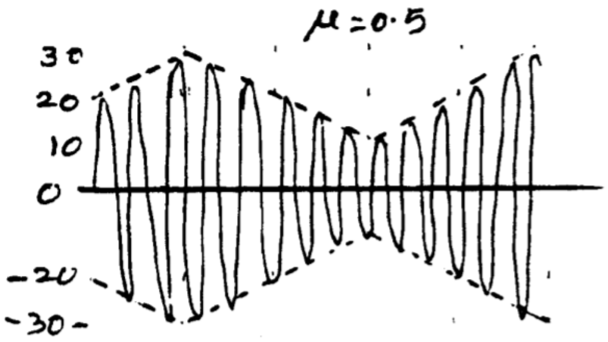
When this signal is passed through a dc block, the dc term A is suppressed, yielding the output $m(t)$. This shows that the system can demodulate the AM signal regardless of the value of A. This is a synchronous or coherent demodulation.

P.10 ➡ *Solution*

With $\mu = 0.5$, amplitude A is found as

$$\mu = 0.5 = \frac{m_p}{A} = \frac{10}{A}$$
$$\therefore A = 20$$

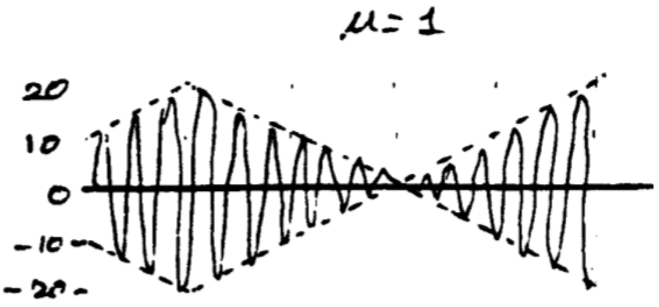
The AM signal is sketched below.



With $\mu = 1.0$, we have

$$\mu = 1.0 = \frac{m_p}{A} = \frac{10}{A}$$
$$\therefore A = 10$$

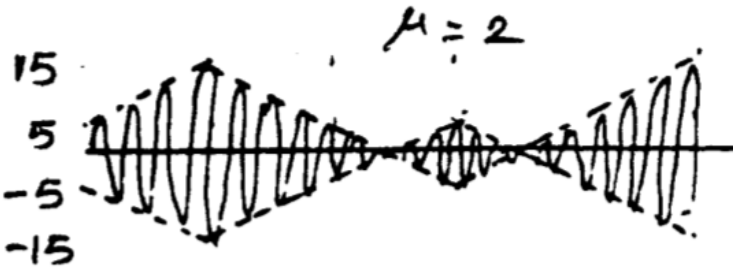
The AM signal is sketched below.



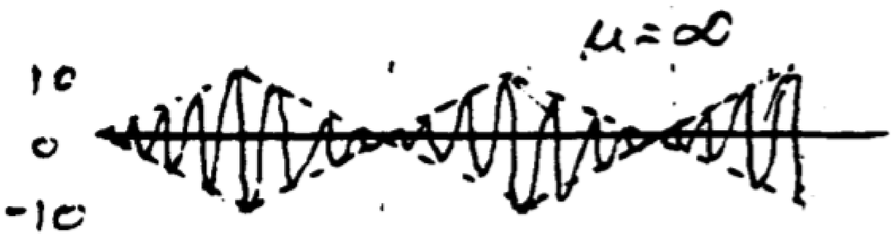
With $\mu = 2$, we have

$$\mu = 2.0 = \frac{m_p}{A} = \frac{10}{A}$$
$$\therefore A = 5.0$$

The AM signal is sketched below.



When $\mu \to \infty, A = 0$ and the modulation corresponds to the DSB-SC case. The modulated waveform is shown below.



P.11 ➡ *Solution*

Problem 11.1: The carrier amplitude is $A = 10/0.8 = 12.5$. The carrier power is $P_c = A^2/2 = 12.5^2/2 = 78.1$.

Problem 11.2: The sideband power is $m^2(t)/2$. Because of symmetry of amplitude values every quarter cycle, the power of $m(t)$ may be computed by averaging the signal energy over a quarter cycle only. Over a quarter cycle *m(t)* can be represented by the straight line equation *m(t)* = 40*t*/*T*$_0$, as illustrated below. Mathematically,

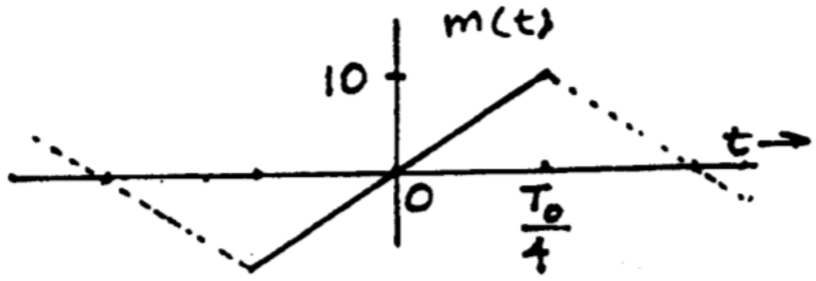
$$\overline{m^2(t)} = \frac{1}{T_0/4}\int_0^{T_0/4}\left(\frac{40t}{T_0}\right)^2 dt = \frac{1}{T_0/4}\times\frac{25T_0}{3} = 33.3$$

The sideband power is

$$P_s = \frac{\overline{m^2(t)}}{2} = \boxed{16.7}$$

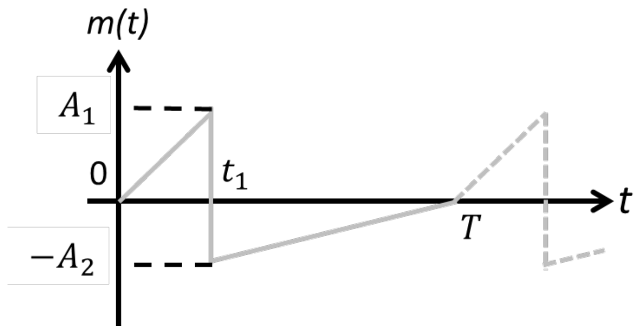
The efficiency is

$$\eta = \frac{P_s}{P_c+P_s}\times 100\% = \frac{16.6}{78.1+16.6}\times 100\% = \boxed{17.5\%}$$



P.12 ➡ *Solution*

Refer to the figure below.



Since the message has zero mean, the upper triangle that constitutes the initial portion of the wave must have the same area as the inverted triangle that constitutes the later portion of the wave; in mathematical terms,

$$\frac{1}{2}t_1A_1 = \frac{1}{2}(T-t_1)A_2 \rightarrow \frac{1}{2}t_1A_1 = \frac{1}{2}A_2T - \frac{1}{2}t_1A_2$$

$$\therefore \frac{1}{2}t_1(A_1+A_2) = \frac{1}{2}A_2T$$

$$\therefore t_1 = \frac{A_2T}{A_1+A_2} \quad \text{(I)}$$

where t_1 is indicated in the foregoing sketch of the signal. Now, for a general AM signal given by

$$s(t) = A_c\left[1+k_am(t)\right]\cos(2\pi f_ct)$$

the power efficiency is

$$\eta = \frac{A_c^2k_a^2P_m/2}{A_c^2/2 + A_c^2k_a^2P_m/2} = \frac{k_a^2P_m}{1+k_a^2P_m} \quad \text{(II)}$$

where P_m denotes the message power, which can be found from the integral

$$P_m = \frac{1}{T}\int_{t=0}^{T} m^2(t)\,dt \quad \text{(III)}$$

In the present case, the message signal can be represented by a piecewise function consisting of two straight lines,

$$m(t) = \begin{cases} A_1t/t_1 \ , \ 0<t<t_1 \\ A_2(t-T)/(T-t_1) \ , \ t_1<t<T \end{cases}$$

Substituting in (III) brings to

$$P_m = \frac{1}{T}\int_{t=0}^{T} m^2(t)\,dt = \frac{1}{T}\int_0^{t_1}\left(\frac{A_1 t}{t_1}\right)^2 dt + \frac{1}{T}\int_{t_1}^{T}\frac{A_2^2(t-T)^2}{(T-t_1)^2}dt$$

$$\therefore P_m = \frac{A_1^2 t_1}{3T} + \frac{A_2^2(T-t_1)}{3T}$$

Using (I),

$$\therefore P_m = \frac{A_1^2}{3T}\times\frac{A_2 T}{A_1+A_2} + \frac{A_2^2\left(T - \frac{A_2 T}{A_1+A_2}\right)}{3T}$$

$$\therefore P_m = \frac{A_1 A_2}{3}$$

Lastly, we substitute into (II) to obtain the power efficiency

$$\eta = \frac{k_a^2 P_m}{1+k_a^2 P_m} = \frac{k_a^2\frac{A_1A_2}{3}}{1+k_a^2\frac{A_1A_2}{3}} = \boxed{\frac{k_a^2 A_1 A_2}{3+k_a^2 A_1 A_2}}$$

P.13 ➡ Solution

The multiplier output can be written as

$$s_1(t) = s(t)\cos\left[2\pi(f_c+\Delta f)t\right] = A_c\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]\cos\left[2\pi(f_c+\Delta f)t\right]$$

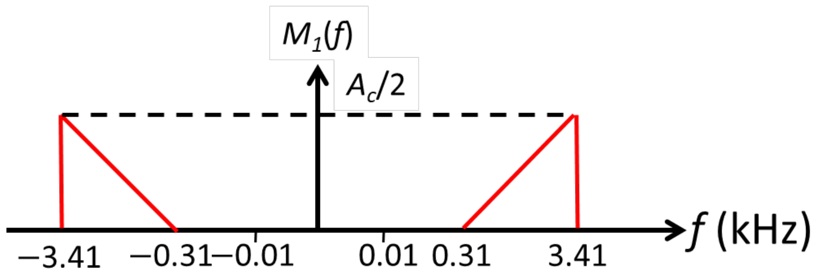
$$\therefore s_1(t) = \frac{A_c}{2}m(t)\left\{\cos(2\pi\Delta f\,t) + \cos\left[2\pi(2f_c+\Delta f)t\right]\right\}$$

$$+\frac{A_c}{2}\hat{m}(t)\left\{\sin\left[2\pi(2f_c+\Delta f)t\right] - \sin(2\pi\Delta f t)\right\}$$

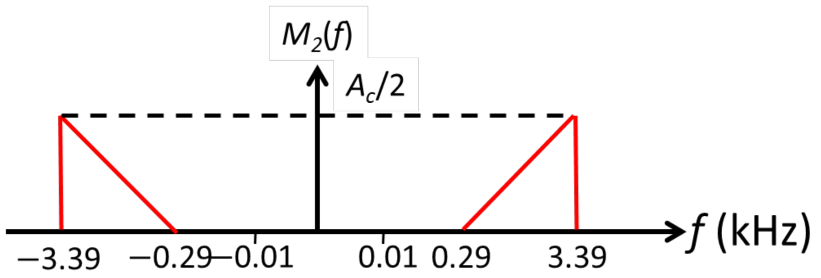
The output of the lowpass filter is

$$s_2(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi\Delta f t) - \hat{m}(t)\sin(2\pi\Delta f t)\right]$$

When Δf is positive, $s_2(t)$ is an SSB signal with carrier frequency Δf and upper sideband transmitted. This is illustrated below for Δf = 10 Hz.



When Δf is negative, $s_2(t)$ is an SSB signal with carrier frequency Δf and lower sideband transmitted. This is illustrated below for Δf = −10 Hz.



P.14 ➡ Solution

The FM-modulated signal in question has the general form

$$x_c(t) = A_c\cos\left[2\pi f_c t + \beta_f\sin(2\pi f_m t)\right]$$

Comparing this expression with the FM signal we were given, it is easy to see that the carrier frequency is

$$f_c = 10^8 = \boxed{100\,\text{MHz}}$$

Comparing the general $x_c(t)$ to the FM signal at hand, the modulating signal frequency is found to be

$$2\pi f_m = 600\pi \rightarrow f_m = \frac{600\pi}{2\pi}$$

$$\therefore \boxed{f_m = 300\,\text{Hz}}$$

Equating $x_c(t)$ to the FM signal in question a third time, it is apparent that the modulation index equals 6.

$$\boxed{\beta_f = 6}$$

Noting that modulation index equals the ratio of peak frequency deviation to modulating signal frequency, we may write

$$\beta_f = \frac{\text{Peak freq. deviation}}{\text{Modulating signal frequency}} = 6 \rightarrow \text{Peak freq. deviation} = f_m \times 6$$

$$\therefore \text{Peak freq. deviation} = 300 \times 6 = \boxed{1.8\,\text{kHz}}$$

P.15 ➡ *Solution*

Per Carson's rule, the bandwidth can be estimated as

$$\text{BW} = 2\left(\beta_f + 1\right) f_m = 35 \times 10^3 \rightarrow \beta_f + 1 = \frac{35 \times 10^3}{2 f_m} \quad \text{(I)}$$

From the definition of modulation index,

$$\beta_f = \frac{\Delta f}{f_m} = \frac{12 \times 10^3}{f_m}$$

Substituting in (I) and solving for modulating signal frequency,

$$\beta_f + 1 = \frac{35 \times 10^3}{2 f_m} \rightarrow \frac{12 \times 10^3}{f_m} + 1 = \frac{35 \times 10^3}{2 f_m}$$

$$\therefore 1 = \frac{35 \times 10^3}{2 f_m} - \frac{12 \times 10^3}{f_m}$$

$$\therefore 1 = \frac{5.5 \times 10^3}{f_m}$$

$$\therefore \boxed{f_m = 5.5\,\text{kHz}}$$

P.16 ➡ *Solution*

Problems 16.1 and 16.2: The PM signal can be stated as

$$m(t) = A_c \cos\left[2\pi f_c t + k_p A_m \cos\left(2\pi f_m t\right)\right]$$

The phase deviation is $k_p A_m$. The instantaneous frequency due to the message is

$$\frac{1}{2\pi}\frac{d}{dt}\left[k_p A_m \cos\left(2\pi f_m t\right)\right] = -k_p A_m f_m \sin\left(2\pi f_m t\right)$$

so the frequency deviation is $\Delta f_1 = k_p A_m f_m$, which indicates that it is directly proportional to f_m. Now, the PM signal described by the first equation can be considered to be an FM signal with message given by

$$m_1(t) = -\left(k_p / k_f\right) A_m f_m \sin\left(2\pi f_m t\right)$$

The bandwidth of the PM signal we began with is then

$$B_{T,PM} = 2\left(\Delta f_1 + f_m\right) = 2 f_m \left(k_p A_m + 1\right)$$

$$\therefore B_{T,PM} \approx 2 f_m k_p A_m$$

which shows that $B_{T,PM}$ is linearly proportional to modulation frequency. In contrast, in FM the frequency deviation is $\Delta f_2 = k_f A_m$, which is independent of f_m. The bandwidth of the FM signal is

$$B_{T,FM} = 2\left(\Delta f_2 + f_m\right) = 2\left(k_f A_m + f_m\right)$$

P.17 ➡ Solution

Problem 17.1: The modulation index for a FM modulation scheme is given by

$$\beta_f = \frac{k_f A_m}{f_m}$$

Here, $k_f = 14\times10^3$ Hz/V as stated, $A_m = 7$ is the amplitude of the modulating signal, and $f_m = 12$ kHz is the linear frequency of the modulating signal; accordingly,

$$\beta_f = \frac{\left(14\times10^3\right)\times7}{12\times10^3} = \boxed{8.17}$$

Appealing to Carson's rule gives the bandwidth *BW*,

$$BW = 2\left(\beta_f + 1\right)f_m = 2\times\left(8.17+1\right)\times12 = \boxed{220\,\text{kHz}}$$

Problem 17.2: The modulation index for a PM modulating scheme is given by

$$\beta_p = k_p A_m = 1.2\times7 = \boxed{8.4}$$

The corresponding bandwidth is

$$BW = 2\left(\beta_p + 1\right)f_m = 2\times\left(8.4+1\right)\times12 = \boxed{226\,\text{kHz}}$$

P.18 ➡ Solution

At first, $\Delta f_1 = 36$ kHz and $f_m = 3.2$ kHz, giving a modulation index $\beta_{f,1}$ such that

$$\beta_{f,1} = \frac{\Delta f_1}{f_m} = \frac{36}{3.2} = 11.3$$

By Carson's rule, the required bandwidth is

$$BW_1 = 2\left(\beta_{f,1} + 1\right)f_m = 2\times\left(11.3+1\right)\times3.2 = \boxed{78.7\,\text{kHz}}$$

Now, if the deviation is reduced to 20 kHz, the modulation index becomes $\beta_{f,2} = 20/3.2 = 6.25$ and the bandwidth required is calculated to be

$$BW_2 = 2\left(\beta_{f,2} + 1\right)f_m = 2\times\left(6.25+1\right)\times3.2 = \boxed{46.4\,\text{kHz}}$$

P.19 ➡ Solution

Problem 19.1: The average transmitted power can be obtained by squaring the amplitude and dividing the result by 2,

$$\text{Average transmitted power} = \frac{120^2}{2} = \boxed{7200\text{ W}}$$

Since $3.6\sin(2100\pi t)$ represents the phase deviation at any instant t and since $\sin(2100\pi t)$ has a peak value of 1, the peak phase deviation equals 3.6 radians.

To find the peak frequency deviation, first note that the instantaneous frequency is given by

$$f_i(t) = \frac{1}{2\pi}\times\left\{\frac{d}{dt}\left[2\pi f_c t + 3.6\sin\left(2100\pi t\right)\right]\right\} = \frac{1}{\cancel{2\pi}}\times\cancel{2\pi} f_c + \frac{1}{2\pi}\times3.6\times2100\pi\cos\left(2100\pi t\right)$$

$$\therefore f_i(t) = f_c + 3780\cos\left(2100\pi t\right)$$

The frequency deviation at instant t is $3780\cos(2100\pi t)$, and the peak frequency deviation is 3780 Hz.

Problem 19.2: The signal can be considered a PM signal with $\beta_p = 3.6$ and a modulating signal of $\sin(2100\pi t)$, or a FM signal with $\beta_f = 3.6$ and a modulating signal of $\cos(2100\pi t)$.

P.20 ➡ Solution

Problem 20.1: For FM, the frequency deviation is $\Delta f = k_f m_p/2\pi$ = 200,000π × 1/2π = 100 kHz and the baseband signal bandwidth is B = 2000π/2π = 1 kHz. The corresponding bandwidth is

$$BW_{FM} = 2\left(\Delta f + B\right) = 2\times\left(100+1\right) = \boxed{202\,\mathrm{kHz}}$$

Similarly, for FM we have $\Delta f = k_p m'_p/2\pi$ = 10 × 2000π/2π = 10 kHz and

$$BW_{PM} = 2\left(\Delta f + B\right) = 2\times\left(10+1\right) = \boxed{22\,\mathrm{kHz}}$$

Problem 20.2: The message signal is restated as $m(t) = 2\sin(2000\pi t)$. The baseband signal bandwidth is unchanged at 1 kHz. Also, m_p = 2 and, deriving the new message signal, m'_p = 4000π. It follows that, for FM, Δf = $k_f m_p/2\pi$ = 200,000π × 2/2π = 200 kHz, giving a bandwidth such that

$$BW'_{FM} = 2\left(\Delta f + B\right) = 2\times\left(200+1\right) = \boxed{402\,\mathrm{kHz}}$$

Likewise, for PM, $\Delta f = k_p m'_p/2\pi$ = 10 × 4000π/2π = 20 kHz, leading to a bandwidth such that

$$BW'_{PM} = 2\left(\Delta f + B\right) = 2\times\left(20+1\right) = \boxed{42\,\mathrm{kHz}}$$

Problem 20.3: The message signal is restated as $m(t) = \sin(4000\pi t)$. The baseband signal bandwidth is now B = 4000π/2π = 2 kHz. Also, m_p = 1 and m'_p = 4000π. Accordingly, for FM, $\Delta f = k_f m_p/2\pi$ = 200,000π × 1/2π = 100 kHz, so that

$$BW''_{FM} = 2\left(\Delta f + B\right) = 2\times\left(100+2\right) = \boxed{204\,\mathrm{kHz}}$$

Similarly, for PM, $\Delta f = k_p m'_p/2\pi$ = 10 × 4000π/2π = 20 kHz and

$$BW''_{PM} = 2\left(\Delta f + B\right) = 2\times\left(20+2\right) = \boxed{44\,\mathrm{kHz}}$$

Problem 20.4: Doubling the amplitude of $m(t)$ roughly doubles the bandwidth of both FM and PM. Doubling the frequency of $m(t)$ (i.e., expanding the spectrum of $M(\omega)$ by a factor of 2) has hardly any effect on the FM bandwidth but roughly doubles the bandwidth of PM, indicating that the PM spectrum is sensitive to the shape of the baseband spectrum. The FM spectrum is relatively insensitive to the nature of the spectrum $M(\omega)$.

P.21 ➡ Solution

Problem 21.1: The carrier component of the FM signal is reduced to zero at values of β such that the Bessel function $J_0(\beta)$ = 0. These are β = 2.44, 5.52, 8.65, 11.8, and so forth; the first β is 2.44.

Problem 21.2: For tone modulation, $\beta = k_f A_m/f_m$. Solving for k_f and substituting brings to

$$\beta = \frac{k_f A_m}{f_m} \rightarrow k_f = \frac{f_m \beta}{A_m}$$

$$\therefore k_f = \frac{1.0\times 2.44}{2} = \boxed{1.22\,\mathrm{kHz/V}}$$

The value of A_m for which the carrier component goes to zero for the second time is

$$A_m = \frac{\beta f_m}{k_f} = \frac{5.52\times 1.0}{1.22} = \boxed{4.52\,\mathrm{V}}$$

P.22 ➡ Solution

Problems 22.1 and 22.2: Switching from time domain to frequency domain, the message signal becomes $M(\omega) = \sqrt{\pi}e^{-\omega^2/4}$. The spectrum $M(\omega)$ = $\sqrt{\pi}e^{-\omega^2/4}$ is a Gaussian pulse that decays rapidly. Its 3-dB bandwidth is 1.178 rad/s or 0.187 Hz, which is a very small bandwidth compared to Δf. Deriving with respect to time yields $\dot{m}(t) = -2te^{-t^2/2}$. The spectrum of $\dot{m}(t)$ is $M'(\omega)$ = $j\omega M(\omega) = j\sqrt{\pi}\omega e^{-\omega^2/4}$. This spectrum also decays rapidly away from the origin, and its bandwidth can also be assumed to be negligible compared to Δf. For FM, the frequency deviation is

$$\Delta f = \frac{k_f m_p}{2\pi} = \frac{6000\pi \times 1}{2\pi} = \boxed{3\,\text{kHz}}$$

and the bandwidth can be approximated as

$$\text{BW} \approx 2\Delta f = 2 \times 3.0 = \boxed{6.0\,\text{kHz}}$$

To find m'_p, we first set the derivative of $\dot{m}(t)$ equal to zero and solve for time; doing so yields

$$\ddot{m}(t) = -2e^{-t^2/2} + 4t^2 e^{-t^2/2} = 0 \to t = \frac{1}{\sqrt{2}} = 0.707$$

(watch out for the two tiny dots above $\ddot{m}(t)$), so that

$$m'_p = \dot{m}(0.707) = 0.858$$

Finally,

$$\Delta f = \frac{k_p m'_p}{2\pi} = \frac{8000\pi \times 0.858}{2\pi} = 3430\,\text{Hz}$$

and

$$\text{BW} \approx 2\Delta f = 2 \times 3.43 = \boxed{6.86\text{ kHz}}$$

P.23 ➡ *Solution*

Problems 23.1: The maximum frequency of the output wideband FM signal is 75 kHz. The maximum frequency deviation of the narrowband FM signal is 1 kHz. It follows that

$$n_1 = \frac{75}{1.0} = \boxed{75}$$

Thus, the carrier frequency at the output of the first frequency multiplier is 75 kHz. However, the required carrier frequency is 104 MHz, which means that a frequency translation in order. The value of n_2 is then

$$1 \times n_2 - 75 = 104 \to \boxed{n_2 = 179}$$

Problems 23.2: Let us assume that the error in f_0 is α Hz. The error α in the final output carrier frequency is

$$n_2\alpha - n_1\alpha = \pm 200 \to (n_2 - n_1)\alpha = \pm 200$$

$$\alpha = \frac{\pm 200}{n_2 - n_1} = \frac{\pm 200}{179 - 75} = \boxed{\pm 1.92\,\text{Hz}}$$

P.24 ➡ *Solution*

The modulation index can be interpreted as the maximum phase change of $s(t)$. In the case at hand,

$$\text{Modulation index} = \max\left[10\cos(2\pi \times 1500t) + 15\sin(2\pi \times 2500t)\right] = 10 + 15 = \boxed{25\,\text{rad}}$$

▶ The correct answer is **C**.

P.25 ➡ *Solution*

The baseband signal bandwidth is $B = 2000\pi/2\pi = 1000$ Hz. Further, the instantaneous frequency $\omega_i(t)$ is

$$\omega_i(t) = \omega_c + 20{,}000\pi\cos(1000\pi t) + 20{,}000\pi\cos(2000\pi t)$$

The maximum value of this expression is $\Delta\omega = 20{,}000\pi + 20{,}000\pi = 40{,}000\pi$, which corresponds to a linear frequency deviation $\Delta f = 20{,}000$ Hz. It remains to determine the bandwidth BW,

$$BW = 2(\Delta f + B) = 2 \times (20{,}000 + 1000) = \boxed{42\,\text{kHz}}$$

P.26 ➡ *Solution*

The maximum phase deviation in PM is

$$\text{Phase deviation in PM} = k_p \times \max\left[m(t)\right] = 3k_p]$$

where we have used the fact that the maximum amplitude of the *m(t)* waveform is 3. Similarly, the maximum phase deviation in FM is

$$\text{Phase deviation in FM} = 2\pi k_f \times \max\left[\int m(t)\,dt\right] = 2\pi k_f \times (4\times 3) = 24\pi k_f$$

where we have used the fact that the maximum area enclosed by the *m(t)* waveform is 4 × 3 = 12. If the maximum phase deviation is the same in both schemes, ratio k_p/k_f is calculated to be

$$3k_p = 24\pi k_f \rightarrow \frac{k_p}{k_f} = \frac{24\pi}{3} = \boxed{8\pi}$$

▶ The correct answer is **C**.

P.27 ➡ *Solution*

Inspecting the Fourier transform spectrum, the bandwidth of the modulating signal is clearly W = 10 kHz. Also, k_f is given as 75 kHz/V and amplitude A_m is taken as 1 V because $|x(t)| \le 1$. The modulation index is then

$$\beta_f = \frac{k_f A_m}{W} = \frac{75\times 1.0}{10} = 7.5$$

The required bandwidth is computed as

$$BW = 2(\beta_f + 1)W = 2\times(7.5+1)\times 10 = \boxed{170\,\text{kHz}}$$

P.28 ➡ *Solution*

Problem 28.1: The instantaneous frequency for FM modulation can be written as

$$f_i = f_c + k_f x(t)$$

Referring to the waveform we were given, the maximum value of *x(t)* is 25 V, so that

$$f_{i,\max} = f_c + k_f\left[x(t)\right]_{\max} = 5\times 10^6 + \left(6\times 10^3\right)\times 25 = \boxed{5.15\,\text{MHz}}$$

Problem 28.2: For a phase modulation scheme, the instantaneous frequency is described by

$$f_i = f_c + \frac{1}{2\pi}\frac{d}{dt}\left[\phi(t)\right]$$

where phase deviation $\phi(t) = k_p x(t)$, so that

$$f_i = f_c + \frac{1}{2\pi}\frac{d}{dt}\left[\phi(t)\right] = f_c + \frac{k_p}{2\pi}\left[\frac{d}{dt}x(t)\right]$$

or, at the extremum,

$$(f_i)_{\max} = f_c + \frac{k_p}{2\pi}\left[\frac{d}{dt}x(t)\right]_{\max}$$

The maximum instantaneous frequency occurs when the slope of the modulating waveform is maximum. In the interval from 0 to 1 ms, $dx(t)/dt = 25/(1\times 10^{-3}) = 25{,}000$; in the interval from 1 to 4 ms, $dx(t)/dt = 0$; in the interval from 4 to 5 ms, $dx(t)/dt = -25/(1\times 10^{-3}) = -25{,}000$; after t = 5 ms, $dx(t)/dt = 0$. Accordingly, the maximum dx/dt is 25,000 and the corresponding instantaneous frequency is

$$(f_i)_{\max} = f_c + \frac{k_p}{2\pi}\left[\frac{d}{dt}x(t)\right]_{\max} = 5\times 10^6 + \frac{6}{2\pi}\times 25{,}000 = \boxed{5.024\,\text{MHz}}$$

P.29 ➡ *Solution*

We first determine the modulation index β_f,

$$2f_m(1+\beta_f) = \text{BW} \rightarrow 2\times\left(2\times 10^3\right)\times(1+\beta_f) = 0.1\times 10^6$$

$$\therefore \beta_f = 24$$

Now, if a FM signal can be represented as

$$s(t) = A_c \cos\left[2\pi f_c t + \beta_f \sin(2\pi f_m t)\right]$$

it follows that a signal *y(t)* = *s*³*(t)* becomes

$$y(t) = s^3(t) = A_c^3 \cos^3\left[2\pi f_c t + \beta_f \sin(2\pi f_m t)\right] = \frac{A_c^3}{4}\cos\left[6\pi f_c t + 3\beta_f \sin(2\pi f_m t)\right]$$

Comparing this modified signal with the general FM waveform, it is easy to see that the modulating frequency stands unchanged at $f_m' = f_m = 2$ kHz and the modulation index $\beta_f' = 3\beta_f = 72$. Lastly, we employ Carson's rule to obtain

$$\text{BW}' = 2f_m'\left(1+\beta_f'\right) = 2\times 2\times(1+72) = 292\,\text{kHz} \approx \boxed{0.3\,\text{MHz}}$$

▶ The correct answer is **C**.

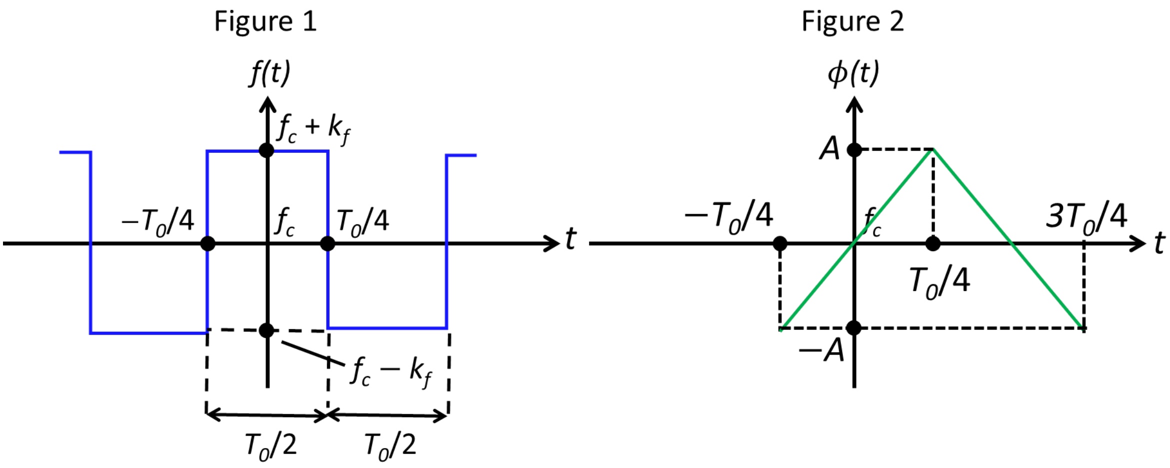
P.30 ➡ *Solution*

Problems 30.1 and 30.2: The total instantaneous frequency is depicted in figure 1 below. Note that

$$\phi(t) = 2\pi k_f \int m(\tau)\,d\tau$$

The variation of *ϕ(t)* is shown in figure 2, where

$$-A + 2\pi k_f \frac{T_0}{2} = A \rightarrow A = \pi k_f \frac{T_0}{2}$$



Problem 30.3: The complex envelope of *s(t)* is

$$\boxed{\tilde{s}(t) = A_c \exp\left[j\phi(t)\right]}$$

Problem 30.4: To compute the Fourier coefficient c_n, first note that *ϕ(t)* can be represented by the straight-line equations

$$\phi(t) = \begin{cases} 2\pi k_f t, & -T_0/4 \le t \le T_0/4 \\ 2\pi k_f(-t + T_0/2), & T_0/4 \le t \le 3T_0/4 \end{cases}$$

so that

$$c_n = \frac{1}{T_0}\int_{t=-T_0/4}^{3T_0/4} \tilde{s}(t)\exp(-j2\pi nt/T_0)\,dt$$

$$\therefore c_n = \underbrace{\frac{A_c}{T_0}\int_{t=-T_0/4}^{T_0/4} \exp\left[j2\pi t\left(k_f - n/T_0\right)\right]dt}_{=\text{Integral 1}} + \underbrace{\frac{A_c}{T_0}e^{j\pi\beta}\int_{t=T_0/4}^{3T_0/4}\exp\left[-j2\pi t\left(k_f + n/T_0\right)\right]dt}_{=\text{Integral 2}} \quad \text{(I)}$$

In the expression above, integral 1 evaluates to

$$\text{Integral 1} = \frac{A_c}{2}\,\text{sinc}\left(\frac{\beta - n}{2}\right)$$

while integral 2 gives

$$\text{Integral 2} = A_c\left[\frac{e^{-j\beta\pi/2}e^{-j3n\pi/2} - e^{j\beta\pi/2}e^{-jn\pi/2}}{-j2\pi(\beta+n)}\right] \quad \text{(II)}$$

Now, note that

$$-\frac{3n\pi}{2}=\left(-\frac{3\pi}{2}+\pi\right)n-n\pi=-\frac{n\pi}{2}-n\pi$$

Likewise,

$$-\frac{3n}{2}=\left(-\frac{\pi}{2}+\pi\right)n-n\pi=\frac{n\pi}{2}-n\pi$$

These two identities can be used to restate (II) as simply

$$\text{Integral } 2=\frac{A_c}{2}e^{-jn\pi}\,\text{sinc}\left(\frac{\beta+n}{2}\right)=\frac{A_c}{2}(-1)^n\,\text{sinc}\left(\frac{\beta+n}{2}\right)$$

Finally, we substitute the two integrals in (I) to obtain the final form of coefficient c_n,

$$c_n=\left[\frac{A_c}{2}\text{sinc}\left(\frac{\beta-n}{2}\right)+\frac{A_c}{2}(-1)^n\,\text{sinc}\left(\frac{\beta+n}{2}\right)\right]$$

Comparing this with the expression for c_n given in the problem statement, it is clear that

$$\boxed{a_n=\frac{A_c}{2}\ ;\ b_n=\frac{A_c}{2}(-1)^n}$$

P.31 ➡ *Solution*

Problem 31.1: The input FM signal can be written as

$$s(t)=A_c\cos\left[2\pi f_ct+\beta\cos(2\pi f_mt)-\pi/2\right]=\text{Re}\left[\tilde{s}(t)\exp(j2\pi f_ct)\right]$$

where

$$\tilde{s}(t)=A_c\exp\left[j\beta\cos(2\pi f_mt)-j\pi/2\right]=-jA_c\exp\left[j\beta\cos(2\pi f_mt)\right]$$

which is a periodic function with period equal to $1/f_m$. It follows that $\tilde{s}(t)$ can be expanded as a Fourier series such that

$$\tilde{s}(t)=\sum_{n=-\infty}^{\infty}c_n\exp(j2\pi nf_mt)\quad\text{(I)}$$

where

$$c_n=f_m\int_{t=-1/(2f_m)}^{1/(2f_m)}\tilde{s}(t)\exp(-j2\pi nf_mt)\,dt=-jA_cf_m\int_{t=-1/(2f_m)}^{1/(2f_m)}\exp\left[j\beta\cos(2\pi f_mt)\right]\exp(-j2\pi nf_mt)\,dt$$

Now, let

$$2\pi f_mt=\frac{\pi}{2}-x$$

so that

$$2\pi f_mt=-dx$$

and c_n can be restated as

$$c_n=\frac{jA_c}{2\pi}\int_{x=3\pi/2}^{-\pi/2}\exp\left[j\left(\beta\sin(x)-n(\pi/2-x)\right)\right]dx$$

Noting that the integrand is periodic with respect to the newly introduced variable x with a period 2π, we can interchange the limits and integrate from $-\pi$ to π,

$$c_n=-\frac{jA_c}{2\pi}\int_{x=-\pi}^{\pi}\exp\left[j\left(\beta\sin(x)-n(\pi/2-x)\right)\right]dx$$

$$\therefore c_n=\frac{A_c}{2\pi}e^{-j(n+1)\pi/2}\int_{x=-\pi}^{\pi}\exp\left[j\left(\beta\sin(x)-nx\right)\right]dx$$

$$\therefore c_n=A_cJ_{-n}(\beta)e^{-j(n+1)\pi/2}$$

Substituting in (I),

$$\tilde{s}(t)=A_c\sum_{n=-\infty}^{\infty}J_{-n}(\beta)\exp\left[j2\pi nf_mt-(n+1)\pi/2\right]$$

That is,

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_{-n}(\beta)\cos\left[2\pi f_c t + 2\pi n f_m t - (n+1)\pi/2\right]$$

The output of the bandpass filter is

$$z(t) = A_c J_0(\beta)\cos(2\pi f_c t - \pi/2) + A_c J_{-1}(\beta)\cos(2\pi f_c t + 2\pi f_m t - \pi)$$
$$+ A_c J_1(\beta)\cos(2\pi f_c t - 2\pi f_m t)$$
$$\therefore z(t) = A_c J_0(\beta)\sin(2\pi f_c t) + A_c J_1(\beta)\cos(2\pi f_c t + 2\pi f_m t)$$
$$+ A_c J_1(\beta)\cos(2\pi f_c t - 2\pi f_m t)$$
$$\therefore z(t) = A_c J_0(\beta)\sin(2\pi f_c t) + 2A_c J_1(\beta)\cos(2\pi f_c t)\cos(2\pi f_m t)$$

Problem 31.2: Assuming that *z(t)* is of the form

$$z(t) = a(t)\cos\left[2\pi f_c t + \theta(t)\right]$$
$$\therefore z(t) = a(t)\cos(2\pi f_c t)\cos\left[\theta(t)\right] - a(t)\sin(2\pi f_c t)\sin\left[\theta(t)\right]$$

it can be seen that

$$a(t)\cos\left[\theta(t)\right] = 2A_c J_1(\beta)\cos(2\pi f_m t)$$
$$a(t)\sin\left[\theta(t)\right] = -A_c J_0(\beta)$$

Accordingly, the envelope of the output is

$$\boxed{a(t) = A_c\sqrt{\left[2J_1(\beta)\cos(2\pi f_m t)\right]^2 + J_0^2(\beta)}}$$

while the phase is

$$\boxed{\theta(t) = -\tan^{-1}\left[\frac{J_0(\beta)}{2J_1(\beta)\cos(2\pi f_m t)}\right]}$$

P.32 ➡ *Solution*

A single-tone FM signal such as the present one can be represented by the general form

$$s(t) = A\sum_{n=0}^{\infty}\cos\left[2\pi(f_c + n f_m)t\right]J_n(\beta)$$

If the peak frequency deviation equals 5 times the maximum frequency, the modulation index β = 5. The cosine term we were given can be conveniently restated as

$$\cos\left[2\pi\left(10^8 + 3\times\left(2\times10^3\right)\right)t\right]$$

which, comparing with the general form of *s(t)*, indicates that n = 3. Thus, the term $cos[2\pi(1006\times10^3)t]$ is accompanied by a Bessel function such that

$$J_n(\beta) = \boxed{J_3(5)}$$

▶ The correct answer is **C**.

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