Quiz HD201

Analysis of Pipe Flow

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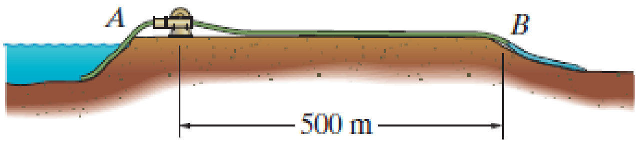
Problems

Problem 1 (Hibbeler, 2017, w/ permission)

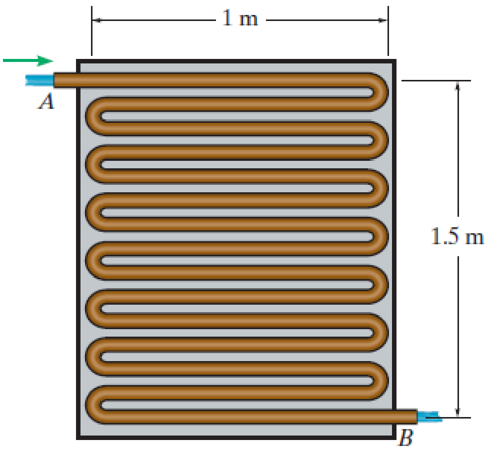
Water is to be delivered at 0.04 m^3/s to point B on the ground, 500 m away from the reservoir. Determine the smallest diameter of pipe that can be used.



A) D = 6.8 cm
B) D = 10.5 cm
C) D = 14.7 cm
D) D = 18.6 cm

Problem 2 (Hibbeler, 2017, w/ permission)

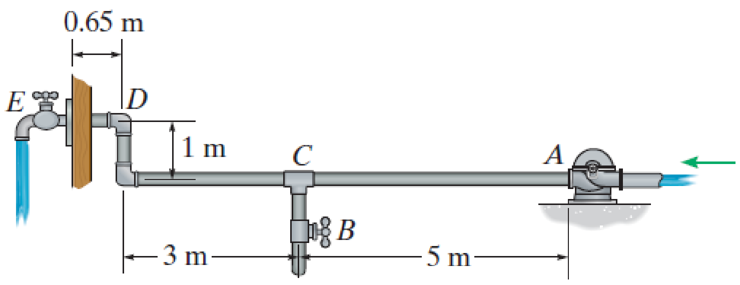
Water enters the radiator at A with an average velocity of 1.5 m/s and a pressure of 400 kPa. Determine the pressure at B. The pipe is made of copper and has a diameter of 8 mm. Include the minor losses due to the 180° bends. The radiator is in the vertical plane. **Use $\nu = 1\times10^{-6}$ m^2/s in this problem and all the following problems.**



A) p_B = 183.4 kPa
B) p_B = 274.3 kPa
C) p_B = 328.1 kPa
D) p_B = 439.2 kPa

Problem 3 (Hibbeler, 2017, w/ permission)

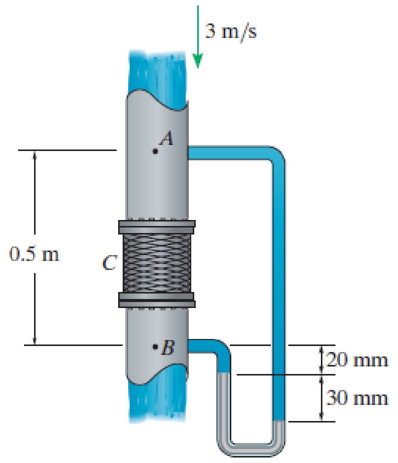
If the faucet (gate valve) at A is fully opened and the pump produces a pressure of 350 kPa at A, determine the pressure just to the right of the tee connection C. The valve at B remains closed. The pipe and faucet both have an inner diameter of 30 mm, and f = 0.04. Include the minor losses of the tee, the two elbows, and the gate valve.



A) p_C = 104.5 kPa
B) p_C = 155.4 kPa
C) p_C = 201.3 kPa
D) p_C = 252.2 kPa

Problem 4 (Hibbeler, 2017, w/ permission)

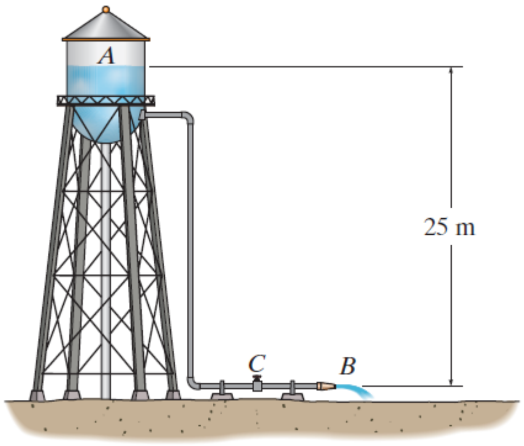
Water flows down through the vertical pipe at a rate of 3 m/s. If the differential elevation of the mercury manometer is 30 mm as shown, determine the loss coefficient K_L for the filter C contained within the pipe. Use ρ_{Hg} = 13,350 kg/m^3.



A) K_L = 0.19
B) K_L = 0.30
C) K_L = 0.81
D) K_L = 1.12

Problem 5 (Hibbeler, 2017, w/ permission)

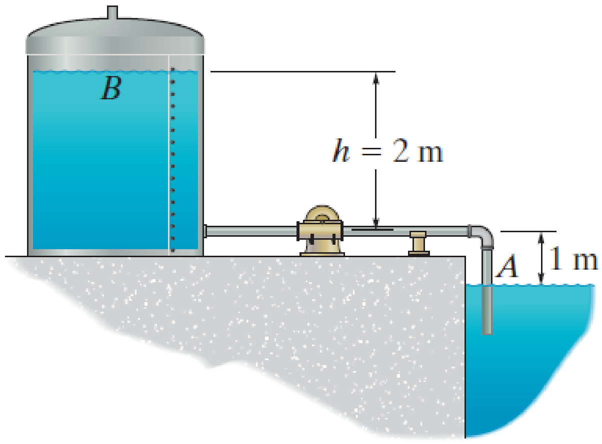
The large tank is filled with water to the depth shown. If the gate valve at C is fully opened, determine the power of the water flowing from the end of the nozzle at B. The asbestos cement pipe is 36 m long and has a diameter of 75 mm. Include the minor losses at the flush entrance, the two elbows, and the gate valve.



A) P = 208 W
B) P = 313 W
C) P = 422 W
D) P = 509 W

Problem 6 (Hibbeler, 2017, w/ permission)

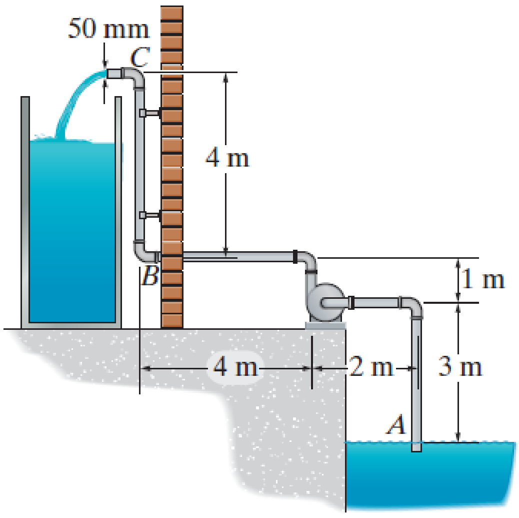
Water from the reservoir A is pumped into the large tank B. If the top of the tank is open, and the power output of the pump is 500 W, determine the volumetric flow into the tank when h = 2 m. The copper pipe has a total length of 6 m and a diameter of 50 mm. Include the minor loss for the elbow, and take K_L = 1.0 for the sudden expansion.



A) Q = 2.36 L/s
B) Q = 4.44 L/s
C) Q = 6.27 L/s
D) Q = 8.12 L/s

Problem 7 (Hibbeler, 2017, w/ permission)

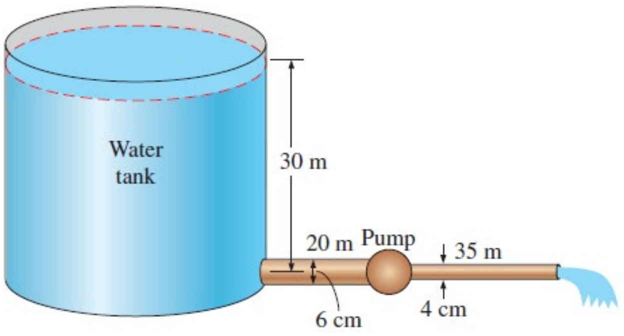
Water is pumped from the reservoir A using a pump that supplies 3 kW of power. Determine the discharge at C if the pipe is made of wrought iron and has a diameter of 150 mm. Include the minor losses of the four elbows.



A) Q = 6 L/s
B) Q = 12 L/s
C) Q = 18 L/s
D) Q = 24 L/s

Problem 8 (Çengel & Cimbala, 2014, w/ permission)

Water is to be discharged from a reservoir at a rate of 18 L/s using two horizontal carbon steel pipes connected in series with a pump between them. The first pipe is 20 m long and has a 6 cm diameter, while the second pipe is 35 m long and has a diameter of 4 cm. The water level in the reservoir is 30 m above the centerline of the pipe. The pipe entrance is sharp-edged, and losses associated with the connection to the pump are negligible. Determine the minimum pumping power required to maintain the indicated flow rate.



A) P = 7.21 kW

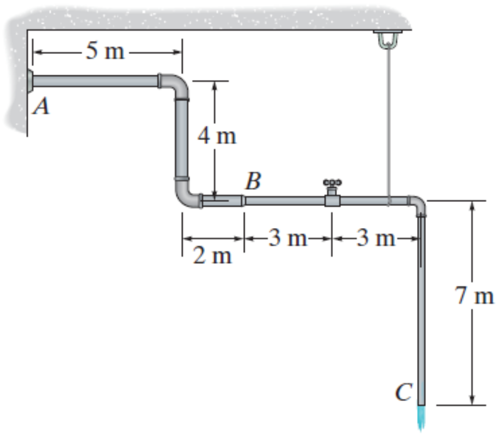
B) P = 13.1 kW

C) P = 23.3 kW

D) P = 33.4 kW

Problem 9 (Hibbeler, 2017, w/ permission)

When the globe valve is fully opened, water is discharged at 3 L/s from C. Determine the pressure at A. The lead pipes AB and BC have diameters of 60 mm and 30 mm, respectively.



A) p_A = 69 kPa

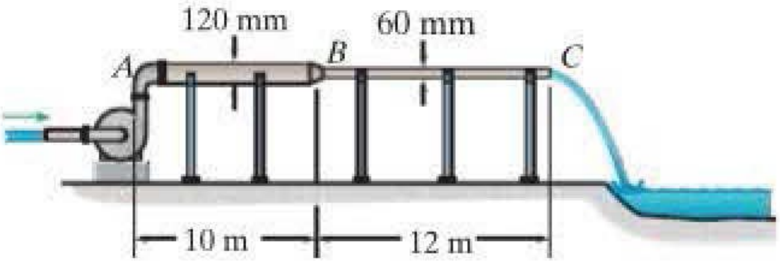
B) p_A = 77 kPa

C) p_A = 85 kPa

D) p_A = 94 kPa

Problem 10 (Hibbeler, 2017, w/ permission)

Water is pumped through the two aluminum pipes having the lengths and diameters shown. If the pressure developed at A is 230 kPa, determine the discharge at C. Neglect minor losses.



A) Q_{BC} = 8.2 L/s

B) Q_{BC} = 12.5 L/s

C) Q_{BC} = 16.7 L/s

D) Q_{BC} = 21.0 L/s

Problem 11

Two pipes of identical diameter and material are connected in parallel. The length of pipe A is five times the length of pipe B. Assuming the flow is fully turbulent in both pipes (and thus the friction factor is independent of the Reynolds number) and disregarding minor losses, determine the ratio of the flow rates of the two pipes. Below, Q_B is the smaller flow rate and Q_A is the larger flow rate.

A) $Q_B/Q_A = 0.238$

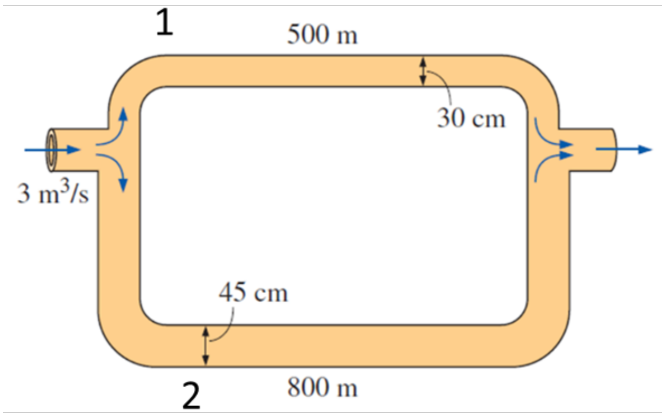
B) $Q_B/Q_A = 0.356$

C) $Q_B/Q_A = 0.447$

D) $Q_B/Q_A = 0.515$

Problem 12 (Çengel & Cimbala, 2014, w/ permission)

A pipeline that transports water at a rate of 3 m^3/s branches out into two parallel pipes made of steel in rusted condition that reconnect downstream. Pipe 1 is 500 m long has a diameter of 30 cm while pipe 2 is 800 m long and a diameter of 45 cm. The minor losses are considered to be negligible. Determine the flow rate through each of the parallel pipes.



A) $Q_1 = 0.94$ m^3/s and $Q_2 = 2.06$ m^3/s

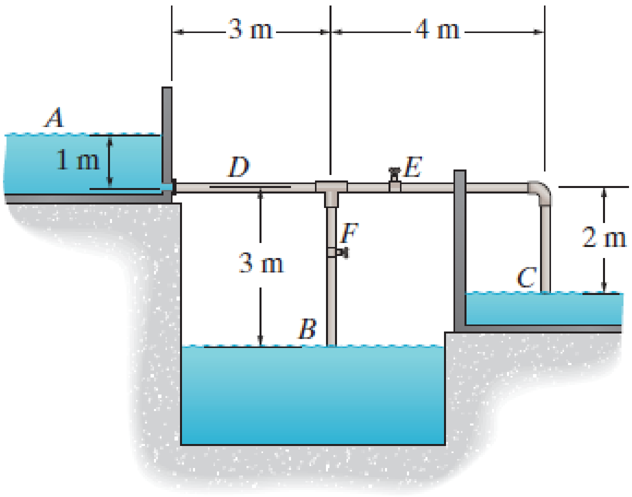
B) $Q_1 = 0.82$ m^3/s and $Q_2 = 2.18$ m^3/s

C) $Q_1 = 0.73$ m^3/s and $Q_2 = 2.27$ m^3/s

D) $Q_1 = 0.65$ m^3/s and $Q_2 = 2.35$ m^3/s

Problem 13 (Hibbeler, 2017, w/ permission)

Water from the reservoir at A drains through the 30-mm diameter pipe assembly. If brass pipe is used, determine the initial flow into pipe D from reservoir A when both valves E and F are fully opened. Neglect any minor losses.



A) $Q_D = 2.36$ L/s

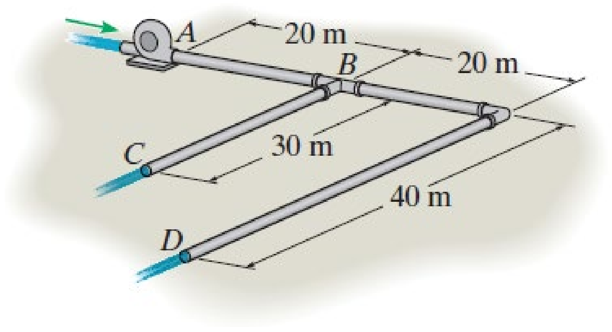
B) $Q_D = 4.43$ L/s

C) $Q_D = 6.51$ L/s

D) $Q_D = 8.65$ L/s

Problem 14 (Hibbeler, 2017, w/ permission)

The horizontal galvanized iron pipe system is used for irrigation purposes and delivers water to two different outlets. If the pump delivers a flow of 0.01 m³/s in the pipe at A, determine the discharge at each outlet, C and D. Don't forget to consider the minor losses of the elbow and tee.



A) Q_{BC} = 0.034 m³/s and Q_{BD} = 0.066 m³/s

B) Q_{BC} = 0.045 m³/s and Q_{BD} = 0.055 m³/s

C) Q_{BC} = 0.058 m³/s and Q_{BD} = 0.042 m³/s

D) Q_{BC} = 0.067 m³/s and Q_{BD} = 0.033 m³/s

Additional Information

Table 1 Absolute roughness for common materials

Material	Roughness (mm)
Asbestos cement	0.07
Asphalted cast iron	0.3
Brass	0.0014
Carbon steel	0.05
Cast iron (new)	0.26
Cast iron (old, sandblasted)	1
Copper	0.0015
Galvanized iron	0.15
Rusted steel	0.7
Riveted steel	0.91
Smooth cement	0.5
Wrought iron	0.045
Aluminum	0.002
Lead	0.0015

Table 2 Minor loss coefficients for commonly used components

Component	Minor Loss Coefficient
180° return bend, threaded	1.5
45° bend	0.4
90° elbow	0.9
Flush entrance	0.5
Gate valve – fully opened	0.19
Globe valve – fully opened	10
Sharp-edged entrance	0.5
Tee with pipe run	0.4

Solutions

P.1 ■ Solution

The average flow velocity in the pipe can be obtained from the equation

$$V=\frac{Q}{A}=\frac{Q}{\frac{\pi}{4}D^2}=\frac{0.04}{\frac{\pi}{4}\times D^2}=\frac{0.0509}{D^2}\ \text{(I)}$$

Next, the pump head can be obtained from the relation that provides its power $\dot{W}$,

$$\dot{W}=\gamma Q h_{\text{pump}}$$

$$\therefore 40{,}000=9810\times 0.04 h_{\text{pump}}$$

$$\therefore h_{\text{pump}}=\frac{40{,}000}{9810\times 0.04}=101.94\,\text{m}$$

The head loss in the pipe follows from the Darcy-Weisbach equation. Substituting the data we have and the expression obtained in (I), it follows that

$$h_L=f\frac{L}{D}\frac{V^2}{2g}=0.02\times\frac{500}{D}\times\frac{\left(\frac{0.0509}{D^2}\right)^2}{2\times 9.81}=\frac{0.00132}{D^5}=101.94$$

Finally, solving for D,

$$\frac{0.00132}{D^5}=101.94\rightarrow D=\left(\frac{0.00132}{101.94}\right)^{\frac{1}{5}}=0.105\text{ m}=\boxed{10.5\text{ cm}}$$

★ The correct answer is **B**.

P.2 ■ Solution

The Reynolds number for flow in the radiator is

$$\text{Re}=\frac{VD}{\nu}=\frac{1.5\times\left(8\times 10^{-3}\right)}{1\times 10^{-6}}=1.2\times 10^4$$

Plugging this value and the relative roughness ε/D = 0.0015/8 = 0.000188 in the Colebrook equation, the friction factor is determined to be f = 0.0298. The major head loss in the pipe can be obtained with the Darcy-Weisbach equation,

$$(h_L)_M=f\frac{L}{D}\frac{V^2}{2g}=0.0298\times\frac{(15\times 1)}{\left(8\times 10^{-3}\right)}\times\frac{1.5^2}{2\times 9.81}=6.41\text{ m}$$

The minor head losses in the system account for the fourteen 180° bends,

$$(h_L)_m=14\times K_L\times\frac{V^2}{2g}=14\times 1.5\times\frac{1.5^2}{2\times 9.81}=2.41\text{m}$$

Thence, the outlet pressure at B, p_{out}, can be determined with the Bernoulli equation,

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_{\text{pump}}=\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}+h_{turb}+(h_L)_M+(h_L)_m$$

$$\therefore\frac{p_{\text{out}}}{9810}+\frac{1.5^2}{2\times 9.81}+0+0+6.41+2.41=\frac{400\times 10^3}{9810}+\frac{1.5^2}{2\times 9.81}+1.5+0$$

$$\therefore\frac{p_{\text{out}}}{9810}+0.115+8.82=40.77+0.115+1.5$$

$$\therefore\boxed{p_{\text{out}}=p_B=328.1\text{kPa}}$$

★ The correct answer is **C**.

P.3 ■ Solution

We first calculate the major head loss in the pipe segment that links *A* to *E*.

$$(h_L)_M = f\frac{L}{D}\frac{V^2}{2g} = 0.04\times\frac{9.65}{0.03}\times\frac{V^2}{2\times 9.81} = 0.656V^2$$

Thence, we compute the minor head loss in the entire segment AE.

$$(h_L)_m = \Sigma K_L\frac{V^2}{2g} = 2K_{L,90^\circ\text{elbow}}\frac{V^2}{2g} + K_{L,\text{Tee}}\frac{V^2}{2g} + K_{L,\text{gate valve}}\frac{V^2}{2g}$$

$$\therefore (h_L)_m = 2\times 0.9\times\frac{V^2}{2\times 9.81} + 0.4\times\frac{V^2}{2\times 9.81} + 0.19\times\frac{V^2}{2\times 9.81} = 0.122V^2$$

The Bernoulli equation can be used to calculate the flow velocity *V*,

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + (h_L)_M + (h_L)_m$$

$$\therefore \frac{0}{9810} + \frac{V^2}{2\times 9.81} + 1 + 0.656V^2 + 0.122V^2 = \frac{350\times 10^3}{9810} + \frac{V^2}{2\times 9.81} + 0 + 0$$

$$\therefore 0.778V^2 + 1 = 35.68$$

$$\therefore V = \sqrt{\frac{34.68}{0.778}} = 6.68 \text{ m/s}$$

The major head loss in portion AC of the system follows from the Darcy-Weisbach equation,

$$(h_L)_M = f\frac{L}{D}\frac{V^2}{2g} = 0.04\times\frac{5}{0.03}\times\frac{6.68^2}{2\times 9.81} = 15.16\,\text{m}$$

Finally, we apply the Bernoulli equation to section AC in isolation to determine the outlet pressure p_{in} at C,

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + (h_L)_M + (h_L)_m$$

$$\therefore \frac{p_{\text{out}}}{9810} + \frac{6.68^2}{2\times 9.81} + z + 0 + 15.16 = \frac{350\times 10^3}{9810} + \frac{6.68^2}{2\times 9.81} + z + 0$$

$$\therefore \frac{p_{\text{out}}}{9810} + 2.27 + z + 0 + 15.16 = 35.68 + 2.27 + z + 0$$

$$p_{\text{out}} = p_C = (35.68 - 15.16)\times 9810 = \boxed{201.3 \text{ kPa}}$$

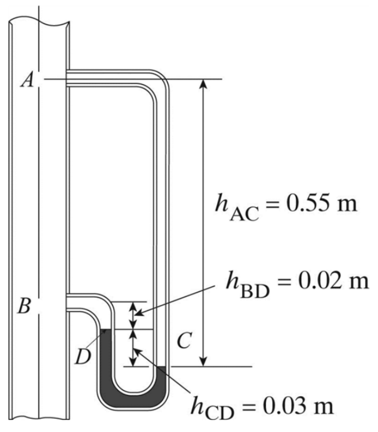
★ The correct answer is **C**.

P.4 ■ Solution

The head loss caused by the filter can be obtained with the usual relation

$$h_L = K_L\frac{V^2}{2g} = K_L\times\frac{3^2}{2\times 9.81} = 0.459K_L$$

Consider the following figure.



The pressure difference between points *A* and *B*, $p_{\text{out}} - p_{\text{in}}$, can be established from fluid statics,

$$p_{\text{in}} + \rho_w g\left(h_L\right)_{\text{AC}} = p_{\text{out}} + \rho_{\text{Hg}} g\left(h_L\right)_{\text{CD}} + \rho_w g\left(h_L\right)_{\text{BD}}$$

$$\therefore p_{\text{in}} + 1000\times 9.81\times 0.55 = p_{\text{out}} + 13{,}350\times 9.81\times 0.03 + 1000\times 9.81\times 0.02$$

$$\therefore p_{\text{in}} + 5395.5 = p_{\text{out}} + 3928.9 + 196.2$$

$$\therefore p_{\text{in}} - p_{\text{out}} = 4125.1 - 5395.5 = -1270.4$$

$$\therefore p_{\text{out}} - p_{\text{in}} = 1270.4\ \text{Pa}$$

Coefficient K_L can be found with the Bernoulli equation; that is,

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + \left(h_L\right)_M + \left(h_L\right)_m$$

$$\therefore \frac{p_{\text{out}} - p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + \left(h_L\right)_M + \left(h_L\right)_m = \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}}$$

$$\therefore \frac{1270.4}{9810} + \frac{3^2}{2\times 9.81} + 0 + 0 + 0.459K_L = \frac{3^2}{2\times 9.81} + 0.5 + 0$$

$$\therefore 0.13 + \cancel{0.459} + 0.459K_L = \cancel{0.459} + 0.5$$

$$\therefore \boxed{K_L = \frac{0.37}{0.459} = 0.81}$$

★ The correct answer is **C**.

P.5 ■ Solution

We begin by computing the Reynolds number for flow in the pipe system,

$$\text{Re} = \frac{VD}{\nu} = \frac{V\times 0.075}{1\times 10^{-6}} = 7.5\times 10^4 V \quad \text{(I)}$$

The head loss in the pipe is the sum of major loss and minor loss; that is,

$$h_L = f\frac{L}{D}\frac{V^2}{2g} + \Sigma K_L\frac{V^2}{2g}$$

$$\therefore h_L = f\frac{L}{D}\frac{V^2}{2g} + K_{L,\text{flush entrance}}\frac{V^2}{2g} + 2K_{L,90^\circ\ \text{elbow}}\frac{V^2}{2g} + K_{L,\text{gate valve}}\frac{V^2}{2g}$$

$$\therefore h_L = f\times\frac{36}{0.075}\times\frac{V^2}{2\times 9.81} + 0.5\times\frac{V^2}{2\times 9.81} + 2\times 0.9\times\frac{V^2}{2\times 9.81} + 0.19\times\frac{V^2}{2\times 9.81}$$

$$\therefore h_L = (0.127 + 24.46f)V^2 \quad \text{(A)}$$

Now, let us apply the Bernoulli equation between the two ends of the system,

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_{\text{pump}} = \frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} + h_{turb} + h_L$$

$$\therefore \frac{0}{9810} + \frac{V^2}{2\times 9.81} + 0 + 0 + (0.127 + 24.46f)V^2 = \frac{0}{9810} + \frac{0}{2\times 9.81} + 25 + 0$$

$$\therefore 0.051V^2 + (0.127 + 24.46f)V^2 = 25$$

$$\therefore V = \frac{111.8}{\sqrt{89 + 12{,}230f}} \quad \text{(II)}$$

Note that equations (I) and (II) contain three unknowns, namely, the Reynolds number *Re*, the velocity *V*, and the friction factor *f*. Ordinarily, we would solve this problem by trial-and-error: we posit an arbitrary friction factor *f*, substitute it into equation (II) to obtain *V*, compute the corresponding Reynolds number, and obtain the friction factor from the Moody diagram; finally, we verify whether the *f* taken from the Moody diagram coincides with the supposed value. The procedure is repeated until we get a definitive value of *f*. However, a much more efficient procedure, and the one that we will be using in these problems, is to introduce the Colebrook-White equation and solve all expressions simultaneously with a computer software. The Colebrook equation associates the friction factor to the Reynolds number in an implicit relationship and thus suppresses the need to use the Moody diagram altogether. The equation in question is

$$\frac{1}{\sqrt{f}}=-2\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Note that the only additional information that we need to apply this equation is the relative roughness; in the present problem, the pipe material is asbestos cement, for which we have ε = 0.07 mm (Table 1) and therefore ε/D = 0.07/75 = 9.33×10^{-4}. Thus,

$$\frac{1}{\sqrt{f}}=-2\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)=\frac{1}{\sqrt{f}}=-2\log\left(\frac{9.33\times10^{-4}}{3.7}+\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)\ \text{(III)}$$

We now have 3 equations: the Reynolds number equation, the expression that arose from use of the Bernoulli equation, and the newly introduced Colebrook equation. There are also 3 unknowns; the problem has become tractable. All we need is a computer program capable of solving systems of nonlinear equations. In Mathematica, this is easily done with the function *FindRoot*,

$$\mathrm{FindRoot}\left[\left\{\frac{1}{\sqrt{f}}==-2*\mathrm{Log10}\left[\frac{9.33*10^{-4}}{3.7}+\frac{2.51}{\mathrm{RE}\sqrt{f}}\right],0.051V^2+(0.127+24.46f)V^2=\right.\right.$$
$$\left.\left.=25,\mathrm{RE}==75000V\right\},\{\{f,0.01\},\{\mathrm{RE},10000\},\{V,3\}\}\right]$$

This yields the solutions f = 0.020, *Re* = 4.59×10^5, V = 6.12 m/s. We are now able to compute the flow rate Q,

$$Q=VA=6.12\times\left(\frac{\pi}{4}\times0.075^2\right)=0.027\ \mathrm{m^3/s}$$

The head loss in pipe section AB can be obtained from equation (A),

$$\left(h_L\right)_{A-B}=\left(0.127+24.46\times0.020\right)\times6.12^2=23.08\ \mathrm{m}$$

We aim for the energy head at B; that is,

$$\left(h_L\right)_{A-B}=H_A-H_B$$
$$\therefore 23.08=25-H_B$$
$$\therefore H_B=25-23.08=1.92\ \mathrm{m}$$

Finally, we calculate the power of water flow at the end of the nozzle in B,

$$P=\gamma QH_B=9810\times0.027\times1.92=\boxed{509\ \mathrm{W}}$$

★ The correct answer is **D**.

P.6 ■ Solution

We begin by computing the Reynolds number,

$$\mathrm{Re}=\frac{VD}{\nu}=\frac{V\times0.05}{1\times10^{-6}}=5\times10^4V\ \ \text{(I)}$$

The flow rate Q for this system is

$$Q=AV=\frac{\pi}{4}\times0.05^2\times V=0.00197V\ \text{(A)}$$

The pump head of the system, in turn, is

$$\dot{W} = \gamma Q h_{\text{pump}}$$

$$\therefore 500 = 9810 \times 0.00197V \times h_{\text{pump}}$$

$$\therefore h_{\text{pump}} = \frac{25.87}{V}$$

The total head loss in the pipe is

$$h_L = f\frac{L}{D}\frac{V^2}{2g} + \Sigma K_L \frac{V^2}{2g}$$

$$\therefore h_L = f\frac{L}{D}\frac{V^2}{2g} + K_{L,90^{\circ}\text{ elbow}}\frac{V^2}{2g} + K_{L,\text{sudden expansion}}\frac{V^2}{2g}$$

$$\therefore h_L = f \times \frac{6}{0.05} \times \frac{V^2}{2\times 9.81} + 0.9 \times \frac{V^2}{2\times 9.81} + 1 \times \frac{V^2}{2\times 9.81}$$

$$\therefore h_L = (0.097 + 6.12f)V^2$$

We then appeal to the Bernoulli equation,

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + (h_L)_M + (h_L)_m$$

$$\therefore \frac{0}{9810} + \frac{0}{2\times 9.81} + \frac{25.87}{V} = \frac{0}{9810} + \frac{0}{2\times 9.81} + 3 + 0 + (0.097 + 6.12f)V^2$$

$$\therefore \frac{25.87}{V} = 3 + (0.097 + 6.12f)V^2$$

$$\therefore (0.097 + 6.12f)V^3 + 3V = 25.87 \text{ (II)}$$

We have two equations and three unknowns (namely, *Re*, *V*, and *f*). The Colebrook equation can be used to relate the friction factor to the Reynolds number. Applying the *FindRoot* command in Mathematica, we have

$$\text{FindRoot}\left[\left\{\text{RE} == 5 * 10^4 V, (0.097 + 6.12f)V^3 + 3V == 25.87, \frac{1}{\sqrt{f}} = = -2\text{Log10}\left[\frac{0.0015/50}{3.7} + \frac{2.51}{\text{RE}\sqrt{f}}\right]\right\}, \{\{\text{RE}, 50000\}, \{V, 1\}, \{f, 0.02\}\}\right]$$

This returns *Re* = 205,870, *V* = 4.12 m/s, and *f* = 0.0158. Substituting the velocity value in equation (A), we can determine the volumetric flow into the tank,

$$Q = 0.00197 \times 4.12 = 0.00812 \text{ m}^3/\text{s} = \boxed{8.12 \text{ L/s}}$$

★ The correct answer is **D**.

P.7 ■ Solution

We begin by computing the Reynolds number for flow in the pipe system,

$$\text{Re} = \frac{VD}{\nu} = \frac{V \times 0.05}{1\times 10^{-6}} = 50{,}000V \text{ (I)}$$

The major head loss in the pipe is

$$(h_L)_M = f\frac{L}{D}\frac{V^2}{2g} = f \times \frac{14}{0.05} \times \frac{V^2}{2\times 9.81} = 14.271fV^2$$

while the minor head loss follows as

$$(h_L)_m = \Sigma K_L \frac{V^2}{2g} = 4K_{L,90^{\circ}\text{ elbow}}\frac{V^2}{2g} = 4 \times 0.9 \times \frac{V^2}{2\times 9.81} = 0.183V^2$$

Then, we apply the Bernoulli equation to pipe segment AC,

$$\frac{p_{\text{in}}}{\gamma}+\frac{V_{\text{in}}^2}{2g}+z_{\text{in}}+h_{\text{pump}}=\frac{p_{\text{out}}}{\gamma}+\frac{V_{\text{out}}^2}{2g}+z_{\text{out}}+h_{\text{turb}}+h_L$$

$$\therefore \frac{0}{9810}+\frac{0}{2\times 9.81}+0+h_{\text{pump}}=\frac{0}{9810}+\frac{V^2}{2\times 9.81}+8+0+14.271fV^2+0.183V^2$$

$$\therefore h_{\text{pump}}=0.051V^2+8+14.271fV^2+0.183V^2=8+0.234V^2+14.271fV^2$$

$$\therefore h_{\text{pump}}=8+0.234V^2+14.271fV^2$$

The flow rate in the pipe system is the variable we seek,

$$Q=AV=\frac{\pi\times 0.05^2}{4}\times V=0.00196V\ (\text{A})$$

Another relationship to consider is the expression for power developed by the pump,

$$\dot{W}=\gamma Q h_{\text{pump}}$$

$$\therefore 3000=9810\times 0.00196V\times\left(8+14.271fV^2+0.234V^2\right)$$

$$\therefore (4.50+274.4f)V^3+153.82V=3000\ \ (\text{II})$$

We have two equations and three unknowns – namely, the Reynolds number *Re*, the flow velocity *V*, and the friction factor *f*. The Colebrook equation can serve as the third relation. In Mathematica, the equations can be solved simultaneously with the *FindRoot* function,

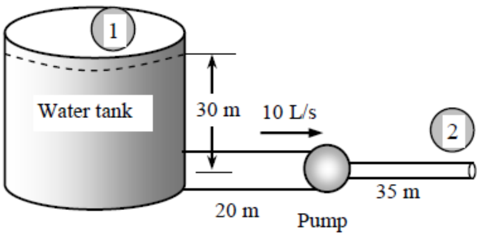
$$\text{FindRoot}\left[\left\{\text{RE}==50000V,(4.5+274.4f)V^3+153.82V==3000,\frac{1}{\sqrt{f}}=\right.\right.$$
$$\left.\left.=-2*\text{Log10}\left[\frac{0.045/50}{3.7}+\frac{2.51}{\text{RE}\sqrt{f}}\right]\right\},\{\{\text{RE},50000\},\{V,3\},\{f,0.015\}\}\right]$$

This returns *Re* = 306,000, *V* = 6.13 m/s, and *f* = 0.0162. Having obtained the velocity *V*, we are able to compute the discharge in the pipe system via equation (A),

$$Q=0.00196\times 6.13=0.012\,\text{m}^3/\text{s}=\boxed{12\ \text{L/s}}$$

★ The correct answer is **B**.

P.8 ■ Solution



The system can be described with the Bernoulli equation. We take one extreme as the free surface of the tank, and the other extreme as the reference level at the centerline of the pipe. Thus,

$$\cancel{\frac{p_1}{\gamma}}+\cancel{\frac{V_1^2}{2g}}+z_1+h_{\text{pump}}=\cancel{\frac{p_2}{\gamma}}+\frac{V_2^2}{2g}+\cancel{z_2}+\cancel{h_{\text{turb}}}+h_L$$

$$\therefore z_1+h_{\text{pump}}=\frac{V_2^2}{2g}+h_L$$

Noting that the two pipes are connected in series, and that therefore the flow rate through each of them is the same, the head loss for each pipe can be determined as follows. We designate the first pipe as 1 and the second pipe as 2. For pipe 1, the flow velocity is

$$V_1 = \frac{Q}{A_1} = \frac{0.018}{\left(\pi \times 0.06^2/4\right)} = 6.37 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{6.37 \times 0.06}{1 \times 10^{-6}} = 3.82 \times 10^5$$

The relative roughness of the pipe is 0.05/60 = 8.33×10^{-4}. Substituting the previous results in the Colebrook equation, we can establish the friction factor f_1 for the first pipe, which is found as f_1 = 0.0197. Next, we compute the major and minor losses for the first pipe, recalling that the minor loss coefficient for a sharp-edged entrance is K_L = 0.5.

$$h_{L,1} = \left(f_1 \frac{L_1}{D_1} + \Sigma K_L\right)\frac{V_1^2}{2g} = 0.0197 \times \frac{20}{0.06} \times \frac{6.37^2}{2 \times 9.81} + 0.5 \times \frac{6.37^2}{2 \times 9.81} = 14.61 \text{ m}$$

Let us proceed to pipe 2. The velocity of flow is

$$V_2 = \frac{Q_2}{A_2} = \frac{0.018}{\left(\pi \times 0.04^2/4\right)} = 14.32 \text{ m/s}$$

The corresponding Reynolds number is

$$\text{Re}_2 = \frac{VD}{\nu} = \frac{14.32 \times 0.04}{1 \times 10^{-6}} = 5.73 \times 10^5$$

The relative roughness of the pipe is 0.05/40 = 1.25×10^{-3}. Using the Colebrook equation, we obtain f_2 = 0.0212. The second pipe involves no minor losses. Its head loss is then

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = 0.0194 \times \frac{35}{0.04} \times \frac{14.32^2}{2 \times 9.81} = 193.88 \text{ m}$$

The total head loss for two pipes connected in series is the sum of the head losses of the two pipes, namely,

$$\left(h_L\right)_{\text{Total}} = h_{L,1} + h_{L,2} = 14.61 + 193.88 = 208.49 \text{ m}$$

Then, the pumping head and the minimum power required will be

$$h_{\text{pump}} = \frac{V_2^2}{2g} + h_L - z_1 = \frac{14.32^2}{2 \times 9.81} + 208.49 - 30 = 188.94 \text{ m}$$

$$\therefore P = Q\Delta p = \rho Q g h_{\text{pump}} = 1000 \times 0.018 \times 9.81 \times 188.94 = \boxed{33.4 \text{ kW}}$$

The pump must supply a minimum of 33.4 kW of useful mechanical energy to the water.

★ The correct answer is **D**.

P.9 ■ Solution

The flow velocity in pipe segment AB can be obtained with the usual equation

$$V_{AB} = \frac{Q}{A_{AB}} = \frac{0.003}{\frac{\pi \times 0.06^2}{4}} = 1.06 \text{ m/s}$$

Likewise, the velocity of flow in pipe segment BC is

$$V_{BC} = \frac{Q}{A_{BC}} = \frac{0.003}{\frac{\pi \times 0.03^2}{4}} = 4.24 \text{ m/s}$$

We also require the Reynolds number in segments AB and BC,

$$\mathrm{Re}_{AB} = \frac{V_{AB}D_{AB}}{\nu} = \frac{1.06\times0.06}{1\times10^{-6}} = 6.36\times10^4$$

$$\mathrm{Re}_{BC} = \frac{V_{BC}D_{BC}}{\nu} = \frac{4.24\times0.03}{1\times10^{-6}} = 1.27\times10^5$$

The relative roughness of pipe segment AB is ε/D_{AB} = 0.0015/60 = 2.5×10^{-5}. Plugging this value and Reynolds number Re_{AB} in the Colebrook equation and solving for the friction factor, we obtain f_{AB} = 0.0199. Similarly, substituting the relative roughness of pipe segment BC, ε/D_{BC} = 0.0015/30 = 5×10^{-5}, and the Reynolds number Re_{BC} = 1.27×10^5 in the Colebrook equation and solving for f, we get f_{BC} = 0.0174. We then proceed to calculate the major head loss in the pipe,

$$(h_L)_M = f_{AB}\frac{L_{AB}}{D_{AB}}\frac{V_{AB}^2}{2g} + f_{BC}\frac{L_{BC}}{D_{BC}}\frac{V_{BC}^2}{2g}$$

$$\therefore (h_L)_M = 0.0199\times\frac{9}{0.06}\times\frac{1.06^2}{2\times9.81} + 0.0174\times\frac{13}{0.03}\times\frac{4.24^2}{2\times9.81} = 7.08\text{ m}$$

To compute the minor head loss in the pipe, we must account for the 3 elbows and the fully opened globe valve,

$$(h_L)_m = \Sigma K_L\frac{V^2}{2g} = 2K_{L,90^\circ\text{ elbow}}\frac{V_{AB}^2}{2g} + K_{L,90^\circ\text{ elbow}}\frac{V_{BC}^2}{2g} + K_{L,\substack{\text{fully opened}\\\text{globe valve}}}\frac{V_{BC}^2}{2g}$$

$$\therefore (h_L)_m = 2\times0.9\times\frac{1.06^2}{2\times9.81} + 0.9\times\frac{4.24^2}{2\times9.81} + 10\times\frac{4.24^2}{2\times9.81} = 10.09\text{ m}$$

Finally, we apply the Bernoulli equation between ends A and C of the pipe to obtain the pressure at inlet A (p_{in}),

$$\frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + h_L$$

$$\therefore \frac{p_A}{9810} + \frac{1.06^2}{2\times9.81} + 11 + 0 = \frac{0}{9810} + \frac{4.24^2}{2\times9.81} + 0 + 0 + 7.08 + 10.09$$

$$\therefore \frac{p_A}{9810} + 0.0573 + 11 = 0.92 + 7.08 + 10.09$$

$$\therefore \frac{p_A}{9810} = 7.03$$

$$\therefore \boxed{p_A = 69\text{ kPa}}$$

★ The correct answer is **A**.

P.10 ■ Solution

The Reynolds number for pipe segment AB is

$$\mathrm{Re}_{AB} = \frac{V_{AB}D_{AB}}{\nu} = \frac{V_{AB}\times0.12}{1\times10^{-6}} = 1.2\times10^5 V_{AB}\quad\text{(I)}$$

The Reynolds number in pipe section BC, in turn, is

$$\mathrm{Re}_{BC} = \frac{V_{BC}D_{BC}}{\nu} = \frac{V_{BC}\times0.06}{1\times10^{-6}} = 6\times10^4 V_{BC}\quad\text{(II)}$$

Next, we write the continuity equation,

$$V_{AB}A_{AB} - V_{BC}A_{BC} = 0$$

$$\therefore V_{AB}A_{AB} = V_{BC}A_{BC}$$

$$\therefore V_{AB}\times\frac{\pi D_{AB}^2}{4} = V_{BC}\times\frac{\pi D_{BC}^2}{4}$$

$$\therefore V_{AB}\times\frac{\pi\times0.12^2}{4} = V_{BC}\times\frac{\pi\times0.06^2}{4}$$

$$\therefore 0.0113V_{AB} = 0.00283V_{BC}$$

$$\therefore V_{AB} = 0.25V_{BC} \quad \text{(III)}$$

Thence, we compute the major head loss, accounting for pipe sections AB and BC,

$$\left(h_L\right)_M = f_{AB}\frac{L_{AB}}{D_{AB}}\frac{V_{AB}^2}{2g} + f_{BC}\frac{L_{BC}}{D_{BC}}\frac{V_{BC}^2}{2g}$$

$$\therefore \left(h_L\right)_M = f_{AB}\times\frac{10}{0.12}\times\frac{V_{AB}^2}{2\times 9.81} + f_{BC}\times\frac{12}{0.06}\times\frac{V_{BC}^2}{2\times 9.81}$$

$$\therefore \left(h_L\right)_M = 4.247 f_{AB}V_{AB}^2 + 10.194 f_{BC}V_{BC}^2$$

We apply the Bernoulli equation at points *A* (*in*) and *C* (*out*) to obtain velocity V_{BC},

$$\frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_{\text{pump}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_{\text{turb}} + h_L$$

$$\therefore \frac{0}{9810} + \frac{V_{BC}^2}{2\times 9.81} + z + 0 + 0.266 f_{AB}V_{BC}^2 + 10.194 f_{BC}V_{BC}^2 = \frac{230{,}000}{9810} + \frac{\left(0.25V_{BC}\right)^2}{2\times 9.81} + z + 0$$

$$\therefore \left(0.051 + 0.266 f_{AB} + 10.194 f_{BC}\right)V_{BC}^2 = 23.45 + 0.00319V_{BC}^2$$

$$\therefore \left(0.0478 + 0.266 f_{AB} + 10.194 f_{BC}\right)V_{BC}^2 = 23.45$$

$$\therefore V_{BC} = \frac{4.84}{\sqrt{0.0478 + 0.266 f_{AB} + 10.194 f_{BC}}} \quad \text{(IV)}$$

In addition to the expressions gathered heretofore, we write the Colebrook equation for pipe segments AB and BC,

$$\frac{1}{\sqrt{f_{AB}}} = -2\log\left(\frac{0.002/0.12}{3.7} + \frac{2.51}{\text{Re}_{AB}\sqrt{f_{AB}}}\right) \quad \text{(V)}$$

$$\frac{1}{\sqrt{f_{BC}}} = -2\log\left(\frac{0.002/0.06}{3.7} + \frac{2.51}{\text{Re}_{BC}\sqrt{f_{BC}}}\right) \quad \text{(VI)}$$

We have 6 equations and 6 variables – namely, Reynolds numbers Re_{AB} and Re_{BC}, velocities V_{AB} and V_{BC}, and friction factors f_{AB} and f_{BC}. In Mathematica, these equations can be solved simultaneously with the *FindRoot* command,

$$\text{FindRoot}\Bigg[\Bigg\{\text{REAB} == 120000\text{VAB}, \text{REBC} == 60000\text{VBC}, \text{VAB} == 0.25\text{VBC}, \text{VBC} =$$
$$= \frac{4.84}{\sqrt{0.0478 + 0.266\text{fAB} + 10.194\text{fBC}}}, \frac{1}{\sqrt{\text{fAB}}} =$$
$$= -2\text{Log10}\left[\frac{0.002/0.12}{3.7} + \frac{2.51}{\text{REAB}\sqrt{\text{fAB}}}\right], \frac{1}{\sqrt{\text{fBC}}} =$$
$$= -2\text{Log10}\left[\frac{0.002/0.06}{3.7}\right.$$
$$\left.+ \frac{2.51}{\text{REBC}\sqrt{\text{fBC}}}\right]\Bigg\}, \{\{\text{REAB}, 10000\}, \{\text{REBC}, 10000\}, \{\text{VAB}, 2\}, \{\text{VBC}, 8\}, \{\text{fAB}, 0.01\}, \{\text{fBC}, 0.01\}\}\Bigg]$$

This returns Re_{AB} = 177,000, Re_{BC} = 355,000, V_{AB} = 1.48 m/s, V_{BC} = 5.91 m/s, f_{AB} = 0.0457, f_{BC} = 0.0598. Having obtained the velocity in pipe segment *BC* and knowing its diameter, we can calculate the flow rate therein; that is,

$$Q_{BC} = V_{BC}A_{BC} = 5.91\times\left(\frac{\pi\times 0.06^2}{4}\right) = 0.0167\ \text{m}^3/\text{s} = \boxed{16.7\ \text{L/s}}$$

★ The correct answer is **C**.

P.11 ■ Solution

When two pipes are parallel in a piping system, the head loss must be the same for each pipe. Disregarding minor losses, the head loss for turbulent flow in a pipe of length L and diameter D can be arranged as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{Q}{A}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{Q}{\pi D^2/4}\right)^2$$

$$\therefore h_L = f\frac{L}{D}\frac{1}{2g}\frac{16Q^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{Q^2}{D^5}$$

Solving for the flow rate Q, we obtain

$$h_L = 8f\frac{L}{g\pi^2}\frac{Q^2}{D^5} \rightarrow Q = \sqrt{\frac{g\pi^2 h_L D^5}{8fL}}$$

With the pipe diameter, friction factor, and head loss held constant (the latter being the case for an in-parallel connection of pipes), we are able to write

$$Q = \frac{k}{\sqrt{L}}$$

Clearly, the flow rate becomes inversely proportional to the square root of the length L. When the length is multiplied by five, the flow rate will decrease by a factor of $5^{1/2}$ = 2.24. That is, if

$$Q_A = \frac{k}{\sqrt{L_A}}$$

then

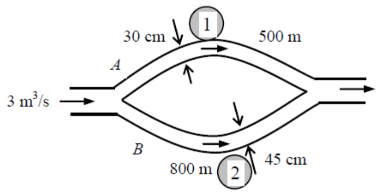
$$Q_B = \frac{k}{\sqrt{L_B}} = \frac{k}{\sqrt{5L_A}} = \frac{k}{\sqrt{5}\sqrt{L_A}} = \frac{1}{\sqrt{5}}\underbrace{\frac{k}{\sqrt{L_A}}}_{=Q_A} = \frac{1}{\sqrt{5}}Q_A = 0.447Q_A$$

$$\therefore \boxed{\frac{Q_B}{Q_A} = 0.447}$$

The flow in one pipe will be just below half the flow in the other pipe.

★ The correct answer is **C**.

P.12 ■ Solution



The head loss in two parallel branches must be the same, and the total flow rate must be the sum of the flow rates in the parallel branches. Accordingly,

$$h_{L,1} = h_{L,2} \quad \text{(I)}$$

$$Q_1 + Q_2 = 3 \quad \text{(II)}$$

The velocities are related to the flow rates by the following equations,

$$V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\left(\pi D_1^2/4\right)} = \frac{Q_1}{\left(\pi \times 0.30^2/4\right)} = 14.147Q_1 \quad \text{(III)}$$

$$V_2 = \frac{Q_2}{A_2} = \frac{Q_2}{\left(\pi D_2^2/4\right)} = \frac{Q_2}{\left(\pi \times 0.45^2/4\right)} = 6.288Q_2 \quad \text{(IV)}$$

The Reynolds number for flow in each pipe segment is

$$\mathrm{Re}_1 = \frac{V_1 D_1}{\nu} = \frac{0.30\times V_1}{1\times 10^{-6}} = 3\times 10^5 V_1 \;\; \text{(V)}$$

$$\mathrm{Re}_2 = \frac{V_2 D_2}{\nu} = \frac{0.45\times V_2}{1\times 10^{-6}} = 4.5\times 10^5 V_2 \;\; \text{(VI)}$$

The major head loss in each pipe segment is given by the Darcy-Weisbach formula,

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_1 \times \frac{500}{0.3}\times\frac{V_1^2}{2\times 9.81} = 84.947 f_1 V_1^2 \;\; \text{(VII)}$$

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = f_2 \times \frac{800}{0.45}\times\frac{V_2^2}{2\times 9.81} = 90.611 f_2 V_2^2 \;\; \text{(VIII)}$$

Finally, we relate the Reynolds number to the friction factor with the Colebrook equation, recalling that the relative roughness is 0.7/300 = 0.00233 for pipe segment 1 and 0.7/450 = 0.00156 for pipe segment 2. Thus,

$$\frac{1}{\sqrt{f_1}} = -2\log\left(\frac{0.0233}{3.7} + \frac{2.51}{\mathrm{Re}_1\sqrt{f_1}}\right) \;\; \text{(IX)}$$

$$\frac{1}{\sqrt{f_2}} = -2\log\left(\frac{0.00156}{3.7} + \frac{2.51}{\mathrm{Re}_2\sqrt{f_2}}\right) \;\; \text{(X)}$$

We have gleaned 10 equations for 10 unknowns – namely, velocities V_1 and V_2, flow rates Q_1 and Q_2, Reynolds numbers Re_1 and Re_2, head losses $h_{L,1}$ and $h_{L,2}$, and friction factors f_1 and f_2. They can be solved simultaneously in Mathematica with the *FindRoot* function,

$$\mathrm{FindRoot}\Big[\Big\{\mathrm{hL1} == \mathrm{hL2}, \mathrm{Q1}+\mathrm{Q2} == 3, \mathrm{V1} == 14.147\mathrm{Q1}, \mathrm{V2} == 6.288\mathrm{Q2}, \mathrm{RE1} =$$
$$= 3 * 10^5\mathrm{V1}, \mathrm{RE2} == 4.5 * 10^5\mathrm{V2}, \mathrm{hL1} == 84.947 * \mathrm{f1} * \mathrm{V1}^2, \mathrm{hL2} =$$
$$= 90.611 * \mathrm{f2} * \mathrm{V2}^2, \frac{1}{\sqrt{\mathrm{f1}}} == -2\mathrm{Log10}\left[\frac{0.0233}{3.7} + \frac{2.51}{\mathrm{RE1}\sqrt{\mathrm{f1}}}\right], \frac{1}{\sqrt{\mathrm{f2}}} =$$
$$= -2\mathrm{Log10}\left[\frac{0.0233}{3.7}\right.$$
$$\left.+\frac{2.51}{\mathrm{RE2}\sqrt{\mathrm{f2}}}\right]\Big\}, \{\{\mathrm{V1},1\},\{\mathrm{V2},1\},\{\mathrm{Q1},1\},\{\mathrm{Q2},1\},\{\mathrm{RE1},10000\},\{\mathrm{RE2},10000\},\{\mathrm{hL1},1\},\{\mathrm{hL2},1\},\{\mathrm{f1},0.01\},\{\mathrm{f2},0.01\}\}\Big]$$

This yields V_1 = 13.35 m/s, V_2 = 12.93 m/s, Q_1 = 0.94 m^3/s, Q_2 = 2.06 m^3/s, $Re_1 = 4\times 10^6$, $Re_2 = 5.8\times 10^6$, $h_{L,1}$ = 781.85 m, $h_{L,2}$ = 781.85 m, f_1 = 0.0516, f_2 = 0.0516. The flow rate in branch 1 is just below 1 m^3/s, while the flow rate in branch 2 is just above 2 m^3/s.

★ The correct answer is **A**.

P.13 ■ Solution

Applying the continuity equation to the pipe system, we have

$$-V_D A + V_E A + V_F A = 0$$

$$\therefore -V_D \times \cancel{\frac{\pi D^2}{4}} + V_E \times \cancel{\frac{\pi D^2}{4}} = V_F \times \cancel{\frac{\pi D^2}{4}}$$

$$\therefore V_D = V_E + V_F \;\; \text{(I)}$$

The major head loss for pipe segment *D* is

$$\left(h_L\right)_D = f_D \frac{L_D}{D}\frac{V_D^2}{2g} = f_D \times \frac{3}{0.03}\times\frac{V_D^2}{2\times 9.81} = 5.097 f_D V_D^2$$

Next, we compute the major head loss for pipe *E*,

$$\left(h_L\right)_E = f_E \frac{L_E}{D_E}\frac{V_E^2}{2g} = f_E \times \frac{6}{0.03}\times\frac{V_E^2}{2\times 9.81} = 10.194 f_E V_E^2$$

The major head loss for pipe *F*, in turn, is

$$\left(h_L\right)_F = f_F \frac{L_F}{D_F}\frac{V_F^2}{2g} = f_F \times \frac{3}{0.03}\times\frac{V_F^2}{2\times 9.81} = 5.097 f_F V_F^2$$

Let us apply the Bernoulli equation to pipe section DFB,

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_{\text{pump}}=\frac{p_{\text{out}}}{\gamma}+\frac{V_{\text{out}}^2}{2g}+z_{\text{out}}+h_{\text{turb}}+\left(h_L\right)_D+\left(h_L\right)_F$$

$$\therefore 0+\frac{0}{2\times 9.81}+0+0+5.097 f_D V_D^2+5.097 f_F V_F^2=0+\frac{0}{2\times 9.81}+4+0$$

$$\therefore 5.097 f_D V_D^2+5.097 f_F V_F^2=4$$

$$\therefore f_D V_D^2+f_F V_F^2=0.785 \quad \text{(II)}$$

Thence, we apply the Bernoulli equation to pipe section DEC,

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_{\text{pump}}=\frac{p_{\text{out}}}{\gamma}+\frac{V_{\text{out}}^2}{2g}+z_{\text{out}}+h_{\text{turb}}+\left(h_L\right)_D+\left(h_L\right)_E$$

$$\therefore 0+\frac{0}{2\times 9.81}+0+0+5.097 f_D V_D^2+10.194 f_E V_E^2=0+\frac{0}{2\times 9.81}+3+0$$

$$\therefore 5.097 f_D V_D^2+10.194 f_E V_E^2=3$$

$$\therefore f_D V_D^2+2 f_E V_E^2=0.589 \quad \text{(III)}$$

The Reynolds number for pipe section *D* is

$$\text{Re}_D=\frac{V_D D}{\nu}=\frac{0.03 V_D}{1\times 10^{-6}}=30,000 V_D \quad \text{(IV)}$$

The Reynolds number for pipe section *E* is

$$\text{Re}_E=\frac{V_E D}{\nu}=\frac{0.03 V_E}{1\times 10^{-6}}=30,000 V_E \quad \text{(V)}$$

The Reynolds number for pipe section *F* is

$$\text{Re}_F=\frac{V_F D}{\nu}=\frac{0.03 V_F}{1\times 10^{-6}}=30,000 V_F \quad \text{(VI)}$$

In addition to these relations, we apply the Colebrook equation to each of the pipe segments, noting that the relative roughness is 0.0014/30 = 4.67×10^{-5},

$$\frac{1}{\sqrt{f_D}}=-2\log\left(\frac{4.67\times 10^{-5}}{3.7}+\frac{2.51}{\text{Re}_D\sqrt{f_D}}\right) \quad \text{(VII)}$$

$$\frac{1}{\sqrt{f_E}}=-2\log\left(\frac{4.67\times 10^{-5}}{3.7}+\frac{2.51}{\text{Re}_E\sqrt{f_E}}\right) \quad \text{(VIII)}$$

$$\frac{1}{\sqrt{f_F}}=-2\log\left(\frac{4.67\times 10^{-5}}{3.7}+\frac{2.51}{\text{Re}_F\sqrt{f_F}}\right) \quad \text{(IX)}$$

There are 9 equations and 9 unknowns – namely, V_D, V_E, V_F, f_D, f_E, f_F, Re_D, Re_E, and Re_F. In Mathematica, the solution can yet again be obtained through *FindRoot*,

$$\text{FindRoot}\Big[\Big\{\text{VD}==\text{VE}+\text{VF},\text{fD}*\text{VD}^2+\text{fF}*\text{VF}^2==0.785,\text{fD}*\text{VD}^2+2*\text{fE}*\text{VE}^2=$$
$$=0.589,\text{RED}==30000\text{VD},\text{REE}==30000\text{VE},\text{REF}==30000\text{VF},\frac{1}{\sqrt{\text{fD}}}=$$
$$=-2\text{Log10}\left[\frac{0.5/30}{3.7}+\frac{2.51}{\text{RED}\sqrt{\text{fD}}}\right],\frac{1}{\sqrt{\text{fE}}}==-2\text{Log10}\left[\frac{0.5/30}{3.7}+\frac{2.51}{\text{REE}\sqrt{\text{fE}}}\right],\frac{1}{\sqrt{\text{fF}}}=$$
$$=-2\text{Log10}\Bigg[\frac{0.5/30}{3.7}$$
$$+\frac{2.51}{\text{REF}\sqrt{\text{fF}}}\Bigg]\Big\},\{\{\text{VD},1\},\{\text{VE},1\},\{\text{VF},1\},\{\text{fD},0.01\},\{\text{fE},0.01\},\{\text{fF},0.01\},\{\text{RED},10000\},\{\text{REE},10000\},\{\text{REF},10000\}\}\Big]$$

This yields the solutions V_D = 3.34 m/s, V_E = 0.91 m/s, V_F = 2.44 m/s, f_D = 0.0458, f_E = 0.0470, f_F = 0.0460, Re_D = 100,254, Re_E = 27,167, Re_F = 73,086. It remains to compute the flow rate Q_D into pipe *D*, which follows as

$$Q_D=\frac{\pi D^2}{4}\times V_D=\frac{\pi\times 0.03^2}{4}\times 3.34=0.00236\text{ m}^3/\text{s}=\boxed{2.36\text{ L/s}}$$

★ The correct answer is **A**.

P.14 ■ Solution

The flow velocity of fluid at inlet A is

$$V=\frac{Q}{\frac{\pi D^2}{4}}=\frac{0.01}{\frac{\pi\times 0.03^2}{4}}=14.15\text{ m/s}$$

The Reynolds number Re_{AB} of flow in pipe section AB is

$$\mathrm{Re}_{\mathrm{AB}}=\frac{V_A D}{\nu}=\frac{14.15\times 0.03}{1\times 10^{-6}}=4.25\times 10^5$$

The relative roughness ε/D in the pipe is 0.15/30 = 0.005. The friction factor that conforms to this data is 0.0306. The major head loss in pipe section AB is

$$\left(h_L\right)_{\mathrm{AB}}=f_{AB}\frac{L_{AB}}{D}\frac{V_A^2}{2g}=0.0306\times\frac{20}{0.03}\times\frac{14.15^2}{2\times 9.81}=208.18\text{ m}$$

The head loss in pipe section BC, in turn, is

$$\left(h_L\right)_{BC}=f_{BC}\frac{L_{BC}}{D}\frac{V_C^2}{2g}+\Sigma K_L\frac{V_C^2}{2g}=f_{BC}\times\frac{30}{0.03}\times\frac{V_C^2}{2\times 9.81}+1.8\times\frac{V_C^2}{2\times 9.81}$$

$$\therefore\left(h_L\right)_{BC}=50.97 f_{BC}V_C^2+0.092V_C^2$$

Thence, we compute the head loss in pipe section BD,

$$\left(h_L\right)_{\mathrm{BD}}=f_{BD}\frac{L_{BD}}{D}\frac{V_D^2}{2g}+\Sigma K_L\frac{V_D^2}{2g}=f_{BD}\frac{L_{BD}}{D}\frac{V_D^2}{2g}+K_{L,\text{Tee with pipe run}}\frac{V_D^2}{2g}+K_{L,90^\circ\text{ elbow}}\frac{V_D^2}{2g}$$

$$\therefore\left(h_L\right)_{BD}=f_{BD}\times\frac{60}{0.03}\times\frac{V_D^2}{2\times 9.81}+0.4\times\frac{V_D^2}{2\times 9.81}+0.9\times\frac{V_D^2}{2\times 9.81}$$

$$\therefore\left(h_L\right)_{BD}=101.94 f_{BD}V_D^2+0.066V_D^2$$

The pressure p_{in} at inlet A in pipe section AC can be obtained with the Bernoulli equation,

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_{\text{pump}}=\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}+h_{turb}+\left(h_L\right)_{\mathrm{AB}}+\left(h_L\right)_{\mathrm{BC}}$$

$$\therefore\frac{p_{in}}{9810}+\frac{14.15^2}{2\times 9.81}+z+0=\frac{0}{9810}+\frac{V_C^2}{2\times 9.81}+z+210.90+50.97 f_{BC}V_C^2+0.092V_C^2$$

$$\therefore 2\times 10^{-3}p_{in}=\left(1000.03 f_{BC}+2.81\right)V_C^2+3937.64\ \ \text{(I)}$$

Similarly, we compute the pressure p_{in} at inlet A in pipe section AD,

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}+h_{\text{pump}}=\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}+h_{turb}+\left(h_L\right)_{\mathrm{AB}}+\left(h_L\right)_{\mathrm{BD}}$$

$$\therefore\frac{p_{in}}{9810}+\frac{14.15^2}{2\times 9.81}+z+0=\frac{0}{9810}+\frac{V_D^2}{2\times 9.81}+z+210.90+101.94 f_{BD}V_D^2+0.066V_D^2$$

$$\therefore 2\times 10^{-3}p_{in}=\left(2000.06 f_{BD}+2.29\right)V_D^2+3937.64\ \ \text{(II)}$$

From the continuity equation, the flow rates are related as

$$Q_A=Q_C+Q_D=V_C A+V_D A$$

$$\therefore V_C\frac{\pi}{4}\times 0.03^2+V_D\times\frac{\pi}{4}\times 0.03^2=0.01$$

$$\therefore 0.00071V_C+0.00071V_D=0.01$$

$$\therefore V_C+V_D=14.08\ \ \text{(III)}$$

Heretofore, we have 3 equations and 5 unknowns – namely, p_{in}, f_{BC}, f_{BD}, V_C, and V_D. We can introduce expressions for the Reynolds number in pipe segments BC and BD,

$$\text{Re}_{BC} = \frac{V_C D}{\nu} = \frac{V_C \times 0.03}{1\times 10^{-6}} = 30,000V_C \quad \text{(IV)}$$

$$\text{Re}_{BD} = \frac{V_D D}{\nu} = \frac{V_D \times 0.03}{1\times 10^{-6}} = 30,000V_D \quad \text{(V)}$$

We now have 5 equations and 7 unknowns. As in previous problems, we take advantage of the Colebrook equation, applying it to pipe segments BC and BD,

$$\frac{1}{\sqrt{f_{BC}}} = -2\log\left(\frac{0.15/30}{3.7} + \frac{2.51}{\text{Re}_{BC}\sqrt{f_{BC}}}\right) \quad \text{(VI)}$$

$$\frac{1}{\sqrt{f_{BD}}} = -2\log\left(\frac{0.15/30}{3.7} + \frac{2.51}{\text{Re}_{BD}\sqrt{f_{BD}}}\right) \quad \text{(VII)}$$

A favorable condition is reached as the number of unknowns and equations is now the same. Solving the equations simultaneously, we apply the code

$$\text{FindRoot}\left[\left\{2*10^{-3}p == (1000.03\text{fBC} + 2.81)\text{VC}^2 + 3937.64, 2*10^{-3}p = = (2000.06\text{fBD} + 2.29)\text{VD}^2 + 3937.64, \text{VC} + \text{VD} == 14.08, \text{REBC} == 30000\text{VC}, \text{REBD} = = 30000\text{VD}, \frac{1}{\sqrt{\text{fBC}}} == -2\text{Log}10\left[\frac{0.15/30}{3.7} + \frac{2.51}{\text{REBC}\sqrt{\text{fBC}}}\right], \frac{1}{\sqrt{\text{fBD}}} = = -2\text{Log}10\left[\frac{0.15/30}{3.7} + \frac{2.51}{\text{REBD}\sqrt{\text{fBD}}}\right]\right\}, \{\{p, 10^5\}, \{\text{fBC}, 0.01\}, \{\text{fBD}, 0.01\}, \{\text{REBC}, 10000\}, \{\text{REBD}, 10000\}, \{\text{VC}, 1\}, \{\text{VD}, 1\}\}\right]$$

This yields p_{in} = 309 kPa, f_{BC} = 0.0308, f_{BD} = 0.0309, Re_{BC} = 245,000, Re_{BD} = 177,000, V_C = 8.17 m/s, V_D = 5.91 m/s. We finally have enough data to compute the flow rate in pipe segments BC and BD. Q_{BC} is such that

$$Q_{BC} = V_C A = 8.17 \times \frac{\pi \times 0.03^2}{4} = \boxed{0.0058 \text{ m}^3/\text{s}}$$

while Q_{BD} easily follows from the continuity equation,

$$Q_{BC} + Q_{BD} = 0.01 \rightarrow Q_{BD} = 0.01 - 0.0058 = \boxed{0.0042 \text{ m}^3/\text{s}}$$

★ The correct answer is **C**.

Answer Summary

Problem 1	B
Problem 2	C
Problem 3	C
Problem 4	C
Problem 5	D
Problem 6	D
Problem 7	B
Problem 8	D
Problem 9	A
Problem 10	C
Problem 11	C
Problem 12	A
Problem 13	A
Problem 14	C

References

- ÇENGEL, Y. and CIMBALA, J. (2014). *Fluid Mechanics: Fundamentals and Applications*. 3rd edition. New York: McGraw-Hill.
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