Montogue

Quiz SM201

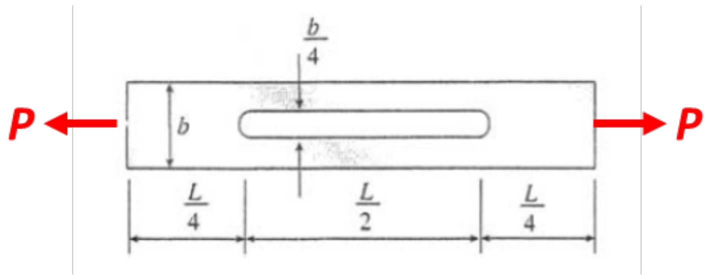
AXIAL LOADS

Lucas Montogue

PROBLEMS

Problem 1A (Gere & Goodno, 2009, w/ permission)

A rectangular bar of length L has a slot in the middle half of its length, as shown. The bar has width b, thickness t, and modulus of elasticity E. Obtain a formula for the elongation δ of the bar due to the axial loads P.



A) $\delta = (4PL)/(5Ebt)$

B) $\delta = (7PL)/(6Ebt)$

C) $\delta = (3PL)/(2Ebt)$

D) $\delta = (9PL)/(4Ebt)$

Problem 1B

Calculate the elongation of the bar if the material is high-strength steel (E = 200 GPa), the axial stress in the middle region is 160 MPa, the length is 750 mm, and the modulus of elasticity is 210 GPa.

A) δ = 0.500 mm

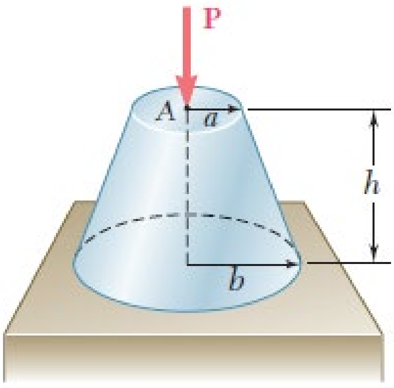
B) δ = 0.750 mm

C) δ = 1.000 mm

D) δ = 1.250 mm

Problem 2 (Beer et al., 2012, w/ permission)

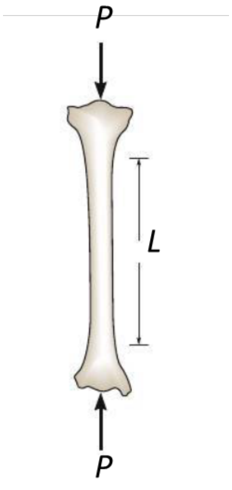
The vertical load P is applied at the center A of the upper section of a homogeneous frustum of a circular cone of height h, minimum radius a, and maximum radius b. Denoting by E the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point A.



A) $\delta = \dfrac{Ph}{2\pi Eab}$
B) $\delta = \dfrac{Ph}{\pi Eab}$
C) $\delta = \dfrac{Ph}{\pi E(b^2-a^2)}$
D) $\delta = \dfrac{Ph}{\pi E(b-a)^2}$

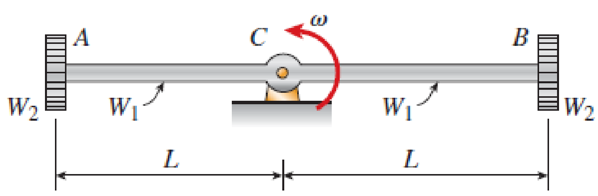
Problem **3** (Hibbeler, 2014, w/ permission)

Bone material has a stress-strain diagram that can be defined by the relation $\sigma = E[\varepsilon/(1 + kE\varepsilon)]$, where *k* and *E* are constants. Determine the compression within the length *L* of the bone, where it is assumed that the cross-sectional area *A* of the bone is constant.



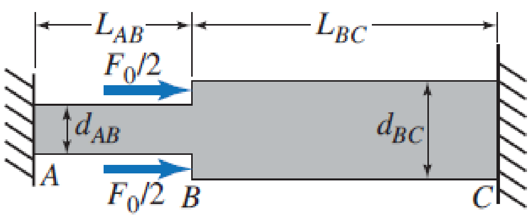
Problem **4** (Gere & Goodno, 2009, w/ permission)

A bar ABC revolves in a horizontal plane about a vertical axis at the midpoint *C* (see figure). The bar, which has length 2*L* and cross-sectional area *A*, revolves at a constant angular speed ω. Each half of the bar (*AC* and *BC*) has a weight W_1 and supports a weight W_2 at its end. Derive a formula for the elongation of one-half of the bar (that is, the elongation of either *AC* or *BC*). Denote the modulus of elasticity of the material that constitutes the bar and the acceleration of gravity as *E* and g, respectively.



Problem **5A** (Steif, 2012, w/ permission)

The bar of uniform elastic modulus *E* has a step in diameter as shown, and is subjected to an axial force of F_o acting at the shoulder. Determine the displacement of the cross-section located midway between *B* and *C*. Use the following dimensions and properties: $F_o = 10$ kN, $E = 200$ GPa, $L_{BC} = 300$ mm, $D_{AB} = 15$ mm, and $D_{BC} = 25$ mm.



A) $|\delta_{BC}| = 4.8\ \mu\text{m}$

B) $|\delta_{BC}| = 9.9\ \mu\text{m}$

C) $|\delta_{BC}| = 15.7\ \mu\text{m}$

D) $|\delta_{BC}| = 22.6\ \mu\text{m}$

Problem **5B**

Determine the lengths of the individual segments L_{AB} and L_{BC} of the bar if the total length $L_{AB} + L_{BC} = 18$ in. and the stress in the right section is to have half the stress in the left section. Use as diameters $d_{AB} = 0.75$ in. and $d_{BC} = 1$ in.

A) $L_{AB} = 2$ in. and $L_{BC} = 16$ in.

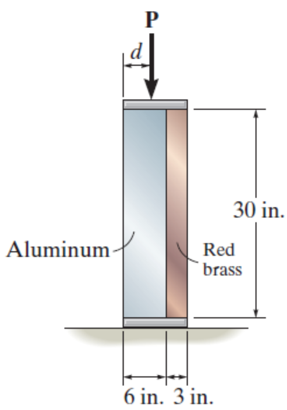
B) $L_{AB} = 3$ in. and $L_{BC} = 15$ in.

C) $L_{AB} = 6$ in. and $L_{BC} = 12$ in.

D) $L_{AB} = 12$ in. and $L_{BC} = 6$ in.

Problem **6** (Hibbeler, 2014, w/ permission)

The assembly consists of an aluminum member and a red brass member that rest on rigid plates. Determine the distance *d* where the vertical load *P* should be placed on the plates so that the plates remain horizontal when the materials deform. Each member has a width of 8 in.



A) $d = 2.7$ in.

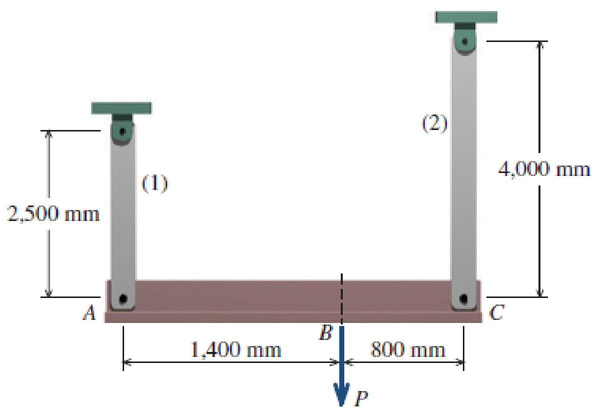
B) $d = 4.9$ in.

C) $d = 6.8$ in.

D) $d = 8.3$ in.

Problem **7** (Philpot, 2013, w/ permission)

In the following figure, aluminum ($E = 70$ GPa) links (1) and (2) support rigid beam ABC. Link (1) has a cross-sectional area of 300 mm² and link (2) has a cross-sectional area of 450 mm². For an applied load of $P = 55$ kN, determine the rigid beam deflection at point *B*.



A) $v_B = 1.14$ mm

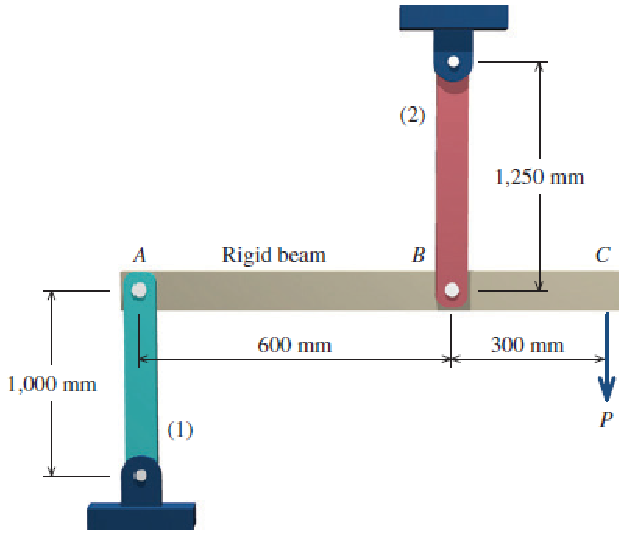
B) $v_B = 2.48$ mm

C) $v_B = 3.69$ mm

D) $v_B = 4.55$ mm

Problem 8 (Philpot, 2013, w/ permission)

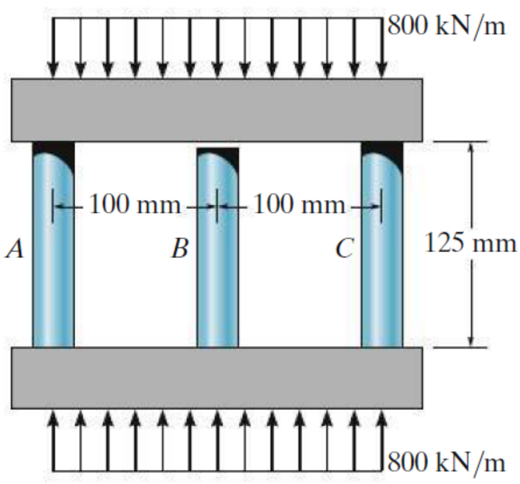
The rigid beam shown is supported by links (1) and (2), which are made from a polymer material ($E = 16$ GPa). Link (1) has a cross-sectional area of 400 mm², and link (2) has a cross-sectional area of 800 mm². Determine the maximum load P that may be applied if the deflection of the beam is not to exceed 20 mm at point C.



A) $P = 18.4$ kN
B) $P = 36.5$ kN
C) $P = 55.2$ kN
D) $P = 77.3$ kN

Problem 9 (Hibbeler, 2014, w/ permission)

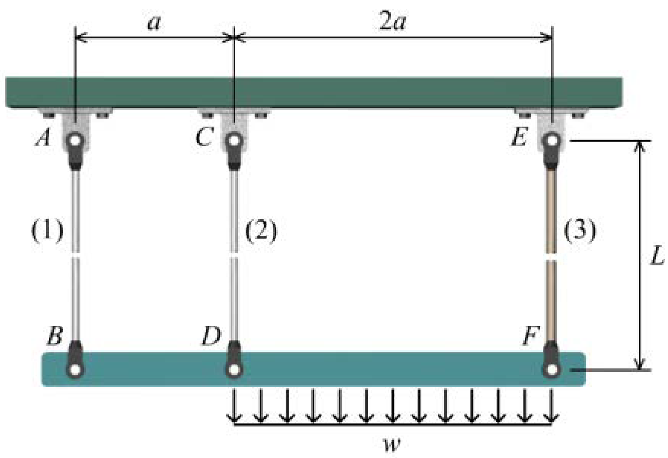
The center post B of the assembly has an original length of 124.7 mm, while posts A and C have a length of 125 mm. If the caps on the top and bottom can be considered rigid, determine the average normal stress in each post. What is the difference $\Delta\sigma$ between the normal stress at posts A or C and that of post B? The posts are made of aluminum and have a cross-sectional area of 400 mm². Use $E_{Al} = 70$ GPa.



A) $\Delta\sigma = [0\,;25)$ MPa
B) $\Delta\sigma = [25\,;50)$ MPa
C) $\Delta\sigma = [50\,;75)$ MPa
D) $\Delta\sigma = [75\,;+\infty)$ MPa

Problem **10** (Philpot, 2013, w/ permission)

A uniformly distributed load w is supported by a structure consisting of rigid bar BDF and three rods, as shown in the next figure. Rods (1) and (2) are 15-mm-diameter stainless steel rods that have an elastic modulus $E = 193$ GPa and a yield strength $\sigma_Y = 330$ MPa. Use $a = 1.5$ m and $L = 3$ m. If a minimum factor of safety of 2.5 is specified for the normal stress in each rod, calculate the maximum distributed load magnitude w that may be supported.



A) $w = 20.4$ kN/m

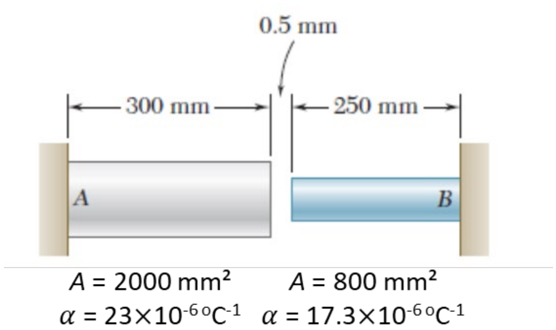
B) $w = 30.6$ kN/m

C) $w = 40.5$ kN/m

D) $w = 50.3$ kN/m

Problem **11** (Beer et al., 2012, w/ permission)

At room temperature (20°C), a 0.5-mm gap exists between the ends of the rods shown. At a later time, when the temperature has reached 140°C, determine the elongation of the aluminum rod. Use $E_{st} = 200$ GPa and $E_{Al} = 70$ GPa.



A) $\delta_A = 0.102$ mm

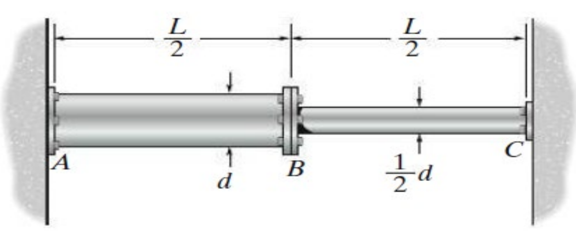
B) $\delta_A = 0.215$ mm

C) $\delta_A = 0.371$ mm

D) $\delta_A = 0.494$ mm

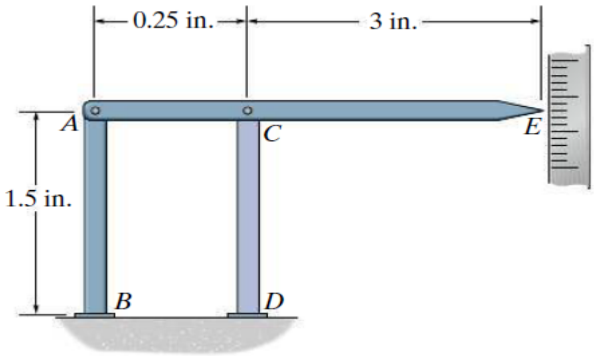
Problem **12** (Hibbeler, 2014, w/ permission)

If the assembly fits snugly between the two supports A and C when the temperature is at T_1, determine the normal stress developed in segment BC when the temperature rises to T_2 (such that $T_2 > T_1$). Both segments are made of the same material having a modulus of elasticity E and coefficient of thermal expansion α. The flexible supports at A and C each have a stiffness k.



Problem **13** (Hibbeler, 2014, w/ permission)

The device is used to measure a change in temperature. Bars AB and CD are made of steel and aluminum, respectively. When the temperature is at 75°F, ACE is in the horizontal position. Determine the vertical displacement of the pointer at *E* when the temperature rises to 150°F. Take the coefficients of thermal expansion $\alpha_{st} = 6.6 \times 10^{-6}$ °F⁻¹ and $\alpha_{Al} = 12.8 \times 10^{-6}$ °F⁻¹.



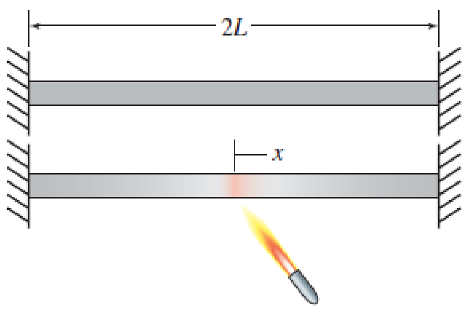
A) δ_E = 0.0039 in.
B) δ_E = 0.0067 in.
C) δ_E = 0.0098 in.
D) δ_E = 0.0126 in.

Problem **14** (Hibbeler, 2014, w/ permission)

A torch causes concentrated heating of the central portion of a bar that is initially unstressed at uniform temperature. The bar has uniform properties *E*, *A*, α, and length *2L*. Approximate the remaining structure to which the ends are attached as rigid and immovable. Say the heating causes the bar's temperature to increase by an amount

$$\Delta T = \Delta T_0 \exp\left(-\frac{|x|}{c}\right)$$

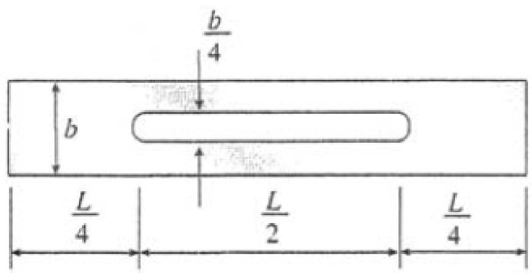
where *x* is the distance from the center and parameter *c* captures the length of the heated zone. Determine the compressive stress in the bar as a function of the parameters.



SOLUTIONS

P.1 Solution

Part A: The bar can be divided into three segments, namely, two prismatic segments of length *L*/2 and a slotted segment of length *L*/2, as shown.



Since the system is one of simple axial loading, the elongation of the bar is simply the sum of the elongations of each segment; mathematically,

$$\delta = \sum\left(\frac{P_i L_i}{E_i A_i}\right)$$

where P is the force applied on the i-th segment, E_i is the modulus of elasticity of the material, and A_i is its cross-sectional area. Summing the elongations associated with each segment, we obtain

$$\delta = \frac{P\times(L/4)}{E\times(bt)} + \frac{P\times(L/2)}{E\times\left(\frac{3}{4}bt\right)} + \frac{P\times(L/4)}{E\times(bt)}$$

$$\therefore \delta = \frac{PL}{4Ebt} + \frac{2PL}{3Ebt} + \frac{PL}{4Ebt}$$

$$\therefore \delta = \frac{PL}{Ebt}\left(\frac{1}{4} + \frac{2}{3} + \frac{1}{4}\right)$$

$$\therefore \boxed{\delta = \frac{7PL}{6Ebt}}$$

The correct answer is **B**.

Part B: The stress in the middle region can be easily determined from the *P*/*A* ratio,

$$\sigma = \frac{P}{A} = \frac{P}{\left(\frac{3}{4}bt\right)} = \frac{4P}{3bt} \rightarrow \frac{P}{bt} = \frac{3\sigma}{4}$$

Manipulating the expression for elongation obtained above, we have

$$\delta = \frac{7PL}{6Ebt} = \frac{7L}{6E}\times\left(\frac{P}{bt}\right) = \frac{7L}{6E}\times\left(\frac{3\sigma}{4}\right)$$

$$\therefore \delta = \frac{7\sigma L}{8E}$$

Substituting the pertaining variables with the numerical data we were given, it follows that

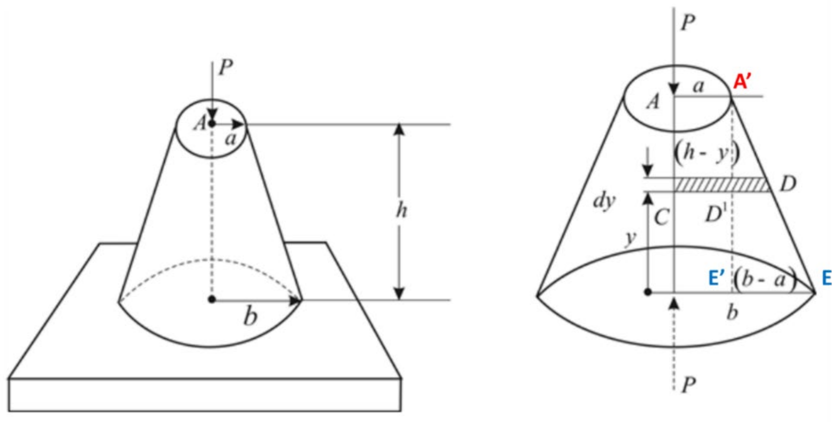
$$\delta = \frac{7\sigma L}{8E} = \frac{7\times 160\times 750}{8\times 210{,}000} = \boxed{0.500\text{ mm}}$$

The bar will deform by one half of a millimeter.

The correct answer is **A**.

P.2 ➡ *Solution*

Consider the following illustration of the cone frustum.



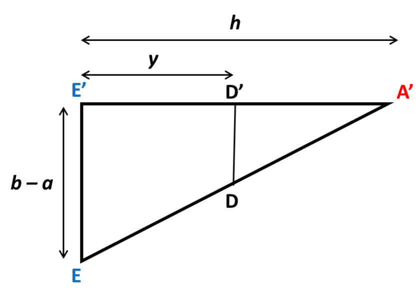
The deflection of point A, u_A, is the overall deformation of the member, δ. Such a deformation is determined with the usual formula $\delta = PL/AE$; however, the section of the body varies vertically from the upper cross-section to the lower cross-section. In this case, the deformation is given by

$$\delta = \frac{P}{E}\int_0^h \frac{dy}{A(y)} \text{ (I)}$$

We must determine the variation of cross-sectional area A with the vertical position. The area A_y in an arbitrary section of the body, as shown above, is

$$A_y = \pi \times \overline{CD}^2 = \pi \times \left(\overline{CD'} + \overline{D'D}\right)^2 \text{ (II)}$$

Clearly, $\overline{CD'} = a$. Determining $\overline{D'D}$ is a little more involved. Refer to triangle $A'E'E$ below.



From similar triangles,

$$\frac{\overline{E'E}}{\overline{D'D}} = \frac{\overline{A'E'}}{\overline{A'D'}}$$

However, $\overline{E'E} = b - a$, $\overline{A'E'} = h$, and $\overline{A'D'} = h - y$. Substituting, we get

$$\frac{\overline{E'E}}{\overline{D'D}} = \frac{\overline{A'E'}}{\overline{A'D'}} \rightarrow \frac{b-a}{\overline{D'D}} = \frac{h}{h-y}$$

$$\therefore \overline{D'D} = \frac{1}{h}(b-a)(h-y)$$

Returning to the equation for A_y, (II), we have

$$A_y = A(y) = \pi\left[a + \frac{(b-a)}{h}(h-y)\right]^2 = \pi\left[b + \frac{(a-b)}{h}y\right]^2$$

This equation provides the area of the frustum's cross-section as a function of y. We now have all the information we need to determine the deformation from equation (I). Thus,

$$\delta = \frac{P}{E}\int_0^h \frac{1}{A(y)}dy = \frac{P}{E}\int_0^h \frac{1}{\pi\left[b + \frac{(a-b)}{h}y\right]^2}dy$$

$$\therefore \delta = \frac{Ph}{\pi E(a-b)}\left\{-\frac{1}{\left[b+\frac{(a-b)}{h}y\right]}\right\}\Bigg|_{y=0}^{y=h} = \frac{Ph}{\pi E(b-a)}\left(-\frac{1}{a}+\frac{1}{b}\right)$$

$$\therefore \delta = \frac{Ph}{\pi E\left(\cancel{a-b}\right)}\left(\frac{\cancel{a-b}}{ab}\right)$$

$$\therefore \boxed{\delta = \frac{Ph}{\pi Eab}}$$

The correct answer is **B**.

P.3 ➡ *Solution*

The normal stress and the strain are respectively defined as

$$\sigma = \frac{P}{A}\;;\;\varepsilon = \frac{\delta x}{dx}$$

We can adjust the relationship we were given for axial stress,

$$\sigma = E\left(\frac{\varepsilon}{1+kE\varepsilon}\right) \rightarrow \frac{P}{A} = \frac{E\frac{\delta x}{dx}}{1+kE\frac{\delta x}{dx}}$$

$$\therefore \frac{P}{A} + \frac{PkE}{A}\left(\frac{\delta x}{dx}\right) = E\left(\frac{\delta x}{dx}\right)$$

$$\therefore \frac{P}{A} = \left(E - \frac{PkE}{A}\right)\left(\frac{\delta x}{dx}\right)$$

$$\therefore \int_0^L \frac{P}{AE\left(1-\frac{Pk}{A}\right)}dx = \int_0^\delta \delta x$$

Evaluating the integrals on both sides, we conclude that

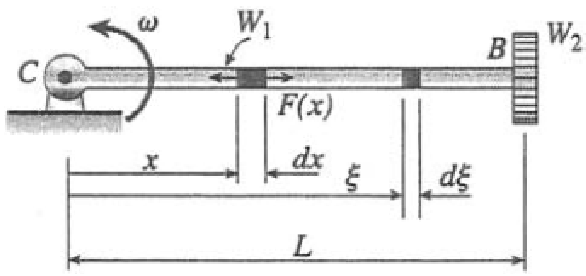
$$\int_0^L \frac{P}{AE\left(1-\frac{Pk}{A}\right)}dx = \int_0^\delta \delta x \rightarrow \delta = \frac{PL}{AE\left(1-\frac{Pk}{A}\right)}$$

$$\therefore \boxed{\delta = \frac{PL}{E(A-Pk)}}$$

Note that the deformation, in this case, is not given by the simple force-displacement relationship $\delta = PL/EA$, because the cross-section area A, albeit constant, is replaced by the factor $(A - Pk)$. Assuming that k is a positive constant, we surmise that the deformation would be smaller than the compression expected for a member with a linear stress-strain relation $\sigma = E\varepsilon$.

P.4 ➡ *Solution*

Consider segment CB illustrated below.



F(x) is the axial force in the bar at a distance *x* from the center of rotation, point C. To find such a force, we must obtain the inertia force of the part of the bar from distance *x* to distance *L* and the inertia force of the weight W_2. Noting that the inertia force varies with the distance from point *C*, we take an element of length $d\xi$ at a distance ξ from this point (see above). The mass of element $d\xi$ is

$$\text{Mass of element } d\xi = \frac{d\xi}{L} \times \left(\frac{W_1}{g}\right)$$

The element in question has acceleration $\xi\omega^2$, and, consequently, the centrifugal force produced by the element is

$$dF = \text{mass} \times \text{acceleration} = \frac{d\xi W_1}{gL} \times \xi\omega^2$$

$$\therefore dF = \frac{W_1\omega^2}{gL}\xi d\xi$$

The centrifugal force produced by weight W_2, in turn, is such that

$$F_{W_2} = \left(\frac{W_2}{g}\right) \times L\omega^2 = \frac{W_2 L\omega^2}{g}$$

Accordingly, axial force *F(x)* is the sum of the centrifugal force due to the bar and that owing to weight W_2; that is,

$$F(x) = \int_{\xi=x}^{\xi=L} \frac{W_1\omega^2}{gL}\xi d\xi + \frac{W_2 L\omega^2}{g}$$

$$\therefore F(x) = \frac{W_1\omega^2}{gL}\frac{\xi^2}{2}\bigg|_{\xi=x}^{\xi=L} + \frac{W_2 L\omega^2}{g}$$

$$\therefore F(x) = \frac{W_1\omega^2}{2gL}\left(L^2 - x^2\right) + \frac{W_2 L\omega^2}{g}$$

Having established force *F(x)*, we can determine the elongation of the bar with the force-displacement relation $\delta = F(x)L/AE$, as follows,

$$\delta = \frac{1}{EA}\left[\int_0^L \frac{W_1\omega^2}{2gL}\left(L^2 - x^2\right)dx + \int_0^L \frac{W_2 L\omega^2}{g}dx\right]$$

$$\therefore \delta = \frac{1}{EA}\frac{W_1\omega^2}{2gL}\underbrace{\left[\int_0^L (L^2 - x^2)dx\right]}_{=2L^3/3} + \frac{1}{EA}\frac{W_2 L\omega^2}{g}\int_0^L dx$$

$$\therefore \delta = \frac{1}{EA}\frac{W_1\omega^2}{2gL} \times \frac{2L^3}{3} + \frac{1}{EA}\frac{W_2 L^2\omega^2}{g}$$

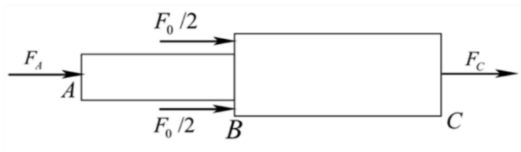
$$\therefore \delta = \frac{W_1\omega^2 L^2}{3gEA} + \frac{W_2\omega^2 L^2}{gEA}$$

$$\therefore \boxed{\delta = \frac{\omega^2 L^2}{3gEA}\left(W_1 + 3W_2\right)}$$

Clearly, the deformation of the member is substantially proportional to the angular velocity of the structure (doubling it would increase the deformation four-fold), and more dependent on weight W_2 than on the weight of the bar segment.

P.5 ➡ *Solution*

Part A: The free-body diagram of the bar is shown below.



Applying an equilibrium of forces in the horizontal direction gives

$$F_A - \frac{F_0}{2} - \frac{F_0}{2} - F_C = 0$$

$$\therefore F_A - F_0 - F_C = 0$$

$$\therefore F_A - F_C = 10 \quad \text{(I)}$$

since $F_0 = 10$ kN. In view of the fact that this the only feasible equation of equilibrium from elementary statics, the system is statically indeterminate. As a compatibility equation, we can state that the deformation of the bar, δ_{AC}, is such that

$$\delta_{AC} = 0 \rightarrow \delta_{AB} + \delta_{BC} = 0$$

Here, δ_{AB} is the deformation of segment AB and δ_{BC} is the deformation of segment BC. For axial loading, such deformations can be determined with the usual relationship $\delta = FL/AE$; that is,

$$\delta_{AB} + \delta_{BC} = 0 \rightarrow \frac{F_A L_{AB}}{A_{AB}E} + \frac{F_C L_{BC}}{A_{BC}E} = 0$$

$$\therefore \frac{F_A L_{AB}}{A_{AB}} + \frac{F_C L_{BC}}{A_{BC}} = 0$$

Factor E cancels out because both segments of the bar are made of the same material. Substituting other pertaining variables, we get

$$\frac{F_A \times 200}{\left(\frac{\pi \times 15^2}{4}\right)} + \frac{F_C \times 300}{\left(\frac{\pi \times 25^2}{4}\right)} = 0$$

$$\therefore 1.13F_A + 0.61F_C = 0 \quad \text{(II)}$$

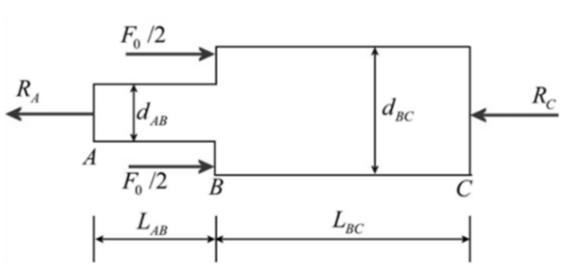
Equations (I) and (II) can be solved simultaneously for F_A and F_C, yielding F_A = 3.51 kN and F_C = −6.49 kN. (The negative sign means that F_C is a compressive force.) We are now able to compute the displacement of the cross-section located midway between B and C,

$$|\delta_{BC}| = \frac{|F_C| \times (L_{AB}/2)}{A_{BC}E} = \frac{6490 \times 300/2}{\left(\frac{\pi \times 25^2}{4}\right) \times 200{,}000} \times 1000 = \boxed{9.9\ \mu\text{m}}$$

The deformation of the midway section will be close to 10 micrometers.

The correct answer is **B**.

Part B: The free-body diagram for the bar is shown below.



Since the bar is fixed at both ends, the total deformation along the shaft equals zero; mathematically,

$$\delta_{AB} + \delta_{BC} = 0$$

where δ_{AB} and δ_{BC} are the deformations in sections AB and BC, respectively. For a material in the elastic regime, Hooke's law can be written as $\sigma = E\varepsilon$; since the strain ε is related to deformation by the relationship $\varepsilon = \delta/L$, it follows that $\sigma = E\delta/L$, or

$$\sigma = \frac{E\delta}{L} \rightarrow \delta = \frac{\sigma L}{E}$$

Backsubstituting in the sum of deformations, we obtain

$$\delta_{AB} + \delta_{BC} = 0 \rightarrow \frac{\sigma_{AB} L_{AB}}{E} + \frac{(-\sigma_{BC}) L_{BC}}{E} = 0$$

$$\therefore \frac{\sigma_{AB} L_{AB}}{\cancel{E}} - \frac{\sigma_{BC} L_{BC}}{\cancel{E}} = 0$$

$$\therefore \sigma_{AB} L_{AB} = \sigma_{BC} L_{BC}$$

$$\therefore \frac{\sigma_{AB}}{\sigma_{BC}} = \frac{L_{BC}}{L_{AB}}$$

A negative sign accompanies σ_{BC} because we are assuming that the stress in segment BC is compressive. Young's modulus *E* cancels out because both segments are made of the same material. Since the stress in the right section must equal half the stress in the left section, we have $\sigma_{AB}/\sigma_{BC} = 2$ and, consequently,

$$\frac{\sigma_{AB}}{\sigma_{BC}} = \frac{L_{BC}}{L_{AB}} = 2$$

$$\therefore L_{BC} = 2L_{AB}$$

We know that the sum of segments AB and BC must equal 18 in., or, in mathematical terms, $L_{BC} + L_{AB} = 18$. Accordingly,

$$L_{AB} + L_{BC} = 18$$

$$\therefore L_{AB} + (2L_{AB}) = 18$$

$$\therefore 3L_{AB} = 18$$

$$\therefore \boxed{L_{AB} = 6 \text{ in.}}$$

In turn, $L_{BC} = 12$ in. The length of the segment with smaller diameter is 6 inches, while that of the segment with larger diameter is 12 inches.

The correct answer is **C**.

P.6 ➡ *Solution*

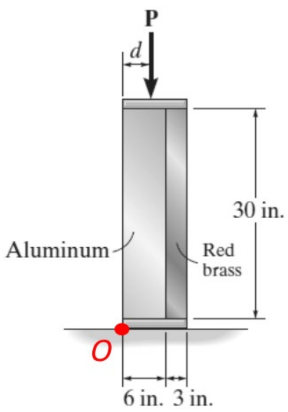
Summing forces in the vertical direction, we obtain

$$\Sigma F_y = 0 \rightarrow -P + F_{\text{Al}} + F_{\text{RB}} = 0 \quad \text{(I)}$$

where F_{Al} is the force exerted on the aluminum plate, and F_{RB} is the force on the red brass plate. Taking moments about point *O* (see below), we have

$$\Sigma M_O = 0 \rightarrow 3F_{\text{Al}} + 7.5F_{\text{RB}} - P \times d = 0$$

$$\therefore 3F_{\text{Al}} + 7.5F_{\text{RB}} = P \times d \quad \text{(II)}$$



These are the only equations we can infer from statics. To formulate a compatibility equation, note that, if the plates are to remain horizontal when the bars are acted upon by load *P*, their deformations should be the same. Mathematically,

$$\delta_{\mathrm{RB}} - \delta_{\mathrm{Al}} = 0 \rightarrow \delta_{RB} = \delta_{\mathrm{Al}}$$

Applying the formula $\delta = FL/AE$, we see that

$$\delta_{\mathrm{RB}} = \delta_{\mathrm{Al}} \rightarrow \frac{F_{\mathrm{RB}}\cancel{L}}{A_{\mathrm{RB}}E_{\mathrm{RB}}} = \frac{F_{\mathrm{Al}}\cancel{L}}{A_{\mathrm{Al}}E_{\mathrm{Al}}}$$

$$\therefore \frac{F_{\mathrm{RB}}}{A_{\mathrm{RB}}E_{\mathrm{RB}}} = \frac{F_{\mathrm{Al}}}{A_{\mathrm{Al}}E_{\mathrm{Al}}}$$

$$\therefore F_{\mathrm{RB}} = A_{\mathrm{RB}}E_{\mathrm{RB}} \times \frac{F_{\mathrm{Al}}}{A_{\mathrm{Al}}E_{\mathrm{Al}}}$$

$$\therefore F_{\mathrm{RB}} = \left(\frac{A_{\mathrm{RB}}E_{\mathrm{RB}}}{A_{\mathrm{Al}}E_{\mathrm{Al}}}\right)F_{\mathrm{Al}}$$

$$\therefore F_{\mathrm{RB}} = \left(\frac{l_{\mathrm{RB}}wE_{\mathrm{RB}}}{l_{\mathrm{Al}}wE_{\mathrm{Al}}}\right)F_{\mathrm{Al}}$$

$$\therefore F_{\mathrm{RB}} = \left(\frac{l_{\mathrm{RB}}E_{\mathrm{RB}}}{l_{\mathrm{Al}}E_{\mathrm{Al}}}\right)F_{\mathrm{Al}}$$

where *l* and *w* are dimensions of the plates' transverse cross-section. Substituting the pertaining variables, we get

$$F_{\mathrm{RB}} = \frac{3\times\left(14.6\times10^{3}\right)}{6\times\left(10\times10^{3}\right)}F_{\mathrm{Al}} \rightarrow F_{\mathrm{RB}} = 0.73F_{\mathrm{Al}} \quad \text{(III)}$$

Substituting F_{RB} in equation (I) gives

$$-P + F_{\mathrm{Al}} + F_{\mathrm{RB}} = 0 \rightarrow -P + F_{\mathrm{Al}} + 0.73F_{\mathrm{Al}} = 0$$

$$\therefore P = 1.73F_{\mathrm{Al}}$$

Finally, backsubstituting in equation (II), we obtain

$$3F_{\mathrm{Al}} + 7.5F_{\mathrm{RB}} = P\times d$$

$$\therefore 3F_{\mathrm{Al}} + 7.5\times\left(0.73F_{\mathrm{Al}}\right) = \left(1.73F_{\mathrm{Al}}\right)\times d$$

Dividing by F_{Al} on both sides, we ultimately obtain

$$3 + 7.5\times0.73 = 1.73d$$

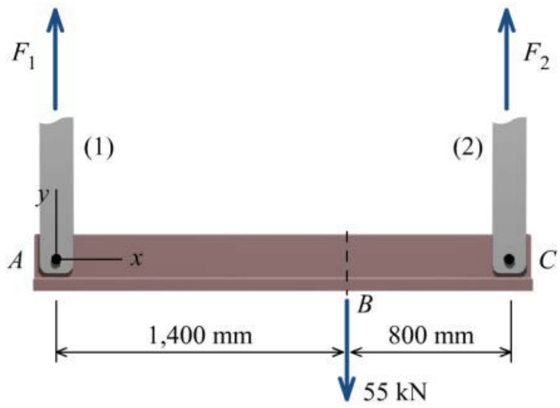
$$\therefore \boxed{d = 4.9\text{ in.}}$$

That is to say, the vertical load *P* should be placed nearly 5 inches away from the left extremity of the structure, as seen from the perspective given in the previous figure.

The correct answer is **B**.

P.7 ➡ *Solution*

Consider the free-body diagram of the rigid beam.



The magnitude of force F_1 can be determined by taking moments about point C,

$$-(1400+800)\times F_1 + 800\times 55 = 0$$

$$\therefore F_1 = 20 \text{ kN}$$

By inspection, $F_2 = 55 - 20 = 35$ kN. The deformations in links (1) and (2) easily follow,

$$\delta_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{20,000\times 2500}{300\times 70,000} = 2.381 \text{ mm}$$

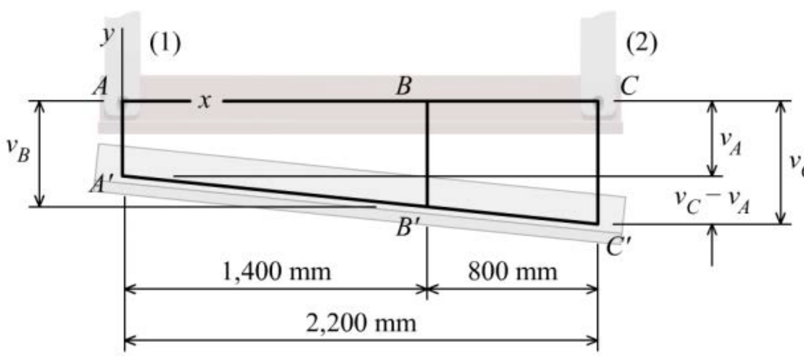
$$\delta_2 = \frac{F_2 L_2}{A_2 E_2} = \frac{35,000\times 4000}{450\times 70,000} = 4.444 \text{ mm}$$

Since the connections at A and C are perfect, the rigid beam deflections at these joints are equal to the deformations of links (1) and (2), respectively,

$$v_A = \delta_1 = 2.381 \text{ mm}$$

$$v_C = \delta_2 = 4.444 \text{ mm}$$

Consider the following diagram of deformations.



The deflection at B can be determined from similar triangles,

$$\frac{v_C - v_A}{2200} = \frac{v_B - v_A}{1400}$$

Substituting $v_A = 2.381$ mm and $v_C = 4.444$ mm, we obtain

$$\frac{4.444 - 2.381}{2200} = \frac{v_B - 2.381}{1400}$$

$$\therefore \frac{1400}{2200}(4.444 - 2.381) = v_B - 2.381$$

$$\therefore 1.313 = v_B - 2.381$$

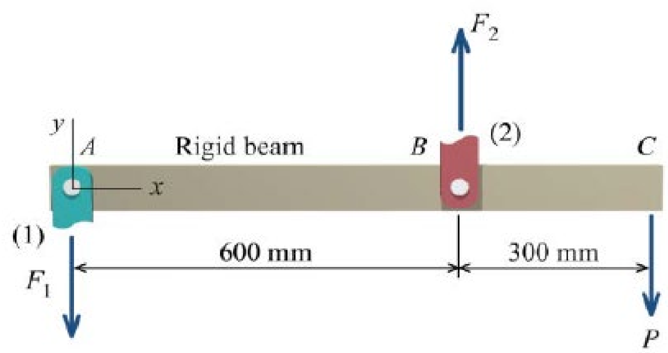
$$\therefore \boxed{v_B = 3.69 \text{ mm}}$$

Point B will deflect more than one third of a centimeter.

The correct answer is **C**.

P.8 ➡ *Solution*

First, we consider a free-body diagram of the rigid beam and assume that each link is under tension.



Applying the second condition of equilibrium to point *A*, we have

$$\Sigma M_A = 0 \rightarrow 600 \times F_2 - (600+300) \times P = 0$$

$$\therefore 600 \times F_2 = 900 \times P$$

$$\therefore F_2 = \frac{900}{600} \times P$$

$$\therefore F_2 = 1.5P$$

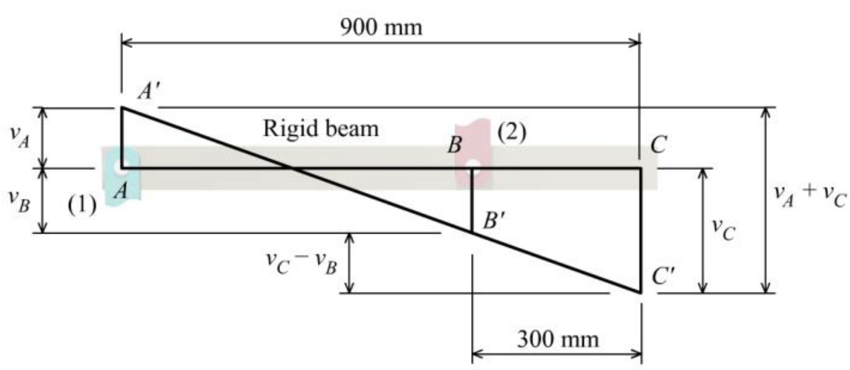
Then, applying the first condition of equilibrium to the rigid body, we have

$$\Sigma F_y = 0 \rightarrow \underbrace{F_2}_{=1.5P} - F_1 - P = 0$$

$$\therefore 1.5P - F_1 - P = 0$$

$$\therefore 0.5P - F_1 = 0 \rightarrow F_1 = 0.5P$$

Consider a deformation diagram of the rigid beam, as illustrated below.



Using similar triangles, one way to express the relation between v_A, v_B, and v_C is

$$\frac{v_A + v_C}{900} = \frac{v_C - v_B}{300} \quad \text{(I)}$$

The rigid beam deflection at *A* will equal the deformation that occurs in link (1), that is,

$$v_A = \delta_1$$

while the deflection at *B* equals the deformation associated with link (2),

$$v_B = \delta_2$$

Backsubstituting in equation (I), we get

$$\frac{\delta_1+v_C}{900}=\frac{v_C-\delta_2}{300}\rightarrow\delta_1+v_C=\frac{900}{300}\times(v_C-\delta_2)$$

$$\therefore\delta_1+v_C=3\times(v_C-\delta_2)$$

$$\therefore\delta_1+v_C=3v_C-3\delta_2$$

$$\therefore-2v_C=-\delta_1-3\delta_2$$

$$\therefore v_C=0.5\delta_1+1.5\delta_2\quad\text{(II)}$$

The relationship between internal force and deformation comes from the usual formula,

$$\delta_1=\frac{F_1L_1}{A_1E_1}\;;\;\delta_2=\frac{F_2L_2}{A_2E_2}$$

However, we know that $F_1=0.5P$ and $F_2=1.5P$; consequently,

$$\delta_1=\frac{0.5PL_1}{A_1E_1}\;;\;\delta_2=\frac{1.5PL_2}{A_2E_2}$$

Substituting in the equation for v_C, equation (II), we obtain

$$v_C=0.5\times\left(\frac{0.5PL_1}{A_1E_1}\right)+1.5\times\left(\frac{1.5PL_2}{A_2E_2}\right)$$

$$\therefore v_C=\frac{0.25PL_1}{A_1E_1}+\frac{2.25PL_2}{A_2E_2}=P\left(\frac{0.25L_1}{A_1E_1}+\frac{2.25L_2}{A_2E_2}\right)$$

The deflection of the beam shall not exceed 20 mm at point C, i.e., $v_C=20$ mm at most. Substituting this variable and other pertaining quantities in the equation above, we can determine the maximum load P,

$$v_C=P\left(\frac{0.25L_1}{A_1E_1}+\frac{2.25L_2}{A_2E_2}\right)\rightarrow P=\frac{v_C}{\left(\frac{0.25L_1}{A_1E_1}+\frac{2.25L_2}{A_2E_2}\right)}$$

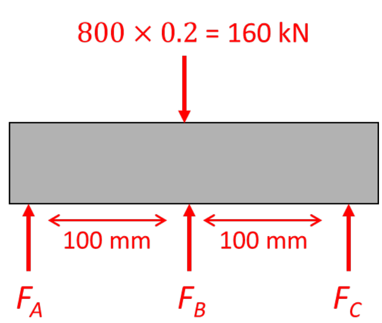
$$\therefore P=\frac{20}{\left(\frac{0.25\times1000}{400\times16,000}+\frac{2.25\times1250}{800\times16,000}\right)}=\boxed{77.3\text{ kN}}$$

The load P should be no greater than about 77 kilonewtons.

The correct answer is **D**.

P.9 ➡ *Solution*

Consider the following free-body diagram for the upper rigid bar.



Note that we have replaced the distributed load with a concentrated load of intensity 800×0.2 = 160 kN acting on the middle of the rigid bar. Summing moments about the middle of the rigid bar gives

$$\Sigma M_B=0\rightarrow-F_A\times100+F_C\times100=0$$

$$\therefore F_C=F_A$$

Then, applying the first condition of equilibrium in the vertical direction, we obtain

$$\Sigma F_y = 0 \rightarrow F_A + F_B + F_C - 160 = 0$$

$$\therefore F_A + F_B + F_C = 160$$

$$\therefore 2F_A + F_B = 160 \quad \text{(I)}$$

Post B has a length of 124.7 mm, while posts A and C have a length of 125 mm. It follows that the deformation of post A should equal the deformation of post B plus 0.3 mm. In mathematical terms,

$$\delta_A = \delta_B + 0.3$$

Applying the force-displacement relationship $P = FL/AE$, the equation above becomes

$$\frac{F_A L_A}{A_A E_{\text{Al}}} = \frac{F_B L_B}{A_B E_{\text{Al}}} + 0.3$$

Substituting the pertaining variables, we get

$$\frac{F_A \times 125}{400 \times 70,000} = \frac{F_B \times 124.7}{400 \times 70,000} + 0.3$$

$$\therefore 125F_A - 124.7F_B = 8400 \quad \text{(II)}$$

Equations (I) and (II) can be solved simultaneously for forces F_A and F_B, yielding $F_A = 75.7$ kN and $F_B = 8.55$ kN. Also, $F_C = F_A = 75.7$ kN. The normal stresses in posts *A* and *C* are both such that

$$\sigma_A = \sigma_C = \frac{75.7}{\left(400 \times 10^{-6}\right)} = 189.3 \text{ MPa}$$

while the normal stress in post *B* is

$$\sigma_B = \frac{8.55}{\left(400 \times 10^{-6}\right)} = 21.4 \text{ MPa}$$

The difference $\Delta\sigma$ between stresses in posts A and C and that of post B equals

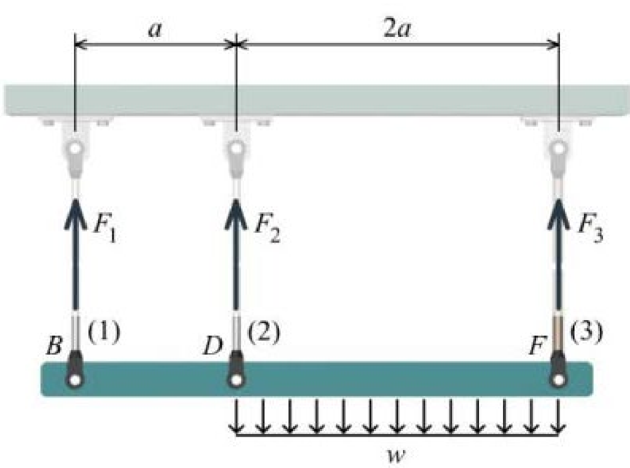
$$\Delta\sigma = \sigma_A - \sigma_B = 189.3 - 21.4 = \boxed{167.9 \text{ MPa}}$$

which is located inside the interval $[75\,;+\infty)$ MPá. It is noteworthy that, in this hypothetical situation, the existence of a small difference in length of one post relative to the others causes one of them to have a stress as much as 189.3/21.4 = 8.8 times greater than its counterparts.

The correct answer is **D**.

P.10 ➡ *Solution*

Consider a free-body diagram of the rigid bar.



Applying the first condition of equilibrium in the vertical direction, we have

$$\Sigma F_y = 0 \rightarrow F_1 + F_2 + F_3 - w \times 2a = 0$$

$$\therefore F_1 + F_2 + F_3 = w \times 2a \quad \text{(I)}$$

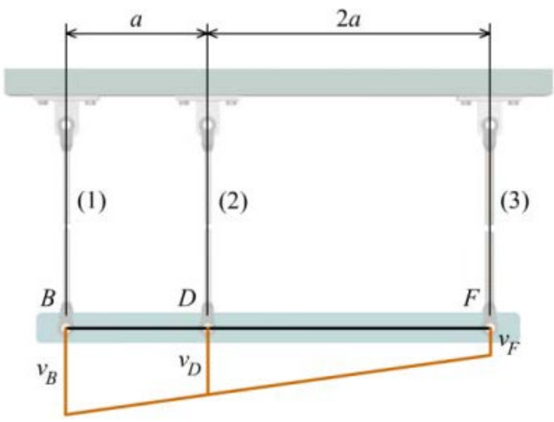
Then, applying the second condition of equilibrium to point F, we get

$$\Sigma M_y = 0 \rightarrow (w \times 2a) \times a - F_2 \times 2a - F_1 \times 3a = 0$$

$$\therefore (w \times 2a) - 2F_2 - 3F_1 = 0$$

$$\therefore 3F_1 + 2F_2 = w \times 2a \quad \text{(II)}$$

Once the load is applied to the rigid bar, it will deflect downward but will not remain horizontal. Although rods (1), (2) and (3) are not evenly spaced, we can use similar triangles to obtain a relationship between the deformations.



From the geometry of the figure above, we have

$$\frac{v_B - v_F}{3a} = \frac{v_D - v_F}{2a} \rightarrow \frac{v_B - v_F}{3} = \frac{v_D - v_F}{2}$$

$$\therefore 2(v_B - v_F) = 3(v_D - v_F)$$

$$\therefore 2v_B - 2v_F = 3v_D - 3v_F$$

$$\therefore 3v_D = 2v_B + v_F \quad \text{(III)}$$

Since the rods are attached to the rigid bar with perfect connections (that is, without gaps or clearances), we can equate the displacements v to the deformations δ,

$$\delta_1 = v_B \;\; ; \;\; \delta_2 = v_D \;\; ; \;\; \delta_3 = v_F$$

Substituting in equation (III), we see that

$$3\delta_2 = 2\delta_1 + \delta_3$$

At this point, we evoke the relationship for deformation under axial loads, $\delta = PL/AE$, so that the equation above becomes

$$3\frac{F_2 L_2}{A_2 E_2} = 2\frac{F_1 L_1}{A_1 E_1} + \frac{F_3 L_3}{A_3 E_3} \quad \text{(IV)}$$

The cross-sectional area of the stainless steel rods is $A_1 = A_2 = (\pi \times 15^2)/4 = 176.7$ mm². For a factor of safety *FS* = 2.5, the allowable stress imparted on the rods will be $\sigma_{\text{allow}} = 250/2.5 = 100$ MPa, and the corresponding load will be such that

$$F_{\text{allow},1} = F_{\text{allow},2} = 100 \times 176.7 = 17.7 \text{ kN}$$

The same process applies to the bronze rod, which has area $A_3 = (\pi \times 20^2)/4 = 314.2$ mm² and is associated with an allowable stress $\sigma_{\text{allow}} = 330/2.5 = 132$ MPa. The allowable load is, in this case,

$$F_{\text{allow},3} = 132 \times 314.2 = 41.5 \text{ kN}$$

Then, we attempt to solve the equations. The two expressions that we derived from statics, (I) and (II), can be combined to yield

$$2F_1 + F_2 = F_3 \quad \text{(V)}$$

Backsubstituting in the compatibility equation, (IV), it follows that

$$3\frac{F_2L_2}{A_2E_2} = 2\frac{F_1L_1}{A_1E_1} + \frac{(2F_1+F_2)L_3}{A_3E_3}$$

$$\therefore 3\frac{F_2L_2}{A_2E_2} = 2\frac{F_1L_1}{A_1E_1} + 2\frac{F_1L_3}{A_3E_3} + \frac{F_2L_3}{A_3E_3}$$

$$\therefore 2F_1\left(\frac{L_1}{A_1E_1} + \frac{L_3}{A_3E_3}\right) = F_2\left(\frac{3L_2}{A_2E_2} - \frac{L_3}{A_3E_3}\right)$$

The equation is further simplified if we note that the rods all have the same length,

$$2F_1\left(\frac{1}{A_1E_1} + \frac{1}{A_3E_3}\right) = F_2\left(\frac{3}{A_2E_2} - \frac{1}{A_3E_3}\right)$$

Solving for F_1, the relation above becomes

$$F_1 = \frac{\frac{F_2}{2}\left(\frac{3}{A_2E_2} - \frac{1}{A_3E_3}\right)}{\left(\frac{1}{A_1E_1} + \frac{1}{A_3E_3}\right)} = \frac{\frac{F_2}{2}\left(3 - \frac{A_2E_2}{A_3E_3}\right)}{\left(1 + \frac{A_1E_1}{A_3E_3}\right)}$$

Substituting each pertaining variable, it follows that

$$F_1 = \frac{\frac{F_2}{2}\left(3 - \frac{176.7\times193}{314.2\times105}\right)}{\left(1 + \frac{176.7\times193}{314.2\times105}\right)} = 0.483F_2$$

However, we know that the maximum allowable value of F_2 for the prescribed factor of safety is 17.7 kN. Therefore,

$$F_1 = 0.483\times17.7 = 8.55 \text{ kN}$$

which is less than the allowable value of F_1 (also 17.7 kN). The only remaining force is F_3, which may be determined with equation (V),

$$F_3 = 2F_1 + F_2 = 2\times8.55 + 17.7 = 34.8 \text{ kN}$$

This is also less than $F_{allow,3}$ = 41.5 kN. The forces in all three rods have been determined, and F_2 controls the capacity of the system. We can now determine the magnitude of the maximum distributed load *w* supported by this arrangement, using, say, equation (I),

$$F_1 + F_2 + F_3 = w\times2a$$

$$\therefore w = \frac{F_1+F_2+F_3}{2a} = \frac{8.55+17.7+34.8}{2\times1.5} = \boxed{20.4 \text{ kN/m}}$$

The correct answer is **A**.

P.11 ➡ *Solution*

The elongation due to temperature rise is given by the equation $\delta_T = L\alpha\Delta T$. The total elongation due to thermal stresses in both members is given by

$$\delta_T = L\alpha\Delta T \rightarrow \delta_T = L_{\text{Al}}\alpha_{\text{Al}}\Delta T + L_{\text{st}}\alpha_{\text{st}}\Delta T$$

$$\therefore \delta_T = \left(L_{\text{Al}}\alpha_{\text{Al}} + L_{\text{st}}\alpha_{\text{st}}\right)\Delta T$$

$$\therefore \delta_T = \left[300\times\left(23\times10^{-6}\right) + 250\times(17.3\times10^{-6})\right]\times\left(140-20\right) = 1.347 \text{ mm}$$

Inspecting the geometry of the structure, we see that the elongation due to mechanical and thermal stresses in members A and B should be no greater than 0.5 mm. This means that a compressive load *P* should be applied to the system so that δ_P = 1.347 – 0.5 = 0.847 mm. Accordingly,

$$\delta_P = \sum_i \frac{P_i L_i}{E_i A_i}$$

which becomes

$$\delta_P = \frac{P_{\text{Al}}L_{\text{Al}}}{A_{\text{Al}}E_{\text{Al}}} + \frac{P_{\text{st}}L_{\text{st}}}{A_{\text{st}}E_{\text{st}}} \rightarrow \delta_P = \left(\frac{L_{\text{Al}}}{A_{\text{Al}}E_{\text{Al}}} + \frac{L_{\text{st}}}{A_{\text{st}}E_{\text{st}}}\right)P$$

$$\therefore P = \frac{\delta_P}{\left(\dfrac{L_{\text{Al}}}{A_{\text{Al}}E_{\text{Al}}} + \dfrac{L_{\text{st}}}{A_{\text{st}}E_{\text{st}}}\right)}$$

where *P* is the common compressive load applied to both members. Substituting the pertaining variables, we get

$$P = \frac{0.847}{\left(\dfrac{300}{2000\times70,000} + \dfrac{250}{800\times200,000}\right)} = 228.6 \text{ kN}$$

The change in length of the aluminum rod is equal to the elongation due to temperature rise minus the shortening due to the compressive load; that is,

$$\delta_{\text{A}} = \delta_{\text{T}} - \delta_{\text{P}} \rightarrow \delta_{\text{A}} = L_{\text{A}}\alpha_{\text{st}}\Delta T - \frac{PL_{\text{st}}}{A_{\text{st}}E_{\text{st}}}$$

$$\therefore \delta_{\text{A}} = 300\times\left(23\times10^{-6}\right)\times\left(140-20\right) - \frac{228,600\times300}{2000\times75,000}$$

$$\therefore \delta_A = 0.828 - 0.457$$

$$\therefore \boxed{\delta_A = 0.371 \text{ mm}}$$

The aluminum rod will elongate by over a third of a millimeter.

The correct answer is **C**.

P.12 ➡ *Solution*

When the assembly is constrained, it has a free expansion δ_T given by

$$\delta_T = \alpha\Delta TL = \alpha\left(T_2 - T_1\right)L$$

where α is the coefficient of thermal expansion of the material, ΔT is the variation in temperature, *L* is the length of the pipe, T_1 is the initial temperature of the assembly, and T_2 is its final temperature. The deflection due to mechanical stress, in turn, is given by

$$\delta_F = \left[\left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \frac{F}{k}\right]$$

The deformation of the assembly can be determined by superposition. That is to say, the deformation at point *C* should equal

$$\delta_C = \delta_T - \delta_F$$

where $\delta_C = F/k$. Substituting the equations we have for δ_T and δ_F, the expression above becomes

$$\delta_C = \delta_T - \delta_F \rightarrow \frac{F}{k} = \alpha\left(T_2 - T_1\right)L - \left[\left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \frac{F}{k}\right]$$

$$\therefore \alpha\left(T_2 - T_1\right)L - \left(\frac{PL}{AE}\right)_{AB} - \left(\frac{PL}{AE}\right)_{BC} = \frac{2F}{k}$$

At this point, we substitute the pertaining variables for the load-displacement relationship as applied to segments AB and BC. For *AB*, we have $P_{AB} = F$ (an arbitrary internal force), $L_{AB} = L/2$, $A_{AB} = \pi d^2/4$, and $E_{AB} = E$; similar terms apply to segment *BC*. Substituting above, we get

$$\alpha\left(T_2 - T_1\right)L - \left(\frac{PL}{AE}\right)_{AB} - \left(\frac{PL}{AE}\right)_{BC} = \frac{2F}{k}$$

$$\therefore \alpha\left(T_2 - T_1\right)L - \frac{F \times (L/2)}{\left(\dfrac{\pi \times d^2}{4}\right) \times E} - \frac{F \times (L/2)}{\left(\dfrac{\pi \times (d/2)^2}{4}\right) \times E} = \frac{2F}{k}$$

$$\therefore \alpha\left(T_2 - T_1\right)L - \frac{2FL}{\pi d^2 E} - \frac{8FL}{\pi d^2 E} = \frac{2F}{k}$$

$$\therefore \alpha\left(T_2 - T_1\right)L - \frac{10FL}{\pi d^2 E} = \frac{2F}{k}$$

$$\therefore \alpha\left(T_2 - T_1\right)L = F\left(\frac{10L}{\pi d^2 E} + \frac{2}{k}\right)$$

$$\therefore F\left(\frac{10kL + 2\pi d^2 E}{\pi d^2 Ek}\right) = \alpha\left(T_2 - T_1\right)L$$

$$\therefore F = \frac{\pi d^2 kE\alpha\left(T_2 - T_1\right)L}{10kL + 2\pi d^2 E}$$

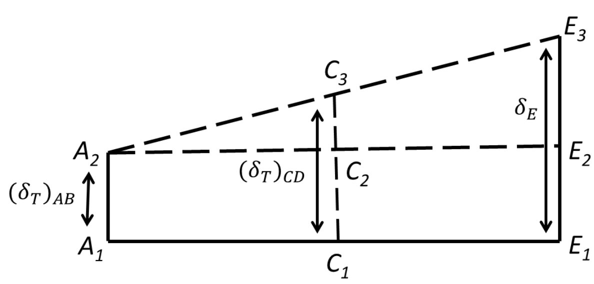
We can then obtain the normal stress developed in pipe segment *BC*; all we have to do is divide *F* as given above by the area of the segment in question, $A_{BC} = \pi \times (d/2)^2/4 = \pi d^2/16$; that is,

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{\dfrac{\pi d^2 kE\alpha\left(T_2 - T_1\right)L}{10kL + 2\pi d^2 E}}{\dfrac{\pi d^2}{16}} = \frac{16}{\cancel{\pi d^2}}\left(\frac{\cancel{\pi d^2} kE\alpha\left(T_2 - T_1\right)L}{10kL + 2\pi d^2 E}\right) = \frac{16}{2}\left(\frac{kE\alpha\left(T_2 - T_1\right)L}{5kL + \pi d^2 E}\right)$$

$$\therefore \boxed{\sigma_{BC} = \frac{8kE\alpha\left(T_2 - T_1\right)L}{5kL + \pi d^2 E}}$$

P.13 ➡ *Solution*

Consider the following deformation diagram for member ACE.



The thermal deformation of bar AB is obtained via the relationship $\delta_T = \alpha(\Delta T)L$, which in this case becomes

$$\left(\delta_T\right)_{AB} = \alpha_{\text{st}}\left(\Delta T\right)L_{AB} = \left(6.6 \times 10^{-6}\right) \times \left(150 - 75\right) \times 1.5 = 7.43 \times 10^{-4} \text{ in.}$$

Similarly, the deformation of bar CD is determined as

$$(\delta_T)_{CD} = \alpha_{\text{Al}}(\Delta T)L_{CD} = (12.8\times10^{-6})\times(150-75)\times1.5 = 1.44\times10^{-3}\text{ in.}$$

The deformation diagram can be used to establish the pointer deflection δ_E. From similar triangles, we have

$$\frac{E_2E_3}{A_1E_1} = \frac{C_2C_3}{A_2C_2}$$

$$\therefore \frac{E_1E_3 - E_1E_2}{A_1E_1} = \frac{C_1C_3 - C_1C_2}{A_2C_2}$$

$$\therefore \frac{\delta_E - (\delta_T)_{AB}}{3.25} = \frac{(\delta_T)_{CD} - (\delta_T)_{AB}}{0.25}$$

Substituting $(\delta_T)_{AB} = 7.43\times10^{-4}$ in. and $(\delta_T)_{CD} = 1.44\times10^{-3}$ in., we get

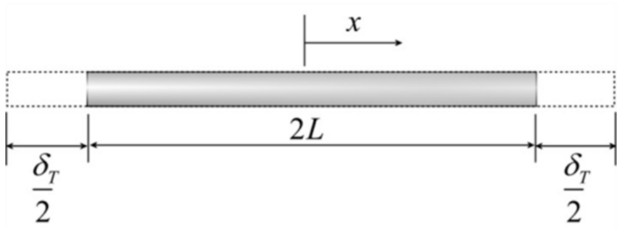
$$\frac{\delta_E - (7.43\times10^{-4})}{3.25} = \frac{(1.44\times10^{-3}) - (7.43\times10^{-4})}{0.25} \rightarrow \boxed{\delta_E = 0.0098\text{ in.}}$$

The displacement of the pointer when the temperature rises is close to one hundredth of an inch.

The correct answer is **C**.

P.14 ➡ *Solution*

Consider the following diagram for the bar.



Then, take a differential element of length *dx* located at a distance *x* from the center of the bar. The free expansion of the element due to the temperature raise is

$$d\delta_T = \alpha\Delta T dx$$

The total deformation δ_T due to the temperature raise can be determined by integrating over the length of the bar.

$$\int d\delta_T = 2\int_0^L \alpha\Delta T dx$$

However, $\Delta T = \Delta T_0 \exp(-|x|/c)$. Thus,

$$\int d\delta_T = 2\int_0^L \alpha\Delta T_0 e^{-|x|/c}dx = 2\alpha\Delta T_0 \times \left(-ce^{-|x|/c}\right)\Big|_0^L$$

$$\therefore \delta_T = 2\alpha c\Delta T_0\left(1 - e^{-|L|/c}\right)$$

Then, we determine the displacement due to load,

$$\delta_P = \frac{\sigma\times2L}{E}$$

Since the bar is fixed on both ends, the total deformation should equal zero, i.e.,

$$\delta_T - \delta_P = 0 \rightarrow \delta_T = \delta_P$$

We have already prepared equations for δ_T and δ_P. Substituting above, we see that

$$\delta_T = \delta_P \rightarrow \frac{2\sigma L}{E} = 2\alpha c\Delta T_0 \times \left(1 - e^{-|L|/c}\right)$$

Solving for the compressive stress σ, we finally obtain

$$\frac{\cancel{2}\sigma L}{E} = \cancel{2}\alpha c \Delta T_0 \times \left(1 - e^{-|L|/c}\right)$$

$$\therefore \boxed{\sigma = \frac{cE\alpha\Delta T_0}{L}\left(1 - e^{-|L|/c}\right)}$$

Note that this situation is a departure from the typical thermal stress $E\alpha\Delta T$ expected in an elementary temperature-displacement analysis.

ANSWER SUMMARY

Problem 1	**1A**	**B**
	1B	**A**
Problem 2		**B**
Problem 3		Open-ended pb.
Problem 4		Open-ended pb.
Problem 5	**5A**	**B**
	5B	**C**
Problem 6		**B**
Problem 7		**C**
Problem 8		**D**
Problem 9		**D**
Problem 10		**A**
Problem 11		**C**
Problem 12		Open-ended pb.
Problem 13		**C**
Problem 14		Open-ended pb.

REFERENCES

- BEER, F., JOHNSTON, E., DEWOLF, J., and MAZUREK, D. (2012). *Mechanics of Materials*. 6th edition. New York: McGraw-Hill.
- GERE, J. and GOODNO, B. (2009). *Mechanics of Materials*. 7th edition. Stamford: Cengage Learning.
- HIBBELER, R. (2014). *Mechanics of Materials*. 9th edition. Upper Saddle River: Pearson.
- PHILPOT, T. (2013). *Mechanics of Materials: An Integrated Learning System*. 3rd edition. Hoboken: John Wiley and Sons.
- STEIF, P. (2012). *Mechanics of Materials*. Upper Saddle River: Pearson.

Got any questions related to this quiz? We can help!
Send a message to contact@montogue.com and we'll
answer your question as soon as possible.