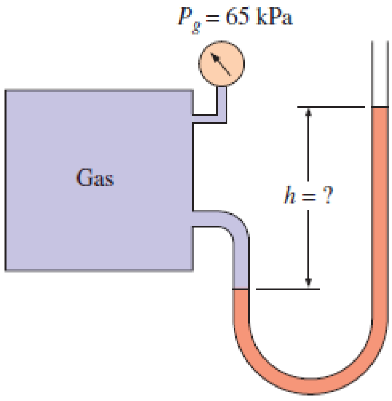
Montogue

Quiz FM101
Basic Fluid Statics
Lucas Montogue

PROBLEMS

Problem 1 (Çengel & Cimbala, 2014, w/ permission)

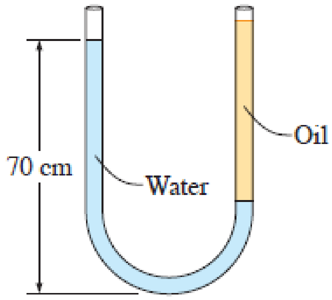
Both a gage and a manometer are attached to a gas tank to measure its pressure. If the reading on the pressure gage is 65 kPa, determine the distance between the two fluid levels of the manometer if the fluid is mercury (ρ = 13,600 kg/m^3).



A) h = 18 cm
B) h = 25 cm
C) h = 33 cm
D) h = 49 cm

Problem 2 (Çengel & Cimbala, 2014, w/ permission)

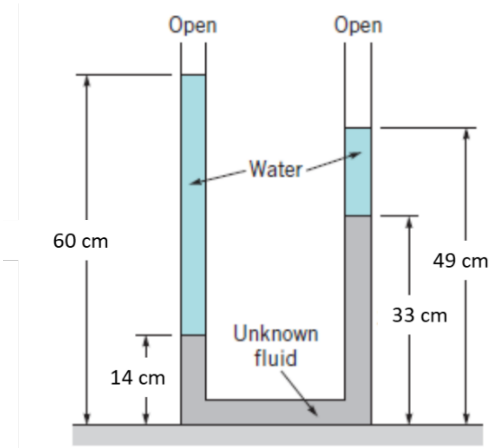
Consider a U-tube whose arms are open to the atmosphere. Water is poured into the U-tube from one arm, and light oil (ρ_{oil} = 790 kg/m^3) into the other. One arm contains 70-cm-high water, while the other arm contains both fluids with an oil-to-water height ratio of 6. Determine the height of oil in the right arm.



A) h_{oil} = 64.4 cm
B) h_{oil} = 73.2 cm
C) h_{oil} = 85.3 cm
D) h_{oil} = 96.1 cm

Problem 3 (Munson, 2009, w/ permission)

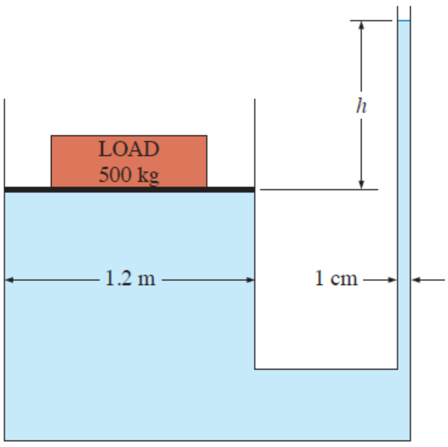
For the configuration shown in the figure below, what must be the value of the specific weight of the unknown fluid?



A) γ = 8.5 kN/m³
B) γ = 12.0 kN/m³
C) γ = 15.5 kN/m³
D) γ = 19.0 kN/m³

Problem 4 (Çengel & Cimbala, 2014, w/ permission)

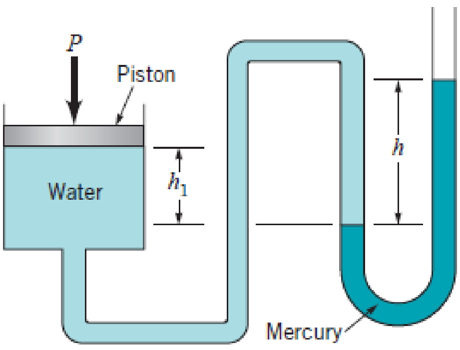
The 500-kg load on the hydraulic lift shown is to be raised by pouring oil (ρ = 780 kg/m³) into a thin tube. Determine how high h should be in order to begin to raise the weight.



A) h = 32.2 cm
B) h = 56.7 cm
C) h = 78.1 cm
D) h = 94.5 cm

Problem 5 (Munson et al., 2009, w/ permission)

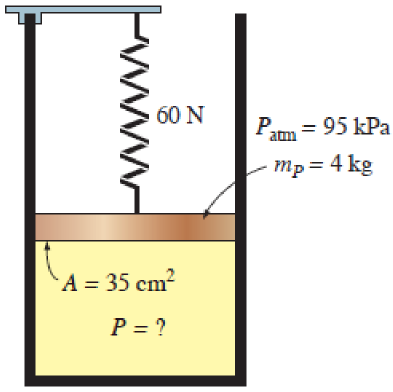
A piston having a cross-sectional area of 0.07 m² is located in a cylinder containing water as shown. An open U-tube manometer is connected to the cylinder. For h_1 = 60 mm and h = 100 mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.



A) P = 654 N
B) P = 739 N
C) P = 893 N
D) P = 971 N

Problem 6 (Çengel & Cimbala, 2014, w/ permission)

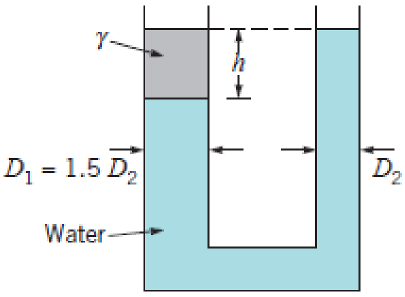
A gas is contained in a vertical, frictionless piston-cylinder device. The piston has a mass of 4 kg and a cross-sectional area of 35 cm^2. A compressed spring above the piston exerts a force of 60 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.



A) p = 95.5 kPa
B) p = 103.6 kPa
C) p = 118.7 kPa
D) p = 123.4 kPa

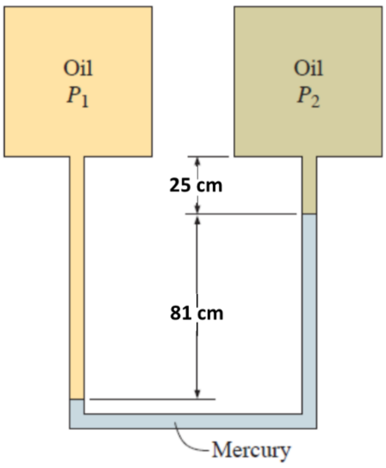
Problem 7 (Munson et al., 2009, w/ permission)

The U-shaped tube shown initially contains water only. A second liquid of specific weight γ, smaller than that of water, is placed on top of the water with no mixing occurring. Can the height, h, of the second liquid be adjusted so that the left and right levels are at the same height? Provide proof of your answer.



Problem 8 (Çengel & Cimbala, 2014, w/ permission)

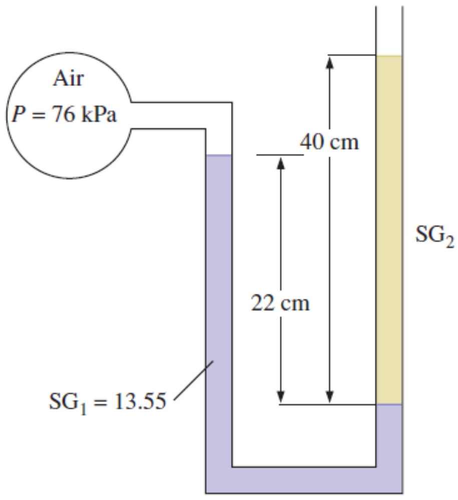
Two oil tanks are connected to each other through a manometer. If the difference between the mercury levels in the two arms is 81 cm, determine the pressure difference between the two tanks. The densities of oil and mercury are 800 kg/m^3 and 13,600 kg/m^3, respectively.



A) Δp = 101.7 kPa
B) Δp = 125.5 kPa
C) Δp = 144.9 kPa
D) Δp = 167.9 kPa

Problem 9 (Çengel & Cimbala, 2014, w/ permission)

Consider a double-fluid manometer attached to an air pipe as shown. If the specific gravity of one fluid is 13.55, determine the specific gravity of the other fluid for the indicated air pressure. Take the atmospheric pressure to be 100 kPa.



A) $SG_2 = 0.91$

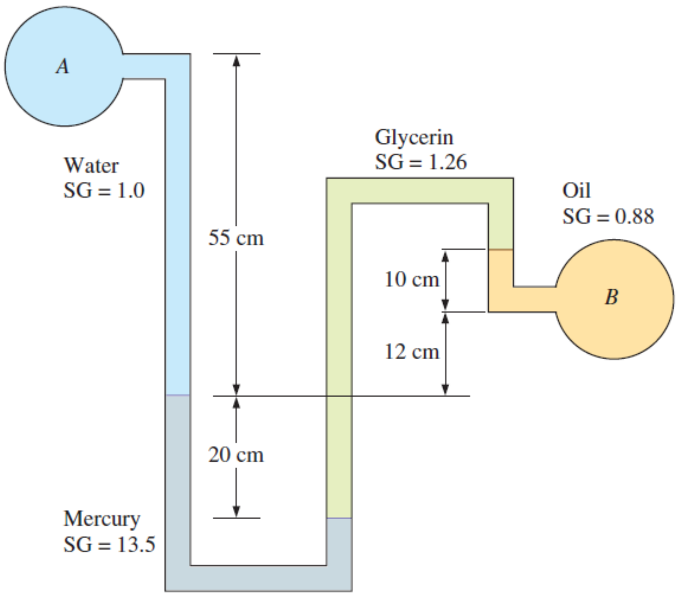
B) $SG_2 = 1.34$

C) $SG_2 = 1.75$

D) $SG_2 = 2.08$

Problem 10 (Çengel & Cimbala, 2014, w/ permission)

The pressure difference between an oil pipe and water pipe is measured by a double-fluid manometer, as shown. For the given fluid heights and specific gravities, calculate the pressure difference $\Delta p = p_B - p_A$.



A) $\Delta p = 9.51$ kPa

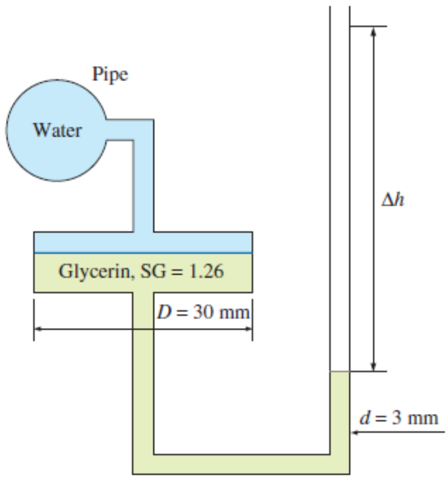
B) $\Delta p = 27.6$ kPa

C) $\Delta p = 44.9$ kPa

D) $\Delta p = 66.8$ kPa

Problem 11 (Çengel & Cimbala, 2014, w/ permission)

The system shown in the figure is used to accurately measure the pressure changes when the pressure is increased by Δp in the water pipe. When Δh = 70 mm, what is the change in the pipe pressure?



A) Δp = 233 Pa

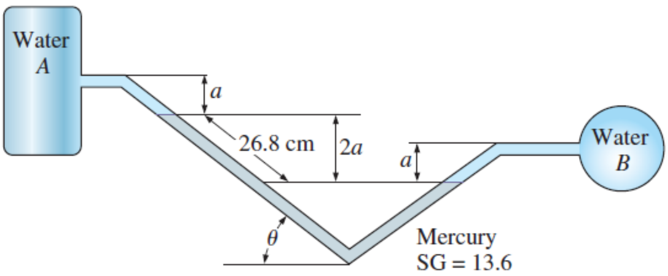
B) Δp = 479 Pa

C) Δp = 651 Pa

D) Δp = 867 Pa

Problem 12 (Çengel & Cimbala, 2014, w/ permission)

Two water tanks are connected to each other through a mercury manometer with inclined tubes, as shown. If the pressure difference between the two tanks is 20 kPa, calculate θ.



A) θ = 10°

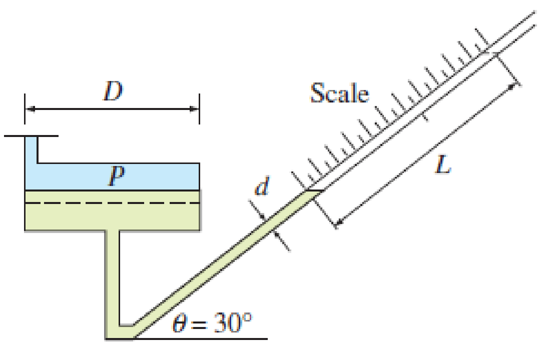
B) θ = 21°

C) θ = 34°

D) θ = 43°

Problem 13 (Çengel & Cimbala, 2014, w/ permission)

The manometer shown in the figure is designed to measure pressures of up to a maximum of 100 Pa. If the reading error is estimated to be ±0.5 mm, what should the ratio d/D be in order for the error associated with pressure measurement not to exceed 2.5% of the full scale?



A) d/D = 0.04

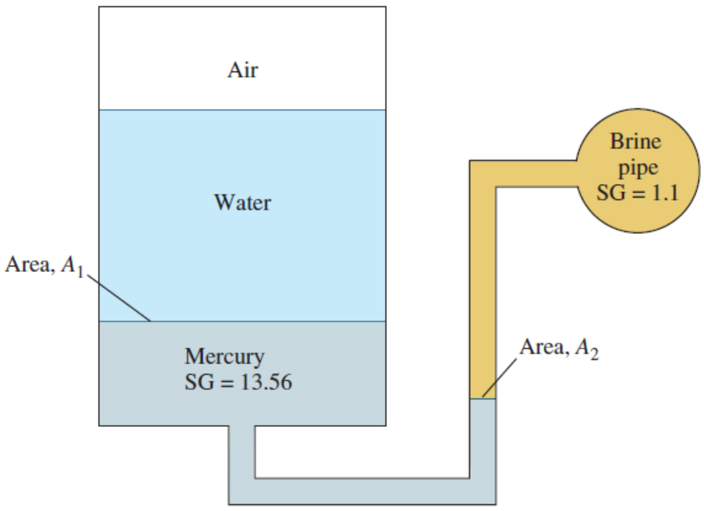
B) d/D = 0.10

C) d/D = 0.16

D) d/D = 0.22

Problem 14 (Çengel & Cimbala, 2014, w/ permission)

Consider the system illustrated below. If a change in 0.9 kPa in the pressure of air causes the brine-mercury interface in the right column to drop by 5 mm while the pressure in the brine pipe remains constant, determine the ratio A_1/A_2.



A) $A_2/A_1 = 0.151$

B) $A_2/A_1 = 0.245$

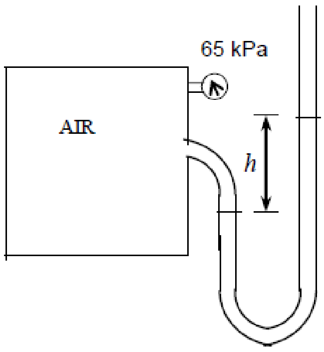
C) $A_2/A_1 = 0.322$

D) $A_2/A_1 = 0.434$

SOLUTIONS

P.1 Solution

The system is illustrated below.



The gage pressure is related to the vertical distance h between the two fluid levels by

$$p_{\text{gage}} = \rho g h$$

All we have to do is solve for h,

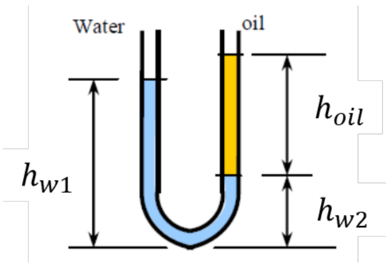
$$p_{\text{gage}} = \rho g h \rightarrow h = \frac{p_{\text{gage}}}{\rho g}$$

$$\therefore h = \frac{65{,}000}{13{,}600 \times 9.81} = 0.49\,\text{m} = \boxed{49\ \text{cm}}$$

➡ The correct answer is **D**.

P.2 Solution

The system is illustrated below.



The height of water column in the left arm of the manometer is $h_{w,1}$ = 0.70 m. As for the right arm, we let the height of water and oil therein be $h_{w,2}$ and h_{oil}, respectively. According to the problem statement, $h_{oil} = 6h_{w,2}$. Both arms are open to the atmosphere and the atmospheric pressure is denoted as p_{atm}. The pressure at the bottom of the tube is p_{bottom}. We can write, for the left half of the tube,

$$p_{\text{bottom}} = p_{\text{atm}} + \rho_w g h_{w,1}$$

and, for the right half,

$$p_{\text{bottom}} = p_{\text{atm}} + \rho_w g h_{w,2} + \rho_{\text{oil}} g h_{\text{oil}}$$

Equating the two expressions and manipulating, we obtain

$$\begin{cases} p_{\text{bottom}} = p_{\text{atm}} + \rho_w g h_{w,1} \quad (\text{I}) \\ p_{\text{bottom}} = p_{\text{atm}} + \rho_w g h_{w,2} + \rho_{\text{oil}} g h_{\text{oil}} \quad (\text{II}) \end{cases}$$

$$\therefore (\text{I}) = (\text{II}) \rightarrow \cancel{p_{\text{atm}}} + \rho_w g h_{w,1} = \cancel{p_{\text{atm}}} + \rho_w g h_{w,2} + \rho_{\text{oil}} g h_{\text{oil}}$$

$$\therefore \rho_w g h_{w,1} = \rho_w g h_{w,2} + \rho_{\text{oil}} g h_{\text{oil}}$$

$$\therefore \rho_w g h_{w,1} = \rho_w g h_{w,2} + 6\rho_{\text{oil}} g h_{w,2}$$

$$\therefore \rho_w \cancel{g} h_{w,1} = \rho_w \cancel{g} h_{w,2} + 6\rho_{\text{oil}} \cancel{g} h_{w,2}$$

$$\therefore \rho_w h_{w,1} = \rho_w h_{w,2} + 6\rho_{\text{oil}} h_{w,2} = \left(\rho_w + 6\rho_{\text{oil}}\right) h_{w,2}$$

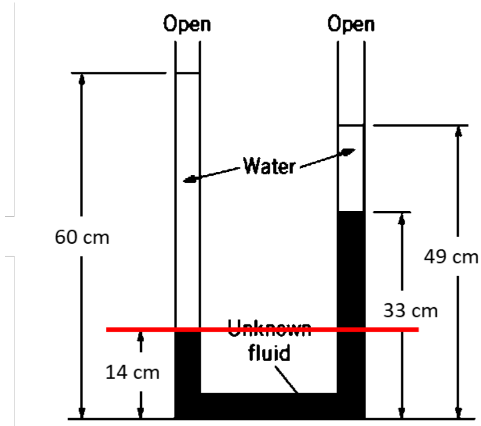
$$\therefore h_{w,2} = \frac{\rho_w}{\rho_w + 6\rho_{\text{oil}}} h_{w,1} = \frac{1000}{1000 + 6 \times 790} \times 70 = 12.2\,\text{cm}$$

Finally, noting that $h_{\text{oil}} = 6h_{w,2}$, we conclude that the height of oil in the right arm is h_{oil} = 6 × 12.2 = 73.2 cm.

➡ The correct answer is **B**.

P.3 ● Solution

All we have to do is balance pressures in a specified level of the U-tube at, say, the level established in the figure below.



Let ρ_{fluid} be the density of the unknown fluid, and γ the specific weight we're looking for. Pressures in the left arm will be accounted for in the left-hand side of the equation, while pressures in the right arm are computed in the right-hand side. Accordingly, we write

$$\rho_w g\left(0.60-0.14\right) = \underbrace{\rho_{\text{fluid}} g}_{=\gamma}\left(0.33-0.14\right) + \rho_w g\left(0.49-0.33\right)$$

$$\therefore \gamma\left(0.33-0.14\right) = \rho_w g\left(0.60-0.14\right) - \rho_w g\left(0.49-0.33\right)$$

$$\therefore \gamma = \frac{\rho_w g\left(0.60-0.14\right) - \rho_w g\left(0.49-0.33\right)}{\left(0.33-0.14\right)}$$

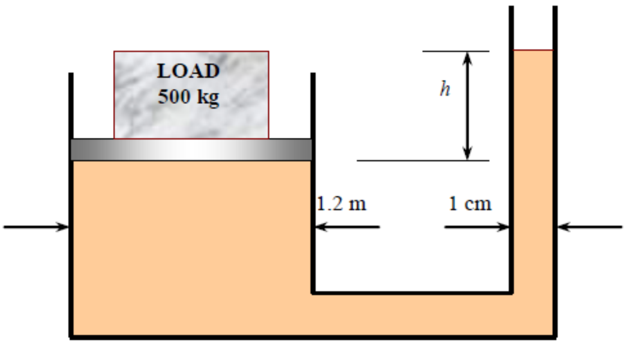
$$\therefore \gamma = \frac{1000 \times 9.81 \times \left(0.60-0.14\right) - 1000 \times 9.81 \times \left(0.49-0.33\right)}{0.33-0.14} = \boxed{15.5\ \text{kN/m}^3}$$

➡ The correct answer is **C**.

P.4 ● Solution

Noting that pressure is force per unit area, the gage pressure in the fluid under the load is given by the ratio of the weight to the circular area of the lift. Mathematically,

$$p_{\text{gage}} = \frac{W}{A} = \frac{mg}{\pi D^2/4} = \frac{500 \times 9.81}{\pi \times 1.2^2/4} = 4.34 \text{ kPa}$$



Next, the required oil height h that will cause a pressure increase of 4.34 kPa is

$$p_{\text{gage}} = \rho g h \rightarrow h = \frac{p_{\text{gage}}}{\rho g}$$

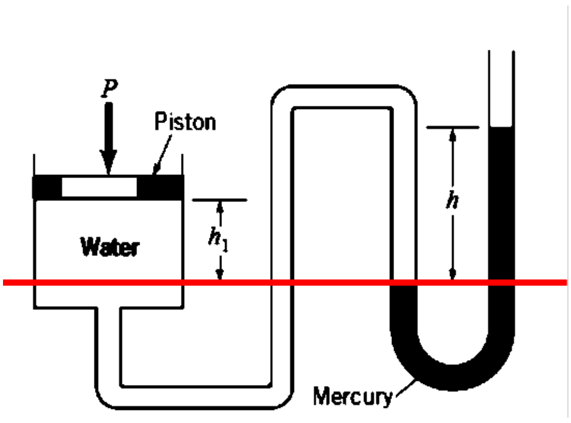
$$\therefore h = \frac{4340}{780 \times 9.81} = 0.567 = \boxed{56.7 \text{ cm}}$$

Thus, the 500 kg load can be raised by simply raising the oil level in the tube by little more than half a meter.

➡ The correct answer is **B**.

P.5 ● Solution

We establish the level defined by the red line as a datum.



The force imparted on the piston is $P = p_p A_p$, where p_p is the pressure acting upon it and A_p is the surface area. Balancing pressures in the leftmost, wider region of the system and the rightmost arm, we can state that

$$p_p + \rho_w g h_1 = \rho_{\text{Hg}} g h$$

$$\therefore p_p = \rho_{\text{Hg}} g h - \rho_w g h_1 = \left(\rho_{\text{Hg}} h - \rho_w h_1\right) g$$

$$\therefore p_p = (13{,}600 \times 0.1 - 1000 \times 0.06) \times 9.81 = 12.75 \text{ kPa}$$

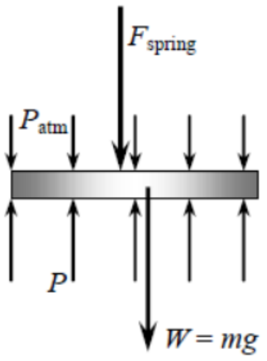
The force acting on the piston follows as

$$P = p_p A_p = 12{,}750 \times 0.07 = \boxed{893 \text{ N}}$$

➡ The correct answer is **C**.

P.6 ● Solution

The free body diagram of the piston is shown below.



Balancing forces in the vertical direction, we write

$$pA = p_{\text{atm}}A + W + F_{\text{spring}}$$

$$\therefore p = p_{\text{atm}} + \frac{mg + F_{\text{spring}}}{A}$$

Substituting p_{atm} = 95 kPa, m = 4 kg, g = 9.81 m/s^2, F_{spring} = 60 N, and A = 35×10^{-4} m^2, it follows that

$$pA = p_{\text{atm}}A + W + F_{\text{spring}}$$

$$\therefore p = 95 + \frac{4\times 9.81 + 60}{35\times 10^{-4}} \times \frac{1\text{ kPa}}{1000\text{ N/m}^2} = \boxed{123.4\text{ kPa}}$$

➡ The correct answer is **D**.

P.7 ● Solution

The pressure at point A must be equal to the pressure at point B since the pressures at equal elevations in a continuous mass of fluid must be the same. We have

$$p_1 = \gamma h$$

and

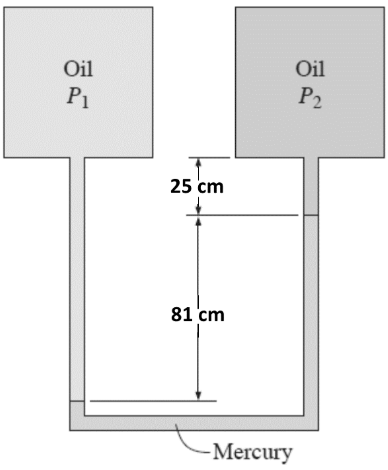
$$p_2 = \gamma_w h$$

It is easy to see that these two pressures can only be equal if $\gamma = \gamma_w$. However, the specific weight of the fluid is less than that of water, that is, $\gamma \neq \gamma_w$. Consequently, the configuration shown in the figure is not possible.

P.8 ● Solution

Starting with the pressure at the bottom of tank 1 (where the pressure is p_1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the bottom of tank 2 (where the pressure is p_2), we write

$$p_1 + \rho_{\text{oil}}g(h_1 + h_2) - \rho_{\text{Hg}}gh_2 - \rho_{\text{oil}}gh_1 = p_2$$



where h_1 = 25 cm. and h_2 = 81 cm. Manipulating the relation above, pressure difference Δp is found to be

$$p_1 + \rho_{\text{oil}} g (h_1 + h_2) - \rho_{\text{Hg}} g h_2 - \rho_{\text{oil}} g h_1 = p_2$$

$$\therefore p_1 + \rho_{\text{oil}} g h_1 + \rho_{\text{oil}} g h_2 - \rho_{\text{Hg}} g h_2 - \rho_{\text{oil}} g h_1 = p_2$$

$$\therefore p_1 - p_2 = \cancel{\rho_{\text{oil}} g h_1} + \rho_{\text{Hg}} g h_2 - \rho_{\text{oil}} g h_2 - \cancel{\rho_{\text{oil}} g h_1}$$

$$\therefore \underbrace{p_1 - p_2}_{=\Delta p} = (\rho_{\text{Hg}} - \rho_{\text{oil}}) g h_2$$

$$\therefore \Delta p = (\rho_{\text{Hg}} - \rho_{\text{oil}}) g h_2$$

Lastly, we substitute ρ_{Hg} = 13,600 kg/m^3, ρ_{oil} = 800 kg/m^3, g = 9.81 m/s^2, and h_2 = 81 cm, with the result that

$$\Delta p = (\rho_{\text{Hg}} - \rho_{\text{oil}}) g h_2 = (13{,}600 - 800) \times 9.81 \times 0.81 = \boxed{101.7 \text{ kPa}}$$

➡ The correct answer is **A**.

P.9 ● Solution

Starting with the pressure of air in the tank, and moving along the tube by adding (as we go down) or subtracting (as we go up), we account for the ρgh terms and other pressure terms, including the atmospheric pressure. The balance is

$$p_{\text{air}} + \rho_1 g h_1 - \rho_2 g h_2 = p_{\text{atm}}$$

Denoting the density of water as ρ_w, we can write

$$p_{\text{air}} + \rho_1 g h_1 - \rho_2 g h_2 = p_{\text{atm}}$$

$$\therefore p_{\text{air}} + SG_1 \rho_w g h_1 - SG_2 \rho_w g h_2 = p_{\text{atm}}$$

$$\therefore SG_2 \rho_w g h_2 = p_{\text{air}} - p_{\text{atm}} + SG_1 \rho_w g h_1 \quad \left(\times \frac{1}{\rho_w g h_2} \right)$$

$$\therefore SG_2 = \frac{p_{\text{air}} - p_{\text{atm}}}{\rho_w g h_2} + SG_1 \times \frac{h_1}{h_2}$$

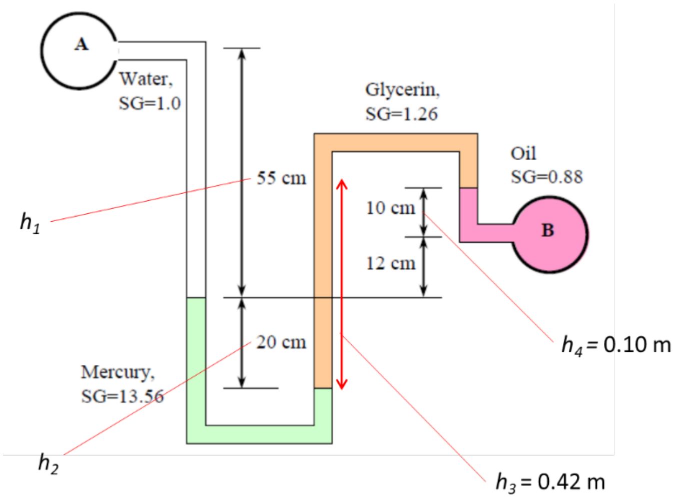
Finally, we substitute p_{air} = 76 kPa, p_{atm} = 100 kPa, ρ_w = 1000 kg/m^3, g = 9.81 m/s^2, h_1 = 0.22 m, h_2 = 0.40 m, and SG_1 = 13.55 to obtain

$$SG_2 = \frac{p_{\text{air}} - p_{\text{atm}}}{\rho_w g h_2} + SG_1 \times \frac{h_1}{h_2} = \frac{76{,}000 - 100{,}000}{1000 \times 9.81 \times 0.40} + 13.55 \times \frac{0.22}{0.40} = \boxed{1.34}$$

➡ The correct answer is **B**.

P.10 ● Solution

The system in question is illustrated below.



Starting with the pressure in the water pipe (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) to account for the ρgh terms until we reach the oil pipe (point B), we can state that

$$p_A+\rho_w gh_1+\rho_{\text{Hg}} gh_2-\rho_{\text{gly}} gh_3+\rho_{\text{oil}} gh_4=p_B$$

Each height h is indicated in the previous figure. Rearranging and introducing the definition of specific gravity, we can obtain the desired pressure difference,

$$p_A+\rho_w gh_1+\rho_{\text{Hg}} gh_2-\rho_{\text{gly}} gh_3+\rho_{\text{oil}} gh_4=p_B$$

$$\therefore p_B-p_A=\rho_w gh_1+\rho_{\text{Hg}} gh_2-\rho_{\text{gly}} gh_3+\rho_{\text{oil}} gh_4$$

$$\therefore p_B-p_A=\underbrace{SG_w}_{=1}\rho_w gh_1+SG_{\text{Hg}}\rho_w gh_2-SG_{\text{gly}}\rho_w gh_3+SG_{\text{oil}}\rho_w gh_4$$

$$\therefore p_B-p_A=\left(h_1+SG_{\text{Hg}}h_2-SG_{\text{gly}}h_3+SG_{\text{oil}}h_4\right)\rho_w g$$

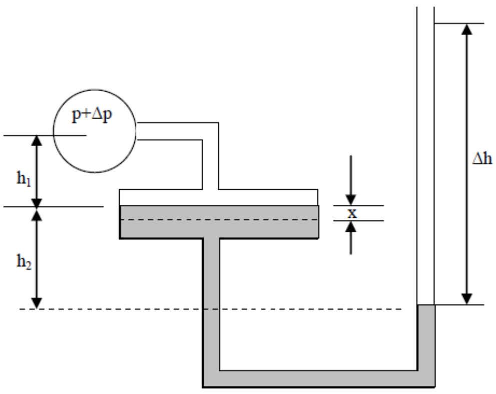
$$\therefore \Delta p=\left(0.55+13.5\times 0.20-1.26\times 0.42+0.88\times 0.10\right)$$

$$\times 1000\times 9.81\left(\times\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)=\boxed{27.6\text{ kPa}}$$

➡ The correct answer is **B**.

P.11 ● Solution

The system is illustrated below.



Initially, equilibrium of pressures enables us to write

$$p+\gamma_w h_1+\gamma_{\text{gly}} h_2=0$$

After the pressure is applied, we have

$$p+\Delta p+\gamma_w\left(h_1+x\right)+\gamma_{\text{gly}}\left(h_2-x\right)-\gamma_{\text{gly}}\Delta h=0$$

On the other hand, due to continuity, the following relation between volumes is possible,

$$\frac{\pi D^2}{4}x=\frac{\pi d^2}{4}\Delta h$$

$$\therefore x=\frac{\cancel{\pi}d^2}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{\pi}D^2}\Delta h$$

$$\therefore x=\left(\frac{d}{D}\right)^2\Delta h=\left(\frac{3}{30}\right)^2\Delta h$$

$$\therefore x=0.01\Delta h$$

The first equation can be rearranged as

$$p+\gamma_w h_1+\gamma_{\text{gly}} h_2=0\rightarrow p=-\gamma_w h_1-\gamma_{\text{gly}} h_2$$

Then, we can substitute in the second equation and expand,

$$p+\Delta p+\gamma_w\left(h_1+x\right)+\gamma_{\text{gly}}\left(h_2-x\right)-\gamma_{\text{gly}}\Delta h=0$$

$$\therefore\left(-\gamma_w h_1-\gamma_{\text{gly}} h_2\right)+\Delta p+\gamma_w\left(h_1+x\right)+\gamma_{\text{gly}}\left(h_2-x\right)-\gamma_{\text{gly}}\Delta h=0$$

$$\therefore -\cancel{\gamma_w h_1}-\cancel{\gamma_{\text{gly}} h_2}+\Delta p+\cancel{\gamma_w h_1}+\gamma_w x+\cancel{\gamma_{\text{gly}} h_2}-\gamma_{\text{gly}} x-\gamma_{\text{gly}}\Delta h=0$$

$$\therefore \Delta p + \gamma_w x - \gamma_{gly} x - \gamma_{gly} \Delta h = 0$$

$$\therefore \Delta p + \gamma_w x - \gamma_{gly} (x + \Delta h) = 0$$

$$\therefore \Delta p = \gamma_{gly} (x + \Delta h) - \gamma_w x$$

Recalling that $x = 0.01\Delta h$ and substituting, pressure difference Δp is determined to be

$$\Delta p = \gamma_{gly} (x + \Delta h) - \gamma_w x$$

$$\therefore \Delta p = \gamma_{gly} (0.01\Delta h + \Delta h) - \gamma_w \times 0.01\Delta h$$

$$\therefore \Delta p = \left[\gamma_{gly} (0.01+1) - 0.01\gamma_w\right]\Delta h$$

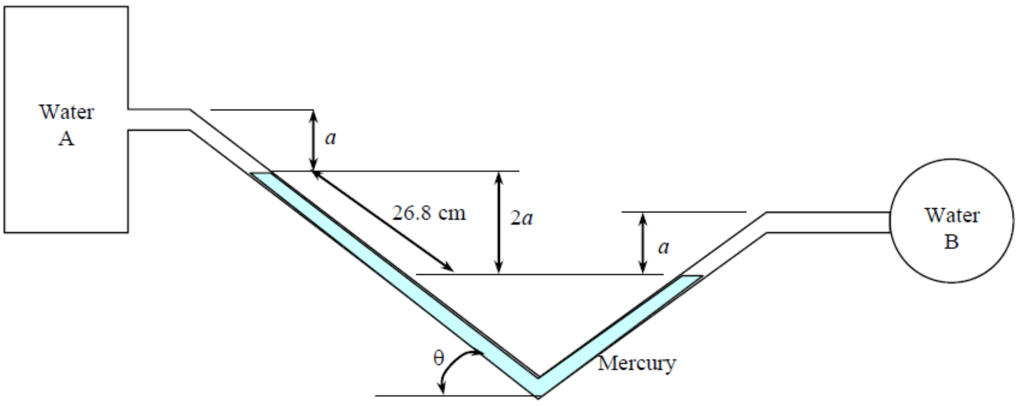
$$\therefore \Delta p = \left(1.01\gamma_{gly} - 0.01\gamma_w\right)\Delta h$$

$$\therefore \Delta p = (1.01 \times 1.26 \times 9810 - 0.01 \times 9810) \times 0.07 = \boxed{867 \text{ Pa}}$$

The correct answer is **D**.

P.12 ● Solution

The system is illustrated below.



Starting with the pressure in the tank A and moving along the tube by adding (as we go down) or subtracting (as we go up) the pressure (=ρgh) terms until we reach tank B, and setting the result equal to p_B, we obtain

$$p_A + \cancel{\rho_w ga} + \rho_{Hg} g (2a) - \cancel{\rho_w ga} = p_B$$

$$\therefore p_A + \rho_{Hg} g (2a) = p_B$$

$$\therefore p_B - p_A = 2\rho_{Hg} ga$$

Solving for length a and substituting the known values, we see that

$$p_B - p_A = 2\rho_{Hg} ga \rightarrow a = \frac{p_B - p_A}{2\underbrace{\rho_{Hg}}_{=13.6\rho_w} g} = \frac{\overbrace{p_B - p_A}^{=20{,}000}}{27.2\rho_w g} = \frac{20{,}000}{27.2 \times 1000 \times 9.81} = 0.075 \text{ m} = 7.5 \text{ cm}$$

Finally, from the geometry of the left branch of the manometer, we have

$$\sin\theta = \frac{2a}{26.8} = \frac{2 \times 7.5}{26.8} = 0.56$$

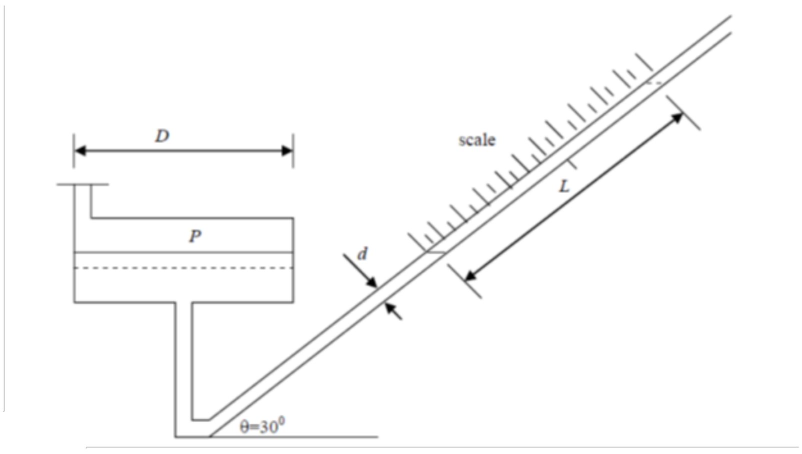
$$\therefore \theta = \arcsin 0.56$$

$$\therefore \boxed{\theta = 34^\circ}$$

The correct answer is **C**.

P.13 ● Solution

The system in question is illustrated below.



Equating pressures, we can write

$$\frac{p}{\gamma}=x+L\sin\theta$$

From the geometry of the device, we have

$$x\times\frac{\pi D^2}{4}=L\times\frac{\pi d^2}{4}$$

$$\therefore x=\left(\frac{d}{D}\right)^2 L$$

Substituting in the first equation gives

$$\frac{p}{\gamma}=x+L\sin\theta$$

$$\therefore \frac{p}{\gamma}=\left(\frac{d}{D}\right)^2 L+L\sin\theta$$

$$\therefore \frac{p}{\gamma}=L\left[\left(\frac{d}{D}\right)^2+\sin\theta\right]$$

$$\therefore p=\gamma L\left[\left(\frac{d}{D}\right)^2+\sin\theta\right]$$

In order to obtain the reading error in "L", we differentiate p with respect to L,

$$p=\gamma L\left[\left(\frac{d}{D}\right)^2+\sin\theta\right]$$

$$\therefore \frac{dp}{dL}=\gamma\left[\left(\frac{d}{D}\right)^2+\sin\theta\right]$$

$$\therefore dp=\gamma\left[\left(\frac{d}{D}\right)^2+\sin\theta\right]dL$$

Dividing the result above by p gives

$$\frac{dp}{p}=\left[\left(\frac{d}{D}\right)^2+\sin\theta\right]\frac{dL}{p/\gamma}$$

From the given data, dp/p = 0.025, $\sin\theta=\sin 30^{\circ}=0.5$, $p/\gamma=100/9810$ = 0.0102 = 10.2 mm, and dL = 0.5 mm. Accordingly,

$$\frac{dp}{p}=\left[\left(\frac{d}{D}\right)^2+\sin\theta\right]\frac{dL}{p/\gamma}$$

$$\therefore 0.025=\left[\left(\frac{d}{D}\right)^2+0.5\right]\frac{0.5}{10.2}$$

$$\therefore \left(\frac{d}{D}\right)^2+0.5=\frac{0.025\times 10.2}{0.5}=0.51$$

$$\therefore \left(\frac{d}{D}\right)^2=0.01$$

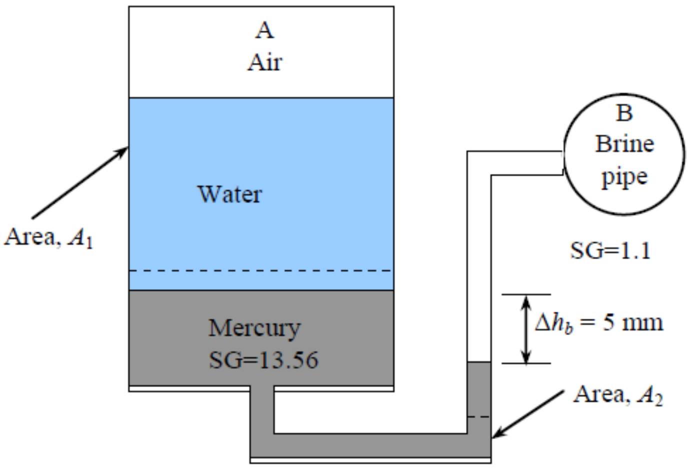
$$\therefore \boxed{\frac{d}{D}=0.1}$$

That is to say, the diameter of the thin tube section should be one tenth of the diameter of the wide region.

➡ The correct answer is **B**.

P.14 ● Solution

The system is illustrated in continuation.



It is clear from the problem statement and the figure provided that the brine pressure is much higher than the air pressure, and when the air pressure drops by, say, 0.9 kPa, the pressure difference between the brine and the air space also increases by the same amount. Starting with the air pressure (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the pressure terms until we reach the brine pipe (point B), and setting the result equal to p_B before and after the pressure change of air, we have

$$\text{Before: } p_{\mathrm{A},1}+\rho_w g h_w+\rho_{\mathrm{Hg}} g h_{\mathrm{Hg},1}-\rho_{\mathrm{Br}} g h_{\mathrm{Br},1}=p_B$$

$$\text{After: } p_{A,2}+\rho_w g h_w+\rho_{\mathrm{Hg}} g h_{\mathrm{Hg},2}-\rho_{\mathrm{Br}} g h_{\mathrm{Br},2}=p_B$$

Subtracting one equation from the other gives

$$p_{\mathrm{A},1}+\cancel{\rho_w g h_w}+\rho_{\mathrm{Hg}} g h_{\mathrm{Hg},1}-\rho_{\mathrm{Br}} g h_{\mathrm{Br},1}-\left(p_{A,2}+\cancel{\rho_w g h_w}+\rho_{\mathrm{Hg}} g h_{\mathrm{Hg},2}-\rho_{\mathrm{Br}} g h_{\mathrm{Br},2}\right)$$

$$=p_B-p_B=0$$

$$\therefore p_{\mathrm{A},1}-p_{\mathrm{A},2}+\rho_{\mathrm{Hg}} g\left(h_{\mathrm{Hg},1}-h_{\mathrm{Hg},2}\right)+\rho_{\mathrm{Br}} g\left(h_{\mathrm{Br},2}-h_{\mathrm{Br},1}\right)=0$$

$$\therefore p_{\mathrm{A},1}-p_{\mathrm{A},2}-\rho_{\mathrm{Hg}} g\Delta h_{\mathrm{Hg}}+\rho_{\mathrm{Br}} g\Delta h_{\mathrm{Br}}=0$$

Substituting $\rho_{\mathrm{Hg}}=SG_{\mathrm{Hg}}\rho_w$, $\rho_{\mathrm{Br}}=SG_{\mathrm{Br}}\rho_w$, and solving for the pressure difference $p_{\mathrm{A},1}-p_{\mathrm{A},2}$, it follows that

$$p_{\mathrm{A},1}-p_{\mathrm{A},2}-\rho_{\mathrm{Hg}} g\Delta h_{\mathrm{Hg}}+\rho_{\mathrm{Br}} g\Delta h_{\mathrm{Br}}=0$$

$$\therefore p_{\mathrm{A},1}-p_{\mathrm{A},2}=SG_{\mathrm{Hg}}\rho_w g\Delta h_{\mathrm{Hg}}-SG_{\mathrm{Br}}\rho_w g\Delta h_{\mathrm{Br}}=0\ \left(\times\frac{1}{\rho_w g}\right)$$

$$\therefore \frac{p_{\mathrm{A},1}-p_{\mathrm{A},2}}{\rho_w g}=SG_{\mathrm{Hg}}\Delta h_{\mathrm{Hg}}-SG_{\mathrm{Br}}\Delta h_{\mathrm{Br}}=0\ \ (\mathrm{I})$$

Here, Δh_{Hg} and Δh_{Br} are the changes in the differential mercury and brine column heights, respectively, due to the drop in air pressure. Both of these are positive quantities since, as the mercury-brine interface drops, the differential fluid heights for both mercury and brine increase. Noting also that the volume of mercury is constant and hence we can state, from conservation of mass, that $A_1 \Delta h_{\text{Hg,left}} = A_2 \Delta h_{\text{Hg,right}}$. In addition,

$$p_{A,2} - p_{A,1} = -0.9 \text{ kPa} = -900 \text{ N/m}^2$$

and Δh_{Br} = 0.005 m. The variation on the level of mercury, in turn, is

$$\Delta h_{\text{Hg}} = \Delta h_{\text{Hg,right}} + \Delta h_{\text{Hg,left}} = \Delta h_{\text{Br}} + \Delta h_{\text{Br}} \times \frac{A_2}{A_1} = \Delta h_{\text{Br}} \left(1 + \frac{A_2}{A_1}\right)$$

Inserting these results into equation (I) and manipulating, we obtain

$$\frac{p_{A,1} - p_{A,2}}{\rho_w g} = SG_{\text{Hg}} \Delta h_{\text{Hg}} - SG_{\text{Br}} \Delta h_{\text{Br}} = 0$$

$$\therefore \frac{900}{1000 \times 9.81} = 13.56 \times \left[\Delta h_{\text{Br}} \left(1 + \frac{A_2}{A_1}\right)\right] - 1.1 \times \Delta h_{\text{Br}} = 0$$

$$\therefore \frac{900}{1000 \times 9.81} = \left[13.56\left(1 + \frac{A_2}{A_1}\right) - 1.1\right] \Delta h_{\text{Br}} = \left[13.56\left(1 + \frac{A_2}{A_1}\right) - 1.1\right] \times 0.005$$

$$\therefore 13.56\left(1 + \frac{A_2}{A_1}\right) - 1.1 = \frac{900}{1000 \times 9.81 \times 0.005}$$

$$\therefore \boxed{\frac{A_2}{A_1} = 0.434}$$

That is to say, the area of the right column should be about 40% of the area of the wider column.

➡ The correct answer is **D**.

ANSWER SUMMARY

Problem 1	**D**
Problem 2	**B**
Problem 3	**C**
Problem 4	**B**
Problem 5	**C**
Problem 6	**D**
Problem 7	Open-ended pb.
Problem 8	**A**
Problem 9	**B**
Problem 10	**B**
Problem 11	**D**
Problem 12	**C**
Problem 13	**B**
Problem 14	**D**

REFERENCES

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