Quiz EL201

Basic Semiconductor Physics

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PROBLEM DISTRIBUTION

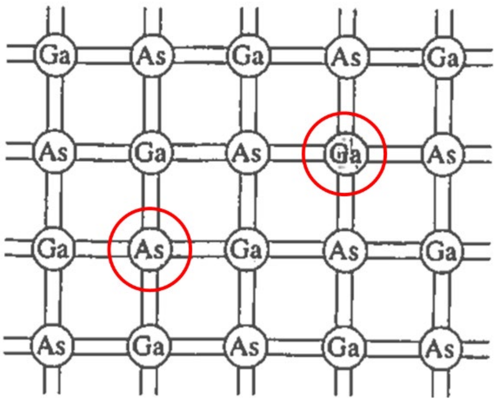
Problems	Subject
1	Basic semiconductor structure
2 - 3	Fermi-Dirac distribution
4 - 9	Carrier concentration and transport
10 - 18	More on semiconductor characteristics (conductivity, electron mobility, etc.)

Refer to the Additional Information section for semiconductor data.

PROBLEMS

Problem 1 (Pierret, 1996)

Problem 1.1: The bonding model for gallium arsenide is illustrated below. Draw the bonding model for GaAs depicting the removal of the circled Ga and As atoms. Bear in mind that Ga and As take their bonding electrons with them when they are removed from the lattice.



Problem 1.2: Redraw the bonding model for GaAs showing the insertion of silicon atoms into the missing Ga and As atom sites.

Problem 1.3: Is the GaAs doped *p*- or *n*-type when Si atoms replace Ga atoms? Explain.

Problem 1.4: Is the GaAs doped *p*- or *n*-type when Si atoms replace As atoms? Explain.

■ *Background for Problem 2*

In general, the carrier concentrations *n* (electrons) and *p* (holes) can be determined from the integrals

$$n = \int_{E_c}^{E_{\text{top}}} g_c(E) f_P(E)\, dE$$

and

$$p = \int_{E_{\text{bottom}}}^{E_V} g_v(E)\left[1 - f_P(E)\right] dE$$

Here, $g_c(E)$ and $g_v(E)$ are the densities of states in the conduction and valence bands, respectively; $f_P(E)$ is the Fermi-Dirac function; E_c and E_v are the energies in the conduction and valence bands, respectively; and E_{bottom} and E_{top} are the lowermost and uppermost energy levels in the range under consideration, respectively. As the reader may know, the densities of states *g(E)* are actually condensed parameters obtained from quantum-mechanical considerations, and can be written as

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \;\; ; \;\; E \geq E_c$$

$$g_v(E) = \frac{m_p^*\sqrt{2m_p^*(E_v - E)}}{\pi^2\hbar^3} \;\; ; \;\; E \le E_v$$

Here, m_n^* and m_p^* are the effective electron and hole masses and $\hbar$ is Planck's constant. Substituting these definitions of $g_c(E)$ and $g_v(E)$ into the expressions for n and p, we can carry out the integrations to obtain the familiar results

$$n = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

and

$$p = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

where E_F is Fermi energy, k is Boltzmann's constant, and N_c and N_v are the effective density of states functions for conduction and valence band states, which are defined as

$$N_c = 2\left(\frac{m_n^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{m_p^* kT}{2\pi\hbar^2}\right)^{3/2}$$

Problem 2 (Pierret, 1996)

With the results and definitions above in mind, this problem considers what would happen if we began the derivation of the electron distribution with a slightly different expression for the density of states $g_c(E)$, namely:

$$g_c(E) = \text{constant} = \frac{N_c}{kT} \;\; ; \;\; E \ge E_c$$

where N_c is defined above.

Problem 2.1: Assuming $E_F < E_c - 3kT$, sketch the electron distribution in the conduction band of the hypothetical semiconductor with $g_c(E)$ defined by the relationship above.

Problem 2.2: Establish relationships for the electron concentration in the hypothetical semiconductor.

Problem 3 (Pierret, 1996)

Let us continue our analysis of carrier energy distributions.

Problem 3.1: The carrier distributions or number of carriers as a function of energy in the conduction and valence bands peak at an energy very close to the band edges. Taking the semiconductor to be nondegenerate, show that the energies at which the carrier distributions peak are $E_c + kT/2$ and $E_v - kT/2$ for the conduction and valence bands, respectively.

Problem 3.2: As indicated in Problem 3.1, for a nondegenerate semiconductor the peak in the electron distribution versus energy in the conduction band occurs at $E_c + kT/2$. Expressed as a fraction of the electron population at the peak energy, what is the electron population in a nondegenerate semiconductor at $E = E_c + 5kT$?

Problem 4 (Neamen, 2003, w/ permission)

Plot the intrinsic carrier concentration, n_i, for a temperature range of $200 \le T \le 600$ K for silicon, germanium, and gallium arsenide. Use a log scale for n_i. Recall that Boltzmann's constant $k = 8.62\times10^{-5}$ eV/K.

Problem 5 (Neamen, 2003, w/ permission)

In a particular semiconductor material, the effective density of states functions are given by $N_c = N_{c0}(T/300)^{3/2}$ and $N_v = N_{v0}(T/300)^{3/2}$, where N_{c0} and N_{v0} are constants independent of temperature. Experimentally determined intrinsic carrier concentrations are found to be $n_i = 1.40\times10^2$ cm^{-3} at $T = 200$ K and $n_i = 7.70\times10^{10}$ cm^{-3} at $T = 400$ K. Determine the product $N_{c0} \times N_{v0}$ and the bandgap energy E_g. (Assume E_g is constant over this temperature range.)

Problem 6 (Neamen, 2003, w/ permission)

Silicon at $T = 300$ K is doped with arsenic atoms such that the thermal-equilibrium concentration of electrons is $n_0 = 7\times10^{15}$ cm^{-3}.

Problem 6.1: Find the difference between the conduction band energy and the Fermi energy, $E_c - E_F$.

Problem 6.2: Determine the difference between the Fermi energy and the valence band energy, $E_F - E_v$.

Problem 6.3: Calculate the thermal-equilibrium concentration of holes, p_0.

Problem 6.4: Which carrier is the minority carrier?

Problem 6.5: Find the difference between Fermi energy and intrinsic Fermi energy, $E_F - E_{F,i}$.

Problem 7

The thermal-equilibrium concentration of holes in a silicon semiconductor at $T = 300$ K is $p_0 = 2\times10^{16}$ cm^{-3}.

Problem 7.1: Determine the difference between the Fermi energy and the valence band energy, $E_F - E_v$.

Problem 7.2: Determine the difference between the conduction band energy and the Fermi energy, $E_c - E_F$.

Problem 7.3: What is the value of the thermal-equilibrium concentration of electrons, n_0?

Problem 7.4: Determine the difference between intrinsic Fermi energy and Fermi energy, $E_{F,i} - E_F$.

Problem 8

Problem 8.1: For gallium arsenide at 375 K, the difference between the conduction band energy and the Fermi energy is $E_c - E_F = 0.28$ eV. Calculate the concentrations of electrons, n_0, and holes, p_0.

Problem 8.2: Assuming the value of n_0 in Problem 8.1 remains constant, determine the difference $E_c - E_F$ and the value of p_0 at $T = 300$ K.

Problems 9 and 10 require knowledge of both the Fermi-Dirac distribution, which we studied in Problems 2 and 3, and charge carrier concentrations, which we studied in Problems 4 to 8.

Problem 9

The following statements concern the Fermi-Dirac probability function for modelling of charge carriers.

Problem 9.1: Under equilibrium conditions and temperature $T > 0$ K, what is the probability of an electron state being occupied if it is located at the Fermi level?

Problem 9.2: If the Fermi energy E_F is positioned at the conduction band energy E_c, determine the probability of finding electrons in states at $E_c + kT$.

Problem 9.3: The probability that a state is filled at $E_c + kT$ is equal to the probability that a state is empty at $E_c + kT$. In view of this observation, determine a simple expression for the Fermi level.

Problem 10

The Fermi level in n-type silicon at $T = 300$ K is 245 meV below the conduction band and 200 meV below the donor level. Determine the probability of finding an electron:

Problem 10.1: In the donor level;

Problem 10.2: In a state in the conduction band kT above the conduction-band edge.

Problem 11 (Neamen, 2003, w/ permission)

The concentration of donor impurity atoms in silicon is $N_d = 10^{15}$ cm^{-3}. Assume an electron mobility of $\mu_n = 1300$ cm^2/V·s and a hole mobility of $\mu_p = 450$ cm^2/V·s. Calculate the conductivity and resistivity of the material.

Problem 12 (Neamen, 2003, w/ permission)

A p-type silicon material is to have a conductivity of $\sigma = 1.80$ (Ω-cm)$^{-1}$. If the mobility values are $\mu_n = 1250$ cm^2/V·s for electrons and $\mu_p = 380$ cm^2/V·s for holes, what must be the acceptor impurity concentration in the material?

Problem 13 (Neamen, 2003, w/ permission)

A silicon sample is 2.5 cm long and has a cross-sectional area of 0.1 cm^2. The silicon is n-type with a donor impurity concentration of $N_d = 2\times10^{15}$ cm^{-3}. The resistance of the sample was measured to be 70 Ω. What is the electron mobility?

Problem 14 (Sze and Lee, 2012)

For a semiconductor with constant mobility ratio $b = \mu_n/\mu_p > 1$ independent of impurity concentration, find the maximum resistivity $\rho_{\max}$ in terms of the intrinsic resistivity ρ_i and the mobility ratio.

Problem 15 (Neamen, 2003, w/ permission)

In a particular semiconductor material, the mobility values are μ_n = 1000 cm^2/V·s and μ_p = 600 cm^2/V·s, and the effective density of states functions are $N_c = N_v = 10^{19}$ cm^{-3}. The measured conductivity of the intrinsic material is $\sigma = 10^{-6}$ (Ω-cm)$^{-1}$ at T = 300 K. Find the conductivity at T = 500 K. Consider N_c, N_v, μ_n and μ_p to be constant with temperature.

Problem 16 (Neamen, 2003, w/ permission)

The total current in a semiconductor is constant and is composed of electron drift current and hole diffusion current. The electron concentration is constant and equal to 10^{16} cm^{-3}. The hole concentration is given by

$$p(x) = 10^{15} \exp\left(\frac{-x}{L}\right) \left[\mathrm{cm}^{-3}\right] \; ; \; (x \geq 0)$$

where L = 12 μm. The hole diffusion coefficient is D_p = 12 cm^2/s and the electron mobility is μ_n = 1000 cm^2/V·s. The total current density is J = 4.8 A/cm^2. Calculate:

Problem 16.1: The hole diffusion current density as a function of x.

Problem 16.2: The electron current density as a function of x.

Problem 16.3: The electric field as a function of x.

Problem 17 (Neamen, 2003, w/ permission)

The conductivity of a semiconductor layer varies with depth y as $\sigma(y) = \sigma_o exp(-y/D)$, where σ_0 = 20 (Ω-cm)$^{-1}$ and D = 0.3 μm. If the thickness of the semiconductor layer is t = 1.5 μm, determine the average conductivity of this layer.

Problem 18 (Neamen, 2003, w/ permission)

An n-type silicon resistor has a length L = 150 μm, width W = 7.5 μm, and thickness T = 1 μm. A voltage of 2 V is applied across the length of the resistor. The donor impurity concentration varies linearly through the thickness of the resistor with $N_d = 2\times10^{16}$ cm^{-3} at the top surface and $N_d = 2\times10^{15}$ cm^{-3} at the bottom surface. Assume an average electron mobility μ_n = 750 cm^2/V·s.

Problem 18.1: What is the electric field per unit length in the resistor?

Problem 18.2: Determine the average conductivity of the silicon.

Problem 18.3: Calculate the current in the resistor.

Problem 18.4: Calculate the current density near the top surface and the current density near the bottom surface.

Problem 19 (Sze and Lee, 2012)

Assume that a conduction electron in Si (electron mobility μ_n = 1350 cm^2/V-s) has a thermal energy kT, related to its mean thermal velocity by $E_{th} = m_0 v_{\mathrm{th}}^2/2$, where $m_0 = 9.1\times10^{-31}$ kg is the rest mass of an electron. This electron is placed in an electric field of 100 V/cm. Show that the drift velocity of the electron in this case is small compared to its thermal velocity. Repeat for a field of 10,000 V/cm, using the same value of μ_n. Comment on the actual mobility effects at this higher value of field.

ADDITIONAL INFORMATION

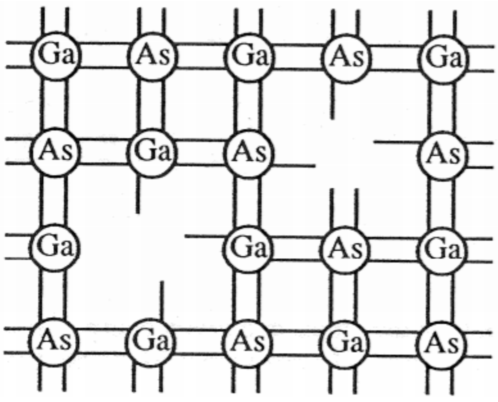
Table 1 Silicon, gallium arsenide, and germanium properties (T = 300 K)

Property	**Si**	**GaAs**	**Ge**
Atoms (cm^{-3})	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.6
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^3)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Effective density of states in conduction band, N_c (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration, n_i (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Electron mobility, μ_n (cm^2/V·s)	1350	8500	3900
Hole mobility, μ_p (cm^2/V·s)	480	400	1900

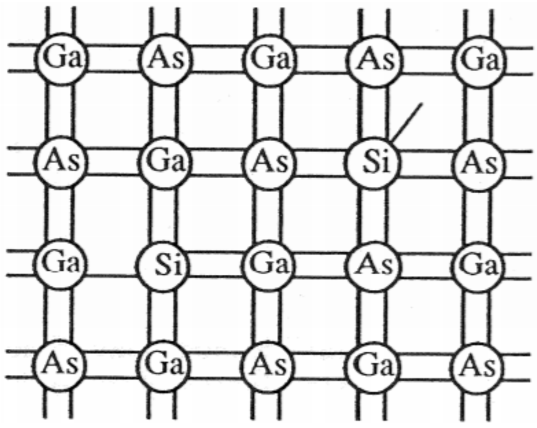
SOLUTIONS

P.1 ➡ *Solution*

Problem 1.1: Gallium is a column III element and as such carries 3 valence electrons. Removing a Ga atom will leave 8 – 3 = 5 dangling bonds in the vicinity of the vacancy, as illustrated below. Arsenic is a column V element and hence possesses 5 valence electrons. Removing an As atom will leave 8 – 5 = 3 dangling bonds in the vicinity of the vacancy, as shown.



Problem 1.2: When a Si atom with four valence electrons is inserted into the missing Ga site, there is one extra electron that does not fit snugly into the bonding pattern; to illustrate this, we draw one extra straight line protruding from the newly inserted silicon dopant. Now, when a Si atom is inserted into the missing As site, there are one too few bonds to complete the bonding scheme and a hole is formed; such a substitution is represented by a missing straight line, as shown.



Problem 1.3: The extra electron afforded by the replacement of Ga with Si yields a n-type semiconductor.

Problem 1.4: The extra hole formed by the replacement of As with Si leads to a p-type semiconductor.

P.2 ➡ *Solution*

Problem 2.1: To determine the electron distribution, simply multiply the density of states $g_c(E)$ defined in the problem statement by the Fermi-Dirac function,

$$f_P(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

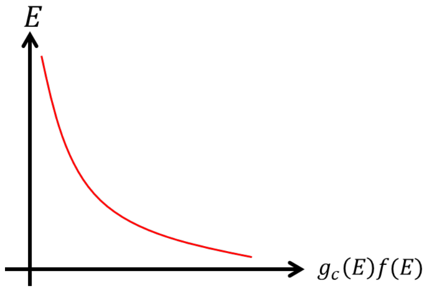
which, for a nondegenerate semiconductor ($E_F < E_c - 3kT$), can be simplified as

$$f_P(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} \approx e^{-(E-E_F)/kT}$$

so that

$$\text{Electron distribution} = g_c(E)f_P(E) = \frac{N_c}{kT}e^{-(E-E_F)/kT}$$

The ensuing electron distribution has $g_c(E)f_P(E)$ decaying exponentially with increasing energy, as shown.



Problem 2.2: To establish the concentration of electrons, we set up the integral

$$n = \int_{E_c}^{E_{\text{top}}} g_c(E)f_P(E)\,dE = \frac{N_c}{kT}\int_{E_c}^{E_{\text{top}}} \frac{dE}{1+e^{(E-E_F)/kT}}$$

Now, let

$$\eta = \frac{E-E_c}{kT}\ ;\ \eta_c = \frac{E_F-E_c}{kT}$$

and, for simplicity, we extend the upper bound of integration from $E = E_{\text{top}}$ to $E \to \infty$; accordingly,

$$n = N_c\int_0^{\infty} \frac{d\eta}{1+e^{\eta\eta_c}}$$

The integral above can be written in closed form as

$$\int_0^{\infty} \frac{d\eta}{1+e^{\eta\eta_c}} = \left[\eta_c + \ln\left(1+e^{-\eta_c}\right)\right]$$

so that

$$\boxed{n = N_c\left[\eta_c + \ln\left(1+e^{-\eta_c}\right)\right]}\ \text{(I)}$$

Now, if the semiconductor is degenerate, $\eta_c \leq 3$. Thus, considering the logarithm above in isolation,

$$\ln\left(1+e^{-\eta_c}\right) = \ln\left[e^{-\eta_c}\left(1+e^{\eta_c}\right)\right] = -\eta_c + \ln\left(1+e^{\eta_c}\right) \approx -\eta_c + e^{\eta_c}$$

where we used, in the last passage,

$$\exp(\eta_c) \ll 1\ ;\ \ln(1+x) \approx x \text{ if } x \ll 1$$

Finally, substituting the approximation above in (I) brings to

$$n = N_c\left[\eta_c + \ln\left(1+e^{-\eta_c}\right)\right] = N_c\left(\eta_c - \eta_c + e^{\eta_c}\right) = N_c\exp(\eta_c)$$

$$\therefore \boxed{n = N_c\exp\left(\frac{E_F-E_c}{kT}\right)}$$

Notice that the electron distribution turned out to have the same form as the result posited in the background box.

P.3 ➡ *Solution*

Problem 3.1: The distribution of electrons in the conduction band is given by $g_c(E)f_P(E)$, while the distribution of holes in the valence band is given by $g_v(E)(1 - f(E))$. Working with the electron distribution, we first note that for a nondegenerate semiconductor, for all $E \geq E_c$,

$$f_P(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \approx e^{-(E-E_F)/kT}$$

We proceed to write

$$g_c(E) f_P(E) \approx \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} e^{-(E-E_F)/kT} = \underbrace{\frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3}}_{=\kappa} (E - E_c)^{1/2} e^{-(E-E_F)/kT}$$

$$\therefore g_c(E) f_P(E) = \kappa (E - E_c)^{1/2} e^{-(E-E_F)/kT}$$

where κ is constant. To find the extrema of this relationship, we differentiate it with respect to energy and set the resulting expression to zero,

$$\frac{d}{dE}\left[g_c(E) f_P(E)\right] = \frac{\kappa}{2(E - E_c)^{1/2}} e^{-(E-E_F)/kT} - \frac{\kappa (E - E_c)^{1/2}}{kT} e^{-(E-E_F)/kT} = 0$$

$$\therefore \frac{\cancel{\kappa}}{2(E - E_c)^{1/2}} \cancel{e^{-(E-E_F)/kT}} - \frac{\cancel{\kappa}(E - E_c)^{1/2}}{kT} \cancel{e^{-(E-E_F)/kT}} = 0$$

$$\therefore \frac{1}{2(E - E_c)^{1/2}} - \frac{(E - E_c)^{1/2}}{kT} = 0$$

$$\therefore E_{\text{peak}} - E_c = \frac{kT}{2}$$

$$\therefore \boxed{E_{\text{peak}} = E_c + \frac{kT}{2}}$$

as we intended to show. The development for holes in the valence band is completely analogous and leads to

$$\boxed{E_{\text{peak}} = E_v - \frac{kT}{2}}$$

Problem 3.2: Using the simplified notation introduced in Problem 3.1, we may write

$$g_c(E_c + kT/2) f_P(E_c + kT/2) = \kappa\left[(E_c + kT/2) - E_c\right]^{1/2} e^{-\left[(E_c + kT/2) - E_F\right]/kT}$$

and

$$g_c(E_c + 5kT) f_P(E_c + 5kT) = \kappa\left[(E_c + 5kT) - E_c\right]^{1/2} e^{-\left[(E_c + 5kT) - E_F\right]/kT}$$

so that

$$\frac{g_c(E_c + 5kT) f_P(E_c + 5kT)}{g_c(E_c + kT/2) f_P(E_c + kT/2)} = \frac{\cancel{\kappa}\left[(\cancel{E_c} + 5kT) - \cancel{E_c}\right]^{1/2} e^{-\left[(E_c + 5kT) - E_F\right]/kT}}{\cancel{\kappa}\left[(\cancel{E_c} + kT/2) - \cancel{E_c}\right]^{1/2} e^{-\left[(E_c + kT/2) - E_F\right]/kT}}$$

$$\therefore \frac{g_c(E_c + 5kT) f_P(E_c + 5kT)}{g_c(E_c + kT/2) f_P(E_c + kT/2)} = \frac{\sqrt{5\cancel{kT}}}{\sqrt{\cancel{kT}/2}} \exp\left[-(E_c + 5kT - E_F) + (E_c + kT/2 - E_F)\right]$$

$$\therefore \frac{g_c(E_c + 5kT) f_P(E_c + 5kT)}{g_c(E_c + kT/2) f_P(E_c + kT/2)} = \frac{\sqrt{5}}{\sqrt{1/2}} \exp\left[-\frac{9}{2}\right] = \boxed{0.0351}$$

P.4 ➡ Solution

First, elementary charge carrier theory can be used to state that the thermal-equilibrium electron concentration n_0 in the conduction band of a semiconductor is given by

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

where N_c is the effective density of states function in the conduction band, E_c is the conduction-band energy, E_F is the Fermi energy, k is Boltzmann's constant, and T is temperature. In a similar manner, the thermal-equilibrium concentration p_0 of holes in the valence band is written as

$$p_0 = N_v \exp\left[\frac{-\left(E_F - E_v\right)}{kT}\right]$$

where N_v is the effective density of states function in the valence band and E_v is the valence-band energy. For an intrinsic semiconductor, the concentration of electrons in the conduction band is equal to the concentration of holes in the valence band. We may denote n_i and p_l as the electron and hole concentrations, respectively, in such a semiconductor; importantly, $n_i = p_i$, so normally we simply use the parameter n_i as the intrinsic carrier concentration, which refers to either the intrinsic electron or hole concentration. The Fermi energy level for the intrinsic semiconductor is called the intrinsic Fermi energy, or $E_F = E_{F_i}$. With an intrinsic SC in mind, the two preceding equations can be restated as

$$\left(n_0\right)_i = n_i = N_c \exp\left[\frac{-\left(E_c - E_{F,i}\right)}{kT}\right]$$

$$\left(p_0\right)_i = p_i = N_v \exp\left[\frac{-\left(E_{F,i} - E_v\right)}{kT}\right]$$

Multiplying one equation by the other, we obtain

$$n_i \times \underbrace{p_i}_{=n_i} = n_i^2 = N_c \exp\left[\frac{-\left(E_c - E_{F,i}\right)}{kT}\right] \times N_v \exp\left[\frac{-\left(E_{F,i} - E_v\right)}{kT}\right]$$

$$\therefore n_i^2 = N_c N_v \exp\left[\frac{-E_c + E_{F,i} - E_{F,i} + E_v}{kT}\right] = N_c N_v \exp\left[\frac{\underbrace{-\left(E_c - E_v\right)}_{=E_g}}{kT}\right]$$

$$\therefore n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

where E_g is the bandgap energy. Now, if N_c and N_v each vary with temperature to the 3/2 power, their product will obviously be proportional to T^3. Taking a temperature of 300 K as the reference value, at which the density of states functions attain values denoted by N_{c0} and N_{v0}, respectively, we may write

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right) = N_{c0} N_{v0} \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

Taking the square root,

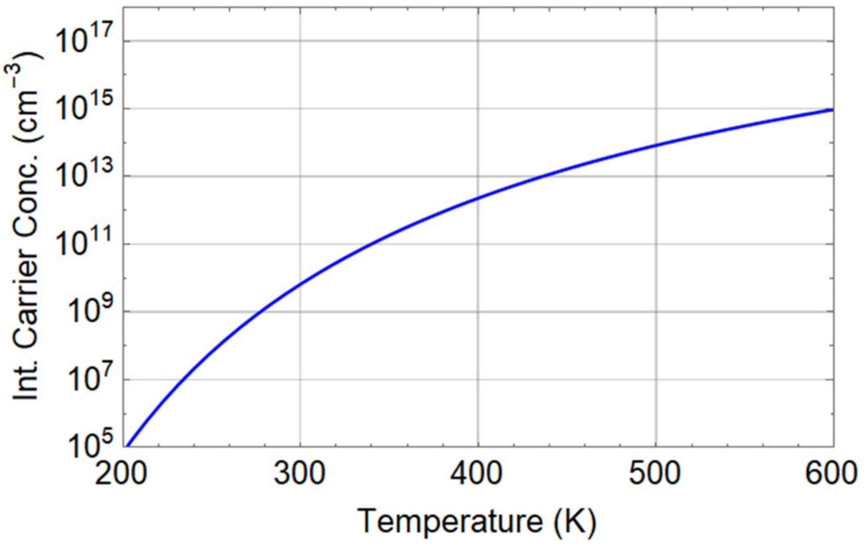
$$n_i^2 = N_{c0} N_{v0} \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right) \rightarrow n_i = \sqrt{N_{c0} N_{v0} \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)}$$

This is the basic equation we have to plot. For silicon, we refer to the Additional Information section and read $N_{e0} = 2.8 \times 10^{19}$ cm^{-3}, $N_{v0} = 1.04 \times 10^{19}$ cm^{-3} and $E_g = 1.12$ eV; also, Boltzmann's constant is 8.62×10^{-5} eV/K. Substituting above brings to

$$n_i = \sqrt{\left(2.8 \times 10^{19}\right) \times \left(1.04 \times 10^{19}\right) \times \left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-1.12}{\left(8.62 \times 10^{-5}\right) \times T}\right]}$$

$$\therefore n_i = 3.28 \times 10^{15} T^{3/2} \exp\left(\frac{-6.50 \times 10^3}{T}\right)$$

The equation is plotted below for the range $T \in [200, 600]$ K, using a log scale for n_i as prescribed.

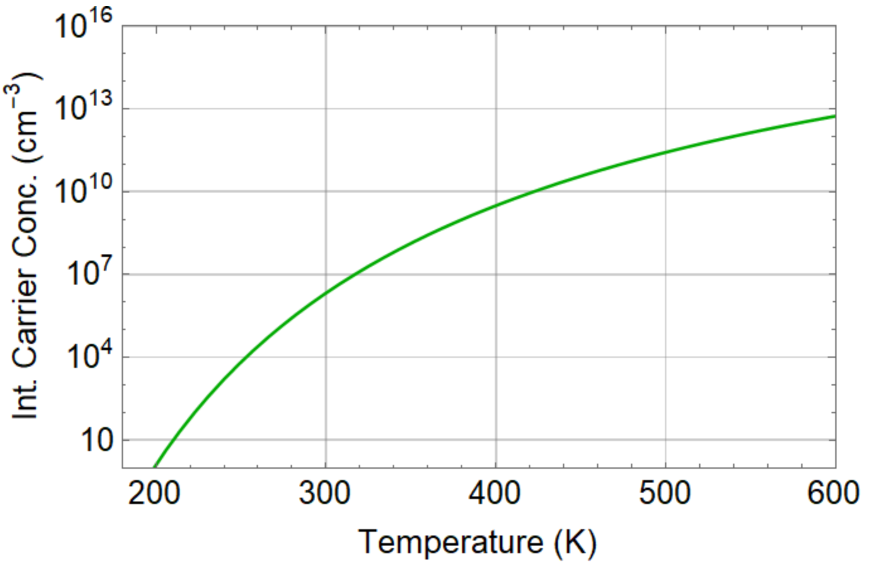


We now turn to gallium arsenide, for which N_{e0} = 4.7×10^{17} cm^{-3}, N_{v0} = 7.0×10^{18} cm^{-3} and E_g = 1.42 eV, giving

$$n_i = \sqrt{\left(4.7\times10^{17}\right)\times\left(7.0\times10^{18}\right)\times\left(\frac{T}{300}\right)^3\times\exp\left(\frac{-1.42}{\left(8.62\times10^{-5}\right)\times T}\right)}$$

$$\therefore n_i = 3.49\times10^{14}T^{3/2}\exp\left(\frac{-8.24\times10^3}{T}\right)$$

The equation is plotted below.

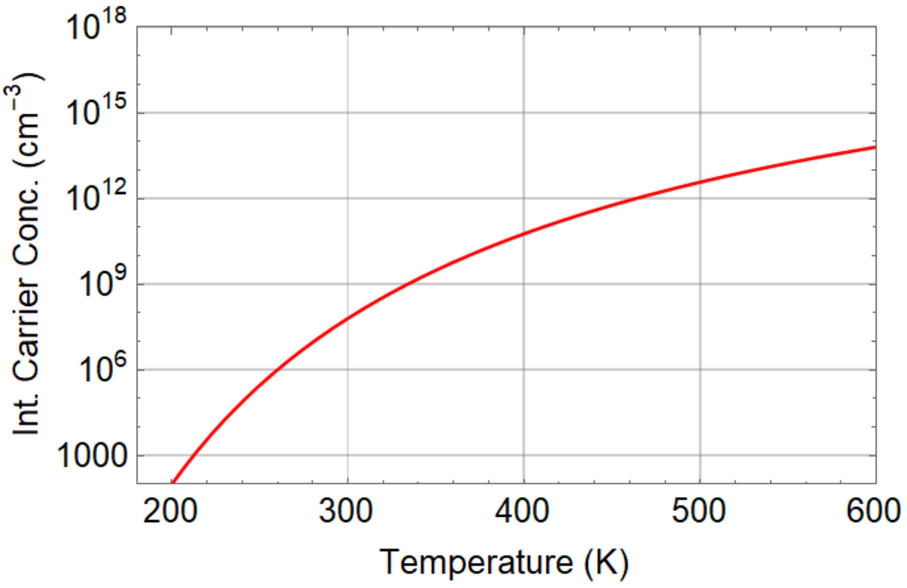


The third and last material to consider is germanium, for which N_{e0} = 1.04×10^{19} cm^{-3}, N_{v0} = 6.0×10^{18} cm^{-3} and E_g = 0.66 eV, giving

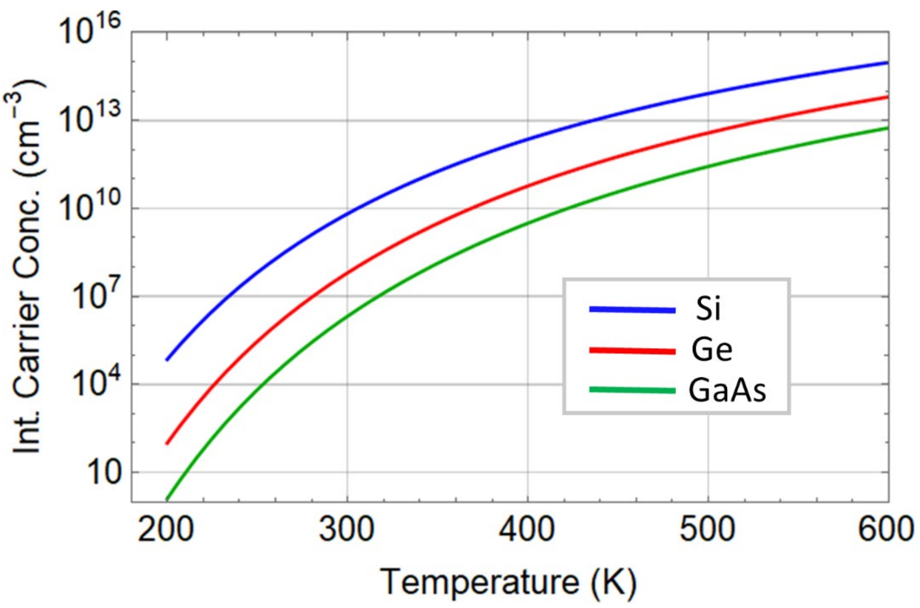
$$n_i = \sqrt{\left(1.04\times10^{19}\right)\times\left(6.0\times10^{18}\right)\times\left(\frac{T}{300}\right)^3\times\exp\left(\frac{-0.66}{\left(8.62\times10^{-5}\right)\times T}\right)}$$

$$\therefore n_i = 1.52\times10^{15}T^{3/2}\exp\left(\frac{-7.66\times10^3}{T}\right)$$

The equation is plotted below.



We close this problem by plotting the three curves in a single chart:



P.5 ➡ *Solution*

Let $n_{i,1}$ and $n_{i,2}$ denote the ICC at T_1 = 200 K and T_2 = 400 K, respectively; their ratio is

$$\frac{n_{i.2}^2}{n_{i,1}^2} = \frac{\left(7.70\times10^{10}\right)^2}{\left(1.40\times10^{2}\right)^2} = 3.03\times10^{17}$$

or

$$\frac{n_{i.2}^2}{n_{i,1}^2} = \frac{N_{c0}N_{v0}\left(T_2/300\right)^3\exp\left(-E_g/kT_2\right)}{N_{c0}N_{v0}\left(T_1/300\right)^3\exp\left(-E_g/kT_1\right)} \rightarrow 3.03\times10^{17} = \left(\frac{T_2}{T_1}\right)^3\exp\left[-\frac{E_g}{kT_2}-\left(-\frac{E_g}{kT_1}\right)\right]$$

$$\therefore 3.03\times10^{17} = \left(\frac{T_2}{T_1}\right)^3\exp\left[E_g\left(\frac{1}{kT_1}-\frac{1}{kT_2}\right)\right]$$

$$\therefore \ln\left[\frac{3.03\times10^{17}}{\left(T_2/T_1\right)^3}\right] = E_g\left(\frac{1}{kT_1}-\frac{1}{kT_2}\right)$$

$$\therefore E_g = \frac{\ln\left[\frac{3.03\times10^{17}}{\left(T_2/T_1\right)^3}\right]}{\left(\frac{1}{kT_1}-\frac{1}{kT_2}\right)} = \frac{\ln\left[\frac{3.03\times10^{17}}{\left(400/200\right)^3}\right]}{\frac{1}{8.62\times10^{-5}}\left(\frac{1}{200}-\frac{1}{400}\right)} = \boxed{1.32\,\text{eV}}$$

Lastly, product $N_{c0}\times N_{v0}$ is expressed as

$$n_{i,1}^2 = N_{c0}N_{v0}\left(\frac{T_1}{300}\right)^3\exp\left(-\frac{E_g}{kT_1}\right) \rightarrow \left(1.40\times10^2\right)^2 = N_{c0}N_{v0}\times\left(\frac{200}{300}\right)^3\times\exp\left[-\frac{1.32}{\left(8.62\times10^{-5}\right)\times200}\right]$$

$$\therefore N_{c0}N_{v0} = \frac{\left(1.40\times10^2\right)^2}{\left(\frac{200}{300}\right)^3\times\exp\left[-\frac{1.32}{\left(8.62\times10^{-5}\right)\times200}\right]} = \boxed{1.18\times10^{38}\,\text{cm}^{-6}}$$

P.6 ➡ *Solution*

Problem 6.1: Recall that the thermal-equilibrium concentration of electrons, n_0, can be stated as

$$n_0 = N_c\exp\left[\frac{-\left(E_c-E_F\right)}{kT}\right] \quad \text{(I)}$$

For silicon, $N_c = 2.8\times10^{19}$ cm^{-3}; solving for $E_c - E_F$ brings to

$$n_0 = N_c\exp\left[\frac{-\left(E_c-E_F\right)}{kT}\right] \rightarrow \frac{n_0}{N_c} = \exp\left[\frac{-\left(E_c-E_F\right)}{kT}\right]$$

$$\therefore \ln\left(\frac{n_0}{N_c}\right) = -\frac{\left(E_c-E_F\right)}{kT}$$

$$\therefore E_c-E_F = -kT\ln\left(\frac{n_0}{N_c}\right) = -\left(8.62\times10^{-5}\right)\times300\times\ln\left(\frac{7\times10^{15}}{2.8\times10^{19}}\right) = \boxed{0.214\,\text{eV}}$$

Problem 6.2: To find $E_c - E_F$, we appeal to the definition of bandgap energy, which equals 1.12 eV for silicon,

$$E_g = E_c - E_v \to -E_v = E_g - E_c$$
$$\therefore -E_v + E_F = E_g - E_c + E_F$$
$$\therefore E_F - E_v = E_g - (E_c - E_F)$$
$$\therefore E_F - E_v = 1.12 - 0.214 = \boxed{0.906\,\mathrm{eV}}$$

Problem 6.3: Equipped with $E_F - E_v$, we can easily determine the thermal-equilibrium concentration of holes, p_0,

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = (1.04\times10^{19})\times\exp\left[-\frac{0.906}{(8.62\times10^{-5})\times300}\right] = \boxed{6.33\times10^{3}\ \mathrm{cm}^{-3}}$$

Problem 6.4: Holes are the minority carriers.

Problem 6.5: To determine the difference $E_F - E_{F,i}$, we manipulate the slightly modified equation for the thermal-equilibrium concentration of electrons,

$$n_0 = n_i \exp\left[\frac{E_F - E_{F,i}}{kT}\right] \to E_F - E_{F,i} = kT\ln\left(\frac{n_0}{n_i}\right)$$
$$\therefore E_F - E_{F,i} = (8.62\times10^{-5})\times300\times\ln\left(\frac{7\times10^{15}}{1.5\times10^{10}}\right) = \boxed{0.338\,\mathrm{eV}}$$

P.7 ➡ *Solution*

Problem 7.1: Note that the thermal-equilibrium concentration of holes, p_0, can be stated as

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

For silicon, $N_v = 1.04\times10^{19}$ cm^{-3}; solving for $E_F - E_v$ brings to

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] \to E_F - E_v = kT\ln\left(\frac{N_v}{p_0}\right)$$
$$\therefore E_F - E_v = (8.62\times10^{-5})\times300\times\ln\left(\frac{1.04\times10^{19}}{2.0\times10^{16}}\right) = \boxed{0.162\,\mathrm{eV}}$$

Problem 7.2: To find $E_F - E_v$, we write

$$E_g = E_c - E_v \to E_c = E_g + E_v$$
$$\therefore E_c - E_F = E_g + E_v - E_F$$
$$\therefore E_c - E_F = E_g - (E_F - E_v)$$
$$\therefore E_c - E_F = 1.12 - 0.162 = \boxed{0.958\,\mathrm{eV}}$$

Problem 7.3: Equipped with $E_c - E_F$, we can determine the thermal-equilibrium concentration of electrons, n_0,

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = (2.8\times10^{19})\times\exp\left[\frac{-0.958}{(8.62\times10^{-5})\times300}\right] = \boxed{2.28\times10^{3}\ \mathrm{cm}^{-3}}$$

Problem 7.4: To determine the difference $E_F - E_{F,i}$, we manipulate the slightly modified equation for the thermal-equilibrium concentration of holes,

$$p_0 = n_i \exp\left[\frac{-(E_F - E_{F,i})}{kT}\right] \to E_F - E_{F,i} = kT\ln\left(\frac{p_0}{n_i}\right)$$
$$\therefore E_F - E_{F,i} = (8.62\times10^{-5})\times300\times\ln\left(\frac{2.0\times10^{16}}{1.5\times10^{10}}\right) = \boxed{0.365\,\mathrm{eV}}$$

P.8 ➡ Solution

Problem 8.1: From the Additional Information section, we read a density of states function $N_c = 4.7\times10^{17}$ cm^{-3} for GaAs at $T = 300$ K. Using the 3/2-power dependence of carrier concentration on temperature, we may write, for the concentration of electrons,

$$n_0 = N_{c0}\left(\frac{T}{300}\right)^{3/2}\exp\left[\frac{-(E_c-E_F)}{kT}\right] = \left(4.7\times10^{17}\right)\times\left(\frac{375}{300}\right)^{3/2}\times\exp\left[\frac{-0.28}{\left(8.62\times10^{-5}\right)\times375}\right] = \boxed{1.14\times10^{14}\text{ cm}^{-3}}$$

To compute the concentration of holes, we first note that $N_v = 7.0\times10^{18}$ cm^{-3} for GaAs at 300 K. To estimate the energy difference $E_F - E_v$, some quick manipulation gives

$$E_F - E_v = E_g - \left(E_c - E_F\right) = 1.42 - 0.28 = 1.14\,\text{eV}$$

so that

$$p_0 = N_{v0}\left(\frac{T}{300}\right)^{3/2}\exp\left[\frac{-(E_F-E_v)}{kT}\right] = \left(7.0\times10^{18}\right)\times\left(\frac{375}{300}\right)^{3/2}\times\exp\left[\frac{-1.14}{\left(8.62\times10^{-5}\right)\times375}\right] = \boxed{4.72\times10^{3}\text{ cm}^{-3}}$$

Problem 8.2: To determine $E_c - E_F$, take the equation for n_0 and solve for this energy difference, giving

$$E_c - E_F = -kT\ln\left(\frac{n_0}{N_c}\right) = -\left(8.62\times10^{-5}\right)\times300\times\ln\left(\frac{1.14\times10^{14}}{4.7\times10^{17}}\right) = \boxed{0.215\,\text{eV}}$$

Before computing p_0, note that difference $E_F - E_V$ at 300 K is stated as

$$E_F - E_v = E_g - \left(E_c - E_F\right) = 1.42 - 0.215 = 1.21\,\text{eV}$$

so that

$$p_0 = N_v\exp\left[\frac{-(E_F-E_v)}{kT}\right] = \left(7.0\times10^{18}\right)\times\exp\left[\frac{-1.21}{\left(8.62\times10^{-5}\right)\times300}\right] = \boxed{3.34\times10^{-2}\text{ cm}^{-3}}$$

P.9 ➡ Solution

Problem 9.1: The probability of electrons occupying states at a given energy under equilibrium conditions is given by the Fermi function,

$$f_P(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

Per the problem statement, we're looking for $f_P(E = E_F)$, namely

$$f_P(E = E_F) = \frac{1}{\underbrace{1+\exp\left(\dfrac{E_F-E_F}{kT}\right)}_{=1}} = \frac{1}{1+1} = \boxed{0.5}$$

Problem 9.2: In this case, we are told that $E_F = E_c$ and the energy of interest is $E = E_c + kT$, so that

$$f_P(E = E_c + kT) = \frac{1}{1+\exp\left[\dfrac{(E_c+kT)-E_c}{kT}\right]} = \frac{1}{1+\exp\left(\dfrac{kT}{kT}\right)}$$

$$\therefore f_P(E = E_c + kT) = \frac{1}{1+e^1} = \boxed{0.269}$$

Problem 9.3: In view of the problem statement, we may write $f_P(E_c + kT) = 1 - f_P(E_c + kT)$ or, evoking the Fermi-Dirac function again,

$$f_P(E_c+kT) = 1 - f_P(E_c+kT) \rightarrow \frac{1}{1+\exp\left[\dfrac{(E_c+kT)-E_F}{kT}\right]} = 1 - \frac{1}{1+\exp\left[\dfrac{(E_c+kT)-E_F}{kT}\right]}$$

$$\therefore \frac{1}{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}=\frac{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}-\frac{1}{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}$$

$$\therefore \frac{1}{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}=\frac{\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}$$

$$\therefore \frac{1}{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}=\frac{1}{\frac{1}{\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}+\frac{\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}{\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}}$$

$$\therefore \frac{1}{1+\exp\left[\frac{(E_c+kT)-E_F}{kT}\right]}=\frac{1}{1+\exp\left[\frac{E_F-E_c-kT}{kT}\right]}$$

Equating exponents,

$$E_c+kT-E_F=E_F-E_c-kT$$
$$\therefore 2E_F=2E_c+2kT$$
$$\therefore \boxed{E_F=E_c+kT}$$

P.10 ➡ *Solution*

Problem 10.1: It can be shown that the probability function of electrons occupying the donor state is

$$n_d=\frac{N_d}{1+\frac{1}{2}\exp\left(\frac{E_d-E_F}{kT}\right)}$$

where n_d is the density of electrons occupying the donor level, N_d is the concentration of donors, E_d is the energy of the donor level, and E_F is the energy of the Fermi level. According to the problem statement, $E_d - E_F = 200$ meV; the probability we aim for is then

$$\frac{n_d}{N_d}=\frac{1}{1+\frac{1}{2}\exp\left(\frac{E_d-E_F}{kT}\right)}=\frac{1}{1+\frac{1}{2}\times\exp\left[\frac{0.2}{\left(8.62\times10^{-5}\right)\times300}\right]}=8.75\times10^{-4}$$

$$\therefore \boxed{\frac{n_d}{N_d}\approx0.09\%}$$

Problem 10.2: Now, we're looking for the probability of finding an electron in the conduction band with energy kT above the conduction-band edge, that is, such that $E - E_c = kT$. Using this difference and the fact that the Fermi level is 245 meV below the conduction band, we write

$$E-E_F=(E-E_c)+(E_c-E_F)=kT+0.245$$

so that, evoking the Fermi-Dirac probability function,

$$f_P(E)=\frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}=\frac{1}{1+\exp\left[\frac{\left(8.62\times10^{-5}\right)\times300+0.245}{\left(8.62\times10^{-5}\right)\times300}\right]}=2.83\times10^{-5}$$

$$\therefore \boxed{\frac{n_d}{N_d}\approx0.003\%}$$

P.11 ➡ Solution

The conductivity is given by the product of elementary charge, electron mobility, and dopant concentration,

$$\sigma = e\mu_n N_d = \left(1.60\times10^{-19}\right)\times1300\times10^{15} = \boxed{0.208\ (\Omega\text{-cm})^{-1}}$$

The resistivity is simply the reciprocal of conductivity,

$$\rho = \frac{1}{e\mu_n N_d} = \frac{1}{0.208} = \boxed{4.81\ \Omega\text{-cm}}$$

P.12 ➡ Solution

Noting that the conductivity of a *p*-type material is calculated on the basis of the hole mobility μ_p, we write

$$\sigma = e\mu_p N_d \rightarrow N_d = \frac{\sigma}{e\mu_p}$$

$$\therefore N_d = \frac{1.80}{\left(1.60\times10^{-19}\right)\times380} = \boxed{2.96\times10^{16}\ \text{cm}^{-3}}$$

P.13 ➡ Solution

All we have to do is write Ohm’s law of resistivity and solve for electron mobility,

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} \rightarrow \sigma = \frac{L}{RA}$$

$$\therefore e\mu_n N_d = \frac{L}{RA}$$

$$\therefore \mu_n = \frac{L}{eN_d RA}$$

$$\therefore \mu_n = \frac{2.5}{\left(1.6\times10^{-19}\right)\times\left(2.0\times10^{15}\right)\times70\times0.1} = \boxed{1120\,\text{cm}^2/\text{V}\cdot\text{s}}$$

P.14 ➡ Solution

Noting that $\mu_n = b\mu_p$ and $p = n_i^2/n$, we restate the conductivity σ,

$$\sigma = e\left(\underbrace{\mu_n}_{=b\mu_p} n + \mu_p \underbrace{p}_{=n_i^2/n}\right) = e\left(b\mu_p n + \frac{\mu_p n_i^2}{n}\right) = e\mu_p\left(bn + \frac{n_i^2}{n}\right)\ \text{(I)}$$

With $n = n_i$, the corresponding intrinsic value is

$$\sigma_i = e\mu_p\left(bn_i + \frac{n_i^2}{n_i}\right) = e\mu_p n_i\left(b+1\right)\ \text{(II)}$$

We’re interested in the maximum resistivity ρ_{max}, which is essentially equivalent to looking for the minimum conductivity σ_{min}; accordingly, we differentiate (I) and set the result to zero, giving

$$\sigma = e\mu_p bn + \frac{e\mu_p n_i^2}{n} \rightarrow \frac{d\sigma}{dn} = e\mu_p b - \frac{e\mu_p n_i^2}{n^2}$$

$$\therefore \cancel{e\mu_p} b = \frac{\cancel{e\mu_p} n_i^2}{n^2}$$

$$\therefore n = \frac{n_i}{\sqrt{b}}$$

Differentiating a second time yields

$$\frac{d^2\sigma}{dn^2} = \frac{e\mu_p n_i^2}{n^3}$$

Substituting $n = n_i/\sqrt{b}$ would give a positive result; hence, $\sigma(n_i/\sqrt{b})$ indeed constitutes a local minimum. Evaluating (I) at $n = n_i/\sqrt{b}$ brings to

$$\sigma_{\min} = e\mu_p\left[b\times\left(\frac{n_i}{\sqrt{b}}\right)+\frac{n_i^2}{\left(n_i/\sqrt{b}\right)}\right] = e\mu_p\left(\sqrt{b}n_i+\sqrt{b}n_i\right) = e\mu_p\times\left(2\sqrt{b}n_i\right)$$

Finally,

$$\frac{\rho_{\max}}{\rho_i} = \frac{1/\sigma_{\min}}{1/\sigma_i} = \frac{\dfrac{1}{e\mu_p\times 2\sqrt{b}n_i}}{\dfrac{1}{e\mu_p n_i\left(b+1\right)}} = \frac{\cancel{e\mu_p}\,\cancel{n_i}\left(b+1\right)}{\cancel{e\mu_p}\times 2\sqrt{b}\,\cancel{n_i}} = \boxed{\frac{b+1}{2\sqrt{b}}}$$

P.15 ➡ Solution

Using the conductivity of the intrinsic material at 300 K, we can determine the corresponding intrinsic carrier concentration n_i,

$$\sigma_{i,300} = en_{i,300}\left(\mu_n+\mu_p\right) \to n_{i,300} = \frac{\sigma_{i,300}}{e\left(\mu_n+\mu_p\right)}$$

$$\therefore\ n_{i,300} = \frac{10^{-6}}{\left(1.60\times10^{-19}\right)\times\left(1000+600\right)} = 3.91\times10^{9}\ \text{cm}^{-3}$$

The conductivity of the material at 500 K can be established from the same equation,

$$\sigma_{i,500} = en_{i.500}\left(\mu_n+\mu_p\right)\ \ \text{(I)}$$

Before proceeding, we need the intrinsic carrier concentration of the material at 500 K; this in turn requires the bandgap energy of the semiconductor material, namely

$$n_{i,300}^2 = N_cN_v\exp\left(\frac{-E_g}{kT}\right) \to E_g = -kT\ln\left(\frac{n_{i,300}^2}{N_cN_v}\right)$$

$$\therefore\ E_g = -\left(8.62\times10^{-5}\right)\times300\times\ln\left[\frac{\left(3.91\times10^{9}\right)^2}{10^{19}\times10^{19}}\right] = 1.12\,\text{eV}$$

so that

$$n_i^2 = N_cN_v\exp\left(\frac{-E_g}{kT}\right) = 10^{19}\times10^{19}\times\exp\left[\frac{-1.12}{\left(8.62\times10^{-5}\right)\times500}\right] = 5.18\times10^{26}\ \text{cm}^{-3}$$

Substituting in (I),

$$\sigma_{i,500} = en_{i.500}\left(\mu_n+\mu_p\right) = \left(1.60\times10^{-19}\right)\times\sqrt{5.18\times10^{26}}\times\left(1000+600\right) = \boxed{5.83\times10^{-3}\left(\Omega\text{-cm}\right)^{-1}}$$

P.16 ➡ Solution

Problem 16.1: The hole diffusion current is given by the product

$$J_{p,\text{diff}} = -eD_p\frac{dp(x)}{dx} = -eD_p\frac{d}{dx}\left[10^{15}\exp\left(\frac{-x}{L}\right)\right] = \frac{10^{15}eD_p}{L}\exp\left(\frac{-x}{L}\right)$$

$$\therefore\ J_{p,\text{diff}} = \frac{10^{15}\times\left(1.6\times10^{-19}\right)\times12}{12\times10^{-4}}\times\exp\left(\frac{-x}{12\times10^{-4}}\right)$$

$$\therefore\ \boxed{J_{p,\text{diff}} = 1.6\exp\left(-833x\right)\ \left[\text{A/cm}^2\right]}$$

with x given in cm.

Problem 16.2: Given the total current density J = 4.8 A/cm^2 and the hole diffusion current density obtained just now, the electron current density can be established with the simple difference

$$J = J_{n,\text{drift}} + J_{p,\text{diff}} \to J_{n,\text{drift}} = J - J_{p,\text{diff}}$$

$$\therefore \boxed{J_{n,\text{drift}} = 4.8 - 1.6\exp(-833x)\ \left[\text{A/cm}^2\right]}$$

Problem 16.3: Recall that electron current density can be expressed as $J_{n,drift} = env_d$, where e is elementary charge, n is electron concentration, and v_d is drift velocity,

$$J_{n,\text{drift}} = env_d$$

Also, $v_d = \mu_n E$. Substituting and solving for E, we obtain

$$J_{n,\text{drift}} = en\mu_n E \rightarrow E = \frac{J_{n,\text{drift}}}{en\mu_n}$$

$$\therefore E = \frac{4.8 - 1.6\exp(-833x)}{\left(1.6\times10^{-19}\right)\times10^{16}\times1000} = \boxed{3 - 1.0\exp(-833x)\ [\text{V/cm}]}$$

P.17 ➡ *Solution*

Appealing to the mean value theorem for integrals, integrate from $y_{min} = 0$ to $y_{max} = t$, then divide by $y_{max} - y_{min} = t - 0 = t$,

$$\sigma_{\text{avg}} = \frac{1}{y_{\max} - y_{\min}}\int_{y_{\min}}^{y_{\max}} \sigma_0 e^{(-y/D)}dy = \frac{\sigma_0}{t}\int_0^t e^{(-y/D)}dy$$

$$\therefore \sigma_{\text{avg}} = -\frac{D\sigma_0}{t}\left[e^{(-y/D)}\Big|_0^t\right] = -\frac{D\sigma_0}{t}\left[e^{(-t/D)} - 1\right] = \frac{D\sigma_0}{t}\left[1 - e^{-(t/D)}\right]$$

$$\therefore \sigma_{\text{avg}} = \frac{0.3\times20}{1.5}\left[1 - e^{-(1.5/0.3)}\right] = \boxed{3.97\,(\Omega\text{-cm})^{-1}}$$

P.18 ➡ *Solution*

Problem 18.1: This conversation starter should be obvious to the student:

$$E = \frac{V}{L} = \frac{2.0}{150\times10^{-6}} = 13{,}300\ \text{V/m} = \boxed{133\ \text{V/cm}}$$

Problem 18.2: The conductivity is written as

$$\sigma(x) = e\mu_n N_d(x)\ \text{(I)}$$

where x is the depth along the thickness of the resistor. If the dopant concentration $N_d(x) = b - ax$ varies linearly from $N_d = 2\times10^{16}$ cm^{-3} at $x = 0$ to $N_d = 2\times10^{15}$ cm^{-3} at $x = 1$ μm = 10^{-4} cm, we may write, for these two data points,

$$N_d(x) = b - ax \rightarrow \begin{cases} 2\times10^{16} = b - a\times0 \\ 2\times10^{15} = b - a\times10^{-4} \end{cases}$$

From the first equation,

$$b = 2\times10^{16}\ \text{cm}^{-3}$$

Substituting in the second,

$$2\times10^{15} = 2\times10^{16} - a\times10^{-4} \rightarrow a = \frac{2\times10^{16} - 2\times10^{15}}{10^{-4}} = 1.8\times10^{20}\ \text{cm}^{-3}$$

Accordingly,

$$N_d(x) = 2\times10^{16} - 1.8\times10^{20}x$$

Substituting in (I) and making use of the mean value theorem, we get

$$\sigma_{\text{avg}} = e\mu_n\left[\frac{1}{10^{-4} - 0}\int_0^{10^{-4}} N_d(x)\right]$$

$$\therefore \sigma_{\text{avg}} = \left(1.6\times10^{-19}\right)\times750\times\left[\frac{1}{10^{-4}}\int_0^{10^{-4}}\left(2\times10^{16} - 1.8\times10^{20}x\right)dx\right]$$

$$\therefore \sigma_{\text{avg}} = \left(1.6\times10^{-19}\right)\times750\times\left[\frac{1}{10^{-4}}\left(2\times10^{16}x - 0.9\times10^{20}x^2\right)\Big|_0^{10^{-4}}\right]$$

$$\therefore \sigma_{\text{avg}} = \left(1.6\times10^{-19}\right)\times750\times\left[\frac{1}{10^{-4}}\left(2\times10^{16}\times10^{-4} - 0.9\times10^{20}\times10^{-8}\right)\right] = \boxed{1.32\,(\Omega\text{-cm})^{-1}}$$

Problem 18.3: The resistance of the resistor is

$$R = \frac{L}{\sigma_{\text{avg}} A} = \frac{150\times10^{-4}}{1.32\times\left[\left(7.5\times10^{-4}\right)\times10^{-4}\right]} = 152\,\text{k}\Omega$$

and the current follows from Ohm's law,

$$I = \frac{V}{R} = \frac{2.0}{152\times10^{3}} = 1.32\times10^{-5}\ \text{A} = \boxed{13.2\,\mu\text{A}}$$

Problem 18.4: The current density can be found by multiplying the conductivity σ by the electric field E. The conductivity at the top surface, wherein $x = 0$, is

$$\sigma(0) = e\mu_n N_d(0) = \left(1.6\times10^{-19}\right)\times750\times\left[2\times10^{16} - 1.8\times10^{20}\times(0)\right] = 2.4(\Omega\text{-cm})^{-1}$$

so that

$$J(0) = \sigma(0)E = 2.4\times133 = \boxed{319\ \text{A/cm}^2}$$

Likewise, at the bottom surface, $x = 10^{-4}$ cm and

$$\sigma\left(10^{-4}\right) = e\mu_n N_d\left(10^{-4}\right) = \left(1.6\times10^{-19}\right)\times750\times\left[2\times10^{16} - 1.8\times10^{20}\times10^{-4}\right] = 0.24(\Omega\text{-cm})^{-1}$$

giving

$$J\left(10^{-4}\right) = \sigma\left(10^{-4}\right)E = 0.24\times133 = \boxed{31.9\ \text{A/cm}^2}$$

P.19 ➡ *Solution*

Taking a standard temperature of 300 K, defining $E_{th} = kT$ and solving for thermal velocity, we obtain

$$E_{th} = kT = \frac{mv_{th}^2}{2} \to v_{th} = \sqrt{\frac{2kT}{m}}$$

$$\therefore v_{th} = \sqrt{\frac{2\times\left(1.38\times10^{-23}\right)\times300}{9.1\times10^{-31}}} = 9.54\times10^{4}\ \text{m/s} = 9.54\times10^{6}\ \text{cm/s}$$

The corresponding drift velocity under an electric field $E = 100$ V/cm is

$$v_d = \mu_n E = 1350\times100 = 1.35\times10^{5}\ \text{cm}$$

Thus, the thermal velocity, v_{th}, is about 70.6 times greater than the drift velocity, v_d. Now, under an electric field $E' = 10{,}000$ V/cm, the theoretical drift velocity becomes

$$v_d' = \mu_n E' = 1350\times10{,}000 = 1.35\times10^{7}\ \text{cm}$$

That is, in this case the drift velocity and the thermal velocity are within approximately 40% of each other; as a result, the linear relationship between drift velocity and electric field intensity is no longer appropriate.

REFERENCES

- NEAMEN, D.A. (2003). *Semiconductor Physics and Devices.* 4th edition. New York: McGraw-Hill.
- PIERRET, R.F. (1996). *Semiconductor Device Fundamentals.* Reading: Addison-Wesley.
- SZE, S.M. and LEE, M.K. (2012). *Semiconductor Devices.* 3rd edition. Hoboken: John Wiley and Sons.