Quiz SM203

BENDING AND SHEAR
PART 1: BASIC PROBLEMS

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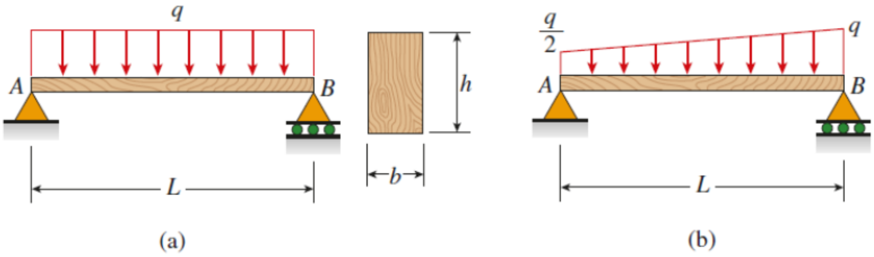
PROBLEMS

Problem 1A (Gere & Goodno, 2009, w/ permission)

A simply supported wood beam *AB* with span length $L = 4$ m carries a uniform load of intensity $q = 5.8$ kN/m (see figure). Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross-section with width $b = 140$ mm and height $h = 240$ mm.



A) $\sigma_{max} = 2.96$ MPa

B) $\sigma_{max} = 4.85$ MPa

C) $\sigma_{max} = 6.74$ MPa

D) $\sigma_{max} = 8.63$ MPa

Problem 1B

Repeat Part A but use the trapezoidal distributed load shown in figure (b).

A) $\sigma_{max} = 2.71$ MPa

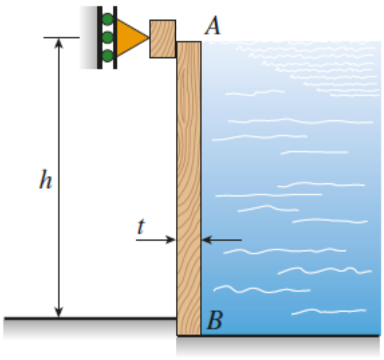
B) $\sigma_{max} = 4.88$ MPa

C) $\sigma_{max} = 6.51$ MPa

D) $\sigma_{max} = 8.45$ MPa

Problem 2 (Gere & Goodno, 2009, w/ permission)

A small dam of height $h = 2.0$ m is constructed of vertical wood beams *AB* of thickness $t = 120$ mm, as shown in the next figure. Consider the beams to be simply supported at the top and bottom. Determine the maximum bending stress σ_{max} in the beams to be simply supported at the top and bottom. Use $\gamma = 9.81$ kN/m³ for water.



A) $\sigma_{\max} = 2.10$ MPa

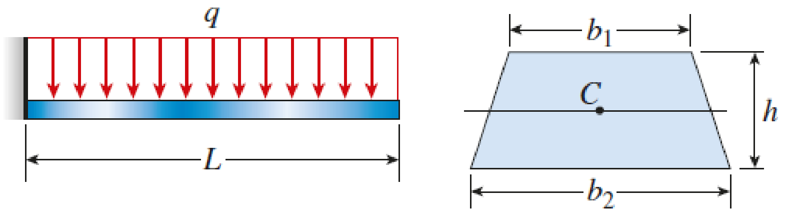
B) $\sigma_{\max} = 4.40$ MPa

C) $\sigma_{\max} = 6.30$ MPa

D) $\sigma_{\max} = 8.20$ MPa

Problem **3A** (Gere & Goodno, 2009, w/ permission)

A cantilever beam *AB* of isosceles trapezoidal cross-section has length L = 0.8 m, dimensions b_1 = 80 mm, b_2 = 90 mm, and height h = 110 mm (see figure). The beam is made of brass weighting 85 kN/m³. Determine the maximum tensile stress σ_t and maximum compressive stress σ_c due to the beam's own weight.



A) $\sigma_t = 1.12$ MPa and $\sigma_c = 0.97$ MPa

B) $\sigma_t = 1.12$ MPa and $\sigma_c = 1.46$ MPa

C) $\sigma_t = 1.51$ MPa and $\sigma_c = 0.97$ MPa

D) $\sigma_t = 1.51$ MPa and $\sigma_c = 1.46$ MPa

Problem **3B**

Consider the following statements.

Statement 1: If the width b_1 is doubled, the maximum tensile stress will be reduced by more than 20%.

Statement 2: If the height h is doubled, the maximum compressive stress will be reduced by more than 20%.

A) Both statements are true.

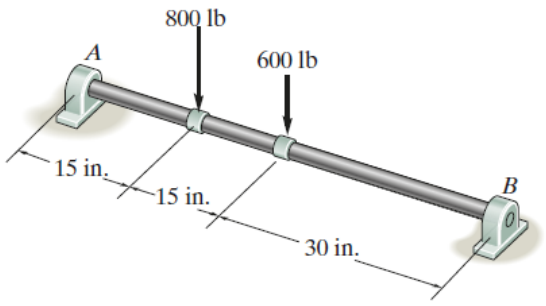
B) Statement 1 is true and statement 2 is false.

C) Statement 1 is false and statement 2 is true.

D) Both statements are false.

Problem **4** (Hibbeler, 2014, w/ permission)

Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The journal bearings at *A* and *B* only support vertical forces. The allowable bending stress is σ_{allow} = 22 ksi.



A) $d = 1.42$ in.

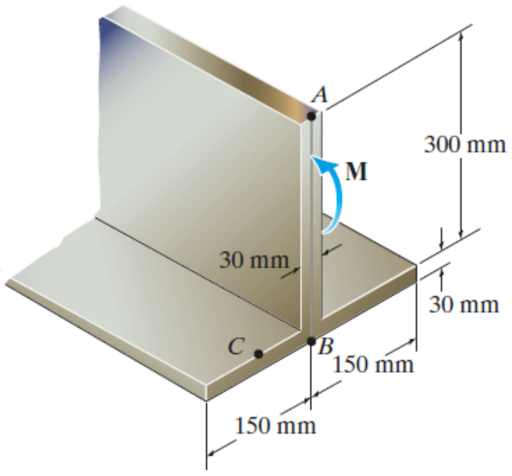
B) $d = 1.91$ in.

C) $d = 2.44$ in.

D) $d = 2.93$ in.

Problem 5A (Hibbeler, 2014, w/ permission)

If the beam is subjected to an internal moment of $M = 100$ kN·m, determine the bending stresses developed at points A, B, and C. True or false?



1. ( ) Point A has a compressive stress of 120.4 MPa.
2. ( ) Point B has a tensile stress of 35.4 MPa.
3. ( ) Point C has a tensile stress of 51.0 MPa.

Problem 5B

If the beam in the previous part is made of material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 125$ MPa and $(\sigma_{\text{allow}})_c = 150$ MPa, respectively, determine the maximum allowable internal moment M that can be applied to the beam.

A) $M = 64.7$ kN·m

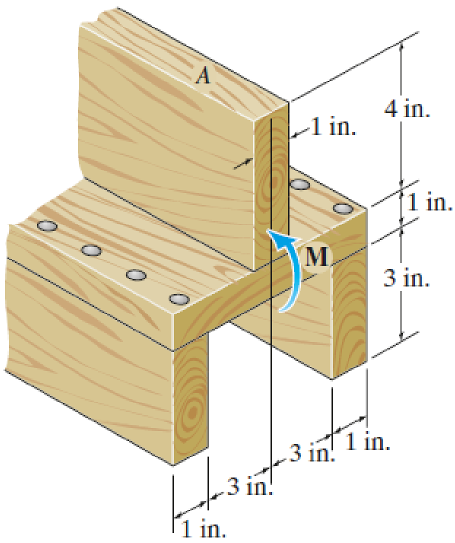
B) $M = 124.6$ kN·m

C) $M = 184.8$ kN·m

D) $M = 244.9$ kN·m

Problem 6A (Hibbeler, 2014, w/ permission)

If the allowable tensile and compressive stress for the beam are $(\sigma_{\text{allow}})_t = 2$ ksi and $(\sigma_{\text{allow}})_c = 3$ ksi, respectively, determine the maximum allowable internal moment M that can be applied to the beam.



A) $M = 1.98$ kip-ft

B) $M = 2.85$ kip-ft

C) $M = 3.77$ kip-ft

D) $M = 4.69$ kip-ft

Problem 6B

If the beam is subjected to an internal moment of $M = 2$ kip-ft, determine the resultant force acting on the top vertical board *A*.

A) $F_R = 1.65$ kip

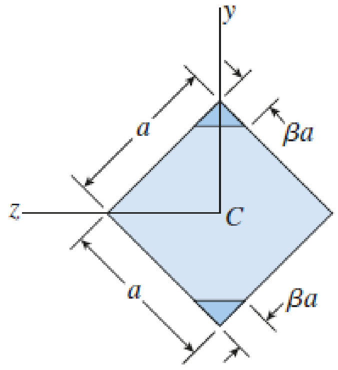
B) $F_R = 2.54$ kip

C) $F_R = 3.43$ kip

D) $F_R = 4.32$ kip

Problem 7 (Gere & Goodno, 2009, w/ permission)

A beam of square cross-section (a = length of each side) is bent in the plane of a diagonal (see figure). By removing a small amount of material at the top and bottom corners, as shown by the shaded triangles in the next figure, we can increase the section modulus and obtain a stronger beam, even though the area of the cross-section is reduced. Determine the ratio β defining the areas that should be removed in order to obtain the strongest cross-section. By what percent is the section modulus increased when the areas are removed?



A) % increase in section modulus = 5.3%

B) % increase in section modulus = 10.2%

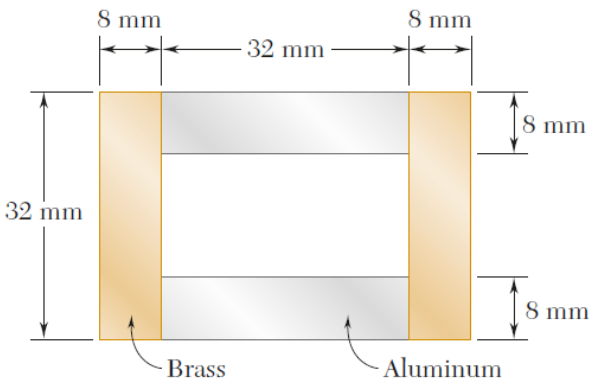
C) % increase in section modulus = 15.1%

D) % increase in section modulus = 20.0%

Problem 8 (Beer et al., 2012, w/ permission)

A bar having a cross-section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of Elasticity	70 GPa	105 GPa
Allowable Stress	100 MPa	160 MPa



A) $M = 612.5$ N·m

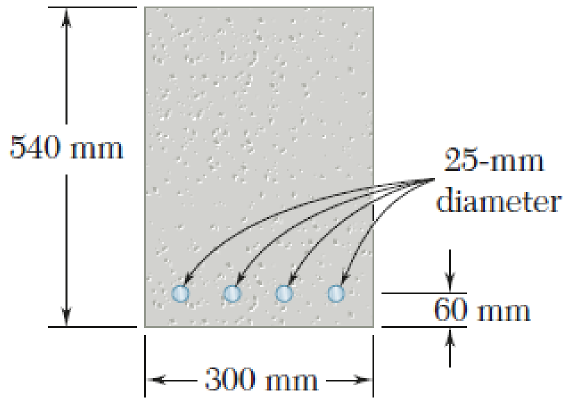
B) $M = 757.5$ N·m

C) $M = 887.5$ N·m

D) $M = 1000.4$ N·m

Problem 9 (Beer et al., 2012, w/ permission)

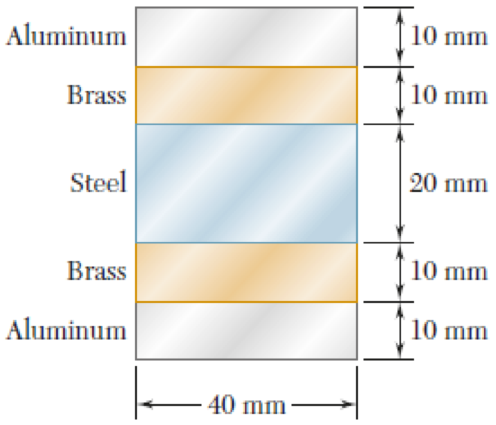
The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine the stress in the steel and the maximum stress in the concrete.



A) $\sigma_s = 104.5$ MPa and $\sigma_c = 15.6$ MPa
B) $\sigma_s = 104.5$ MPa and $\sigma_c = 30.7$ MPa
C) $\sigma_s = 211.2$ MPa and $\sigma_c = 15.6$ MPa
D) $\sigma_s = 211.2$ MPa and $\sigma_c = 30.7$ MPa

Problem 10A (Beer et al., 2012, w/ permission)

Five metal strips, each 40 mm wide, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 1800 N-m, consider the following statements. True or false?



1. ( ) The absolute value of the bending stress in each of the aluminum sections is 62.3 MPa.

2. ( ) The absolute value of the bending stress in each of the brass sections is 93.4 MPa.

3. ( ) The absolute value of the bending stress in the steel section is 124.6 MPa.

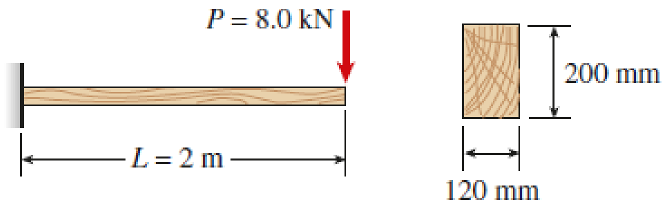
Problem 10B

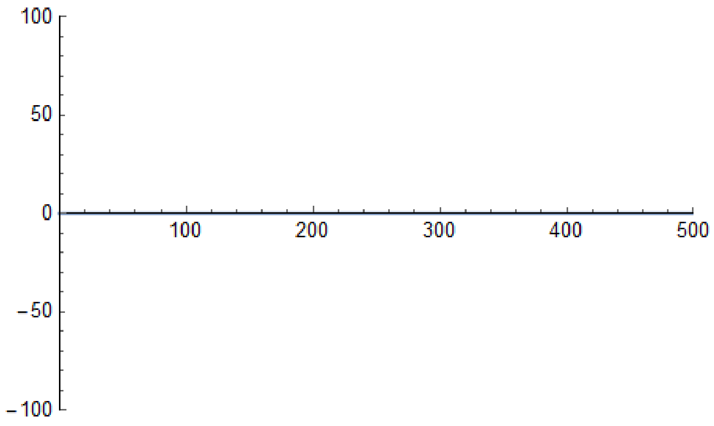
Determine the radius of curvature of the composite beam considered in the previous part.

A) $\rho = 15.9$ m
B) $\rho = 24.8$ m
C) $\rho = 33.7$ m
D) $\rho = 42.6$ m

Problem **11** (Gere & Goodno, 2009, w/ permission)

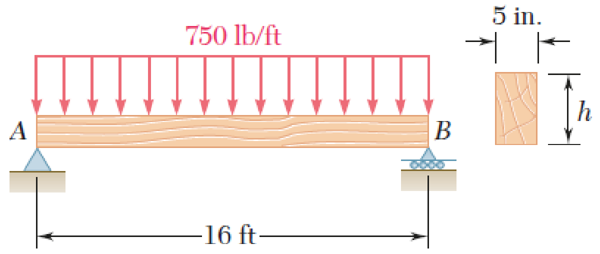
A cantilever beam of length $L = 2$ m supports a load $P = 8.0$ kN (see figure). The beam is made of wood with cross-sectional dimensions 120 mm × 200 mm. Calculate the shear stresses due to load P at points located 25 mm, 50 mm, 75 mm, and 100 mm from the neutral axis of the beam. From these results, plot a graph showing the distribution of shear stresses from top to bottom of the beam.





Problem **12** (Beer et al., 2012, w/ permission)

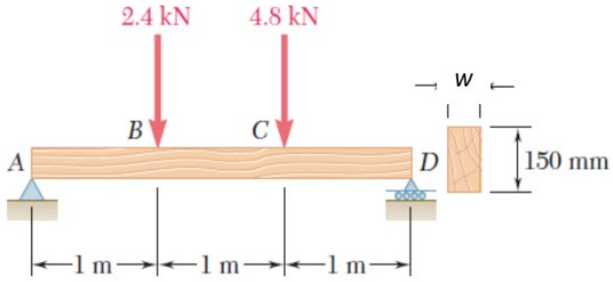
For the beam and loading shown, determine the minimum required depth h, knowing that for the grade of timber used, $\sigma_{allow} = 1750$ psi and $\tau_{allow} = 130$ psi.



A) h_{min} = 6.7 in.
B) h_{min} = 10.6 in.
C) h_{min} = 14.1 in.
D) h_{min} = 18.0 in.

Problem **13** (Beer et al., 2012, w/ permission)

For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{allow} = 12$ MPa and $\tau_{allow} = 825$ kPa.



A) w_{min} = 29;7 mm
B) w_{min} = 48.5 mm
C) w_{min} = 66.6 mm
D) w_{min} = 88.8 mm

SOLUTIONS

P.1 ➡ *Solution*

Part A: The maximum bending moment due to uniform load q is $M_{max} = qL^2/8$. The section modulus for a rectangular beam such as the one in the present case is then

$$S = \frac{I}{h/2} = \frac{bh^3/12}{h/2} = \frac{bh^2}{6}$$

Equipped with M_{max} and S, we can determine the maximum bending stress,

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{qL^2/8}{bh^2/6} = \frac{6qL^2}{8bh^2} \to \sigma_{\max} = \frac{3qL^2}{4bh^2}$$

Substituting the pertaining variables, we ultimately obtain

$$\sigma_{\max} = \frac{3qL^2}{4bh^2} = \frac{3\times 5.8\times 4^2}{4\times 0.14\times 0.24^2} = \boxed{8.63\text{ MPa}}$$

The maximum stress under this distributed load is close to 8.6 megapascals.

The correct answer is **D**.

Part B: From statics, the reaction at support A is determined as $R_A = (1/3)qL$. Now, let the trapezoidal distributed load in question be separated into an uniform load of intensity $q_0/2$ and a triangular load also of intensity $q_0/2$. The maximum bending moment should occur at the location of zero shear. To obtain this position, we consider an equilibrium of forces in the y-direction,

$$\Sigma F_y = 0 \to R_A - \frac{q}{2}x - \frac{1}{2}\left(\frac{x}{L}\times\frac{q}{2}\right)x = 0$$

Substituting R_A and manipulating, we have

$$R_A - \frac{q}{2}x - \frac{1}{2}\left(\frac{x}{L}\times\frac{q}{2}\right)x = 0 \to \frac{1}{3}\cancel{q}L - \frac{1}{2}\cancel{q}x - \frac{1}{4}\frac{\cancel{q}x^2}{L} = 0$$

$$\therefore -\frac{1}{3}L + \frac{1}{2}x + \frac{1}{4}\frac{x^2}{L} = 0$$

$$\therefore 3x^2 + 6Lx - 4L^2 = 0$$

This is a second-degree equation in x. Solving it, we see that

$$x = \frac{-6L \pm \sqrt{84L^2}}{2\times 3} \to x = \frac{-3L + \sqrt{21}L}{3}$$

$$\therefore \frac{x}{L} = \frac{-3+\sqrt{21}}{3} = 0.528$$

$$\therefore x_0 = 0.528L$$

That is, the shear force is zero when x is close to the middle of the beam. Next, we can consider a sum of bending moments and substitute x_0 to obtain the maximum moment M_{max}, i.e.,

$$M_{\max} = R_A \times x_0 - \frac{q}{2} \times \frac{x_0^2}{2} - \frac{1}{2} \times \left(\frac{x_0 \times q}{2L} \right) \times \frac{x_0^2}{3}$$

$$\therefore M_{\max} = \frac{1}{3} qL \times 0.528L - \frac{q}{2} \times \frac{(0.528L)^2}{2} - \frac{1}{2} \times \left(\frac{0.528L \times q}{2L} \right) \times \frac{(0.528L)^2}{3}$$

$$\therefore M_{\max} = 0.176qL^2 - 0.07qL^2 - 0.0123qL^2$$

$$\therefore M_{\max} = 0.0940qL^2$$

Substituting q = 5.8 kN/m and L = 4 m, M_{max} is evaluated as

$$M_{\max} = 0.0940 \times 5.8 \times 4^2 = 8.72 \text{ kN} \cdot \text{m}$$

As before, the section modulus $S = bh^2/6 = 0.14 \times 0.24^2/6 = 0.00134$ m³. Lastly, the maximum bending stress is determined to be

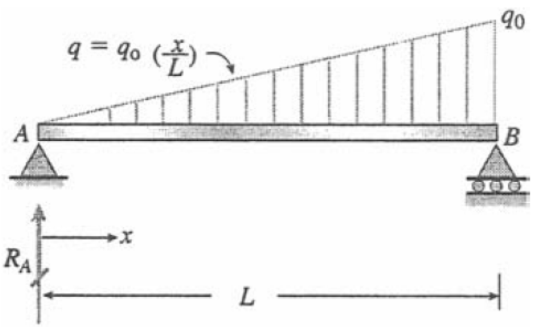
$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{8.72}{0.00134} = \boxed{6.51 \text{ MPa}}$$

That is to say, the maximum stress under the specified trapezoidal load is slightly greater than 6.5 megapascals.

The correct answer is **C**.

P.2 ➡ *Solution*

The intensity of the hydrostatic load increases linearly from top to bottom; thus, the system in question can be equivalently represented by the following loaded beam.



Let $q_0 = \gamma bh$ be the maximum intensity of the distributed load, where γ = 9.81 kN/m³ is the unit weight of water. Reaction $R_A = q_0L/6$ can be obtained from statics. The bending moment in the beam evolves with distance x from point A according to the relationship

$$M = R_A x - \frac{q_0 x^3}{6L}$$

Substituting R_A gives

$$M = \frac{q_0 Lx}{6} - \frac{q_0 x^3}{6L}$$

To find the length x for which the bending moment is maximum, we differentiate this expression with respect to the horizontal coordinate, set it to zero, and solve for x,

$$\frac{dM}{dx} = \frac{q_0 L}{6} - \frac{q_0 x^2}{2L} = 0 \rightarrow \frac{\cancel{q_0} x^2}{2L} = \frac{\cancel{q_0} L}{6}$$

$$\therefore x = \frac{L}{\sqrt{3}}$$

(To verify that this value of x indeed corresponds to a *maximum* bending moment, you can differentiate a second time and verify that $d^2M/dx^2 < 0$ when $x = L/\sqrt{3}$.) Then, the maximum bending moment M follows as

$$M\big|_{x=L/\sqrt{3}} = \frac{q_0 L}{6} \times \left(\frac{L}{\sqrt{3}}\right) - \frac{q_0}{6L} \times \left(\frac{L}{\sqrt{3}}\right)^3 \rightarrow M\big|_{x=L/\sqrt{3}} = \frac{q_0 L^2}{6\sqrt{3}} - \frac{q_0 L^2}{18\sqrt{3}}$$

$$\therefore M\big|_{x=L/\sqrt{3}} = \frac{3q_0 L^2}{18\sqrt{3}} - \frac{q_0 L^2}{18\sqrt{3}} = \frac{q_0 L^2}{9\sqrt{3}}$$

For the horizontal wood beam, the length L becomes equal to the height h of the original beam; that is, $L = h = 2.0$ m. Also, we can replace q_0 with γbh. The maximum bending moment is then

$$M_{\max} = \frac{q_0 L^2}{9\sqrt{3}} \rightarrow M_{\max} = \frac{\gamma bh \times h^2}{9\sqrt{3}} = \frac{\gamma bh^3}{9\sqrt{3}}$$

The section modulus $S = bt^2/6$. Accordingly, the maximum bending stress can be calculated with the flexure formula,

$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{\dfrac{\gamma \not{b} h^3}{9\sqrt{3}}}{\dfrac{\not{b} t^2}{6}} = \frac{6\gamma h^3}{9\sqrt{3}t^2} \rightarrow \sigma_{\max} = \frac{2\gamma h^3}{3\sqrt{3}t^2}$$

Substituting the pertaining variables, we conclude that

$$\sigma_{\max} = \frac{2\gamma h^3}{3\sqrt{3}t^2} \rightarrow \sigma_{\max} = \frac{2 \times 9.81 \times 2.0^3}{3\sqrt{3} \times 0.12^2} = \boxed{2.10 \text{ MPa}}$$

The maximum bending stress in the beams is slightly greater than 2 megapascals.

The correct answer is **A**.

P.3 ➡ *Solution*

Part A: The maximum bending moment M_{max} due to the beam's own weight is

$$M_{\max} = \frac{qL^2}{2}$$

where q equals the product of the unit weight γ of the material of which the beam is made and the cross-section area A of the beam; that is, $q = \gamma A$ = 85,000×½(0.08+0.09)×0.11 = 794.8 N/m. Hence,

$$M_{\max} = \frac{794.8 \times 0.8^2}{2} = 254.3 \text{ N} \cdot \text{m}$$

Next, we proceed to obtain the height $\bar{y}$ of the centroid of the cross-section, which, in the case of a trapezoidal cross-section, is such that

$$\bar{y} = \frac{h(2b_1 + b_2)}{3(b_1 + b_2)} = \frac{0.11 \times (2 \times 0.08 + 0.09)}{3 \times (0.08 + 0.09)} = 0.0539 \text{ m}$$

or 53.9 mm. The moment of inertia, in turn, is

$$I = \frac{h^3 \times (b_1^2 + 4b_1 b_2 + b_2^2)}{36 \times (b_1 + b_2)} = \frac{0.11^3 \times (0.08^2 + 4 \times 0.08 \times 0.09 + 0.09^2)}{36 \times (0.08 + 0.09)} = 9.42 \times 10^{-6} \text{ m}^4$$

The maximum tensile stress at the support occurs in the uppermost fiber of the cross-section and is calculated as

$$\sigma_t = \frac{M_{\max}(h - \bar{y})}{I} = \frac{254.3 \times (0.11 - 0.0539)}{(9.42 \times 10^{-6})} = \boxed{1.51 \text{ MPa}}$$

The maximum compressive stress, in turn, occurs in the bottom fiber of the cross-section and is computed as

$$\sigma_c = \frac{M_{\max}\bar{y}}{I} = \frac{254.3\times0.0539}{\left(9.42\times10^{-6}\right)} = \boxed{1.46\text{ MPa}}$$

The maximum tensile stress is slightly (about 3.4 percent) greater than the maximum compressive stress.

The correct answer is **D**.

Part B: Let b_1 = 2×0.08 = 0.16 m. The cross-sectional area now becomes A = ½(0.16+0.09)×0.11 = 0.0138 m², while the weight of the beam becomes $q = \gamma A$ = 85,000×0.0138 = 1173.0 N/m. The maximum moment is now

$$M_{\max} = \frac{qL^2}{2} = \frac{1173\times0.8^2}{2} = 375.4\text{ N}\cdot\text{m}$$

The position of the centroid of the cross-section is shifted to

$$\bar{y} = \frac{h(2b_1+b_2)}{3(b_1+b_2)} = \frac{0.11\times(2\times0.16+0.09)}{3\times(0.16+0.09)} = 0.0601\text{ m}$$

or 60.1 mm. The moment of inertia also changes,

$$I = \frac{h^3\left(b_1^2+4b_1b_2+b_2^2\right)}{36(b_1+b_2)} = \frac{0.11^3\times\left(0.16^2+4\times0.16\times0.09+0.09^2\right)}{36\times(0.16+0.09)} = 1.35\times10^{-5}\text{ m}^4$$

The maximum tensile stress is updated as

$$\sigma_t = \frac{M_{\max}(h-\bar{y})}{I} = \frac{375.4\times(0.11-0.0601)}{\left(1.35\times10^{-5}\right)} = 1.39\text{ MPa}$$

We therefore have a decrease of 1 − 1.39/1.51 ≈ 7.95% relative to the initial tensile stress. As a result, Statement 1 is false. Suppose now that h = 220 mm instead of 110 mm. The cross-sectional area now becomes A= ½(0.08 + 0.09)×0.22 = 0.0187 m², while the weight of the beam becomes $q = \gamma A$ = 85,000×0.0187 = 1589.5 N/m. The maximum bending moment is then

$$M_{\max} = \frac{qL^2}{2} = \frac{1589.5\times0.8^2}{2} = 508.6\text{ N}\cdot\text{m}$$

The height of the centroid, in turn, is changed to

$$\bar{y} = \frac{h(2b_1+b_2)}{3(b_1+b_2)} = \frac{0.22\times(2\times0.08+0.09)}{3\times(0.08+0.09)} = 0.108\text{ m}$$

or 108 mm. The moment of inertia is also no longer the same,

$$I = \frac{h^3\left(b_1^2+4b_1b_2+b_2^2\right)}{36(b_1+b_2)} = \frac{0.22^3\times\left(0.08^2+4\times0.08\times0.09+0.09^2\right)}{36\times(0.08+0.09)} = 7.53\times10^{-5}\text{ m}^4$$

Finally, the maximum compressive stress becomes

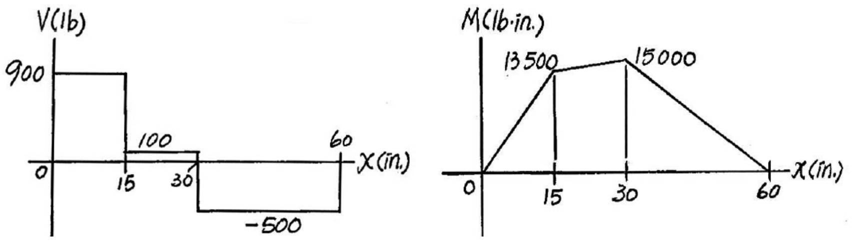
$$\sigma_C = \frac{M_{\max}\bar{y}}{I} = \frac{508.6\times0.108}{\left(7.53\times10^{-5}\right)} = 0.730\text{ MPa}$$

We therefore have a decrease of 1 – 0.730/1.46 =50% relative to the compressive stress calculated in the previous part. Consequently, Statement 2 is true.

The correct answer is **C**.

P.4 ➡ Solution

The shear and moment diagrams are shown in continuation. As indicated in the moment diagram, the maximum bending moment in the structure is M_{max} = 15,000 lb-in.



The maximum moment of inertia of the cross-section about the neutral axis is

$$I = \frac{\pi \times (d/2)^4}{4} = \frac{\pi d^4}{64}$$

To determine the minimum diameter of the shaft, we make use of the flexure formula,

$$\sigma_{\text{allow}} = \frac{M_{\text{max}} c}{I}$$

Here, $c = d/2$. Substituting this and other pertaining variables gives

$$\sigma_{\text{allow}} = \frac{M_{\text{max}} c}{I} \rightarrow 22,000 = \frac{15,000 \times d/2}{\frac{\pi d^4}{64}}$$

$$\therefore 22,000 = \frac{15,000 \times 32}{\pi \times d^3}$$

$$\therefore d = \sqrt[3]{\frac{15,000 \times 32}{\pi \times 22,000}} = \boxed{1.91 \text{ in.}}$$

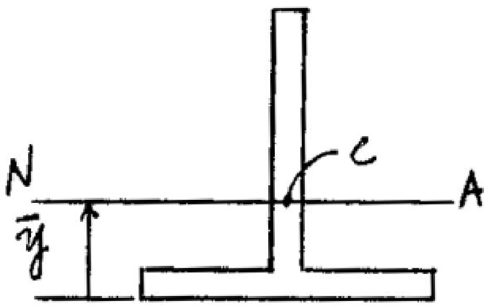
A diameter of 2 inches would be adequate.

The correct answer is **B**.

P.5 ➡ Solution

Part A: The neutral axis passes through the centroid of the cross-section, as shown in the next figure. The location of the centroid is determined as

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015 \times (0.03 \times 0.3) + 0.18 \times (0.3 \times 0.03)}{0.03 \times 0.3 + 0.3 \times 0.03} = 0.0975 \text{ m}$$



Then, the moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} \times (0.3 \times 0.03^3) + 0.3 \times 0.03 \times (0.0975 - 0.015)^2 + \frac{1}{12} \times (0.03 \times 0.3^3) +$$

$$+ 0.03 \times 0.3 \times (0.18 - 0.0975)^2 = 1.91 \times 10^{-4} \text{ m}^4$$

The distance from the neutral axis to points A, B, and C is, in each case,

$$\bar{y}_A = (0.30+0.03) - 0.0975 = 0.23 \text{ m}$$

$$\bar{y}_B = 0.0975 \text{ m}$$

$$\bar{y}_C = 0.0975 - 0.03 = 0.0675 \text{ m}$$

The corresponding bending stresses, labeled as σ_A, σ_B, and σ_C, are such that

$$\sigma_A = \frac{My_A}{I} = \frac{(100\times10^3)\times0.23}{1.91\times10^{-4}} = 120.4 \text{ MPa (C)}$$

$$\sigma_B = \frac{My_B}{I} = \frac{(100\times10^3)\times0.0975}{1.91\times10^{-4}} = 51.0 \text{ MPa (T)}$$

$$\sigma_C = \frac{My_C}{I} = \frac{(100\times10^3)\times0.0675}{1.91\times10^{-4}} = 35.4 \text{ MPa (T)}$$

The first stress is compressive because point A is located above the neutral axis, whereas the latter two stresses are tensile because points B and C are located below the neutral axis.

Statement **1** is true, whereas statements **2** and **3** are false.

Part B: The neutral axis passes through centroid C of the cross-section. The position of the centroid remains as $\bar{y}$ = 0.0975 m and the moment of inertia still equals 1.91×10^{-4} m^4. The maximum compressive and tensile stress occurs at the topmost and bottommost fibers of the cross-section, respectively. For the topmost fiber, we have

$$(\sigma_{\text{allow}})_C = \frac{Mc}{I} \rightarrow 150\times10^6 = \frac{M\times[(0.30+0.03)-0.0975]}{1.91\times10^{-4}}$$

$$\therefore M = \frac{150\times10^6\times1.91\times10^{-4}}{0.23}$$

$$\therefore M = 124.6 \text{ kN}\cdot\text{m}$$

For the bottommost fiber, in turn, we see that

$$(\sigma_{\text{allow}})_t = \frac{Mc}{I} \rightarrow 125\times10^6 = \frac{M\times0.0975}{1.91\times10^{-4}}$$

$$\therefore M = \frac{125\times10^6\times1.91\times10^{-4}}{0.0975}$$

$$\therefore M = 244.9 \text{ kN}\cdot\text{m}$$

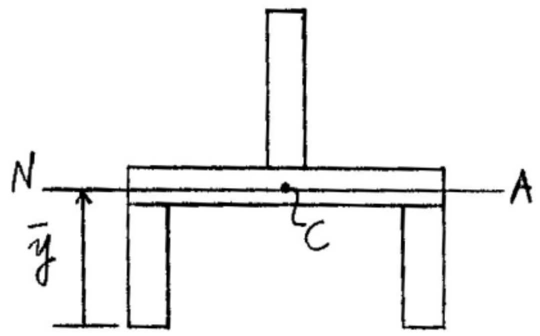
The lower result controls the design of the beam; hence, we take M = 124.6 kN·m as the maximum allowable internal moment.

The correct answer is **B**.

P.6 ➡ *Solution*

Part A: As usual, we consider that the neutral axis passes through the centroid C of the cross-section, as shown in the next figure. The location of C is given by

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{(2\times1.5\times3\times1+3.5\times8\times1+6\times4\times1)}{2\times3\times1+8\times1+4\times1} = 3.39 \text{ in.}$$



The moment of inertia of the cross-section about the neutral axis can be obtained with the parallel axis theorem,

$$I = \Sigma \overline{I} + Ad^2 \rightarrow I = 2\times\left(\frac{1}{12}\times1\times3^3 + 1\times3\times(3.39-1.5)^2\right) - \frac{1}{12}\times8\times1^3 + 8\times(3.5-3.39)^3$$

$$+\frac{1}{12}\times1\times4^3 + 1\times4\times(6-3.39)^2 = 57.9\text{ in.}^4$$

The maximum compressive and tensile stress occurs at the topmost and bottommost fibers of the cross-section, respectively. Both can be determined from the flexure formula. For the maximum allowable compressive stress, we have

$$(\sigma_{\text{allow}})_C = \frac{Mc}{I} \rightarrow (\sigma_{\text{allow}})_C = \frac{M\times[(4+1+3)-3.39]}{57.9} = 3$$

$$\therefore 0.0796M = 3$$

$$\therefore M = 37.7\text{ kip-in.} = 3.14\text{ kip-ft}$$

while for the maximum allowable tensile stress, we have

$$(\sigma_{\text{allow}})_t = \frac{Mc}{I} \rightarrow \frac{M\times3.39}{57.9} = 2$$

$$\therefore 0.0585M = 2$$

$$\therefore M = 34.2\text{ kip-in.} = 2.85\text{ kip-ft}$$

Obviously, the lower value governs the selection of the maximum internal moment. Accordingly, we take M = 2.85 kip-ft.

The correct answer is **B**.

Part B: As before, the neutral axis passes through the centroid *C* of the cross-section. The height of the centroid remains as $\bar{y}$ = 3.39 in., and the moment of inertia still is I = 57.9 in.^{4}. The distances from the neutral axis to the top and bottom of board A are $y_t = 8 - 3.39 = 4.61$ in. and $y_b = 4 - 3.39 = 0.61$ in. The corresponding stresses are, respectively,

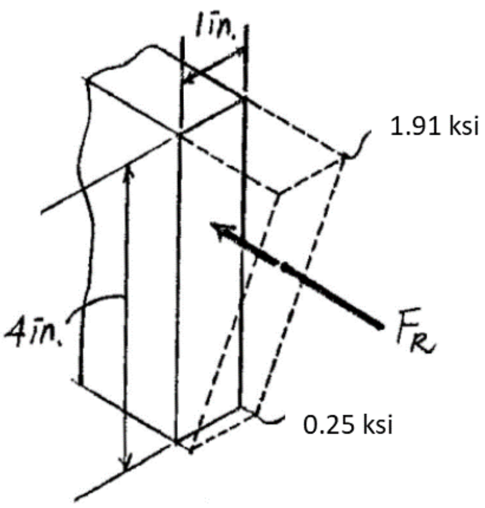
$$\sigma_t = \frac{My_t}{I} = \frac{(2\times12)\times4.61}{57.9} = 1.91\text{ ksi}$$

$$\sigma_c = \frac{My_c}{I} = \frac{(2\times12)\times0.61}{57.9} = 0.25\text{ ksi}$$

The resultant force exerted on board A equals the volume of the trapezoidal element illustrated below. Mathematically,

$$F_R = \left[\frac{1}{2}\times(1.91+0.25)\times4\right]\times1 = \boxed{4.32\text{ kip}}$$

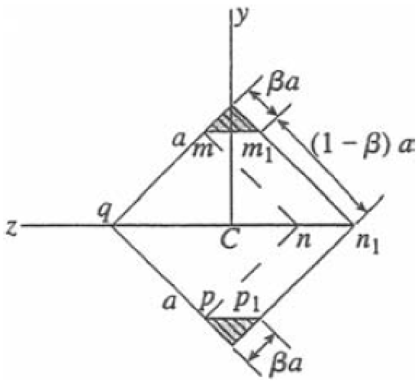
The force in question is close to 4.3 kilopounds.



The correct answer is **D**.

P.7 ➡ *Solution*

Consider the following sketch.



The entire cross-section, which we label as area 0, has parameters

$$I_0 = \frac{a^4}{12} \;;\; c_0 = \frac{a}{\sqrt{2}} \;;\; S_0 = \frac{I_0}{c_0} = \frac{a^4/12}{a/\sqrt{2}} = \frac{\sqrt{2}a^3}{12}$$

Next, square *mnpq*, labeled area 1, has side $(1-\beta)a$ and corresponding moment of inertia given by

$$I_1 = \frac{\left[(1-\beta)a\right]^4}{12} = \frac{1}{12}(1-\beta)^4 a^4$$

The moment of inertia of parallelogram mm_1n_1n, in turn, is established as

$$I_2 = \frac{1}{3}\times\left(\beta a\sqrt{2}\right)\times\left[\frac{(1-\beta)a}{\sqrt{2}}\right]^3 = \frac{1}{3}\times\beta a\cancel{\sqrt{2}}\times\frac{(1-\beta)^3 a^3}{2\cancel{\sqrt{2}}}$$

$$\therefore I_2 = \frac{1}{6}\beta(1-\beta)^3 a^4$$

Reduced cross-section $qmm_1n_1p_1pq$ has a moment of inertia given by

$$I = I_1 + 2I_2 = \frac{1}{12}(1-\beta)^4 a^4 + \frac{1}{3}\beta(1-\beta)^3 a^4 \rightarrow I = \frac{1}{12}(1-\beta)^4 a^4 + \frac{4}{12}\beta(1-\beta)^3 a^4$$

$$\therefore I = \frac{a^4}{12}\times\left[(1-\beta)^4 + 4\beta(1-\beta)^3\right]$$

$$\therefore I = \frac{a^4}{12}\times(1-\beta)^3\times\left[(1-\beta)+4\beta\right]$$

$$\therefore I = \frac{a^4}{12}(1-\beta)^3(1+3\beta)$$

Also, the reduced cross-section has $c = (1-\beta)a/\sqrt{2}$, so that

$$S=\frac{I}{c}=\frac{\frac{a^4}{12}(1-\beta)^3(1+3\beta)}{\frac{(1-\beta)a}{\sqrt{2}}}=\frac{\sqrt{2}a^3}{12}(1+3\beta)(1-\beta)^2$$

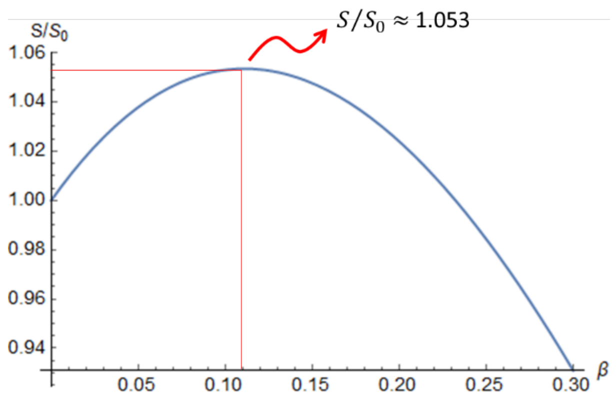
The ratio of the section modulus *S* of the reduced cross section to the section modulus S_0 of the original cross-section is such that

$$\frac{S}{S_0}=\frac{\frac{\sqrt{2}a^3}{12}(1+3\beta)(1-\beta)^2}{\frac{\sqrt{2}a^3}{12}}\rightarrow\frac{S}{S_0}=(1+3\beta)(1-\beta)^2$$

The expression in the right-hand side above is a third-degree polynomial in β. We want to determine the parameter β that produces the maximum section modulus *S* of the modified section relatively to the section modulus S_0 of the initial cross-section. This can be done by optimizing the relation above. To do so, we differentiate the expression and set it to zero, giving

$$\frac{S}{S_0}=(1+3\beta)(1-\beta)^2=3\beta^3-5\beta^2+\beta+1\rightarrow\frac{d}{d\beta}\left(\frac{S}{S_0}\right)=9\beta^2-10\beta+1=0$$

The result is a second-degree equation that can be easily solved for β. The solutions are $\beta=1$, which produces the trivial result $S=S_0$, and $\beta=1/9\approx 0.11$, which is the value we seek. To verify that this value indeed produces a maximum S/S_0 ratio, we can verify the sign of the second derivative or plot S/S_0 over an interval of values of β. Indeed, a graph of S/S_0 versus β shows that this function reaches a local maximum at the domain 0.11; see below.



Substituting this solution into the expression for S/S_0 gives

$$\left.\frac{S}{S_0}\right|_{\beta=0.11}=(1+3\times 0.11)(1-0.11)^2=1.053$$

That is to say, the section modulus is increased by about 5.3 percent after the beam is modified.

The correct answer is **A**.

P.8 ➡ *Solution*

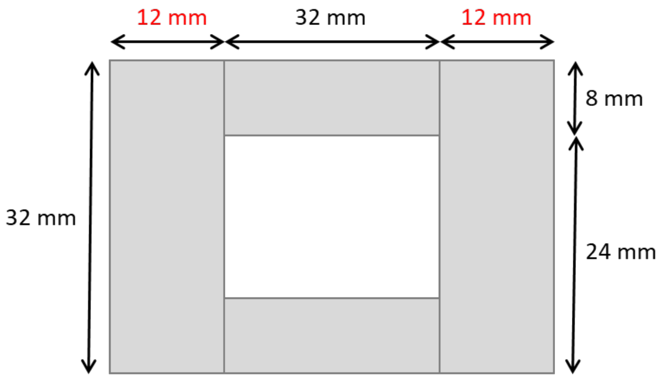
We begin by calculating the ratio of the moduli of elasticity of brass and aluminum,

$$n=\frac{E_b}{E_a}=\frac{105}{70}=1.5$$

Next, we convert the brass member into an equivalent aluminum member by multiplying its width by the modular ratio; that is,

$$b_a=n\times b_b\rightarrow b_a=1.5\times 8=12\text{ mm}$$

The two lateral brass rectangles are replaced with a pair of larger aluminum rectangles, each one having a width of 12 mm rather than 8 mm, as shown.



The moment of inertia of the transformed section can be obtained by considering the full cross-section, including the hollow rectangle in the middle, then subtracting the moment of the hollow rectangle. Mathematically,

$$I=\frac{(32+2\times 12)\times 32^3}{12}-\frac{32\times(32-2\times 8)^3}{12}=142{,}000\ \text{mm}^4=1.42\times 10^{-7}\ \text{m}^4$$

The maximum bending moment for this member can be determined by means of the flexure formula. For the aluminum section, we have

$$(\sigma_{\text{allow}})_{\text{Al}}=\frac{Mc}{I}$$

where $(\sigma_{\text{allow}})_{\text{Al}}=100$ MPa is the allowable stress in the aluminum member; substituting this and other variables gives

$$(\sigma_{\text{allow}})_{\text{Al}}=\frac{Mc}{I}\rightarrow 100\times 10^6=\frac{M\times 0.016}{1.42\times 10^{-7}}$$

$$\therefore M=\frac{100\times 10^6\times 1.42\times 10^{-7}}{0.016}=887.5\ \text{N}\cdot\text{m}$$

Proceeding similarly with the brass section, we have

$$M=\frac{(\sigma_{\text{allow}})_{\text{b}}I}{nc}$$

where $(\sigma_{\text{allow}})_b=160$ MPa is the allowable stress for the brass. Note that we have added the modular ratio *n* in the denominator. Substituting the pertaining variables yields

$$M=\frac{(\sigma_{\text{allow}})_{\text{b}}c}{nI}\rightarrow M=\frac{(160\times 10^6)\times 0.016}{1.5\times(1.42\times 10^{-7})}$$

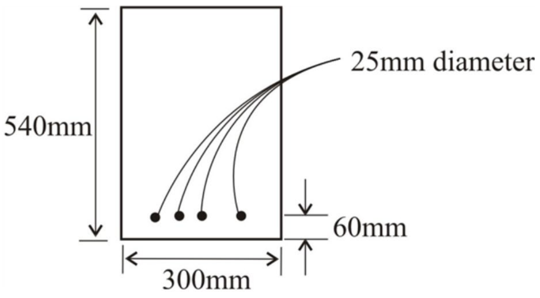
$$\therefore M=\frac{(160\times 10^6)\times(1.42\times 10^{-7})}{1.5\times 0.016}=946.7\ \text{N}\cdot\text{m}$$

Beam design and analysis dictates that the smaller moment should be taken as the final answer. Accordingly, M = 887.5 N·m is the largest permissible bending moment.

The correct answer is **C**.

P.9 ➡ *Solution*

The structure in question is illustrated below.



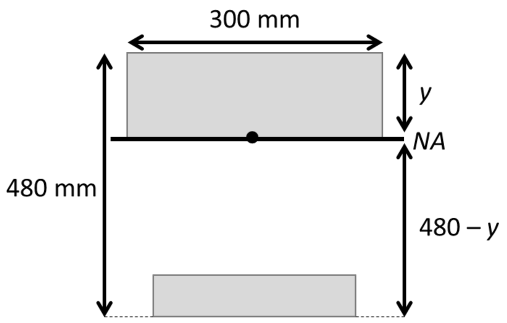
The modular ratio is $n = 200/25 = 8$. The concrete receives the compressive load, while the steel takes the tensile load. Transforming the steel section to a concrete section will increase the area of the steel rods n-fold. In mathematical terms,

$$A_s' = A_s \times n$$

where A_s' is the area of the transformed section and A_s is the area of one of the steel rods. Substituting the pertaining variables brings to

$$A_s' = \left(4 \times \frac{\pi}{4} \times 25^2\right) \times 8 = 15,708 \text{ mm}^2 = 0.0157 \text{ m}^2$$

Thereafter, let y be the distance from the upper fiber of the concrete section to the neutral axis of the beam. The distance from the centroid of the transformed steel section to the neutral axis equals $480 - y$, as shown.



The algebraic sum of moments relative to the neutral axis must equal zero. Thus,

$$300 \times y \times \frac{y}{2} - 15,708 \times (480 - y) = 0 \rightarrow 150y^2 + 15,708y - 7.54 \times 10^6 = 0$$

This expression is a second-degree equation in y. Solving it, one obtains y = 177.9 mm. We can then compute the moment of inertia of the modified section,

$$I = \frac{1}{3}by^3 + A_s' \times (480 - y)^2 = \frac{1}{3} \times 300 \times 177.9^3 + 15,708 \times (480 - 177.9)^2$$

$$\therefore I = 2.00 \times 10^9 \text{ mm}^4 = 0.002 \text{ m}^4$$

The maximum stress in the transformed steel member is

$$\sigma_s' = \frac{M \times (0.48 - y)}{I}$$

where $M = 175$ kN·m is the bending moment to which the concrete beam is subjected. Substituting this and other pertaining variables, we obtain

$$\sigma_s' = \frac{M \times (480 - y)}{I} \rightarrow \sigma_s' = \frac{175,000 \times (0.48 - 0.178)}{0.002} = 26.4 \text{ MPa}$$

The stress in the original steel reinforcement is obtained with the aid of the modular ratio n,

$$\sigma_s = n \times \sigma_s' = 8 \times 26.4 = \boxed{211.2 \text{ MPa}}$$

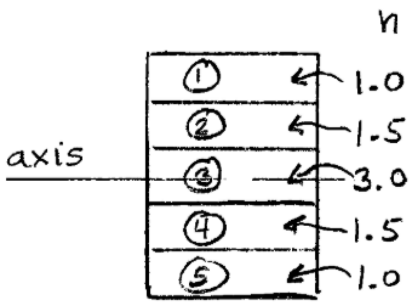
Similarly, the stress in the concrete member is given by

$$\sigma_c = \frac{My}{I} \rightarrow \sigma_c = \frac{175{,}000 \times 0.178}{0.002} = \boxed{15.6 \text{ MPa}}$$

The correct answer is **C**.

P.10 ➡ *Solution*

Part A: Let aluminum be the reference material. We then have $n_1 = 1$ for the aluminum portion of the beam, $n_2 = 105/70 = 1.5$ for brass, and $n_3 = 210/70 = 3$ for steel. Owing to symmetry, the neutral axis is located at the center of the beam, as shown.



Using the parallel-axis theorem, we proceed to determine the moments of inertia for each part of the beam,

$$I_1 = I_5 = \frac{1}{12} \times 40 \times 10^3 + 40 \times 10 \times 25^2 = 253{,}333 \text{ mm}^4$$

$$I_2 = I_4 = 1.5 \times \frac{1}{12} \times 40 \times 10^3 + 1.5 \times 40 \times 10 \times 15^2 = 140{,}000 \text{ mm}^4$$

$$I_3 = 3.0 \times \frac{1}{12} \times 40 \times 20^3 = 80{,}000 \text{ mm}^4$$

$$\therefore I = \Sigma I_i = 866{,}666 \text{ mm}^4 = 8.67 \times 10^{-7} \text{ m}^4$$

The stresses in each part of the beam can be determined with the flexure formula. For the aluminum part of the cross-section, we have the bending stress

$$\sigma_{\text{Al}} = \frac{n_1 My}{I} = \frac{1.0 \times 1800 \times 0.030}{8.67 \times 10^{-7}} = 62.3 \text{ MPa}$$

For the brass portion of the section, in turn, the stress is

$$\sigma_b = \frac{1.5 \times 1800 \times 0.020}{8.67 \times 10^{-7}} = 62.3 \text{ MPa}$$

Finally, the stress in the steel region is

$$\sigma_s = \frac{3 \times 1800 \times 0.010}{8.67 \times 10^{-7}} = 62.3 \text{ MPa}$$

Surprisingly, the stress is the same for all three materials.

Statement **1** is true, whereas statements **2** and **3** are false.

Part B: The radius of curvature can be determined with the moment-curvature relationship,

$$\frac{1}{\rho} = \frac{M}{E_{\text{Al}} I} = \frac{1800}{\left(70 \times 10^9\right) \times \left(8.67 \times 10^{-7}\right)} = 0.0297 \text{ m}^{-1}$$

Inverting this relation, we get $\rho = 33.7$ m.

The correct answer is **C**.

P.11 ➡ *Solution*

As the student surely knows, the shear stress in a rectangular beam such as the one considered herein is

$$\tau = \frac{V}{Ib}\int y\,dA$$

where the integral is the first moment of the area with respect to the neutral axis. In the present case, we have

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

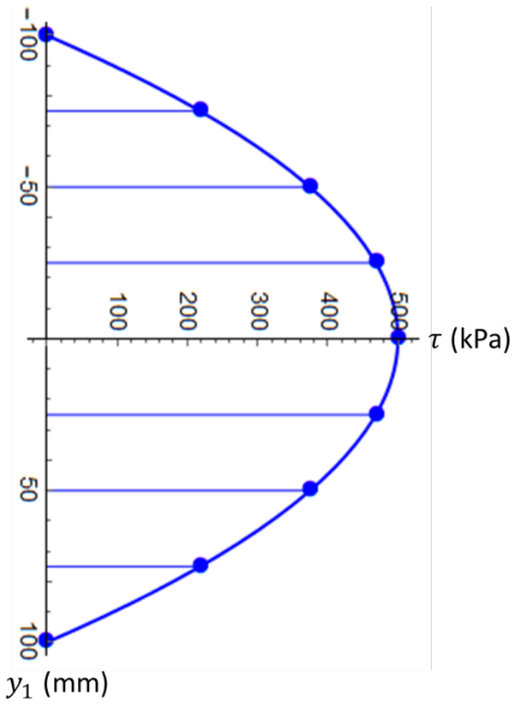
Here, h = 200 mm, V = 8000 N, and I = 120×200³/12 = 8×10⁷ mm⁴, so that

$$\tau = \frac{8000}{2\times\left(8\times10^7\right)}\left(\frac{200^2}{4} - y_1^2\right) \rightarrow \tau = 5\times10^{-5}\left(10{,}000 - y_1^2\right)\ \left[\mathrm{N/mm^2}\right]$$

Note that the equation above gives the shear stress in N/mm², which is numerically equivalent to MPa. We can convert the results to a more convenient unit, such as kPa. Calculations for the distribution of shear stresses from the neutral axis to the top (or bottom) of the beam are performed below.

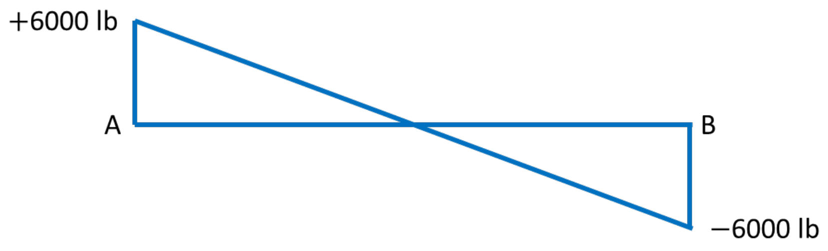
Distance from the top surface (mm)	y_1 (mm)	τ (MPa)	τ (kPa)
0	100	0.000	0.0
25	75	0.219	218.8
50	50	0.375	375.0
75	25	0.469	468.8
100	0	0.500	500.0

The plot we are looking for is one of depth (the blue column) versus shear stress (the red column). The graph in question is shown below.



P.12 ➡ *Solution*

From statics, the vertical reactions at points A and B are determined as R_A = 6000 lb and R_B = −6000 lb. The shear force diagram for is illustrated in continuation.



The cross-sectional area of the beam is $A = bh$, where b is width and h is depth; since b =5 in., we have $A = 5h$. Then, note that the maximum bending moment is located where the shear force equals zero, which, in accordance with the shear force diagram, occurs in the middle of the beam. This moment is calculated as

$$M_{\max} = R_A \times 8 - 750 \times (16/2) \times (16/4)$$
$$\therefore M_{\max} = 6000 \times 8 - 750 \times 8 \times 4 = 24,000 \text{ lb-ft}$$

The section modulus for a rectangular cross-section such as the present one equals

$$S = \frac{1}{6} wh^2 = \frac{1}{6} \times 5 \times h^2 \rightarrow S = 0.83h^2$$

The allowable bending stress, maximum bending moment, and section modulus are related by the flexure formula. Substituting the pertaining variables and solving for h gives

$$\sigma_{\text{all}} = \frac{M_{\max}}{S} \rightarrow S = \frac{M_{\max}}{\sigma_{\text{all}}}$$
$$\therefore 0.83h^2 = \frac{(24,000 \times 12)}{1750}$$
$$\therefore h_{\min} = \sqrt{\frac{(24,000 \times 12)}{0.83 \times 1750}} = 14.1 \text{ in.}$$

Before we can take this value as the definitive height of the beam, we must verify whether the maximum shear stress is exceeded or not. Such a stress is given by

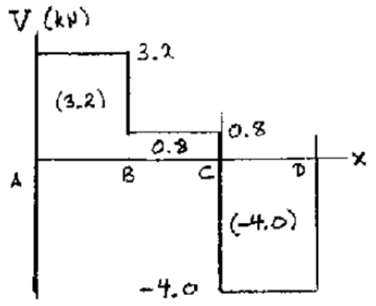
$$\tau_{\text{all}} = \frac{3V_{\max}}{2A} \rightarrow \tau = \frac{3V_{\max}}{2 \times 5h}$$
$$\therefore \tau = \frac{3 \times 6000}{2 \times 5 \times 14.1} = 127.7 \text{ psi} < 130 \text{ psi}$$

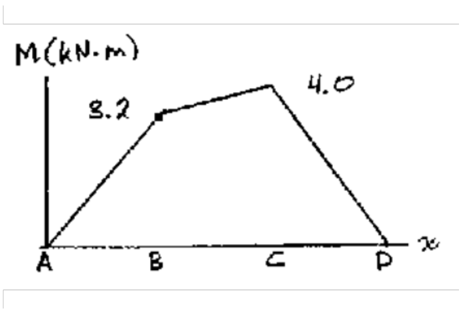
Hence, the depth calculated is acceptable. The beam must have a height of at least 14.1 inches.

The correct answer is **C**.

P.13 ➡ *Solution*

The vertical reactions at A and D can be obtained from statics, and are such that $R_A = 2.4$ kN and $R_B = 4.8$ kN. The shear force and bending moment diagrams are shown below.





Clearly, the maximum shear force is $|V_{max}| = 4.0$ kN and the maximum bending moment is $|M_{max}| = 4.0$ kN·m. The section modulus for a rectangular section such as the one considered here is

$$S = \frac{1}{6}wh^2 = \frac{1}{6}\times w \times 0.15^2 = 0.00375w$$

However, from the flexure formula, we have

$$\sigma_{all} = \frac{M_{max}}{S} \rightarrow S = \frac{M_{max}}{\sigma_{all}}$$

$$\therefore S = \frac{4000}{12\times 10^6} = 3.33\times 10^{-4}\ \text{m}^3$$

Equating the two previous expressions and solving for w, we get

$$0.00375w = 3.33\times 10^{-4} \rightarrow w_{min} = \frac{3.33\times 10^{-4}}{0.00375} = 0.0888\ \text{m} = 88.8\ \text{mm}$$

Next, consider the shear stress in the beam, τ. Applying the pertaining equation and solving for the width w, we obtain

$$\tau_{all} = \frac{3V_{max}}{2A} \rightarrow A = \frac{3V_{max}}{2\tau_{all}}$$

$$\therefore w = \frac{3V_{max}}{2\tau_{all}h} = \frac{3\times 4000}{2\times\left(825\times 10^3\right)\times 0.15}$$

$$\therefore w_{min} = 0.0485\ \text{m} = 48.5\ \text{mm}$$

The required value of w is the larger result, so that $w = 88.8$ mm would be an adequate beam width. The beam should have a minimum width close to 9 centimeters.

The correct answer is **D**.

ANSWER SUMMARY

Problem 1	**1A**	**D**
	1B	**C**
Problem 2		**A**
Problem 3	**3A**	**D**
	3B	**C**
Problem 4		**B**
Problem 5	**5A**	T/F
	5B	**B**
Problem 6	**6A**	**B**
	6B	**D**
Problem 7		**A**
Problem 8		**C**
Problem 9		**C**
Problem 10	**10A**	T/F
	10B	**C**
Problem 11		Open-ended pb.
Problem 12		**C**
Problem 13		**D**

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