Montogue

Quiz SM204

BENDING AND SHEAR: PART 2

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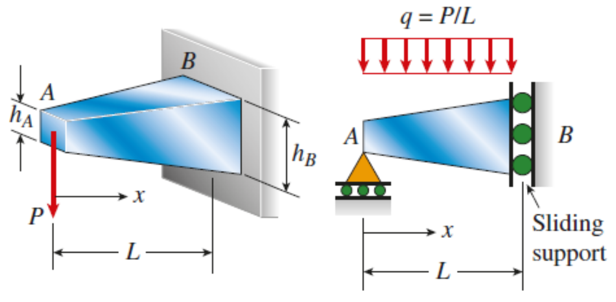
PROBLEMS

Problem 1A (Gere, 2009, w/ permission)

A tapered cantilever beam *AB* of length *L* has square cross-sections and supports a concentrated load *P* at the free end (see figure (a) below). The width and height of the beam vary linearly from h_A at the free end to h_B at the fixed end. Determine the distance *x* from the free end *A* to the cross-section of maximum bending stress if $h_B = 3h_A$. What is the ratio of the maximum stress to the largest stress *B* at the support?



(a) (b)

A) $\sigma_{\max}/\sigma_B = 2.0$
B) $\sigma_{\max}/\sigma_B = 2.5$
C) $\sigma_{\max}/\sigma_B = 3.0$
D) $\sigma_{\max}/\sigma_B = 3.5$

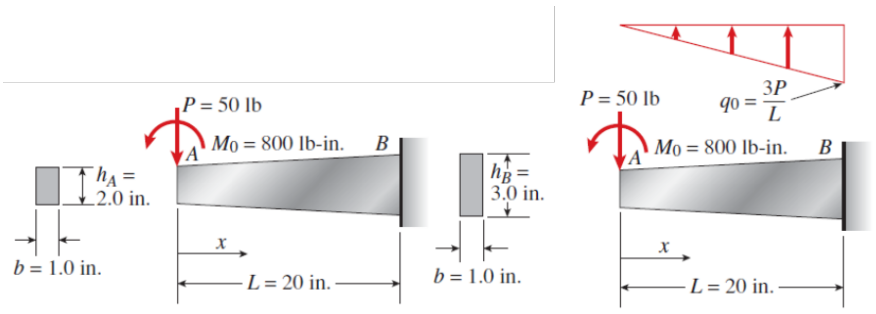
Problem 1B

Repeat Part A if load *P* is now applied as a uniform load of intensity $q = P/L$ over the entire beam, as shown in figure (b) above. Point *A* is restrained by a roller support and *B* is a sliding support.

A) $\sigma_{\max}/\sigma_B = 2.06$
B) $\sigma_{\max}/\sigma_B = 2.55$
C) $\sigma_{\max}/\sigma_B = 3.06$
D) $\sigma_{\max}/\sigma_B = 3.55$

Problem 2A (Gere, 2009, w/ permission)

A tapered cantilever *AB* having rectangular cross-sections is subjected to a concentrated load $P = 50$ lb and a couple $M_0 = 800$ lb-in. acting at the free end. The width *b* of the beam is constant and equal to 1.0 in., but the height varies linearly from $h_A = 2.0$ in. at the loaded end to $h_B = 3.0$ in. at the support. Determine the distance from the free end at which the maximum bending stress occurs. What is the ratio of the maximum stress to the largest stress σ_B at the support?



(a) (b)

A) $\sigma_{\max}/\sigma_B = 1.04$
B) $\sigma_{\max}/\sigma_B = 1.52$
C) $\sigma_{\max}/\sigma_B = 2.08$
D) $\sigma_{\max}/\sigma_B = 2.51$

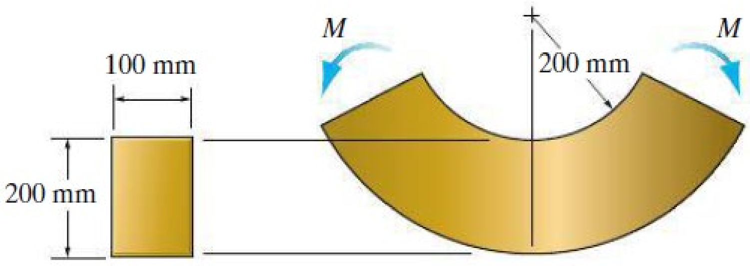
Problem **2B**

Repeat Part A if, in addition to P and M_o, a triangular distributed load with peak intensity $q_o = 3P/L$ acts upward over the entire beam, as shown in figure (b) above. Find the ratio of the maximum stress to the stress at the location of maximum moment.

A) $\sigma_{\max}/\sigma_M = 0.882$
B) $\sigma_{\max}/\sigma_M = 1.21$
C) $\sigma_{\max}/\sigma_M = 1.65$
D) $\sigma_{\max}/\sigma_M = 2.03$

Problem **3** (Hibbeler, 2014, w/ permission)

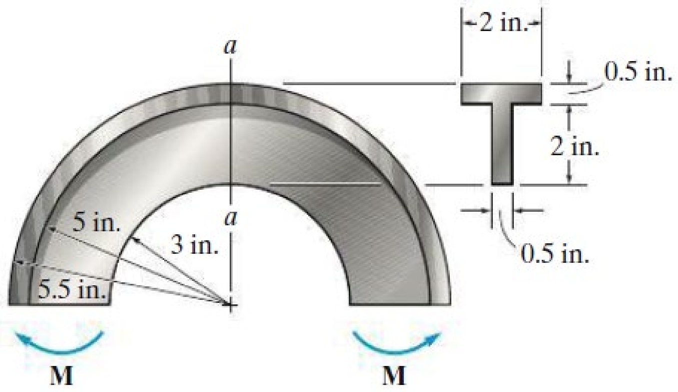
The curved member is made from material having an allowable bending stress of $\sigma_{\text{allow}} = 100$ MPa. Determine the maximum allowable internal moment M that can be applied to the member.



A) $M = 51.8$ kN·m
B) $M = 82.3$ kN·m
C) $M = 113.0$ kN·m
D) $M = 140.2$ kN·m

Problem **4** (Hibbeler, 2014, w/ permission)

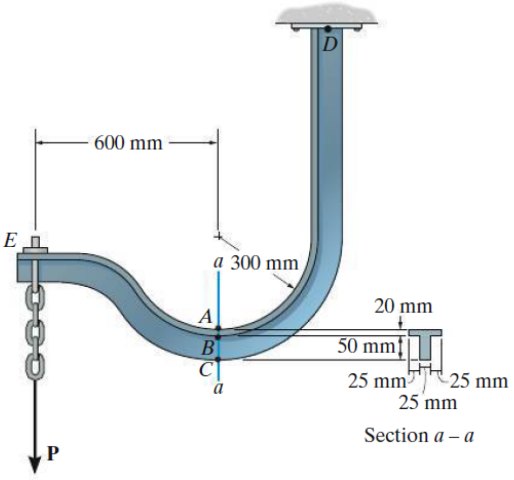
The curved beam is subjected to a bending moment of $M = 40$ lb-ft. Determine the maximum bending stress in the beam.



A) $\sigma_{\max} = 314.4$ psi
B) $\sigma_{\max} = 578.2$ psi
C) $\sigma_{\max} = 842.0$ psi
D) $\sigma_{\max} = 1105.8$ psi

Problem 5 (Hibbeler, 2014, w/ permission)

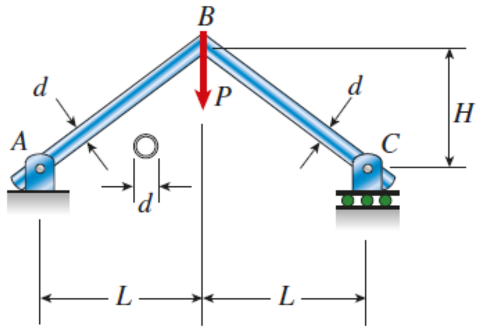
If the maximum bending stress at section *a*-*a* is not allowed to exceed $\sigma_{allow} = 150$ MPa, determine the maximum allowable force *P* that can be applied to end *E*.



A) $P_{max} = 3.55$ kN
B) $P_{max} = 6.91$ kN
C) $P_{max} = 10.3$ kN
D) $P_{max} = 13.8$ kN

Problem 6 (Gere, 2009, w/ permission)

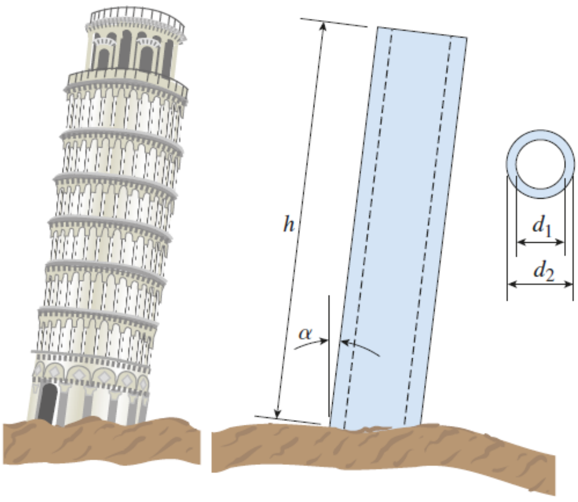
A rigid frame *ABC* is formed by welding two steel pipes at *B*. Each pipe has cross-sectional area $A = 11.31 \times 10^3$ mm², moment of inertia $I = 46.37 \times 10^6$ mm⁴, and outside diameter $d = 200$ mm. Find the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the frame due to the load $P = 8.0$ kN if $L = H = 1.4$ m.



A) $\sigma_t = 7.39$ MPa and $\sigma_c = 7.71$ MPa
B) $\sigma_t = 7.39$ MPa and $\sigma_c = 12.33$ MPa
C) $\sigma_t = 11.83$ MPa and $\sigma_c = 7.71$ MPa
D) $\sigma_t = 11.83$ MPa and $\sigma_c = 12.33$ MPa

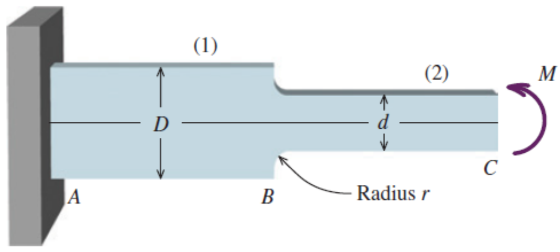
Problem 7 (Gere, 2009, w/ permission)

Because of its foundation settlement, a circular tower is leaning at an angle α to the vertical (see figure). The structural core of the tower is a circular cylinder of height h, outer diameter d_2, and inner diameter d_1. For simplicity in the analysis, assume that the weight of the tower is uniformly distributed along the height. Obtain a formula for the maximum permissible angle α if there is to be no tensile stress in the tower.



Problem 8 (Philpot, 2013, w/ permission)

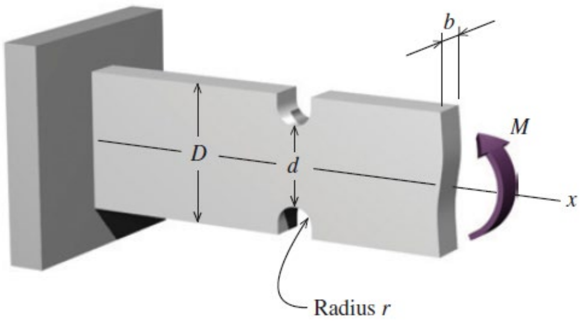
The stainless-steel spring shown below has a thickness of ¾ in. and a change in depth from $D = 1.50$ in. to $d = 1.25$ in. The radius of the fillet between the two sections is $r = 0.125$ in. If the bending moment applied to the spring is $M = 2000$ lb-in, determine the maximum normal stress in the spring.



A) $\sigma_{max} = 7.44$ ksi
B) $\sigma_{max} = 12.1$ ksi
C) $\sigma_{max} = 17.4$ ksi
D) $\sigma_{max} = 22.3$ ksi

Problem 9 (Philpot, 2013, w/ permission)

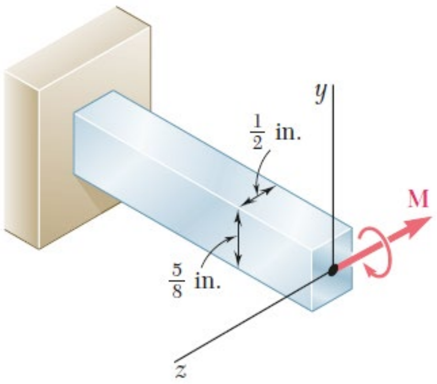
The notched bar shown in the next figure is subjected to a bending moment of $M = 300$ N·m. The major bar width is $D = 75$ mm, the minor bar width at the notches is $d = 50$ mm, and the radius of each notch is $r = 10$ mm. If the maximum bending stress in the bar is not to exceed 90 MPa, determine the minimum required bar thickness b.



A) $b = 9$ mm
B) $b = 14$ mm
C) $b = 19$ mm
D) $b = 24$ mm

Problem 10A (Beer et al., 2012, w/ permission)

The prismatic bar shown, made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi, is subjected to a couple of 1350 lb-in. parallel to the z-axis. Determine the thickness of the elastic core.



A) $t = 0.103$ in.
B) $t = 0.314$ in.
C) $t = 0.409$ in.
D) $t = 0.524$ in.

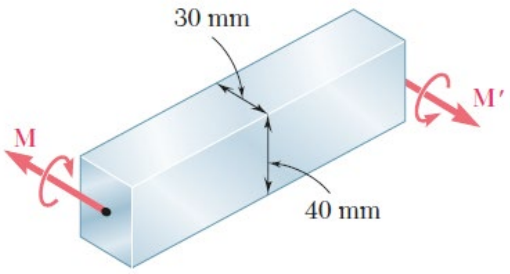
Problem 10B

Determine the radius of curvature of the bar considered in the previous part.

A) $\rho = 17.6$ ft
B) $\rho = 34.5$ ft
C) $\rho = 51.4$ ft
D) $\rho = 68.3$ ft

Problem 11A (Beer et al., 2012, w/ permission)

A bar of rectangular cross-section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa. Determine the bending moment for which yield first occurs.



A) $M_Y = 1.2$ kN·m
B) $M_Y = 2.4$ kN·m
C) $M_Y = 3.6$ kN·m
D) $M_Y = 4.8$ kN·m

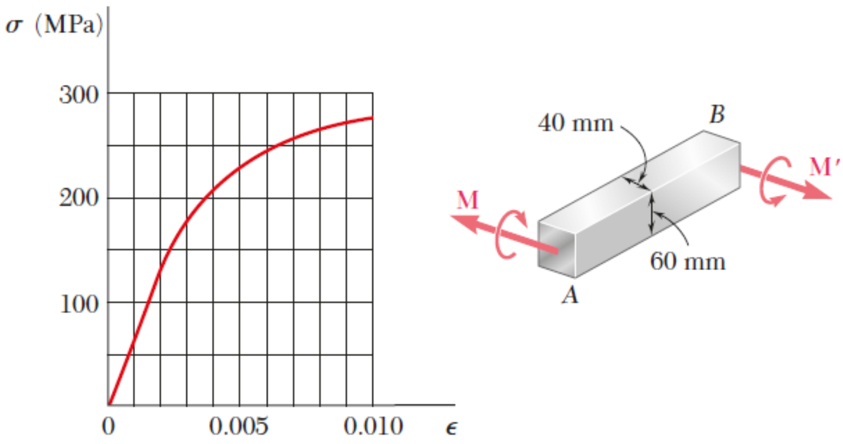
Problem 11B

Determine the bending moment for which the plastic zones at the top and bottom are 12 mm thick.

A) $M = 1.74$ kN·m
B) $M = 2.55$ kN·m
C) $M = 3.41$ kN·m
D) $M = 4.76$ kN·m

Problem 12A (Beer et al., 2012, w/ permission)

The prismatic bar is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine the radius of curvature of the bar when the maximum stress is 250 MPa.



A) $\rho = 2.82$ m
B) $\rho = 4.69$ m
C) $\rho = 6.75$ m
D) $\rho = 8.54$ m

Problem 12B

Find the corresponding value of the bending moment in the previous part.

A) $M = 1.89$ kN·m
B) $M = 4.57$ kN·m
C) $M = 7.29$ kN·m
D) $M = 9.27$ kN·m

ADDITIONAL INFORMATION

Table 1 Values of the integral $\int dA/r$ for different cross-section shapes

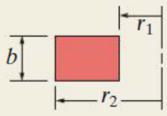
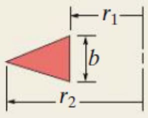
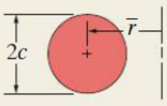
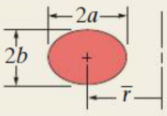
Shape	$\int_A \frac{dA}{r}$
	$b \ln \frac{r_2}{r_1}$
	$\frac{b\, r_2}{(r_2 - r_1)}\left(\ln \frac{r_2}{r_1}\right) - b$
	$2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$
	$\frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$

Figure 1 Stress-concentration factors K for bending of a flat bar with shoulder fillets.

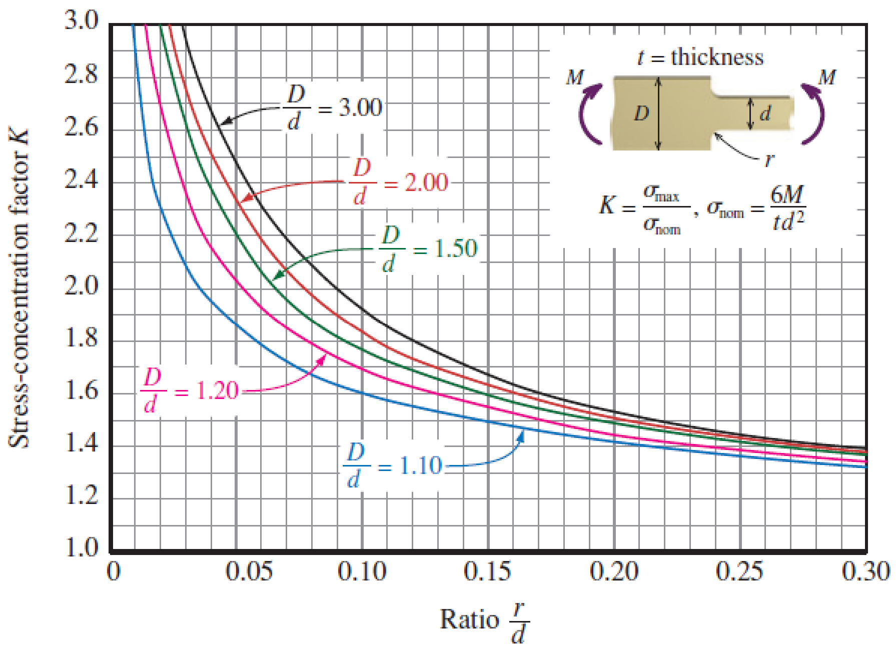
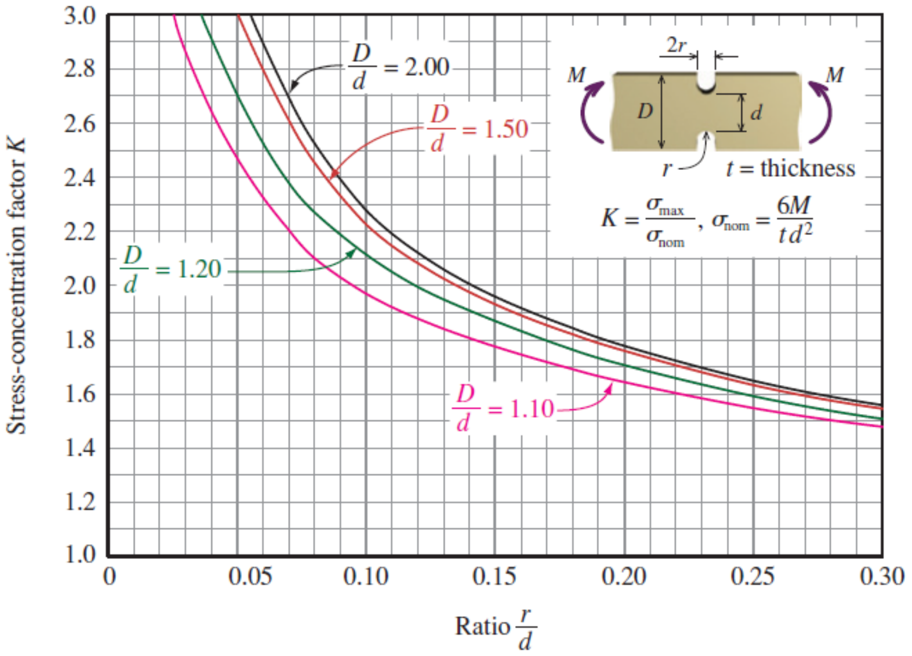


Figure 2 Stress-concentration factors K for bending of a flat bar with opposite U-shaped notches.



SOLUTIONS

P.1 ➡ *Solution*

Part A: The section modulus of the beam varies with *x* following the equation

$$S(x)=\frac{[h(x)]^3}{6}$$

Here, the depth *h(x)* increases with distance *x* from the free end in accordance with the equation

$$h(x)=h_A\left(1+\frac{2x}{L}\right)$$

The bending stress is given by the ratio of bending moment to section modulus, both of which are functions of *x*; thus,

$$\sigma(x)=\frac{M(x)}{S(x)}$$

Substituting $M(x) = Px$ and $S(x)$ gives

$$\sigma(x)=\frac{M(x)}{S(x)}=\frac{6Px}{\left[h_A\left(1+\frac{2x}{L}\right)\right]^3}\rightarrow\sigma(x)=\frac{6PL^3x}{h_A^3(L+2x)^3}$$

In order to obtain the position of maximum bending stress, we differentiate the equation above with respect to *x* and set the result to zero; that is,

$$\frac{d}{dx}\left(\frac{6PL^3x}{h_A^3(L+2x)^3}\right)=0\rightarrow 6P\frac{L^3}{h_A^3(L+2x)^3}-36Px\frac{L^3}{h_A^3(L+2x)^4}=0$$

$$\therefore\frac{-L+4x}{h_A^3(L+2x^4)}=0$$

The expression above equals zero when the numerator is zero, i.e., when $-L+4x=0$, which implies that the optimum bending stress occurs when $x = L/4$. (To verify that this point indeed corresponds to a maximum stress, we could differentiate a second time, substitute $x = L/4$, and verify that $d^2M/dx^2 = -128P/27Lh_A^3 < 0$.) Substituting $x = L/4$ in the equation for $\sigma(x)$ gives

$$\sigma\left(\frac{L}{4}\right)=\frac{6PL^3\times(L/4)}{h_A^3\times(L+2\times L/4)^4}\rightarrow\sigma_{\max}=\frac{4PL}{9h_A^3}$$

Noting that $\sigma_B = 2PL/9h_A^3$ at the fixed end, we can determine the ratio of maximum stress to the stress at point *B*,

$$\frac{\sigma_{\max}}{\sigma_B}=\frac{4\,PL/9h_A^3}{2\,PL/9h_A^3}\rightarrow\boxed{\frac{\sigma_{\max}}{\sigma_B}=2}$$

That is, the maximum bending stress in the beam is twice the bending stress at the fixed end.

The correct answer is **A**.

Part B: From statics, the reaction at point *A* is $R_A = P$. Through an analysis of internal forces and moments, we see that moment *M(x)* is described by the relation

$$M(x)=R_Ax-\frac{P}{L}\times x\times\frac{x}{2}\rightarrow M(x)=Px-\frac{P}{2L}x^2$$

The section modulus is the same as in the previous part. We can then determine an expression for the bending stress,

$$\sigma(x) = \frac{M(x)}{S(x)} \rightarrow \sigma(x) = \frac{Px - \frac{P}{2L}x^2}{\frac{\left[h_A\left(1+\frac{2x}{L}\right)\right]^3}{6}}$$

$$\therefore \sigma(x) = -3xP(-2L+x)\frac{L^2}{h_A^3(L+2x)^3}$$

Differentiating the expression above gives

$$\frac{d}{dx}\sigma(x) = -3P(-2L+x)\times\frac{L^2}{h_A^3(L+2x)^3} - 3xP\times\frac{L^2}{h_A^3(L+2x)^3}$$

$$+18xP(-2L+x)\times\frac{L^2}{h_A^3(L+2x)^4} = 0$$

Simplifying, we arrive at the second-degree equation

$$x^2 - 5Lx + L^2 = 0$$

which can be easily solved for x_{max},

$$\frac{x}{L} = \frac{5-\sqrt{5^2-4}}{2} \approx 0.209 \rightarrow x = 0.209L$$

That is to say, the maximum bending stress should occur at a distance of about 0.2*L* from the free end. Substituting the value of *x* just obtained into the expression for $\sigma(x)$, we get

$$\sigma_{\max} = \sigma(0.209L) = 0.394\frac{PL}{h_A^3}$$

Also, the bending stress at end *B* is $\sigma_B = PL/9h_A^3$. Finally, the ratio of stresses $\sigma_{\max}/\sigma_B$ is such that

$$\frac{\sigma_{\max}}{\sigma_B} = \frac{0.394\,PL/h_A^3}{PL/9h_A^3} \rightarrow \boxed{\frac{\sigma_{\max}}{\sigma_B} = 3.55}$$

That is to say, the maximum stress is about than three and a half times the stress at end *B*.

The correct answer is **D**.

P.2 ➡ *Solution*

Part A: The section modulus of the beam is given by

$$S(x) = \frac{b[h(x)]^2}{6}$$

where the depth *h(x)* is denoted as such because it varies with *x*; that is,

$$S(x) = \frac{b[h(x)]^2}{6} = \frac{b\left[h_A\left(1+\frac{x}{2L}\right)\right]^2}{6}$$

The bending moment $M(x) = Px + M_0$ is also a function of *x*. The bending stress is then obtained by dint of the flexure formula,

$$\sigma(x) = \frac{M(x)}{S(x)} = \frac{Px + M_0}{b\left[h_A\left(1+\frac{x}{2L}\right)\right]^2} \rightarrow \sigma(x) = 24(Px+M_0)\frac{L^2}{bh_A^2(2L+x)^2}$$

Differentiating this expression with respect to x and setting the result to zero, we get

$$\frac{d}{dx}\sigma(x)=0\rightarrow 24P\frac{L^2}{bh_A^2(2L+x)^2}-2(24Px+24M_0)\frac{L^2}{bh_A^2(2L+x)^3}=0$$

which, upon simplification, becomes

$$-24L^2\times\frac{-PL+Px+2M_0}{bh_A^2(2L+x)^3}=0$$

For the expression above to yield zero, the numerator of the fraction on the left-hand side must equal zero. Accordingly,

$$-24L^2\times\frac{-2PL+Px+2M_0}{bh_A^2(2L+x)^3}=0\rightarrow -2PL+Px+2M_0=0$$

$$\therefore x=\frac{2(PL-M_0)}{P}$$

Substituting numerical values, we have P = 50 lb, L = 20 in., and M_0 = 800 lb-in., so that

$$x=\frac{2(PL-M_0)}{P}=\frac{2\times(50\times20-800)}{50}=8\text{ in.}$$

To verify whether this value of x indeed corresponds to a maximum stress, the student is invited to take the second derivative of $\sigma(x)$ and substitute x = 8 in. in the ensuing equation. We can now evaluate the maximum stress by substituting x = 8 in. in the expression for $\sigma(x)$, along with P = 50 lb, M_0 = 800 lb-in., L = 20 in., b = 1.0 in., and h_A = 2.0 in. Thus,

$$\sigma_{\max}=\sigma(8)=24\times(50\times8+800)\times\frac{20^2}{1\times2^2\times(2\times20+8)^2}=1250\text{ psi}$$

Similarly, we can determine the bending stress σ_B at point B by substituting x = L = 20 in. and other numerical data into the expression for $\sigma(x)$. Thus,

$$\sigma_B=\sigma(20)=24\times(50\times20+800)\times\frac{20^2}{1\times2^2\times(2\times20+20)^2}=1200\text{ psi}$$

Finally, the ratio $\sigma_{\max}/\sigma_B$ is such that

$$\frac{\sigma_{\max}}{\sigma_B}=\frac{1250}{1200}=\boxed{1.04}$$

That is to say, the maximum bending stress in the beam is only slightly (about 4 percent) greater than the bending stress at the fixed end.

The correct answer is **A**.

Part B: As before, the section modulus is described with the equation

$$S(x)=\frac{b\left[h_A\left(1+\frac{x}{2L}\right)\right]^2}{6}$$

The bending moment, in turn, is now slightly more complex due to the existence of the triangular distributed load,

$$M(x)=Px+M_0-\frac{1}{2}\times\left(\frac{x}{L}q_0\right)\times x\times\frac{x}{3}$$

The bending stress is equal to the ratio of bending moment to section modulus,

$$\sigma(x)=\frac{Px+M_0-q_0x^3/6L}{\dfrac{b\left[h_A\left(1+\dfrac{x}{2L}\right)\right]^2}{6}}\rightarrow\sigma(x)=-4\left(q_0x^3-6PLx-6M_0L\right)\times\frac{L}{bh_A^2(2L+x)^2}=0$$

As we have done in the previous part, the position of the maximum stress section is determined by optimizing $\sigma(x)$. The derivative of $\sigma(x)$ is

$$\frac{d}{dx}\sigma(x)=\left(-12q_0x^2+24PL\right)\times\frac{L}{bh_A^2(2L+x)^2}$$

$$-2\times\left(-4q_0x^3+24PLx+24M_0L\right)\times\frac{L}{bh_A^2(2L+x)^3}=0$$

Simplifying, the expression above becomes

$$q_0x^3+6q_0Lx^2+6PLx-12PL^2+12M_0L=0$$

which is a third-degree equation in *x*. Substituting $q_0 = 3P/L = 3\times 50/20 = 7.5$ lb/in., $L = 20$ in., $P = 50$ lb, and $M_0 = 800$ lb-in., it follows that

$$q_0x^3+6q_0Lx^2+6PLx-12PL^2+12M_0L=0$$

$$\rightarrow 7.5x^3+6\times7.5\times20x^2+6\times50\times20x-12\times50\times20^2+12\times800\times20=0$$

$$\therefore 7.5x^3+900x^2+6000x-48{,}000=0$$

This equation can be solved by means of a CAS such as Mathematica, yielding two negative solutions, which mean nothing, and $x = 4.64$ in., which is a feasible solution. Hence, we conclude that the maximum stress occurs at a distance of about 4.6 inches from the free end of the beam. The maximum stress is obtained by inserting this value into the expression for bending stress, along with other numerical data. The result is

$$\sigma_{\max}=\sigma(4.64)$$

$$\therefore\sigma_{\max}=-4\times\left(7.5\times4.64^3-6\times20\times4.64-6\times800\times20\right)\times\frac{20}{1\times2.0^2\times(2\times20+4.64)^2}=1235.4\text{ psi}$$

We want to compare this moment with the bending stress at the location of maximum moment. To obtain the position of maximum moment, we differentiate *M(x)* and equate the result to zero; that is,

$$M(x)=Px+M_0-\frac{q_0x^3}{6L}\rightarrow\frac{dM}{dx}=P-\frac{q_0x^2}{2L}=0$$

$$\therefore P=\frac{q_0x^2}{2L}$$

$$\therefore x=\sqrt{\frac{2PL}{q_0}}=\sqrt{\frac{2\times50\times20}{7.5}}=16.3\text{ in.}$$

Substituting this value of *x* into the expression for bending stress, the stress σ_M at the location of maximum moment is computed as

$$\sigma_M=\sigma(16.3)=-4\times\left(7.5\times16.3^3-6\times20\times16.3-6\times800\times20\right)\times\frac{20}{1\times2.0^2\times(2\times20+16.3)^2}=1017.9\text{ psi}$$

Finally, the ratio of maximum bending stress to stress at the location of maximum moment is

$$\frac{\sigma_{\max}}{\sigma_{\text{M}}}=\frac{1235.4}{1017.9}=\boxed{1.21}$$

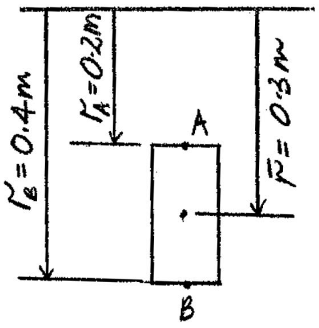
The ratio in question is close to 1.2.

The correct answer is **B**.

P.3 ➡ *Solution*

Since the distance between the centroid and the neutral axis is very small, we must make use of more than two or three decimal points to maximize the accuracy of this curved beam analysis. Referring to the figure below, the location of the neutral surface from the center of curvature of the curved beam is

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.1\times 0.2}{0.1\times \ln\frac{0.4}{0.2}} = 0.288539 \text{ m}$$



Note that we have used one of the results from Table 1 of the Additional Information section. The distance *e* between the neutral axis and the centroid is $e = \bar{r} - R = 0.3 - 0.288539 = 0.011461$ m. The maximum bending stress occurs at either point *A* or *B* (that is, at either the topmost or bottommost fiber of the cross-section). For point *A*, which is in tension, we have

$$\sigma_{\text{allow}} = \frac{M(R-r_A)}{Aer_A} \rightarrow 100\times 10^6 = \frac{M\times(0.288539-0.2)}{(0.1\times 0.2)\times 0.011461\times 0.2}$$

$$\therefore M = 51.8 \text{ kN}\cdot\text{m}$$

while for point *B*, which is in compression,

$$\sigma_{\text{allow}} = \frac{M(R-r_B)}{Aer_B} \rightarrow -100\times 10^6 = \frac{M\times(0.288539-0.4)}{(0.1\times 0.2)\times 0.011461\times 0.4}$$

$$\therefore M = 82.3 \text{ kN}\cdot\text{m}$$

The smaller value controls; thus, the maximum allowable internal moment is $M = 51.8$ kN·m.

The correct answer is **A**.

P.4 ➡ *Solution*

Much like the previous problem, the bending stress can be determined as

$$\sigma = \frac{M(R-r)}{Ar(\bar{r}-R)}$$

Here, *M* is the bending moment, *r* is the radial distance from the center of curvature, *A* is the cross-sectional area, $\bar{r}$ is the radial distance to the centroid of the cross-section, and *R* is the location of the neutral surface from the center of the beam's curvature, given by

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

We realize that the integral in the denominator pertains to the first case in Table 1 of the Additional Information section. Thus,

$$\Sigma\int_A \frac{dA}{r} = 0.5\ln\frac{5}{3} + 2\ln\frac{5.5}{5} = 0.446033 \text{ in.}$$

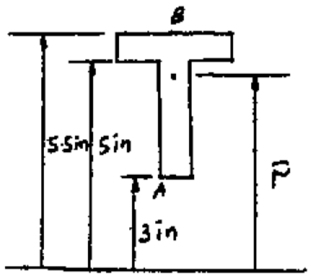
Also, the area $A = 2\times 0.5 + 0.5\times 2 = 2$ in.^{2}, so that

$$R = \frac{2}{0.446033} = 4.4840 \text{ in.}$$

In addition, the radial distance $\bar{r}$ to the centroid of the section is determined as

$$\bar{r} = \frac{4 \times 2 \times 0.5 + 5.25 \times 2 \times 0.5}{2 \times 0.5 + 2 \times 0.5} = 4.625 \text{ in.}$$

We now have all the variables needed to establish the bending stress, which reaches a maximum value either at point *A*, in the bottommost fiber, or point *B*, in the topmost fiber; see below.



For the lowermost fiber, at point *A*, we have1

$$\sigma_A = \frac{M(R - r_A)}{A r_A (\bar{r} - R)} = \frac{(40 \times 12) \times (4.4840 - 3)}{2 \times 3 \times (4.625 - 4.4840)} = 842.0 \text{ psi (T)}$$

whereas for the uppermost fiber, at point *B*, it follows that

$$\sigma_B = \frac{M(R - r_B)}{A r_B (\bar{r} - R)} = \frac{(40 \times 12) \times (4.4840 - 5.5)}{2 \times 5.5 \times (4.625 - 4.4840)} = 314.4 \text{ psi (C)}$$

Clearly, the bending stress reaches a maximum equal to 842.0 psi at point *A*.

The correct answer is **C**.

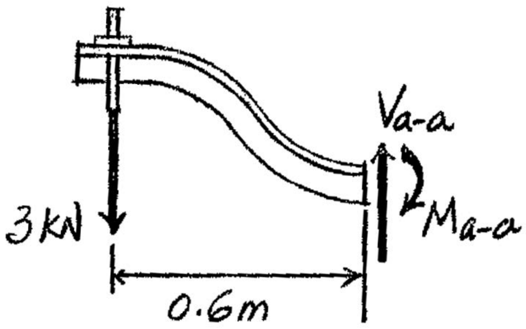
P.5 ➡ *Solution*

The internal moment developed at section *a-a* can be determined by writing the moment equation of equilibrium about the neutral axis of the cross-section at *a-a*,

$$\Sigma M_{a-a} = 0 \rightarrow P \times 0.6 - M_{a-a} = 0$$

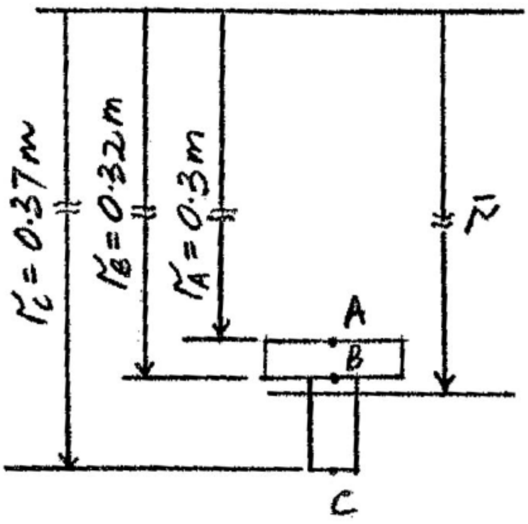
$$\therefore M_{a-a} = 0.6P$$

Here, M_{a-a} is considered positive since it tends to decrease the curvature of the curved segment of the beam.



Referring to the next figure, the location $\bar{r}$ of the centroid of the cross-section from the center of the beam's curvature is such that

$$\bar{r} = \frac{\Sigma \tilde{r} A}{\Sigma A} = \frac{0.31 \times (0.02 \times 0.075) + 0.345 \times (0.05 \times 0.025)}{0.02 \times 0.075 + 0.05 \times 0.025} = 0.325909 \text{ m}$$



The location of the neutral surface from the center of the beam's curvature can be determined from

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

where $A = 0.02 \times 0.075 + 0.05 \times 0.025 = 0.00275$ m². The integral in the denominator pertains to the first case in Table 1 of the Additional Information section, so that

$$\int_A \frac{dA}{r} = 0.075 \times \ln\frac{0.32}{0.3} + 0.025 \times \ln\frac{0.37}{0.32} = 0.00846994 \text{ m}$$

and, therefore,

$$R = \frac{0.00275}{0.00846994} = 0.324678 \text{ m}$$

Consequently, the distance *e* between the position of the centroid and the position of the neutral axis is

$$e = \bar{r} - R = 0.325909 - 0.324678 = 0.001231 \text{ m}$$

or about 1.23 mm. The distance between the centroid and the neutral axis is small enough to justify the use of more than two or three decimal points. The maximum normal stress occurs at either point *A* (the uppermost fiber) or *C* (the lowermost one); noting that $M = 0.6P$, we have, for point *A*,

$$\sigma_{\text{allow}} = \frac{M(R - r_A)}{Aer_A} \rightarrow 150 \times 10^6 = \frac{0.6P \times (0.324678 - 0.3)}{0.00275 \times 0.001231 \times 0.3}$$

$$\therefore 150 \times 10^6 = 14{,}579.7P$$

$$\therefore P = 10.3 \text{ kN}$$

and, for point *C*,

$$\sigma_{\text{allow}} = \frac{M(R - r_C)}{Aer_C} \rightarrow 150 \times 10^6 = \frac{0.6P \times (0.324678 - 0.37)}{0.00275 \times 0.001231 \times 0.37}$$

$$\therefore 150 \times 10^6 = -21{,}710.4P$$

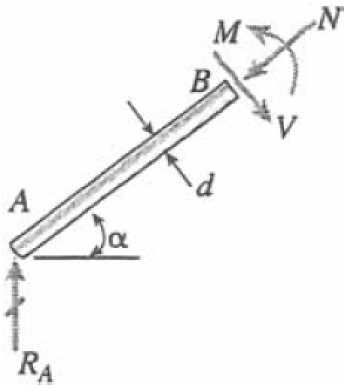
$$\therefore |P| = 6.91 \text{ kN}$$

The lower value controls the analysis of the beam; hence, we take P_{max} = 6.91 kN as our answer.

The correct answer is **B**.

P.6 ➡ *Solution*

Consider the following free-body diagram of one half of the frame.



Due to symmetry, the reaction at *A* is $R_A = P/2$. The axial force $N = R_A \sin\theta$ = $(P/2)\sin\theta$, and the bending moment $M = R_A L = PL/2$. Both the axial force and the bending moment contribute to the tensile stress in the beam, which is given by

$$\sigma_t = -\frac{N}{A} + \frac{Mc}{I} \rightarrow \sigma_t = -\frac{P\sin\alpha}{2A} + \frac{PLd}{4I}$$

Noting that the cross-sectional area $A = 11.31\times10^3$ mm², the moment of inertia $I = 46.37\times10^6$ mm⁴, and substituting the pertaining data, we obtain

$$\sigma_t = -\frac{P\sin\alpha}{2A} + \frac{PLd}{4I}$$

$$\therefore \sigma_t = \frac{-8000\times 1/\sqrt{2}}{2\times(11.31\times10^3)} + \frac{8000\times1400\times200}{4\times(46.37\times10^6)} = \boxed{11.83 \text{ MPa}} \ \text{(T)}$$

Similarly, the compressive stress σ_c has contributions from the axial load and the bending moment; that is,

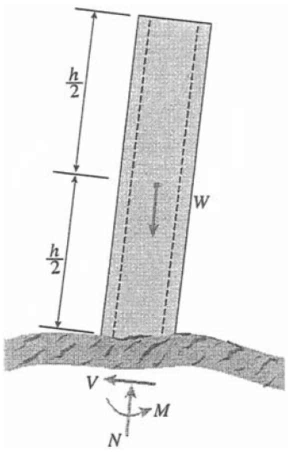
$$\sigma_c = -\frac{N}{A} - \frac{Mc}{I} \rightarrow \sigma_c = -0.250 - 12.08 = \boxed{12.33 \text{ MPa}} \ \text{(C)}$$

Note that the maximum tensile and compressive stresses are within less than 5 percent of each other.

The correct answer is **D**.

P.7 ➡ *Solution*

A free body diagram for the tower is illustrated below.



The cross-sectional area and moment of inertia of the tower section are, respectively,

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right)$$

$$I = \frac{\pi}{64}\times\left(d_2^4 - d_1^4\right) = \frac{\pi}{64}\times\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right)$$

and the ratio of the latter to the former is such that

$$\frac{I}{A}=\frac{\frac{\cancel{\pi}}{64}\times\cancel{\left(d_2^2-d_1^2\right)}\left(d_2^2+d_1^2\right)}{\frac{\cancel{\pi}}{4}\cancel{\left(d_2^2-d_1^2\right)}}=\frac{\left(d_2^2+d_1^2\right)}{16}$$

At the base of the tower, the axial load and the bending moment are, respectively,

$$N=W\cos\alpha\ ;\ M=W\left(\frac{h}{2}\right)\sin\alpha$$

Noting that $c = d_2/2$, the tensile stress in the tower is computed as

$$\sigma_t=-\frac{N}{A}+\frac{Mc}{I}=-\frac{W\cos\alpha}{A}+\frac{W}{I}\left(\frac{h}{2}\sin\alpha\right)\frac{d_2}{2}$$

This tensile stress must equal zero; thus, setting $\sigma_t = 0$ and solving for the angle α, it follows that

$$-\frac{\cancel{W}\cos\alpha}{A}+\frac{\cancel{W}}{I}\left(\frac{h}{2}\sin\alpha\right)\frac{d_2}{2}=0\rightarrow\frac{\cos\alpha}{A}=\frac{hd_2}{4I}\sin\alpha$$

$$\therefore\ \tan\alpha=\frac{4}{hd_2}\times\left(\frac{I}{A}\right)=\frac{4}{hd_2}\times\frac{\left(d_2^2+d_1^2\right)}{16}$$

$$\therefore\ \boxed{\alpha=\arctan\left(\frac{d_2^2+d_1^2}{4hd_2}\right)}$$

This is the angle α needed for the tower to develop no tensile bending stresses.

P.8 ➡ *Solution*

The ratio of the radius of the fillet, *r*, to the depth after the choke, *d*, is *r*/*d* = 0.125/1.25 = 0.10. Similarly, the ratio of the depth before the choke, *D*, to the depth after the choke, *d*, is *D*/*d* = 1.50/1.25 = 1.20. Entering these two values into Figure 1, we read the stress concentration factor $K \approx 1.70$. The moment of inertia of the minimum depth cross-section is

$$I=\frac{0.75\times1.25^3}{12}=0.122\text{ in.}^4$$

Then, the nominal bending stress at the minimum depth section is

$$\sigma_{\text{nom}}=\frac{My}{I}=\frac{2000\times1.25/2}{0.122}=10,246\text{ psi}$$

and the maximum bending stress, finally, is calculated as

$$\sigma_{\max}=K\sigma_{\text{nom}}=1.70\times10,246=\boxed{17.4\text{ ksi}}$$

The correct answer is **C**.

P.9 ➡ *Solution*

The ratio of the radius of each notch, *r*, to the minor bar depth, *d*, is *r*/*d* = 10/50 = 0.20, while the ratio of the major bar depth, *D*, to the minor depth, *d*, is *D*/*d* = 75/50 = 1.5. Mapping these two quantities onto Figure 2, we read the stress concentration factor $K \approx 1.75$. We were given the maximum bending stress $\sigma_{\max}$ = 90 MPa; the maximum nominal bending stress can then be obtained via the relationship $\sigma_{\max} = K\sigma_{\text{nom}}$,

$$\sigma_{max} = K\sigma_{nom} \rightarrow \sigma_{nom} = \frac{\sigma_{max}}{K}$$

$$\sigma_{nom} = \frac{90}{1.75} = 51.4 \text{ MPa}$$

The nominal bending moment is related to the geometry of the beam by the flexure formula,

$$\sigma_{nom} = \frac{My}{I} = \frac{M\times(d/2)}{(bd^3/12)} = \frac{6M}{bd^2}$$

Solving for the width b gives

$$\sigma_{nom} = \frac{6M}{bd^2} \rightarrow b = \frac{6M}{\sigma_{nom}d^2}$$

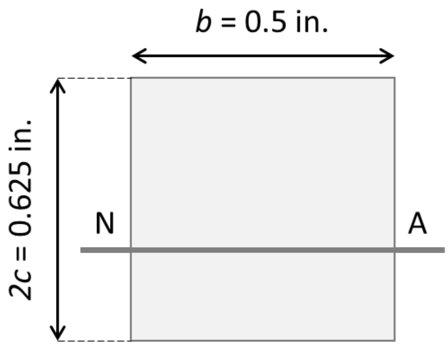
$$\therefore b = \frac{6\times300}{51.4\times50^2}\times1000 = \boxed{14 \text{ mm}}$$

The width of the beam should be no less than 14 millimeters.

The correct answer is **B**.

P.10 ➡ *Solution*

Part A: The cross-section of the structure in question is illustrated below.



The moment of inertia of the section about the neutral axis is

$$I = \frac{0.5\times0.625^3}{12} = 0.0102 \text{ in.}^4$$

The maximum bending moment that can be applied to the section is given by

$$M_Y = \frac{\sigma_Y I}{c}$$

where $\sigma_Y = 36\times10^3$ ksi and $c = 0.313$ in., so that

$$M_Y = \frac{\sigma_Y I}{c} \rightarrow M = \frac{(36\times10^3)\times0.0102}{0.313} = 1173.2 \text{ lb-in.}$$

Let y_Y be the half-thickness of the elastic core. The bending moment is related to the yield moment M_Y and the elastic core half-thickness by the equation

$$M = \frac{3}{2}M_Y\left(1-\frac{1}{3}\frac{y_Y^2}{c^2}\right)$$

Substituting $M = 1350$ lb-in., $M_Y = 1173.2$ lb-in., and $c = 0.313$ in., it follows that

$$M = \frac{3}{2}\times1173.2\times\left(1-\frac{1}{3}\times\frac{y_Y^2}{0.313^2}\right) \rightarrow 1350 = 1759.8(1-3.40y_Y^2)$$

$$\therefore y_Y = 0.262 \text{ in.}$$

The thickness t of the elastic core is then

$$t = 2y_Y = \boxed{0.524\text{ in.}}$$

The thickness of the elastic core is close to half an inch.

The correct answer is **D**.

Part B: Let ρ_Y be the radius of curvature corresponding to the bending moment at yield M_Y. Such a radius can be determined via the moment-curvature relationship,

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI} \to \rho_Y = \frac{EI}{M_Y}$$

$$\therefore \rho_Y = \frac{\left(29\times10^6\right)\times0.0102}{1173.2} = 252.1\text{ in.}$$

Now, suppose that ρ is the radius of curvature corresponding to moment M. This radius follows from the relation

$$\frac{\rho}{\rho_Y} = \frac{y_y}{c} \to \rho = \frac{\rho_Y y_y}{c}$$

$$\therefore \rho = \frac{252.1\times0.262}{0.313} = 211.0\text{ in.} = \boxed{17.6\text{ ft}}$$

The radius of curvature is just above 17.5 feet.

The correct answer is **A**.

P.11 ➡ *Solution*

Part A: The moment of inertia about the neutral axis of a rectangular cross-section such as the one considered herein is

$$I = \frac{bd^3}{12} = \frac{0.03\times0.04^3}{12} = 1.6\times10^{-7}\text{ m}^4$$

The bending moment that corresponds to yield conditions can be easily determined with the aid of the flexure formula,

$$\sigma_Y = \frac{M_Y c}{I} \to M_Y = \frac{\sigma_Y I}{c}$$

$$\therefore M_Y = \frac{\left(300\times10^6\right)\times\left(1.6\times10^{-7}\right)}{0.02} = 2400\text{ N}\cdot\text{m} = \boxed{2.4\text{ kN}\cdot\text{m}}$$

The bending moment for which yield first occurs is 2.4 kN·m, or 2400 N·m.

The correct answer is **B**.

Part B: The depths of the elastic and plastic zones are related by the simple expression

$$y_Y = c - t$$

where y_Y is the half-thickness of the elastic core and t is the thickness of the plastic zones at the top and bottom of the beam. Substituting c = 40/2 = 20 mm and t = 12 mm gives

$$y_Y = \frac{40}{2} - 12 = 8\text{ mm}$$

We now have all the necessary variables to calculate the bending moment under the specified conditions, at which point we make use of the formula

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right)$$

Substituting M_Y = 2.4 kN-m, y_T = 0.008 m and c = 0.02 m yields

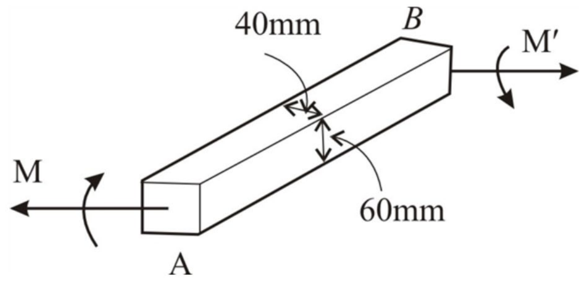
$$M = \frac{3}{2} \times 2.4 \times \left(1 - \frac{1}{3}\frac{0.008^2}{0.02^2}\right) = \boxed{3.41 \text{ kN} \cdot \text{m}}$$

That is, in order for the plastic zones to have the specified dimensions, the bending moment must be equal to 3.41 kN·m.

The correct answer is **C**.

P.12 ➡ *Solution*

Part A: The structure in question is illustrated below.



The depth from the neutral axis to the topmost (or bottommost) fiber is c = (1/2)d = (1/2)×60 = 30 mm. The maximum stress specified is σ = 250 MPa; entering this value into the graph we were given, we find that the maximum strain is about ε = 0.0064. The radius of curvature ρ is related to the strain by the expression

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.213 \text{ m}^{-1}$$

which, when inverted, gives

$$\frac{1}{\rho} = 0.213 \text{ m}^{-1} \rightarrow \boxed{\rho = 4.69 \text{ m}}$$

The radius of curvature is close to 4.7 meters.

The correct answer is **B**.

Part B: The normal strain varies linearly with the distance y from the neutral surface, as per the relationship

$$\varepsilon = -\varepsilon_m \frac{y}{c}$$

The bending moment can be obtained by means of the integral

$$M = -b\int_{-c}^{c} y\sigma dy$$

or, equivalently,

$$M = 2b\int_0^c y\sigma dy = 2bc^2\int_0^c \frac{y}{c}|\sigma|\frac{dy}{c}$$

Letting $y/c = x$, we obtain

$$M = 2bc^2\int_0^c \frac{y}{c}|\sigma|\frac{dy}{c} = 2bc^2\int_0^1 x|\sigma|dx$$

Suppose that the integral above is equal to a quantity F such that

$$F = \int_0^1 x|\sigma|dx$$

The integral can be evaluated by means of a numerical integration algorithm such as Simpson's rule, in which case we'd have

$$F = \frac{\Delta h}{3} \times \left[y_0 + 4(y_1 + y_3) + 2y_2 + y_4\right]$$

Here, Δh is the width of each trapezoidal element, and can be calculated as

$$\Delta h = \frac{\text{upper limit} - \text{lower limit}}{4} = \frac{1-0}{4} = 0.25$$

The calculations associated with each y_i term are provided below.

x	$\varepsilon = \varepsilon_m = 0.0064x$	σ (MPa) (From Graph)	$x \times \sigma$	
0	0	0	0	→ y_0
0.25	0.0016	110	27.5	→ y_1
0.5	0.0032	180	90	→ y_2
0.75	0.0048	225	168.75	→ y_3
1	0.0064	250	250	→ y_4

With these results, *F* is determined as

$$F = \frac{0.25}{3} \times \left[0 + 4\times\left(27.5 + 168.75\right) + 2\times 90 + 250\right] = 101.25 \text{ MPa}$$

Lastly, we can calculate the bending moment *M*,

$$M = 2bc^2F = 2\times 0.04 \times 0.03^2 \times \left(101.25\times 10^6\right) = \boxed{7.29 \text{ kN}\cdot\text{m}}$$

The correct answer is **C**.

ANSWER SUMMARY

Problem 1	**1A**	**A**
	1B	**D**
Problem 2	**2A**	**A**
	2B	**B**
Problem 3		**A**
Problem 4		**C**
Problem 5		**B**
Problem 6		**D**
Problem 7		Open-ended pb.
Problem 8		**C**
Problem 9		**B**
Problem 10	**10A**	**D**
	10B	**A**
Problem 11	**11A**	**B**
	11B	**C**
Problem 12	**12A**	**B**
	12B	**C**

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