Quiz SM205

BENDING AND SHEAR
PART 3: MOSTLY SHEAR!

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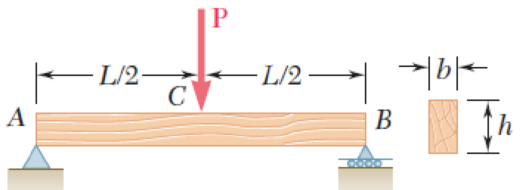
PROBLEMS

Problem 1A (Beer et al., 2012, w/ permission)

A timber beam of length L and rectangular cross-section carries a single concentrated load P at its midpoint C. Show that the ratio τ_m/σ_m of the maximum values of the shear and normal stresses in the beam is equal to $h/2L$, where h and L are the depth and length of the beam, respectively.



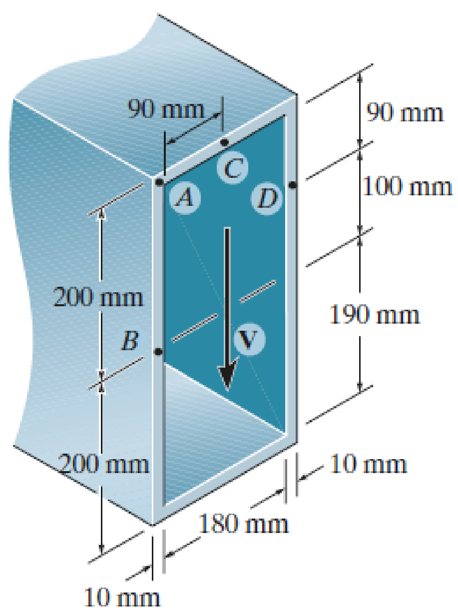
Problem 1B

Determine the depth h and the width b of the beam, knowing that L = 2.5 m, P = 50 kN, τ_m = 900 kPa, and σ_m = 12 MPa.

A) h = 375 mm and b = 111 mm
B) h = 375 mm and b = 205 mm
C) h = 450 mm and b = 111 mm
D) h = 450 mm and b = 205 mm

Problem 2 (Hibbeler, 2014, w/ permission)

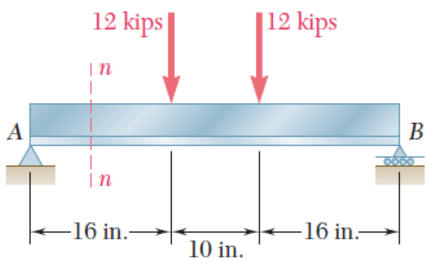
Consider the box girder illustrated below. True or false?

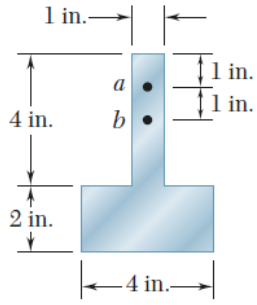


1. ( ) Suppose that V = 300 kN. In this case, the shear flow at point B is more than twice the shear flow at point A.
2. ( ) Suppose that V = 450 kN. In this case, the shear flow at point C is zero.
3. ( ) Suppose that V = 450 kN. In this case, the shear flow at point D is less than 500 kN/m.

Problem 3 (Beer et al., 2012, w/ permission)

For the beam and loading shown, consider section *n-n*. True or false?

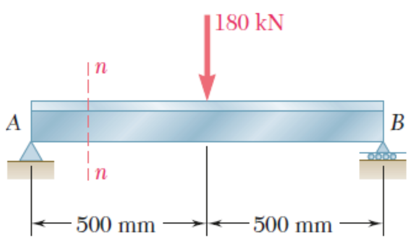


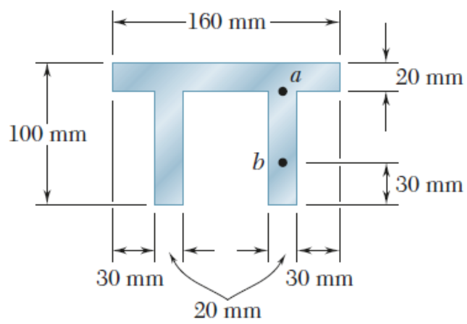


1. ( ) The shear stress at point *a* is greater than 1 ksi.
2. ( ) The shear stress at point *b* is less than 2 ksi.
3. ( ) The maximum shear stress is greater than 2.5 ksi.

Problem 4 (Beer et al., 2012, w/ permission)

For the beam and loading shown, consider section *n-n*. True or false?

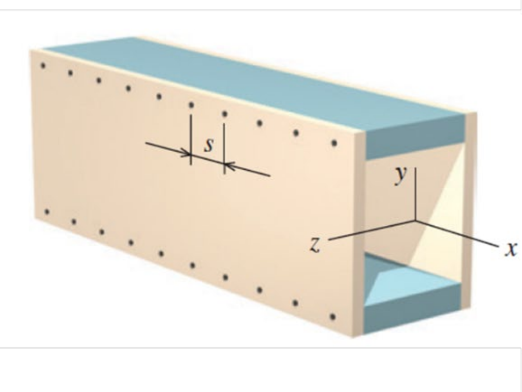


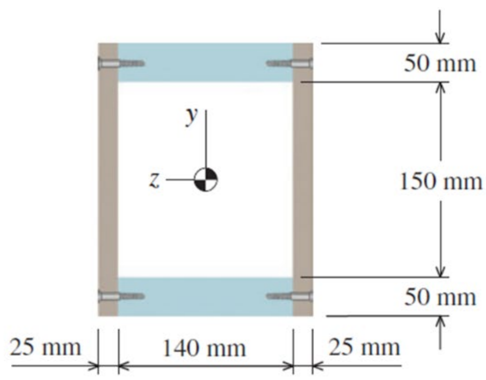


1. ( ) The shear stress at point *a* is less than 25 MPa.
2. ( ) The shear stress at point *b* is less than 20 MPa.
3. ( ) The maximum shear stress in the beam section is less than 40 MPa.

Problem 5 (Philpot, 2013, w/ permission)

A box beam is fabricated from four boards, which are fastened together with screws, as illustrated in the next figure. Each screw will be installed so that bending occurs about the *z*-axis, and the maximum shear force in the beam will be 9 kN. Determine the maximum spacing *s* for the screws.

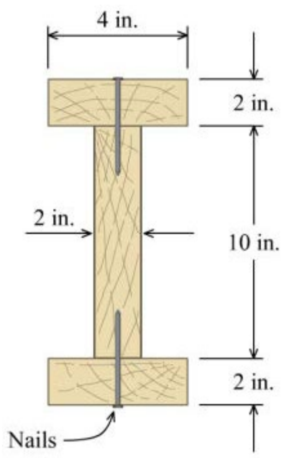




A) $s = 32$ mm
B) $s = 53$ mm
C) $s = 71$ mm
D) $s = 90$ mm

Problem **6A** (Philpot, 2013, w/ permission)

A wooden beam is fabricated from one 2 × 10 and two 2 × 4 pieces of dimension lumber to form the I-beam cross-section shown in the next figure. The I-beam will be used as a simply-supported beam to carry a concentrated load *P* at the center of a 20-ft span. The wood has an allowable bending stress of 1200 psi and an allowable shear stress of 90 psi. The flanges of the beam are fastened to the web with nails that can safely transmit a force of 120 lb in direct shear. If the nails are uniformly spaced at an interval of $s = 4.5$ in. along the span, what is the maximum concentrated load *P* that can be supported by the beam? Demonstrate that the maximum bending and shear stresses produced by *P* are acceptable.



Problem **6B**

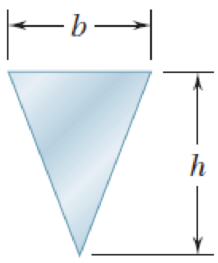
Determine the magnitude of load *P* that produces the allowable bending stress in the span (i.e., $\sigma_b = 1200$ psi). What nail spacing *s* is required to support this load magnitude? Demonstrate that the maximum horizontal shear stresses produced by *P* are acceptable.

Problem **7** (Beer et al., 2012, w/ permission)

A beam having the cross-section shown is subjected to a vertical shear *V*. Determine the horizontal line along which the shear stress is maximum and the constant *k* in the expression for maximum shearing stress

$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-section of the beam.



A) $k = 6/5$

B) $k = 4/3$

C) $k = 7/5$

D) $k = 3/2$

Problem **8** (Beer et al., 2012, w/ permission)

A beam having the cross-section shown is subjected to a vertical shear *V*. Determine the horizontal line along which the shear stress is maximum and the constant *k* in the expression for maximum shearing stress

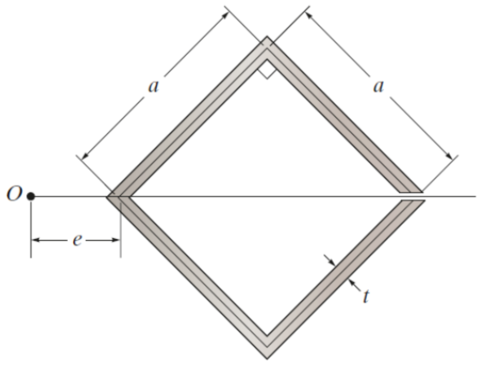
$$\tau_{\max} = k\frac{V}{A}$$

where *A* is the cross-section of the beam.

A) $k = 6/5$
B) $k = 4/3$
C) $k = 7/5$
D) $k = 3/2$

Problem **9** (Hibbeler, 2014, w/ permission)

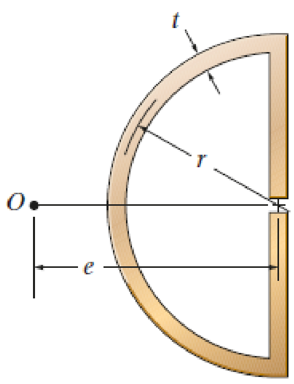
A thin plate of thickness t is bent to form the beam having the cross-section shown. Point O is the shear center of the section. If $a = 40$ mm, what is the value of length e?



A) $e = 7$ mm
B) $e = 14$ mm
C) $e = 20$ mm
D) $e = 25$ mm

Problem **10** (Hibbeler, 2014, w/ permission)

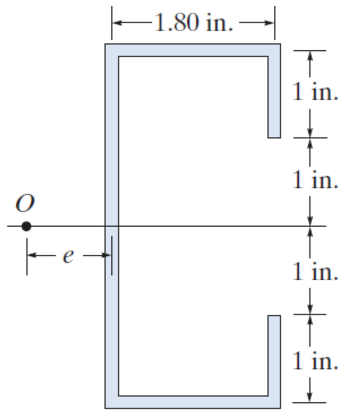
A thin plate of thickness t is bent to form the beam having the cross-section shown. Point O is the shear center of the section. If $r = 25$ mm, what is the value of distance e?



A) $e = 30$ mm
B) $e = 35$ mm
C) $e = 40$ mm
D) $e = 45$ mm

Problem **11** (Hibbeler, 2014, w/ permission)

Determine the location e of the shear center, point O, for the thin-walled member having the cross-section shown. What is the value of distance e? The member segments have the same thickness t.



A) $e = 0.29$ in.

B) $e = 0.54$ in.

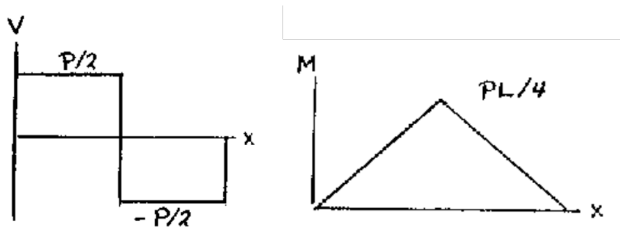
C) $e = 1.06$ in.

D) $e = 1.51$ in.

SOLUTIONS

P.1 ➡ *Solution*

Part A: The shear force and bending moment diagrams for the beam are shown in continuation.



Clearly, reactions R_A and R_B are both equal to $P/2$. The area of the rectangular cross-section is $A = bh$. The maximum shear stress in the beam is such that

$$\tau_m = \frac{3V_{\max}}{2A} \rightarrow \tau_m = \frac{3P}{4bh}$$

Given the maximum bending moment $M_{max} = PL/4$, the maximum normal stress can be determined with the flexure formula,

$$\sigma_m = \frac{M_{\max}}{S} = \frac{PL/4}{bh^3/6} = \frac{3PL}{2bh^3}$$

Finally, dividing τ_m by σ_m gives

$$\frac{\tau_m}{\sigma_m} = \frac{\frac{3P}{4bh}}{\frac{3PL}{2bh^2}} = \frac{3P}{4bh} \times \frac{2bh^2}{3PL} \rightarrow \boxed{\frac{\tau_m}{\sigma_m} = \frac{h}{2L}}$$

Part B: Solving the equation obtained above for the depth h yields

$$\frac{\tau_m}{\sigma_m} = \frac{h}{2L} \rightarrow h = \frac{2\tau_m L}{\sigma_m}$$

$$\therefore h = \frac{2 \times 0.9 \times 2.5}{12} = 0.375$$

$$\therefore \boxed{h = 375 \text{ mm}}$$

We then utilize the equation for τ_m to determine the width b,

$$\tau_m = \frac{3P}{4bh} \rightarrow b = \frac{3P}{4\tau_m h}$$

$$\therefore b = \frac{3 \times 50}{4 \times 900 \times 0.375} = 0.111$$

$$\therefore \boxed{b = 111 \text{ mm}}$$

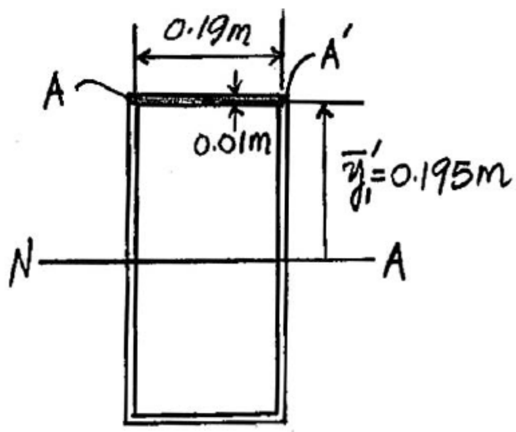
The correct answer is **A**.

P.2 ➡ *Solution*

1. True. The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}\times 0.2\times 0.4^3 - \frac{1}{12}\times 0.18\times 0.38^3 = 2.44\times 10^{-4}\ \text{m}^4$$

Refer to the following figure.



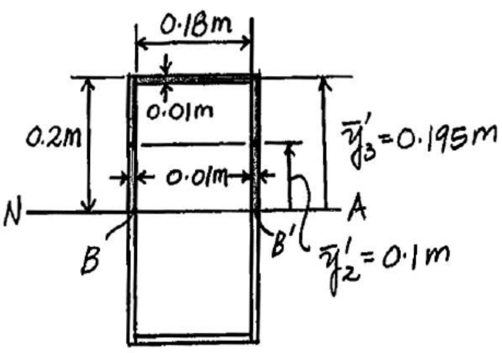
The first moment of area relative to the fiber at which point A is located follows as

$$Q_A = \bar{y}_1' A_1' = 0.195\times(0.01\times 0.19) = 3.71\times 10^{-4}\ \text{m}^3$$

Due to symmetry, the shear flow is the same for points *A* and *A'*. Shear flow q_A is then calculated as

$$q_A = \frac{1}{2}\left(\frac{VQ_A}{I}\right) = 0.5\times\left[\frac{(300\times 10^3)\times(3.71\times 10^{-4})}{(2.44\times 10^{-4})}\right] = 228\ \text{kN/m}$$

We know antecedently that the closer the point is to the neutral axis, the more intense the shear flow will be. Therefore, shear flow q_B is certainly greater than q_A, and it remains to calculate how much so. Refer to the following figure.



The moment relative to the location of point *B* is given by

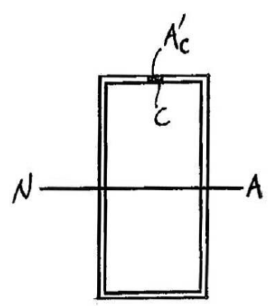
$$Q_B = 2\bar{y}_2' A_2' + \bar{y}_3' A_3' = 2\times 0.1\times(0.2\times 0.01) + 0.195\times(0.01\times 0.18) = 7.51\times 10^{-4}\ \text{m}^3$$

Due to symmetry, the shear flow at points *B* and *B'* is the same. Thus, shear flow q_B is determined to be

$$q_B = \frac{1}{2}\left(\frac{VQ_B}{I}\right) = 0.5\times\left[\frac{(300\times 10^3)\times(7.51\times 10^{-4})}{(2.44\times 10^{-4})}\right] = 462\ \text{kN/m}$$

which is more than twice the value of the shear flow at point *A*.

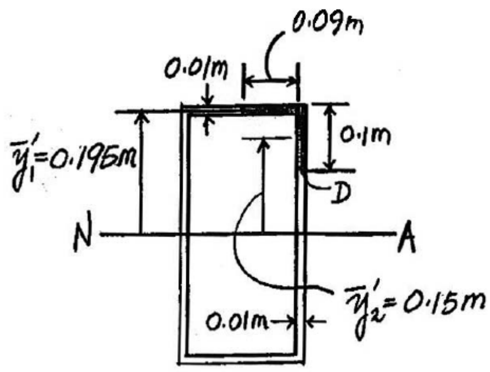
2. True. Consider the following illustration.



Due to symmetry, we clearly have $A_C' = 0$ and, consequently, $Q_C = 0$. In this case, the shear flow is trivial,

$$q_C = \frac{VQ_C}{I} = 0$$

3. False. Consider the following illustration.



The first moment of area relative to the neutral axis is given by

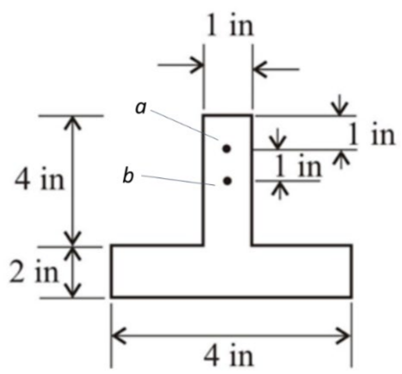
$$Q_D = \bar{y}_1' A_1' + \bar{y}_2' A_2' = 0.195 \times (0.01 \times 0.09) + 0.15 \times (0.1 \times 0.01) = 3.26 \times 10^{-4} \text{ m}^3$$

The shear flow at point D is then

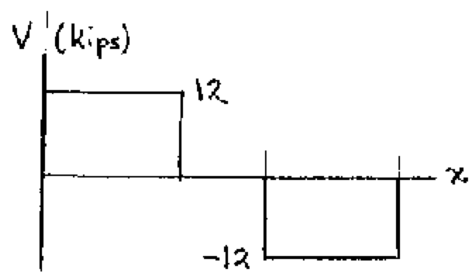
$$q_D = \frac{VQ_D}{I} = \frac{(450 \times 10^3) \times (3.26 \times 10^{-4})}{(2.44 \times 10^{-4})} = 601 \text{ kN/m}$$

P.3 ➡ *Solution*

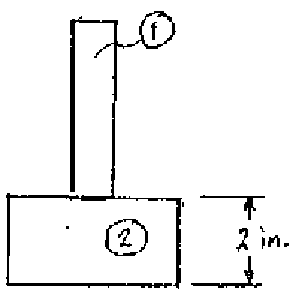
1. True. The cross-section of the beam is illustrated below.



An analysis of the forces acting on the beam leads to the following shear force distribution.



Clearly, the shear force acting on section *n-n* is $V = 12$ kips. We then divide the cross-section into two segments, as shown.



The quantities we require are tabulated below.

Part	A (in.²)	$\bar{y}$ (in.)	$A\bar{y}$ (in.³)	d (in.)	Ad^2 (in.⁴)	$\bar{I}$ (in.⁴)
1	1×4 = 4.0	4.0	16.0	2.0	16.0	$1\times4^3/12 = 5.33$
2	4×2 = 8.0	1.0	8.0	-1.0	8.0	$4\times2^3/12 = 2.67$
Σ	12.0		24.0		24.0	8.0

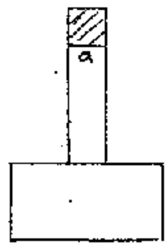
We proceed to determine the centroid $\bar{y}$ of the beam section,

$$\bar{y} = \frac{\Sigma A\tilde{y}}{\Sigma A} = \frac{24.0}{12.0} = 2.0 \text{ in.}$$

The moment of inertia, in turn, is given by

$$I = \Sigma\bar{I} + \Sigma Ad^2 = 8.0 + 24.0 = 32 \text{ in.}^4$$

In pursuance of the shear stress at point a, consider the shaded portion of the cross-section.



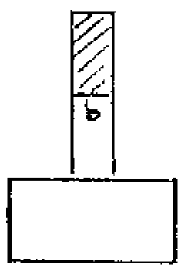
The area of this region is $A = 1\times1 = 1$ in.², and the distance from the neutral axis to the center of the region is $\bar{y} = 3.5$ in. The moment of area about the neutral axis is then

$$Q_a = A\bar{y} = 1\times3.5 = 3.5 \text{ in.}^3$$

In addition, the thickness of the region is $t = 1.0$ in. The shear stress can be determined with the shear formula,

$$\tau_a = \frac{VQ_a}{It} = \frac{12\times3.5}{32\times1.0} = 1.3 \text{ ksi}$$

2. False. In order for us to determine the shear at point b, consider another shaded portion of the beam section.



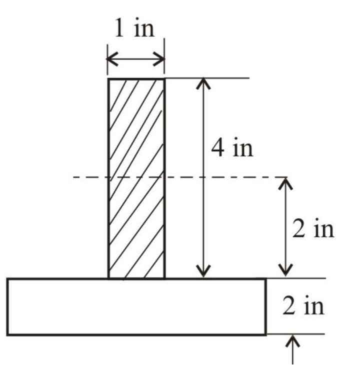
The area of this region is $A = 1\times2 = 2$ in,², and the distance from the neutral axis to the center of the region is $\bar{y} = 3$ in. The moment of area about the neutral axis is then

$$Q_b = A\bar{y} = 2\times3 = 6.0 \text{ in.}^3$$

As before, the thickness of the region is $t = 1.0$ in. Lastly, the shear stress is calculated as

$$\tau_b = \frac{VQ_b}{It} = \frac{12\times6.0}{32\times1.0} = 2.3 \text{ ksi}$$

3. True. To find the maximum shearing stress, refer to the figure below.



The moment of the shaded area with respect to the neutral axis is

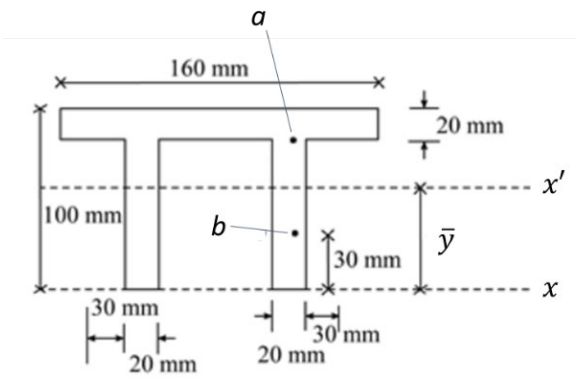
$$Q_{\max} = 4\times1\times2 = 8 \text{ in.}^3$$

The corresponding shear stress is then

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{12\times8}{32\times1} = 3 \text{ ksi}$$

P.4 ➡ *Solution*

1. False. The cross-section of the beam is illustrated below.



The reactions at points *A* and *B* are easily seen to be equal to 90 kN. Thus, the shear force at section *n-n* has an intensity of 90 kN. We proceed to determine the centroid $\bar{y}$ of the beam section,

$$\bar{y} = \frac{(160\times20)\times\left(100-\frac{20}{2}\right)+2\times\left[20\times(100-20)\right]\times\left(\frac{100-20}{2}\right)}{160\times20+2\times\left[20\times(100-20)\right]} = 65 \text{ mm}$$

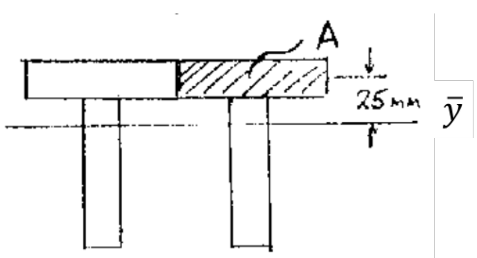
We also require the moment of inertia about the *x*-axis, which follows from the parallel-axis theorem,

$$I_x = I + Ay^2A$$

$$\therefore I_x = \left\{\frac{160\times20^3}{12}+\left[160\times20\times\left(100-65-\frac{20}{2}\right)^2\right]\right\}+2\left\{\frac{20\times80^3}{12}+\left[20\times80\times\left(\frac{80}{2}-65\right)^2\right]\right\}$$

$$\therefore I_x = 2.11\times10^6 + 3.71\times10^6 = 5.82\times10^6 \text{ mm}^4$$

In order for us to determine the shear at point *a*, consider the shaded portion of the cross-section.



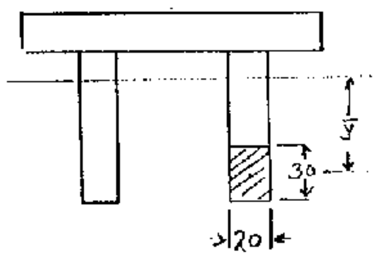
The area of this region is $A = 80\times20 = 1600$ mm², and the distance from the neutral axis to the center of the region is $\bar{y} = 25$ mm. The moment of area about the neutral axis is then

$$Q_a = A\bar{y} = 1600\times25 = 40{,}000 \text{ mm}^3 = 40\times10^{-6} \text{ m}^3$$

The corresponding shear stress follows from the shear formula,

$$\tau_a = \frac{VQ_a}{It} = \frac{\left(90\times10^3\right)\times\left(40\times10^{-6}\right)}{\left(5.82\times10^{-6}\right)\times\left(20\times10^{-3}\right)} = 30.9 \text{ MPa}$$

2. False. Next, to determine the shear at point *b*, consider another shaded portion of the cross-section.



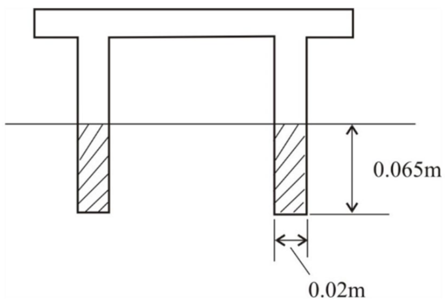
The area of this region is $A = 30 \times 20 = 600$ mm², and the distance from the neutral axis to the center of the region is $\bar{y} = 65 - 15 = 50$ mm. The moment of area about the neutral axis is then

$$Q_b = A\bar{y} = 600 \times 50 = 30{,}000 \text{ mm}^3 = 30 \times 10^{-6} \text{ m}^3$$

The corresponding shear stress is calculated as

$$\tau_b = \frac{VQ_b}{It} = \frac{(90 \times 10^3) \times (30 \times 10^{-6})}{(5.81 \times 10^{-6}) \times (20 \times 10^{-3})} = 23.2 \text{ MPa}$$

3. True. The largest shear stress occurs in the region below the centroidal axis. Refer to the figure below.



The moment of the shaded area with respect to the neutral axis is

$$Q_{\max} = A\bar{y} = (0.065 \times 0.02) \times \left(\frac{0.065}{2} \times 2\right) = 8.45 \times 10^{-5} \text{ m}^3$$

The corresponding shear stress is computed as

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{(90 \times 10^3) \times (8.45 \times 10^{-5})}{(5.82 \times 10^{-6}) \times (0.02 \times 2)} = 32.7 \text{ MPa}$$

P.5 ➡ *Solution*

The moment of inertia of the beam about the *z*-axis is

$$I_z = \frac{190 \times 250^3}{12} - \frac{140 \times 150^3}{12} = 2.08 \times 10^8 \text{ mm}^4$$

The first moment of area Q is

$$Q = 140 \times 50 \times 100 = 700{,}000 \text{ mm}^3$$

The shear flow *q* based on the beam shear force *V* is given by

$$q = \frac{VQ}{I} = \frac{9000 \times 700{,}000}{(2.08 \times 10^8)} = 30.3 \text{ N/mm}$$

The maximum spacing interval *s* follows from the inequality

$$qs \le n_f V_f \rightarrow s \le \frac{n_f V_f}{q}$$

$$\therefore s \le \frac{2 \text{ screws} \times 800 \text{ N/screw}}{30.3 \text{ N/mm}} = \boxed{53 \text{ mm}}$$

The nails must be spaced no more than 5.3 centimeters apart.

○ The correct answer is **B**.

P.6 ➡ *Solution*

Part A: To begin, we compute the moment of inertia of the section about the z-axis.

Shape	Width b (in.)	Height h (in.)	I_C (in.^{4})	$d = y_i - \bar{y}$ (in.)	d^2A (in.^{4})	$I_C + d^2A$ (in.^{4})
Top flange	4	2	$4\times2^3/12 = 2.67$	6.0	$6.0^2\times(4\times2) = 288.0$	290.67
Web	2	10	$2\times10^3/12 = 166.67$	0	0	166.67
Bottom flange	4	2	$4\times2^3/12 = 2.67$	−6.0	$6.0^2\times(4\times2) = 288.0$	290.67
					Moment of inertia about the z-axis (in.^{4})	748.0

The moment of area for the system at hand is $Q = 2\times4\times6 = 48$ in.^{3} For the loading to be acceptable, the shear flow q must be such that

$$qs \le n_f V_f \rightarrow q \le \frac{n_f V_f}{s}$$

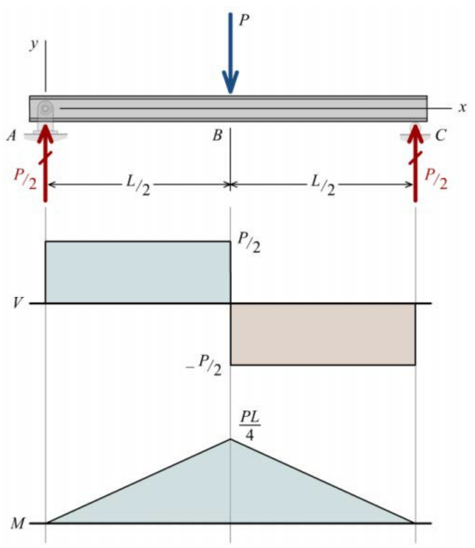
$$\therefore q \le \frac{1\text{ nail}\times120\text{ lb/nail}}{4.5\text{ in.}} = 26.67\text{ lb/in.}$$

Using the shear formula, we can determine the corresponding shear force V,

$$q = \frac{VQ}{I} \rightarrow V = \frac{qI}{Q}$$

$$\therefore V = \frac{26.67\times748}{48} = 416\text{ lb}$$

The loading pattern of the beam, the shear force and bending moment diagrams are illustrated below.



From the diagrams above, we see that $V = P/2$ and, accordingly,

$$V = \frac{P}{2} \rightarrow P_{\max} = 2V$$

$$\therefore P_{\max} = 2\times416 = \boxed{832\text{ lb}}$$

We must verify that this load does not produce normal and shear stresses greater than the limiting values proposed. The maximum normal stress is

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{(PL/4)\times c}{I} = \frac{(832\times240/4)\times7}{748} = 467\text{ psi} < 1200\text{ psi}$$

which is a valid inequality. To determine the maximum shear stress, we first compute the maximum moment of area,

$$Q_{\max} = 2\times5\times2.5 + 4\times2\times6 = 73\text{ in.}^3$$

Then,

$$\tau_{\max} = \frac{VQ}{It} = \frac{416\times73}{748\times2} = 20.3\text{ psi} < 90\text{ psi}$$

which is also a valid inequality. We conclude that the bending and shear stresses produced by P are both acceptable.

Part B: We turn to the magnitude of load P that would produce the allowable bending stress in the span. Using the flexure formula, we have

$$\sigma_b = \frac{Mc}{I} \leq 1200 \rightarrow M_{\max} = \frac{1200 \times I}{c}$$

$$\therefore M_{\max} = \frac{1200 \times 748}{7} = 128,200 \text{ lb-in.}$$

From the bending moment diagram presented just now, we see that $M_{max} = PL/4$. Therefore,

$$M_{\max} = \frac{PL}{4} \rightarrow P_{\max} = \frac{4M_{\max}}{L}$$

$$\therefore P_{\max} = \frac{4 \times 128,200}{(20 \times 12)} = 2137 \text{ lb}$$

To determine the required nail spacing s, note that $V_{max} = P_{max}/2 = 2137/2 = 1069$ lb. The corresponding shear flow is

$$q = \frac{VQ}{I} = \frac{1069 \times 48}{748} = 68.6 \text{ lb/in.}$$

The required nail spacing s is then

$$qs \leq n_f V_f \rightarrow s \leq \frac{1 \text{ nail} \times 120 \text{ lb}}{68.6 \text{ lb/in.}} = \boxed{1.75 \text{ in.}}$$

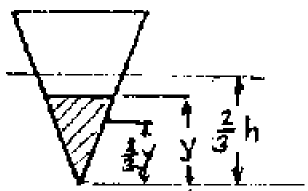
It remains to show that the maximum horizontal shear stress produced by P is acceptable; that is,

$$\tau_{\max} = \frac{VQ}{It} = \frac{(P/2) \times Q}{It} = \frac{(2137/2) \times 73}{748 \times 2} = 52.1 \text{ psi} < 90 \text{ psi}$$

The inequality is true and hence the loading conditions are tolerable.

P.7 ➡ *Solution*

The area of the cross-section is $A = (1/2)bh$, and the moment of inertia about the neutral axis is $I = (1/36)bh^3$. Suppose the section were cut at a location y, as shown below.



The thickness t of the section evolves from bottom to top as

$$t(y) = \frac{by}{h}$$

The area of the shaded section, in turn, is calculated as

$$A(y) = \frac{1}{2} \times \frac{by}{h} \times y = \frac{by^2}{2h}$$

The centroidal depth $\bar{y}$ is given by

$$\bar{y}(y) = \frac{2}{3}h - \frac{2}{3}y$$

The first moment of area is the product of the two preceding quantities,

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h - y)$$

Equipped with these relations, we can determine the shear stress τ,

$$\tau(y)=\frac{VQ}{It}=\frac{V\times\frac{by^2}{3}(h-y)}{\frac{1}{36}bh^3\times\frac{by}{h}}=\frac{12V}{bh^3}\left(hy-y^2\right)$$

To find the location of maximum τ, we differentiate the relation above with respect to *y* and put it to zero,

$$\frac{d\tau}{dy}=\frac{12V(h-y)}{bh^2}-\frac{12Vy}{bh^2}=\frac{12V(h-2y)}{bh^2}=0\rightarrow h-2y=0$$

$$\therefore y_m=\frac{h}{2}$$

That is, the shear stress is maximum at mid-height. Evaluating $\tau(y_m)$, we get

$$\tau(y_m)=\frac{12V}{bh^2}\times\left[h\times\frac{h}{2}-\left(\frac{h}{2}\right)^2\right]=\frac{12V}{bh^2}\times\left(\frac{h^2}{2}-\frac{h^2}{4}\right)$$

$$\therefore \tau(y_m)=\tau_{\max}=\frac{12V}{bh^3}\times\frac{h^2}{4}=\frac{3V}{bh}$$

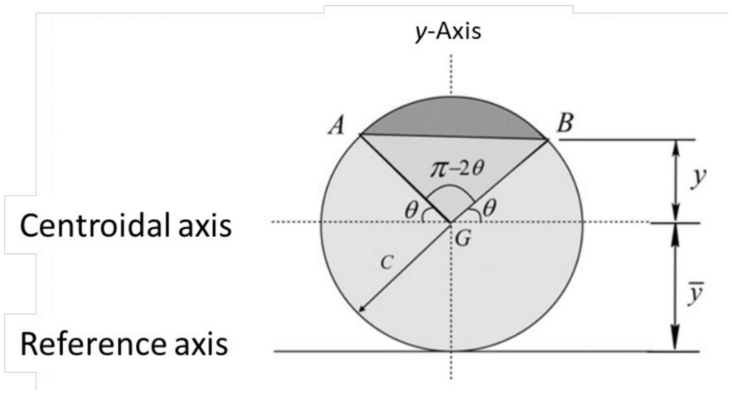
$$\therefore \tau_{\max}=\frac{3}{2}\frac{V}{A}$$

The value of constant *k* in the expression for maximum shearing stress, $\tau_{\max}=k(V/A)$, is 3/2.

The correct answer is **D**.

P.8 ➡ *Solution*

Consider an axis *AB* parallel to the centroidal axis, as shown.



The area of the section above *AB* (that is, the area shaded in dark gray) is given by the difference

$$A=\text{Area of sector ABG}-\text{Area of triangle ABG}$$

$$\therefore A=\frac{(\pi-2\theta)}{2\pi}\times\pi c^2-\frac{1}{2}\times 2c\times\cos\theta\times c\times\sin\theta$$

$$\therefore A=\left[\frac{(\pi-2\theta)}{2\pi}-\left(\frac{\sin 2\theta}{2}\right)\right]c^2$$

We proceed to determine the moment of area above *AB*,

$$Q=\Sigma A\bar{y}$$

$$\therefore Q=\left\{\frac{[\pi-2\theta]}{2}c^2\times\left[\frac{2c\sin(90-\theta)}{3\left(\frac{\pi}{2}-\theta\right)}\right]\right\}-\left[\left(\frac{\sin 2\theta}{2}\right)c^2\left(\frac{2}{3}c\sin\theta\right)\right]$$

$$\therefore Q=\frac{c^3}{3}\left[2\left(\frac{\pi-2\theta}{\pi-2\theta}\right)\cos\theta-2\sin^2\theta\cos\theta\right]$$

$$\therefore Q=\frac{c^3}{3}\left[2\cos\theta-2\sin^2\theta\cos\theta\right]$$

The shear stress in the beam is given by the shear formula, $\tau = VQ/Ib$. Substituting the moment obtained just now and the section width $b = 2c\cos\theta$ gives

$$\tau = \frac{V}{I}\left[\frac{\frac{c^3}{3}\left(2\cos\theta - 2\sin^2\theta\cos\theta\right)}{2c\cos\theta}\right]$$

$$\therefore \tau = \frac{V}{I} \times \frac{c^2}{3} \times \left(1 - \sin^2\theta\right) \;\; \text{(I)}$$

For the shear stress to attain a maximum value, we differentiate this relation with respect to θ and set it to zero; that is,

$$\frac{d\tau}{d\theta} = -2\sin\theta\cos\theta = 0$$

$$\therefore \sin 2\theta = 0$$

$$\therefore \theta = 0$$

The maximum shear stress occurs at $\theta = 0$, i.e., at the centroidal axis. The value of such a stress can be determined by substituting $\theta = 0$ in equation (I) and noting that the moment of inertia $I = \pi c^4/4$, namely,

$$\tau_{\max} = \frac{V}{I} \times \frac{c^2}{3} \times \left(1 - \underbrace{\sin^2 0}_{=0}\right)$$

$$\therefore \tau_{\max} = \frac{V}{\left(\pi c^4/4\right)} \times \frac{c^2}{3} = \frac{4}{3}\frac{V}{\pi c^2}$$

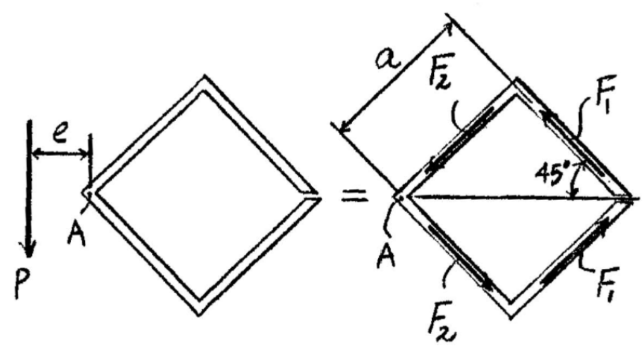
$$\therefore \tau_{\max} = \frac{4}{3}\frac{V}{A}$$

The value of constant k in the expression for maximum shearing stress $\tau_{\max} = k(V/A)$ is 4/3.

The correct answer is **B**.

P.9 ➡ *Solution*

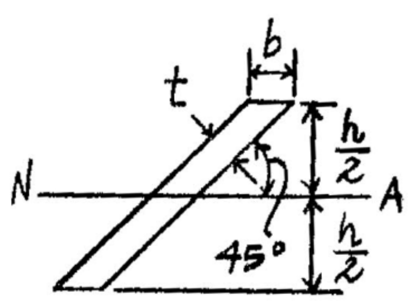
Refer to the figure below.



Summing moments about point A, we have

$$\Sigma M_A = 0 \rightarrow P \times e = 2F_1 \times a \;\; \text{(I)}$$

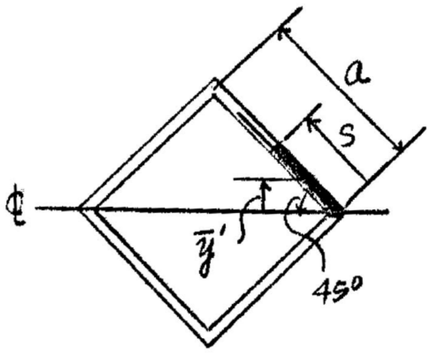
Next, consider the inclined segment shown below.



The moment of inertia of this segment about the neutral axis is given by the usual formula $I = bh^3/12$. In this case, however, we have $b = t/\sin 45^{\circ} = (2/\sqrt{2})t$ and $h = 2a\sin 45^{\circ} = \sqrt{2}a$. Accordingly, the moment of inertia of the section is

$$I = 2\times\left[\frac{1}{12}\times\left(\frac{2t}{\sqrt{2}}\right)\left(\sqrt{2}a\right)^3\right] = \frac{2}{3}ta^3$$

Consider now the following figure.



The distance from the axis of symmetry to the centroid of the shaded segment of the section is $y = (s/2)\sin 45^{\circ} = (\sqrt{2}/4)s$. The first moment of area of this segment about the axis of symmetry is then

$$Q = \bar{y}'A' = \frac{\sqrt{2}}{4}s\times st = \frac{\sqrt{2}}{4}ts^2$$

The shear flow easily follows,

$$q = \frac{VQ}{I} = \frac{P\times\frac{\sqrt{2}}{4}ts^2}{\frac{2}{3}a^3t} = \frac{3\sqrt{2}}{8a^3}Ps^2$$

The shear force resisted by the open-ended segment is obtained by integrating q,

$$F_1 = \int_0^a q\,ds = \int_0^a \frac{3\sqrt{2}P}{8a^3}s^2\,ds = \frac{3\sqrt{2}P}{8a^3}\left(\frac{s^3}{3}\right)\Bigg|_0^a = \frac{\sqrt{2}P}{8}$$

Backsubstituting this result into equation (I) gives

$$P\times e = 2F_1\times a \rightarrow \cancel{P}\times e = 2\times\frac{\sqrt{2}\cancel{P}}{8}\times a$$

$$\therefore e = \frac{\sqrt{2}}{4}a$$

With a = 40 mm, we ultimately have

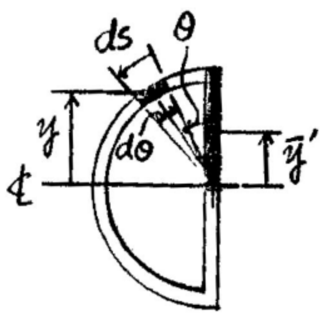
$$e = \frac{\sqrt{2}}{4}\times 40 = \boxed{14\text{ mm}}$$

The shear center is located 14 mm away from the nearest corner of the section.

The correct answer is **B**.

P.10 ➡ *Solution*

For the infinitesimal arc segment shown in the next figure, we have $y = r\cos\theta$, and the area is $dA = tds = trd\theta$.



Then, the moment of inertia of the entire cross-section about the axis of symmetry is

$$I = \frac{1}{12} \times t \times (2r)^3 + \int y^2 dA$$

$$\therefore I = \frac{2}{3} r^3 t + \int_0^{\pi} (r\cos\theta)^2 \, trd\theta$$

$$\therefore I = \frac{2}{3} r^3 t + \frac{r^3 t}{2} \times \left(\frac{1}{2}\sin 2\theta + \theta \right)\Big|_0^{\pi}$$

$$\therefore I = \frac{r^3 t}{6}(4 + 3\pi)$$

Referring to the figure above, we have $\bar{y}' = r/2$. The moment of area Q as a function of *s* is then

$$Q = \bar{y}'A' + \int y dA = \frac{r}{2} \times rt + \int_0^{\theta} r\cos\theta \times trd\theta$$

$$\therefore Q = \frac{1}{2} r^2 t + r^2 t \int_0^{\theta} \cos\theta d\theta$$

$$\therefore Q = \frac{r^2 t}{2} \times (1 + 2\sin\theta)$$

Appealing to the equation for shear flow, we obtain

$$q = \frac{VQ}{I} = \frac{P \times \frac{r^2 \cancel{t}}{2}(1 + 2\sin\theta)}{\frac{r^3 \cancel{t}}{2}(4 + 3\pi)} = \frac{3P}{(4 + 3\pi)r} \times (1 + 2\sin\theta)$$

The shear force resisted by the arc segment follows as

$$F = \int q ds = \int_0^{\pi} qrd\theta$$

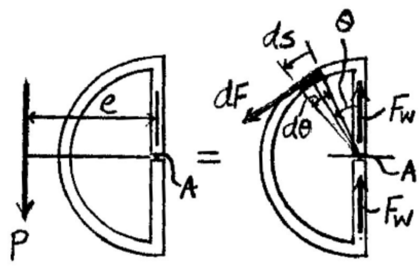
$$\therefore F = \int_0^{\pi} \frac{3P}{(4 + 3\pi)r} \times (1 + 2\sin\theta) rd\theta$$

$$\therefore F = \frac{3P}{(4 + 3\pi)} \times (\theta - 2\cos\theta)\Big|_0^{\pi}$$

$$\therefore F = \frac{3P}{(4 + 3\pi)} \times \left[\pi - 2 \times (-1) - (-2) \right]$$

$$\therefore F = \frac{3P(\pi + 4)}{(4 + 3\pi)}$$

Refer to the figure below.



The location of the shear center can be obtained by taking moments about point *A*,

$$\Sigma M_A = 0 \rightarrow P \times e = r \int dF$$

$$\therefore \cancel{P} \times e = r \times \frac{3\cancel{P}(\pi + 4)}{(4 + 3\pi)}$$

$$\therefore e = \frac{3(\pi + 4)}{4 + 3\pi} r$$

Substituting r = 25 mm yields

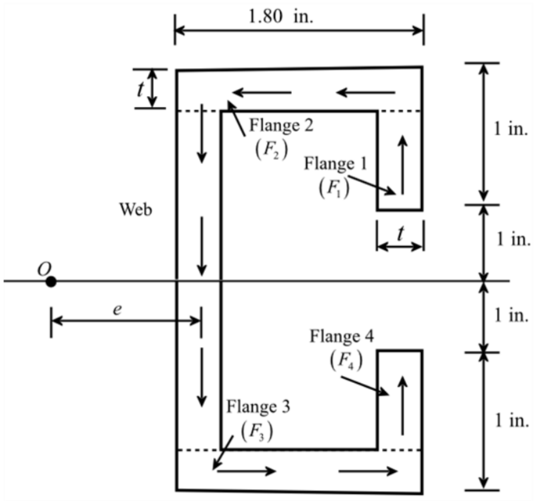
$$e = \frac{3(\pi + 4)}{4 + 3\pi} \times 25 = \boxed{40 \text{ mm}}$$

The shear center is located 40 millimeters away from point *A*.

The correct answer is **C**.

P.11 Solution

The cross-section of the beam is divided in a web and four flange segments, as shown in continuation.



Given the dimensions b and t of the web, the corresponding moment of inertia is

$$I_w = \frac{tb^3}{12}$$

$$\therefore I_w = \frac{t(4-2t)^3}{12} = \frac{8t^4 + 48t^3 - 96t^2 + 64t}{12}$$

$$\therefore I_w = \frac{64t}{12} = 5.33t$$

where we have neglected all terms containing thickness with powers greater than 1. Proceeding similarly with flanges 1 and 2, the moments of inertia are determined to be $I_{F,1} = 2.33t$ and $I_{F,2} = 7.2t$. Due to symmetry, the moments at flanges 3 and 4 are such that

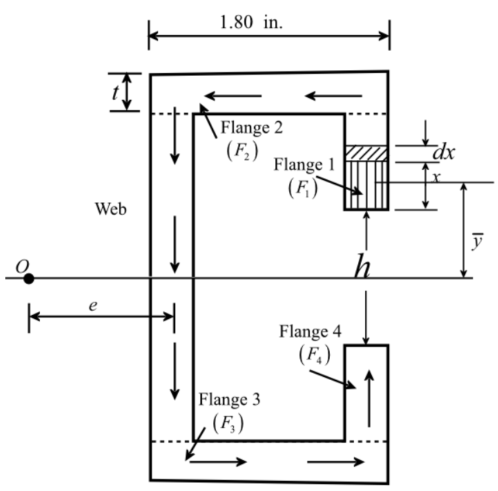
$$I_{F,3} = I_{F,2} = 7.2t$$
$$I_{F,4} = I_{F,1} = 2.33t$$

The total moment of inertia is then

$$I = I_{\text{web}} + I_{F,1} + I_{F,2} + I_{F,3} + I_{F,4}$$

$$\therefore I = 5.33t + 2.33t + 7.2t + 7.2t + 2.33t = 24.4t$$

Consider now an infinitesimal element of thickness dx in flange 1, located at a distance x from the base of the flange.



Owing to symmetry, the neutral axis is located in the middle of the cross-section. The distance $\bar{y}$ of the element from the neutral axis is

$$\bar{y} = 1 + \frac{x}{2}$$

The first moment of area about the neutral axis for flange 1 is then

$$Q_{F,1} = A_{F,1}\bar{y}_1 = (x \times t) \times \left(1 + \frac{x}{2}\right) = t\left(x + \frac{x^2}{2}\right)$$

The shear flow in this flange is obtained with the shear formula,

$$q_{F,1} = \frac{VQ_1}{I} = \frac{Vt\left(x + \frac{x^2}{2}\right)}{24.4t}$$

$$\therefore q_{F,1} = \frac{2Vx + Vx^2}{48.8}$$

The shear flow can be converted to a force by integration,

$$V_{F,1} = \int q_{F,1} dA = \int_0^{1-t} \frac{2Vx + Vx^2}{48.8} dx$$

$$\therefore V_{F,1} = \frac{2V}{48.8}\int_0^{1-t} x dx + \frac{V}{48.8}\int_0^{1-t} x^2 dx$$

$$\therefore V_{F,1} = \frac{2V}{48.8}\left(\frac{x^2}{2}\right)\Bigg|_0^{1-t} + \frac{V}{48.8}\left(\frac{x^3}{3}\right)\Bigg|_0^{1-t}$$

$$\therefore V_{F,1} = \frac{V}{48.8}(1-t)^2 + \frac{V}{146.4}(1-t)^3$$

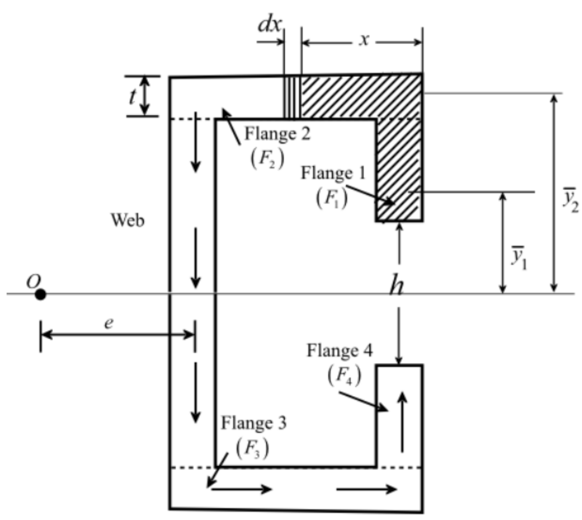
$$\therefore V_{F,1} = \frac{V}{48.8} \times \left(1 - \cancel{2t + t^2}\right) + \frac{V}{146.4} \times \left(1 - \cancel{3t + 3t^2 - t^3}\right)$$

$$\therefore V_{F,1} = 0.0273V$$

where, for simplicity, we have neglected all terms involving the thickness t. The shear force on flange 4 is the same as that on flange 1; that is,

$$V_{F,4} = V_{F,1} = 0.0273V$$

Consider now an infinitesimal element of thickness dx in flange 2, located a distance x from the rightmost fiber of the cross-section.



The first moment of area about the neutral axis for flange 2 is

$$Q_{F,2} = A_{F,1}\bar{y}_1 + A_{F,2}\bar{y}_2$$

where $\bar{y}_1$ is the distance from the neutral axis to the centroid of flange 1 and $\bar{y}_2$ is the distance to the centroid of flange 2. Substituting the pertaining quantities, we obtain

$$Q_{F,2} = \left[t\times(1-t)\right]\times\left[(1-t)+0.5\right]+xt\times\left[1+(1-t)+\frac{t}{2}\right]$$

$$\therefore Q_{F,2} = \left(t^3-2.5t^2+1.5t\right)+x\left(2t-\frac{t^2}{2}\right)$$

$$\therefore Q_{F,2} = t(1.5+2x)$$

where we have neglected terms with t to powers greater than 1. The shear flow in this flange follows as

$$q_{F,2} = \frac{VQ_{F,2}}{I} = \frac{V\times t(1.5+2x)}{24.4t}$$

$$\therefore q_{F,2} = \frac{V(1.5+2x)}{24.4}$$

As before, this shear flow can be converted to a force by integration,

$$V_{F,2} = \int q_{F,2}dA = \frac{V\times 1.5}{24.4}\int_0^{1.8}dx+\frac{2V}{24.4}\int_0^{1.8}xdx$$

$$\therefore V_{F,2} = \int q_{F,2}dA = \frac{V\times 1.5}{24.4}\times(x)\Big|_0^{1.8}+\frac{2V}{24.4}\left(\frac{x^2}{2}\right)\Bigg|_0^{1.8}$$

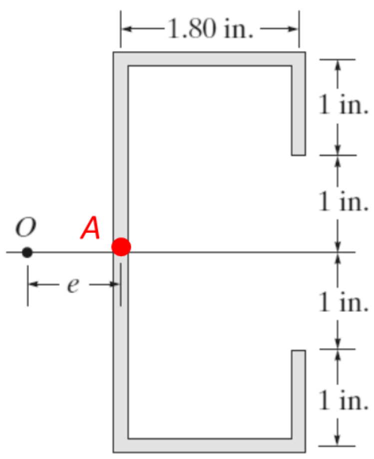
$$\therefore V_{F,2} = \frac{V\times 1.5}{24.4}\times 1.8+\frac{2V}{24.4}\times\frac{1}{2}\times 1.8^2$$

$$\therefore V_{F,2} = 0.11V+0.13V = 0.24V$$

The shear force on flange 3 is the same as that on flange 2; that is,

$$V_{F,3} = V_{F,2} = 0.24V$$

Lastly, we shall locate the shear center by taking moments about point *A*, which is indicated below.



Thus,

$$V\times e = V_{F,1}\times 1.80+V_{F,2}\times 2+V_{\text{web}}\times 0+V_{F,3}\times 2+V_{F,4}\times 1.80$$

$$\therefore e\times V = 0.0273V\times 1.80+0.24V\times 2+0+0.24V\times 2+0.0273V\times 1.80$$

$$\therefore \boxed{e = 1.06\text{ in.}}$$

The shear center is about 1 inch away from the leftmost fiber of the section.

The correct answer is **C**.

ANSWER SUMMARY

Problem 1	**1A**	Open-ended pb.
	1B	**A**
Problem 2		T/F
Problem 3		T/F
Problem 4		T/F
Problem 5		**B**
Problem 6	**6A**	Open-ended pb.
	6B	Open-ended pb.
Problem 7		**D**
Problem 8		**B**
Problem 9		**B**
Problem 10		**C**
Problem 11		**C**

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