Quiz SM210

BUCKLING OF COLUMNS

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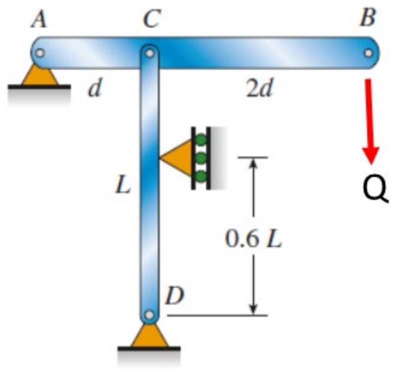
PROBLEMS

Problem 1 (Gere & Goodno, 2009, w/ permission)

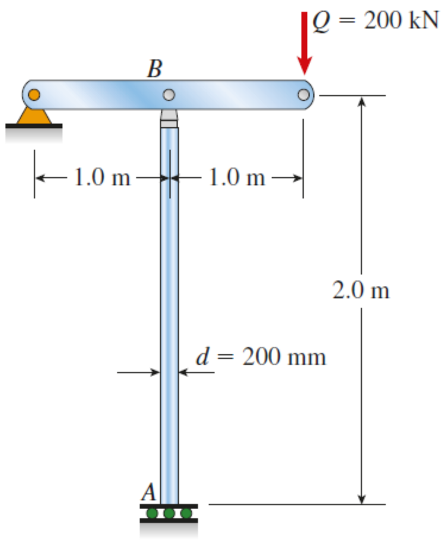
A horizontal beam *AB* is pin supported at end *A* and carries a clockwise moment *M* at joint *B*, as shown in the figure. The beam is also supported at *C* by a pinned column of length *L*; the column is restrained laterally at 0.6*L* from the base at *D*. Assume the column can only buckle in the plane of the frame. The column is a solid aluminum bar ($E = 10\times10^6$ psi) of square cross-section having length $L = 30$ in. and side dimensions $b = 1.5$ in. Let dimension $d = L/2$. Based upon the critical load of the column, determine the allowable force *Q* if the factor of safety with respect to buckling is $FS = 2.0$.



A) $Q_{allow} = 10.4$ k
B) $Q_{allow} = 21.5$ k
C) $Q_{allow} = 30.4$ k
D) $Q_{allow} = 41.5$ k

Problem 2 (Gere & Goodno, 2009, w/ permission)

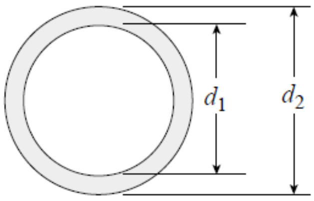
An aluminum ($E = 72$ GPa) tube *AB* of circular cross-section has a guided support at the base and is pinned at the top to a horizontal beam supporting a load $Q = 200$ kN (see figure). Determine the required thickness *t* of the tube if its outside diameter *d* is 200 mm and the desired factor of safety with respect to Euler buckling is $FS = 3.0$.



A) t = 10 mm
B) t = 17 mm
C) t = 24 mm
D) t = 31 mm

Problem **3** (Gere & Goodno, 2009, w/ permission)

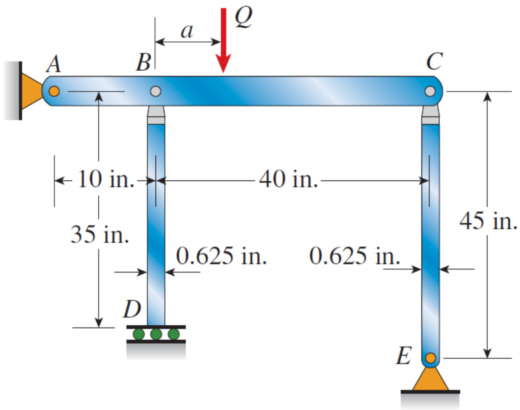
An aluminum pipe column (E = 10,400 ksi) with length L = 10.0 ft has inside and outside diameters d_1 = 5.0 in. and d_2 = 6.0 in, respectively (see figure). The column is supported only at the ends and may buckle in any direction. True or false? Consider lowest critical loads only.



1. ( ) The critical load if the column were pinned at both ends would be greater than 200 k.
2. ( ) The critical load if the column were fixed at one end and free at the other would be greater than 50 k.
3. ( ) The critical load if the column were fixed at one end and pinned at the other would be greater than 500 k.
4. ( ) The critical load if the column were fixed at both ends would be greater than 1000 k.

Problem **4A** (Gere & Goodno, 2009, w/ permission)

The horizontal beam ABC in the figure is supported by columns BD and CE. The beam is prevented from moving horizontally by the pin support at end A. Each column is pinned at its upper end to the beam, but at the lower ends, support D is a guided support and support E is pinned. Both columns are solid steel bars (E = 30×10^6 psi) of square cross-section with width of 0.625 in. A load Q acts at distance a from column BD. If the distance a = 12 in., what is the critical value Q_{cr} of the load?



A) Q_{cr} = 2980 lb
B) Q_{cr} = 3540 lb
C) Q_{cr} = 4050 lb
D) Q_{cr} = 4570 lb

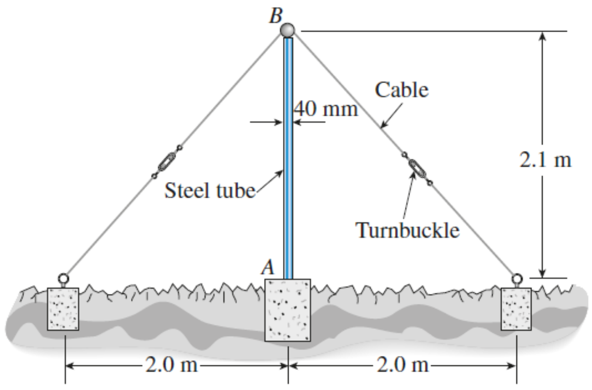
Problem **4B**

If the distance a can be varied between 0 and 40 in., what is the maximum value of Q_{cr}?

A) $Q_{cr,max}$ = 5580 lb
B) $Q_{cr,max}$ = 7040 lb
C) $Q_{cr,max}$ = 8550 lb
D) $Q_{cr,max}$ = 10,050 lb

Problem **5** (Gere & Goodno, 2009, w/ permission)

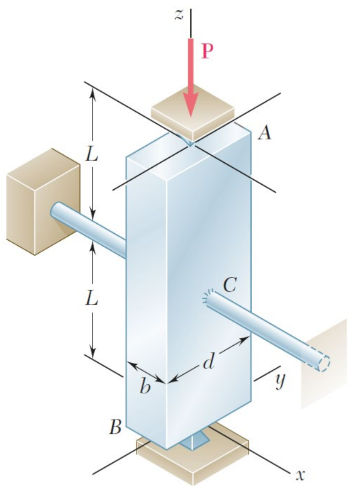
A vertical post *AB* is embedded in a concrete foundation and held at the top by two cables (see figure). The post is a hollow steel tube with modulus of elasticity 200 GPa, outer diameter 40 mm, and thickness 5 mm. The cables are tightened equally by turnbuckles. If a factor of safety of 3.0 against Euler buckling in the plane of the figure is desired, what is the maximum allowable tensile force T_{allow} in the cables?



A) $T_{allow} = 8.3$ kN
B) $T_{allow} = 13.2$ kN
C) $T_{allow} = 18.1$ kN
D) $T_{allow} = 23.0$ kN

Problem **6** (Beer et al., 2012, w/ permission)

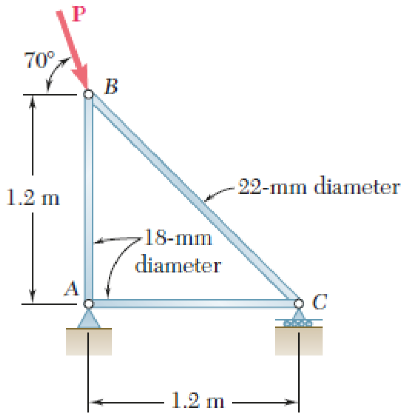
Column ABC has a uniform cross-section and is braced in the *xz*-plane at its midpoint *C*. Determine the ratio *b*/*d* for which the factor of safety is the same with respect to buckling in the *xz* and *yz* planes.



A) $b/d = 1/4$
B) $b/d = 1/3$
C) $b/d = 1/2$
D) $b = d$

Problem **7** (Beer et al., 2012, w/ permission)

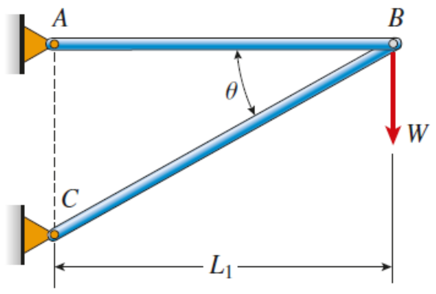
Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



A) $FS = 2.27$
B) $FS = 3.13$
C) $FS = 4.05$
D) $FS = 5.20$

Problem **8** (Gere & Goodno, 2009, w/ permission)

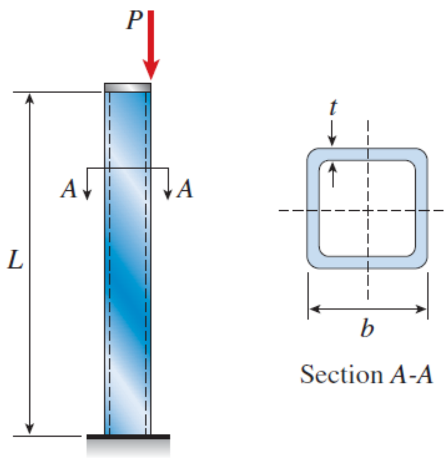
A truss ABC supports a load W at joint B, as shown in the figure. The length L_1 of member AB is fixed, but the length of strut BC varies as the angle θ is changed. Strut BC has a solid circular cross-section. Joint B is restrained against displacement perpendicular to the plane of the truss. Assuming that collapse occurs by Euler buckling of the strut, determine the angle θ for minimum weight of the strut.



A) $\theta = 19.8°$
B) $\theta = 26.6°$
C) $\theta = 32.5°$
D) $\theta = 38.1°$

Problem **9** (Gere & Goodno, 2009, w/ permission)

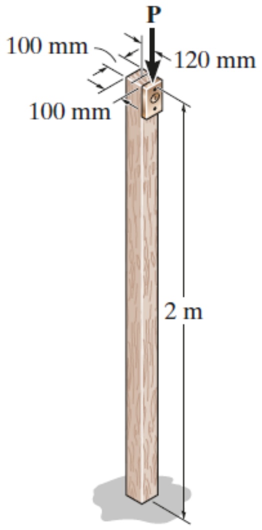
An aluminum box column ($E = 73$ GPa) of square cross-section is fixed at the base and free at the top (see figure). The outside dimension b of each side is 100 mm and the thickness of the wall is 8 mm. The resultant of the compressive loads on the top of the column is a force $P = 50$ kN acting on the outer edge of the column at the midpoint of one side. What is the longest permissible length L_{max} of the column if the deflection at the top is not to exceed 30 mm?



A) $L_{max} = 1.07$ m
B) $L_{max} = 1.42$ m
C) $L_{max} = 1.88$ m
D) $L_{max} = 2.21$ m

Problem 10 (Hibbeler, 2014, w/ permission)

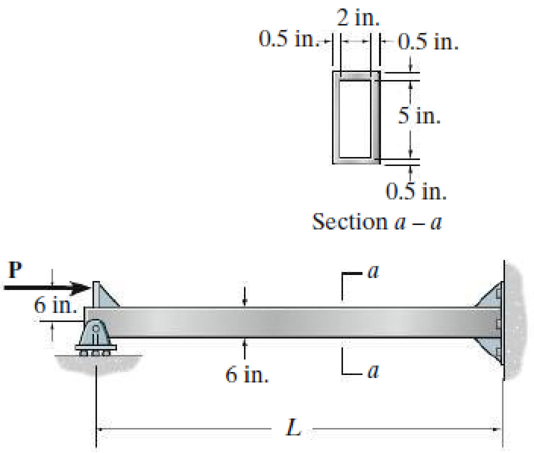
The wood column ($E = 12$ GPa) has a square cross-section with dimensions 100 mm by 100 mm. It is fixed at the base and free at the top. Determine the load P that can be applied to the edge of the column without causing the column to fail either by buckling or yielding.



A) $P_{max} = 10.6$ kN
B) $P_{max} = 20.5$ kN
C) $P_{max} = 31.4$ kN
D) $P_{max} = 41.3$ kN

Problem 11A (Hibbeler, 2014, w/ permission)

The steel ($E = 30 \times 10^3$ psi) rectangular hollow section column is pinned at both ends. If it has a length of $L = 14$ ft, determine the maximum allowable eccentric force it can support without causing it to buckle or yield. Use $\sigma_Y = 50$ ksi.



A) $P_{max} = 45.4$ k
B) $P_{max} = 61.7$ k
C) $P_{max} = 76.3$ k
D) $P_{max} = 90.6$ k

Problem 11B (Hibbeler, 2014, w/ permission)

The steel ($E = 30 \times 10^3$ psi) hollow rectangular column with the same cross-section as in the previous part is pinned at both ends. If it is subjected to the eccentric force $P = 45$ kip, determine the maximum allowable length without causing it to either buckle or yield. Use $\sigma_Y = 50$ ksi.

A) $L_{max} = 12.3$ ft
B) $L_{max} = 21.6$ ft
C) $L_{max} = 33.2$ ft
D) $L_{max} = 45.2$ ft

SOLUTIONS

P.1 ➡ *Solution*

The moment of inertia of the column is $I = b^4/12 = 1.5^4/12 = 0.422$ in^4. The Euler load follows as

$$P_{cr} = \frac{\pi^2 EI}{(0.6L)^2} = \frac{\pi^2 \times (10\times10^6)\times 0.422}{(0.6\times30)^2} = 129 \text{ k}$$

For a factor of safety of 2.0, the allowable load is found as

$$P_{allow} = \frac{P_{cr}}{FS} = \frac{129}{2} = 64.5 \text{ k}$$

Refer to beam ACB. Taking moments about point *A*, it is easily seen that

$$\Sigma M_A = 0 \rightarrow Q = \frac{P}{3}$$

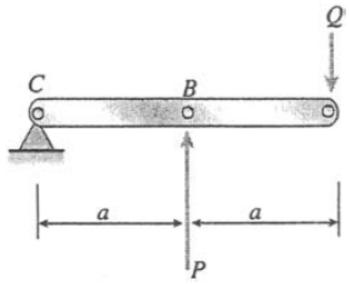
The allowable value of Q is then

$$Q_{allow} = \frac{P_{allow}}{3} = \frac{64.5}{3} = \boxed{21.5 \text{ k}}$$

The correct answer is **B**.

P.2 ➡ *Solution*

The free body diagram for the beam is shown below.



Taking moments about point *C*, it is easy to see that

$$\Sigma M_C = 0 \rightarrow P = 2Q$$

$$\therefore P = 2\times200 = 400 \text{ kN}$$

Given $FS = 3.0$, the corresponding critical load is

$$P_{cr} = P\times FS \rightarrow P_{cr} = 400\times3.0 = 1200 \text{ kN}$$

Since the column is fixed at one end and fixed at the other, the critical load is determined as

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

Solving for the moment of inertia gives

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \rightarrow I = \frac{4P_{cr}L^2}{\pi^2 E}$$

$$\therefore I = \frac{4\times1200,000\times2^2}{\pi^2\times(72\times10^9)}\times10^{12} = 2.70\times10^7 \text{ mm}^4$$

Recall that the moment of inertia for a hollow circular section is given by

$$I = \frac{\pi}{64}\times\left[d^4 - (d-2t)^4\right]$$

Substituting the pertaining data, we arrive at a fourth degree equation in t, which can be easily solved with a CAS such as Mathematica; that is,

$$I=\frac{\pi}{64}\times\left[200^4-(200-2t)^4\right]=2.70\times10^7$$

$$\therefore \boxed{t=10\text{ mm}}$$

The tube should have a minimum thickness of 1 centimeter.

The correct answer is **A**.

P.3 ➡ *Solution*

1. True. The moment of inertia of the column is

$$I=\frac{\pi}{64}\times\left(d_2^4-d_1^4\right)=\frac{\pi}{64}\times\left(6.0^4-5.0^4\right)=32.9\text{ in.}^4$$

Furthermore, L = 10.0 ft = 120 in. If the column is pinned at both ends, the critical load is given by the Euler formula,

$$P_{\text{cr}}=\frac{n^2\pi^2EI}{L^2}=\frac{1^2\times\pi^2\times10{,}400\times32.9}{120^2}=235\text{ k}$$

2. True. If the column were fixed at one end and free at the other, the critical load would be given by

$$P_{\text{cr}}=\frac{n^2\pi^2EI}{4L^2}\rightarrow P_{\text{cr}}=\frac{1^2\times\pi^2\times10{,}400\times32.9}{4\times120^2}=58.6\text{ k}$$

3. False. If the column were fixed at one end and pinned at the other, the critical load would be given by

$$P_{\text{cr}}=\frac{2.046\pi^2EI}{L^2}=\frac{2.046\times\pi^2\times10{,}400\times32.9}{120}=480\text{ k}$$

4. False. If the column were fixed at both ends, the critical load would be given by

$$P_{\text{cr}}=\frac{4\pi^2EI}{L^2}=\frac{4\times\pi^2\times10{,}400\times32.9}{120}=938\text{ k}$$

P.4 ➡ *Solution*

Part A: The moment of inertia of both columns is $I = 0.625^4/12 = 0.0127$ in.^{4} Column BD is fixed at the base and free at the top, and hence its critical load is given by

$$P_{\text{cr,BD}}=\frac{\pi^2EI}{4L^2}=\frac{\pi^2\times\left(30\times10^6\right)\times0.0127}{4\times35^2}=767\text{ lb}$$

Column CE, in turn, is pinned at both ends, and hence its critical load is determined with the Euler formula,

$$P_{\text{cr,CE}}=\frac{\pi^2EI}{L^2}=\frac{\pi^2\times\left(30\times10^6\right)\times0.0127}{45^2}=1857\text{ lb}$$

We must find Q_{cr} when a = 12 in. Critical conditions are attained when both columns buckle. Thus, taking moments about point A, we have

$$\Sigma M_A=0\rightarrow P_{\text{cr,BD}}\times10+P_{\text{cr,CE}}\times50-Q_{\text{cr}}\times(a+10)=0$$

$$\therefore Q_{\text{cr}}=\frac{P_{\text{cr,BD}}\times10+P_{\text{cr,CE}}\times50}{a+10}$$

$$\therefore Q_{cr} = \frac{767 \times 10 + 1857 \times 50}{12 + 10} = \boxed{4570 \text{ lb}}$$

The correct answer is **D**.

Part B: Inspecting the relation we obtained when taking moments about point *A*, it is easy to see that Q_{cr} is greatest when $a = 0$. Accordingly,

$$Q_{cr,max} = \frac{767 \times 10 + 1857 \times 50}{0 + 10} = \boxed{10{,}050 \text{ lb}}$$

The correct answer is **D**.

P.5 ➡ *Solution*

The moment of inertia of the post is

$$I = \frac{\pi}{64} \times \left(40^4 - 30^4\right) = 85{,}900 \text{ mm}^4$$

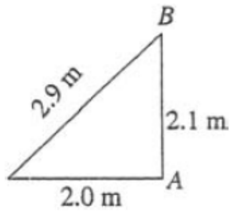
Buckling in the plane of the figure means fixed-pinned end conditions. The critical load, accordingly, is given by

$$P_{cr} = \frac{2.046\pi^2 EI}{L^2} = \frac{2.046\pi^2 \times \left(200 \times 10^9\right) \times (85{,}900 \times 10^{-12})}{2.1^2} = 78.7 \text{ kN}$$

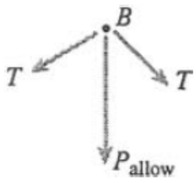
so that, with $FS = 3.0$,

$$P_{allow} = \frac{P_{cr}}{FS} = \frac{78.7}{3.0} = 26.2 \text{ kN}$$

From the geometry of the figure, we have the following schematic.



The free body diagram of joint *B* is shown below.



In this figure, *T* is the tensile force in each cable and P_{allow} is the compressive force in the tube. From the equilibrium of forces in the vertical direction, we can write

$$\Sigma F_{vert} = 0 \rightarrow P_{allow} - 2T_{allow} \times \left(\frac{2.1}{2.9}\right) = 0$$

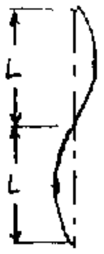
$$\therefore P_{allow} - 1.45 T_{allow} = 0$$

$$\therefore T_{allow} = \frac{26.2}{1.45} = \boxed{18.1 \text{ kN}}$$

The correct answer is **C**.

P.6 ➡ *Solution*

Consider first buckling in the *xz*-plane. In this case, the column will buckle as shown, so that $L_{e,1} = L$.



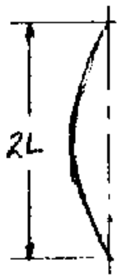
The moment of inertia to be used in this case is $I_1 = db^3/12$. Therefore,

$$P_{\mathrm{cr},1} = \frac{\pi^2 EI_1}{L_{e,1}^2} \rightarrow P_{\mathrm{cr},1} = \frac{\pi^2 Edb^3}{12L^2}$$

and the corresponding factor of safety is

$$FS_1 = \frac{P_{\mathrm{cr},1}}{P} = \frac{\pi^2 Edb^3}{12PL^2}$$

Consider now buckling in the *yz*-plane. In this case, the column will buckle as shown, which means that $L_{e,2} = 2L$.



The moment of inertia to be used in this case is $I_2 = bd^3/12$. Thus,

$$P_{\mathrm{cr},2} = \frac{\pi^2 EI_2}{L_{e,2}} = \frac{\pi^2 Ebd^3}{48L^2}$$

and the corresponding *FS* is

$$FS_2 = \frac{P_{\mathrm{cr},2}}{P} = \frac{\pi^2 Ebd^3}{48PL^2}$$

Equating the two factors of safety, it follows that

$$FS_1 = FS_2 \rightarrow \frac{\pi^2 Edb^3}{12PL^2} = \frac{\pi^2 Ebd^3}{48PL^2}$$

$$\therefore \frac{db^3}{12} = \frac{bd^3}{48}$$

$$\therefore \frac{b^2}{12} = \frac{d^2}{48}$$

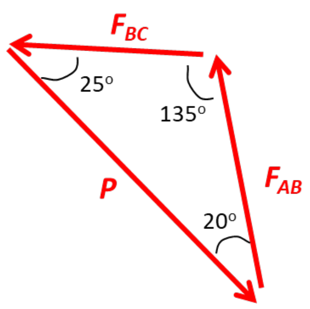
$$\therefore \boxed{\frac{b}{d} = \frac{1}{2}}$$

In order for the factor of safety to be the same in both buckling directions, one of the dimensions of the cross-section must be half as large than the other.

The correct answer is **C**.

P.7 ➡ *Solution*

The force triangle for joint *B* is shown in continuation.



Applying the law of sines, we see that

$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$\therefore F_{AB} = \frac{\sin 25^\circ}{\sin 135^\circ} \times 5.2 = 3.11 \text{ kN}$$

and

$$F_{BC} = \frac{\sin 20^\circ}{\sin 135^\circ} \times 5.2 = 2.52 \text{ kN}$$

The moment of inertia of segment *AB* is

$$I_{AB} = \frac{\pi \times 18^4}{64} = 5.15 \times 10^3 \text{ mm}^4 = 5.15 \times 10^{-9} \text{ m}^4$$

The Euler load for segment AB follows as

$$F_{\text{AB, cr}} = \frac{\pi^2 \times \left(200 \times 10^9\right) \times \left(5.15 \times 10^{-9}\right)}{1.2^2} = 7060 \text{ N}$$

The factor of safety for this segment is then

$$FS_{AB} = \frac{F_{\text{AB, cr}}}{F_{AB}} = \frac{7060}{3110} = 2.27$$

Consider now segment *BC*. The moment of inertia is

$$I_{BC} = \frac{\pi \times 22^4}{64} = 11{,}500 \text{ mm}^4 = 11.5 \times 10^{-9} \text{ m}^4$$

From the Pythagorean theorem, $L_{BC}^2 = 2.88$ m. The Euler load for segment *BC* follows as

$$F_{BC,\text{cr}} = \frac{\pi^2 \times \left(200 \times 10^9\right) \times \left(11.5 \times 10^{-9}\right)}{2.88} = 7880 \text{ N}$$

The factor of safety for this segment is then

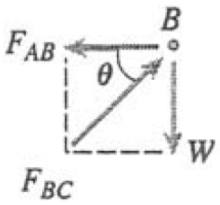
$$FS_{BC} = \frac{F_{BC,\text{cr}}}{F_{BC}} = \frac{7880}{2520} = 3.13$$

The smaller factor of safety governs the analysis; thus, $FS = FS_{AB} = 2.27$.

The correct answer is **A**.

P.8 ➡ *Solution*

The lengths of the members are $L_{AB} = L_1$, which is fixed, and $L_{BC} = L_1/\cos\theta$, which varies with the angle θ. The free body diagram of joint *B* is shown below.



Summing forces in the vertical direction, we can write

$$\Sigma F_{\text{vert}} = 0 \rightarrow F_{BC} \sin\theta - W = 0$$

$$\therefore \frac{W}{\sin\theta} = F_{\text{BC}}$$

Strut *BC* has a cross-sectional area $A = \pi d^2/4$ and moment of inertia $I = \pi d^4/64$, so that

$$I = \frac{A^2}{4\pi}$$

The Euler load is then

$$P_{cr} = \frac{\pi^2 EI}{L_{BC}^2} = \frac{\pi E A^2 \cos^2\theta}{4L_1^2}$$

When strut *BC* buckles, we have $F_{BC} = P_{cr}$; that is,

$$F_{BC} = P_{cr} \rightarrow \frac{W}{\sin\theta} = \frac{\pi E A^2 \cos^2\theta}{4L_1^2}$$

$$\therefore A = \frac{2L_1}{\cos\theta}\left(\frac{W}{\pi E \sin\theta}\right)^{1/2}$$

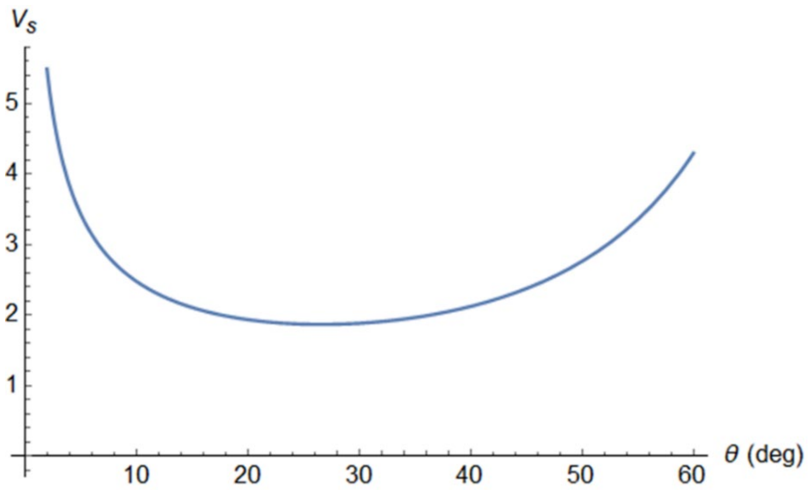
For the strut to have a minimum weight, its volume $V_S = AL_{BC}$ must be minimum. Hence,

$$V_S = AL_{BC} = \frac{2L_1}{\cos\theta}\left(\frac{W}{\pi E \sin\theta}\right)^{1/2} \times \frac{L_1}{\cos\theta} = \frac{2L_1^2}{\cos^2\theta}\left(\frac{W}{\pi E \sin\theta}\right)^{1/2}$$

With the exception of $\cos^2\theta$ and $\sin\theta$, the remaining terms in the equation above are constant. We can write

$$V_S = \frac{2L_1^2}{\cos^2\theta}\left(\frac{W}{\pi E \sin\theta}\right)^{1/2} = C \times \sec^2\theta\sqrt{\csc\theta}$$

Volume V_S will be minimum when $\sec^2\theta\sqrt{\csc\theta}$ is minimum. Let us plot this trigonometric function to get an idea of its shape.



Clearly, there is an angle θ in the first trigonometric quadrant for which the product $\sec^2\theta\sqrt{\csc\theta}$, and hence the volume V_S, will be minimum. To find this angle, we differentiate the function with respect to θ and set it to zero, as follows,

$$\frac{d}{d\theta}(V_S) = -\frac{C}{4}(-3 + 5\cos 2\theta)\csc^{3/2}\theta\sec^3\theta = 0$$

One way to satisfy the equation above is to have the expression in parentheses be equal to zero. Thus,

$$(-3 + 5\cos 2\theta) = 0 \rightarrow \cos 2\theta = \frac{3}{5}$$

$$\therefore \theta = \frac{1}{2}\arccos\frac{3}{5} = \boxed{26.6^\circ}$$

The angle for which the strut will have a minimum weight is close to 27 degrees.

The correct answer is **B**.

P.9 ➡ Solution

The deflection δ for a column under an eccentric load such as the one considered here is

$$\delta = e(\sec kL - 1)$$

With some algebra, we can solve this equation for L,

$$\delta = e(\sec kL - 1)$$

$$\therefore \frac{\delta}{e} = \sec kL - 1$$

$$\therefore \sec kL = \frac{\delta}{e} + 1$$

$$\therefore kL = \text{arcsec}\left(\frac{\delta}{e} + 1\right)$$

$$\therefore L_{\max} = \frac{1}{k}\text{arcsec}\left(\frac{\delta + e}{e}\right)$$

Since $k = \sqrt{P/EI}$, we ultimately obtain

$$L_{\max} = \sqrt{\frac{EI}{P}}\text{arcsec}\left(\frac{\delta + e}{e}\right)$$

We can then proceed to substitute the numerical data for this column. The moment of inertia is

$$I = \frac{1}{12}\times\left[b^4 - (b-2t)^4\right] = \frac{1}{12}\times\left[100^4 - (100 - 2\times 8)^4\right] = 4.18\times 10^6 \text{ mm}^4 = 4.18\times 10^{-6} \text{ m}^4$$

so that

$$\sqrt{\frac{EI}{P}} = \sqrt{\frac{(73\times 10^9)\times(4.18\times 10^{-6})}{50{,}000}} = 2.47$$

and, noting that $e = b/2 = 100/2 = 50$ mm, we find that

$$L_{\max} = 2.47\times\text{arcsec}\left(\frac{30+50}{50}\right) = \boxed{2.21 \text{ m}}$$

The column should be no taller than about 2.2 meters.

○ The correct answer is **D**.

P.10 ➡ Solution

The cross-sectional area of the column is $A = 0.1\times 0.1 = 0.01$ m², the moment of inertia is $I = (1/12)(0.01\times 0.01^3) = 8.33\times 10^{-6}$ m⁴, and the radius of gyration is $r = (I/A)^{1/2} = 0.0289$ m. Noting that the column is fixed at one end and free at the other, the critical load is determined to be

$$P_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2\times(12\times 10^9)\times(8.33\times 10^{-6})}{4\times 2^2} = 61.7 \text{ kN}$$

The critical stress should be less than the yield stress. Mathematically,

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{61{,}700}{0.01} = 6.17 \text{ MPa} < \sigma_Y = 55 \text{ MPa}$$

The inequality checks out, and the column will fail by yielding, not buckling. To determine the maximum tolerable load P, we set take the secant formula and set $\sigma_{max} = \sigma_Y$, as follows.

$$\sigma_{max} = \sigma_Y = \frac{P}{A}\left[1+\frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\therefore 55\times10^6 = \frac{P}{0.01}\times\left[1+\frac{0.12\times0.05}{0.0289^2}\sec\left(\frac{2\times2}{2\times0.0289}\sqrt{\frac{P}{\left(12\times10^9\right)\times0.01}}\right)\right]$$

$$\therefore 55\times10^4 = P\times\left[1+7.18\sec\left(0.00632\sqrt{P}\right)\right]$$

The equation above is transcendental and requires a numerical method to be solved. In Mathematica, one way to go is the *FindRoot* function,

$$\text{FindRoot}[55*10^4 - P(1+7.18\text{Sec}[0.00632\sqrt{P}]),\{P,1\}]$$

This returns $P = P_{max} = 31.4$ kN. As expected, this value is below the critical load for buckling.

The correct answer is **C**.

P.11 ➡ *Solution*

Part A: The area of the section is $A = 3\times6 - 2\times5 = 8$ in.², and the moments of inertia are

$$I_x = \frac{1}{12}\times3\times6^3 - \frac{1}{12}\times2\times5^3 = 33.2\text{ in.}^4$$

$$I_y = \frac{1}{12}\times3^3\times6 - \frac{1}{12}\times5\times2^3 = 10.2\text{ in.}^4$$

The radius of gyration about the x-axis is $r_x = (33.2/8)^{1/2} = 2.04$ in. Since $I_y < I_x$, there will be buckling in the y-direction and yielding in the x-direction. In the former case, noting that the column is pinned at both ends, the formula to use for the critical load is

$$P_{cr} = \frac{\pi^2 EI_y}{L^2} = \frac{\pi^2\times\left(30\times10^3\right)\times10.2}{\left(14\times12\right)^2} = 107\text{ k}$$

For the Euler formula to be valid, we must have $\sigma_{cr} < \sigma_Y$; that is,

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{107}{8} = 13.4\text{ ksi} < \sigma_Y = 50\text{ ksi}$$

Consider now yielding about the strong axis. The maximum eccentric load P_{max} can be determined with the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1+\frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P_{max}}{EA}}\right)\right]$$

$$\therefore 50 = \frac{P_{max}}{8}\times\left[1+\frac{6\times3}{2.04^2}\sec\left(\frac{\left(14\times12\right)}{2\times2.04}\sqrt{\frac{P_{max}}{\left(30\times10^3\right)\times8}}\right)\right]$$

$$\therefore 50 = 0.125P_{max}\left[1+4.33\sec\left(0.0841\sqrt{P_{max}}\right)\right]$$

Solving this equation for P_{max}, we obtain $P_{max} = 61.7$ k. Since this value is less than the critical load P_{cr}, we take it as the maximum allowable eccentric force. The maximum load is close to 60 kilopounds.

The correct answer is **B**.

Part B: As before, the cross-sectional area of the beam is $A = 8$ in.², the moments of inertia are $I_x = 33.2$ in.⁴ and $I_y = 10.2$ in.⁴, and the radius of gyration with respect to the x-axis is $r_x = 2.04$ in. Consider buckling about the weak axis. We take $P_{cr} = 45$ kip as the Euler load, so that

$$P_{cr} = \frac{\pi^2 EI_y}{L_y^2} \rightarrow 45 = \frac{\pi^2 \times (30\times 10^3)\times 10.2}{L^2}$$

$$\therefore L = 259.1 \text{ in.} = 21.6 \text{ ft}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$; that is,

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{45}{8} = 5.63 \text{ ksi} < \sigma_Y = 50 \text{ ksi}$$

Consider now yielding about the strong axis. We apply the secant formula and solve it for the length *L*,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\therefore 50 = \frac{45}{8}\left[1 + \frac{6\times 3}{2.03^2}\sec\left(\frac{1\times L}{2\times 2.03}\times\sqrt{\frac{45}{(30\times 10^3)\times 8}}\right)\right]$$

$$\therefore 50 = 5.63 + 24.6\sec\left(3.37\times 10^{-3} L\right)$$

$$\therefore L = 291.7 \text{ in.} = 24.3 \text{ ft}$$

We've obtained two values of *L*, and the smaller one governs our choice of minimum *L*; hence, we take L_{min} = 21.6 ft. The member should be no shorter than about 21 and a half feet.

The correct answer is **B**.

ANSWER SUMMARY

Problem 1		**B**
Problem 2		**A**
Problem 3		T/F
Problem 4	**4A**	**D**
	4B	**D**
Problem 5		**C**
Problem 6		**C**
Problem 7		**A**
Problem 8		**B**
Problem 9		**D**
Problem 10		**C**
Problem 11	**11A**	**B**
	11B	**B**

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