

# **Problems**

PROBLEM (1) (Bedford & Fowler, 2008, w/ permission)

The cable supports a distributed load w = 12,000 lb/ft. Determine the maximum tension in the cable.



**A)**  $T_{\text{max}} = 561.7$  kip **B)**  $T_{\text{max}} = 656.8$  kip **C)**  $T_{\text{max}} = 758.9$  kip **D)**  $T_{\text{max}} = 834.0$  kip

 $I_{\text{max}} = 0.54.0 \text{ klp}$ 

PROBLEM (Merriam & Kraige., 2002, w/ permission)

A cable weighing 25 N/m is suspended from point *A* and passes over the small pulley at *B*. Calculate the mass of the attached cylinder which will produce a sag of 9 m.



**A)** m = 215.5 kg **B)** m = 269.5 kg **C)** m = 312.5 kg **D)** m = 355.5 kg

# PROBLEM (Beer et al., 2013, w/ permission)

The mass of cable ACB is 20 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine the sag of the cable and the slope at point *A*.



**A)**  $h = 50.6 \text{ mm and } \theta_A = 2.63^\circ$  **B)**  $h = 50.6 \text{ mm and } \theta_A = 5.26^\circ$  **C)**  $h = 91.8 \text{ mm and } \theta_A = 2.63^\circ$ **D)**  $h = 91.8 \text{ mm and } \theta_A = 5.26^\circ$ 

# PROBLEM (Beer et al., 2013, w/ permission)

Two cables of the same gauge are attached to a transmission tower at *B*. Since the tower is slender, the horizontal component of the forces exerted by the cables at *B* is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine the sag and the maximum tension in each cable.



**A)** h = 3.68 m and T<sub>m</sub> = 388.4 N **B)** h = 3.68 m and T<sub>m</sub> = 613,9 N **C)** h = 6.75 m and T<sub>m</sub> = 388.4 N **D)** h = 6.75 m and T<sub>m</sub> = 613.9 N

# PROBLEM (Beer et al., 2013, w/ permission)

The total mass of cable *AC* is 25 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine the sag *h* of the cable and the slope at point *C*.



**A)**  $h = 14 \text{ mm and } \theta_c = 15.5^{\circ}$  **B)**  $h = 14 \text{ mm and } \theta_c = 27.6^{\circ}$  **C)**  $h = 28 \text{ mm and } \theta_c = 15.5^{\circ}$ **D)**  $h = 28 \text{ mm and } \theta_c = 27.6^{\circ}$ 

#### PROBLEM (6) (Bedford & Fowler, 2008, w/ permission)

The stationary balloon's tether is horizontal at point *O* where it is attached to the truck. The mass per unit length of the tether is 0.45 kg/m. The tether exerts a 50-N horizontal force on the truck. The horizontal distance from point *O* to point *A* where the tether is attached to the balloon is 20 m. What is the height of point *A* relative to point *O*?



A) h = 12.8 m
B) h = 17.8 m
C) h = 22.8 m
D) h = 27.8 m

#### PROBLEM 🕖 (Bedford & Fowler, 2008, w/ permission)

The mass per unit length of lines *AB* and *BC* is 2 kg/m. The tension at the lowest point of cable AB is 1.8 kN. The two lines exert equal horizontal forces at *B*. Assume the cables to be catenaries. Regarding this system, which of the following is *not* true?



**A)** The sag  $h_1$  of cable AB is 6.25 m.

**B)** The sag  $h_2$  of cable BC is 2.19 m.

**C)** The maximum tension for cable *AB* is 1897 N.

**D)** The maximum tension for cable *BC* is 1843 N.

PROBLEM 8A (Beer et al., 2013, w/ permission)

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at *A* and to a collar at *B*, as shown in the figure below. Neglecting the effect of friction, determine the force *P* for which h = 8 m and the corresponding span *L*.



A) P = 4.41 N and L = 9.88 m
B) P = 4.41 N and L = 13.4 m
C) P = 8.82 N and L = 9.88 m

**D)** *P* = 8.82 N and *L* = 13.4 m

# PROBLEM 8B

Reconsider the previous figure. A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine the force *P* for which L = 15 m and the corresponding sag *h*.

**A)** *P* = 5.61 N and *h* = 5.89 m

**B)** *P* = 5.61 N and *h* = 8.68 m

**C)** *P* = 10.9 N and *h* = 5.89 m

**D)** *P* = 10.9 N and *h* = 8.68 m

PROBLEM (Beer et al., 2013, w/ permission)

A 40-m cable is strung between two buildings as shown. The maximum tension is found to be 350 N, and the lowest point of the cable is observed to be 6 m above the ground. Determine the minimum distance between the buildings and the total mass of the cable.



A) L = 17.8 m and m = 49.2 kg
B) L = 17.8 m and m = 63.5 kg
C) L = 35.6 m and m = 49.2 kg
D) L = 35.6 m and m = 63.5 kg

# PROBLEM 10

The following are statements related to standards *Structural Applications of Steel Cables for Buildings* (ASCE 19-10), by the American Society of Civil Engineers, and *Recommendations for Stay Cable Design*, by the Post-Tensioning Institute (PTI). True or false?

**1. (** ) ASCE 19-10 recommends reductions in nominal cable strength and modulus of elasticity for temperatures above 93°C.

**2. (**) According to the PTI's *Recommendations*, stay cable specimens shall have a minimum length of 3.5 m. The anchorages of the stay cable specimens shall be supported on wedge-shaped shim plates, creating angular deviations of 10<sup>-2</sup> radians.

**3. (**) Also according to the PTI's *Recommendations*, after fatigue loading, a test specimen shall be reloaded, with the wedge-shaped shim plates remaining in place, and shall develop a minimum tensile force of 95% of the actual tensile strength of the cable or 92% of the minimum ultimate tensile strength of the cable, whichever is greater.

**4. (**) In consonance with ASCE 19-10, the prestretching force of the cable may not exceed 55% of the minimum breaking force.

**5. (**) Cable vibrations in bridges may be excited by dynamic wind forces acting on the cable. Such forces are caused by turbulence in the oncoming air flow (galloping), vortex shedding in the wake behind the cable, self-induction (buffeting), fluid-elastic interaction between neighboring cables (wake galloping), or by interaction between rain, wind, and cable.

**6. ()** According to ASCE 19-10, cables exposed to atmospheric conditions shall have corrosion protection at least equivalent to Class B zinc coating as defined in ASTM standards.

**7. ()** According to the PTI's *Recommendations*, non-destructive evaluation and monitoring techniques include the vibration method, ultrasonic testing, and the magnetic perturbation method.

**8. ()** According to ASCE 19-10, If required in the contract documents, assemblies with attached sockets shall be proof loaded to the tension force specified in order to verify the integrity of socketing.

**9. (**) According to ASCE 19-10, all cables and their associated elements shall receive a routine inspection on a 3-year cycle to determine their physical and functional condition, and to identify any changes that deviate from the original installation.

**10. ()** A section of the PTI's *Recommendations* establishes that the fatigue strength of the stay cable system be demonstrated by tests of at least three stay cable specimens. Passage of these tests ensures similar properties for all of the strand produced for the project.

# **Solutions**

## P.1 Solution

The equation  $y = (1/2)ax^2$ , where  $a = w/T_0$ , must be satisfied for both attachment points. Hence,

$$y_L = 40 = \frac{1}{2}ax_L^2$$
;  $90 = \frac{1}{2}ax_R^2$ 

Dividing the second equation by the first yields

$$\frac{x_R^2}{x_L^2} = \frac{9}{4}$$

The horizontal span of the bridge is  $x_R - x_L = 100$  ft. Solving this equation along with the previous one yields  $x_R = 60$  ft and  $x_L = -40$  ft. Substituting the coordinates on the expression for the parabola, we obtain

$$y_R = \frac{1}{2}ax_R^2 \to 90 = \frac{1}{2}a(60)^2$$
  
 $\therefore a = 0.05 \text{ ft}^{-1}$ 

Therefore, the tension on the lowest point is

$$T_0 = \frac{w}{a} = \frac{12,000 \text{ lb/ft}}{0.05 \text{ ft}^{-1}} = 240,000 \text{ lb}$$

The maximum tension on the cable occurs at its right end. Substituting the pertaining variables in the equation for the tension, *T*, it follows that

$$T = T_0 \sqrt{1 + a^2 x^2} = 240 \times \sqrt{1 + 0.05^2 \times 60^2} = 758.9 \text{ kip}$$

The maximum tension in the cable is close to 759 kips and occurs at x = 60

ft.

The correct answer is C.

P.2 Solution

Consider the following illustration of the cable.



Given the equation that describes the cable,  $y = (wx^2/2T_0)$ , we have, at point *B*,

$$21 = \frac{wx_B^2}{2T_0} \quad (I)$$

while at point A,

$$9 = \frac{w(100 - x_B)^2}{2T_0}$$
 (II)

The two previous equations can be combined to yield  $x_B^2 - 350x_B + 17,500 = 0$ , which is a quadratic equation in  $x_B$ . Solving it, we get  $x_B = 289.6$  m, which is unfeasible, and  $x_B = 60.4$  m, which is the proper result. To obtain the tension  $T_0$ , we substitute the foregoing value of  $x_B$  into Equation (I), giving

$$21 = \frac{25(60.4)^2}{2T_0} \to T_0 = 2171.5 \text{ N}$$

To obtain the weight of the cylinder, we sum forces in segment OB,

$$(mg)^2 = T_0^2 + (wx_B)^2$$

Substituting the pertaining variables, we get

$$(m \times 9.81)^2 = 2170^2 + (25 \times 60.4)^2$$

$$\therefore m = 269.5 \text{ kg}$$

The correct answer is **B**.

P.3 Solution

To begin, we consider equilibrium for the entire frame.



The sum of moments relative to point D is such that

$$\Sigma M_D = 0 \rightarrow A_x(4.5) - 20(9.81)(4) - 150(9.81)(6) = 0$$
  
 $\therefore A_x = 2136.4 \text{ N}$ 

Similarly, reaction  $A_y$  can be obtained by considering the free body diagram for the entire cable.



The sum of moments relative to point B should equal zero; that is,

$$\Sigma M_B = 0 \to A_y(8) - 196.2(4) = 0$$
  
 $\therefore A_y = 98.1 \text{ N}$ 

Next, consider segment AC of the cable.



The sum of forces in the *x*-direction and the sum of moments relative to point *A* should both equal zero. Accordingly,

$$\Sigma F_x = 0 \to T_0 - A_x = 0$$
  

$$\therefore T_0 = A_x = 2136.4 \text{ kN}$$
  

$$\Sigma M_A = 0 \to T_0 h - 98.1(2) = 0$$
  

$$\therefore h = \frac{98.1(2)}{2136.4} = 0.0918 = \boxed{91.8 \text{ mm}}$$

Next, in order to determine the slope of the cable, consider equilibrium relative to end *A*.

The cable makes an angle  $\theta_{\rm A}$  relative to the horizontal, which may be obtained as

$$\tan \theta_A = \frac{A_x}{A_y} = \frac{98.1}{2136.4} \to \theta_A = \arctan 0.0459 = \boxed{2.63^\circ}$$

The sag of the cable is 91.8 mm, and the slope of the cable at A is 2.63°.

The correct answer is **C**.

### P.4 Solution

The weight *W* of cable AB between an arbitrary point *D* and its rightmost point, *B*, is  $W = wx_B$ .



The sum of moments relative to point B is such that

$$\Sigma M_B = 0 \to T_0 y_B - w x_B \left(\frac{x_B}{2}\right) = 0$$

The horizontal component  $T_0$  is then

$$T_0 = \frac{w x_B^2}{2 y_B} \quad (I)$$

For cable AB, we have  $x_B = 45$  m and  $T_0$  follows as

$$T_0 = \frac{w(45)^2}{2h} = 1012.5 \frac{w}{h} \quad (\text{II})$$

Next, for cable BC, with  $x_B$  = 30 m and  $y_B$  = 3 m, we have

$$T_0 = \frac{w(30)^2}{2(3)} = 150w \quad \text{(III)}$$

Because the horizontal component of the forces exerted by the cables is to be zero, Equations (II) and (III) should yield the same result; that is,

$$(T_0)_{II} = (T_0)_{III} \rightarrow 1012.5 \frac{\#}{h} = 150 \frac{}{}$$
  
$$\therefore h = \frac{1012.5}{150} = \boxed{6.75 \text{ m}}$$

The required sag is 6.75 m. Next, we appeal to the following expression for tension  $T_m$ ,

$$T_m^2 = T_0^2 + W^2$$
 (IV)

For cable AB, the distributed weight is w = 0.4(9.81) = 3.92 N/m. Then, substituting  $x_B = 45$  m and  $y_B = h = 6.75$  m into Equation (I), we obtain

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{3.92(45)^2}{2(6.75)} = 588 \text{ N}$$

Now, the weight *W* is such that  $W = wx_B = 3.92(45) = 176.4$  N. Substituting these results into Equation (IV), we have

$$T_m = \sqrt{588^2 + 176.4^2} = 613.9 \,\mathrm{N}$$

The maximum tension is close to 614 N.

The correct answer is **D**.

P.5 Solution

The weight of the cable is W = 25(9.81) = 245.3 N, while the weight of the 450 kg block is  $W_B = 450(9.81) = 4414.5$  N.



Summing moments relative to point *B*, we have

$$\Sigma M_B = 0 \rightarrow 245.3(2.5) + 4414.5(3) - C_x(2.5) = 0$$
  
 $\therefore C_x = 5542.7 \text{ N}$ 

Because  $F_x$  and  $C_x$  are the only forces in the *x*-direction, we can easily conclude that  $|A_x| = |C_x| = 5542.7$  N. Now, taking moments relative to point *A*, we have

$$\Sigma M_A = 0 \rightarrow C_y(5) - 5542.7(2.5) - 245.3(2.5) = 0$$
  
 $\therefore C_y = 2894 \text{ N}$ 

We can then obtain the slope of the cable at point *C*,

$$\tan \theta_C = \frac{C_y}{C_x} = \frac{2894}{5542.7} \to \theta_C = \arctan 0.522 = \boxed{27.6^\circ}$$

Consider the following free body diagram of the cable.

$$\begin{array}{c} C_{y} \\ C_{x} = SS^{+}2N \\ C_{x} = SS^{+}2N$$

Summing forces in the y-direction, we have

$$\Sigma F_y = 0 \rightarrow C_y - A_y - 245.3 = 0$$

$$\therefore A_y = C_y - 245.3 = 2894 - 245.3 = 2648.7 \text{ N}$$

Next, consider equilibrium of one half of the cable.

The weight of the cable segment is W = 12.5(9.81) = 122.6 N. Summing moments relative to hypothetical point O, we have

$$\Sigma M_0 = 0 \to 122.6(1.25) + 2649(2.5) - 5543y_d = 0$$

$$\therefore y_d = 1.222 \text{ m}$$

The sag *h* of the cable is, then,

$$h = 1.250 - 1.222 = 0.028 = 28 \text{ mm}$$

The correct answer is **D**.

### P.6 Solution

A cable hanging against its own weight can be described by the equation

$$y = \frac{1}{a}(\cosh ax - 1)$$

where  $a = (w/T_0) = [9.81(0.45)/50] = 0.0883 \text{ m}^{-1}$ .



Substituting this value along with x = 20 m, we can obtain the value of the height h, namely,

$$h = \frac{1}{0.0883} \left[ \cosh(0.0883 \times 20) - 1 \right] = \boxed{22.8 \text{ m}}$$

The correct answer is C.

#### P.7 Solution

The lines meet the conditions for a catenary. The weight density is w = 2(9.81) = 19.62 N/m. Parameter *a* for segment *AB* is found as

$$a_1 = \frac{w}{T_{AB}} = \frac{19.62}{1800} = 0.0109$$

The sag of cable AB is then

$$h_1 = \left(\frac{1}{a_1}\right)(\cosh 30a_1 - 1) = 4.95 \text{ m}$$

Next, consider segment BC. The horizontal component of the tension in *B* is  $T_{AB} = 1.8$  kN. Thus the tension of the lowest point in BC is 1.8 kN, and parameter *a* for line BC is equal to  $a_1$ . The sag is

$$h_2 = \left(\frac{1}{a_2}\right)(\cosh 20a_1 - 1) = 2.19 \text{ m}$$

The maximum tension for cable AB is

 $T_{AB,\max} = T_{AB} \cosh 30a_1 = 1897.1 \text{ N}$ 

while the maximum tension for cable BC is

$$T_{BC,\max} = T_{BC} \cosh 20a_1 = 1842.9 \text{ N}$$

Because sag  $h_1$  = 4.95  $\neq$  6.25 m, the first statement is incorrect.

The false statement is **A**.

P.8 Solution

**Part A:** Consider the free body diagram for the cable.





$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

which can be solved for *c* to give

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{10^2 - 8^2}{2(8)} = 2.25 \text{ m}$$

Now, assuming that the cable is a catenary, its length can be obtained from the relation

$$s_B = c \sinh \frac{x_B}{c}$$

which can be easily solved for the horizontal span  $x_{Br}$ 

$$x_B = c \sinh^{-1} \frac{s_B}{c} = 2.25 \sinh^{-1} \frac{10}{2.25} = 4.94 \text{ m}$$

Hence,

$$L = 2x_B = 2(4.94) = 9.88 \text{ m}$$

Finally, the force *P* that corresponds to a sag h = 8 m is such that

$$P = T_0 = wc = 1.96(2.25) = 4.41 \text{ N}$$

The correct answer is A.

**Part B:** Once again, consider the following free body diagram.



The length of either half of the cable is  $s_B = 20/2 = 10$  m, while its weight per unit length is w = 0.2(9.81) = 1.96 N/m. Assuming the cable to be a catenary, it can be described with the equation

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{L}{2c}$$

Substituting the values for  $s_B$  (= 10 m) and L (= 7.5 m), we arrive at the relation

$$10 = c \sinh \frac{7.5}{c}$$

This can be solved using a CAS such as Mathematica, in which case we could apply the *FindRoot* command with an initial guess of c = 1 m. The pertaining code is

FindRoot 
$$\left[c * \operatorname{Sinh}\left[\frac{7.5}{c}\right] - 10, \{c, 1\}\right]$$

This returns c = 5.55 m. To obtain the sag of the cable, we first resort to the equation

$$y_B = c \cosh \frac{x_B}{c} = 5.55 \cosh \frac{7.5}{5.55} = 11.44 \text{ m}$$

Accordingly, the sag is

$$h = y_B - c = 11.44 - 5.55 = 5.89 \text{ m}$$

Lastly, the force P exerted on the cable is given by

$$P = wc = 1.96(5.55) = 10.9$$
 N

The correct answer is **C**.

© 2019 Montogue Quiz

## P.9 Solution

Consider the figure below.



We have  $s_B = 20$  m,  $T_m = 350$  N, and h = 14 - 6 = 8 m. Distance  $y_B$  is such that  $y_B = h + c = 8 + c$ . We know that

$$y_B^2 - s_B^2 = c^2$$

which, upon substituting the corresponding values, becomes

$$(8+c)^2 - 20^2 = c^2$$

with the result that c = 21.0 m. The expression for the length of the catenary,  $s_{B}$ , can be used to obtain the value of  $x_{B}$ ,

$$s_B = c \sinh \frac{x_B}{c} \rightarrow 20 = 21.0 \sinh \frac{x_B}{21.0}$$
  
 $\therefore x_B = 17.8 \text{ m}$ 

The distance between the buildings is

$$L = 2x_B = 2(17.8) = 35.6 \text{ m}$$

Now, we appeal to the expression for maximum tension in the cable,

$$T_m = wy_B \rightarrow 350 = w(8 + 21.0)$$
$$\therefore w = 12.1 \text{ N/m}$$

The weight *W* is such that

$$W = 2s_B w = 40 \times 12.07 = 482.8 \text{ N}$$

The mass of the cable is then

$$m = \frac{W}{g} = \frac{482.8}{9.81} = \boxed{49.2 \text{ kg}}$$

The correct answer is C.

## P.10 Solution

- True. Indeed, ASCE 19-10 establishes a reduction in nominal cable strength if temperatures surpass 200°F (or 93°C).
- 2. **True.** In fact, according to Section 4.2 of the *Recommendations*, "stay cable specimens shall have a minimum length of 3.5 m. The anchorages of the stay cable specimens shall be supported on wedge-shaped shim plates, creating angular deviations of 10<sup>-2</sup> radians, and oriented such as to create a S-shaped cable profile."
- 3. False. The statement has switched percentages. In fact, Section 4.2, "Acceptance Testing of Stay Cables," establishes that, after fatigue loading, the cable shall be reloaded and "shall develop a minimum tensile force equal to 92% of the actual ultimate tensile strength of the cable or 95% of the minimum ultimate tensile strength of the cable, whichever is greater."
- 4. True. Indeed, Section 4.2 of ASCE 19-10 establishes that "prestretching force shall not exceed 55% of the minimum breaking force." Furthermore, "for cables more than 2.5 in. (63 mm) in diameter, consultation with cable manufacturers during structural design is recommended."
- 5. **False**. The excerpt has switched the definitions of *galloping* and *buffeting*. In fact, galloping is "vortex shedding in the wake behind the cable", whereas buffeting is the turbulence in oncoming air flow.
- 6. **False**. The statement is wrong in that it establishes Class B as the minimum requirement for corrosion protection of cable material. In fact, "cables exposed to atmospheric conditions shall have corrosion protection at least

equivalent to Class A zinc coating on all wires defined in ASTM A586 and A603, except those composed of stainless steel wires."

- 7. **True**. These methods are listed in pages 81 and 82 of *Recommendations*.
- 8. **True**. This provision is established in Section 7.2. of ASCE 19-10.
- False. According to Section 9.2, "Routine Inspections", of ASCE 19-10, "all cables and their associated elements shall receive a routine inspection on a 2- [not 3-] year cycle to determine their physical and functional condition."
- 10. False. As per Appendix A of *Recommendations*, "passage of these tests does *not* ensure similar properties for all of the strand produced for the project." The authors thus recommend application of the so-called one-pin test, which gives much better assurance as for the ductility of a length of strand.

# **Answer Summary**

Problem 1		С
Problem 2		В
Problem 3		С
Problem 4		D
Problem 5		D
Problem 6		С
Problem 7		Α
Problem 8	8 <b>A</b>	Α
FIODICITIO	8B	C
Problem 9		С
Problem 10		T/F

# References

- American Society of Civil Engineers. (2010). *ASCE Standard 19-10: Structural Applications of Steel Cables for Buildings*. Reston: ASCE Press.
- BEDFORD, A. and FOWLER, W. (2008). *Engineering Mechanics: Statics*. 5th edition. Upper Saddle River: Pearson.
- BEER, F. et al. (2013). *Vector Mechanics for Engineers.* 10th edition. New York: McGraw-Hill.
- MERIAM, J. and KRAIGE, L. (2002). *Engineering Mechanics*: Statics. 5th edition. Hoboken: John Wiley and Sons.
- Post-Tensioning Institute. (2007). *Recommendations for Stay Cable Design, Testing and Installation.* Phoenix: Post-Tensioning Institute.



Got any questions related to this quiz? We can help! Send a message to <u>contact@montogue.com</u> and we'll answer your question as soon as possible.