Montogue

Quiz SM212

STRAIN ENERGY AND CASTIGLIANO'S THEOREM

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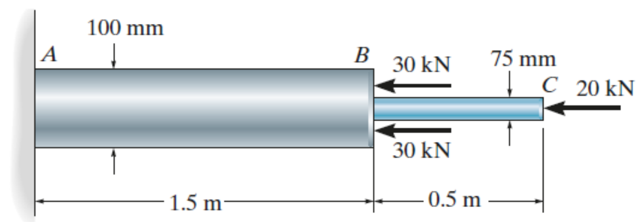
PROBLEMS – STRAIN ENERGY

Problem 1A (Hibbeler, 2014, w/ permission)

Determine the strain energy in the stepped rod assembly. Portion *AB* is steel and *BC* is brass. E_{br} = 101 GPa, E_{st} = 200 GPa, $(\sigma_Y)_{br}$ = 410 MPa, and $(\sigma_Y)_{st}$ = 250 MPa.



A) U = 0.674 J

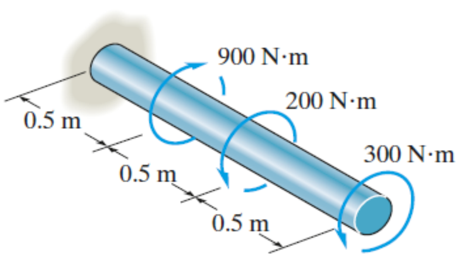
B) U = 3.28 J

C) U = 5.77 J

D) U = 8.20 J

Problem 1B (Hibbeler, 2014, w/ permission)

Determine the torsional strain energy in the steel shaft. The shaft has a diameter of 40 mm. Use G = 75 GPa.



A) U = 1.58 J

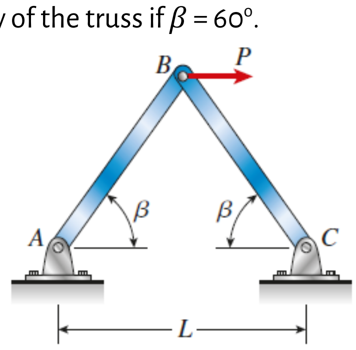
B) U = 4.15 J

C) U = 6.64 J

D) U = 9.13 J

Problem 2A (Gere & Goodno, 2009, w/ permission)

The truss ABC shown in the figure is subjected to a horizontal load *P* at point *B*. The two bars are identical with cross-sectional area *A* and modulus of elasticity *E*. Determine the strain energy of the truss if β = 60°.



A) $U = \dfrac{P^2L}{3EA}$
B) $U = \dfrac{P^2L}{2EA}$
C) $U = \dfrac{P^2L}{EA}$
D) $U = \dfrac{2P^2L}{EA}$

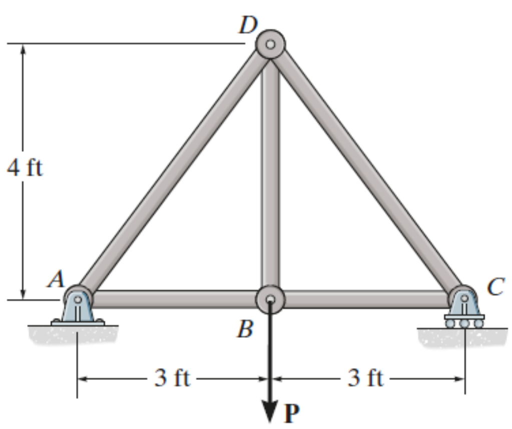
Problem 2B

Determine the horizontal displacement of joint *B* by equating the strain energy of the truss to the work done by the load.

A) $\delta_B = \dfrac{PL}{3EA}$
B) $\delta_B = \dfrac{PL}{2EA}$
C) $\delta_B = \dfrac{PL}{EA}$
D) $\delta_B = \dfrac{2PL}{EA}$

Problem 3 (Hibbeler, 2014, w/ permission)

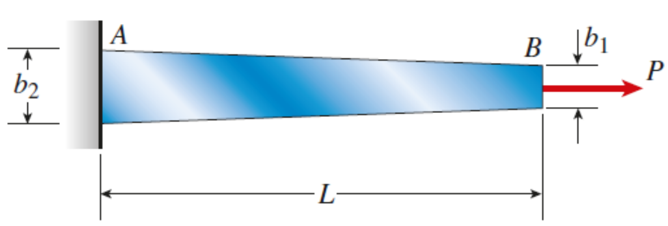
Determine the maximum force *P* and the corresponding maximum total strain energy that can be stored in the truss without causing any of the members to have permanent deformation. Each member of the truss has a diameter of 2 in. and is made of steel with $E = 29 \times 10^3$ ksi and $\sigma_Y = 36$ ksi.



A) $U = 4.65$ in.· kip
B) $U = 7.36$ in.· kip
C) $U = 10.2$ in.· kip
D) $U = 13.1$ in.· kip

Problem 4A (Gere & Goodno, 2009, w/ permission)

A slightly tapered bar *AB* of rectangular cross-section and length *L* is acted upon by a force *P* (see figure). The width of the bar varies uniformly from b_2 at end *A* to b_1 at end *B*. The thickness *t* is constant. Determine the strain energy of the bar.



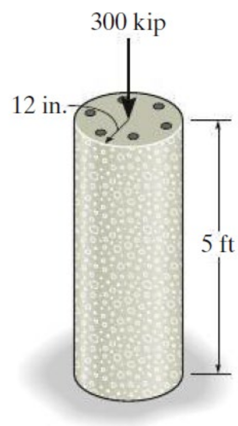
A) $U = \dfrac{P^2L}{3Et(b_2-b_1)}\ln(b_2/b_1)$
B) $U = \dfrac{P^2L}{2Et(b_2-b_1)}\ln(b_2/b_1)$
C) $U = \dfrac{P^2L}{Et(b_2-b_1)}\ln(b_2/b_1)$
D) $U = \dfrac{2P^2L}{Et(b_2-b_1)}\ln(b_2/b_1)$

Problem 4B

Determine the elongation of the bar by equating the strain energy to the work done by the force *P*.

Problem 5 (Hibbeler, 2014, w/ permission)

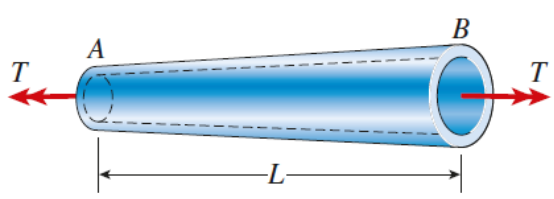
The concrete column contains six 1-in.-diameter steel reinforcing rods. If the column supports a load of 300 kip, determine the strain energy in the column. Use E_{st} = 29×10^3 ksi and $E_c = 3.6 \times 10^3$ ksi.

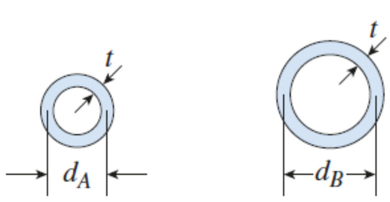


A) $U = 1.55$ in.· kip
B) $U = 3.52$ in.· kip
C) $U = 5.69$ in.· kip
D) $U = 7.48$ in.· kip

Problem 6A (Gere & Goodno, 2009, w/ permission)

A thin-walled hollow tube *AB* of conical shape has constant thickness t and average diameters d_A and d_B at the ends (see figure). Determine the strain energy of the tube when it is subjected to pure torsion by torques *T*. Use the approximate formula $J = \pi d^3 t/4$ for a thin circular ring.





A) $U = \dfrac{T^2 L}{4\pi G t}\left(\dfrac{d_A+d_B}{d_A^2 d_B^2}\right)$

B) $U = \dfrac{T^2 L}{3\pi G t}\left(\dfrac{d_A+d_B}{d_A^2 d_B^2}\right)$

C) $U = \dfrac{T^2 L}{2\pi G t}\left(\dfrac{d_A+d_B}{d_A^2 d_B^2}\right)$

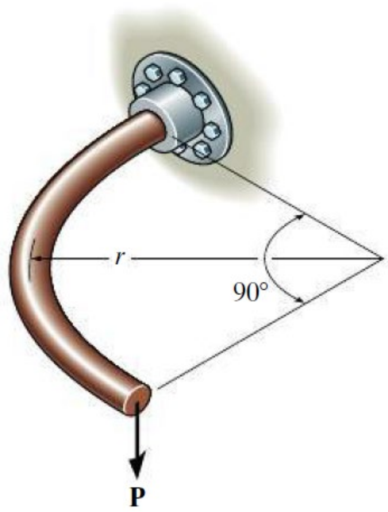
D) $U = \dfrac{T^2 L}{\pi G t}\left(\dfrac{d_A+d_B}{d_A^2 d_B^2}\right)$

Problem 6B

Determine the angle of twist of the tube by equating the strain energy to the work done by the torque *T*.

Problem 7 (Hibbeler, 2014, w/ permission)

Determine the strain energy in the horizontal curved bar due to torsion. There is a vertical force *P* acting at its end. *GJ* is constant.



A) $U = \frac{P^2r^3}{GJ}\left(\frac{3\pi}{8} - 1\right)$

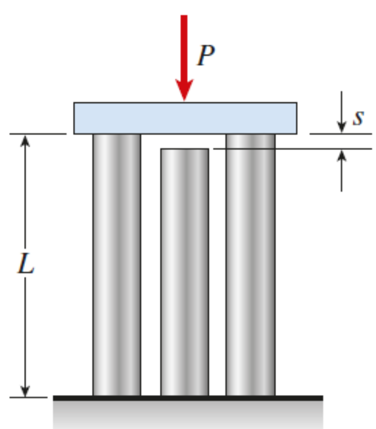
B) $U = \frac{P^2r^3}{GJ}\left(\frac{3\pi}{4} - 2\right)$

C) $U = \frac{P^2r^3}{GJ}\left(\frac{\pi}{2} - 1\right)$

D) $U = \frac{P^2r^3}{GJ}\left(\frac{3\pi}{4} - 1\right)$

Problem 8A (Gere & Goodno, 2009, w/ permission)

A compressive load *P* is transmitted through a rigid plate to three magnesium-alloy bars that are identical except that initially the middle bar is slightly shorter than the other bars (see figure). The dimensions and properties of the assembly are as follows: length $L = 1.0$ m, cross-sectional area of each bar $A = 3000$ mm², modulus of elasticity $E = 45$ GPa, and gap $s = 1.0$ mm. Calculate the load P_G required to close the gap and the downward displacement δ of the rigid plate when $P = 400$ kN.



A) $P_G = 270$ kN and $\delta = 1.32$ mm

B) $P_G = 270$ kN and $\delta = 1.89$ mm

C) $P_G = 365$ kN and $\delta = 1.32$ mm

D) $P_G = 365$ kN and $\delta = 1.89$ mm

Problem 8B

Calculate the total strain energy of the three bars when $P = 400$ kN.

A) $U = 64.5$ J

B) $U = 144$ J

C) $U = 242$ J

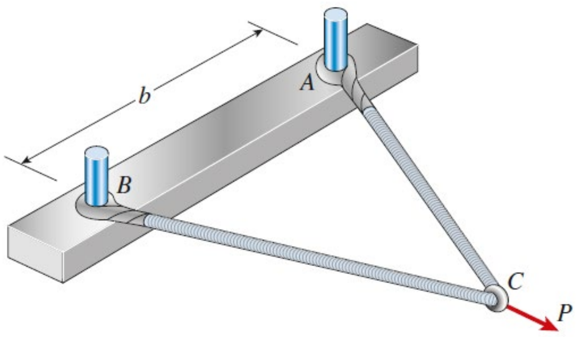
D) $U = 321$ J

Problem 8C

Explain why the strain energy is *not* equal to $P\delta/2$.

Problem 9A (Gere & Goodno, 2009, w/ permission)

A bungee cord that behaves linearly elastically has an unstretched length L_0 = 760 mm and a stiffness k = 140 N/m. The cord is attached to two pegs, a distance b = 180 mm apart, and pulled at its midpoint by a force P = 80 N (see figure). How much strain energy is stored in the cord?



A) $U = 3.25$ J
B) $U = 6.51$ J
C) $U = 9.48$ J
D) $U = 12.4$ J

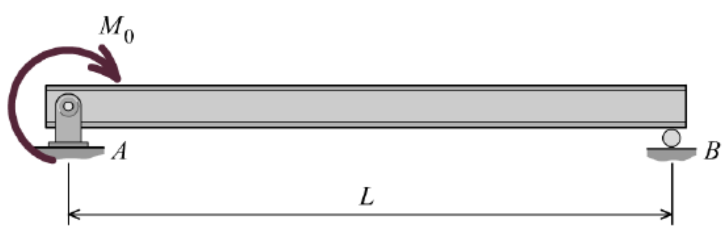
Problem 9B

What is the displacement δ_C of the point where the load is applied? Compare the strain energy obtained in the previous part with the quantity $P\delta_C/2$.

PROBLEMS – CASTIGLIANO'S THEOREM

Problem 10 (Philpot, 2013, w/ permission)

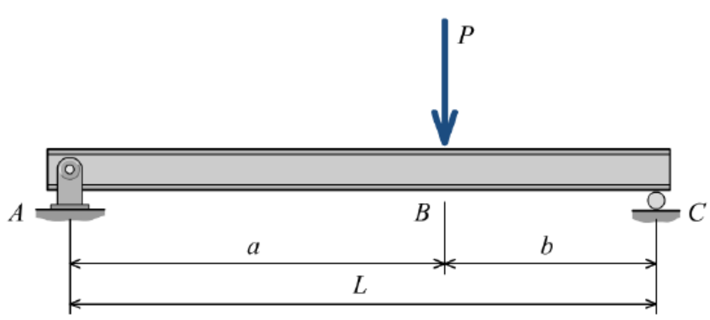
Employing Castigliano's second theorem, calculate the slope of the beam at *A* for the loading shown in the figure. Assume that *EI* is constant for the beam.



A) $\theta_A = \dfrac{M_0L}{4EI}$
B) $\theta_A = \dfrac{M_0L}{3EI}$
C) $\theta_A = \dfrac{M_0L}{2EI}$
D) $\theta_A = \dfrac{M_0L}{EI}$

Problem 11 (Philpot, 2013, w/ permission)

Employing Castigliano's second theorem, determine the deflection of the beam at *B*. Assume that *EI* is constant for the beam.



A) $\delta_B = \dfrac{Pa^2b^2}{4LEI}$

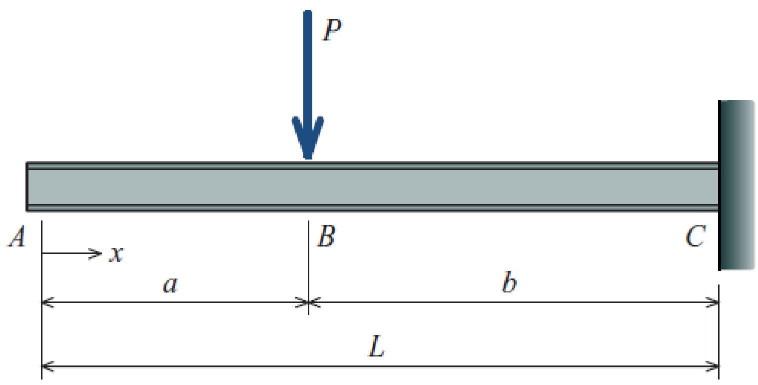
B) $\delta_B = \dfrac{Pa^2b^2}{3LEI}$

C) $\delta_B = \dfrac{Pa^2b^2}{2LEI}$

D) $\delta_B = \dfrac{Pa^2b^2}{LEI}$

Problem **12** (Philpot, 2013, w/ permission)

Employing Castigliano's second theorem, determine the deflection of the beam at *A*. Assume that *EI* is constant for the beam.



A) $\delta_A = \dfrac{Pb^2}{8EI}(2L - b)$

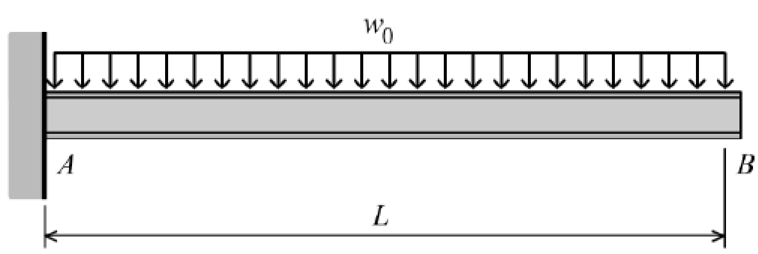
B) $\delta_A = \dfrac{Pb^2}{8EI}(3L - b)$

C) $\delta_A = \dfrac{Pb^2}{6EI}(2L - b)$

D) $\delta_A = \dfrac{Pb^2}{6EI}(3L - b)$

Problem **13** (Philpot, 2013, w/ permission)

Employing Castigliano's second theorem, determine the deflection of the beam at *B*. Assume that *EI* is constant for the beam.



A) $\theta_B = \dfrac{w_0L^3}{6EI}$ and $\delta_B = \dfrac{w_0L^4}{8EI}$

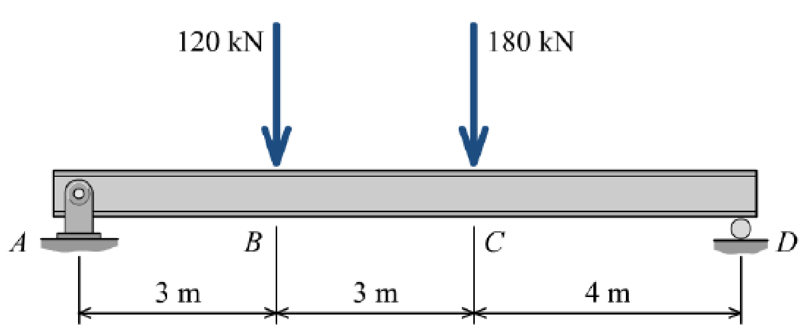
B) $\theta_B = \dfrac{w_0L^3}{6EI}$ and $\delta_B = \dfrac{w_0L^4}{4EI}$

C) $\theta_B = \dfrac{w_0L^3}{3EI}$ and $\delta_B = \dfrac{w_0L^4}{8EI}$

D) $\theta_B = \dfrac{w_0L^3}{3EI}$ and $\delta_B = \dfrac{w_0L^4}{4EI}$

Problem **14** (Philpot, 2013, w/ permission)

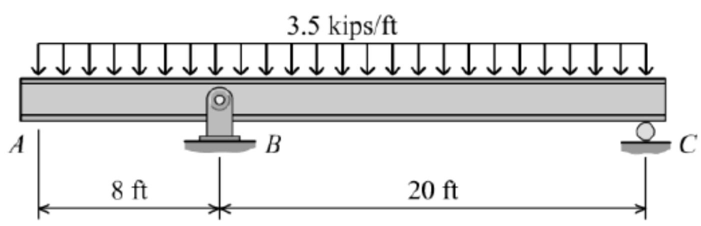
Apply Castigliano's second theorem to compute the deflection of the beam at *C* for the loading in the next figure. Assume that $EI = 1.72 \times 10^5$ kN·m² for the beam.



A) $\delta_C = 6.54$ mm
B) $\delta_C = 18.4$ mm
C) $\delta_C = 30.5$ mm
D) $\delta_C = 42.5$ mm

Problem **15** (Philpot, 2013, w/ permission)

Apply Castigliano's second theorem to compute the deflection of the beam at *A* and the slope at *C* for the loading in the next figure. Assume that $EI = 1.72 \times 10^5$ kN·m² for the beam. In the following values of δ_A, the negative sign indicates an upward displacement.



A) $\delta_A = -0.0921$ in. and $\theta_C = 0.00761$ rad
B) $\delta_A = -0.0921$ in. and $\theta_C = 0.0108$ rad
C) $\delta_A = -0.181$ in. and $\theta_C = 0.00761$ rad
D) $\delta_A = -0.181$ in. and $\theta_C = 0.0108$ rad

SOLUTIONS

P.1 ➡ *Solution*

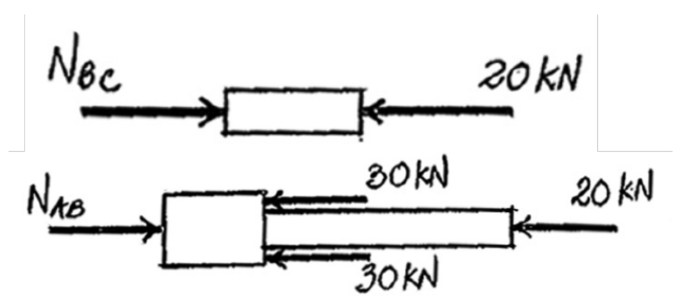
Part A: Refer to the free-body diagrams shown below. N_{BC} is the normal force exerted on segment *BC* and N_{AB} is the normal force imparted on segment *AB*. From equilibrium of forces in the *x*-direction, we have, in each case,

$$\Sigma F_x = 0 \to N_{BC} - 20 = 0$$

$$\therefore N_{BC} = 20 \text{ kN}$$

$$\therefore \Sigma F_x = 0 \to N_{AB} - 30 - 30 - 20 = 0$$

$$\therefore N_{AB} = 80 \text{ kN}$$



The cross-sectional areas of segments *AB* and *BC* are $A_{AB} = \pi \times 0.1^2/4 = 7.85 \times 10^{-3}$ m² and $A_{BC} = \pi \times 0.075^2/4 = 4.42 \times 10^{-3}$ m². Applying the formula for axial strain energy leads to

$$U = \Sigma \frac{N^2 L}{2AE} = \frac{N_{AB}^2 L_{AB}}{2A_{AB}E_{\text{st}}} + \frac{N_{BC}^2 L_{BC}}{2A_{BC}E_{\text{br}}}$$

$$\therefore U = \frac{\left(80\times10^3\right)^2 \times 1.5}{2\times\left(7.85\times10^{-3}\right)\times\left(200\times10^9\right)} + \frac{\left(20\times10^3\right)^2 \times 0.5}{2\times\left(4.42\times10^{-3}\right)\times\left(101\times10^9\right)} = 3.06 + 0.224 = \boxed{3.28 \text{ J}}$$

It should be borne in mind that this analysis is only valid if the normal stress in each member does not exceed the yield stress of the material. Considering the stresses in segments *AB* and *BC* respectively, we have

$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{80\times10^3}{7.85\times10^{-3}} = 10.2 \text{ MPa} < (\sigma_Y)_{\text{st}} = 250 \text{ MPa (OK)}$$

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{20\times10^3}{4.42\times10^{-3}} = 4.52 \text{ MPa} < (\sigma_Y)_{\text{br}} = 410 \text{ MPa (OK)}$$

The correct answer is **B**.

Part B: Refer to the following free-body diagrams. From equilibrium of moments about the longitudinal axis of the shaft, we have, in each case,

$$\Sigma M_x = 0 \rightarrow T_{AB} - 300 = 0$$

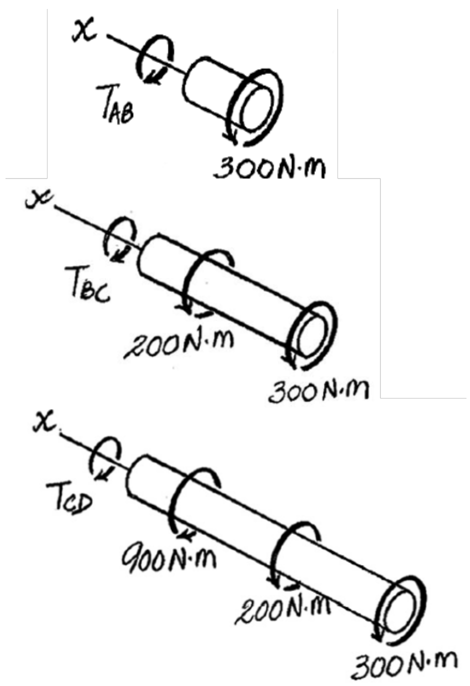
$$\therefore T_{AB} = 300 \text{ N}\cdot\text{m}$$

$$\Sigma M_x = 0 \rightarrow T_{BC} - 200 - 300 = 0$$

$$\therefore T_{BC} = 500 \text{ N}\cdot\text{m}$$

$$\Sigma M_x = 0 \rightarrow T_{CD} - 200 - 300 + 900 = 0$$

$$\therefore T_{CD} = -400 \text{ N}\cdot\text{m}$$



The shaft has a constant circular cross-section and its polar moment of inertia is $J = \pi \times 0.04^4/32 = 2.51 \times 10^{-7}$ m^4. Applying the formula for torsional strain energy brings to

$$U = \Sigma\frac{T^2L}{2GJ} = \frac{T_{AB}^2L_{AB}}{2GJ} + \frac{T_{BC}^2L_{BC}}{2GJ} + \frac{T_{CD}^2L_{CD}}{2GJ}$$

$$\therefore U = \frac{1}{2\times(75\times10^9)\times(2.51\times10^{-7})} \times \left[300^2\times0.5 + 500^2\times0.5 + (-400)^2\times0.5\right] = \boxed{6.64 \text{ J}}$$

The correct answer is **C**.

P.2 ➡ *Solution*

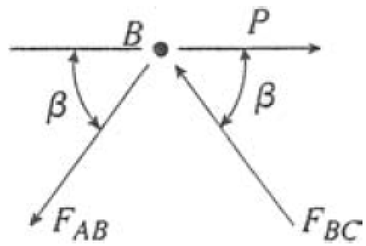
Part A: Consider a free-body diagram of joint *B*, as shown. From the equilibrium of forces in the vertical and horizontal directions, we have respectively

$$\Sigma F_x = 0 \rightarrow -F_{AB}\sin\beta + F_{BC}\sin\beta = 0$$

$$\therefore F_{AB} = F_{BC}$$

$$\Sigma F_y = 0 \rightarrow -F_{AB}\cos\beta - F_{BC}\cos\beta + P = 0$$

$$\therefore F_{AB} = F_{BC} = \frac{P}{2\cos\beta} = \frac{P}{2\times1/2} = P$$



Therefore, the axial forces acting on the truss members are $N_{AB} = P$ (tension) and $N_{BC} = -P$ (compression). The total strain energy of the truss is obtained by adding up the contributions of members *AB* and *BC*, so that

$$U = \Sigma \frac{N^2 L}{2EA} = \frac{F_{AB}^2 L}{2EA} + \frac{F_{BC}^2 L}{2EA}$$

$$\therefore \boxed{U = \frac{P^2 L}{EA}}$$

The correct answer is **C**.

Part B: All we have to do is equate the work done by load *P* to the strain energy of the system,

$$P \times \frac{\delta_B}{2} = U \rightarrow P \times \frac{\delta_B}{2} = \frac{P^2 L}{EA}$$

$$\therefore \boxed{\delta_B = \frac{2PL}{EA}}$$

The correct answer is **D**.

P.3 ➡ *Solution*

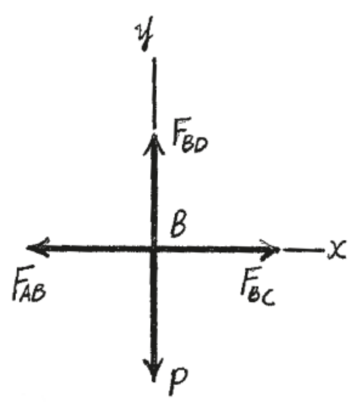
The normal force developed in each member of the truss can be determined with the method of joints. Summing forces at joint *B*, we have

$$\Sigma F_x = 0 \rightarrow F_{BC} - F_{AB} = 0$$

$$\therefore F_{BC} = F_{AB}$$

$$\Sigma F_y = 0 \rightarrow F_{BD} - P = 0$$

$$\therefore F_{BD} = P \ \ \text{(T)}$$



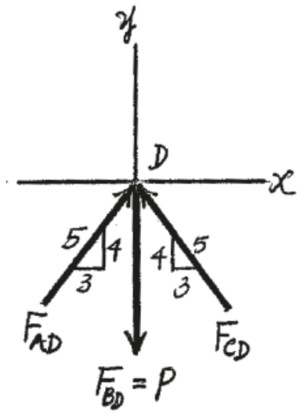
Likewise, summing forces at joint *D* gives

$$\Sigma F_x = 0 \rightarrow F_{AD} \times 0.6 - F_{CD} \times 0.6 = 0$$

$$\therefore F_{AD} = F_{CD} = F$$

$$\Sigma F_y = 0 \rightarrow 2 \times F \times 0.8 - P = 0$$

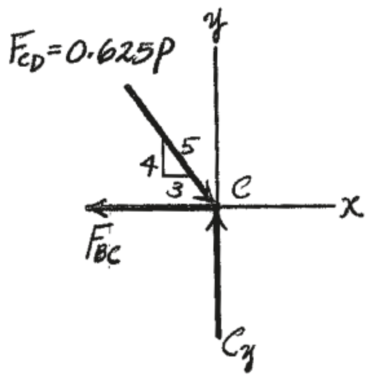
$$\therefore F_{AD} = F_{CD} = F = 0.625P \ \ \text{(C)}$$



Finally, we sum horizontal forces at joint *C*.

$$\Sigma F_x = 0 \rightarrow 0.625P \times 0.6 - F_{BC} = 0$$

$$\therefore F_{BC} = 0.375P \ \text{(T)}$$



Member *BD* is critical since it is subjected to the greatest normal force. Equating the normal stress in this member to the yield stress of steel, we have

$$\sigma_Y = \frac{F_{BD}}{A} \rightarrow 36 = \frac{F_{BD}}{\left(\frac{\pi \times 2^2}{4}\right)}$$

$$\therefore F_{BD} = P = 113 \text{ kip}$$

Substituting *P* in the equilibrium equations, we obtain $F_{AD} = F_{CD} = 70.6$ kip and $F_{BC} = F_{AB} = 42.4$ kip. The energy stored in the truss can be determined by summing the contribution of each member to strain energy; that is,

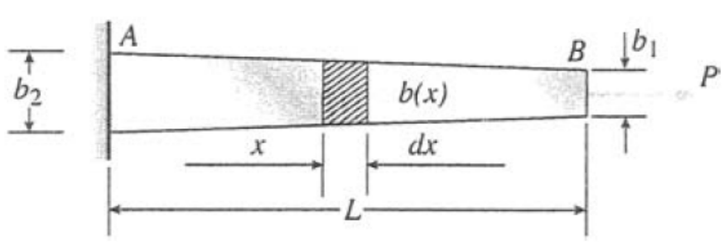
$$U = \Sigma \frac{N^2 L}{2AE} = \frac{1}{2 \times 3.14 \times \left(29 \times 10^3\right)} \times \left[113^2 \times (4 \times 12) + 2 \times 70.6^2 \times (5 \times 12) + 2 \times 42.4^2 \times (3 \times 12)\right]$$

$$\therefore \boxed{U = 7.36 \text{ in.} \cdot \text{kip}}$$

The correct answer is **B**.

P.4 ➡ *Solution*

Part A: Refer to the figure below.



The width of the bar varies uniformly according to the relation

$$b(x) = b_2 - \frac{(b_2 - b_1)x}{L}$$

The cross-sectional area of the bar follows as

$$A(x)=tb(x)=t\left[b_2-\frac{(b_2-b_1)x}{L}\right]$$

The strain energy of the bar is calculated as

$$U=\int\frac{[N(x)]^2\,dx}{2EA(x)}$$

Inserting the relation we derived for *A(x)* gives

$$U=\int_0^L\frac{P^2dx}{2Etb(x)}=\frac{P^2}{2Et}\int_0^L\frac{dx}{b_2-(b_2-b_1)x/L}$$

At this point, we can employ the standard integral

$$\int\frac{dx}{a+bx}=\frac{1}{b}\ln(a+bx)$$

with the result that

$$U=\frac{P^2}{2Et}\times\left\{\frac{1}{-[b_2-b_1]/L}\ln\left[b_2-\frac{(b_2-b_1)x}{L}\right]\right\}\Bigg|_0^L$$

$$\therefore U=\frac{P^2}{2Et}\times\left\{\frac{-L}{[b_2-b_1]}\ln b_1-\left[\frac{-L}{(b_2-b_1)}\ln b_2\right]\right\}$$

$$\therefore \boxed{U=\frac{P^2L}{2Et(b_2-b_1)}\ln\left(\frac{b_2}{b_1}\right)}$$

The correct answer is **B**.

Part B: All we have to do is equate the strain energy obtained just now to the work done by the force *P*,

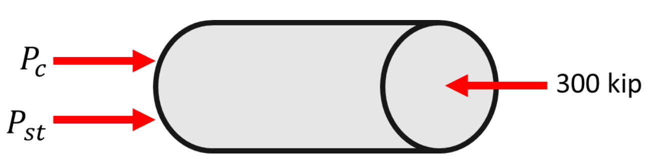
$$\delta=\frac{2U}{P}=\frac{\cancel{2}}{\cancel{P}}\times\frac{P^{\cancel{2}}L}{\cancel{2}Et(b_2-b_1)}\ln\left(\frac{b_2}{b_1}\right)$$

$$\therefore \boxed{\delta=\frac{PL}{Et(b_2-b_1)}\ln\left(\frac{b_2}{b_1}\right)}$$

P.5 ➡ *Solution*

Consider the free-body diagram of the column. The sum of the axial forces exerted on the concrete, P_c, and on the steel, P_{st}, must equal 300 kip; that is,

$$P_c+P_{st}=300$$



As a compatibility condition, we note that the axial deformation of the concrete and the steel must be the same,

$$\delta_c=\delta_{st}\rightarrow\frac{P_c\cancel{L}}{A_cE_c}=\frac{P_{st}\cancel{L}}{A_{st}E_{st}}$$

$$\therefore\frac{P_c}{(\pi\times12^2-6\times\pi\times0.5^2)\times(3.6\times10^3)}=\frac{P_{st}}{(6\times\pi\times0.5^2)\times(29\times10^3)}$$

$$\therefore\frac{P_c}{1.61\times10^6}=\frac{P_{st}}{1.37\times10^5}$$

$$\therefore P_c=11.8P_{st}$$

Substituting in the first equation gives

$$P_c + P_{st} = 300 \rightarrow 11.8P_{st} + P_{st} = 300$$

$$\therefore P_{st} = \frac{300}{12.8} = 23.4 \text{ kip}$$

and $P_c = 300 - 23.4 = 277$ kip. We are now ready to compute the strain energy of the system,

$$U = \Sigma \frac{N^2 L}{2AE} = \frac{23.4^2 \times (5\times 12)}{2\times(6\times\pi\times 0.5^2)\times(29\times 10^3)} + \frac{277^2 \times (5\times 12)}{2\times(\pi\times 12^2 - 6\times\pi\times 0.5^2)\times(3.6\times 10^3)}$$

$$\therefore U = 0.12 + 1.43 = \boxed{1.55 \text{ in.}\cdot\text{kip}}$$

The correct answer is **A**.

P.6 ➡ *Solution*

Part A: The average diameter at a distance *x* from end *A* is given by

$$d(x) = d_A + \left(\frac{d_B - d_A}{L}\right)x$$

Accordingly, the polar moment of inertia is approximated as

$$J(x) = \frac{\pi[d(x)]^3 t}{4} = \frac{\pi t}{4}\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^3$$

Appealing to the equation for torsional strain energy, we have

$$U = \int_0^L \frac{T^2 dx}{2GJ(x)} \rightarrow U = \frac{2T^2}{\pi G t}\int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^3}$$

At this point, we can employ the standard integral

$$\int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}$$

so that

$$\int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]} = -\frac{1}{\frac{2(d_B - d_A)}{L}\times\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^2}\Bigg|_0^L$$

$$\therefore \int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]} = -\frac{L}{2(d_B - d_A)d_B^2} + \frac{L}{2(d_B - d_A)d_A^2}$$

$$\therefore \int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]} = \frac{L(d_A + d_B)}{2d_A^2 d_B^2}$$

Backsubstituting in the expression for *U*, we obtain

$$U = \frac{2T^2}{\pi G t}\int_0^L \frac{dx}{\left[d_A + \left(\frac{d_B - d_A}{L}\right)x\right]^3} = \frac{\cancel{2}T^2}{\pi G t}\times\frac{L(d_A + d_B)}{\cancel{2}d_A^2 d_B^2}$$

$$\therefore \boxed{U = \frac{T^2 L}{\pi G t}\left(\frac{d_A + d_B}{d_A^2 d_B^2}\right)}$$

The correct answer is **D**.

Part B: The angle of twist can be determined by equating the strain energy U to the work of torque T; that is,

$$W = U \rightarrow \frac{\cancel{T}\phi}{2} = \frac{T^{\cancel{2}}L}{\pi G t}\left(\frac{d_A + d_B}{d_A^2 d_B^2}\right)$$

$$\therefore \boxed{\phi = \frac{2TL}{\pi G t}\left(\frac{d_A + d_B}{d_A^2 d_B^2}\right)}$$

P.7 ➡ *Solution*

The torsional moment due to force P can be shown to be

$$T = Pr(1 - \cos\theta)$$

The strain energy due to torsion is given by

$$U = \int_0^L \frac{T^2 ds}{2GJ}$$

Here, arc length $s = r\theta$ and $ds = rd\theta$. Inserting the pertaining variables and integrating, we find that

$$U = \int_0^{\pi/2} \frac{\left[Pr(1 - \cos\theta)\right]^2}{2GJ} r d\theta$$

$$\therefore U = \frac{P^2 r^3}{2GJ}\int_0^{\pi/2} (1 - \cos\theta)^2 \, d\theta$$

$$\therefore U = \frac{P^2 r^3}{2GJ}\int_0^{\pi/2} \left(1 - 2\cos\theta + \cos^2\theta\right) d\theta$$

$$\therefore U = \frac{P^2 r^3}{2GJ}\int_0^{\pi/2} \left(1 - 2\cos\theta + \frac{\cos 2\theta}{2} + \frac{1}{2}\right) d\theta$$

$$\therefore U = \frac{P^2 r^3}{2GJ}\left(\frac{3\theta}{2} - 2\sin\theta + \frac{\sin 2\theta}{4}\right)\Bigg|_0^{\pi/2}$$

$$\therefore U = \frac{P^2 r^3}{2GJ}\left[\left(\frac{3\pi}{4} - 2 + 0\right) - (0)\right]$$

$$\therefore \boxed{U = \frac{P^2 r^3}{GJ}\left(\frac{3\pi}{8} - 1\right)}$$

The correct answer is **A**.

P.8 ➡ *Solution*

Part A: The load required to close the gap is calculated as

$$\delta = \frac{PL}{EA} \rightarrow P_G = \frac{EA\delta}{L}$$

$$\therefore P_G = 2 \times \frac{\left(45 \times 10^9\right) \times \left(3000 \times 10^{-6}\right) \times 10^{-3}}{1.0} = \boxed{270 \text{ kN}}$$

A factor of 2 was introduced because two bars are acted upon by this force. Since 400 kN > P_G, all three bars will be compressed when P is applied. Load P equals P_G plus the additional force required to compress all three bars by an amount $\delta - s$. In mathematical terms,

$$P = P_G + 3\left(\frac{EA}{L}\right)(\delta - s)$$

$$\therefore 400 \times 10^3 = 270 \times 10^3 + 3 \times \frac{\left(45 \times 10^9\right) \times \left(3000 \times 10^{-6}\right)}{1.0} \times \left(\delta - 1.0 \times 10^{-3}\right)$$

$$\therefore \delta = \boxed{1.32 \text{ mm}}$$

The correct answer is **A**.

Part B: The strain energy follows from the relation

$$U = \Sigma \frac{EA\delta^2}{2L}$$

The displacement of the outer bars is $\delta = 1.32$ mm while that of the middle bar is $1.32 - 1.0 = 0.32$ mm. Thus, U is computed as

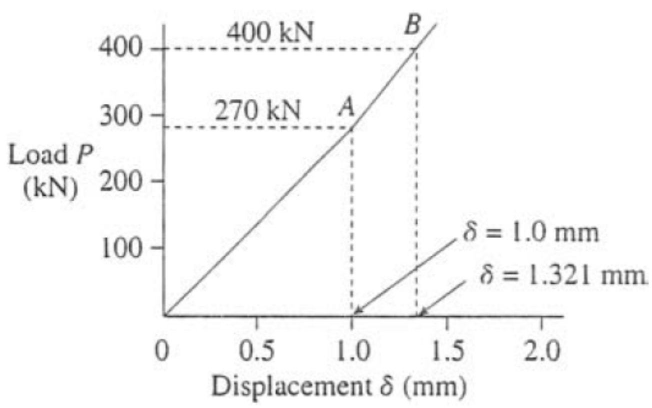
$$U = \Sigma \frac{EA\delta^2}{2L} = \frac{(45\times10^9)\times(3000\times10^{-6})}{2\times1.0}\times\left[2\times(1.32\times10^{-3})^2 + (0.32\times10^{-3})^2\right] = \boxed{242 \text{ J}}$$

The correct answer is **C**.

Part C: Using the typical formula, the strain energy of the assembly would have been

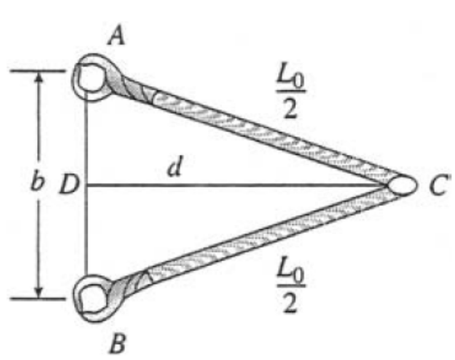
$$U = P\times\frac{\delta}{2} = (400\times10^3)\times\frac{(1.32\times10^{-3})}{2} = 264 \text{ J}$$

which of course is different from our result of 242 J. The difference occurs because the load-displacement relation is not precisely linear. This can be explained by drawing a load-displacement diagram such as the one shown below. The strain energy is the area under line OAB, whereas $P\delta/2$ is the area under a straight line traveling from O to B, which is larger than U.



P.9 ➡ *Solution*

Part A: The dimensions of the cord before load P is applied are illustrated below.

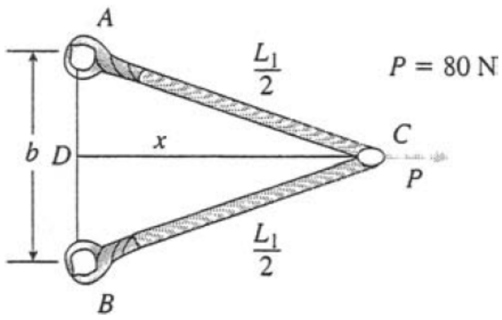


We have $b = 380$ mm and $L_0 = 760$ mm as given. Distance d can be obtained with the Pythagorean theorem,

$$\left(\frac{L_0}{2}\right)^2 = \left(\frac{b}{2}\right)^2 + d^2 \rightarrow d = \frac{1}{2}\sqrt{L_0^2 - b^2}$$

$$\therefore d = \frac{1}{2}\times\sqrt{0.76^2 - 0.38^2} = 0.329 \text{ m}$$

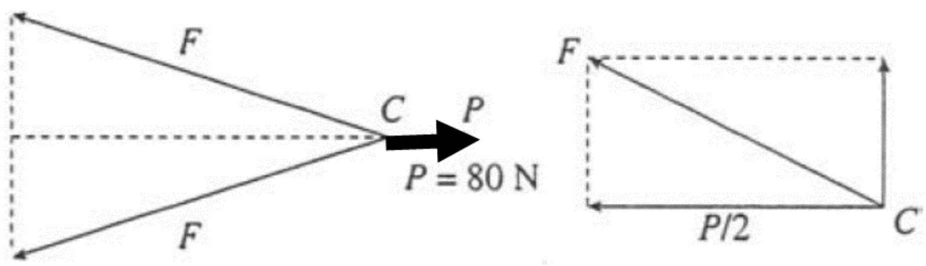
Consider now the dimensions *after* load P is applied.



The dimension of the stretched bungee cord is denoted as L_1. From triangle ACD, we find that

$$\left(\frac{L_1}{2}\right)^2 = \left(\frac{b}{2}\right)^2 + x^2 \rightarrow L_1 = \sqrt{b^2 + 4x^2}$$

Let us now consider the equilibrium of point *C* of the bungee cord. Below, *F* denotes the tensile force in the bungee cord.



From similar triangles, we see that

$$\frac{F}{P/2} = \frac{L_1/2}{x} \rightarrow F = \frac{P}{2} \times \frac{L_1}{2} \times \frac{1}{x}$$

$$\therefore F = \frac{P}{2}\sqrt{1 + \frac{b^2}{4x^2}}$$

The elongation of the bungee cord is denoted as δ. From Hooke's law for springs, we write

$$\delta = \frac{F}{k} = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \quad \text{(I)}$$

Displacement of the bungee cord allows us to state that

$$\delta = L_1 - L_0 \rightarrow L_1 = L_0 + \delta$$

$$\therefore \sqrt{b^2 + 4x^2} = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}}$$

$$\therefore \sqrt{b^2 + 4x^2} = L_0 + \frac{P}{4kx}\sqrt{b^2 + 4x^2}$$

$$\therefore L_0 = \left(1 - \frac{P}{4kx}\right)\sqrt{b^2 + 4x^2}$$

Inserting the pertaining data and solving for *x*, we obtain

$$760 = \left[1 - \frac{80}{4 \times \left(\frac{140}{1000}\right) \times x}\right] \times \sqrt{380^2 + 4x^2}$$

$$\therefore x = 498 \text{ mm}$$

Backsubstituting *x* into equation (I), the elongation of the cord is determined as

$$\delta = \frac{80}{2 \times 140} \times \sqrt{1 + \frac{0.38^2}{4 \times 0.498^2}} = 0.305 \text{ m}$$

We can now proceed to calculate the strain energy of the cord,

$$U = \frac{k\delta^2}{2} = \frac{140 \times 0.305^2}{2} = \boxed{6.51 \text{ J}}$$

The correct answer is **B**.

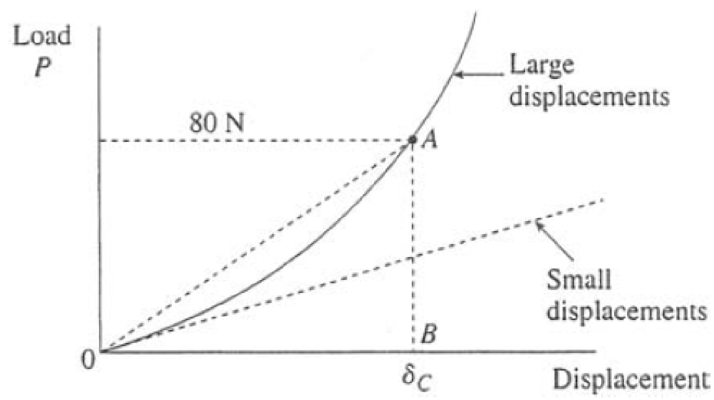
Part B: The displacement δ_C at point C is given by the difference

$$\delta_C = x - d = 0.498 - 0.329 = 0.169 \text{ m}$$

The quantity with which we want to compare our results is

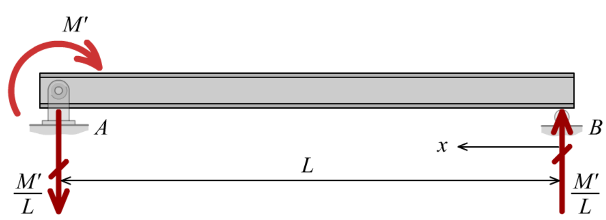
$$U_C = \frac{P\delta_C}{2} = \frac{80 \times 0.169}{2} = 6.76 \text{ J}$$

Our result, U, and U_C are not the same. The work done by the load *P* is *not* equal to $P\delta_C/2$ because the load-displacement relation (see below) is nonlinear when the displacements are large. (The *work* done by the load *P* is equal to the strain energy because the bungee cord behaves elastically and there are no energy losses.) In the load-displacement diagram below, *U* is the area *OAB* under the curve *OA* and $P\delta_C/2$ is the area of triangle *OAB*, which is greater than *U*.

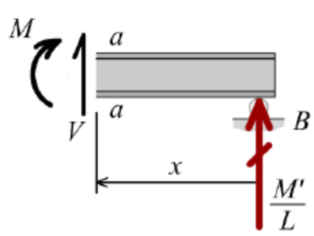


P.10 ➡ *Solution*

Let us designate the concentrated moment at *A* as *M'*. The free-body diagram of the beam is shown in continuation.



We proceed to consider a segment of the beam that goes from fixed end *B* to a section *a-a* somewhere along the beam, as shown.



Referring to this figure, we have, summing moments about section *a-a*,

$$\Sigma M_{a-a} = 0 \rightarrow \frac{M'}{L}x - M = 0$$

$$\therefore M = \frac{M'}{L}x$$

Differentiating this expression with respect to *M'* gives

$$\frac{\partial M}{\partial M'} = \frac{x}{L}$$

Substituting $M' = M_0$ in the bending moment equation gives

$$M = \frac{M_0}{L}x$$

Finally, we can use Castigliano's second theorem to determine the slope at *A*,

$$\theta_A = \int_0^L \left(\frac{\partial M}{\partial M'}\right)\frac{M}{EI} = \int_0^L \frac{x}{L}\times\frac{M_0 x}{LEI}dx$$

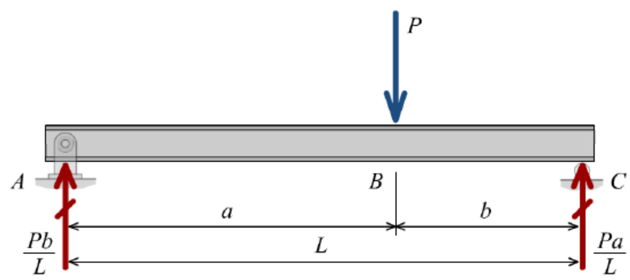
$$\therefore \theta_A = \frac{M_0}{L^2 EI}\int_0^L x^2 dx$$

$$\therefore \boxed{\theta_A = \frac{M_0 L}{3EI}}$$

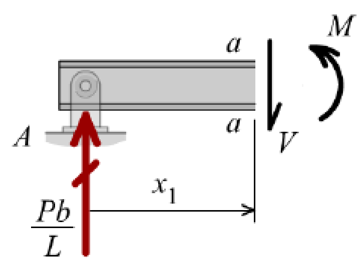
The correct answer is **B**.

P.11 ➡ *Solution*

The free-body diagram of the beam is shown in continuation.



We take a segment of the beam that goes from left end *A* to a section *a-a* somewhere along its span, as illustrated below.



Referring to this figure, we have, from the equilibrium of moments about *a-a*,

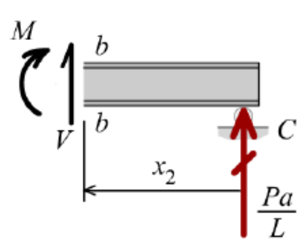
$$\Sigma M_{a-a} = 0 \rightarrow -\frac{Pb}{L}x_1 + M = 0$$

$$\therefore M = \frac{Pb}{L}x_1 \;\; \{0 \le x_1 \le a\}$$

Differentiating this expression with respect to *P* gives

$$\frac{\partial M}{\partial P} = \frac{b}{L}x_1$$

Consider now a beam segment that spans end *C* to a section *b-b*, as shown.



Equilibrium of moments about section *b-b* brings to

$$\Sigma M_{b-b} = 0 \rightarrow \frac{Pa}{L}x_2 - M = 0$$

$$\therefore M = \frac{Pa}{L}x_2 \;\; \{0 \le x_2 < b\}$$

Differentiating this equation with respect to *P* gives

$$\frac{\partial M}{\partial P}=\frac{a}{L}x_2$$

Lastly, we can use Castigliano's second theorem to determine the deflection at *B*,

$$\delta_B=\int_0^L\left(\frac{\partial M}{\partial P}\right)\frac{M}{EI}dx\rightarrow\delta_B=\int_0^a\left(\frac{b}{L}x_1\right)\times\left(\frac{Pb}{LEI}x_1\right)dx_1+\int_0^b\left(\frac{a}{L}x_2\right)\times\left(\frac{Pa}{LEI}x_2\right)dx_2$$

$$\therefore\delta_B=\int_0^a\frac{Pb^2}{L^2EI}x_1^2dx_1+\int_0^b\frac{Pa^2}{L^2EI}x_2^2dx_2$$

$$\therefore\delta_B=\frac{Pa^3b^2}{3L^2EI}+\frac{Pa^2b^3}{3L^2EI}$$

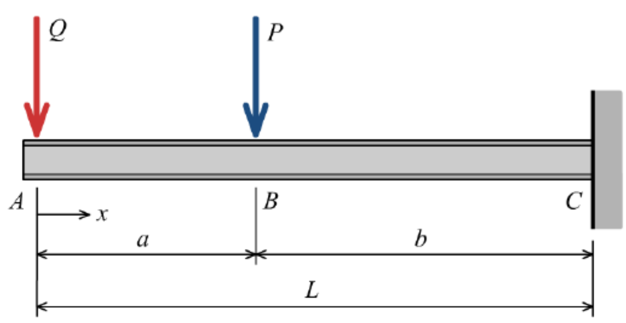
$$\therefore\delta_B=\frac{Pa^2b^2}{L^2EI}\times\underbrace{(a+b)}_{=L}$$

$$\therefore\boxed{\delta_B=\frac{Pa^2b^2}{LEI}}$$

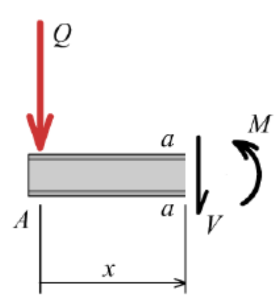
The correct answer is **D**.

P.12 ➡ *Solution*

To determine the deflection of the cantilever beam at *A*, a downward dummy load Q will be applied to the point in question, as illustrated below.



Consider the loads on a beam segment that spans end *A* to a section *a-a* somewhere in the range $0 \leq x < a$.



Referring to this figure, we have, from the equilibrium of moments about *a-a*,

$$\Sigma M_{a-a}=0\rightarrow Qx+M=0$$

$$\therefore M=-Qx\ \{0\leq x<a\}$$

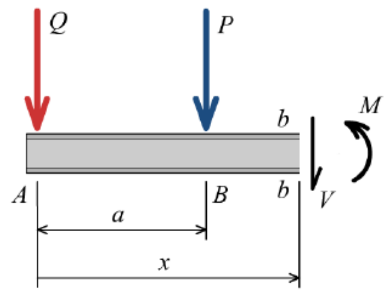
Differentiating this expression with respect to Q gives

$$\frac{\partial M}{\partial Q}=-x$$

Substituting Q = 0 in the bending moment equation, we obtain

$$M=0$$

Consider now a beam segment that encompasses end *A* to a section *b-b*. The range of interest is now $a \leq x < L$.



Equilibrium of moments about section *b-b* brings to

$$\Sigma M_{b-b} = 0 \rightarrow Qx + P(x-a) + M = 0$$

$$\therefore M = -Qx - P(x-a) \quad \{a \le x < L\}$$

Differentiating this equation with respect to Q gives

$$\frac{\partial M}{\partial Q} = -x$$

Substituting Q = 0 into the bending moment equation, we obtain

$$M = -P(x-a)$$

We can now apply Castigliano's second theorem to establish the deflection of point *A*,

$$\delta_A = \int_0^L \left(\frac{\partial M}{\partial Q}\right)\frac{M}{EI}dx \rightarrow \delta_A = \int_0^a (-x)\times 0dx + \int_a^L (-x)\times\left[-\frac{P}{EI}(x-a)\right]dx$$

$$\therefore \delta_A = \frac{P}{EI}\int_a^L (x^2 - xa)dx$$

$$\therefore \delta_A = \frac{P}{EI}\int_a^L x^2dx - \frac{Pa}{EI}\int_a^L xdx$$

$$\therefore \delta_A = \frac{P}{3EI}\times(L^3 - a^3) - \frac{Pa}{2EI}\times(L^2 - a^2)$$

$$\therefore \delta_A = \frac{PL^3}{3EI} - \frac{Pa^3}{3EI} - \frac{PaL^2}{2EI} + \frac{Pa^3}{2EI}$$

$$\therefore \delta_A = \frac{PL^3}{3EI} + \frac{Pa^3}{6EI} - \frac{PaL^2}{2EI}$$

$$\delta_A = \frac{P}{6EI}\times(2L^3 - 3aL^2 + a^3)$$

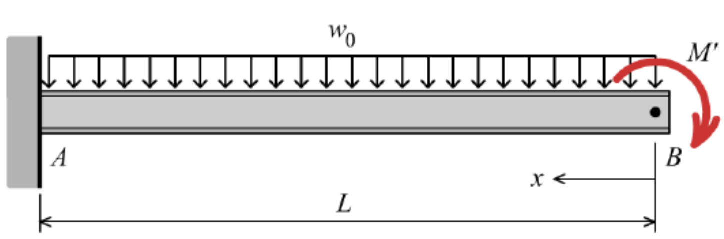
After lengthy manipulations, the result above becomes

$$\boxed{\delta_A = \frac{Pb^2}{6EI}(3L - b)}$$

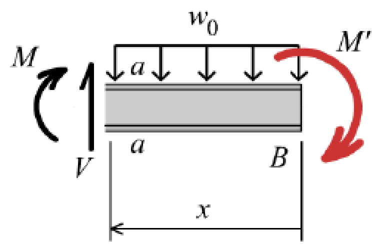
The correct answer is **D**.

P.13 ➡ *Solution*

In order to determine the slope at point *B*, we introduce a clockwise dummy moment *M'* acting at this point of interest, as shown.



Consider the loads acting on a beam segment that goes from free end *B* to a section *a-a* somewhere along the beam span.



With reference to this figure, we have, from the equilibrium of moments about *a-a*,

$$\Sigma M_{a-a} = 0 \rightarrow -M' - \frac{w_0}{2}x^2 - M = 0$$

$$\therefore M = -\frac{w_0}{2}x^2 - M' \ \{0 \le x \le L\}$$

Differentiating this expression with respect to *M'* gives

$$\frac{\partial M}{\partial M'} = -1$$

Substituting *M'* = 0 into the bending moment equation gives

$$M = -\frac{w_0}{2}x^2$$

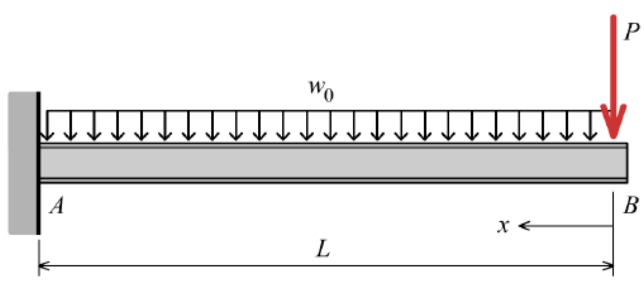
At this point, we resort to Castigliano's second theorem as applied to beam slopes,

$$\theta = \int_0^L \left(\frac{\partial M}{\partial M'}\right)\frac{M}{EI}dx \rightarrow \theta_B = \int_0^L (-1)\times\left(-\frac{w_0 x^2}{2EI}\right)dx$$

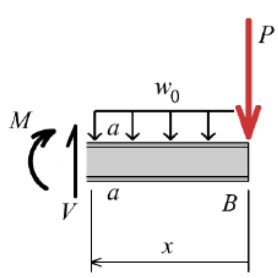
$$\therefore \theta_B = \frac{w_0}{2EI}\int_0^L x^2 dx$$

$$\therefore \boxed{\theta_B = \frac{w_0 L^3}{6EI}}$$

Next, in order to determine the deflection at point *B* of the cantilever beam, we insert a dummy concentrated load at the point in question, as shown.



Consider the loads on a beam segment that spans end *B* to a section *a-a* somewhere in the range $0 \le x < a$.



Equilibrium of moments about section *a-a* brings to

$$\Sigma M_{a-a} = 0 \rightarrow -Px - \frac{w_0}{2}x^2 - M = 0$$

$$\therefore M = -Px - \frac{w_0}{2}x^2$$

Differentiating this equation with respect to P gives

$$\frac{\partial M}{\partial P} = -x$$

Substituting $P = 0$ into the bending moment equation gives

$$M = -\frac{w_0}{2}x^2$$

Finally, we can use Castigliano's second theorem to determine the deflection at B,

$$\delta = \int_0^L \left(\frac{\partial M}{\partial P}\right)\frac{M}{EI}dx \rightarrow \delta_B = \int_0^L (-x)\times\left(-\frac{w_0}{2EI}x^2\right)dx$$

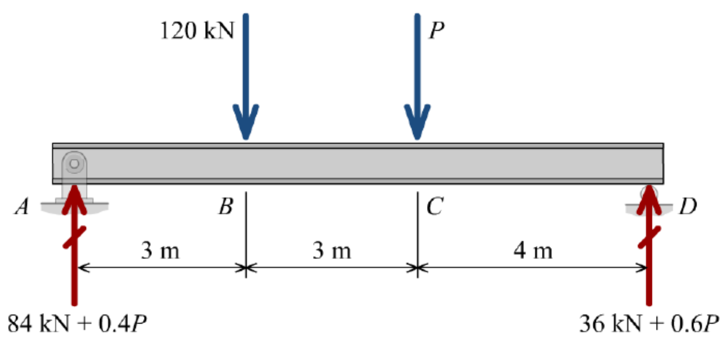
$$\therefore \delta_B = \frac{w_0}{2EI}\int_0^L x^3 dx$$

$$\therefore \boxed{\delta_B = \frac{w_0 L^4}{8EI}}$$

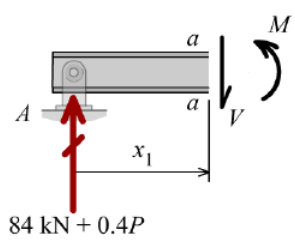
The correct answer is **A**.

P.14 ➡ *Solution*

Let us designate the concentrated load at C as the variable load P. The free-body diagram of the beam is illustrated below.



Consider the loads on a beam segment that spans end A to a section a-a somewhere in the range $0 \le x_1 < 3$ m.



Referring to this figure, we have, from the equilibrium of moments about a-a,

$$\Sigma M_{a-a} = 0 \rightarrow -(84 + 0.4P)x_1 + M = 0$$

$$\therefore M = (84 + 0.4P)x_1 \;\; \{0 \le x_1 < 3\text{ m}\}$$

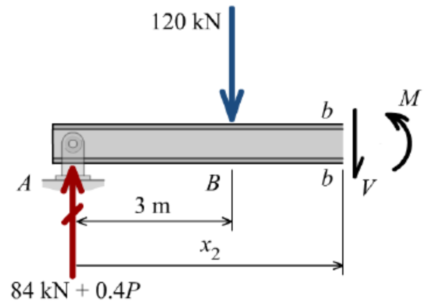
Note that, in this and other developments, forces will be expressed in kN. Differentiating the foregoing relation with respect to P gives

$$\frac{\partial M}{\partial P} = 0.4x_1$$

Substituting $P = 180$ kN into the bending moment equation, we obtain

$$M = (84 + 0.4\times 180)x_1 = 156x_1$$

Consider now a segment that encompasses end A to a section b-b. The range of interest is now $3 \le x_2 < 6$ m.



Equilibrium of moments about section *b-b* brings to

$$\Sigma M_{b-b} = 0 \rightarrow -(84+0.4P)x_2 + 120\times(x_2-3) + M = 0$$

$$\therefore M = (84+0.4P)x_2 - 120\times(x_2-3)\ \ \{a \le x_2 < L\}$$

Differentiating this equation with respect to *P* gives

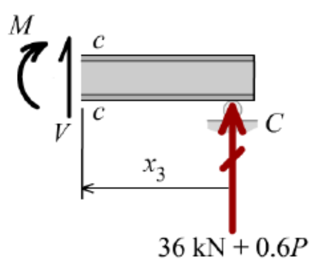
$$\frac{\partial M}{\partial P} = 0.4x_2$$

Substituting $P = 180$ kN into the bending moment equation, we obtain

$$M = (84+0.4\times180)\times x_2 - 120\times(x_2-3)$$

$$\therefore M = 156x_2 - 120\times(x_2-3)$$

As a third free-body diagram, consider a beam segment extending from right end *C* to a section *c-c* in the range $0 < x_3 < 4$ m.



Equilibrium of moments about section *b-b* brings to

$$\Sigma M_{c-c} = 0 \rightarrow (36+0.6P)x_3 - M = 0$$

$$\therefore M = (0.36+0.6P)x_3\ \ \{0 \le x_3 < 4\text{ m}\}$$

Differentiating this equation with respect to *P* gives

$$\frac{\partial M}{\partial P} = 0.6x_3$$

Substituting $P = 180$ kN into the bending moment equation, we obtain

$$M = (36+0.6\times180)\times x_3 = 144x_3$$

We are now ready to apply Castigliano's theorem and determine the deflection at point *C*,

$$\delta_C = \frac{1}{EI}\int_0^3 (0.4x_1)\times(156x_1)x_1dx_1 + \frac{1}{EI}\int_3^6 (0.4x_2)\times\left[156x_2 - 120(x_2-3)\right]dx_2$$

$$+\frac{1}{EI}\int_0^4 (0.6x_3)\times(144x_3)dx_3$$

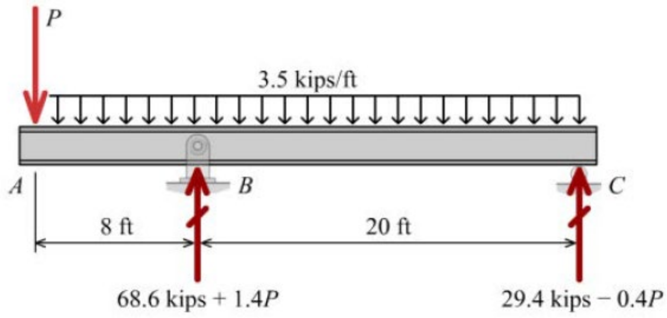
$$\therefore \delta_C = \frac{1}{EI}\times562 + \frac{1}{EI}\times2850 + \frac{1}{EI}\times1840$$

$$\therefore \delta_C = \frac{5250}{1.72\times10^5} = 0.0305 = \boxed{30.5\text{ mm}}$$

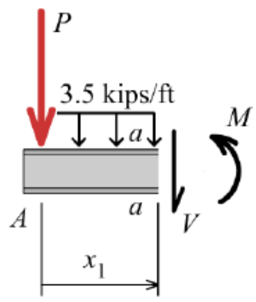
The correct answer is **C**.

P.15 ➡ *Solution*

To determine the deflection of the simply-supported beam, we introduce a dummy concentrated load *P* at point *A*, as shown.



Consider the loads acting on a beam segment that goes from free end *A* to a section *a-a* somewhere in the range $0 < x_1 < 8$ ft.



Referring to this figure, we have, from the equilibrium of moments about *a-a*,

$$\Sigma M_{a-a} = 0 \rightarrow Px_1 + \frac{3.5}{2}x_1^2 + M = 0$$

$$\therefore M = -\frac{3.5}{2}x_1^2 - Px_1 \quad \{0 \le x_1 \le 8 \text{ ft}\}$$

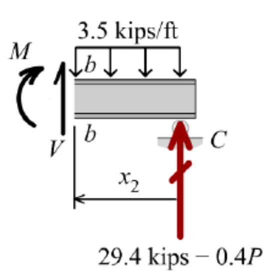
Differentiating this expression with respect to *P* gives

$$\frac{\partial M}{\partial P} = -x_1$$

Substituting $P = 0$ into the bending moment equation gives

$$M = -\frac{3.5}{2}x_1^2$$

Consider now a segment that encompasses end *C* to a section *b-b* in the range $0 \le x_2 < 20$ ft.



Equilibrium of moments about section *b-b* brings to

$$\Sigma M_{b-b} = 0 \rightarrow -\frac{3.5}{2}x_2^2 + (29.4 - 0.4P)x_2 - M = 0$$

$$\therefore M = -\frac{3.5}{2}x_2^2 + (29.4 - 0.4P)x_2 \quad \{0 \le x_2 < 20 \text{ ft}\}$$

Differentiating this equation with respect to *P* gives

$$\frac{\partial M}{\partial P} = -0.4x_2$$

Substituting $P = 0$ into the bending moment equation gives

$$M = -\frac{3.5}{2}x_2^2 + 29.4x_2$$

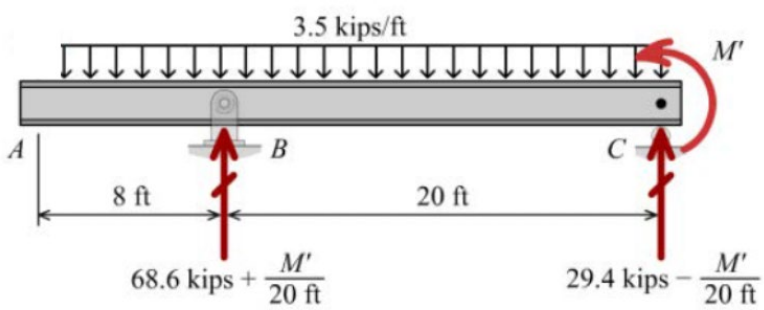
Finally, we can apply Castigliano's theorem to determine the deflection at *A*,

$$\delta = \int_0^L \left(\frac{\partial M}{\partial P}\right)\frac{M}{EI}dx \rightarrow \delta_A = \int_0^8 (-x_1)\times\left(-\frac{3.5}{2EI}x_1^2\right)dx_1 + \int_0^{20}(-0.4x_2)\times\left(-\frac{3.5}{2}x_2^2 + 29.4x_2\right)dx_2$$

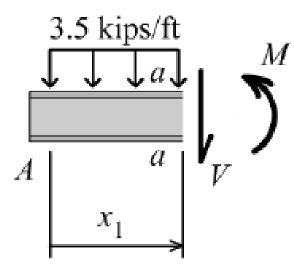
$$\therefore \delta_A = \frac{1}{EI}\times 1790 - \frac{1}{EI}\times 3360$$

$$\therefore \delta_A = -\frac{1570\times 12^3}{15\times 10^6} = \boxed{-0.181\text{ in.}}$$

The 12^3 factor was included to change the dimensions of the numerator from kip-ft^3 to kip-in.^{3}. The negative sign indicates that the displacement of point *A* is upward, not downward. Next, in order to determine the slope at point *C* of the beam, we place a dummy counterclockwise moment *M'* at the point in question, as shown.



As before, consider the loads on the beam segment in the interval $0 \le x_1 < 8$ ft.



Equilibrium of moments about section *a-a* leads to

$$\Sigma M_{a-a} = 0 \rightarrow \frac{3.5}{2}x_1^2 + M = 0$$

$$\therefore M = -\frac{3.5}{2}x_1^2 \quad \{0 \le x_1 < 8 \text{ ft}\}$$

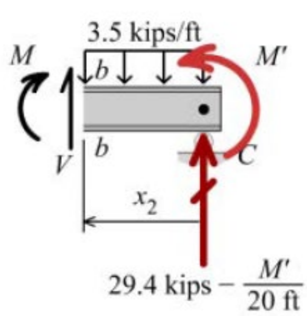
Differentiating this equation with respect to *M'* yields

$$\frac{\partial M}{\partial M'} = 0$$

Substituting *M'* = 0 into the bending moment equation produces no change in the bending moment equation,

$$M = -\frac{3.5}{2}x_1^2$$

Consider the loads on a beam segment that spans end *C* to a section *b-b* somewhere in the range $0 \le x_2 < 20$ ft.



Equilibrium of moments about section *b-b* leads to

$$\Sigma M_{b-b} = 0 \rightarrow -\frac{3.5}{2}x_2^2 + \left(29.4 - \frac{M'}{20}\right)x_2 + M' - M = 0$$

$$\therefore M = -\frac{3.5}{2}x_2^2 + \left(29.4 - \frac{M'}{20}\right)x_2 + M' \ \{0 \le x_2 < 20 \text{ ft}\}$$

Differentiating this equation with respect to *M'* yields

$$\frac{\partial M}{\partial M'} = -\frac{x_2}{20} + 1$$

Substituting *M'* = 0 into the bending moment equation yields

$$M = -\frac{3.5}{2}x_2^2 + 29.4x_2$$

Lastly, we employ Castigliano's second theorem to determine the slope at *C*,

$$\theta = \int_0^L \left(\frac{\partial M}{\partial M'}\right)\frac{M}{EI}dx \rightarrow \theta_C = \int_0^8 (0)\times\left(-\frac{3.5}{2EI}x_1^2\right)dx_1 + \int_0^{20}(-0.05x_2+1)\times\frac{1}{EI}\left(-\frac{3.5}{2}x_2^2 + 29.4x_2\right)dx_2$$

$$\therefore \theta_C = \frac{1}{EI}\times 0 + \frac{793}{EI}$$

$$\therefore \theta_C = \frac{793\times 12^2}{15\times 10^6} = \boxed{0.00761 \text{ rad}}$$

The correct answer is **C**.

ANSWER SUMMARY

Problem 1	**1A**	**B**
	1B	**C**
Problem 2	**2A**	**C**
	2B	**D**
Problem 3		**B**
Problem 4	**4A**	**B**
	4B	Open-ended pb.
Problem 5		**A**
Problem 6	**6A**	**D**
	6B	Open-ended pb.
Problem 7		**A**
Problem 8	**8A**	**A**
	8B	**C**
	8C	Open-ended pb.
Problem 9	**9A**	**B**
	9B	Open-ended pb.
Problem 10		**B**
Problem 11		**D**
Problem 12		**D**
Problem 13		**A**
Problem 14		**C**
Problem 15		**C**

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- GERE, J. and GOODNO, B. (2009). *Mechanics of Materials*. 7th edition. Stamford: Cengage Learning.
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- PHILPOT, T. (2013). *Mechanics of Materials*. 3rd edition. Hoboken: John Wiley and Sons.

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