



# Montogue

## Quiz SM101

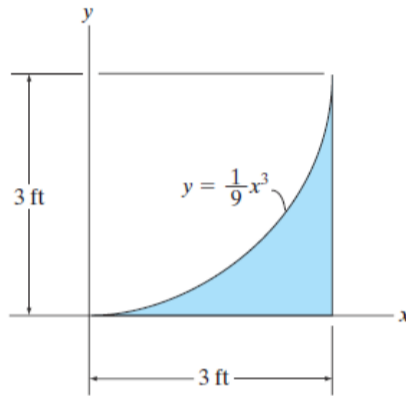
### Centroid and Distributed Forces

Lucas Montogue

#### Problems

##### PROBLEM 1

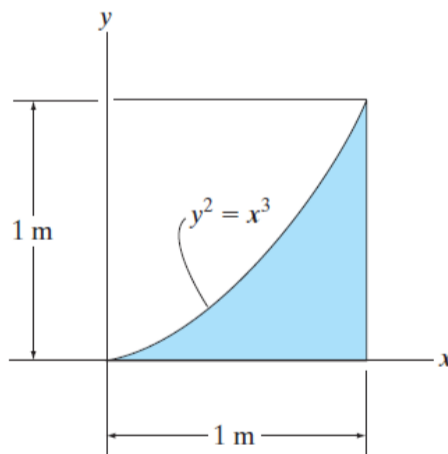
Determine the coordinates of the centroid  $C(\bar{x}, \bar{y})$  of the following area.



- A)  $C(\bar{x}, \bar{y}) = (1.81, 0.86)$  ft
- B)  $C(\bar{x}, \bar{y}) = (1.81, 1.25)$  ft
- C)  $C(\bar{x}, \bar{y}) = (2.39, 0.86)$  ft
- D)  $C(\bar{x}, \bar{y}) = (2.39, 1.25)$  ft

##### PROBLEM 2

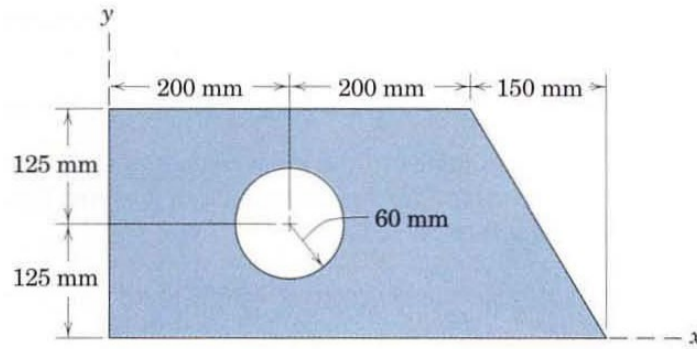
Determine the coordinates of the centroid  $C(\bar{x}, \bar{y})$  of the following area.



- A)  $C(\bar{x}, \bar{y}) = (0.71, 0.31)$  m
- B)  $C(\bar{x}, \bar{y}) = (0.71, 0.56)$  m
- C)  $C(\bar{x}, \bar{y}) = (0.86, 0.31)$  m
- D)  $C(\bar{x}, \bar{y}) = (0.86, 0.56)$  m

**PROBLEM 3** (Merriam & Kraige, 2002, w/ permission)

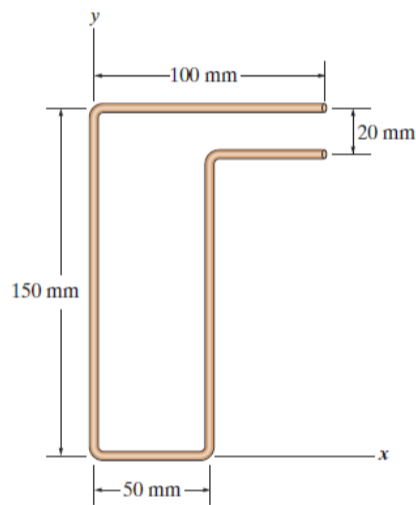
Determine the coordinates of the centroid of the shaded area.



- A)  $C(\bar{x}, \bar{y}) = (243.7, 117.7)$  mm
- B)  $C(\bar{x}, \bar{y}) = (243.7, 141.1)$  mm
- C)  $C(\bar{x}, \bar{y}) = (289.5, 117.7)$  mm
- D)  $C(\bar{x}, \bar{y}) = (289.5, 141.1)$  mm

**PROBLEM 4** (Hibbeler, 2008, w/ permission)

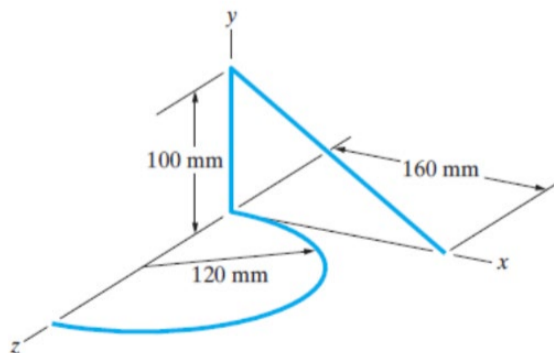
Locate the coordinates of the centroid of the uniform wire bent in the shape shown.



- A)  $C(\bar{x}, \bar{y}) = (34.4, 73.1)$  mm
- B)  $C(\bar{x}, \bar{y}) = (34.4, 85.8)$  mm
- C)  $C(\bar{x}, \bar{y}) = (41.2, 73.1)$  mm
- D)  $C(\bar{x}, \bar{y}) = (41.2, 85.8)$  mm

**PROBLEM 5** (Bedford & Fowler, 2008, w/ permission)

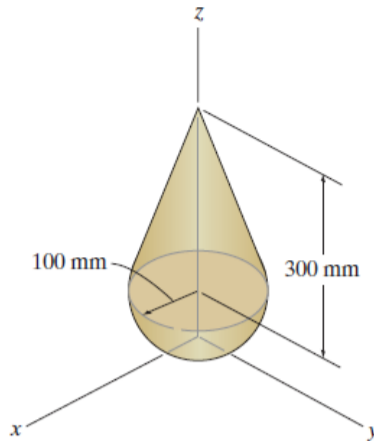
The semicircular part of the line lies in the  $xz$  plane. Determine the centroid of the line. Which of the following is the  $z$ -coordinate of the centroid?



- A)  $\bar{z} = 39$  mm
- B)  $\bar{z} = 47$  mm
- C)  $\bar{z} = 56$  mm
- D)  $\bar{z} = 68$  mm

**PROBLEM 6**

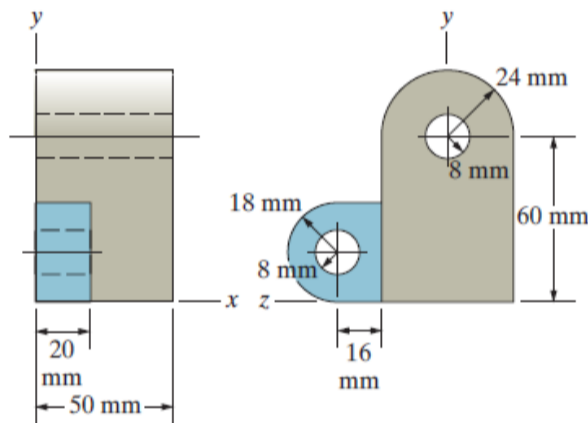
Locate the center of mass  $\bar{z}$  of the assembly shown. The hemisphere and the cone are made from materials having densities of  $8 \text{ Mg/m}^3$  and  $4 \text{ Mg/m}^3$ , respectively.



- A)  $\bar{z} = 90 \text{ mm}$
- B)  $\bar{z} = 111 \text{ mm}$
- C)  $\bar{z} = 125 \text{ mm}$
- D)  $\bar{z} = 151 \text{ mm}$

**PROBLEM 7** (Bedford & Fowler, 2008, w/ permission)

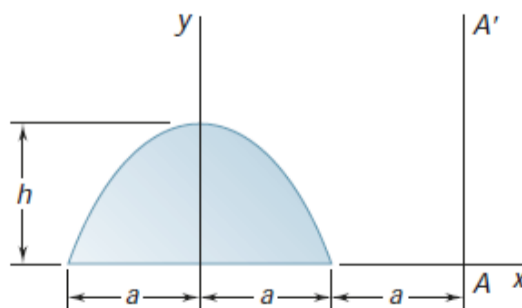
Two views of a machine element are shown. Determine the centroid of its volume. The  $y$ -coordinate of the centroid is:



- A)  $\bar{y} = 17.8 \text{ mm}$
- B)  $\bar{y} = 25.5 \text{ mm}$
- C)  $\bar{y} = 36.6 \text{ mm}$
- D)  $\bar{y} = 48.1 \text{ mm}$

**PROBLEM 8 A** (Beer et al., 2013)

Determine the volume of the solid generated by rotating the parabolic area shown about the  $x$ -axis.



- A)  $V = 4\pi ah^2/5$
- B)  $V = 14\pi ah^2/15$
- C)  $V = \pi ah^2$
- D)  $V = 16\pi ah^2/15$

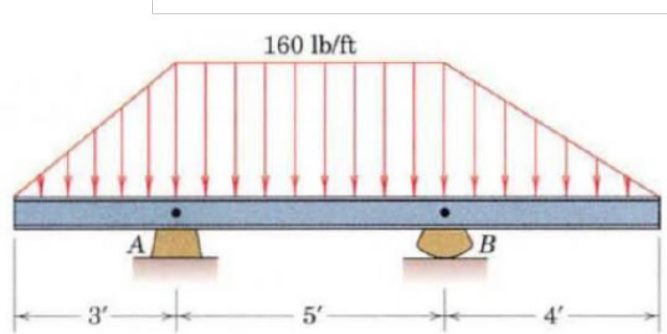
**PROBLEM 8B**

Determine the volume of the solid generated by rotating the preceding parabolic area about axis  $AA'$ .

- A)  $V = 14\pi ah^2/3$
- B)  $V = 5\pi a^2 h$
- C)  $V = 16\pi ah^2/3$
- D)  $V = 6\pi ah^2$

**PROBLEM 9** (Merriam & Kraige, 2002, w/ permission)

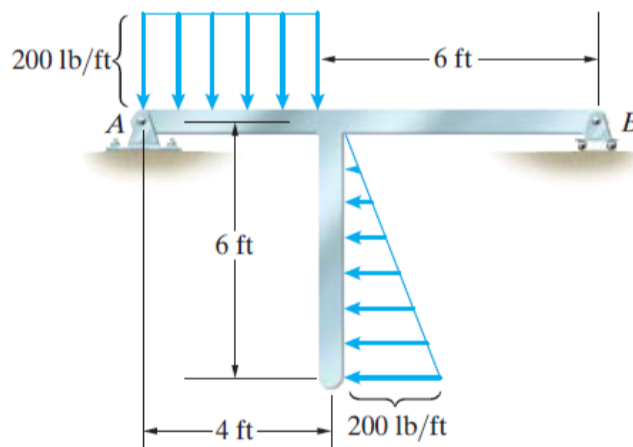
Determine the reactions at A (vertical component) and B for the loaded beam shown.



- A)  $A_y = 525$  lb and  $B_y = 651$  lb
- B)  $A_y = 525$  lb and  $B_y = 757$  lb
- C)  $A_y = 603$  lb and  $B_y = 651$  lb
- D)  $A_y = 603$  lb and  $B_y = 757$  lb

**PROBLEM 10** (Bedford & Fowler, 2008, w/ permission)

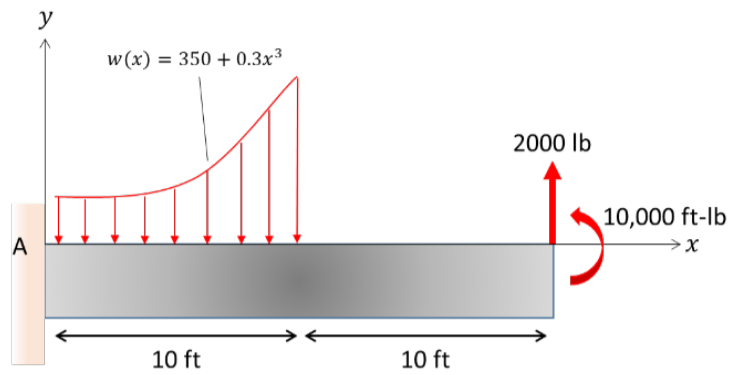
Determine the reactions at A (resultant) and B for the loaded member shown.



- A)  $A = 656$  lb and  $B_y = 300$  lb
- B)  $A = 656$  lb and  $B_y = 400$  lb
- C)  $A = 721$  lb and  $B_y = 300$  lb
- D)  $A = 721$  lb and  $B_y = 400$  lb

**PROBLEM 11 A** (Bedford & Fowler, 2008, w/ permission)

Suppose that the distributed load acting on the beam from  $x = 0$  to  $x = 10$  ft is given by  $w(x) = 350 + 0.3x^3$  lb/ft. Determine the clockwise moment about point A exerted by the distributed load.



- A)  $M_w = 17,600$  lb-ft
- B)  $M_w = 19,300$  lb-ft
- C)  $M_w = 21,400$  lb-ft
- D)  $M_w = 23,500$  lb-ft

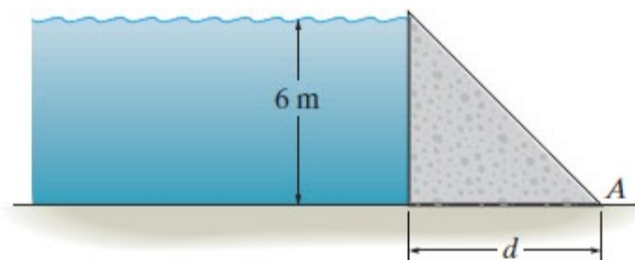
**PROBLEM 11 B**

Considering the system in the previous problem, determine the reaction at the fixed support.

- A)  $A = 1880$  lb
- B)  $A = 2250$  lb
- C)  $A = 2540$  lb
- D)  $A = 2770$  lb

**PROBLEM 12** (Hibbeler, 2010, w/ permission)

The concrete gravity dam is held in place by its own weight. If the density of concrete is  $\rho_c = 2.5$  Mg/m<sup>3</sup>, and water has a density of  $\rho_w = 1.0$  Mg/m<sup>3</sup>, determine the smallest dimension that will prevent the dam from overturning about end A.



- A)  $d = 2.31$  m
- B)  $d = 2.68$  m
- C)  $d = 3.05$  m
- D)  $d = 3.33$  m

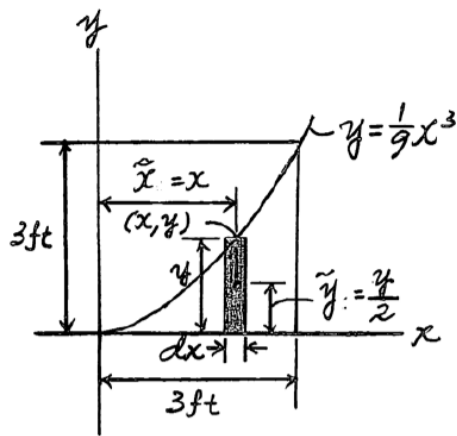
**Solutions**

**P.1 ■ Solution**

The abscissa  $\bar{x}$  and the ordinate  $\bar{y}$  of the area are given by, respectively,

$$\bar{x} = \frac{\int \tilde{x}dW}{\int dW}; \bar{y} = \frac{\int \tilde{y}dW}{\int dW}$$

Consider the shaded infinitesimal area element shown below.



Clearly, the area of the element is

$$dA = y dx = \frac{1}{9} x^3 dx$$

The centroid of the element is located at  $\tilde{x} = x$  and  $\tilde{y} = y/2 = (1/18)x^3$ . Substituting these quantities into the expressions introduced above and integrating, it follows that

$$\bar{x} = \frac{\int_A \tilde{x} dW}{\int_A dW} = \frac{\int_0^3 x \left( \frac{1}{9} x^3 dx \right)}{\int_0^3 \frac{1}{9} x^3 dx} = \frac{\int_0^3 \frac{1}{9} x^4 dx}{\int_0^3 \frac{1}{9} x^3 dx} = \frac{\int_0^3 x^4 dx}{\int_0^3 x^3 dx} = \frac{48.6}{20.3} = 2.39 \text{ ft}$$

$$\bar{y} = \frac{\int_A \tilde{y} dW}{\int_A dW} = \frac{\int_0^3 \left( \frac{1}{18} x^3 \right) \left( \frac{1}{9} x^3 dx \right)}{\int_0^3 \frac{1}{9} x^3 dx} = \frac{\int_0^3 \frac{1}{18} x^6 dx}{\int_0^3 x^3 dx} = \frac{17.4}{20.3} = 0.86 \text{ ft}$$

We conclude that the coordinates of the centroid are

$$C(\bar{x}, \bar{y}) = C(2.39, 0.86) \text{ ft}$$

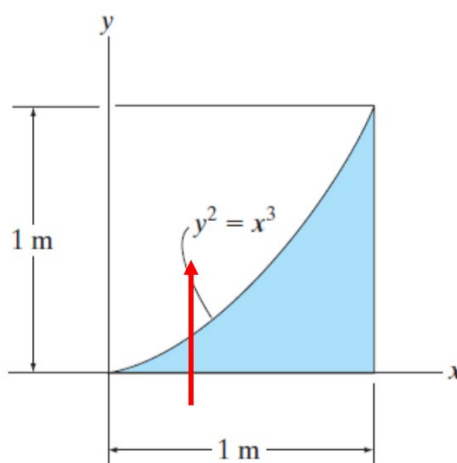
■ The correct answer is **C**.

## P.2 ■ Solution

The abscissa  $\bar{x}$  and the ordinate  $\bar{y}$  of the area are given by

$$\bar{x} = \frac{\int \tilde{x} dW}{\int dW}; \bar{y} = \frac{\int \tilde{y} dW}{\int dW}$$

Instead of using infinitesimal area elements, we could propose a slightly different approach. One way to set up the bounds in the integrals above is to draw hypothetical lines that start perpendicularly to one of the axes and end when they cross one of the boundaries of the surface. For instance, the boundaries of the integral in the relative to the  $y$ -direction are easily obtained by drawing a line parallel to the  $y$ -axis, as shown.



The line “enters” the surface at the horizontal axis, that is, at  $y = 0$ ; therefore,  $y = 0$  is the lower bound of the integral. The line then penetrates the surface and “exits” it at the curve  $y^2 = x^3$ , which corresponds to an ordinate  $y = x^{3/2}$ ; therefore,  $y = x^{3/2}$  is the upper bound of the integral. Then, we are ready to write part of one of the integrals in the foregoing example. Namely,

$$\int_A dW = \int_{x=x_1}^{x=x_2} \int_{y=0}^{y=x^{3/2}} dy dx$$

Once the limits for the first integral have been established by the “hypothetical line” technique, the second integral can be set up quite easily. All we have to do is determine the extreme coordinates encompassed by the surface in the horizontal or vertical, depending on whether we are looking for the boundaries in the  $x$ - or  $y$ -axis, respectively. In the present example, it is easy to see that the area extends itself from  $x = 0$  to  $x = 1$  m; these are the lower and upper bounds of the second integral, respectively. Thus, we can write

$$\int_A dW = \int_{x=0}^{x=1} \int_{y=0}^{y=x^{3/2}} dy dx = \int_0^1 \int_0^{x^{3/2}} dy dx$$

The integral is now ready to be evaluated. The abscissa  $\bar{x}$  of the centroid can be obtained from

$$\bar{x} = \frac{\int_0^1 \int_0^{x^{3/2}} x dy dx}{\int_0^1 \int_0^{x^{3/2}} dy dx}$$

This integral can be easily obtained with a programmable calculator or a CAS such as Mathematica, in which case we could apply the command *Integrate*,

$$\frac{\text{Integrate}[\text{Integrate}[x, \{y, 0, x^{3/2}\}], \{x, 0, 1\}]}{\text{Integrate}[\text{Integrate}[1, \{y, 0, x^{3/2}\}], \{x, 0, 1\}]}$$

The result is  $\bar{x} = 0.71$  m. Proceeding similarly with the vertical coordinate, we have the integrals

$$\bar{y} = \frac{\int_0^1 \int_0^{x^{3/2}} y dy dx}{\int_0^1 \int_0^{x^{3/2}} dy dx}$$

which can be easily evaluated with a programmable calculator or a program such as Mathematica,

$$\frac{\text{Integrate}[\text{Integrate}[y, \{y, 0, x^{3/2}\}], \{x, 0, 1\}]}{\text{Integrate}[\text{Integrate}[1, \{y, 0, x^{3/2}\}], \{x, 0, 1\}]}$$

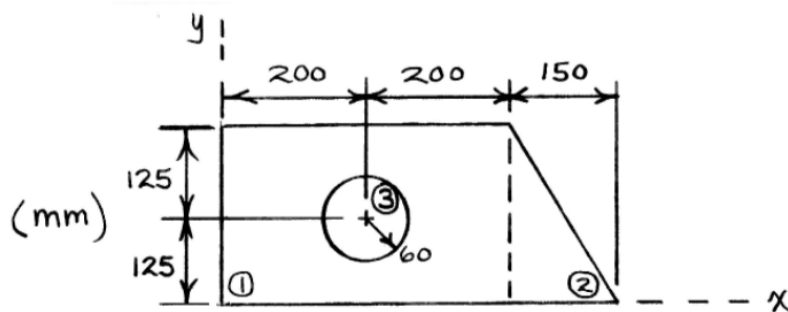
The result is  $\bar{y} = 0.31$  m. We conclude that the coordinates of the centroid are

$$C(\bar{x}, \bar{y}) = (0.71, 0.31) \text{ m}$$

■ The correct answer is **A**.

### P.3 ■ Solution

Consider the following sketch of the section.



The area can be divided into three elementary parts: a rectangle (component 1), a triangle (component 2), and a hollow circle (component 3). The following table is prepared.

Component	Area (mm <sup>2</sup> )	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$A\bar{x}$ (mm)	$A\bar{y}$ (mm)
Rect. 1	100 (10 <sup>3</sup> )	200	125	20 (10 <sup>6</sup> )	12.5 (10 <sup>6</sup> )
Tri. 2	18.75 (10 <sup>3</sup> )	450	83.33	8.44 (10 <sup>6</sup> )	1.56 (10 <sup>6</sup> )
Circ. 3	-11.31 (10 <sup>3</sup> )	200	125	-2.26 (10 <sup>6</sup> )	-1.41 (10 <sup>6</sup> )
<i>Total</i>	$\Sigma A = 107.44 (10^3)$			26.18 (10 <sup>6</sup> )	12.65 (10 <sup>6</sup> )

Because circle 3 is a hole, not a solid region, quantities associated with it have a negative sign. Then, the coordinates of the centroid  $C(\bar{x}, \bar{y})$  can be obtained from the following expressions,

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{26.18 \times 10^6}{107.44 \times 10^3} = 243.7 \text{ mm}$$

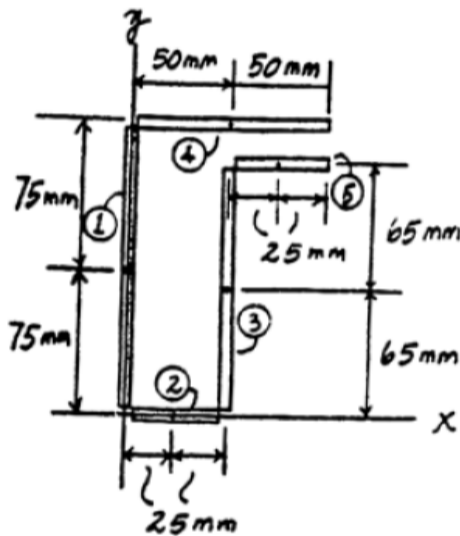
$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{12.65 \times 10^6}{107.44 \times 10^3} = 117.7 \text{ mm}$$

The centroid is located at  $C(\bar{x}, \bar{y}) = (243.7, 117.7)$  mm.

■ The correct answer is **A**.

#### P.4 ■ Solution

Consider the following sketch of the composite wire.



The lengths of each segment and their respective centroids are tabulated below.

Component	Length (mm)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}L$ (mm)	$\bar{y}L$ (mm)
1	150	0	75	0	11,250
2	50	25	0	1250	0
3	130	50	65	6500	8450
4	100	50	150	5000	15,000
5	50	75	130	3750	6500
<i>Total</i>	$\Sigma L = 480$			16,500	41,200

Now, the coordinates of the centroid can be obtained from the expressions

$$\bar{x} = \frac{\Sigma L\bar{x}}{\Sigma L} = \frac{16,500}{480} = 34.4 \text{ mm}$$

$$\bar{y} = \frac{\Sigma L\bar{y}}{\Sigma L} = \frac{41,200}{480} = 85.8 \text{ mm}$$

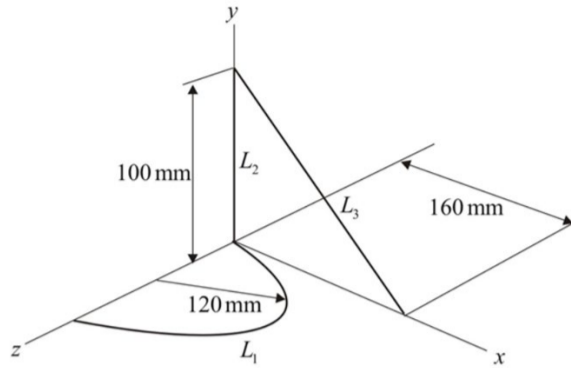
Hence, the centroid is located at  $C(\bar{x}, \bar{y}) = (34.4, 85.8)$  mm.

■ The correct answer is **B**.



### P.5 ■ Solution

The bar is divided into three segments, as shown.



Line  $L_1$  has length  $L_1 = 120\pi = 377$  mm. The  $x$ -coordinate of the centroid of this line segment is  $\bar{x}_1 = 2r/\pi = 2 \times 120/\pi = 76.4$  mm, and the other two coordinates are straightforward, so we ultimately obtain  $(\bar{x}_1, \bar{y}_1, \bar{z}_1) = (76.4, 0, 120)$  mm. Line  $L_2$  is also simple, having a length  $L_2 = 100$  mm and centroid  $(\bar{x}_2, \bar{y}_2, \bar{z}_2) = (0, 50, 0)$  mm. Finally, line  $L_3$  has length  $L_3 = \sqrt{100^2 + 160^2} = 188.7$  mm and centroid  $(\bar{x}_3, \bar{y}_3, \bar{z}_3) = (80, 50, 0)$  mm. The total length of the composite is  $\Sigma L = L_1 + L_2 + L_3 = 377 + 100 + 188.7 = 665.7$  mm. The  $x$ -coordinate of the centroid of the composite is given by

$$\bar{x} = \frac{\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3}{\Sigma L} = \frac{76.4 \times 377 + 0 \times 100 + 80 \times 188.7}{665.7} = 65.9 \text{ mm}$$

The  $y$ -coordinate of the centroid of the composite, in turn, follows as

$$\bar{y} = \frac{\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3}{\Sigma L} = \frac{0 \times 377 + 50 \times 100 + 50 \times 188.7}{665.7} = 21.7 \text{ mm}$$

The  $z$ -coordinate of the centroid of the composite is

$$\bar{z} = \frac{\bar{z}_1 L_1 + \bar{z}_2 L_2 + \bar{z}_3 L_3}{\Sigma L} = \frac{120 \times 377 + 0 \times 100 + 0 \times 188.7}{665.7} = 68 \text{ mm}$$

Thus, the centroid of the line is such that

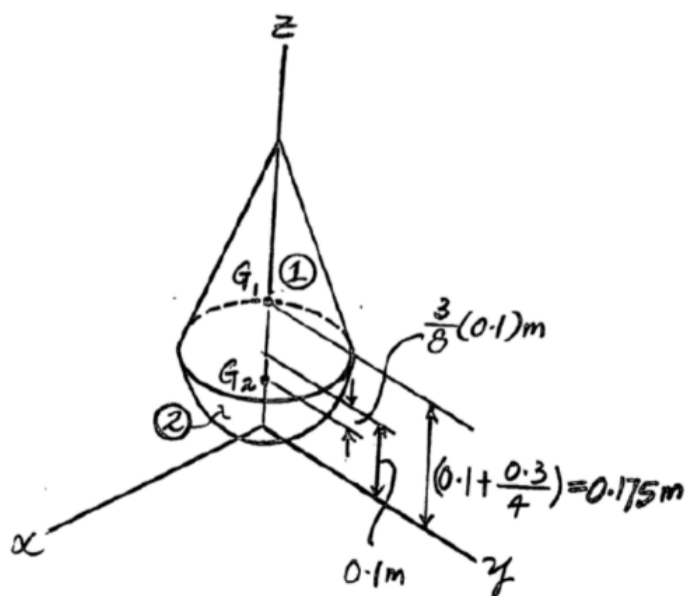
$$C(\bar{x}, \bar{y}, \bar{z}) = C(65.9, 21.7, 68) \text{ mm}$$

The  $z$ -coordinate of the centroid is  $\bar{z} = 68$  mm.

■ The correct answer is **D**.

### P.6 ■ Solution

The center of mass of each composite segment is shown in the figure below.



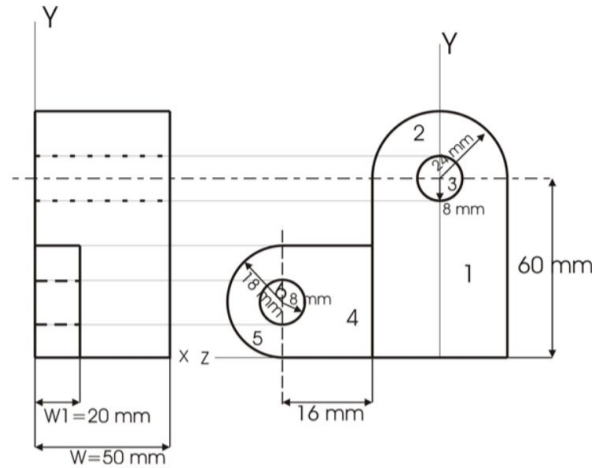
The center of gravity follows from the usual formula,

$$\bar{z} = \frac{\Sigma \bar{z}m}{\Sigma m} = \frac{0.175 \times 4000 \left( \frac{1}{3} \pi \times 0.1^2 \times 0.3 \right) + \left( 0.1 - \frac{3}{8} \times 0.1 \right) \times 8000 \left( \frac{2}{3} \pi \times 0.1^3 \right)}{4000 \left( \frac{1}{3} \pi \times 0.1^2 \times 0.3 \right) + 8000 \left( \frac{2}{3} \pi \times 0.1^3 \right)} = 0.1107 = \boxed{111 \text{ mm}}$$

■ The correct answer is **B**.

### P.7 ■ Solution

The following figure specifies the numbering of each geometric shape involved.



The volumes and coordinates of the centroid for these shapes are summarized in the following table.

Shape	Volume ( $V_i$ )	Centroid x-coordinate ( $\bar{x}_i$ )	Centroid y-coordinate ( $\bar{y}_i$ )	Centroid z-coordinate ( $\bar{z}_i$ )
Rectangular prism 1	$60 \times 48 \times 50$ $= 144,000 \text{ mm}^3$	25 mm	30 mm	0
Semi-cylinder 2	$\frac{\pi r^2 \times W}{2}$ $= \pi \times 24^2 \times 50/2$ $= 45,240 \text{ mm}^3$	25 mm	$60 + \frac{4r}{3\pi} = 70.2 \text{ mm}$	0
Cylinder 3 (hole)	$-\pi r^2 W$ $= -\pi \times 8^2 \times 50$ $= -10,050 \text{ mm}^3$	25 mm	60 mm	0
Rectangular prism 2	$16 \times 36 \times 20$ $= 11,520 \text{ mm}^3$	10 mm	18 mm	$24 + 8 = 32 \text{ mm}$
Semi-cylinder 5	$\frac{\pi r^2 \times W_1}{2}$ $= \pi \times 18^2 \times 20/2$ $= 10,180 \text{ mm}^3$	10 mm	18 mm	$24 + 16 \times \frac{4r_2}{3\pi}$ $= 47.6 \text{ mm}$
Cylinder 6 (hole)	$-\pi r_4^2 W_1$ $= -\pi \times 8^2 \times 20$ $= -4020 \text{ mm}^3$	10 mm	18 mm	40 mm

The total volume is  $\Sigma V_i = 196,870 \text{ mm}^3$ . Then, the x-coordinate of the centroid is

$$\bar{x} = \frac{\bar{x}_1 V_1 + \bar{x}_2 V_2 + \bar{x}_3 V_3 + \bar{x}_4 V_4 + \bar{x}_5 V_5 + \bar{x}_6 V_6}{\Sigma V_i}$$

$$\therefore \bar{x} = \frac{25 \times 144,000 + 25 \times 45,240 - 25 \times 10,050 + 10 \times 11,520 + 10 \times 10,180 - 10 \times 4020}{196,870} = 23.7 \text{ mm}$$

Next, the y-coordinate of the centroid is

$$\bar{y} = \frac{\Sigma \bar{y}_i V_i}{\Sigma V_i} = \frac{30 \times 144,000 + 70.2 \times 45,240 - 60 \times 10,050 + 18 \times 11,520 + 18 \times 10,180 - 18 \times 4020}{196,870} = 36.6 \text{ mm}$$

Finally, the z-coordinate of the centroid is

$$\bar{z} = \frac{\Sigma \bar{z}_i V_i}{\Sigma V_i} = \frac{0 \times 144,000 + 0 \times 45,240 - 0 \times 10,050 + 32 \times 11,520 + 47.6 \times 10,180 - 40 \times 4020}{196,870} = 3.5 \text{ mm}$$

The y-coordinate of the centroid is  $\bar{y} = 36.6 \text{ mm}$  and the overall centroid  $C(\bar{x}, \bar{y}, \bar{z})$  is such that

$$C(\bar{x}, \bar{y}, \bar{z}) = (23.7, 36.6, 3.5) \text{ mm}$$

■ The correct answer is **C**.

**P.8 ■ Solution**

**Part A:** The parabola can be described by the equation

$$y = h \left( 1 - \frac{x^2}{a^2} \right)$$

The area of the parabola is determined as

$$A = \int_{-a}^a h \left( 1 - \frac{x^2}{a^2} \right) dx = \frac{4ah}{3}$$

The height of the centroid  $\bar{y}$  is also be obtained via integration,

$$\bar{y} = \frac{\int_{-a}^a \int_0^{h \left( 1 - \frac{x^2}{a^2} \right)} y \, dy dx}{\int_{-a}^a \int_0^{h \left( 1 - \frac{x^2}{a^2} \right)} dy dx} = \frac{2h}{5}$$

Then, the volume of the solid that results from rotation about an axis is obtained by dint of the second theorem of Pappus-Guldinus,

$$V = 2\pi \bar{y} A = 2\pi \times \frac{2h}{5} \times \frac{4ah}{3} = \boxed{\frac{16}{15} \pi ah^2}$$

■ The correct answer is **D**.

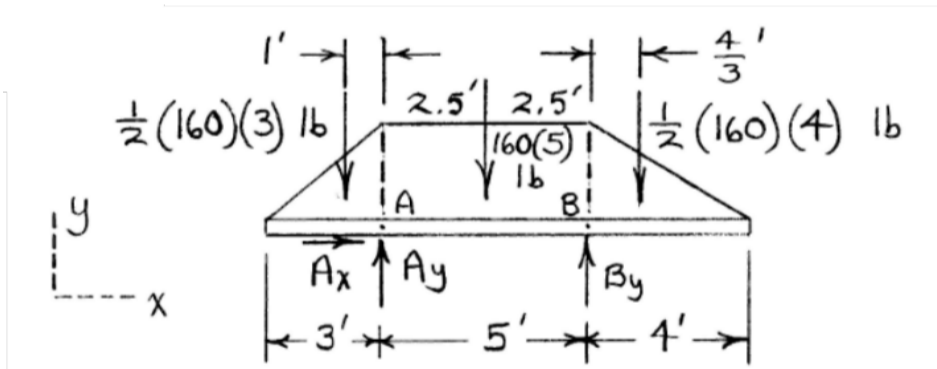
**Part B:** The volume of the solid obtained when the surface is rotated about line AA' is established by substituting  $\bar{y}$  with the perpendicular distance from the centroid to the new axis of revolution. Accordingly,

$$V = 2\pi \bar{y} A = 2\pi \times 2a \times \frac{4}{3} ah = \boxed{\frac{16}{3} \pi a^2 h}$$

■ The correct answer is **C**.

**P.9 ■ Solution**

The free body diagram for the beam is shown below.



The distributed force can be divided in three parts: the triangular load to the left of support A, the rectangular load to located between the two supports, and the triangular load to the right of support B. The force due to the first part has a value of  $(1/2) \times 160 \times 3 = 240$  lb and is concentrated at the centroid of the force distribution triangle, 1 ft to the left of support A; the force due to the second part has a value of  $160 \times 5 = 800$  lb and is located at the centroid of the force distribution rectangle, 2.5 ft away from each of the two supports; finally, the force due to the third distributed load has an intensity of  $(1/2) \times 160 \times 4 = 320$  lb and is concentrated 1.33 ft away from support B. Replacing the distributed loads with their corresponding forces and taking moments about point A, we obtain

$$\sum M_A = 0 \rightarrow 240 \times 1 - 800 \times 2.5 - 320 \times 6.33 + B_y \times 5 = 0$$

$$\therefore \boxed{B_y = 757 \text{ lb}}$$

Next, summing forces in the y-direction, we obtain

$$\Sigma F_y = 0 \rightarrow A_y + 757 - 240 - 800 - 320 = 0$$

$$\therefore \boxed{A_y = 603 \text{ lb}}$$

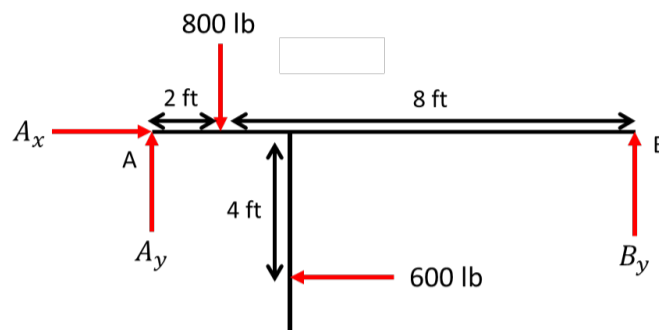
Finally, force equilibrium in the x-direction is elementary,

$$\Sigma F_x = 0 \rightarrow A_x = 0$$

■ The correct answer is **D**.

### P.10 ■ Solution

Replacing the distributed loads with their corresponding concentrated forces, we obtain the following free body diagram.



Consider force equilibrium in the x-direction,

$$\Sigma F_x = 0 \rightarrow A_x - 600 = 0$$

$$A_x = 600 \text{ lb}$$

Next, consider equilibrium of moments relative to point A,

$$\Sigma M_A = 0 \rightarrow -800 \times 2 - 600 \times 4 + 10B_y = 0$$

$$\therefore 10B_y = 1600 + 2400 = 4000$$

$$\therefore B_y = \frac{4000}{10} = \boxed{400 \text{ lb}}$$

Then, consider force equilibrium in the y-direction,

$$\Sigma F_y = 0 \rightarrow A_y + B_y - 800 = 0$$

$$\therefore A_y + 400 - 800 = 0$$

$$\therefore A_y = 400 \text{ lb}$$

The resultant reaction at A follows as

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{600^2 + 400^2} = \boxed{721 \text{ lb}}$$

■ The correct answer is **D**.

### P.11 ■ Solution

**Part A:** The downward force exerted by the distributed load can be obtained by integration,

$$R = \int_0^{10} w dx = \int_0^{10} (350 + 0.3x^3) dx = (350x + 0.075x^4) \Big|_0^{10} = 4250 \text{ lb}$$

The moment about point A exerted by the distributed load follows as

$$M_w = \int_0^{10} xw dx = \int_0^{10} x(350 + 0.3x^3) dx = (175x^2 + 0.06x^5) \Big|_0^{10} = \boxed{23,500 \text{ lb-ft}}$$

■ The correct answer is **D**.

**Part B:** Suppose that  $A_x$  and  $A_y$  are the horizontal and vertical components of the reaction at A, respectively. The former is elementary,

$$\Sigma F_x = 0 \rightarrow A_x = 0$$

The latter follows from force equilibrium in the y-direction,

$$\Sigma F_y = 0 \rightarrow A_y - 4250 + 2000 = 0$$

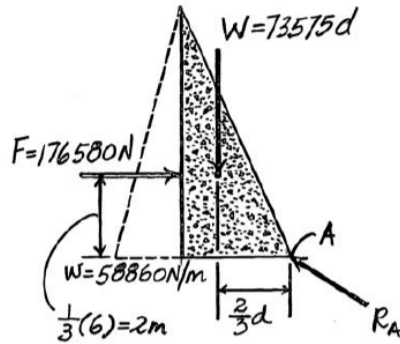
$$\therefore A_y = 4250 - 2000 = \boxed{2250 \text{ lb}}$$

Hence,  $A = A_y = 2250 \text{ lb}$ .

■ The correct answer is **B**.

### P.12 ■ Solution

Consider a 1-m width of dam, as shown



The weight of water per unit length,  $w$ , is given by

$$w = 1000 \times 9.81 \times 6 \times 1 = 58,860 \text{ N/m}$$

The total force due to the water,  $F$ , follows as

$$F = \frac{1}{2} \times 58,860 \times 6 = 176,580 \text{ N}$$

The weight of the dam, in turn, is such that

$$W = \frac{1}{2} \times d \times 6 \times 1 \times 2500 \times 9.81 = 73,580d$$

Now, taking moments about point A, we obtain

$$\Sigma M_A = 0 \therefore -176,580 \times \left(\frac{1}{3} \times 6\right) + 73,580 \times d \times \left(\frac{2}{3}d\right) = 0$$

$$\therefore \boxed{d = 2.68 \text{ m}}$$

In order for the dam not to overturn about point A, dimension  $d$  should be at least equal to 2.68 m.

■ The correct answer is **B**.

## Answer Summary

Problem 1		<b>C</b>
Problem 2		<b>A</b>
Problem 3		<b>A</b>
Problem 4		<b>B</b>
Problem 5		<b>D</b>
Problem 6		<b>B</b>
Problem 7		<b>C</b>
Problem 8	8A	<b>D</b>
	8B	<b>C</b>
Problem 9		<b>D</b>
Problem 10		<b>D</b>
Problem 11	11A	<b>D</b>
	11B	<b>B</b>
Problem 12		<b>B</b>

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