

# Montogue

## QUIZ MS301 Ceramics and Glasses Lucas Montogue

### PROBLEMS

#### Problem 1

Most ceramics fracture

- A) in ductile mode after some plastic deformation.
- B) in ductile mode with no plastic deformation.
- C) in brittle mode after some plastic deformation.
- D) in brittle mode with no plastic deformation.

#### Problem 2 (Modified from Surendranathan, 2015)

Ten rectangular test specimens of alumina were tested in three-point bending. The bars were 1 cm wide and 0.5 cm high and were tested over a 5-cm span. The failure loads for each specimen in ascending are given below. Calculate the average modulus of rupture for this group of samples.

Sample	Failure load (kN)
1	6.67
2	7.11
3	9.23
4	11.4
5	6.54
6	7.55
7	9.55
8	6.74
9	10.8
10	9.4

- A)  $\overline{MOR} = 10.2$  MPa
- B)  $\overline{MOR} = 25.5$  MPa
- C)  $\overline{MOR} = 39.1$  MPa
- D)  $\overline{MOR} = 50.4$  MPa

#### Problem 3.1

Analysis of a sample of specimens for a certain ceramic revealed the following data. Fit the data in a two-parameter Weibull distribution and answer: what is the probability of survival at a stress level of 180 MPa?

Sample	Strength (MPa)
1	215
2	200
3	206
4	222
5	216
6	187
7	186
8	172
9	211
10	175
11	205
12	195

- A)  $P = 56.7\%$
- B)  $P = 68.5\%$
- C)  $P = 88.5\%$
- D)  $P = 98.4\%$

### ■ Problem 3.2

What is the probability of survival if the specimen stress area is one-fifth of the actual component area?

- A)  $P = 16.5\%$
- B)  $P = 28.9\%$
- C)  $P = 40.1\%$
- D)  $P = 53.6\%$

### ■ Problem 4.1 (Modified from Surendranathan, 2015)

The tensile fracture strengths in MPa of chemical vapor deposited silicon carbide are listed below. Calculate the mean strength, the standard deviation of the strength values, and the survival probability of each specimen. Then, plot  $-\ln[-\ln(1/S)]$  versus  $\ln \sigma$ , where  $S$  is the probability of survival and  $\sigma$  is tensile strength. Determine the Weibull modulus from the plot.

Sample	Tensile Strength (MPa)
1	223
2	200
3	309
4	326
5	278
6	300
7	140
8	179
9	268
10	342
11	260
12	297
13	228
14	360
15	388
16	249

### ■ Problem 4.2

Estimate the design stress, i.e., the stress value that corresponds to a 99.9% probability of survival.

- A)  $\sigma = 20.6$  MPa
- B)  $\sigma = 39.8$  MPa
- C)  $\sigma = 60.1$  MPa
- D)  $\sigma = 82.0$  Mpa

## ■ Problem 5

Regarding aspects of ceramic technology, true or false?

- 1.( ) In the Weibull distribution, often used to describe the behavior of brittle materials such as ceramics, one of the specifications is the shape parameter, also known as the Weibull modulus. The larger this parameter, the wider is the range of applied stresses at which failures occurred.
- 2.( ) Description of brittle material strength by Weibull distribution is a “weakest link theory” in that the most serious flaw in the material will control the strength, much like a chain breaking if its weakest link fails. The most serious flaw is always the largest one.
- 3.( ) The strength of ceramic materials is most often characterized by flexure (bend) testing rather than compressive or tensile testing. The stress at failure in such a test is commonly referred to as the modulus of rupture (MOR) and constitutes a specific property of the ceramic being investigated, independent of the size of the specimen and the loading configuration.
- 4.( ) Polycrystalline ceramics are found to be tougher than single crystalline ones. For example, polycrystalline alumina has a fracture toughness substantially greater than that of single crystalline alumina.

**5.( )** Single crystals of noncubic structures have equal thermal expansion along different crystallographic directions and are referred to as isotropic. In contrast, single crystals of noncubic structures have different thermal expansion along different crystallographic directions and are referred to as anisotropic. Some crystalline ceramics have such large differences in thermal expansion along different crystallographic directions that the expansion in one axis may cause the crystal to physically contract along the perpendicular axis in order to accommodate the motion. In practical terms, this implies that the linear thermal expansion coefficient in a given direction may be negative.

**6.( )** Alumina ( $\text{Al}_2\text{O}_3$ ) is used in ceramic products in varying amounts. However, discussion is usually limited to high alumina, which refers to those bodies containing 80 percent or more aluminum oxide. The most common aluminas are those containing 85, 90, 94, 96, 99, 99.8, and 99.9% purity. Properties such as thermal conductivity and dielectric parameters vary sensibly with alumina content. Indeed, the greater the alumina content, the lower will be the thermal conductivity, tensile strength, and Young's modulus of the alumina. This is why low- $\text{Al}_2\text{O}_3$ -content alumina is preferred in sensible applications such as aircraft engine components.



**7.( )** The brittleness index was introduced in the 1970s as a measure of the competition between plastic and brittle deformation modes. It is given by the ratio of hardness to fracture toughness and is usually presented in  $\mu\text{m}^{-1/2}$ . Consider alumina, which has a Vickers hardness of 15.7 GPa and a fracture toughness of 3.5  $\text{MPa}\cdot\text{m}^{1/2}$ . Its brittleness index is greater than 4  $\mu\text{m}^{-1/2}$ .

**8.( )** Although alumina without impurities is a colorless solid, addition of transition metal ions leads to spectacular colors. For instance, the addition of a few hundredths of one percent of iron and titanium leads to the formation of ruby, a precious gemstone.

**9.( )** Properties that set silicon carbide (SiC) apart from other ceramic substrates include its high thermal conductivity and low thermal expansion. Another boon for electrical applications is SiC's low dielectric constant, which is substantially lower than those of, say, aluminum oxide or aluminum nitride.



**10.( )** Another feature of silicon carbide is its high resistance to oxidation and corrosion. Silicon carbide is resistant to most chemicals, efficiently resisting even strong acid solutions such as aqua regia and fuming nitric acid.

**11.( )** Another common carbide ceramic is boron carbide. Joly reported the preparation of boron carbide in 1883, labeling the product as  $\text{B}_3\text{C}$ . Closer to the end of the nineteenth century, Moisson labeled the compound as  $\text{B}_6\text{C}$ . Some 50 years later, Ridgeway postulated that the stoichiometry was in fact 4 to 1, and hence  $\text{B}_4\text{C}$  was the correct notation. Modern methods have shown that Ridgeway was correct.

**12.( )** Boron carbide is one of the hardest known materials. In fact, the hardness of boron carbide at temperatures higher than 1300°C or so is greater than those of cubic boron nitride and even diamond.

**13.( )** Boron nitride occurs in hexagonal (h-BN), cubic (c-BN), and wurtzitic (w-BN) crystalline structures, among others. Cubic and wurtzitic BN are observed to have high anisotropy in mechanical and electrical properties, while hexagonal BN has a more homogeneous behavior.

**14.( )** Cubic boron nitride has a hardness inferior to that of diamond, but can be said to outstrip this material in grinding ferrous alloys, including most steels, and offering a more stable performance at high temperatures (i.e., greater than, say, 700°C).

**15.( )** The Knoop hardness of diamond is greater than that of silicon carbide, which in turn is greater than that of alumina, which in turn is greater than that of cubic boron nitride.

**16.( )** Although the thermal conductivity of aluminum nitride (AlN) is lower than that of beryllia (BeO) at low temperatures (say, lower than 20°C), their thermal conductivities become closer as temperature increases. There is a crossover value at which the thermal conductivity of AlN definitively surpasses that of BeO, and this trend is maintained at higher temperatures.

**17.( )** Beryllium oxide has many characteristics that make it desirable for numerous applications, including an exceptional thermal conductivity, comparable to that of some of the best conductive metals, and the fact that this ceramic is inherently nontoxic, especially in the form of powder or dust.

**18.( )** The availability of fine, pure mullite powders and novel processing routes have made it possible to obtain dense polycrystalline mullite with higher deformation resistance and hardness at higher temperatures than most other ceramics, including alumina. With a low thermal conductivity and thermal expansion coefficient, mullite is an excellent ceramic choice for refractory applications.

**19.( )** Zirconia occurs in three low-pressure phases: monoclinic, tetragonal, and cubic, which are stable at increasingly higher temperatures. Although the room temperature stability of these ceramics is quite low, eliminating impurities such as Y<sub>2</sub>O<sub>3</sub> causes the ceramic to become progressively more stable at mild thermal environments.

**20.( )** In a characterization experiment, it was verified that a beam of light crossed a 1-m thick homogeneous layer of ceramic/glass perpendicularly to its surface in 5.93 μs. With reference to the following table, we surmise that the material in question is a sapphire.

Material	Refractive Index
Boron nitride	2.0
Borosilicate glass	1.47
Diamond	2.42
Fused silica	1.46
Sapphire	1.78

## Problem 6

The following are possible definitions for the four conventional temperature/viscosity levels used in glass production. Associate each point with the correct characteristic.

Point	Possible characteristic
<b>P.</b> Working Point	<b>1.</b> At this viscosity, stresses that form in the glass object are relieved in a matter of hours; viscosity = 10 <sup>14.5</sup> P
<b>Q.</b> Softening Point	<b>2.</b> Viscosity at which a machine is able to work on glass without losing control; viscosity = 10 <sup>4</sup> P
<b>R.</b> Annealing Point	<b>3.</b> At this viscosity, stresses that form in the glass object can be relieved in a matter of minutes; viscosity = 10 <sup>13</sup> P
<b>S.</b> Strain Point	<b>4.</b> Viscosity at which glass will deform under its own weight on the time scale of manufacturing operations; viscosity = 10 <sup>7.6</sup> P

- A) P-2, Q-4, R-3, S-1
- B) P-2, Q-4, R-1, S-3
- C) P-4, Q-2, R-1, S-3
- D) P-3, Q-2, R-4, S-1

## Problem 7

In the 1980s, it was reported that, using the dimensions of radial cracks, the apparent indentation fracture toughness of a glass can be determined by an expression of the form

$$K_{\text{indent}} = \frac{\chi P}{c^{3/2}}$$

Here,  $P$  is the load,  $c$  is characteristic dimension of the mean crack produced by the indenter, and  $\chi$  is a coefficient given by

$$\chi = 0.016 \sqrt{\frac{E}{H}}$$

in which  $E$  is Young's modulus and  $H$  is the Vickers hardness. Consider a soda lime glass specimen with grain size  $E$  of 70 GPa and hardness of 5.5 GPa. Upon applying a peak load of 50 N onto the specimen, a crack with mean dimension of 0.32 mm was produced. Determine the indentation toughness of this glass.

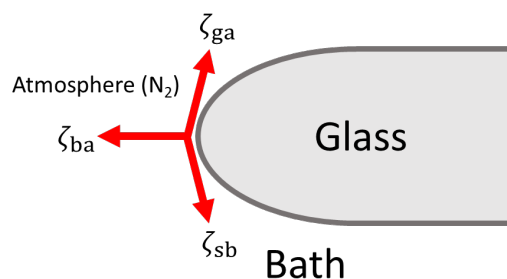
- A)  $K_{\text{indent}} = 0.5 \text{ MPa}\cdot\text{m}^{1/2}$
- B)  $K_{\text{indent}} = 0.75 \text{ MPa}\cdot\text{m}^{1/2}$
- C)  $K_{\text{indent}} = 1.1 \text{ MPa}\cdot\text{m}^{1/2}$
- D)  $K_{\text{indent}} = 1.4 \text{ MPa}\cdot\text{m}^{1/2}$

### Problem 8 (Modified from Bourhis, 2014, w/ permission)

Most flat glass is produced by the float process. This process was introduced by Pilkington Brothers Ltd in the 1950s. Firstly, the raw materials – sand, soda, and lime – are continuously introduced into the furnace, melted at 1500°C, homogenized by convection and fined to eliminate bubbles. The furnace contains typically 2000 tons of glass and produces every day 500 tons of glass. The viscous liquid travels onto the float at a temperature of ~1100°C (viscosity  $10^{2.6-2.8} \text{ Pa}\cdot\text{s}$ ) under a nitrogen atmosphere to prevent corrosion of the tin bath. The ribbon equilibrium allows us to write

$$\frac{gt_H^2\rho_g}{2}\left(1-\frac{\rho_g}{\rho_b}\right)=\zeta_{ga}+\zeta_{gb}-\zeta_{ba}$$

where  $t_H$  is the sheet thickness,  $\rho_g$  and  $\rho_b$  is the density of the glass and bath respectively,  $\zeta$  denotes surface energy, and subscripts  $g$ ,  $b$ , and  $a$  indicate glass, bath, and atmosphere respectively (refer to the illustration below). Let  $\rho_g = 2400 \text{ kg/m}^3$ ,  $\rho_b = 6600 \text{ kg/m}^3$ ,  $\zeta_{ba} = \zeta_{gb} = 0.48 \text{ J/m}^2$  and  $\zeta_{ga} = 0.35 \text{ J/m}^2$ . Determine the equilibrium thickness of the glass sheet.



- A)  $t_H = 3.85 \text{ mm}$
- B)  $t_H = 6.06 \text{ mm}$
- C)  $t_H = 9.02 \text{ mm}$
- D)  $t_H = 12.2 \text{ mm}$

### Problem 9

Regarding aspects of glass technology, true or false?

1. ( ) A glass has weight composition denoted as 65 CaO – 35Al<sub>2</sub>O<sub>3</sub>. A sample of 100 grams of this glass has more than 48 g of aluminum oxide.
2. ( ) For a glass to be formed, a series of conditions must exist, including a low nucleation rate and a high viscosity at or near the melting point.
3. ( ) A soda-lime glass has Young's modulus of 68 GPa, surface energy of 0.7 J/m<sup>2</sup> and Si – O bond length of 0.15 nm. Its theoretical strength is greater than 15 GPa.
4. ( ) Most glass melts are found to exhibit shear-thickening behavior, that is, the coefficient of viscosity increases as shear rate increases.
5. ( ) Rotation viscometers, being quite versatile devices as they are, can measure the viscosity of the entire range of viscosities encompassed by glass melts, including, with the proper adaptations, the annealing point.
6. ( ) The viscosity of glass is expected to decrease with network connectivity, which explains why, among the glasses of practical interest, the tetragonally coordinated SiO<sub>2</sub> glass has one of the lowest viscosities at any temperature.
7. ( ) The Vogel-Fulcher-Tammann formula gives viscosity as

$$\eta(T) = \eta_0 \exp\left(\frac{B}{T - T_0}\right)$$

where  $T$  is temperature in °C and  $\eta_0$ ,  $T_0$ , and  $B$  are parameters. For a lime-silica glass, it was found that  $B = 4300$  K,  $T_0 = 540$  K, and  $\ln \eta_0 = -2.77$ , with  $\eta$  given in Pa·s. The viscosity of this glass at 1000 K is greater than 500 Pa·s.

**8.( )** A glass fiber 20 cm long and 1.5 mm in diameter is found to elongate at a rate of 100  $\mu\text{m}$  per second. The density of the glass is 2.5 g/cm<sup>3</sup>. The viscosity of the glass fiber is greater than  $3 \times 10^6$  Pa·s.

**9.( )** Consider sodium-potassium silicate glass, an example of glass containing two alkali oxides. It can be shown that properties determined by mobile ions, such as electrical conductivity and ionic diffusion, increase linearly as the ratio of sodium (or potassium) to total alkali concentration increases from zero to one.

**10.( )** Although it has been found that water content appreciably affects properties of glasses such as density and dielectric constant, the glass transformation temperature has been verified to be remarkably independent of temperature, especially in the case of glass former oxides.

**11.( )** The molar refractivity of a sodium borate glass was measured to be 9 cm<sup>3</sup>/mol. The molecular mass of the glass is 72.1 g/mol and its density at the temperature of interest is 2.4 g/cm<sup>3</sup>. We conclude that the refractive index of this glass is greater than 1.4.

## SOLUTIONS

### P.1 ■ Solution

Most ceramics fracture in brittle mode with no plastic deformation.

◆ The correct answer is **D**.

### P.2 ■ Solution

The modulus of rupture is given by

$$MOR = \frac{3PL}{2BH^2}$$

where  $P$  is the applied load,  $L = 5 \times 10^{-2}$  m is span,  $B = 1 \times 10^{-2}$  m is width, and  $H = 0.5 \times 10^{-2}$  m is height. For sample 1, we have  $P = 6.67$  kN and

$$MOR = \frac{3PL}{2BH^2} = \frac{3 \times 6670 \times 0.05}{2 \times 0.01 \times 0.05^2} = 20.0 \text{ MPa}$$

The calculations for other data points are summarized below. The average MOR is found to be 25.5 MPa.

Sample	Failure load (kN)	MOR (MPa)
1	6.67	20.0
2	7.11	21.3
3	9.23	27.7
4	11.4	34.2
5	6.54	19.6
6	7.55	22.7
7	9.55	28.7
8	6.74	20.2
9	10.8	32.4
10	9.4	28.2
	Average MOR	25.5

◆ The correct answer is **B**.

### P.3 ■ Solution

**Part 1:** In Mathematica, the data can be easily associated to a Weibull distribution with the command *EstimatedDistribution*,

```
est = EstimatedDistribution[{215,200,206,222,216,187,186,172,211,175,205,195},
    WeibullDistribution[a, b]]
```

This returns a shape parameter  $a = 15.45$  and a scale parameter  $b = 206.2$ . The Weibull formulation for this data is then

$$P = e^{-(\sigma/b)^a} = e^{-(\sigma/206.2)^{15.45}}$$

For a stress level of 180 MPa, the probability of survival is

$$P = e^{-(180/206.2)^{15.45}} = \boxed{88.5\%}$$

♦ The correct answer is **C**.

**Part 2:** In this case, the strength data and scale parameter are modified as

$$\frac{\sigma_s}{\sigma_c} = \left( \frac{A_c}{A_s} \right)^{\frac{1}{a}}$$

where  $\sigma_s$  is the strength of the specimen,  $\sigma_c$  is the strength of the component,  $A_c$  is the area of the component,  $A_s$  is the area of the specimen, and  $a$  is the shape parameter. In the case at hand,  $A_c/A_s = 5$ . Substituting these and other data gives

$$\frac{\sigma_s}{\sigma_c} = 5^{1/15.45} = 1.11$$

$$\therefore \sigma_s = 1.11\sigma_c$$

so that, at the data point in question,  $\sigma_s = 1.11 \times 180 = 200$  MPa. The corresponding probability of survival follows as

$$P = e^{-(200/206.2)^{15.45}} = \boxed{53.6\%}$$

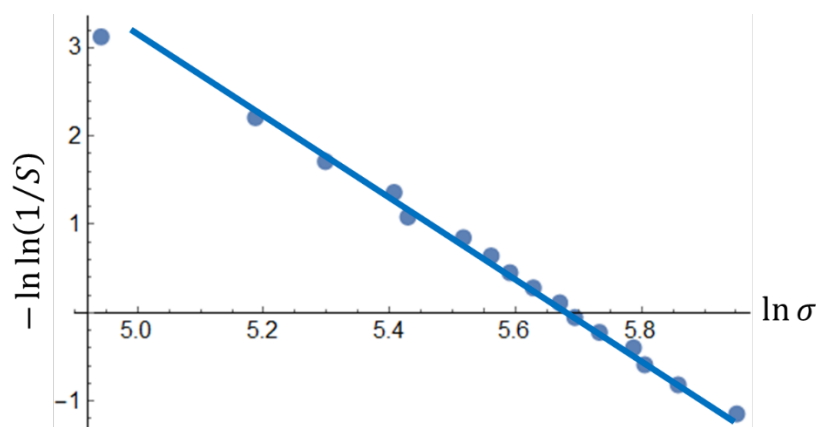
♦ The correct answer is **D**.

## P.4 ■ Solution

**Part 1:** The average stress is determined to be 272 MPa. The pertaining calculations are summarized below.

Sample	Fracture Strength $\sigma$ (MPa)	Deviation from Mean Strength	Square of Deviation	Survival Probability, $S$	$\ln \sigma$	$1/S$	$\ln(1/S)$	$-\ln(\ln(1/S))$
7	140	-132	17424	0.957	4.942	1.045	0.0440	3.125
8	179	-93	8649	0.896	5.187	1.116	0.1098	2.209
2	200	-72	5184	0.835	5.298	1.198	0.1803	1.713
1	223	-49	2401	0.774	5.407	1.292	0.2562	1.362
13	228	-44	1936	0.713	5.429	1.403	0.3383	1.084
16	249	-23	529	0.652	5.517	1.534	0.4277	0.849
11	260	-12	144	0.591	5.561	1.692	0.5259	0.643
9	268	-4	16	0.53	5.591	1.887	0.6349	0.454
5	278	6	36	0.47	5.628	2.128	0.7550	0.281
6	290	18	324	0.409	5.670	2.445	0.8940	0.112
12	297	25	625	0.348	5.694	2.874	1.0556	-0.054
3	309	37	1369	0.287	5.733	3.484	1.2483	-0.222
4	326	54	2916	0.226	5.787	4.425	1.4872	-0.397
10	332	60	3600	0.165	5.805	6.061	1.8018	-0.589
14	350	78	6084	0.104	5.858	9.615	2.2634	-0.817
15	384	112	12544	0.043	5.951	23.256	3.1466	-1.146

The plot of  $-\ln[\ln(1/S)]$  versus  $\ln \sigma$  is shown in continuation.



The fact that the two variables follow a linear trend indicates that the strength of the silicon carbide can indeed be well represented by a Weibull distribution. The absolute value of the Weibull modulus is the slope of the line plotted above. One way to obtain this slope in Mathematica is to fit the data with the *LinearModelFit* function, as follows.

$$\text{data} = \{\{4.942, 3.125\}, \{5.187, 2.209\}, \{5.298, 1.713\}, \{5.407, 1.362\},$$

$$\{5.429, 1.084\}, \{5.517, 0.849\}, \{5.561, 0.643\}, \{5.591, 0.454\}, \{5.628, 0.281\},$$

{5.67,0.112},{5.694,-0.054},{5.733,-0.222},{5.787,-0.397},  
 {5.805,-0.589},{5.858,-0.817},{5.951,-1.146}}

line = LinearModelFit[data,{x,1},x]

This returns a *FittedModel* object of the form  $24.75 - 4.35x$ . Accordingly, the Weibull modulus is 4.35.

**Part 2:** To determine the design stress, we must first establish the scale parameter of the Weibull fit. Doing so, for example by using the *EstimatedDistribution* function in Mathematica, we obtain  $b = 294$  MPa. The design stress follows as

$$P = \exp\left[-(\sigma/b)^a\right] \rightarrow 0.999 = \exp\left[-(\sigma/294)^{4.35}\right]$$

$$\therefore \boxed{\sigma = 60.1 \text{ MPa}}$$

♦ The correct answer is **C**.

## P.5 ■ Solution

**1.False.** A large Weibull modulus indicates that failures occurred at a narrow range of applied stresses. A large value would indicate that the stress required to cause failure is more predictable than it would be if the failures occurred over a wide range of applied stresses, as would be the case for a lower shape parameter. A higher shape parameter value leads to a steeper flaw size distribution and thus to a lower dispersion of the fracture strength. From an engineering point of view, a high Weibull modulus can be as important as a high strength.

**2.False.** The most serious flaw is not necessarily the largest one because its severity also relies on its location and orientation. In fact, strength is controlled by the flaw subjected to the highest stress intensity factor (SIF).

**3.False.** A fundamental limitation of bend testing for modulus of rupture is that the measured strength will significantly vary depending on the size of the specimen tested and whether the load setting is a three-point or four-point configuration. Indeed, four-point bend testing results in lower strength values for a given ceramic material than does three-point bending; these tests in turn produce different values still from the strength obtained by uniaxial tensile testing. Which parameter to use requires significant brainstorming on part of the engineer.

**4.True.** Indeed, the toughness of polycrystalline ceramics is generally higher than that of single crystalline ones. In the case of alumina, it is found that  $K_{IC} = 2.2$  MPa·m<sup>1/2</sup> for the single crystalline form and  $K_{IC} = 4$  MPa·m<sup>1/2</sup> for the polycrystalline form. One of the causes for this difference is crack deflection. Cracks propagate more easily through grain boundaries. Grain boundaries have different orientations. Hence, cracks are deflected to different angles as they propagate. This increases the path length required for the fracture and, consequently, increases the energy to be expended. The stress intensity factor at the crack tip,  $K_{tip}$ , is related to the angle of deflection  $\theta$  and the applied stress intensity  $K_{app}$  by a relation of the form

$$K_{tip} = \cos^2\left(\frac{\theta}{2}\right)K_{app}$$

For an average  $\theta$  value of 45°, the fracture toughness of polycrystalline material should be 1.25 over that of a single crystal. If we apply this value to alumina, the polycrystalline alumina should have a  $K_{IC}$  value of about 3.45 MPa·m<sup>1/2</sup>, which indicates that crack deflection accounts for most, but not all, of the toughness difference between ceramic types. The other two mechanisms are crack bridging and transformation toughening.

**5.True.** Quartz,  $\beta$ -spodumene, and  $\beta$ -eucryptite consist of rings of SiO<sub>4</sub> tetrahedra.  $\beta$ -eucryptite, in particular, resembles a spiral arrangement along the  $c$ -axis that has been likened to the coil of a spring. Such structures are not rigid like close-packed structures or structures made up of edge- or face-shared tetrahedrons or octahedrons. During temperature changes, corner-shared bonds have the flexibility to change angle by tilting or rotating. This can absorb the thermal vibration motion of atoms that would normally (in a close-packed structure) cause bulk expansion. It can, in fact, result in such a high anisotropic behavior that the expansion is negative (contraction) in one direction and positive in the perpendicular direction. Some linear expansion coefficients for anisotropic ceramic



materials are listed below; observe that  $\beta$ -spodumene and  $\beta$ -eucryptite are indeed examples of ceramics that can have  $\alpha < 0$  in a given direction. This anisotropy in thermal expansion can result in the buildup of large thermal residual stresses that can be quite detrimental to the strength and integrity of ceramic parts.

Material	Linear thermal expansion coefficient ( $\alpha$ , $10^{-6} \text{ }^\circ\text{C}^{-1}$ )	
	Normal to $c$ -axis	Parallel to $c$ -axis
SiO <sub>2</sub> (quartz)	14.0	9.0
Graphite	1.0	27.0
CaCO <sub>3</sub> (calcite)	-3.7	25.1
LiAlSi <sub>2</sub> O <sub>6</sub> ( $\beta$ -spodumene)	6.5	-2.0
LiAlSiO <sub>4</sub> ( $\beta$ -eucryptite)	8.2	-17.6

**6.False.** Thermophysical and dielectric properties are in fact directly proportional to the alumina content, as exemplified in the following table. For this reason, high-purity alumina ceramics are preferred in precision applications.

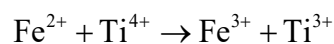
Property	Aluminum oxide, weight percent			
	85	92	96	99.9
Thermal conductivity at 20°C (W/cm·K)	0.15	0.16	0.26	0.39
Young's modulus (10 <sup>6</sup> psi)	32	39	44	56
Tensile strength at 25°C (10 <sup>3</sup> psi)	22	28	28	45

**7.True.** Since  $B = H/K_{IC}$ , we have

$$B = \frac{H}{K_{IC}} = \frac{15,700 \text{ MPa}}{3.5 \text{ MPa} \cdot \text{m}^{1/2}} = 4490 \text{ m}^{-1/2} = 4490 \times (10^6 \mu\text{m})^{-1/2}$$

$$\therefore B = 4.49 \mu\text{m}^{-1/2}$$

**8.False.** Addition of iron and titanium ions yields sapphire, not ruby. The latter is formed by the addition of 1% of Cr<sub>2</sub>O<sub>3</sub>. In the production of sapphire, Fe<sup>2+</sup> and Ti<sup>4+</sup> substitute some of the aluminum atoms and, when light of energy of 2.11 eV is shone on the sapphire, it is absorbed by the charge transfer reaction



**9.False.** While it is true that silicon carbide has a high thermal conductivity (second only to diamond) and low thermal expansion coefficient, which matches that of silicon to a higher degree than any other ceramic, the statement errs by mentioning that SiC has a lower dielectric constant. Silicon carbide has a dielectric current at 1 MHz of 40, which is 4.8 times greater than that of 85%-purity aluminum oxide and 4.6 times greater than that of aluminum nitride under similar conditions. Such a high dielectric constant can produce undesirable effects in electronic applications, including cross-coupling of electronic signals and excessive transmission delay.

**10.True.** Indeed, silicon carbide possesses some of the best corrosion resistance characteristics of any ceramic. Pure SiC is twice as oxidation resistant, even at 1500°C, than even the best superalloys at their maximum service temperature of 1200°C.

**11.False.** Modern methods have shown that boron carbide has no stoichiometric composition but rather ranges from B<sub>4.3</sub>C to B<sub>10.4</sub>C. Accordingly, the three investigators were not entirely correct. Nevertheless, it should be noted that the composition of commercially produced boron carbide in arc furnaces is quite close to B<sub>4</sub>C, which corresponds to the stoichiometric limit on the high-carbon side.

**12.False.** Indeed, the hardness of boron carbide at high temperatures is superior to those of the other two hardest materials known to man. This does not mean that the mechanical performance of this ceramic under extreme conditions is necessarily better than that of cubic boron nitride or diamond; for one, boron carbide, albeit being chemically inert in most situations, can adversely react with oxygen at high temperatures.

**13.False.** In actuality, it is hexagonal boron nitride, not its cubic and wurtzitic counterparts, that possesses the highest degree of anisotropy. Indeed, the hardness, electrical and thermal conductivity of h-BN are each much greater within the planes than perpendicular to them. One explanation is the major difference in

bonding energies in the structure of h-BN – that is, strong covalent bonds within the basal planes and weak bonding between them.

**14.True.** Indeed, cubic boron nitride is superior to diamond in grinding steel, especially at high temperatures. Diamond, being a form of carbon, can react with ferrous materials at temperatures as low as 900°C. Diamond also reacts with carbides at temperatures greater than 700°C, converting back to amorphous carbon. In short, cBN is the abrasive of choice when grinding steels, nickel-based superalloys, and cast iron, whereas diamond works better for nonmetals and nonferrous metals.

**15.False.** While it is true that the hardness of diamond is greater than that of all other ceramics mentioned in the statement, some of the other comparisons are false. The correct order is

$$H(\text{Diamond}) > H(\text{cBN}) > H(\text{SiC}) > (\text{Alumina})$$

The Knoop hardness values and other thermophysical properties of these ceramics at room temperature are summarized below.

Property	Material			
	Diamond	Cubic BN	Silicon carbide	Alumina
Knoop hardness (GPa)	70 – 100	50	25	22
Young's modulus (GPa)	1143	909	410	366
Poisson's ratio	0.07	0.12	0.14	0.22
Coefficient of thermal expansion ( $10^{-6} \text{ } ^\circ\text{C}^{-1}$ )	4.8	5.6	4.5	8.6

**16.True.** Indeed, there is a crossover temperature at which the thermal conductivity of AlN surpasses that of BeO. For the highest grade of AlN, the crossover temperature is about 20°C. Above this temperature, the thermal conductivity of AlN is higher; below 20°C, that of BeO is higher.

**17.False.** While it is true that beryllia has a high thermal conductivity – its value is about 285 W/m·K at room temperature, higher than aluminum metal – the statement errs by saying that powders of this ceramic are nontoxic; chronic exposure to dust containing beryllia may cause berylliosis, a chronic lung disease for which there is no cure. Another limitation of BeO is its high cost, which has limited its use to critical applications such as heat sinks for transistors.

**18.True.** Indeed, mullite offers a stable mechanical performance, including creep resistance, at temperature levels that would be prohibitive for most other ceramics. The low thermal conductivity ( $\sim 0.06 \text{ W/cm}\cdot\text{K}$ ) and low thermal expansion coefficient ( $\sim 4.5 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$ ) make it an excellent choice for refractory applications.

**19.False.** On the contrary, it is the very addition of oxides such as  $\text{Y}_2\text{O}_3$  that promotes the stabilization of the phases of zirconia at low temperatures. Doping of zirconia results in stabilization of the tetragonal phase at lower dopant concentrations (for mechanical toughness) or the cubic phase at higher dopant concentrations (for high ionic conductivity) at room temperature.

**20.True.** The velocity with which the light beam propagated in the material is

$$V = \frac{1.0}{5.93 \times 10^{-6}} = 168,600 \text{ m/s}$$

The refractive index is the ratio of the speed of light to the velocity of propagation in the material, that is,

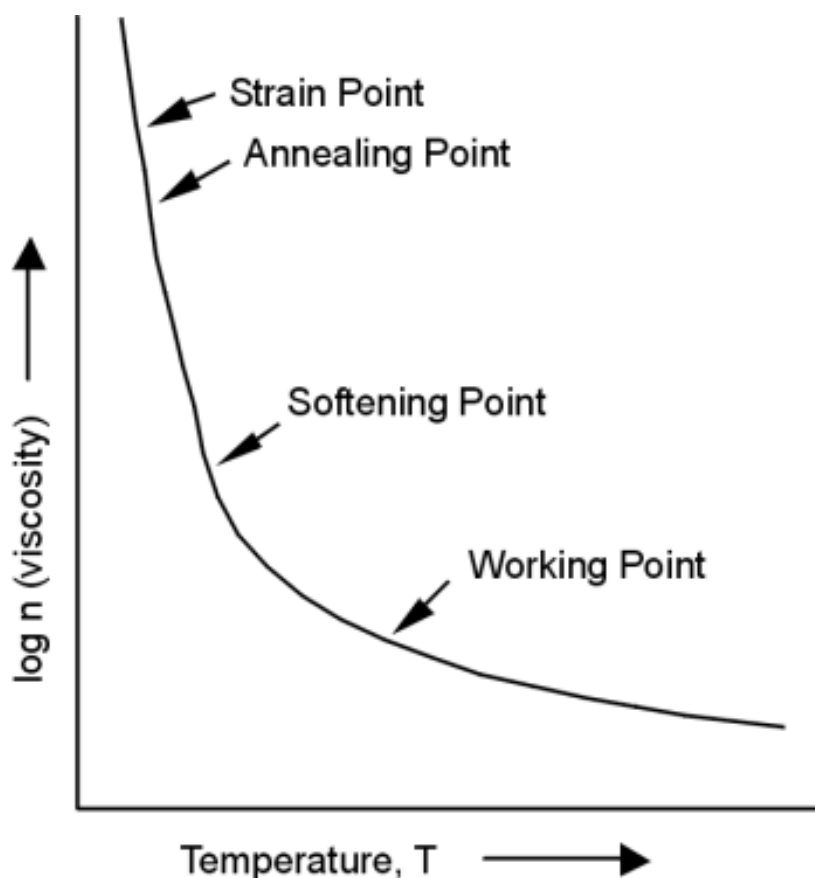
$$n = \frac{c}{V} = \frac{300,000}{168,600} = 1.78$$

With recourse to the table, we conclude that the material in question may indeed be a sapphire.

## P.6 ■ Solution

The correct sequence is P-2, Q-4, R-3, S-1. Some notes on each point are provided below.

Point	Note
Working Point	This point defines the viscosity at which a machine can work on a glass without losing control. At this point, molten glass can be formed and manipulated; viscosity is low enough for some shear processing (pressing, blowing) but high enough to retain shape after shear stress is removed.
Softening Point	This is the viscosity at which glass will deform under its own weight in the time scale of manufacturing operations. More specifically, it may be regarded as the temperature at which a filament of 23.5 cm length and 0.55 – 0.77 mm diameter elongates by 1 mm/min under its own weight.
Annealing Point	This is the temperature to which the glass must be heated to relieve any internal stresses that arose as a result of the forming process. It can be said that, at the annealing point, an artificially strained glass is relaxed to 10% in 15 minutes.
Strain Point	Stresses that form in the glass object are relieved in a matter of hours, not minutes. It can be said that an artificially strained glass is relaxed to 10% of the strain in 15 hours. In addition, the strain point defines the maximum temperature at which a glass can be used for structural applications without undergoing creep.



◆ The correct answer is **A**.

## P.7 ■ Solution

The  $\chi$  coefficient is

$$\chi = 0.016 \sqrt{\frac{E}{H}} = 0.016 \sqrt{\frac{70}{5.5}} = 0.0571$$

Substituting in the formula for  $K_{\text{indent}}$  gives

$$K_{\text{indent}} = \frac{\chi P}{c^{3/2}} = \frac{0.0571 \times 50}{(0.32 \times 10^{-3})^{3/2}} \approx \boxed{0.5 \text{ MPa} \cdot \text{m}^{1/2}}$$

◆ The correct answer is **A**.

## P.8 ■ Solution

All we have to do is substitute the pertaining variables in the formula we were given and solve for  $t_H$ .

$$\frac{9.81 \times t_H^2 \times 2400}{2} \times \left(1 - \frac{2400}{6600}\right) = 0.48 + 0.48 - 0.35$$

$$\therefore 7490 t_H^2 = 0.61$$

$$\therefore \boxed{t_H = 9.02 \text{ mm}}$$

◆ The correct answer is **C**.

## P.9 ■ Solution

**1.True.** The molecular weight of calcium oxide is 56 g/mol, while that of aluminum oxide is 102 g/mol. The molecular weight of the glass is then  $0.65 \times 56 + 0.35 \times 102 = 72.1$  g/mol. The weight fraction of  $\text{Al}_2\text{O}_3$  is  $0.35 \times 102 / 72.1 = 0.495$  and the mass of aluminum oxide contained in 100 g of glass is calculated to be  $100 \times 0.495 = 49.5$  g.

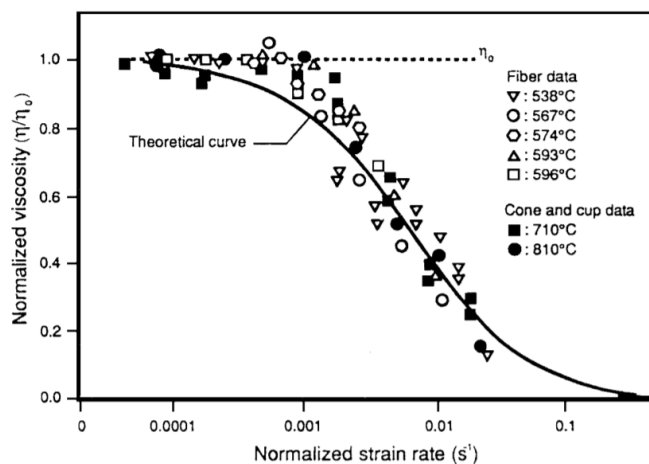
**2.True.** The two conditions mentioned are indeed some of the prerequisites for formation of glass. A low nucleation rate can be achieved by having either a small entropy of fusion or a large crystal/liquid interfacial energy. The lower the entropy of fusion and the higher the crystal/liquid interfacial energy, the more difficult nucleation will be. In addition, a high viscosity ensures that the growth rate will be small.

**3.True.** All we have to do is substitute the pertaining data in the Rowan formula, namely,

$$\sigma_{\text{th}} = \left(\frac{E\zeta}{r_0}\right)^{1/2} = \left[\frac{(68 \times 10^9) \times 0.7}{0.15 \times 10^{-9}}\right]^{1/2} = 17.8 \text{ GPa}$$

Although this suggests that soda-lime glass can achieve substantially high strength, practical measurements are well below this prediction; for example, the strength of flat glass is seldom higher than about 100 MPa. It should be noted that this is but one of the various equations available for theoretical strength; one investigator, for instance, proposes instead that  $\sigma_{\text{th}} = (E\zeta/4r_0)^{1/2}$ .

**4.False.** Glass melts are generally observed to be shear-thinning in nature, that is, their viscosity *decreases* at high shear strain rates. This behavior is evidenced in the following viscosity versus shear rate plot, based on measurements of liquid soda lime glass. Shear thinning is quite important in high shear rate forming processes.



**5.False.** Rotation viscometers are generally restricted to the rheological measurement of liquids in the range of 1 to  $10^4$  Pa·s. Although the viscosity covered by this method can be extended by measuring the time required for the spindle to rotate through a defined angle of deflection (extending the range of measurable viscosities to  $10^{6.5}$  Pa·s at most), or by measuring the torque required to twist the spindle through a small angle (extending the measurable viscosities to almost  $10^9$  Pa·s), none of these techniques enable the measurement of the exceptionally high viscosity –  $10^{12.4}$  Pa·s – associated with the annealing point. Other devices, such as fiber elongation viscometers, are better tailored for this purpose.

**6.False.** In actuality, viscosity tends to increase with increase network connectivity, and  $\text{SiO}_2$  glass has one of the highest viscosities of commercial glasses – its annealing point is around  $1130^\circ\text{C}$ . The triangularly connected  $\text{B}_2\text{O}_3$  glass has the lowest viscosity and features an annealing point as low as about  $230^\circ\text{C}$ . Conversion of triangular boron to tetrahedral boron increases connectivity and hence raises the viscosity.

**7.True.** With  $\ln \eta_0 = -2.77$ , we obtain  $\eta_0 = 0.0627 \text{ Pa}\cdot\text{s}$ . Substituting this and other data in the VFT equation yields

$$\eta(1000) = 0.0627 \times \exp\left(\frac{4300}{1000 - 540}\right) = 719 \text{ Pa}\cdot\text{s}$$

**8.True.** The viscosity in a fiber elongation viscometer apparatus is given by

$$\eta = \frac{LF}{3A(dL/dt)}$$

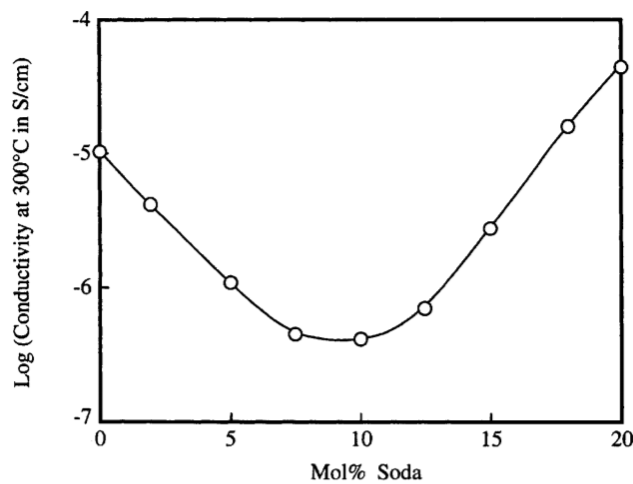
where  $L$  is the length of the fiber,  $F$  is the force applied to the fiber,  $A$  is the cross-sectional area of the fiber, and  $dL/dt$  is the elongation rate of the fiber. In the case at hand, the fiber is elongating due to its own weight, which is given by

$$F = \left\{ \left[ \frac{\pi}{4} \times (1.5 \times 10^{-3})^2 \right] \times 0.2 \right\} \times 2500 \times 9.81 = 8.67 \times 10^{-3} \text{ N}$$

Substituting this and other data in the equation for  $\eta$  yields

$$\eta = \frac{0.2 \times (8.67 \times 10^{-3})}{3 \times \left[ \frac{\pi}{4} \times (1.5 \times 10^{-3})^2 \right] \times (100 \times 10^{-6})} = 3.27 \times 10^6 \text{ Pa}\cdot\text{s}$$

**9.False.** Although a linear variation of properties such as electrical conductivity is indeed expected, in actuality ionic-mobility-based properties of glasses with two or more alkali oxides display an anomalous behavior. For instance, the following graph shows the variation of electrical conductivity at  $300^\circ\text{C}$  of sodium-potassium silicate glass. Although the total alkali oxide content of these glasses is constant, the electrical conductivity passes through a large minimum as the ratio of sodium to total alkali oxide concentration varies from zero to one, rather than experiencing the linear increase expected from simple additivity. This anomalous behavior is known as the mixed-alkali effect. The mechanism underlying this behavior has been the subject of numerous investigations – at least 10 different explanations exist – and is not yet fully understood.



**10.False.** On the contrary, the effect of water content on glass forming temperature can be quite pronounced, *particularly* in the case of oxide glasses. Commercial vitreous silica, for example, is produced with water contents ranging from  $< 1$  to  $> 600$  wtppm of  $\text{H}_2\text{O}$ , and the  $T_g$  value over this range decreases from  $\sim 1200^\circ\text{C}$  to only  $1060^\circ\text{C}$ . Similarly, the  $T_g$  of vitreous boric oxide ranges from  $\sim 200^\circ\text{C}$  for glasses containing as much as 1 wt%  $\text{H}_2\text{O}$  to over  $300^\circ\text{C}$  for the driest glasses, which contain  $< 100$  wtppm  $\text{H}_2\text{O}$ . The effect of water is similar on other glasses, such as soda-lime-silica glasses, but less remarkable.

**11.True.** The molar refractivity is given by

$$R_m = V_m \left( \frac{n^2 - 1}{n^2 + 2} \right)$$

where  $V_m$  is the molar volume, defined as the ratio of molecular weight to density; in the present case,  $V_m = 72.1/2.4 = 30 \text{ cm}^3/\text{mol}$ . Substituting and solving for  $n$ , we get

$$9 = 30 \times \left( \frac{n^2 - 1}{n^2 + 2} \right) \rightarrow n = 1.51$$

## ANSWER SUMMARY

Problem 1		D
Problem 2		B
Problem 3	3.1	C
	3.2	D
Problem 4	4.1	Open-ended pb.
	4.2	C
Problem 5		T/F
Problem 6		A
Problem 7		A
Problem 8		C
Problem 9		T/F

## REFERENCES

- BOURHIS, E. (2014). *Glass: Mechanics and Technology*. 2nd edition. Weinheim: Wiley-VCH.
- HARPER, C. (Ed.) (2001). *Handbook of Ceramics, Glasses, and Diamonds*. 2nd edition. New York: McGraw-Hill.
- RIEDEL, R. and CHEN, I. (Eds.) (2010). *Ceramics Science and Technology*. 4th edition. Hoboken: John Wiley and Sons.
- SHACKELFORD, J. and DOREMUS, R. (Eds.) (2008). *Ceramic and Glass Materials*. Heidelberg: Springer.
- SHELBY, J. (2005). *Introduction to Glass Science and Technology*. 2nd edition. Cambridge: Royal Society of Chemistry.
- SURENDRANATHAN, A. (2015). *An Introduction to Ceramics and Refractories*. Boca Raton: CRC Press.



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