

## Quiz SM206

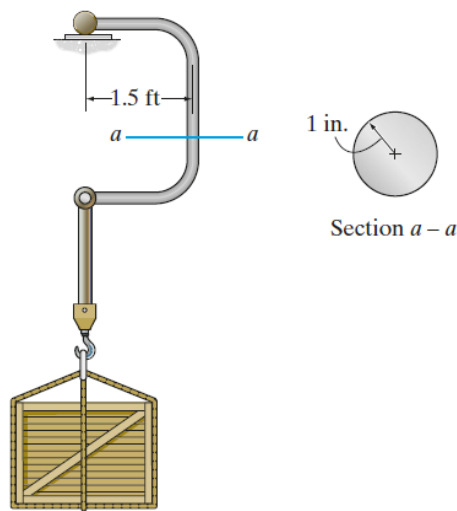
### COMBINED LOADINGS

Lucas Montogue

#### PROBLEMS

##### Problem 1 (Hibbeler, 2014, w/ permission)

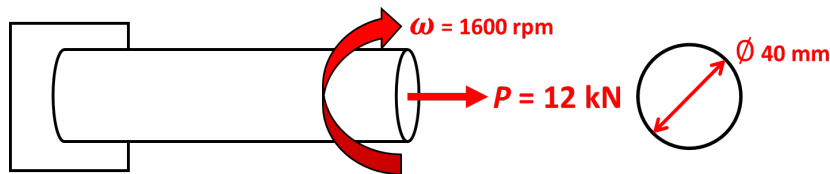
If the load has a weight of 600 lb, determine the maximum normal stress developed in the cross-section of the supporting member at section  $a-a$ .



- A)  $\sigma_{\max} = 8.91$  ksi.
- B)  $\sigma_{\max} = 13.9$  ksi.
- C)  $\sigma_{\max} = 33.1$  ksi.
- D)  $\sigma_{\max} = 54.2$  ksi.

##### Problem 2 (Philpot, 2013, w/ permission)

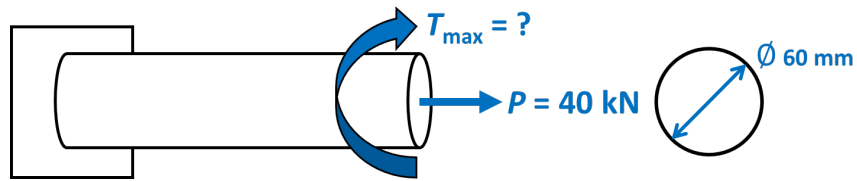
A solid 40-mm diameter shaft is used in an aircraft engine to transmit 100 kW at 1600 rpm to a propeller that develops a thrust of 12 kN. Determine the magnitudes of the maximum tensile stress and the maximum in-plane shear stress.



- A)  $\sigma_1 = 52.6$  MPa and  $\tau_{\max} = 30.2$  MPa
- B)  $\sigma_1 = 52.6$  MPa and  $\tau_{\max} = 47.8$  MPa
- C)  $\sigma_1 = 69.4$  MPa and  $\tau_{\max} = 30.2$  MPa
- D)  $\sigma_1 = 69.4$  MPa and  $\tau_{\max} = 47.8$  MPa

##### Problem 3 (Philpot, 2013, w/ permission)

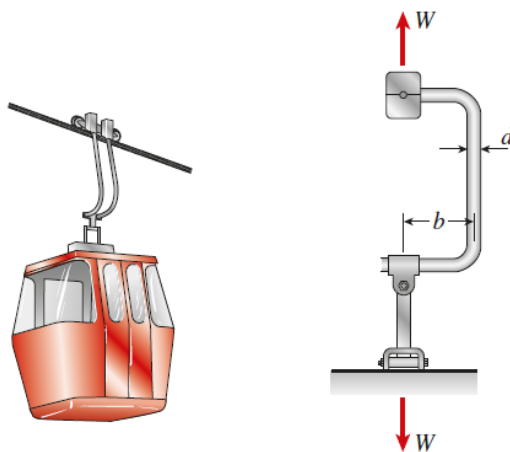
A solid 60-mm diameter shaft must transmit a torque of unknown magnitude while it is supporting an axial tensile load of 40 kN. Determine the maximum allowable value for the torque if the tensile principal stress in the outside surface of the shaft must not exceed 100 MPa.



- A)  $T_{\max} = 3.92 \text{ kN}\cdot\text{m}$
- B)  $T_{\max} = 6.79 \text{ kN}\cdot\text{m}$
- C)  $T_{\max} = 9.88 \text{ kN}\cdot\text{m}$
- D)  $T_{\max} = 12.8 \text{ kN}\cdot\text{m}$

**Problem 4** (Gere & Goodno, 2009, w/ permission)

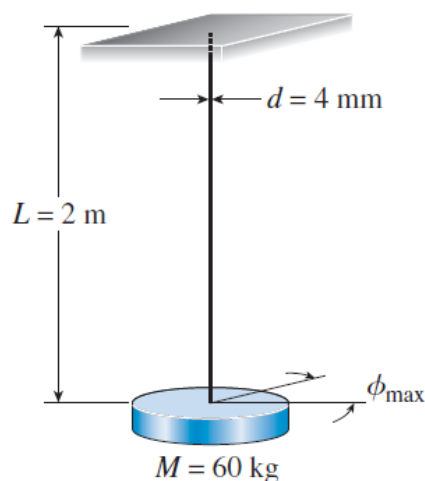
A gondola on a ski lift is supported by two bent arms, as shown in the figure. Each arm is offset by the distance  $b = 180 \text{ mm}$  from the line of action of the weight force  $W$ . The allowable stresses in the arms are  $100 \text{ MPa}$  in tension and  $50 \text{ MPa}$  in shear. If the loaded gondola weighs  $12 \text{ kN}$ , what is the minimum diameter  $d$  of the arms?



- A)  $d_{\min} = 48.4 \text{ mm}$
- B)  $d_{\min} = 88.9 \text{ mm}$
- C)  $d_{\min} = 115 \text{ mm}$
- D)  $d_{\min} = 165 \text{ mm}$

**Problem 5** (Gere & Goodno, 2009, w/ permission)

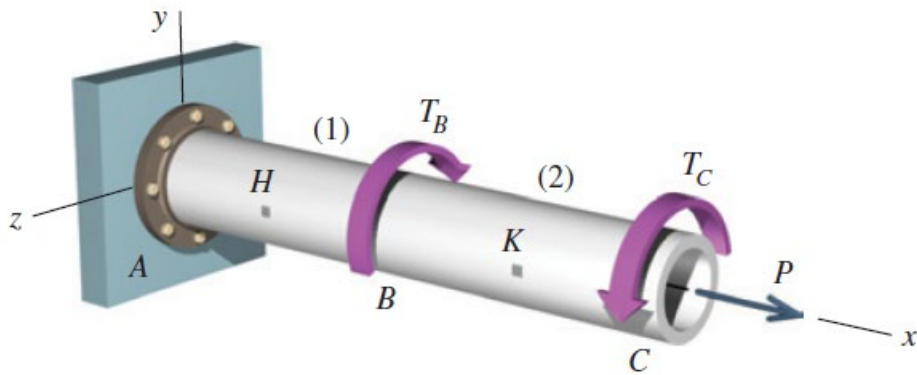
The torsional pendulum shown in the figure consists of a horizontal circular disk of mass  $M = 60 \text{ kg}$  suspended by a vertical steel wire ( $G = 80 \text{ GPa}$ ) of length  $L = 2 \text{ m}$  and diameter  $d = 4 \text{ mm}$ . Calculate the maximum permissible angle of rotation  $\phi_{\max}$  of the disk (that is, the maximum permissible amplitude of torsional vibrations) so that the stresses in the wire do not exceed  $100 \text{ MPa}$  in tension or  $50 \text{ MPa}$  in shear.



- A)  $\phi_{\max} = 5.24^\circ$
- B)  $\phi_{\max} = 18.1^\circ$
- C)  $\phi_{\max} = 31.7^\circ$
- D)  $\phi_{\max} = 52.3^\circ$

**Problem 6** (Philpot, 2013, w/ permission)

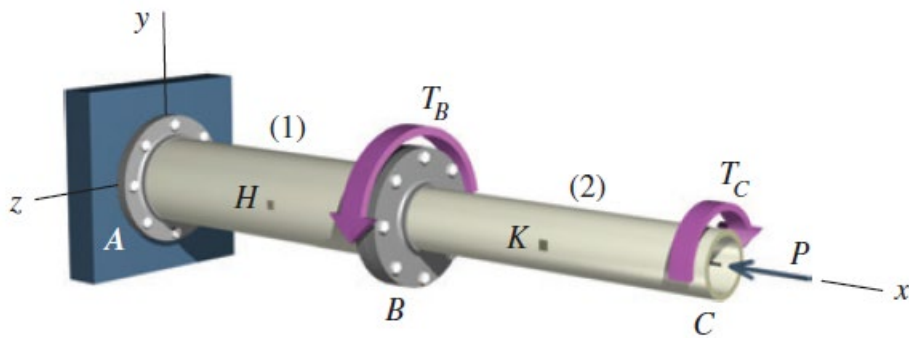
A hollow shaft with an outside diameter of 150 mm and an inside diameter of 130 mm is subjected to an axial tension load of  $P = 75 \text{ kN}$  and torques  $T_B = 16 \text{ kN}\cdot\text{m}$  and  $T_C = 7 \text{ kN}\cdot\text{m}$ , which act in the directions shown in the figure. Which of the following is *false*?



- A) The maximum tensile stress at point  $H$  is greater than 36 MPa.
- B) The maximum in-plane shear stress at point  $H$  is greater than 29 MPa.
- C) The maximum tensile stress at point  $K$  is greater than 32 MPa.
- D) The maximum in-plane shear stress at point  $K$  is greater than 27 MPa.

**Problem 7** (Philpot, 2013, w/ permission)

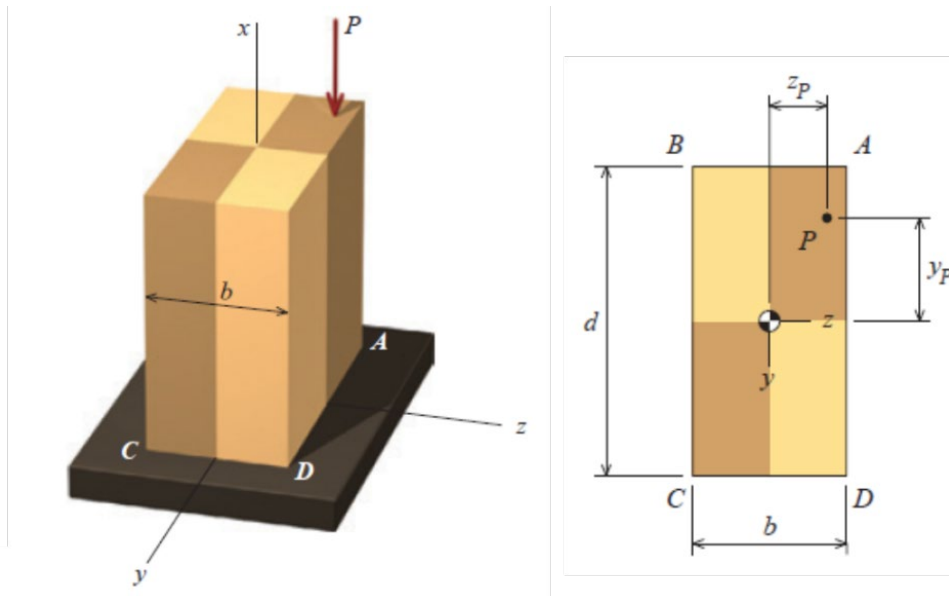
A compound shaft consists of two pipe segments. Segment (1) has an outside diameter of 220 mm and a wall thickness of 10 mm. Segment (2) has an outside diameter of 140 mm and a wall thickness of 15 mm. The shaft is subjected to an axial compression load of  $P = 100 \text{ kN}$  and torques  $T_B = 8 \text{ kN}\cdot\text{m}$  and  $T_C = 12 \text{ kN}\cdot\text{m}$ , which act in the directions shown in the figure. Which of the following is *false*?



- A) The maximum tensile stress at point  $H$  is greater than 1.5 MPa.
- B) The maximum in-plane shear stress at point  $H$  is greater than 8 MPa.
- C) The maximum tensile stress at point  $K$  is greater than 30 MPa.
- D) The maximum in-plane shear stress at point  $K$  is greater than 35 MPa.

**Problem 8** (Philpot, 2013, w/ permission)

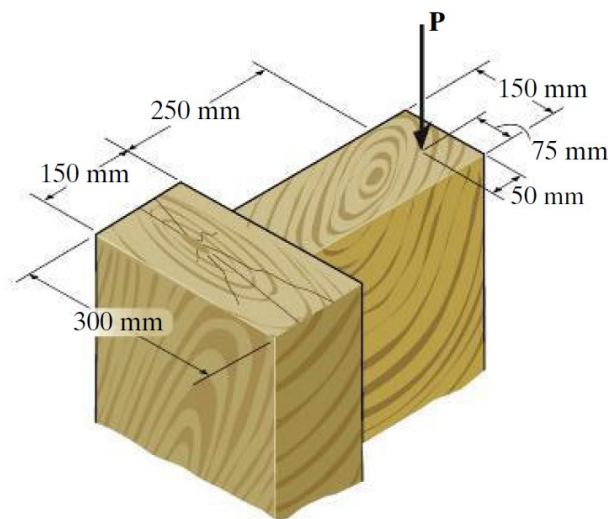
A short rectangular post supports a compressive load of  $P = 2500$  lb as shown in the next figure. A top view of the post showing the location where load  $P$  is applied to the top of the post is shown below. The cross-sectional dimensions of the post are  $b = 5$  in. and  $d = 10$  in. The load  $P$  is applied at offset distances of  $y_p = 3$  in. and  $z_p = 2$  in. from the centroid of the post. Determine the values of the normal stresses at corners A, B, C, and D of the post. Which of the following is *false*?



- A)  $|\sigma_A| = 260$  psi
- B)  $|\sigma_B| = 20$  psi
- C)  $|\sigma_C| = 160$  psi
- D)  $|\sigma_D| = 60$  psi

**Problem 9A** (Hibbeler, 2014, w/ permission)

The cross-section of the vertical member is built by gluing the two identical boards together. Determine the maximum compressive stress developed in the cross-section when the eccentric force of  $P = 50$  kN is applied.



- A)  $\sigma = 2.34$  MPa
- B)  $\sigma = 4.46$  MPa
- C)  $\sigma = 6.51$  MPa
- D)  $\sigma = 8.62$  MPa

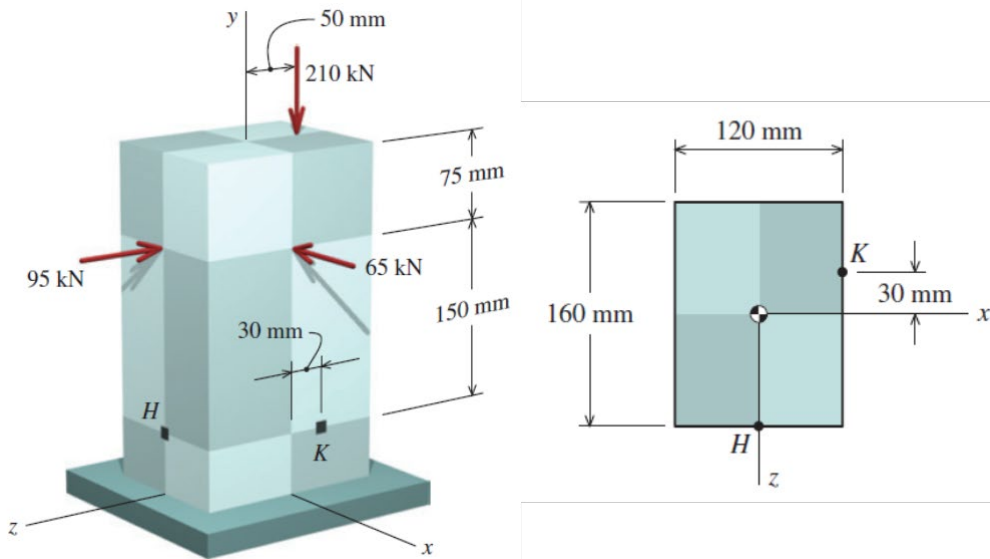
**Problem 9B**

Suppose the wood has an allowable normal stress of  $\sigma_{\text{allow}} = 6$  MPa. Determine the maximum eccentric force that can be applied to the vertical member.

- A)  $P_{\text{max}} = 44.6$  kN
- B)  $P_{\text{max}} = 86.2$  kN
- C)  $P_{\text{max}} = 128$  kN
- D)  $P_{\text{max}} = 165$  kN

**Problem 10** (Philpot, 2013, w/ permission)

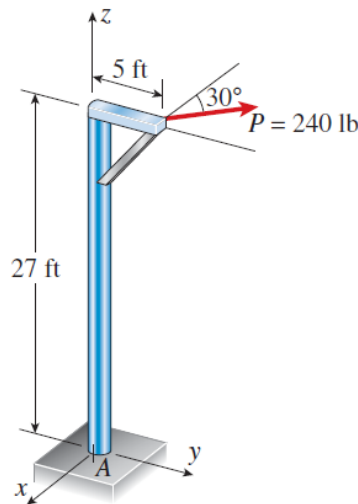
Three loads are applied to the short rectangular post shown in the next figure. The cross-sectional dimensions of the post are shown as well. Determine the normal and shear stresses at points  $H$  and  $K$ . Which of the following is *true*?



- A) The maximum tensile stress at point  $H$  is greater than 40 MPa.
- B) The maximum in-plane shear stress at point  $H$  is greater than 25 MPa.
- C) The maximum tensile stress at point  $K$  is greater than 5 MPa.
- D) The maximum in-plane shear stress at point  $K$  is greater than 5 MPa.

**Problem 11A** (Gere & Goodno, 2009, w/ permission)

A post having a hollow circular cross-section supports a horizontal load  $P = 240$  lb acting at the end of an arm that is 5 ft long (see figure). The height of the post is 27 ft, and its section modulus is  $S = 15$  in.<sup>3</sup> Assume that the outer radius of the post  $r_2 = 4.5$  in. and the inner radius  $r_1 = 4.243$  in. Calculate the maximum tensile stress and maximum in-plane shear stress at point  $A$  on the outer surface of the post along the  $x$ -axis due to the load  $P$ .



- A)  $\sigma_1 = 4.53$  ksi and  $\tau_{\max} = 0.881$  ksi
- B)  $\sigma_1 = 4.53$  ksi and  $\tau_{\max} = 2.29$  ksi
- C)  $\sigma_1 = 6.33$  ksi and  $\tau_{\max} = 0.881$  ksi
- D)  $\sigma_1 = 6.33$  ksi and  $\tau_{\max} = 2.29$  ksi

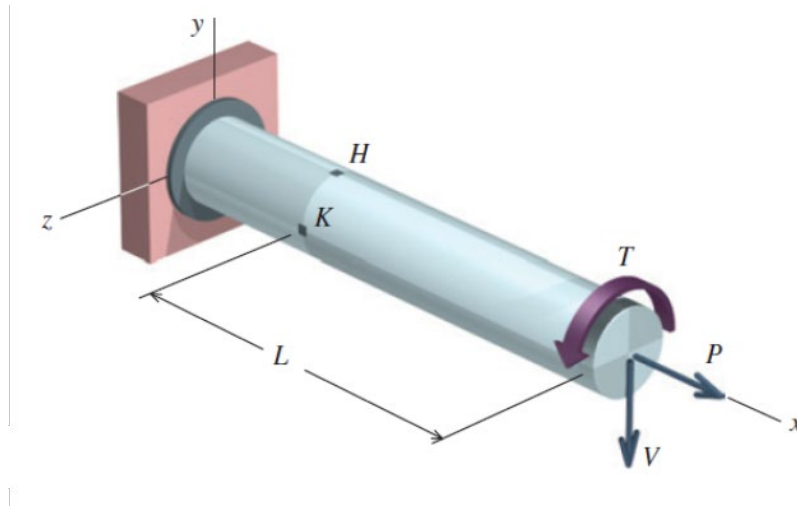
**Problem 11B**

If the maximum tensile stress and maximum in-plane shear stress at point  $A$  are limited to 16,000 psi and 6000 psi respectively, what is the maximum permissible value of the load  $P$ ?

- A)  $P_{\text{allow}} = 629$  lb
- B)  $P_{\text{allow}} = 848$  lb
- C)  $P_{\text{allow}} = 1060$  lb
- D)  $P_{\text{allow}} = 1250$  lb

**Problem 12** (Philpot, 2013, w/ permission)

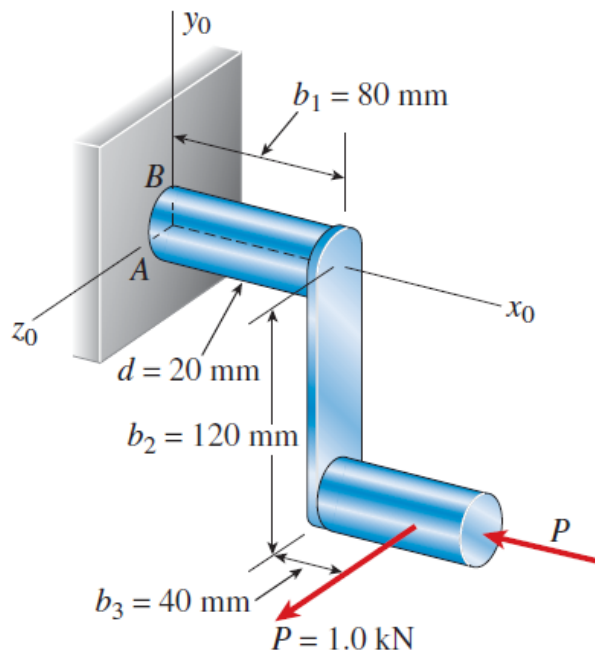
A 40-mm diameter solid shaft is subjected to an axial force of  $P = 2600$  N, a vertical force of  $V = 1700$  N, and a concentrated torque of  $T = 60$  N·m acting on the directions shown in the figure. Assume  $L = 130$  mm. Which of the following is *false*?



- A) The maximum tensile stress at point  $H$  is greater than 40 MPa.
- B) The maximum in-plane shear stress at point  $H$  is greater than 16 MPa.
- C) The maximum tensile stress at point  $K$  is greater than 6 MPa.
- D) The maximum in-plane shear stress at point  $K$  is greater than 6 MPa.

**Problem 13** (Gere & Goodno, 2009, w/ permission)

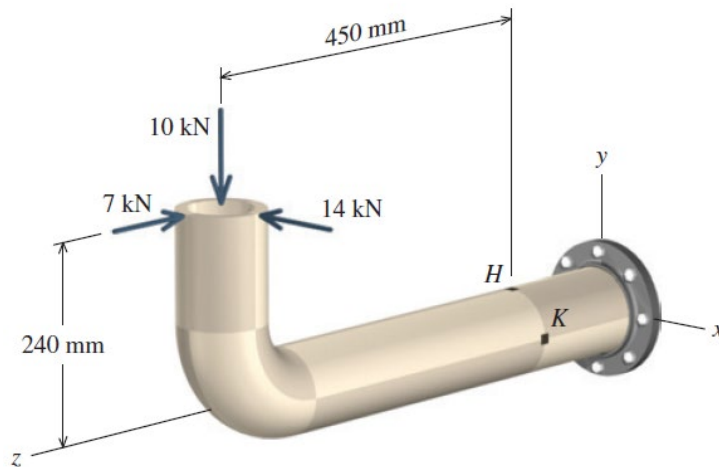
For purposes of analysis, a segment of the crank-shaft in a vehicle is represented as shown in the next figure. Two loads  $P$  act as shown, one parallel to  $-x_0$  and another parallel to  $+z_0$ . Each load  $P$  equals 1.0 kN. The crankshaft dimensions are  $b_1 = 80$  mm,  $b_2 = 120$  mm, and  $b_3 = 40$  mm. The diameter of the upper shaft is  $d = 20$  mm. Investigate the stresses at point  $A$ , which is located on the surface of the upper shaft at the  $z_0$  axis, and point  $B$ , which is located on the surface of the shaft at the  $y_0$  axis. Which of the following is *true*?



- A) The maximum tensile stress at point  $A$  is greater than 40 MPa.
- B) The maximum in-plane shear stress at point  $A$  is greater than 120 MPa.
- C) The maximum tensile stress at point  $B$  is greater than 160 MPa.
- D) The maximum in-plane shear stress at point  $B$  is greater than 120 MPa.

**Problem 14** (Philpot, 2013, w/ permission)

A steel pipe with an outside diameter of 95 mm and an inside diameter of 85 mm supports the loadings shown in the next figure. Investigate the stresses on the top surface of the pipe at point *H* and on the side of the pipe at point *K*. Which of the following is *false*?



- A) The maximum tensile stress at point *H* is greater than 120 MPa.
- B) The maximum in-plane shear stress at point *H* is greater than 80 MPa.
- C) The maximum tensile stress at point *K* is greater than 220 MPa.
- D) The maximum in-plane shear stress at point *K* is greater than 100 MPa.

**SOLUTIONS**

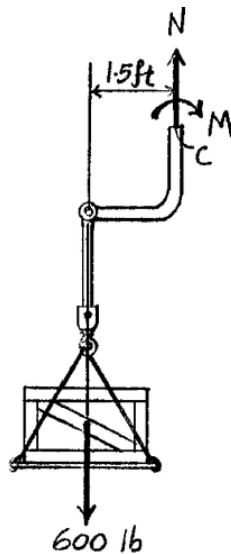
**P.1** → **Solution**

The cross-sectional area and the moment of inertia about the centroidal axis of the member are

$$A = \pi \times 1^2 = 3.14 \text{ in.}^2$$

$$I = \frac{\pi}{4} \times 1^4 = 0.785 \text{ in.}^4$$

Consider the free-body diagram of the bottom cut segment.



Summing forces in the vertical direction gives

$$\Sigma F_y = 0 \rightarrow N - 600 = 0$$

$$\therefore N = 600 \text{ lb}$$

Taking moments about point *C* gives

$$\Sigma M_C = 0 \rightarrow 600 \times 1.5 - M = 0$$

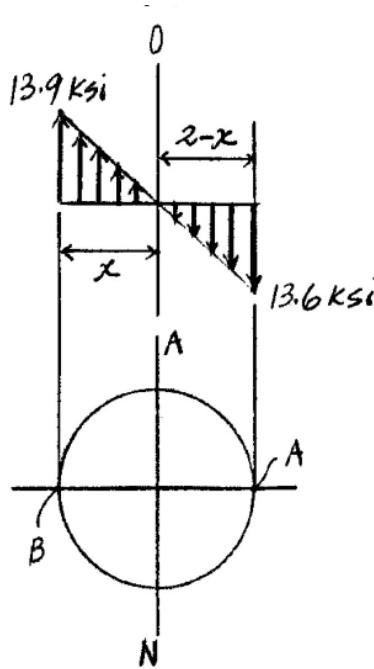
$$\therefore M = 900 \text{ lb-ft}$$

The normal stress is a combination of axial and bending stress. In mathematical terms,

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

By observation, the maximum normal stress occurs at point B in the figure below. Therefore,

$$\sigma_{\max} = \sigma_B = \frac{600}{3.14} + \frac{(900 \times 12) \times 1}{0.785} = \boxed{13.9 \text{ ksi}}$$



If need be, we can also determine the stress at point A, located in the opposite end of the cross-section,

$$\sigma = \frac{600}{3.14} - \frac{(900 \times 12) \times 1}{0.785} = -13.6 \text{ ksi}$$

Using similar triangles, we can also establish the location of the neutral axis,

$$\frac{3.14}{13.9} = \frac{2-x}{13.6} \rightarrow x = -1.07 \text{ in.}$$

That is, the neutral axis is located about 1.07 inches away from the centroid of the section.

ⓘ The correct answer is **B**.

## P.2 → Solution

The area of the section is  $A = \pi \times 0.04^2/4 = 1.26 \times 10^{-3} \text{ m}^2$  and the polar moment of inertia is  $J = \pi \times 0.04^4/32 = 2.51 \times 10^{-7} \text{ m}^4$ . The magnitude of the normal stress is

$$\sigma_x = \frac{P}{A} = \frac{12 \times 10^3}{1.26 \times 10^{-3}} = 9.52 \text{ MPa}$$

The torque in the propeller shaft, in turn, is

$$\dot{W} = T\omega \rightarrow T = \frac{\dot{W}}{\omega}$$

$$\therefore T = \frac{100 \times 10^3}{(2\pi \times 1600/60)} = 597 \text{ N} \cdot \text{m}$$

The shear stress follows as

$$\tau = \frac{Tr}{J} = \frac{597 \times 0.02}{2.51 \times 10^{-7}} = 47.6 \text{ MPa}$$

We can then determine the maximum tensile stress,



$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{9.52 + 0}{2} + \sqrt{\left(\frac{9.52 - 0}{2}\right)^2 + 47.6^2} = \boxed{52.6 \text{ MPa}}$$

The maximum in-plane shear stress is determined as

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{9.52 - 0}{2}\right)^2 + 47.6^2} = \boxed{47.8 \text{ MPa}}$$

☉ The correct answer is **B**.

### P.3 → Solution

The area of the section is  $A = \pi \times 0.06^2/4 = 2.83 \times 10^{-3} \text{ m}^2$  and the polar moment of inertia is  $J = \pi \times 0.06^4/32 = 1.27 \times 10^{-6} \text{ m}^4$ . The magnitude of the normal stress is

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{2.83 \times 10^{-3}} = 14.1 \text{ MPa}$$

The tensile principal stress must not exceed 100 MPa. Mathematically,

$$\begin{aligned} \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} &\leq 100 \text{ MPa} \rightarrow \frac{14.1 + 0}{2} + \sqrt{\left(\frac{14.1 - 0}{2}\right)^2 + \tau_{xy}^2} \leq 100 \\ \therefore 7.05 + \sqrt{49.7 + \tau_{xy}^2} &\leq 100 \\ \therefore \tau_{xy} &\leq 92.7 \text{ MPa} \end{aligned}$$

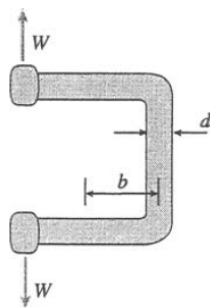
Therefore, the torque applied to the shaft must be limited to

$$\begin{aligned} \tau_{xy} = \frac{T_{\max} r}{J} \rightarrow T_{\max} &= \frac{\tau_{xy} J}{r} \\ \therefore T_{\max} &= \frac{(92.7 \times 10^6) \times (1.27 \times 10^{-6})}{0.03} = \boxed{3.92 \text{ kN} \cdot \text{m}} \end{aligned}$$

☉ The correct answer is **A**.

### P.4 → Solution

Refer to the following figure.



The maximum tensile stress is given by a combination of normal stress and bending stress. In mathematical terms,

$$\sigma_t = \frac{W}{A} + \frac{M}{S} = \frac{4W}{\pi d^2} + \frac{32Wb}{\pi d^3}$$

Manipulating this relation, we obtain

$$\left(\frac{\pi \sigma_t}{4W}\right) d^3 - d - 8b = 0$$

Substituting the numerical data, it follows that

$$\left[ \frac{\pi \times (100 \times 10^6)}{4 \times (12/2 \times 10^3)} \right] d^3 - d - 8 \times 0.18 = 0$$

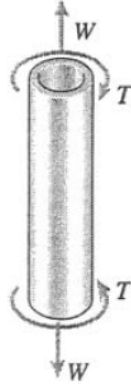
$$\therefore 13,090d^3 - d - 1.44 = 0$$

Solving the equation above numerically, we obtain  $d_{\min} = 0.0484 \text{ m} = 48.4 \text{ mm}$ . We should also verify the shear stress in the arm, which, for uniaxial stress, is written as  $\tau_{\max} = \sigma_t/2$ . Since  $\tau_{\text{allow}}$  is one-half of  $\sigma_{\text{allow}}$ , the minimum diameter for shear is the same as that for tension.

⦿ The correct answer is **A**.

### P.5 → Solution

The loadings on the pendulum are illustrated below.



The tensile loading on the wire is the weight  $W = Mg = 60 \times 9.81 = 589 \text{ N}$ , and the cross-sectional area of the wire is  $A = \pi \times 0.004^2/4 = 1.26 \times 10^{-5} \text{ m}^2$ . The corresponding tensile stress is

$$\sigma_y = \frac{W}{A} = \frac{589}{1.26 \times 10^{-5}} = 46.7 \text{ MPa}$$

The torque exerted on the wire follows from the torque-twist relation

$$T = \frac{GJ\phi_{\max}}{L}$$

which, inserting in the equation for shear stress, yields

$$\tau = -\frac{Tr}{J} = -\frac{G\cancel{J}\phi_{\max}}{L} \times \frac{r}{\cancel{J}} = -\frac{Gr\phi_{\max}}{L}$$

$$\therefore \tau = -\frac{(80 \times 10^9) \times 0.002}{2} \phi_{\max} = -8 \times 10^7 \phi_{\max}$$

or, equivalently,  $\tau = 80\phi_{\max} \text{ MPa}$ . The principal stresses are given by

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The maximum tensile stress has been specified as  $\sigma_{\text{allow}} = 100 \text{ MPa}$ . Equating this quantity to the major principal stress and solving for  $\phi_{\max}$  gives

$$\sigma_1 = \sigma_{\text{allow}} = 100 \text{ MPa} \rightarrow \frac{0 + 46.7}{2} + \sqrt{\left(\frac{0 + 46.7}{2}\right)^2 + (80\phi_{\max})^2} = 100$$

$$\therefore 23.4 + \sqrt{545 + 6400\phi_{\max}^2} = 100$$

$$\therefore \phi_{\max} = 52.3^\circ$$

The maximum allowable shear stress must not exceed  $\tau_{\text{allow}} = 50 \text{ MPa}$ . Since  $\sigma_1$  is positive and  $\sigma_2$  is negative, we surmise that the maximum in-plane shear stress is greater than the maximum out-of-plane shear stress. The maximum angle of rotation based on in-plane shear is then

$$\tau_{xy} = \tau_{\text{allow}} = 50 \text{ MPa} \rightarrow \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2} + \tau_{xy}^2 = 50$$

$$\therefore \sqrt{\left(\frac{0 - 46.7}{2}\right)^2} + (80\phi_{\text{max}})^2 = 50$$

$$\therefore \phi_{\text{max}} = 31.7^\circ$$

The lower result controls, and the maximum permissible angle of rotation is taken as  $31.7^\circ$ .

ⓘ The correct answer is **C**.

### P.6 → Solution

The area of the cross-section is  $A = (\pi/4)(0.15^2 - 0.13^2) = 4.4 \times 10^{-3} \text{ m}^2$  and the polar moment of inertia is  $J = (\pi/32)(0.15^4 - 0.13^4) = 2.17 \times 10^{-5} \text{ m}^4$ . The normal stress at point K is

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{4.4 \times 10^{-3}} = 17 \text{ MPa}$$

Considering equilibrium of moments about the x-direction, the torque exerted on segment (1) is determined as

$$\Sigma M_x = 0 \rightarrow -T_1 - 16 + 7 = 0$$

$$\therefore T_1 = -9 \text{ kN} \cdot \text{m}$$

The shear stress at point H is then

$$\tau = \frac{T_1 r}{J} = \frac{(-9 \times 10^3) \times 0.075}{2.17 \times 10^{-5}} = -31.1 \text{ MPa}$$

The maximum tensile stress at point H is given by

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2} + \tau_{xy}^2 = \frac{17 + 0}{2} + \sqrt{\left(\frac{17 - 0}{2}\right)^2} + (-31.1)^2 = 40.7 \text{ MPa}$$

The maximum in-plane shear stress at point H follows as

$$\tau_{\text{max}} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2} + \tau_{xy}^2 = \sqrt{\left(\frac{17 + 0}{2}\right)^2} + (-31.1)^2 = 32.2 \text{ MPa}$$

The normal stress in point K is the same as that in point H, that is,  $\sigma = P/A = 17 \text{ MPa}$ . From equilibrium in the x-direction, the torque exerted on segment (2) is calculated as

$$\Sigma M_x = 0 \rightarrow -T_2 + 7 = 0$$

$$\therefore T_2 = 7 \text{ kN} \cdot \text{m}$$

Therefore, the shear stress in point K is

$$\tau = \frac{T_2 r}{J} = \frac{(7 \times 10^3) \times 0.075}{2.17 \times 10^{-5}} = 24.2 \text{ MPa}$$

The maximum tensile stress in point K is given by

$$\sigma_1 = \frac{17 + 0}{2} + \sqrt{\left(\frac{17 - 0}{2}\right)^2} + 24.2^2 = 34.1 \text{ MPa}$$

The maximum in-plane shear stress at point K follows as

$$\tau_{\text{max}} = \sqrt{\left(\frac{17 + 0}{2}\right)^2} + 24.2^2 = 25.6 \text{ MPa} < 27 \text{ MPa}$$

ⓘ The false statement is **D**.

### P.7 → Solution

The cross-sectional area of segment (1) is  $A = (\pi/4)(0.22^2 - 0.20^2) = 6.6 \times 10^{-3} \text{ m}^2$  and the polar moment of inertia is  $J = (\pi/32)(0.22^4 - 0.20^4) = 7.29 \times 10^{-5} \text{ m}^4$ . The normal stress in point  $H$  is

$$\sigma = \frac{P}{A} = \frac{-100 \times 10^3}{6.6 \times 10^{-3}} = -15.2 \text{ MPa}$$

Recall that a negative sign denotes a compressive stress. From equilibrium in the  $x$ -direction, the torque exerted on segment (1) is calculated as

$$\begin{aligned}\Sigma M_x = 0 &\rightarrow -T_1 + 8 - 12 = 0 \\ \therefore T_1 &= -4 \text{ kN} \cdot \text{m}\end{aligned}$$

Therefore, the shear stress in point  $H$  is

$$\tau = \frac{T_2 r}{J} = \frac{(-4 \times 10^3) \times 0.11}{7.29 \times 10^{-5}} = -6.04 \text{ MPa}$$

The maximum tensile stress in point  $H$  is then

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-15.2 + 0}{2} + \sqrt{\left(\frac{-15.2 - 0}{2}\right)^2 + (-6.04)^2} = 2.11 \text{ MPa}$$

The maximum in-plane shear stress at point  $H$  follows as

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-15.2 + 0}{2}\right)^2 + (-6.04)^2} = 9.71 \text{ MPa}$$

The cross-sectional area of segment (2) is  $A = (\pi/4)(0.14^2 - 0.11^2) = 5.89 \times 10^{-3} \text{ m}^2$  and the polar moment of inertia is  $J = (\pi/32)(0.14^4 - 0.11^4) = 2.33 \times 10^{-5} \text{ m}^4$ . The normal stress at segment (2) is

$$\sigma = \frac{P}{A} = \frac{-100 \times 10^3}{5.89 \times 10^{-3}} = -17 \text{ MPa}$$

Considering equilibrium of moments about the  $x$ -direction, the torque exerted on segment (2) is determined as

$$\begin{aligned}\Sigma M_x = 0 &\rightarrow -T_2 - 12 = 0 \\ \therefore T_2 &= -12 \text{ kN} \cdot \text{m}\end{aligned}$$

The shear stress in  $K$  is then

$$\tau = \frac{T_2 r}{J} = \frac{(-12 \times 10^3) \times 0.07}{2.33 \times 10^{-5}} = -36.1 \text{ MPa}$$

The maximum tensile stress in  $K$  is given by

$$\sigma_1 = \frac{-17 + 0}{2} + \sqrt{\left(\frac{-17 - 0}{2}\right)^2 + (-36.1)^2} = 28.6 \text{ MPa} < 30 \text{ MPa}$$

The maximum in-plane shear stress in  $K$  follows as

$$\tau_{\max,K} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-17 + 0}{2}\right)^2 + (-36.1)^2} = 37.1 \text{ MPa}$$

⦿ The false statement is **C**.

### P.8 → Solution

The cross-sectional area of the post is  $A = 5 \times 10 = 50 \text{ in}^2$ , the moment of inertia about the  $y$ -axis is  $I_y = 10 \times 5^3 / 12 = 104 \text{ in}^4$  and the moment of inertia about the  $z$ -axis is  $I_z = 5 \times 10^3 / 12 = 417 \text{ in}^4$ . The equivalent loadings at the base of the post are a force  $F = P =$

2500 lb and two bending moments  $M_y = 2500 \times (-2) = -5000$  lb-in. and  $M_z = 2500 \times (-3) = -7500$  lb-in. The normal stress due to force  $F$  is calculated as

$$\sigma = \frac{F}{A} = \frac{2500}{50} = 50 \text{ psi (C)}$$

The bending stress due to  $M_y$  is computed as

$$\sigma_{b,y} = \frac{M_y z}{I_y} = \frac{-5000 \times (\pm 2.5)}{104} = \pm 120 \text{ psi}$$

while that due to  $M_z$  follows as

$$\sigma_{b,z} = \frac{M_z y}{I_z} = \frac{-7500 \times (\pm 5)}{417} = \pm 89.9 \text{ psi}$$

We used plus-or-minus signs in the latter calculations because the sense of the stress acting in each of the specified points, A, B, C, or D, can be determined by inspection. The normal stress in each corner can be established by superposing the appropriate results, as follows. For corner A, we have

$$\sigma_A = 50 \text{ psi (C)} + 120 \text{ psi (C)} + 90 \text{ psi (C)}$$

$$\therefore \sigma_A = -50 - 120 - 90 = -260 \text{ psi}$$

$$\therefore |\sigma_A| = 260 \text{ psi}$$

In a similar manner, for corner B,

$$\sigma_B = 50 \text{ psi (C)} + 120 \text{ psi (T)} + 90 \text{ psi (C)}$$

$$\therefore \sigma_B = -50 + 120 - 90 = -20 \text{ psi}$$

$$\therefore |\sigma_B| = 20 \text{ psi}$$

Similarly, for corner C,

$$\sigma_C = 50 \text{ psi (C)} + 120 \text{ psi (T)} + 90 \text{ psi (T)}$$

$$\therefore \sigma_C = -50 + 120 + 90 = +160 \text{ psi}$$

$$\therefore |\sigma_C| = 160 \text{ psi}$$

Likewise, for corner D,

$$\sigma_D = 50 \text{ psi (C)} + 120 \text{ psi (C)} + 90 \text{ psi (T)}$$

$$\therefore \sigma_D = -50 - 120 + 90 = -80 \text{ psi}$$

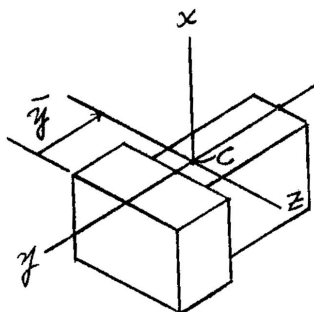
$$\therefore |\sigma_D| = 80 \text{ psi} \neq 60 \text{ psi}$$

⦿ The false statement is D.

### P.9 → Solution

**Part A:** Refer to the figure below. The location of the centroid of the cross-section is determined as

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.075 \times 0.15 \times 0.3 + 0.3 \times 0.3 \times 0.15}{0.15 \times 0.3 + 0.3 \times 0.15} = 0.188 \text{ m}$$

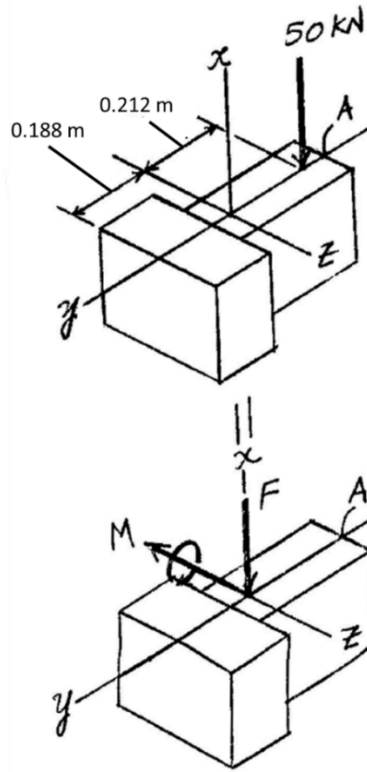


The cross-sectional area and the moment of inertia about the z-axis are

$$A = 0.15 \times 0.3 + 0.3 \times 0.15 = 0.09 \text{ m}^2$$

$$I_z = \frac{1}{12} \times 0.3 \times 0.15^2 + 0.3 \times 0.15 \times (0.188 - 0.075)^2 + \frac{1}{12} \times 0.15 \times 0.3^3 + 0.15 \times 0.3 \times (0.3 - 0.188)^2 = 1.56 \times 10^{-3} \text{ m}^4$$

We replace the 50-kN force with an equivalent force  $F$  and a moment  $M$  acting at the centroid, as shown.



Summing forces in the x-direction gives

$$\begin{aligned} \Sigma F_x &= 0 \rightarrow -50 = -F \\ \therefore F &= 50 \text{ kN} \end{aligned}$$

Taking moments about the z-axis gives

$$\begin{aligned} \Sigma M_z &= 0 \rightarrow -50 \times 0.212 = -M \\ \therefore M &= 10.6 \text{ kN} \cdot \text{m} \end{aligned}$$

The normal stress is a combination of axial and bending stresses. In mathematical terms,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress occurs at points along the edge where  $y = 0.45 - 0.188 = 0.262 \text{ m}$ , such as point A in the illustration above. The stress in question is calculated as

$$\sigma = \frac{-50 \times 10^3}{0.09} - \frac{(10.6 \times 10^3) \times 0.262}{1.56 \times 10^{-3}} = \boxed{2.34 \text{ MPa (C)}}$$

ⓘ The correct answer is **A**.

**Part B:** As before, the location of the centroid is  $\bar{y} = 0.188 \text{ m}$ , the cross-sectional area is  $A = 0.09 \text{ m}^2$ , and the moment of inertia about the z-axis  $I_z = 1.56 \times 10^{-3} \text{ m}^4$ . We propose an equivalent force system similar to the one in the previous part. Summing forces in the x-direction gives

$$\begin{aligned} \Sigma F_x &= 0 \rightarrow -P = -F \\ \therefore P &= F \end{aligned}$$

Taking moments about the z-axis gives

$$\Sigma M_z = 0 \rightarrow -P \times 0.212 = -M$$

$$\therefore M = 0.212P$$

The normal stress is a combination of axial and bending stress; that is,

$$F = \frac{N}{A} \pm \frac{My}{I}$$

By inspection, the maximum normal stress, which is in compression, occurs at the edge of point A in the foregoing illustration. With  $\sigma_{\text{allow}} = 6 \text{ MPa}$ , we have

$$-6 \times 10^6 = -\frac{P}{0.09} - \frac{0.212P \times 0.262}{1.56 \times 10^{-3}}$$

$$\therefore -6 \times 10^6 = -11.1P - 35.6P$$

$$\therefore \boxed{P_{\text{max}} = 128 \text{ kN}}$$

⦿ The correct answer is **C**.

### P.10 → Solution

The cross-sectional area of the post is  $A = 0.12 \times 0.16 = 0.0192 \text{ m}^2$ , the moment of inertia about the  $x$ -axis is  $I_x = 0.12 \times 0.16^3 / 12 = 4.1 \times 10^{-5} \text{ m}^4$  and the moment of inertia about the  $z$ -axis is  $I_z = 0.16 \times 0.12^3 / 12 = 2.3 \times 10^{-5} \text{ m}^4$ . The equivalent loadings at  $K$  are a normal force  $F = 210 \text{ kN}$ , a shearing force  $V_x = -65 \text{ kN}$  in the  $x$ -direction, a shearing force  $V_z = -95 \text{ kN}$  in the  $z$ -direction, and two bending moments  $M_x$  and  $M_z$ , which are respectively determined as

$$M_x = -95 \times 0.15 - 210 \times 0.05 = -24.8 \text{ kN} \cdot \text{m}$$

and

$$M_z = 65 \times 0.15 = 9.75 \text{ kN} \cdot \text{m}$$

We shall use the absolute values of these and other loadings and attribute signs to the calculated stresses by inspection. The axial stress at point  $H$  due to  $F$  is given by

$$\sigma = \frac{F}{A} = \frac{210 \times 10^3}{0.0192} = 10.9 \text{ MPa}$$

Given the first moment  $Q_H = 0.16 \times 0.06 \times 0.03 = 2.88 \times 10^{-4} \text{ m}^3$ , the shear stress at point  $H$  due to  $V_x$  is

$$\tau = \frac{V_x Q_H}{I_z b} = \frac{(65 \times 10^3) \times (2.88 \times 10^{-4})}{(2.3 \times 10^{-5}) \times 0.16} = 5.09 \text{ MPa}$$

Since element  $H$  lies on the reference axis,  $Q_H = 0$  and force  $V_z$  does not contribute to the shear stress at that point. Next, the bending stress at  $H$  due to bending moment  $M_x$  follows from the flexure formula,

$$\sigma_b = \frac{M_x z}{I_x} = \frac{(24.8 \times 10^3) \times 0.08}{4.1 \times 10^{-5}} = 48.4 \text{ MPa}$$

Finally, bending moment  $M_z$  would also produce a bending stress in point  $H$ , given by  $\sigma_b = M_z x / I_z$ , but the point in question is positioned such that  $x = 0$ , and hence  $\sigma_b = 0$ . Cleaning the calculated stresses, we have  $\sigma_x = 0$ ,  $\sigma_y = -10.9 + 48.4 = 37.5 \text{ MPa}$ , and  $\tau_{xy} = -5.09 \text{ MPa}$ . We are now ready to compute the principal stress at point  $H$ ,

$$\sigma_1 = \frac{0 + 37.5}{2} + \sqrt{\left(\frac{0 - 37.5}{2}\right)^2 + (-5.09)^2} = 38.2 \text{ MPa}$$

The maximum in-plane shear stress follows as

$$\tau_{\max} = \sqrt{\left(\frac{0+37.5}{2}\right)^2 + (-5.09)^2} = 19.4 \text{ MPa}$$

We can then proceed to investigate the stresses at point K. The axial stress due to force  $F$  is the same as before; that is,  $\sigma = F/A = 10.9 \text{ MPa}$ . Since element K lies on the reference axis,  $Q_K = 0$  and force  $V_x$  provides no contribution to the shear stress at the point in question. Next, given the first moment  $Q_K = 0.12 \times 0.05 \times 0.055 = 3.3 \times 10^{-4} \text{ m}^3$ , the shear stress at K due to shearing force  $V_z$  is determined as

$$\tau = \frac{V_z Q_K}{I_x b} = \frac{(95 \times 10^3) \times (3.3 \times 10^{-4})}{(4.1 \times 10^{-5}) \times 0.12} = 6.37 \text{ MPa}$$

The bending stress at K due to moment  $M_x$  is

$$\sigma_b = \frac{M_x z}{I_x} = \frac{(24.8 \times 10^3) \times 0.03}{4.1 \times 10^{-5}} = 18.1 \text{ MPa}$$

The bending stress at K due to moment  $M_z$  is

$$\sigma_b = \frac{M_z x}{I_z} = \frac{(9.75 \times 10^3) \times 0.06}{2.3 \times 10^{-5}} = 25.4 \text{ MPa}$$

Gathering the calculated stresses, we have  $\sigma_x = 0$ ,  $\sigma_y = -10.9 - 18.1 + 25.4 = -3.6 \text{ MPa}$ , and  $\tau_{xy} = -6.37 \text{ MPa}$ . We can now compute the major principal stress at K,

$$\sigma_1 = \frac{0-3.6}{2} + \sqrt{\left[\frac{0-(-3.6)}{2}\right]^2 + (-6.37)^2} = 4.82 \text{ MPa}$$

The maximum in-plane shear stress follows as

$$\tau_{\max} = \sqrt{\left[\frac{0+(-3.6)}{2}\right]^2 + (-6.37)^2} = 6.62 \text{ MPa} > 5 \text{ MPa}$$

☉ The correct answer is **D**.

## P.11 → Solution

**Part A:** The cross-sectional area of the shaft is  $A = \pi \times 0.04^2/4 = 1.26 \times 10^{-3} \text{ in}^2$ , the first moment of area is  $Q = 2/3 \times (4.5^3 - 4.243^3) = 9.83 \text{ in}^3$ , the polar moment of inertia is  $J = \pi/2 \times (4.5^4 - 4.243^4) = 135 \text{ in}^4$ , and the moments of inertia about the  $y$ - and  $z$ -axes are both  $I_y = I_z = \pi/4 \times (4.5^4 - 4.243^4) = 67.5 \text{ in}^4$ . The loadings at the support are a bending moment  $M = 240 \times (27 \times 12) \times \cos 30^\circ = 6.73 \times 10^4 \text{ lb-in.}$ , a torque  $T = -240 \times (5 \times 12) \times \cos 30^\circ = -1.25 \times 10^4 \text{ lb-in.}$ , a shearing force  $V_x = 240 \times \cos 30^\circ = 208 \text{ lb}$ , and a shearing force  $V_y = -240 \times \sin 30^\circ = -120 \text{ lb}$ . We shall use the absolute values of these loadings and attribute signs to the calculated stresses by inspection. The bending stress due to  $M$  follows from the flexure formula,

$$\sigma_b = \frac{M r_o}{I_y} = \frac{(6.73 \times 10^4) \times 4.5}{67.5} = 4.49 \times 10^3 \text{ psi}$$

The contribution of torque  $T$  to shear stress is obtained with the torsion formula,

$$\tau = \frac{T r_o}{J} = \frac{(1.25 \times 10^4) \times 4.5}{135} = 417 \text{ psi}$$

The contribution of shearing force  $V_y$  to shear stress is computed with the shear formula,

$$\tau = \frac{V_y Q}{2 I_z t} = \frac{120 \times 9.83}{2 \times 67.5 \times (4.5 - 4.243)} = 34 \text{ psi}$$



Gathering the calculated stresses, we have  $\sigma_x = 0$ ,  $\sigma_y = 4490$  psi, and  $\tau_{xy} = -417 - 34 = -451$  psi. We can now compute the maximum tensile stress at point A,

$$\sigma_1 = \frac{0 + 4490}{2} + \sqrt{\left(\frac{0 - 4490}{2}\right)^2 + (-451)^2} = \boxed{4.53 \text{ ksi}}$$

The maximum in-plane shear stress follows as

$$\tau_{\max} = \sqrt{\left(\frac{0 + 4490}{2}\right)^2 + (-451)^2} = \boxed{2.29 \text{ ksi}}$$

☐ The correct answer is **B**.

**Part B:** Since the stresses at point A are proportional to the load  $P$ , we write, on the basis of tensile stress,

$$\frac{P_{\text{allow}}}{P} = \frac{\sigma_{\text{allow}}}{\sigma_{\max}} \rightarrow P_{\text{allow}} = P \times \frac{\sigma_{\text{allow}}}{\sigma_{\max}}$$

$$\therefore P_{\text{allow}} = 240 \times \frac{16,000}{4530} = 848 \text{ lb}$$

Similarly, we have, on the basis of shear stress,

$$\frac{P_{\text{allow}}}{P} = \frac{\tau_{\text{allow}}}{\tau_{\max}} \rightarrow P_{\text{allow}} = P \times \frac{\tau_{\text{allow}}}{\tau_{\max}}$$

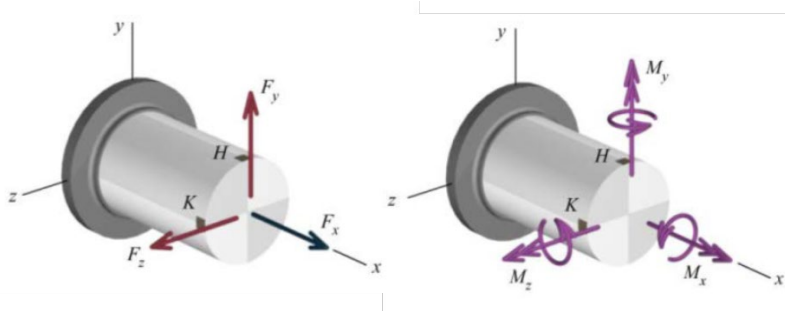
$$\therefore P_{\text{allow}} = 240 \times \frac{6000}{2290} = 629 \text{ lb}$$

The lower result controls, and hence  $P_{\text{allow}} = 629$  lb.

☐ The correct answer is **A**.

## P.12 → Solution

The cross-sectional area of the shaft is  $A = \pi \times 0.04^2/4 = 1.26 \times 10^{-3} \text{ m}^2$ , the first moment of area is  $Q = 0.04^3/12 = 5.33 \times 10^{-6} \text{ m}^3$ , the polar moment of inertia is  $J = \pi \times 0.04^4/32 = 2.51 \times 10^{-7} \text{ m}^4$ , and the moments of inertia about the  $y$ - and  $z$ -axes are both  $I_y = I_z = \pi \times 0.04^4/64 = 1.26 \times 10^{-7} \text{ m}^4$ . The forces acting at points H and K are  $F_x = 2600 \text{ N}$ ,  $F_y = -1700 \text{ N}$ , and  $F_z = 0$ , and the equivalent moments are  $M_x = 60 \text{ N}\cdot\text{m}$ ,  $M_y = 0$ , and  $M_z = -1700 \times 0.13 = 221 \text{ N}\cdot\text{m}$ , as illustrated below. We shall use the absolute values of these loadings and attribute signs to the calculated stresses by inspection.



Each of the nonzero forces and moments will be evaluated to determine whether they create stresses at the points of interest. To begin, consider point H. Force  $F_x$  creates an axial stress at H given by the usual relation

$$\sigma = \frac{F_x}{A} = \frac{2600}{1.26 \times 10^{-3}} = 2.06 \text{ MPa}$$

Moment  $M_x$  creates a torsional shear stress in the  $xz$ -plane at H, and its magnitude is calculated as

$$\tau = \frac{M_x r}{J} = \frac{60 \times 0.02}{2.51 \times 10^{-7}} = 4.78 \text{ MPa}$$

Moment  $M_z$  creates a bending stress at H, and its magnitude is calculated as

$$\sigma_b = \frac{M_z y}{I_z} = \frac{221 \times 0.02}{1.26 \times 10^{-7}} = 35.1 \text{ MPa}$$

Thus, the normal stress in the x-direction for point H is  $\sigma_x = 2.06 + 35.1 = 37.2$  MPa in tension, while the shear stress is  $\tau_{xy} = 4.78$  MPa. There is no normal stress in the y-direction. The maximum tensile stress at point H is then

$$\sigma_1 = \frac{37.2 + 0}{2} + \sqrt{\left(\frac{37.2 - 0}{2}\right)^2 + 4.78^2} = 37.8 \text{ MPa} < 40 \text{ MPa}$$

The maximum in-plane shear stress at the point in question, in sequence, is

$$\tau_{\max} = \sqrt{\left(\frac{37.2 + 0}{2}\right)^2 + 4.78^2} = 19.2 \text{ MPa}$$

Consider now the stresses in point K. Force  $F_x$  creates an axial stress at K, and its magnitude is  $\sigma = 2.06$  MPa as calculated above. Next, force  $F_y$  creates a transverse shear stress in the xy-plane at K, and its magnitude is given by the shear formula,

$$\tau = \frac{F_y Q}{I_z b} = \frac{1700 \times (5.33 \times 10^{-6})}{(1.26 \times 10^{-7}) \times 0.04} = 1.8 \text{ MPa}$$

Moment  $M_x$  causes a torsional shear stress in the xy-plane at K, and its magnitude is given by

$$\tau = \frac{M_x r}{J} = \frac{60 \times 0.02}{2.51 \times 10^{-7}} = 4.78 \text{ MPa}$$

Moment  $M_z$  produces no bending stress at K because K is located on the neutral axis for bending about the z-axis. Thus, the normal stress in the x-direction for element K is  $\sigma_x = 2.06$  MPa, while the shear stress is  $\tau = -1.80 - 4.78 = -6.56$  MPa. There is no stress in the y-direction. The maximum tensile stress at point K is then

$$\sigma_1 = \frac{2.06 + 0}{2} + \sqrt{\left(\frac{2.06 - 0}{2}\right)^2 + (-6.56)^2} = 7.67 \text{ MPa}$$

The maximum in-plane shear stress at the point in question, in sequence, is

$$\tau_{\max} = \sqrt{\left(\frac{2.06 + 0}{2}\right)^2 + (-6.56)^2} = 6.64 \text{ MPa}$$

⦿ The false statement is A.

### P.13 → Solution

The cross-sectional area of the shaft is  $A = \pi \times 0.02^2 / 4 = 3.14 \times 10^{-4} \text{ m}^2$ , the first moment of area is  $Q = 2/3 \times 0.01^3 = 6.67 \times 10^{-7} \text{ m}^3$ , the polar moment of inertia is  $J = \pi/32 \times 0.02^4 = 1.57 \times 10^{-8} \text{ m}^4$ , and the moments of inertia about the  $y_0$ - and  $z_0$ -axes are both  $I_y = I_z = \pi/64 \times 0.02^4 = 7.85 \times 10^{-9} \text{ m}^4$ . The loadings at the support are as follows. There is a shearing force in the x-direction,  $V_x = P = 1.0 \text{ kN}$  and a shearing force in the z-direction,  $V_z = P = 1.0 \text{ kN}$ . There is a torsional moment in the x-direction,  $M_x = 1000 \times b_2 = 120 \text{ N}\cdot\text{m}$ , a bending moment in the y-direction,  $M_y = 1000 \times (b_1 + b_2) = 200 \text{ N}\cdot\text{m}$ , and a bending moment in the z-direction,  $M_z = 1000 \times b_2 = 120 \text{ N}\cdot\text{m}$ . We shall use the absolute values of these loadings and attribute signs to the calculated stresses by inspection. Consider the stresses at point A. The normal stress due to shearing force  $V_x$  is calculated as

$$\sigma = -\frac{V_x}{A} = \frac{1000}{3.14 \times 10^{-4}} = 3.18 \text{ MPa}$$

The normal stress caused by bending moment  $M_y$  follows from the flexure formula,

$$\sigma_b = \frac{M_y r}{I_y} = -\frac{200 \times 0.01}{7.85 \times 10^{-9}} = 153 \text{ MPa}$$

The shear stress caused by torsional moment  $M_x$  is computed with the torsion formula,

$$\tau = \frac{M_x \times d/2}{J} = \frac{120 \times 0.02/2}{1.57 \times 10^{-8}} = 76.4 \text{ MPa}$$

Gathering the calculated stresses, we have  $\sigma_x = -3.18 - 153 = -156 \text{ MPa}$ ,  $\sigma_y = 0$ , and  $\tau_{xy} = 76.4 \text{ MPa}$ . We can now compute the maximum tensile stress at point A,

$$\sigma_1 = \frac{-156 + 0}{2} + \sqrt{\left(\frac{-156 - 0}{2}\right)^2 + 76.4^2} = 31.2 \text{ MPa}$$

The maximum in-plane shear stress, in sequence, is determined as

$$\tau_{\max} = \sqrt{\left(\frac{-156 + 0}{2}\right)^2 + 76.4^2} = 109 \text{ MPa}$$

Let us proceed to determine the stresses at point B. The normal stress due to shearing force  $V_x$  is calculated as before, so that  $\sigma = V_x/A = 3.18 \text{ MPa}$ . The bending stress due to bending moment  $M_z$  is determined as

$$\sigma_b = \frac{M_z r}{I_z} = \frac{120 \times 0.01}{7.85 \times 10^{-9}} = 153 \text{ MPa}$$

The shear stress caused by torsional moment  $M_x$  is computed with the torsion formula,

$$\tau = \frac{120 \times 0.02/2}{1.57 \times 10^{-8}} = 76.4 \text{ MPa}$$

There is also a contribution to shear stress caused by force  $V_z$ ,

$$\tau = \frac{V_z Q}{I_y d} = \frac{1000 \times (6.67 \times 10^{-7})}{(7.85 \times 10^{-9}) \times 0.02} = 4.25 \text{ MPa}$$

Gathering the calculated stresses, we have  $\sigma_x = -3.18 + 153 = 150 \text{ MPa}$ ,  $\sigma_y = 0$ , and  $\tau_{xy} = 76.4 + 4.25 = 80.7 \text{ MPa}$ . We can now compute the maximum tensile stress at the point in question,

$$\sigma_1 = \frac{150 + 0}{2} + \sqrt{\left(\frac{150 - 0}{2}\right)^2 + 80.7^2} = 185 \text{ MPa} > 160 \text{ MPa}$$

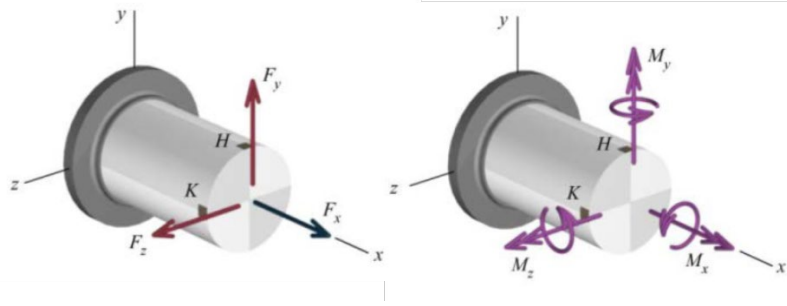
The maximum in-plane shear stress, in sequence, is determined as

$$\tau_{\max} = \sqrt{\left(\frac{150 + 0}{2}\right)^2 + 80.7^2} = 110 \text{ MPa}$$

☉ The correct answer is **C**.

## P.14 → Solution

The cross-sectional area of the pipe is  $A = \pi \times (0.095^2 - 0.085^2) / 4 = 1.41 \times 10^{-3} \text{ m}^2$ , the first moment of area is  $Q = (0.095^3 - 0.085^3) / 12 = 2.03 \times 10^{-5} \text{ m}^3$ , the polar moment of inertia is  $J = \pi \times (0.095^4 - 0.085^4) / 32 = 2.87 \times 10^{-6} \text{ m}^4$ , and the moments of inertia about the  $x$ - and  $y$ -axes are both  $I_x = I_y = \pi \times (0.095^4 - 0.085^4) / 4 = 1.44 \times 10^{-6} \text{ m}^4$ . The forces acting at H and K are  $F_x = -14 \text{ kN}$ ,  $F_y = -10 \text{ kN}$ , and  $F_z = -7 \text{ kN}$ , and the equivalent moments are  $M_x = 10,000 \times 0.45 - 7,000 \times 0.24 = 2820 \text{ N}\cdot\text{m}$ ,  $M_y = -14,000 \times 0.45 = -6300 \text{ N}\cdot\text{m}$ , and  $M_z = 14,000 \times 0.24 = 3360 \text{ N}\cdot\text{m}$ , as illustrated below. We shall use the absolute values of these loadings and attribute signs to the calculated stresses by inspection.



Each of the nonzero forces and moments will be evaluated to determine whether they create stresses at the points of interest. To begin, consider point  $H$ . Force  $F_x$  creates a transverse shear stress in the  $yz$ -plane at  $H$ , and the magnitude of this stress is

$$\tau = \frac{F_x Q}{I_y b} = \frac{14,000 \times (2.03 \times 10^{-5})}{(1.44 \times 10^{-6}) \times (0.095 - 0.085)} = 19.7 \text{ MPa}$$

Force  $F_z$  creates an axial stress at  $H$ , and its magnitude is calculated with the elementary formula

$$\sigma = \frac{F_z}{A} = \frac{7000}{1.41 \times 10^{-3}} = 4.96 \text{ MPa}$$

Moment  $M_x$  creates a bending stress at  $H$ , and its magnitude is calculated with the flexure formula,

$$\sigma_b = \frac{M_x y}{I_x} = \frac{2820 \times 0.095/2}{1.44 \times 10^{-6}} = 93 \text{ MPa}$$

Moment  $M_z$ , which is a torque, creates a torsion shear stress in the  $xz$ -plane at  $H$ , and its magnitude is calculated with the torsion formula,

$$\tau = \frac{M_z r}{J} = \frac{3360 \times 0.095/2}{2.87 \times 10^{-6}} = 55.6 \text{ MPa}$$

Finally, note that force  $F_y$  and moment  $M_y$  create no stresses at the point in question. Gleaning the calculated stresses, we have  $\sigma_x = 0$ ,  $\sigma_z = -4.96 + 93 = 88 \text{ MPa}$ , and  $\tau_{xy} = -19.7 - 55.6 = -75.3 \text{ MPa}$ . We are now ready to compute the maximum tensile stress at point  $H$ ,

$$\sigma_1 = \frac{0 + 88}{2} + \sqrt{\left(\frac{0 - 88}{2}\right)^2 + (-75.3)^2} = 131 \text{ MPa}$$

The maximum in-plane shear stress follows as

$$\tau_{\max} = \sqrt{\left(\frac{0 + 88}{2}\right)^2 + (-75.3)^2} = 87.2 \text{ MPa}$$

We can then proceed to investigate the stresses in point  $K$ . Force  $F_y$  creates a transverse shear stress in the  $yz$ -plane at  $K$ , and its magnitude is given by

$$\tau = \frac{F_y Q}{I_z b} = \frac{10,000 \times (2.03 \times 10^{-5})}{(1.44 \times 10^{-6}) \times (0.095 - 0.085)} = 14.1 \text{ MPa}$$

Force  $F_z$  creates an axial stress at  $K$ , and its magnitude is  $\sigma = F_z/A = 4.96 \text{ MPa}$  as calculated just now. Moment  $M_y$  creates a bending stress at  $K$ , and its magnitude is given by

$$\sigma_b = \frac{M_y x}{I_y} = \frac{6300 \times 0.095/2}{1.44 \times 10^{-6}} = 207 \text{ MPa}$$

Moment  $M_z$  creates a torsional shear stress in the  $yz$ -plane at  $K$ , and its magnitude is  $\tau = 55.6 \text{ MPa}$  as calculated for point  $H$ . Finally, note that force  $F_x$  and moment  $M_x$  create no stresses in the point in question. Gleaning the calculated

stresses, we have  $\sigma_y = 0$ ,  $\sigma_z = -4.96 + 207 = 202$  MPa, and  $\tau_{xy} = -14.1 + 55.6 = 41.5$  MPa. We are now ready to compute the maximum tensile stress at point K,

$$\sigma_1 = \frac{0 + 202}{2} + \sqrt{\left(\frac{0 - 202}{2}\right)^2 + 41.5^2} = 210 \text{ MPa} < 220 \text{ MPa}$$

The maximum in-plane shear stress follows as

$$\tau_{\max} = \sqrt{\left(\frac{0 - 202}{2}\right)^2 + 41.5^2} = 109 \text{ MPa}$$

⦿ The false statement is **C**.

## ANSWER SUMMARY

<b>Problem 1</b>		<b>B</b>
<b>Problem 2</b>		<b>B</b>
<b>Problem 3</b>		<b>A</b>
<b>Problem 4</b>		<b>A</b>
<b>Problem 5</b>		<b>C</b>
<b>Problem 6</b>		<b>D</b>
<b>Problem 7</b>		<b>C</b>
<b>Problem 8</b>		<b>D</b>
<b>Problem 9</b>	<b>9A</b>	<b>A</b>
	<b>9B</b>	<b>C</b>
<b>Problem 10</b>		<b>D</b>
<b>Problem 11</b>	<b>11A</b>	<b>B</b>
	<b>11B</b>	<b>A</b>
<b>Problem 12</b>		<b>A</b>
<b>Problem 13</b>		<b>C</b>
<b>Problem 14</b>		<b>C</b>

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