Montogue

Quiz FM105

Control Volume Analysis – Momentum

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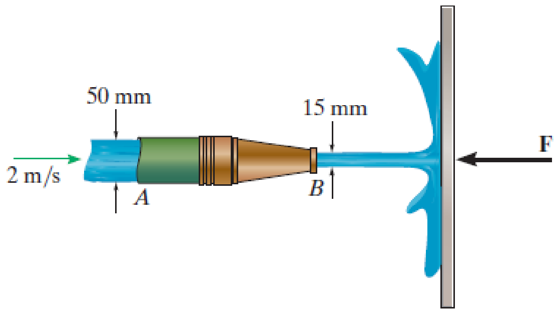
PROBLEMS

Problem 1 (Hibbeler, 2017, w/ permission)

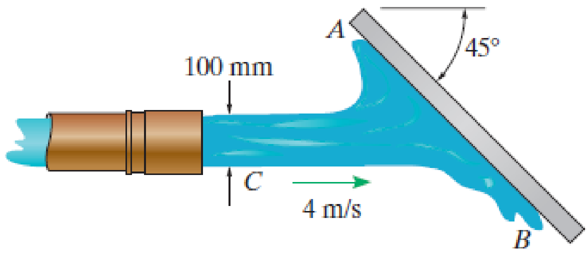
Water flows through the hose with a velocity of 2 m/s. Determine the force *F* needed to keep the circular plate moving to the left at 2 m/s.



A) F = 22.3 N
B) F = 47.4 N
C) F = 71.9 N
D) F = 96.8 N

Problem 2 (Hibbeler, 2017, w/ permission)

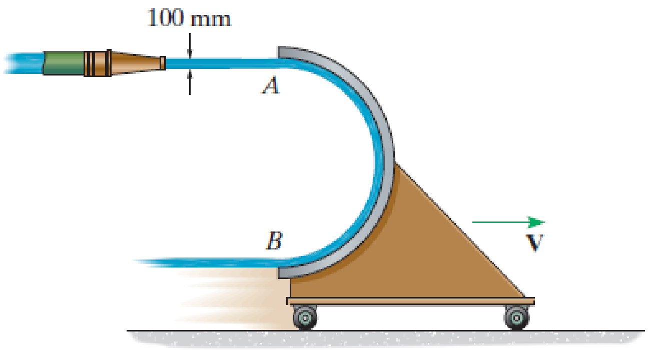
The jet of water flows from the 100-mm-diameter pipe at 4 m/s. If it strikes the fixed vane and is deflected as shown, determine the volume flow towards *A* and towards *B* if the tangential component of the force that the water exerts on the vane is zero.



A) Q_A = 0.00455 m³/s and Q_B = 0.02690 m³/s
B) Q_A = 0.02690 m³/s and Q_B = 0.00455 m³/s
C) Q_A = 0.01111 m³/s and Q_B = 0.02029 m³/s
D) Q_A = 0.02029 m³/s and Q_B = 0.01111 m³/s

Problem 3 (Hibbeler, 2017, w/ permission)

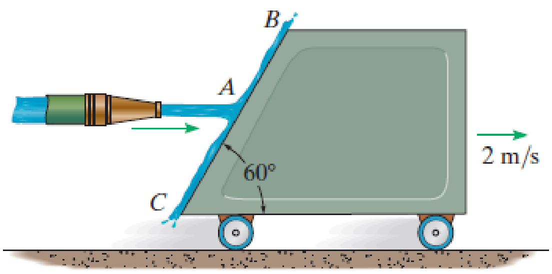
Water flows at 0.1 m^3/s through the 100-mm-diameter nozzle and strikes the vane on the 150-kg cart, which is originally at rest. Determine the acceleration of the cart when it attains a velocity of 2 m/s.



A) a = 3.4 m/s^2
B) a = 12.1 m/s^2
C) a = 20.6 m/s^2
D) a = 28.5 m/s^2

Problem 4 (Hibbeler, 2017, w/ permission)

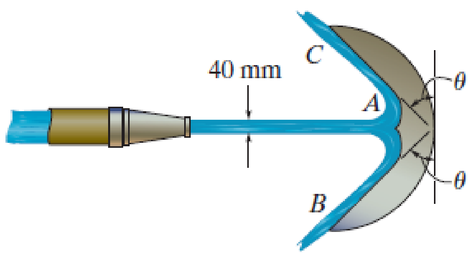
Flow from the water stream strikes the inclined surface of the cart. Determine the power produced by the stream if, due to rolling friction, the cart moves to the right with a constant velocity of 2 m/s. One-fourth of the discharge flows down the incline, and three-fourths flows up the incline.



A) P = 533 W
B) P = 994 W
C) P = 1492 W
D) P = 1995 W

Problem 5 (Hibbeler, 2017, w/ permission)

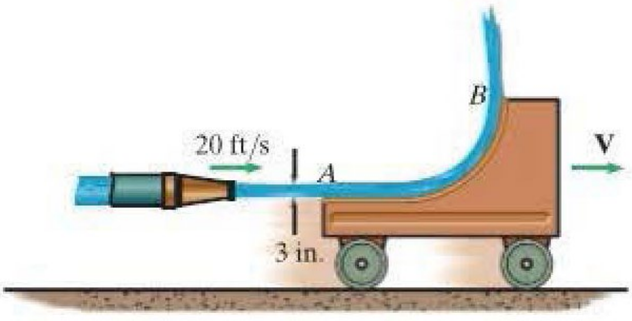
The nozzle has a diameter of 40 mm. If it discharges water with a velocity of 20 m/s against the fixed blade, which of the following expressions provides the horizontal force exerted by the water on the blade when $\theta = 75^{p}$?



A) $F(75^{o})$ = 713 N
B) $F(75^{o})$ = 983 N
C) $F(75^{o})$ = 1254 N
D) $F(75^{o})$ = 1544 N

Problem 6 (Hibbeler, 2017, w/ permission)

Determine the velocity of the 50-lb cart in 3 s starting from rest if a stream of water, flowing from the nozzle at 20 ft/s, strikes the vane and is deflected upward. The stream has a diameter of 3 in. Neglect the rolling resistance of the wheels.



A) V = 14.6 ft/s

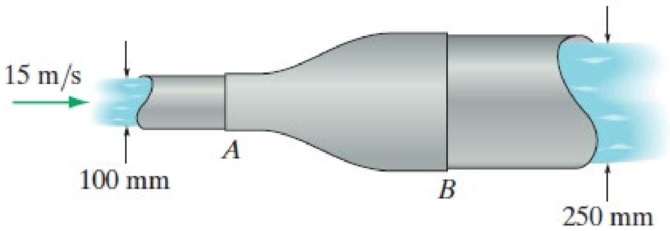
B) V = 23.8 ft/s

C) V = 35.7 ft/s

D) V = 46.5 ft/s

Problem 7 (Hibbeler, 2017, w/ permission)

Air at a temperature of 30°C flows through the expansion fitting such that its velocity at A is 15 m/s and the absolute pressure is 250 kPa. If heat and frictional loss due to the expansion cause the temperature and absolute pressure of the air to become 20°C and 7.50 kPa, determine the resultant force needed to hold the fitting in place. Use R = 286.9 J/kg·K.



A) $|F|$ = 0.54 kN

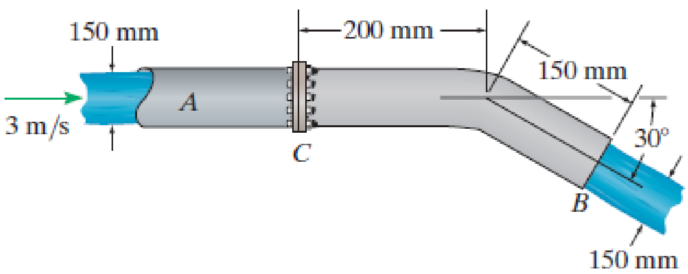
B) $|F|$ = 1.06 kN

C) $|F|$ = 1.57 kN

D) $|F|$ = 2.05 kN

Problem 8 (Hibbeler, 2017, w/ permission)

Water flows into the bend fitting with a velocity of 3 m/s. If the water exits at B into the atmosphere, determine the reaction at C needed to hold the fitting in place. Neglect the weight of the fitting and of the water flowing within it.



A) C = 82.3 N

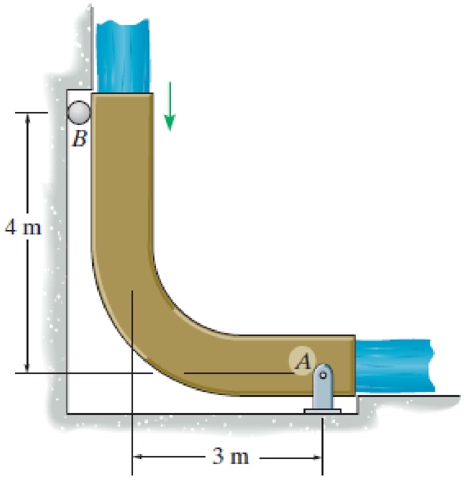
B) C = 115.6 N

C) C = 148.9 N

D) C = 182.2 N

• Problem 9A (Hibbeler, 2017, w/ permission)

The chute is used to divert the flow of water. If the flow is 0.4 m^3/s and it has a cross-sectional area of 0.03 m^2, determine the horizontal force at the roller B necessary for equilibrium.



A) B_x = 2.0 kN

B) B_x = 4.0 kN

C) B_x = 6.0 kN

D) B_x = 8.0 kN

• Problem 9B

Considering the chute illustrated in the previous figure, compute the resultant force at pin *A*.

A) *A* = 2.2 kN

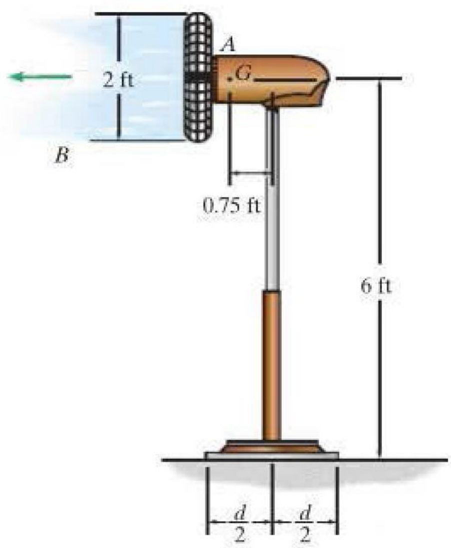
B) *A* = 3.3 kN

C) *A* = 4.4 kN

D) *A* = 5.5 kN

Problem 10 (Hibbeler, 2017, w/ permission)

The fan blows air at 6000 ft^3/min. If the fan has a weight of 40 lb and center of gravity at *G*, determine the smallest diameter *d* of its base so that it will not tip over. Assume the air stream through the fan has a diameter of 2 ft. The specific weight of the air is γ_a = 0.076 lb/ft^3.



A) *d* = 3.2 in.

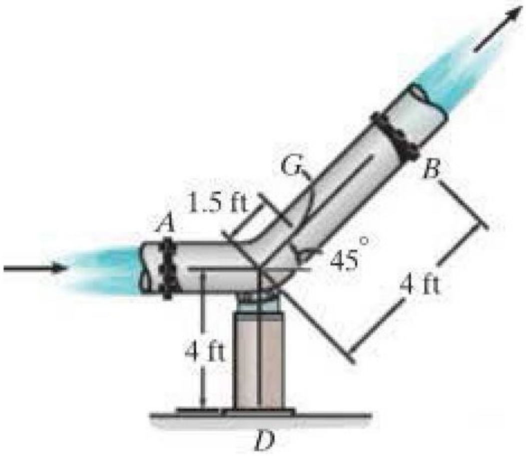
B) *d* = 6.3 in.

C) *d* = 9.1 in.

D) *d* = 12.0 in.

● Problem 11A (Hibbeler, 2017, w/ permission)

The bend is connected to the pipe at flanges *A* and *B* as shown. If the diameter of the pipe is 1 ft and it carries a volumetric flow of 50 ft³/s, determine the resultant force exerted at the fixed base *D* of the support. The total weight of the bend and the water within it is 500 lb, with a mass center at point *G*. The pressure of the water at A is 15 psi. Assume that no force is transferred to the flanges at *A* and *B*.



A) *D* = 1.1 kips

B) *D* = 3.9 kips

C) *D* = 6.4 kips

D) *D* = 8.8 kips

● Problem 11B

Considering the system in the previous problem, find the resultant moment exerted on point *D*.

A) M_D = 1.5 kip-ft

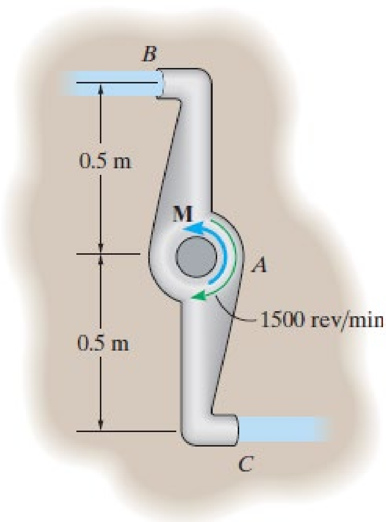
B) M_D = 6.9 kip-ft

C) M_D = 11.4 kip-ft

D) M_D = 16.8 kip-ft

Problem 12 (Hibbeler, 2017, w/ permission)

Air enters into the hollow propeller tube at *A* with a mass flow of 3 kg/s and exits at the ends *B* and *C* with a velocity of 400 m/s, measured relative to the tube. If the tube rotates at 1500 rev/min, determine the frictional torque *M* on the tube.



A) |*M*| = 122 N·m

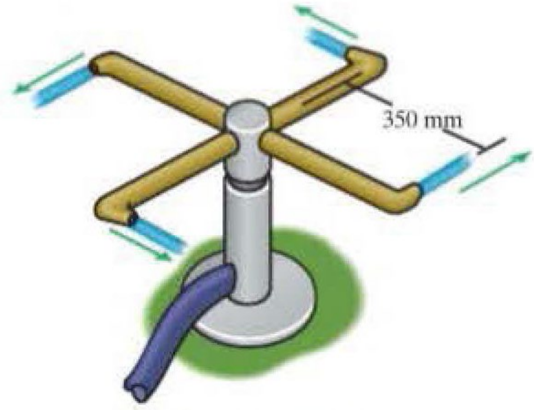
B) |*M*| = 305 N·m

C) |*M*| = 482 N·m

D) |*M*| = 665 N·m

Problem 13 (Hibbeler, 2017, w/ permission)

The lawn sprinkler consists of four arms that rotate in the horizontal plane. The cross-section of each nozzle is 7.85×10^{-5} m². The water is supplied through the hose at 0.008 m³/s and is ejected horizontally through the four arms. Determine the steady-state angular velocity of the arms. Neglect friction.

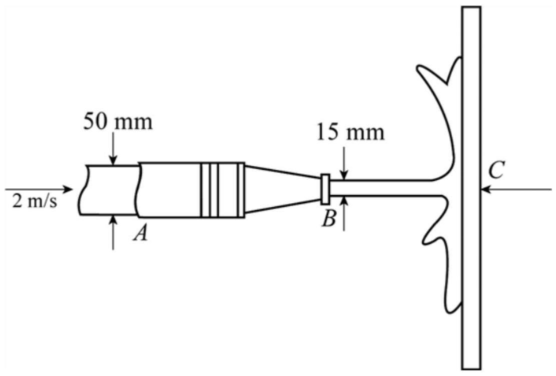


A) ω = 12.9 rad/s
B) ω = 33.8 rad/s
C) ω = 51.9 rad/s
D) ω = 72.8 rad/s

SOLUTIONS

P.1 Solution (Modified from Chegg)

The free body diagram for the system is illustrated below.



The cross-sectional area of the nozzle at section A is $A_A = \pi \times 0.05^2/4$ = 0.00196 m², while that of B is $A_B = \pi \times 0.015^2/4$ = 0.000177 m². The velocity of flow at B can be determined with the continuity equation,

$$-V_B A_B + V_A A_A = 0$$

$$\therefore V_A A_A = V_B A_B$$

$$\therefore V_B = \frac{A_A}{A_B} V_A = \frac{0.00196}{0.000177} \times 2 = 22.15 \text{ m/s}$$

The velocity of flow in the control surface is given by

$$V_{f/\text{cs}} = V_f - V_{\text{cv}}$$

in which V_f is the velocity of flow, which in this case is $V_f = V_B$ = 22.15 m/s, and V_{cv} is the velocity of the control volume, which we establish as 2 m/s. Hence,

$$V_{f/\text{cs}} = V_f - V_{\text{cv}} = 22.15 - 2 = 20.15 \text{ m/s}$$

The flow $Q_{f/cs}$ onto the plate is then

$$Q_{f/\text{cs}} = A_B V_{f/\text{cs}} = 0.000177 \times 20.15 = 0.00357 \, \text{m}^3/\text{s}$$

The force F required to keep the circular plate moving at a velocity of 2 m/s is obtained from an equilibrium of forces in the x-direction,

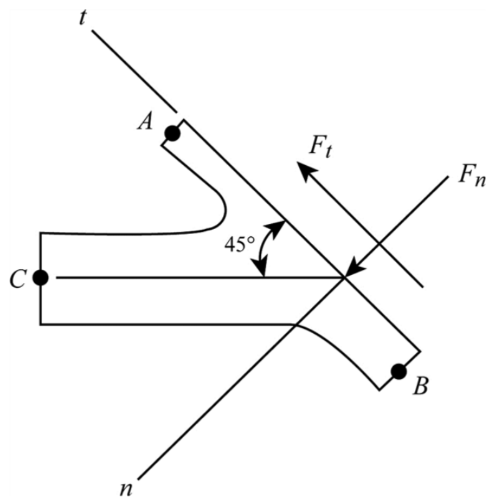
$$\Sigma F_x = 0 \rightarrow F - \rho\left(V_{f/cs}\right)_B Q_{f/cs} = 0$$

$$\therefore F = 1000 \times 20.15 \times 0.00357 = \boxed{71.9 \text{ N}}$$

➡ The correct answer is **C**.

P.2 ● Solution (Modified from Chegg)

The free body diagram for the control volume is illustrated in continuation.



Because we have free flow at points A, B, and C, we can surmise that the pressures therein are all zero, $p_A = p_B = p_C = 0$. In addition, we neglect elevation changes in the water jet before or after it impinges on the vane. Applying Bernoulli's equation at points A, B and C, we have

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g}$$

Substituting $p_A = p_B = p_C = 0$, $\gamma = 9810$ N/m³, $g = 9.81$ m/s², and $V_C = 4.0$ m/s, it follows that

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g}$$

$$\therefore \frac{0}{9810} + \frac{V_A^2}{2 \times 9.81} = \frac{0}{9810} + \frac{V_B^2}{2 \times 9.81} = \frac{0}{9810} + \frac{4.0^2}{2 \times 9.81}$$

$$\therefore \frac{V_A^2}{19.62} = \frac{V_B^2}{19.62} = 0.815$$

We obtain a $A = B = C$ equality. Using $A = B$, the result is

$$\frac{V_A^2}{19.62} = \frac{V_B^2}{19.62} \rightarrow |V_A| = |V_B|$$

Then, using $B = C$ gives

$$\frac{V_B^2}{19.62} = 0.815 \rightarrow V_B = \sqrt{0.815 \times 19.62} = 4 \text{ m/s}$$

which, in combination with the previous result, yields $V_A = 4$ m/s. The cross-sectional area of the pipe is $A = \pi \times 0.1^2/4 = 0.00785$ m², and the flow rate at its exit is $Q_C = V_C A = 4 \times 0.00785 = 0.0314$ m³/s. We want to compute flow rates at point A, Q_A, and point B, Q_B. From the continuity equation, we have

$$Q_A + Q_B = Q_C \rightarrow Q_A + Q_B = 0.0314 \quad \text{(I)}$$

The tangential component of velocity, $(V_C)_t$, of the jet just before it impinges on the vane is $(V_C)_t = V_C \times \cos 45° = 2.84$ m/s. We proceed to apply the first condition of equilibrium in the tangential direction,

$$\Sigma F_t = 0 \rightarrow \rho_w Q_A V_A + \rho_w Q_B\left(-V_B\right) - \rho_w Q_C\left(-V_C\right)_t = 0$$

Substituting $\rho_w = 1000$ kg/m³ along with other available data, we obtain

$$\rho_w Q_A V_A + \rho_w Q_B(-V_B) - \rho_w Q_C(-V_C)_t = 0$$

$$\therefore \cancel{1000} \times Q_A \times 4 + \cancel{1000} \times Q_B \times (-4) - \cancel{1000} \times 0.0314 \times (-2.84) = 0$$

$$\therefore 4Q_A - 4Q_B + 0.0892 = 0$$

$$\therefore Q_A - Q_B = -0.0223 \quad \text{(II)}$$

Equations (I) and (II) form a simple system of linear equations,

$$\begin{cases} Q_A + Q_B = 0.0314 \quad \text{(I)} \\ Q_A - Q_B = -0.0223 \quad \text{(II)} \end{cases}$$

Summing equations (I) and (II) brings to

$$(Q_A + Q_B) + (Q_A - Q_B) = 0.0314 - 0.0223 = 0.0091$$

$$\therefore 2Q_A = 0.0091$$

$$\therefore Q_A = \frac{0.0091}{2} = \boxed{0.00455 \text{ m}^3/\text{s}}$$

Finally, substituting Q_A in equation (I) (or the second equation), we conclude that

$$0.00455 + Q_B = 0.0314$$

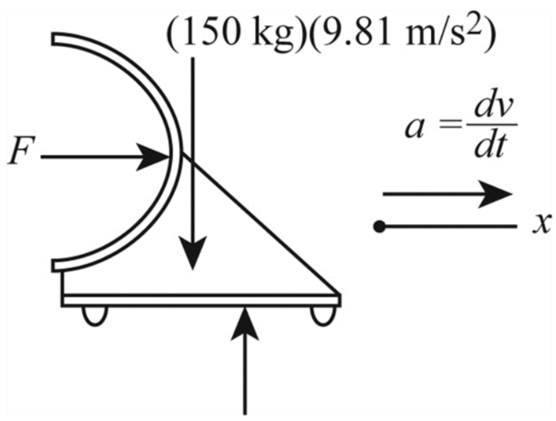
$$\therefore Q_B = 0.0314 - 0.00455 = \boxed{0.02690 \text{ m}^3/\text{s}}$$

We see that most of the water ejected from the pipe (or 0.0269/0.0314 ≈ 86%) is diverted to section B, whereas only a small portion follows section A.

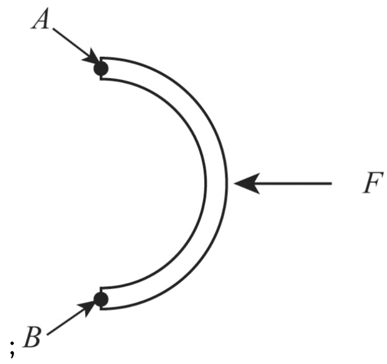
➡ The correct answer is **A**.

P.3 ● Solution (Modified from Chegg)

The free body diagram of the vane is shown below.



We consider an equilibrium of forces in the *x*-direction to determine force *F*.



We suppose that fluid flows steadily across the vane. The control volume is the portion of water that strikes the cart. The velocity V_A at point *A* is $V_A = Q/A_A = 0.1/(\pi \times 0.1^2/4)$ = 12.73 m/s, and the relative velocity $V_{A/cs}$ of the control surface at point *A* is given by

$$V_{\text{A/cs}} = V_A - V_{\text{cs}}$$

Here, V_{cs} is the velocity of the control surface, that is, the velocity of the cart, which we will denote simply as V. Thus,

$$V_{\mathrm{A/cs}} = V_A - V_{\mathrm{cs}} = 12.73 - V$$

The flow rate Q of the control surface at section A (and across the water jet as it impinges on the vane) is

$$Q_A = V_{\mathrm{A/cs}} A_A$$

in which $A_A = \pi \times 0.1^2/4 = 0.00785$ m^2 is the cross-sectional area of section A, so that

$$Q_A = 0.00785(12.73 - V)$$

Force F acting on the cart can be obtained from an equilibrium of forces in the x-direction,

$$\Sigma F_x = 0 \rightarrow F - \rho V_{\mathrm{A/cs}} Q_A - \rho \underbrace{V_{\mathrm{B/cs}}}_{=V_{\mathrm{A/cs}}} \underbrace{Q_B}_{=Q_A} = 0$$

$$\therefore F = 2\rho V_{\mathrm{A/cs}} Q_A$$

where $\rho = 1000$ kg/m^3, $V_{A/cs} = 12.73 - V$, and $Q_A = 0.00785(12.73 - V)$. Hence,

$$F = 2\rho V_{\mathrm{A/cs}} Q_A = 2 \times 1000 \times (12.73 - V) \times \left[0.00785(12.73 - V)\right] = 15.7(12.73 - V)^2$$

The acceleration a of the cart can be obtained from Newton's second law, noting that F is the resultant force on the cart,

$$F = ma \rightarrow 15.7(12.73 - V)^2 = 150 \times a$$

$$\therefore a = \frac{15.7(12.73 - V)^2}{150}$$

Finally, we substitute $V = 2$ m/s as the velocity of the cart,

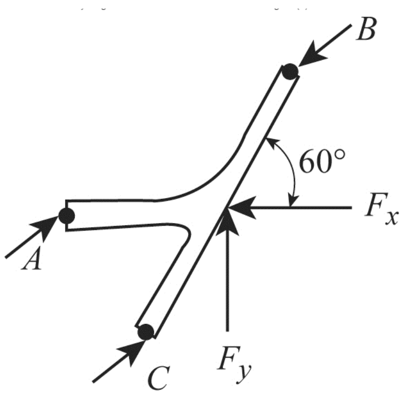
$$a = \frac{15.7 \times \left[12.73 - (2.0)\right]^2}{150} = \boxed{12.1\ \mathrm{m/s^2}}$$

When the cart is being displaced at the prescribed velocity, its acceleration will be close to 12 m/s^2.

➡ The correct answer is **B**.

P.4 ● Solution (Modified from Chegg)

The free body diagram for the system is provided below.



The velocity V_A of flow right before the stream strikes the cart is $V_A = Q/A_A = 0.04/(\pi \times 0.05^2/4) = 20.37$ m/s. The relative velocity $V_{A/cs}$ of the control surface at the exit of the nozzle is given by

$$V_{A/\mathrm{cs}} = V_A - V_{\mathrm{cs}}$$

where $V_{cs} = 2$ m/s. Hence,

$$V_{A/\mathrm{cs}} = 20.37 - 2 = 18.37\ \mathrm{m/s}$$

Accordingly, the relative flow at section A is

$$Q_{\mathrm{A/cs}} = V_{\mathrm{A/cs}} A_A = 18.37 \times \left(\pi \times 0.05^2/4\right) = 0.0361 \ \mathrm{m^3/s}$$

We can then compute the relative flow rate at point B,

$$Q_{\mathrm{B/cs}} = \frac{3}{4} Q_{A/\mathrm{cs}} = \frac{3}{4} \times 0.0361 = 0.0271 \ \mathrm{m^3/s}$$

as well as the flow at point C, $Q_{C/cs} = 0.25 \times 0.0361 = 0.00903$ m³/s. The x-component of the relative velocity at B is $\left(V_{B/cs}\right)_x = V_{A/cs} \cos 60^{\circ} = 18.37 \times 0.5 =$ 9.19 m/s, whereas at C it is $\left(V_{C/cs}\right)_x = -V_{C/cs} \cos 60^{\circ} = -18.37 \times 0.5 = -9.19$ m/s. The horizontal force F_x acting on the cart can be determined from the equilibrium of forces in the x-direction,

$$\Sigma F_x = 0 \rightarrow \rho\left[Q_{\mathrm{B/cs}}\left(V_{\mathrm{B/cs}}\right)_{\mathrm{x}} + Q_{\mathrm{C/cs}}\left(V_{\mathrm{C/cs}}\right)_{\mathrm{x}} - Q_{\mathrm{A/cs}}\left(V_{\mathrm{A/cs}}\right)_{\mathrm{x}}\right] - F_x = 0$$

$$\therefore F_x = \rho\left[Q_{\mathrm{B/cs}}\left(V_{\mathrm{B/cs}}\right)_{\mathrm{x}} + Q_{\mathrm{C/cs}}\left(V_{\mathrm{C/cs}}\right)_{\mathrm{x}} - Q_{\mathrm{A/cs}}\left(V_{\mathrm{A/cs}}\right)_{\mathrm{x}}\right]$$

$$\therefore F_x = 1000 \times \left[0.0271 \times 9.19 + 0.00903 \times (-9.19) - 0.0361 \times 18.37\right] = -497 \ \mathrm{N}$$

$$\therefore \left|F_x\right| = 497 \ \mathrm{N}$$

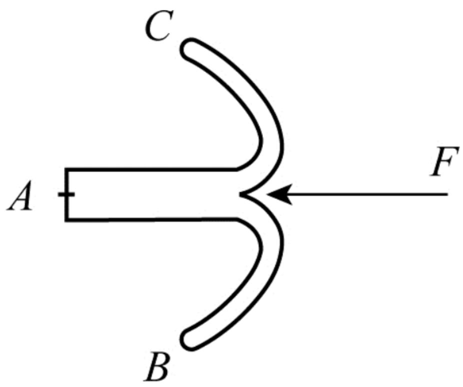
Finally, the power P developed by the jet stream is

$$P = F_x V = 497 \times 2.0 = \boxed{994 \ \mathrm{W}}$$

➡ The correct answer is **B**.

P.5 ● Solution

The cross-sectional area A_A of the jet at point A is $A_A = \pi \times 0.04^2 =$ 0.00126 m², and the flow rate there, given V = 20 m/s, is $Q_A = VA_A = 20 \times 0.00126 =$ 0.0252 m³/s. The free body diagram for the system is quite simple.



Flow at points A, B, and C is considered to be free, so we can surmise the pressure values $p_A = p_B = p_C = 0$. Furthermore, the change in elevation is negligible, and the velocity at points A, B, and C is the same, $V_A = V_B = V_C$ = 20 m/s. The horizontal component of velocity at point B is $(V_B)_x = V_B \sin\theta = 20\sin\theta$. In addition, $(V_C)_x = (V_B)_x$. Continuity enables us to write

$$Q_A = Q_B + Q_C$$

where Q_A is the flow rate at A, Q_B is the flow at B, and Q_C is the flow at C. Substituting Q_A brings to

$$Q_B + Q_C = 0.0252 \ \mathrm{m^3/s}$$

We proceed to apply an equilibrium of forces in the x-direction,

$$\Sigma F_x = 0 \rightarrow \rho\left(-V_A\right)_x\left(-Q_A\right) + \rho\left(V_B\right)_x Q_B + \rho\left(V_C\right)_x Q_C - F = 0$$

$$\therefore F = \rho\left(V_A\right)_x Q_A + \rho\left(V_B\right)_x Q_B + \rho\left(V_C\right)_x Q_C$$

$$\therefore F = \rho\left[\left(V_A\right)_x Q_A + \left(V_B\right)_x Q_B + \left(V_C\right)_x Q_C\right]$$

Substituting ρ = 1000 kg/m³, $(V_A)_x = V_A$ = 20 m/s (the horizontal component of the velocity at A), and $(V_B)_x = (V_C)_x = 20\sin\theta$, it follows that

$$F = 1000 \times [20 \times 0.0252 + 20 \sin\theta \times Q_B + 20 \sin\theta \times Q_C]$$

$$\therefore F = 1000 \times [0.5 + 20 \sin\theta (Q_B + Q_C)]$$

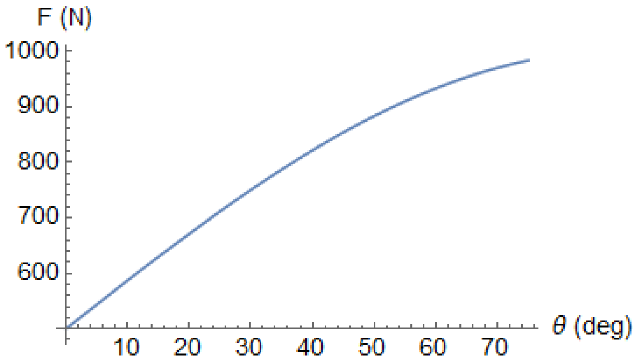
$$\therefore F = 500 + 20,000 \sin\theta (Q_B + Q_C)$$

Substituting $Q_B + Q_C = 0.0252$ m^3/s yields

$$F = 500 + 20,000 \sin\theta \underbrace{(Q_B + Q_C)}_{=0.0252} = 500 + 500 \sin\theta$$

$$\therefore F(\theta) = 500(1 + \sin\theta)$$

The relation above gives the variation of F with θ. This equation is plotted below. The force exerted by the water on the blade increases with the angle θ, i.e., as the extensions of the blade approach the horizontal, making it more "closed" and in greater contact with the oncoming jet. At $\theta = 75°$, we have $F(75°) = 500(1 + \sin 75°) = 983$ N, or 196.6% of the original value.



➡ The correct answer is **B**.

P.6 ● Solution (Modified from Chegg)

The velocity $V_{A/cs}$ of the control surface at point A follows from the formula

$$V_{A/cs} = V_A - V_{cs}$$

in which $V_A = 20$ ft/s is the velocity of the water stream and $V_{cs} = V$ is the velocity of the control surface; that is,

$$V_{A/cs} = V_A - V_{cs} = 20 - V \text{ ft/s}$$

The inlet velocity $(V_{f/cs})_{in}$ of the control surface is equal to the flow velocity $(V_{f/cs})_B$ of the control surface at point B, which in turn equals the velocity $V_{A/cs}$ of the control surface at point A,

$$(V_{f/cs})_{in} = (V_{f/cs})_B = V_{A/cs}$$

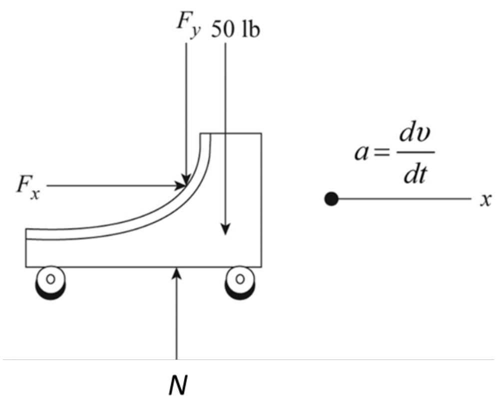
The flow rate $Q_{f/cs}$ of the control surface is such that

$$Q_{f/cs} = V_{A/cs} A_A$$

Substituting $V_{A/cs} = 20 - V$ and $A_A = \pi \times (3/12)^2/4 = 0.0491$ ft^2 gives

$$Q_{f/cs} = (V_{f/cs})_A A_A = (20 - V) \times 0.0491$$

The free body diagram for the cart is provided below.



Summing forces in the x-direction yields

$$\Sigma F_x = 0 \rightarrow F_x - \rho Q_{\mathrm{f/cs}} V_{\mathrm{A/cs}} = 0$$

$$\therefore F_x = \rho Q_{\mathrm{f/cs}} V_{\mathrm{A/cs}} = \left(\frac{62.4}{32.2}\right) \times \left[0.0491(20-V)\right] \times (20-V) = 0.0951(20-V)^2$$

The velocity V of the cart can be determined by applying Newton's second law in the x-direction,

$$\Sigma F_x = \left(\frac{m}{g}\right) \times a$$

where $\Sigma F_x = F_x = 0.0951(20-V)^2$, m = 50 lb is the mass of the cart, and $a = dV/dt$ is the acceleration of the cart. Accordingly,

$$\Sigma F_x = \left(\frac{m}{g}\right) \times a \rightarrow 0.0951(20-V)^2 = \frac{50}{32.2} \times \frac{dV}{dt} = 1.55 \frac{dV}{dt}$$

$$\therefore \frac{0.0951}{1.55} dt = \frac{dV}{(20-V)^2}$$

$$\therefore 0.0614 dt = \frac{dV}{(20-V)^2}$$

We can integrate on both sides to obtain the velocity at t = 3 s,

$$0.0614 \int_0^3 dt = \int_0^V \frac{dV}{(20-V)^2}$$

The integral on left-hand side is elementary, while that in the right-hand side can be unraveled with the help of an integral table, giving

$$\int \frac{dV}{(20-V)^2} = -\frac{1}{V-20} = \frac{1}{20-V}$$

Thus,

$$0.0614 \underbrace{\int_0^3 dt}_{=3} = \int_0^V \frac{dV}{(20-V)^2}$$

$$\therefore \underbrace{0.0614 \times 3}_{=0.18} = \left.\frac{1}{20-V}\right|_0^V = \frac{1}{20-V} - \frac{1}{20-0} = \frac{1}{20-V} - 0.05$$

$$\therefore 0.185 = \frac{1}{20-V}$$

$$\therefore 3.7 - 0.185V = 1$$

$$\therefore 0.185V = 2.7$$

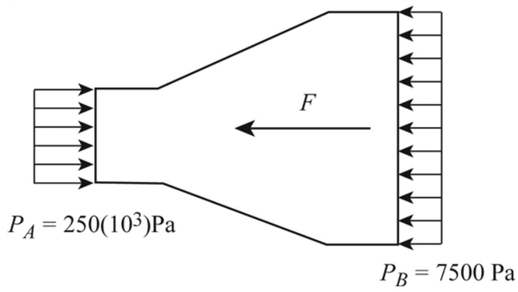
$$\therefore V = \boxed{14.6 \text{ ft/s}}$$

The velocity of the cart after 3 seconds will be close to 15 ft/s.

➡ The correct answer is **A**.

P.7 ● Solution (Modified from Chegg)

The free body diagram of the expansion fitting is shown below.



Flow is steady and the air flowing inside the fitting is an ideal fluid. The density ρ_A of the air that enters the fitting at section A can be obtained from the ideal gas law,

$$p_A = \rho_A RT_A \rightarrow \rho_A = \frac{p_A}{RT_A}$$

in which $p_A = 250\times10^3$ Pa is the pressure at A, $T_A = 30 + 273 = 303$ K is the temperature therein, and $R = 286.9$ J/kg·K, so that

$$\rho_A = \frac{p_A}{RT_A} = \frac{250\times10^3}{286.9\times303} = 2.88 \text{ kg/m}^3$$

Similarly, the density ρ_B of air at section B of the fitting is determined as

$$p_B = \rho_B RT_B \rightarrow \rho_B = \frac{p_B}{RT_B}$$

$$\therefore \rho_B = \frac{7500}{286.9\times(20+273)} = 0.0892 \text{ kg/m}^3$$

The cross-sectional area of the fitting is $A_A = \pi \times 0.1^2/4 = 0.00785$ m² at A and $A_B = \pi \times 0.25^2/4 = 0.0491$ m² at B. We have the velocity $V_A = 15$ m/s with which air enters the fitting at A; the velocity V_B with which it leaves the fitting at section B follows from the continuity equation,

$$Q_A = Q_B \rightarrow V_B = \frac{\rho_A A_A}{\rho_B A_B} V_A$$

$$\therefore V_B = \frac{2.88\times0.00785}{0.0892\times0.0491}\times15 = 77.43 \text{ m/s}$$

Next, the resultant force F needed to hold the fitting in place can be obtained from the equilibrium of forces in the x-direction,

$$\Sigma F_x = 0 \rightarrow p_A\times A_A - F - p_B\times A_B = \rho_A A_A V_A\times(-V_A) + \rho_B A_B V_B\times V_B$$

$$\therefore (250\times10^3)\times0.00785 - F - 7500\times0.0491 = 2.88\times0.00785\times15\times(-15)$$

$$+0.0892\times0.0491\times77.43\times77.43$$

$$\therefore 1962.5 - F - 368.25 = -5.09 + 26.26$$

$$\therefore -F = -5.09 + 26.26 + 368.25 - 1962.5$$

$$\therefore F = -(-5.09 + 26.26 + 368.25 - 1962.5) = 1573.08$$

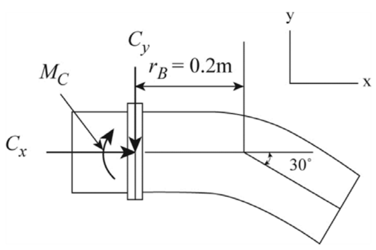
$$\therefore \boxed{|F| = 1.57 \text{ kN}}$$

The force required to hold the fitting in place is just above 1500 newtons.

➡ The correct answer is **C**.

P.8 ● Solution (Modified from Chegg)

The free body diagram of the fitting is shown below.



Pipe B is open to the atmosphere, so the pressure therein can be taken as zero, $p_B = 0$. Since there is no variation in diameter between sections B and C, the velocity of flow in these regions is the same, $V_B = V_C = 3.0$ m/s. In addition, the elevation change between these two regions is small enough for it to have no influence in the momentum analysis, so we surmise that the pressures in B and C are both zero, $p_B = p_C = 0$. The flow rate Q of water in section C is

$$Q = V_C A_C = V_C\times\pi r^2 = 3.0\times\pi\times0.075^2 = 0.0530 \text{ m}^3/\text{s}$$

The horizontal component of velocity at section B is $(V_B)_x = V_A \cos 30^\circ$ = 2.60 m/s, while the vertical component is $(V_B)_y = V_B \sin 30^\circ$ = 1.5 m/s. The horizontal reaction C_x at support C can be obtained from a balance of forces in the x-direction,

$$\Sigma F_x = 0 \rightarrow \rho\left(V_B\right)_x \underbrace{Q_B}_{=Q} + \rho V_C\left(-\underbrace{Q_C}_{=Q}\right) - C_x = 0$$

$$\therefore C_x = \rho\left(V_B\right)_x Q - \rho V_C Q = 1000 \times 2.60 \times 0.053 - 1000 \times 3.0 \times 0.053 = -21.2 \text{ N}$$

Similarly, the vertical reaction C_y can be determined from a balance of forces in the y-direction,

$$\Sigma F_y = 0 \rightarrow 0 + \rho\left(-V_B\right)_y Q_B + C_y = 0$$

$$\therefore C_y = -1000 \times 1.5 \times 0.053 = -79.5 \text{ N}$$

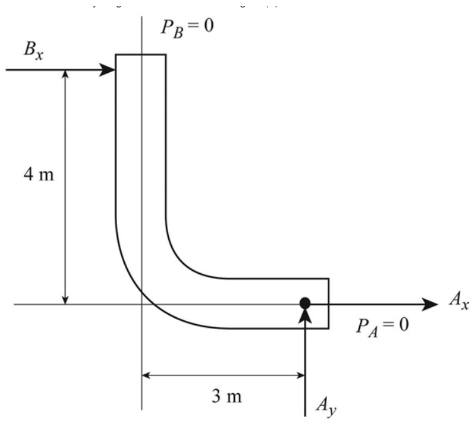
The intensity of the total reaction at C follows as

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-21.2)^2 + (79.5)^2} = \boxed{82.3 \text{ N}}$$

➡ The correct answer is **A**.

P.9 ● Solution (Modified from Chegg)

Part A: The free body diagram of the chute is provided below.



We take the chute as the control volume. Points A and B are open to the atmosphere, so that $p_A = p_B = 0$. The diameter $d = \sqrt{4A/\pi} = \sqrt{(4 \times 0.03)/\pi} = 0.20$ m, and the velocity V with which water flows in the chute is $V = Q/A = 0.4/0.03 =$ 13.33 m/s. Taking moments about A gives

$$\Sigma M_A = 0 \rightarrow B_x \times 4.0 - \rho_w Q V \times 3.0 = 0$$

$$\therefore B_x = \frac{1000 \times 0.4 \times 13.33 \times 3.0}{4.0} = 3999 \text{ N} \approx \boxed{4.0 \text{ kN}}$$

➡ The correct answer is **B**.

Part B: We apply an equilibrium of forces in the x-direction to obtain A_x, the x-component of the reaction at A,

$$\Sigma F_x = 0 \rightarrow A_x + B_x - \rho_w Q V = 0$$

$$\therefore A_x = \rho_w Q V - B_x = 1000 \times 0.4 \times 13.33 - 3999 = 1333 \text{ N}$$

Similarly, we apply the first condition of equilibrium in the y-direction to compute A_y,

$$\Sigma F_y \rightarrow A_y - \rho_w Q V = 0$$

$$\therefore A_y = \rho_w Q V = 1000 \times 0.4 \times 13.33 = 5332 \text{ N}$$

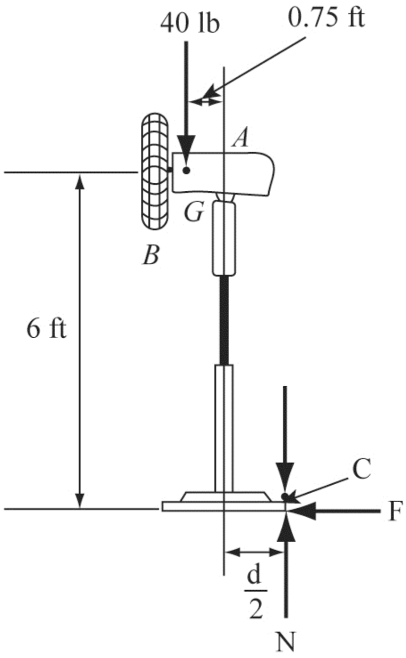
The resultant at pin A is then

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{1333^2 + 5332^2} = 5496 \text{ N} \approx \boxed{5.5 \text{ kN}}$$

➡ The correct answer is **D**.

P.10 ● Solution (Modified from Chegg)

Consider the free body diagram of the fan.



The control volume is simply the fan and the air passing through it. Tipping of the fan will occur about point C. The area A_B of the fan section is $A_B = \pi \times 1^2 = 3.14$ ft^2, the flow rate expressed in ft^3/s rather than ft^3/min is $Q = 6000/60 = 100$ ft^3/s, and the flow velocity is $V_B = Q/A_B = 100/3.14 = 31.85$ ft/s. The smallest diameter d of the base that will not allow the fan to tip over is obtained from equilibrium of angular momentum at point C,

$$W\left(0.75+\frac{d}{2}\right)=\frac{\gamma_a}{g}\times Q\times hV_B$$

where W is the weight of the fan, d is the diameter of the base, h is the height of the fan, γ_a = 0.076 lb/ft^3, and g = 32.2 ft/s^2. Solving for d and substituting yields

$$W\left(0.75+\frac{d}{2}\right)=\frac{\gamma_a}{g}\times Q\times hV_B$$

$$\therefore 0.75+\frac{d}{2}=\frac{\gamma_a QV_B h}{gW}$$

$$\therefore \frac{d}{2}=\frac{\gamma_a QV_B h}{gW}-0.75$$

$$\therefore d=2\left(\frac{\gamma_a QV_B h}{gW}-0.75\right)$$

$$\therefore d=2\left(\frac{0.076\times 100\times 31.85\times 6}{40\times 32.2}-0.75\right)=0.76\,\text{ft}=\boxed{9.1\text{ in.}}$$

The smallest diameter d that the base of the fan must have is about 9 inches.

➡ The correct answer is **C**.

P.11 ● Solution

Part A: The total weight of the bend and the water within it is W = 500 lb. The velocity of flow in either the inlet A or the outlet B is $V_A = V_B = V$, where the flow rate Q = 50 ft^3/s, and the cross-sectional area $A = \pi \times 1^2/4 = 0.79$ ft^2, so that V = 50/0.79 = 63.29 ft/s. We shall take the line of action of force F_A as a horizontal datum, and the elevation of section B will be $z_B = 4 \times \sin 45°$ = 2.83 ft. The pressure p_B at section B can be determined by applying Bernoulli's equation therein and at point A,

$$\frac{p_A}{\gamma}+\frac{V^2}{2}+gz_A=\frac{p_B}{\gamma}+\frac{V^2}{2}+gz_B$$

The pressure of the water at A, as given, is p_A = 15 psi = 15×144 = 2160 lb/ft²; in addition, z_A = 0, γ = 62.4 lb/ft³, and g = 32.2 ft/s². Hence,

$$\frac{p_A}{\left(\frac{\gamma}{g}\right)}+\frac{V^2}{2}+gz_A=\frac{p_B}{\left(\frac{\gamma}{g}\right)}+\frac{V^2}{2}+gz_B$$

$$\therefore \frac{2160}{\left(\frac{62.4}{32.2}\right)}+\cancel{\frac{63.29^2}{2}}+32.2\times 0=\frac{p_B}{\left(\frac{62.4}{32.2}\right)}+\cancel{\frac{63.29^2}{2}}+32.2\times 2.83$$

$$\therefore 1114.62+0=0.52p_B+91.13$$

$$\therefore p_B=\frac{1114.62-91.13}{0.52}=1968.25\text{ lb/ft}^2$$

The x- and y-components of velocity are both $V_x = V_y$ = 63.29× cos 45° = 44.75 ft/s. The force F_A exerted by flow as water enters the bend is

$$F_A=p_A A=2160\times 0.79=1706.4\text{ lb}$$

Similarly, the force F_B due to flow exiting the pipe at B is

$$F_B=p_B A=1968.25\times 0.79=1554.92\text{ lb}$$

The horizontal component D_x of the reaction at D can be obtained by applying Newton's second law in this direction; that is,

$$\Sigma F_x=0\rightarrow F_A-F_B\times\cos 45^\circ-D_x=\rho_w\left(-\underbrace{Q_A}_{=50}\right)\underbrace{V_A}_{=63.29}+\rho_w\left(\underbrace{Q_B}_{=50}\right)\underbrace{\left(V_B\right)_x}_{=V_x=44.75}$$

$$\therefore 1706.4-1554.92\times 0.71-D_x=\frac{62.4}{32.2}\times(-50)\times 63.29+\frac{62.4}{32.2}\times 50\times 44.75$$

$$\therefore 1706.4-1103.99-D_x=-6132.45+4336.02$$

$$\therefore 602.41-D_x=-1796.43$$

$$\therefore D_x=602.41+1796.43=2400\text{ lb}=2.40\text{ kips}$$

The vertical component D_y of the reaction at D, in turn, follows from an equilibrium of forces in the y-direction,

$$\Sigma F_y=0\rightarrow D_y-W-F_B\times\sin 45^\circ=\rho_w\underbrace{Q_B}_{=50}\underbrace{\left(V_B\right)_y}_{=V_y=44.75}$$

$$\therefore D_y-500-1554.92\times 0.71=\frac{62.4}{32.2}\times 50\times 44.75=4336.02$$

$$\therefore D_y-500-1103.99=4336.02$$

$$\therefore D_y=4336.02+500+1103.99=5940.01=5.94\text{ kips}$$

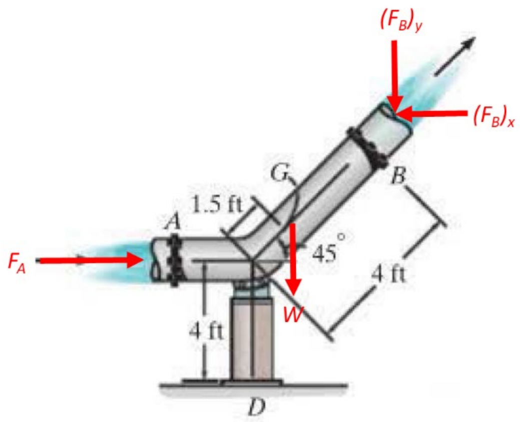
The intensity of the reaction at D is given by the vector sum of D_x and D_y, namely,

$$D=\sqrt{D_x^2+D_y^2}=\sqrt{2.40^2+5.94^2}=\boxed{6.4\text{ kips}}$$

The resultant force at D is 6.4 kilopounds.

➡ The correct answer is **C**.

Part B: The pertaining forces are highlighted below.



To obtain the moment in question, M_D, we apply the second condition of equilibrium at point D, noting that counterclockwise moments are positive,

$$\Sigma M_D = 0 \rightarrow M_D + (F_B)_x \times (4\times\cos 45^\circ + 4) - (F_B)_y \times 4 - F_A \times 4 - W \times (1.5\times\cos 45^\circ)$$

$$= \rho_w(-Q_A)(-V_A r_A) - \rho_w Q_B\left[(V_B)_x r_B\right]$$

The lever arm r_A of the force due to water flow at section A is r_A = 4 ft, which happens to be equal to r_B, the lever arm due of the force due to water flow at section B. As before, the flow velocity at A is V_A = 63.29 ft/s, while $(V_B)_x$, the x-component of velocity at section B, is $(V_B)_x = V_B \times \cos 45^\circ$ = 44.75 ft/s. The flow rates Q_A at section A and Q_B at section B are both $Q_A = Q_B$ = 50 ft³/s. Accordingly,

$$M_D + (F_B)_x \times (4\times\cos 45^\circ + 4) - (F_B)_y \times 4 - F_A \times 4 - W \times (1.5\times\cos 45^\circ)$$

$$= \rho_w(Q_A)(V_A r_A) - \rho_w Q_B\left[(V_B)_x r_B\right]$$

$$\therefore M_D + (1554.92\times\cos 45^\circ)\times(4\times\cos 45^\circ + 4) - (1554.92\times\sin 45^\circ)\times 4 - 1706.4\times 4 - 500\times(1.5\times\cos 45^\circ)$$

$$= \left(\frac{62.4}{32.2}\right)\times 50\times 63.29\times 4 - \left(\frac{62.4}{32.2}\right)\times 50\times[44.75\times 4]$$

$$\therefore M_D + 7507.82 - 4397.98 - 6825.6 - 530.33 = 24{,}529.8 - 17{,}344.1$$

$$\therefore M_D - 4246.09 = 7185.7$$

$$\therefore M_D = 7185.7 + 4246.09 = 11{,}431.8$$

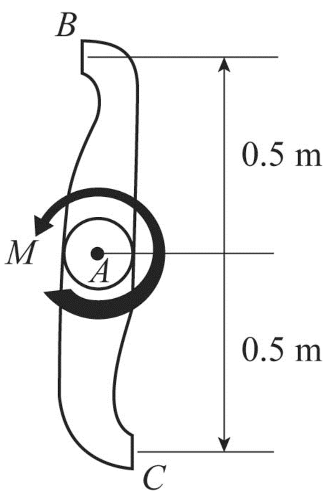
$$\therefore M_D = \boxed{11.4\text{ kips-ft}}$$

The resultant moment at point D is somewhat above 11 kips-ft.

➡ The correct answer is **C**.

P.12 ● Solution (Modified from Chegg)

The free body diagram of the hollow propeller is provided below.



The linear velocity V_B at point B is $V_B = \omega r$, in which ω is the angular speed of the propeller tube and r = 0.5 m. The tube rotates at 1500 rpm, or 157.08 rad/s. Velocity V_B follows as

$$V_B = \omega r = 157.08\times 0.5 = 78.54\text{ m/s}$$

The velocity V_a of air ejected at B results from the formula

$$V_a = V_B + V_{a/B}$$

where V_B = – 78.54 m/s and $V_{a/B}$ = 400 m/s is the exit velocity of ends *B* and *C*. Hence,

$$V_a = V_B + V_{a/B} = -78.54 + 400 = 321.46 \text{ m/s}$$

The mass flow rate impinging over A is 3 kg/s. Due to the symmetry of the device, we can surmise that the mass flows $\dot{m}_A$ and $\dot{m}_B$ exiting the tube at A and B are both equal to 3.0/2 = 1.5 kg/s. Moment *M* can be obtained from the angular momentum equation relative to point *A*,

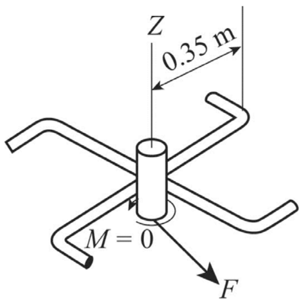
$$\Sigma M_A = 0 \rightarrow M + \dot{m}_B r V_a + \dot{m}_C r V_a = 0$$

$$\therefore |M| = 2\times1.5\times0.5\times321.46 = \boxed{482 \text{ N}\cdot\text{m}}$$

➡ The correct answer is **C**.

P.13 ● Solution (Modified from Chegg)

The free body diagram for the sprinkler is quite simple.



Due to the symmetry of the sprinkler, the discharge from each arm is *Q* = 0.008/4 = 0.002 m³/s. Given the cross-sectional area *A* = 7.85×10⁻⁵ m², the velocity $V_{f/n}$ of water in the nozzle is

$$Q = AV_{f/n} \rightarrow V_{f/n} = \frac{Q}{A}$$

$$\therefore V_{f/n} = \frac{0.002}{7.85\times10^{-5}} = 25.48 \text{ m/s}$$

The linear velocity with which the nozzle spins, V_n, is

$$V_n = \omega r = 0.35\omega$$

The relative velocity of flow V_f, in turn, is

$$V_f = V_n + V_{f/n}$$

Substituting $V_n = -0.35\omega$ and $V_{f/n}$ = 25.48 m/s gives

$$V_f = -0.35\omega + 25.48$$

Finally, we apply the angular momentum equation relative to the center *O* of the device to determine moment *M*,

$$\Sigma M_O = 0 \rightarrow M - 4\times\rho_w Q V_f \times d = 0$$

Here, ρ_w = 1000 kg/m³, *Q* = 0.002 m³/s, $V_f = -0.35\omega + 25.48$, and *d* = 0.35 m is the distance from the end of each arm to the center of the device. Substituting and solving for ω, we obtain

$$M - 4\times\rho_w Q V_f \times d = 0$$

$$\therefore M = 4\rho_w Qd\times V_f = 4\times1000\times0.002\times0.35\times(-0.35\omega + 25.48) = -0.98\omega + 71.34 = 0$$

$$\therefore \omega = \frac{71.34}{0.98} = \boxed{72.8 \text{ rad/s}}$$

➡ The correct answer is **D**.

ANSWER SUMMARY

Problem 1		C
Problem 2		A
Problem 3		B
Problem 4		B
Problem 5		B
Problem 6		A
Problem 7		C
Problem 8		A
Problem 9	9A	B
	9B	D
Problem 10		C
Problem 11	11A	C
	11B	C
Problem 12		C
Problem 13		D

REFERENCES

- HIBBELER, R. (2017). *Fluid Mechanics*. 2nd edition. Upper Saddle River: Pearson.

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