Problems

Problem 1

The intrinsic permeability of a consolidated rock is \(2.7 \times 10^{-3}\) darcy. What is the hydraulic conductivity of a sample of this rock containing water at 20°C? Use \(\nu = 10^{-2}\) cm\(^2\)/s and \(g = 981\) cm/s\(^2\).

A) \(K = 2.61 \times 10^{-6}\) cm/s
B) \(K = 4.55 \times 10^{-6}\) cm/s
C) \(K = 6.49 \times 10^{-6}\) cm/s
D) \(K = 8.37 \times 10^{-6}\) cm/s

Problem 2A

A confined aquifer with an initial thickness of 45 m consolidates (compacts) 0.20 m when the head is lowered by 25 m. What is the vertical compressibility of the aquifer?

A) \(\alpha = 1.8 \times 10^{-8}\) m\(^2\)/N
B) \(\alpha = 2.3 \times 10^{-8}\) m\(^2\)/N
C) \(\alpha = 2.8 \times 10^{-8}\) m\(^2\)/N
D) \(\alpha = 3.3 \times 10^{-8}\) m\(^2\)/N

Problem 2B

If the porosity of the aquifer is 12% after compaction, calculate the storativity of the aquifer. Use \(\beta = 4.6 \times 10^{-10}\) m\(^2\)/N as the compressibility of water.

A) \(S = 2 \times 10^{-3}\)
B) \(S = 4 \times 10^{-3}\)
C) \(S = 6 \times 10^{-3}\)
D) \(S = 8 \times 10^{-3}\)

Problem 3

An aquifer with an area of 7 km\(^2\) experiences a head drop of 0.85 m after 8 years of pumping. If the pumping rate is 5.5 m\(^3\)/day, determine the specific yield of the aquifer.

A) \(S_y = 1.2 \times 10^{-3}\)
B) \(S_y = 2.7 \times 10^{-3}\)
C) \(S_y = 4.2 \times 10^{-3}\)
D) \(S_y = 5.7 \times 10^{-3}\)

Problem 4

A soil sample has a volume of 180 cm\(^3\). The volume of voids in the sample is estimated as 67 cm\(^3\). Out of the volume of the voids, water can move through only 45 cm\(^3\). In the aquifer where the sample was taken, water was pumped at a rate of 6.0 m\(^3\)/day for 5 years, leading to a head drop of 1.0 m. True or False?

1. ( ) The porosity of the soil is greater than 40%.
2. ( ) The specific yield of the soil is less than 30%.
3. ( ) The specific retention of the soil is greater than 10%.
4. ( ) The area of the aquifer is less than 40,000 m\(^2\).
**Problem 5A**
A field sample of an unconfined aquifer is packed in a test cylinder. The length and diameter of the cylinder are 60 cm and 8 cm, respectively. The field sample is tested for a period of 2 min under a constant head difference of 18.5 cm. As a result, 50 cm$^3$ of water is collected at the outlet. Determine the hydraulic conductivity of the aquifer sample.

A) $K = 10.4$ m/day  
B) $K = 23.2$ m/day  
C) $K = 32.1$ m/day  
D) $K = 40.0$ m/day  

**Problem 5B**
The following additional information is given for the aquifer sample of the previous part: the sample has a median grain size of 0.04 cm and a porosity of 0.34. The test is conducted using pure water at 20°C (for which the kinematic viscosity $\nu = 10^{-6}$ m$^2$/s). True or false?

1. (   ) The Darcian velocity is greater than 8 m/day.  
2. (   ) The average linear velocity is less than 20 m/day.  
3. (   ) If a Reynolds number < 1 is the threshold for the validity of Darcy’s law, we conclude that this law is indeed applicable to the present system.

**Problem 6** (Subramanya, 2017)  
Three wells A, B, and C tap the same horizontal aquifer. Consider distances $AB = 1150$ m and $BC = 850$ m. Well B is exactly south of well A and well C lies to the west of well B. The following are the ground surface elevation and depth of water below the ground surface in the three wells. Knowing that the red arrow indicates the direction of flow, determine angle $\theta$.

<table>
<thead>
<tr>
<th>Well</th>
<th>Surface Elevation (m Above Datum)</th>
<th>Depth of Water Table (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>200.00</td>
<td>11.00</td>
</tr>
<tr>
<td>B</td>
<td>197.00</td>
<td>7.00</td>
</tr>
<tr>
<td>C</td>
<td>202.00</td>
<td>14.00</td>
</tr>
</tbody>
</table>

A) $\theta = 10.4^\circ$  
B) $\theta = 20.3^\circ$  
C) $\theta = 30.2^\circ$  
D) $\theta = 40.1^\circ$
Problem 7 (Subramanya, 2017)

A confined aquifer with a horizontal bed has a varying thickness as illustrated below. The aquifer is inhomogeneous with \( K = 12 + 0.0045x \), where \( x = 0 \) at section (1) and the piezometric heads at sections (1) and (2) are 14 m and 19 m, respectively measured above the upper confining layer. Assuming the flow in the aquifer is essentially horizontal, determine the flow rate per unit width.

\[ Q = 0.18 \text{ m}^3/\text{day}/\text{m} \]

Problem 8A (Subramanya, 2017)

A 40-cm diameter well completely penetrates a confined aquifer of coefficient of permeability 45 m/day. The length of the strainer is 18 m. Under steady state of pumping, the drawdown at the well was found to be 3.0 m and the radius of influence was 300 m. Calculate the discharge.

\[ Q = 450 \text{ lpm} \]

Problem 8B

Consider the following hypothetical statements regarding the well in the previous problem.

Statement 1: If the well diameter is changed to 0.60 m and all else remains constant, the discharge is increased by more than 10 percent.

Statement 2: If the drawdown is changed to 4.5 m and all else remains constant, the discharge is increased by more than 25 percent.

A) Both statements are true.
B) Statement 1 is true and Statement 2 is false.
C) Statement 1 is false and Statement 2 is true.
D) Both statements are false.
Problem 9

A 30-cm well completely penetrates an unconfined aquifer of saturated depth of 40 m. After a long period of pumping at a steady rate of 1600 lpm, the drawdowns in two observation wells 30 m and 90 m from the well were found to be 4.0 m and 2.0 m, respectively. Determine the transmissivity of the aquifer and the drawdown at the pumping well.

A) \( T = 2.52 \times 10^{-3} \) m²/s and \( s_w = 13 \) m
B) \( T = 2.52 \times 10^{-3} \) m²/s and \( s_w = 20 \) m
C) \( T = 5.06 \times 10^{-3} \) m²/s and \( s_w = 13 \) m
D) \( T = 5.06 \times 10^{-3} \) m²/s and \( s_w = 20 \) m

Problem 10

A well is located in an aquifer with a conductivity of 15 m/day and a storativity of 0.0057. The aquifer is 20 m thick and is pumped at a rate of 2800 m³/day. What is the drawdown at a distance of 6.5 m from the well after 1 day of pumping?

A) \( s = 2.8 \) m
B) \( s = 5.9 \) m
C) \( s = 7.9 \) m
D) \( s = 10.4 \) m

Problem 11A (Fetter, 2014)

A well that is screened in a confined aquifer is to be pumped at a rate of 165,000 ft³/day for 30 days. If the aquifer transmissivity is 5320 ft²/day, and the storativity is 0.0007, what is the drawdown at a distance of 50 ft from the well?

A) \( s = 10.4 \) ft
B) \( s = 20.3 \) ft
C) \( s = 30.2 \) ft
D) \( s = 40.1 \) ft

Problem 11B

If the aquifer described in the previous problem is not fully confined, but is overlain by a 8.0-ft thick leaky confining layer with a vertical hydraulic conductivity of 0.034 ft²/day, what would be the drawdown value at a distance of 50 ft from the well?

A) \( s = 4.0 \) ft
B) \( s = 8.0 \) ft
C) \( s = 12.0 \) ft
D) \( s = 16.0 \) ft
Problem 12A (Gupta, 2016, w/ permission)

A confined aquifer is pumped at a rate of 1.20 ft$^3$/sec. In an observation well a distance of 200 ft from the well, the following drawdown data were observed. Determine the transmissivity and storage coefficient of the aquifer.

<table>
<thead>
<tr>
<th>Time since Pumping Started (min)</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Drawdown (ft)</td>
<td>0.66</td>
<td>0.87</td>
<td>0.99</td>
<td>1.11</td>
<td>1.21</td>
<td>1.36</td>
<td>1.49</td>
<td>1.75</td>
</tr>
<tr>
<td>Time (min)</td>
<td>10.0</td>
<td>14.0</td>
<td>18.0</td>
<td>24.0</td>
<td>30.0</td>
<td>40.0</td>
<td>50.0</td>
<td>60.0</td>
</tr>
<tr>
<td>Drawdown (ft)</td>
<td>1.86</td>
<td>2.08</td>
<td>2.2</td>
<td>2.36</td>
<td>2.49</td>
<td>2.65</td>
<td>2.78</td>
<td>2.88</td>
</tr>
<tr>
<td>Time (min)</td>
<td>80.0</td>
<td>100.0</td>
<td>120.0</td>
<td>150.0</td>
<td>180.0</td>
<td>210.0</td>
<td>240.0</td>
<td></td>
</tr>
<tr>
<td>Drawdown (ft)</td>
<td>3.04</td>
<td>3.16</td>
<td>3.28</td>
<td>3.42</td>
<td>3.51</td>
<td>3.61</td>
<td>3.67</td>
<td></td>
</tr>
</tbody>
</table>

Problem 12B

Solve the previous problem by the Cooper-Jacob method.

Problem 13

A well fully penetrating an unconfined aquifer of saturated thickness 50 ft is pumped at a rate of 0.8 ft$^3$/s. The drawdowns as measured in an observation well 30 ft from the pumped well are shown below.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Drawdown (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.52</td>
</tr>
<tr>
<td>40</td>
<td>1.77</td>
</tr>
<tr>
<td>60</td>
<td>2.01</td>
</tr>
<tr>
<td>80</td>
<td>2.39</td>
</tr>
<tr>
<td>100</td>
<td>2.88</td>
</tr>
<tr>
<td>200</td>
<td>3.42</td>
</tr>
<tr>
<td>400</td>
<td>4.07</td>
</tr>
<tr>
<td>800</td>
<td>4.39</td>
</tr>
<tr>
<td>1200</td>
<td>4.72</td>
</tr>
<tr>
<td>1700</td>
<td>4.91</td>
</tr>
<tr>
<td>2000</td>
<td>5.03</td>
</tr>
</tbody>
</table>

Problem 14

Barometric pressure head and the observed head values in a well in an aquifer are given in the next table. The measurements started at 08:00 hour and continued until 20:00 hour with 1-hour increments. Using these data, estimate the barometric efficiency of the aquifer. Then, with the estimated barometric efficiency value, determine the undisturbed head values in the well during the measurement period.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>$p_a/\gamma$ (m)</th>
<th>$h$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00</td>
<td>10.256</td>
<td>1604.254</td>
</tr>
<tr>
<td>9:00</td>
<td>10.247</td>
<td>1604.257</td>
</tr>
<tr>
<td>10:00</td>
<td>10.244</td>
<td>1604.260</td>
</tr>
<tr>
<td>11:00</td>
<td>10.241</td>
<td>1604.260</td>
</tr>
<tr>
<td>12:00</td>
<td>10.232</td>
<td>1604.266</td>
</tr>
<tr>
<td>13:00</td>
<td>10.223</td>
<td>1604.269</td>
</tr>
<tr>
<td>14:00</td>
<td>10.217</td>
<td>1604.275</td>
</tr>
<tr>
<td>15:00</td>
<td>10.196</td>
<td>1604.284</td>
</tr>
<tr>
<td>16:00</td>
<td>10.208</td>
<td>1604.278</td>
</tr>
<tr>
<td>17:00</td>
<td>10.214</td>
<td>1604.275</td>
</tr>
<tr>
<td>18:00</td>
<td>10.217</td>
<td>1604.275</td>
</tr>
<tr>
<td>19:00</td>
<td>10.233</td>
<td>1604.269</td>
</tr>
<tr>
<td>20:00</td>
<td>10.232</td>
<td>1604.260</td>
</tr>
</tbody>
</table>
### Table 1 Values of well function $W(u)$ for values of $u$

<table>
<thead>
<tr>
<th>$u$</th>
<th>$1.0$</th>
<th>$2.0$</th>
<th>$3.0$</th>
<th>$4.0$</th>
<th>$5.0$</th>
<th>$6.0$</th>
<th>$7.0$</th>
<th>$8.0$</th>
<th>$9.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\times 1$</td>
<td>0.219</td>
<td>0.049</td>
<td>0.013</td>
<td>0.0033</td>
<td>0.0011</td>
<td>0.00036</td>
<td>0.00012</td>
<td>0.000038</td>
<td>0.0000012</td>
</tr>
<tr>
<td>$\times 10^{-1}$</td>
<td>1.82</td>
<td>1.22</td>
<td>0.91</td>
<td>0.70</td>
<td>0.56</td>
<td>0.45</td>
<td>0.37</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td>$\times 10^{-2}$</td>
<td>4.04</td>
<td>3.35</td>
<td>2.96</td>
<td>2.68</td>
<td>2.47</td>
<td>2.30</td>
<td>2.15</td>
<td>2.03</td>
<td>1.92</td>
</tr>
<tr>
<td>$\times 10^{-3}$</td>
<td>6.33</td>
<td>5.64</td>
<td>5.23</td>
<td>4.95</td>
<td>4.73</td>
<td>4.54</td>
<td>4.39</td>
<td>4.26</td>
<td>4.14</td>
</tr>
<tr>
<td>$\times 10^{-4}$</td>
<td>8.63</td>
<td>7.94</td>
<td>7.33</td>
<td>7.25</td>
<td>7.02</td>
<td>6.84</td>
<td>6.69</td>
<td>6.55</td>
<td>6.44</td>
</tr>
<tr>
<td>$\times 10^{-5}$</td>
<td>10.54</td>
<td>10.24</td>
<td>9.84</td>
<td>9.55</td>
<td>9.33</td>
<td>9.14</td>
<td>8.99</td>
<td>8.86</td>
<td>8.74</td>
</tr>
<tr>
<td>$\times 10^{-8}$</td>
<td>17.84</td>
<td>17.15</td>
<td>16.74</td>
<td>16.46</td>
<td>16.23</td>
<td>16.03</td>
<td>15.80</td>
<td>15.64</td>
<td>15.48</td>
</tr>
<tr>
<td>$\times 10^{-9}$</td>
<td>20.15</td>
<td>19.46</td>
<td>19.05</td>
<td>18.76</td>
<td>18.54</td>
<td>18.33</td>
<td>18.20</td>
<td>18.07</td>
<td>17.93</td>
</tr>
<tr>
<td>$\times 10^{-11}$</td>
<td>24.75</td>
<td>24.06</td>
<td>23.65</td>
<td>23.36</td>
<td>23.14</td>
<td>22.96</td>
<td>22.81</td>
<td>22.67</td>
<td>22.55</td>
</tr>
<tr>
<td>$\times 10^{-12}$</td>
<td>27.05</td>
<td>26.36</td>
<td>25.95</td>
<td>25.67</td>
<td>25.44</td>
<td>25.26</td>
<td>25.11</td>
<td>24.97</td>
<td>24.86</td>
</tr>
<tr>
<td>$\times 10^{-13}$</td>
<td>29.36</td>
<td>28.66</td>
<td>28.26</td>
<td>27.97</td>
<td>27.75</td>
<td>27.56</td>
<td>27.41</td>
<td>27.28</td>
<td>27.16</td>
</tr>
<tr>
<td>$\times 10^{-14}$</td>
<td>31.66</td>
<td>30.97</td>
<td>30.56</td>
<td>30.27</td>
<td>30.05</td>
<td>29.87</td>
<td>29.71</td>
<td>29.58</td>
<td>29.46</td>
</tr>
<tr>
<td>$\times 10^{-15}$</td>
<td>33.96</td>
<td>33.27</td>
<td>32.86</td>
<td>32.58</td>
<td>32.35</td>
<td>32.17</td>
<td>32.02</td>
<td>31.88</td>
<td>31.76</td>
</tr>
</tbody>
</table>

---

**Figure 1** Type curves.
Solutions

P.1 ■ Solution

The intrinsic permeability $K_i$ is linked to the hydraulic conductivity $K$ and other parameters by the equation

$$K = K_i \frac{g}{\nu}$$

Here, $g$ is the acceleration of gravity and $\nu$ is the kinematic viscosity.

Since $1 \text{ D} = 9.87 \times 10^{-9} \text{ cm}^2$, the intrinsic permeability of the rock in question is $K_i = 2.66 \times 10^{-11} \text{ cm}^3$. Noting that $\nu = 10^{-2} \text{ cm}^2/\text{s}$ and $g = 981 \text{ cm/s}^2$, the hydraulic conductivity of the sample is determined to be

$$K = \frac{(2.66 \times 10^{-11}) \times 981}{10^{-2}} = 2.61 \times 10^{-5} \text{ cm/s}$$

★ The correct answer is A.

P.2 ■ Solution

Part A: The given parameter values are the change in head $dP = 25 \text{ m}$, the original aquifer thickness $b = 45 \text{ m}$, and the change in aquifer thickness $db = 0.20 \text{ m}$. A pressure head of 25 m of water can be converted to a fluid pressure by multiplying the pressure head by the density of water times the gravitational constant, that is,
Aquifer compressibility is defined as 
\[ \alpha = \left( \frac{db}{b} \right) / dP \]
where \( dP \) is the change in pressure. Substituting the pertaining variables, we obtain
\[ \alpha = \left( \frac{0.20/45}{245,000} \right) = 1.8 \times 10^{-8} \text{ m}^2 / \text{N} \]
★ The correct answer is A.

**Part B:** The aquifer storativity can be found from the Jacob equation
\[ S = b \left[ \rho_w \cdot g \cdot (\alpha + \eta \beta) \right] \]
where \( b = 45 \text{ m} \) is the initial thickness of the aquifer, \( \rho_w = 1000 \text{ kg/m}^3 \) is the density of water, \( g = 9.81 \text{ m/s}^2 \), \( \alpha = 1.8 \times 10^{-8} \text{ m}^2 / \text{N} \) as computed above, \( n = 0.12 \) is the porosity of the aquifer, and \( \beta = 4.6 \times 10^{-10} \text{ m}^2 / \text{N} \) is the compressibility of water, with the result that
\[ S = 45 \left[ 1000 \times 9.81 \times \left( 1.8 \times 10^{-8} + 0.12 \times 4.6 \times 10^{-10} \right) \right] = 8 \times 10^{-3} \]
★ The correct answer is D.

**P.3 Solution**
All we have to do is apply the equation that defines specific yield, namely,
\[ S_f = \frac{1}{A} \frac{dV}{dh} \]
where \( dV \) is the volume of withdrawn water, \( dh \) is the variation in level of water of the aquifer, and \( A \) is the aquifer area. In the present case, \( \Delta V \) is such that
\[ \Delta V = 5.5 \text{ m}^3 \times 8 \text{ year} \times 365 \text{ day/yr} \times \frac{1 \text{ day}}{24 \text{ hr/day} \times 3600 \text{ sec/hr} \times 3 \text{ day}} = 16,060 \text{ m}^3 \]
while \( \Delta h = 0.85 \text{ m} \) and \( A = 7 \text{ km}^2 = 7 \times 10^6 \text{ m}^2 \) as mentioned in the problem statement. Accordingly,
\[ S_f = \frac{1}{\left( \frac{7 \times 10^6}{0.85} \right)} \frac{16,060}{2.7 \times 10^{-3}} = 2.7 \times 10^{-3} \]
★ The correct answer is B.

**P.4 Solution**
1. False. The porosity of the soil is
\[ n = \frac{V_r}{V} = \frac{67}{180} = 0.37 = 37\% \]
2. True. The specific yield (or effective porosity) of the soil follows as
\[ S_f = \frac{V_r}{V} = \frac{45}{180} = 0.25 = 25\% \]
3. True. Recall that the porosity is given by the sum of specific yield and specific retention; that is,
\[ n = S_f + S_r \]
Rearranging this relation, we can determine the soil’s specific retention,
\[ n = S_f + S_r \Rightarrow S_r = n - S_f \]
\[ \therefore S_r = 0.37 - 0.25 = 0.12 = 12\% \]
4. False. The area of the aquifer can be calculated from the definition of specific yield,
\[ S_f = \frac{V_r}{A} \Delta h \Rightarrow A = \frac{Q \Delta t}{S_f \Delta h} = \frac{6.0 \times \left( 5 \times 365 \right)}{0.25 \times 1.0} = 43,800 \text{ m}^2 \]
P.5 ■ Solution

Part A: The cross-sectional area of the sample is

\[ A = \frac{\pi \times 0.08^2}{4} = 0.00503 \, \text{m}^2 \]

The hydraulic gradient, \( \frac{dh}{dl} \), is given by

\[ \frac{dh}{dl} = \frac{-18.5}{60} = -0.308 \]

and the average flow rate is

\[ Q = \frac{50}{2} = 25 \, \text{cm}^3/\text{min} = 0.036 \, \text{m}^3/\text{day} \]

Using Darcy’s law, the hydraulic conductivity is determined as

\[ Q = -KA \frac{dh}{dl} \rightarrow K = -\frac{Q}{A \times \frac{dh}{dl}} = -\frac{0.036}{0.00503 \times (-0.308)} = 23.2 \, \text{m/day} \]

The correct answer is B.

Part B: 1. False. The Darcian velocity assumes that flow occurs through the entire cross-section of the material without regard to solids and pores. It is determined as

\[ V = -K \frac{dh}{dt} = -23.2 \times (-0.308) = 7.15 \, \text{m/day} \]

2. False. The Darcian velocity is not a true velocity as the cross-sectional area of the soil is partially blocked with soil material. The average linear velocity is different from the Darcian velocity in that it accounts for the fact that flow is limited only to pore space. In mathematical terms, it is equal to the Darcy velocity divided by the porosity; that is,

\[ V_a = \frac{Q}{nA} = \frac{V}{n} = \frac{7.15}{0.34} = 21 \, \text{m/day} \]

3. True. In order to assess the validity of Darcy’s law, we must determine the greatest velocity for which it is applicable, recalling that the threshold for the applicability of this equation is a Reynolds number \( Re < 1 \). This boundary on \( Re \) is related to the fact that Darcy’s law only accurately describes a laminar flow. The Reynolds number is a dimensionless quantity given by

\[ Re = \frac{VD}{\nu} \]

where \( V \) is linear velocity, \( D \) is particle size, and \( \nu \) is the kinematic viscosity. With \( Re = 1 \), we have

\[ Re = \frac{VD}{\nu} \rightarrow 1 = \frac{V}{\nu} \times \left( \frac{0.04 \times 10^{-3}}{10^{-6}} \right) \]

\[ \therefore \, V_{max} = \frac{10^{-4}}{0.04 \times 10^{-3}} = 0.0025 \, \text{m/s} = 216 \, \text{m/day} \]

Thus, Darcy’s law will be valid for Darcy velocities equal to or less than 216 m/day. This value happens to be significantly greater than our calculated Darcian velocity of 7.15 m/day, and hence we conclude that Darcy’s law is a valid expression to model flow in this aquifer.

P.6 ■ Solution

Let \( H \) be the elevation of the water table. We then have

\[ H_A = 200 - 11 = 189.00 \, \text{m} \]
\[ H_B = 197 - 7 = 190.00 \, \text{m} \]
\[ H_C = 202 - 14 = 188.00 \, \text{m} \]

The north direction will be designated as the \( y \)-direction, while the west will be indicated as the \( x \)-direction. Along BA, the change in elevation is
The corresponding hydraulic gradient is

$$i_y = \frac{\Delta H_y}{L_{AB}} = \frac{1.00}{1150} = 8.70 \times 10^{-4}$$

The flow velocity follows from Darcy’s law,

$$V_y = K \times i_y = 8.70 \times 10^{-4} K$$

where $K$ is the hydraulic conductivity. Proceeding similarly with direction BC, we have the change in elevation along wells B and C,

$$\Delta H_y = H_y - H_C = 190 - 188 = 2.00 \text{ m}$$

The corresponding hydraulic gradient is

$$i_y = \frac{\Delta H_y}{L_{BC}} = \frac{2.00}{850} = 2.35 \times 10^{-3}$$

The flow velocity follows from Darcy’s law,

$$V_y = K \times i_y = 2.35 \times 10^{-3} K$$

Angle $\theta$ is then

$$\tan \theta = \frac{V_y}{V_x} = \frac{8.70 \times 10^{-4} K}{2.35 \times 10^{-3} K} = 0.37 \rightarrow \theta = 20.3^\circ$$

The flow velocity makes an angle of about 20 degrees relatively to direction BC.

P.7 Solution

Darcy’s law for an aquifer of constant thickness is written as

$$Q = -KA \frac{dh}{dl}$$

Since the aquifer thickness is variable in this problem, we must also write the cross-sectional area and the hydraulic gradient as functions of the distance $x$. Assuming a unit width, the area $A$ evolves in accordance with the equation

$$A = h_1 + \frac{(b_2 - b_1)}{L} x$$

where $b_1 = 30 \text{ m}$, $b_2 = 75 \text{ m}$, and $L = 3900 \text{ m}$, so that

$$A = 30 + \frac{(75 - 30)x}{3900} = 30 + 0.0115x$$

Substituting the expressions for $A$ and $K$ into the Darcy equation leads to a relation for $Q$ in the following form,

$$Q = (12 + 0.006x)(30 + 0.0115x) \frac{dh}{dx}$$

Rearranging and applying integrals to both sides, we have

$$\int_0^{1000} \frac{1}{(12 + 0.0045x)(30 + 0.0115x)} dx = \int_0^{19} \left( -\frac{1}{Q} \right) dh$$

Performing the integration on the left-hand side by means of a partial fraction decomposition, we get

$$\int_0^{1000} \frac{1}{(12 + 0.0045x)(30 + 0.0115x)} dx = -3.098 - (-7.326) = - \frac{1}{Q} (19 - 14)$$

Finally, solving for the flow rate $Q$, we find that

$$Q = \ldots$$
\[ Q = -1.18 \text{ m}^3/\text{day}/\text{m} \]

The negative sign implies that the flow is from section 2 to section 1, not the contrary.

⭐ The correct answer is C.

P.8 Solution

Part A: With reference to the figure that accompanies the problem statement, we have the well radius \( r_w = 40/2 = 20 \text{ cm} \), the radius of influence \( R = 300 \text{ m} \), the strainer length \( B = 18 \text{ m} \), the hydraulic conductivity \( K = 45/(60 \times 24) = 5.21 \times 10^{-4} \text{ m/s} \), and the transmissivity \( T = KB = (5.21 \times 10^{-4}) \times 18 = 0.00938 \text{ m}^2/\text{s} \). The discharge can be determined with the relation

\[
Q = \frac{2\pi T s_w}{\ln(R/r_w)} = \frac{2\pi \times 0.00938 \times 3.0}{\ln(300/0.20)} = 0.0242 \text{ m}^3/\text{s} = 1450 \text{ lpm}
\]

⭐ The correct answer is C.

Part B: Suppose the well diameter were increased to 0.60 m. If \( T, s_w, \) and \( R \) all remain constant, we can write the ratio of discharges

\[
\frac{Q_1}{Q_2} = \frac{\ln(R/r_{w2})}{\ln(R/r_{w1})} \rightarrow Q_1 = 1450 \times \frac{\ln(300/0.20)}{\ln(300/0.30)} = 1535 \text{ lpm}
\]

That is, increasing the well diameter by 50% will produce an increase of about 6% in the discharge. This modest increase is apparent in the logarithmic relation between discharge and well diameter. Statement 1 is false. Suppose now the drawdown were shifted to 4.5 m. If \( T, R, \) and \( r_w \) all remain constant, we can write the ratio of discharges

\[
\frac{Q_1}{Q_2} = \frac{s_{w1}}{s_{w2}} \rightarrow Q_1 = \frac{4.5}{3.0} \times 1450 = 2175 \text{ lpm}
\]

Note that the discharge increases linearly with drawdown when other data remain unchanged. Statement 2 is true.

⭐ The correct answer is C.

P.9 Solution

The discharge is \( Q = 1600/(60 \times 1000) = 0.0267 \text{ m}^3/\text{s} \). Let the well located at a distance of 25 m from the pumping station be designated as well 1, so that \( r_1 = 30 \text{ m} \) and \( h_1 = 40 - 4.0 = 36 \text{ m} \); similarly, let the well located at a distance of 75 m from the pumping station be designated as well 2, so that \( r_2 = 90 \text{ m} \) and \( h_2 = 40 - 2.0 = 38 \text{ m} \). The hydraulic conductivity of the material can be determined with the relation

\[
Q = \frac{\pi K (h_1^2 - h_2^2)}{\ln(r_1/r_2)} \rightarrow 0.0267 = \frac{\pi \times K \times (38^2 - 36^2)}{\ln(90/30)} \rightarrow K = 6.31 \times 10^{-5} \text{ m/s}
\]

The transmissivity is then

\[
T = KH = (6.31 \times 10^{-5}) \times 40 = 2.52 \times 10^{-3} \text{ m}^2/\text{s}
\]

To establish the value of the drawdown at the pumping well, we utilize the slightly modified equation

\[
Q = \frac{\pi K (h_1^2 - h_w^2)}{\ln(r_1/r_w)}
\]

where \( r_w \) is the radius of the pumping well and \( h_w \) is the water level at the main well, which is the variable we seek. Substituting the appropriate variables and solving for \( h_w \), it follows that

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\[
2.52 \times 10^{-3} = \frac{\pi \left(2.52 \times 10^{-1}\right) \left(38^2 - h_c^2\right)}{\ln(30/0.15)} \rightarrow h_c = 27 \text{ m}
\]

The drawdown is then

\[
s = 40 - h_c \rightarrow s = 13 \text{ m}
\]

★ The correct answer is A.

**P.10 Solution**

The drawdown can be determined via the Theis equation,

\[
h_c - h = \frac{Q}{4\pi T} W(u)
\]

where \(W(u)\) is the so-called well function, a complex integral of which tabulated values can be found in the Additional Information section. Evaluation of this integral requires the well factor \(u\), which is given by

\[
u = \frac{r^2 S}{4T}
\]

The transmissivity is

\[
T = Kb = 15 \times 20 = 300 \text{ m}^2/\text{day}
\]

Coefficient \(u\) then follows as

\[
u = \frac{r^2 S}{4T} = \frac{6.5^2 \times 0.0057}{4 \times 300 \times 1} = 2 \times 10^{-4}
\]

Taking this value to Table 1, the corresponding \(W(u)\) value is read as 7.94.

The drawdown can then be determined with the Theis equation,

\[
h_c - h = s = \frac{Q}{4\pi T} W(u) = \frac{2800}{4\pi \times 300} \times 7.94 = 5.9 \text{ m}
\]

The drawdown is close to six meters.

★ The correct answer is B.

**P.11 Solution**

Part A: The well factor \(u\) for this system is

\[
u = \frac{r^2 S}{4T} = \frac{50^2 \times 0.0007}{4 \times 5320 \times 30} = 2.74 \times 10^{-6}
\]

As in the previous problem, the next step is to enter this value of \(u\) in Table 1 and read the corresponding \(W(u)\). Since the table does not have a value \(W(u)\) for such a specific coefficient, an interpolation is in order. One way to set up this interpolation in Mathematica is to apply the code

\[
\text{InterpolatingPolynomial[}
\{(2 \times 10^{-6}, 12.55), (3 \times 10^{-6}, 12.14)\}, u]/.u \rightarrow 2.74 \times 10^{-6}
\]

This returns \(W(u) = 12.25\). We can now determine the theoretical drawdown after 30 days of pumping by means of the relation

\[
h_c - h = s = \frac{Q}{4\pi T} W(u) = \frac{165000}{4\pi \times 5320} \times 12.25 = 30.2 \text{ ft}
\]

The drawdown is close to thirty feet.

★ The correct answer is C.

Part B: In the presence of an additional leaky confining layer, the drawdown is given by the Hantush-Jacob formula, namely

\[
h_c - h = s = \frac{Q}{4\pi T} W(u, r/B)
\]

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where \(W(u, r/B)\) is the so-called leaky artesian well function, a complex function whose value can be extracted from Table 2. In order for us to utilize this function, we must determine the leakage factor \(B = \left(\frac{Tb'}{K'}\right)^{1/2}\), where \(b'\) is the thickness of the overlying layer and \(K'\) is its hydraulic conductivity; that is,

\[
B = \left(\frac{Tb'}{K'}\right)^{1/2} = \left(\frac{5320 \times 8}{0.034}\right)^{1/2} = 1119 \text{ ft}
\]

We can then compute the ratio \(r/B = 50/1119 = 0.0447\). Recall that \(u = 2.74 \times 10^{-6}\). Referring to Table 2 with these two quantities, the value of \(W(u, r/B)\) is determined to be 6.47. The theoretical drawdown then follows as

\[
h_s - h = \frac{165,000}{4\pi \times 5320} \times 6.47 = 16.0 \text{ ft}
\]

★ The correct answer is **D**.

**P.12 Solution**

**Part A:** The storage and transmissivity of the aquifer can be determined by means of the type-curve method. This procedure is based on the fact that the following equations hold for an impermeable confined aquifer of infinite extent.

\[
\log s = \left[\log \left(\frac{Q}{4\pi T}\right) + \log W(u)\right]
\]

\[
\log \left(\frac{t}{r^2}\right) = \left[\log \left(\frac{S}{4T}\right) + \log \frac{1}{u}\right]
\]

For a constant \(Q\), the bracketed parts of the equations above are constant. Thus if a constant equal to \(\log(\frac{Q}{4\pi T})\) is added to \(\log s\), \(s\) is obtained. Similarly, when \(\log(\frac{S}{4T})\) is added to \(\log(1/u)\), the result is \(\log(t/r^2)\). In other words, a graph between \(\log W(u)\) and \(\log(1/u)\) is similar to a graph between \(\log s\) and \(\log(t/r^2)\). It is offset by constant amounts, as illustrated below.

Having understood the fact that the foregoing equations produce similar graphs, we can propose a method to obtain the parameters necessary for calculation of transmissivity and storage coefficient. The procedure is summarized below.

1. Prepare a plot on log-log paper of \(W(u)\) (on vertical coordinates) versus \(1/u\) (on horizontal coordinates). This is known as the type curve. For various values of \(u\), \(W(u)\) can be determined from Table 1. Three possible type curves are shown in Figure 1. Curve A covers the range of \(1/u\) from \(10^{-1}\) to \(10^{-2}\), curve B from \(10^2\) to \(10^5\), and curve C from \(10^5\) to \(10^8\).

2. From given pumping test data, prepare a plot, on transparent log-log paper, of drawdown, \(s\), versus \(t/r^2\). The length of each cycle of this log-log paper should be the same as that used for the type curve of step 1. This is known as the data curve. Data from this curve can be obtained from a pumping test in which discharge is kept constant. Drawdown can be obtained in an observation well at any distance \(r\) for different time intervals; that is, \(r\) is constant and time varies. Alternatively, drawdowns can be observed at the same time in wells located at different distances,
thus constituting a drawdown-distance analysis. In both cases \( t/r^2 \) is computed and plotted against \( s \) in the form of a data curve.

3. The data plot is superimposed (placed) over the type-curve plot. The data curve plot is moved up or down, right or left, keeping its \( x \) and \( y \) axes parallel to the type-curve axes, until the data curve overlaps a certain portion of the type curve (as illustrated in the previous figure).

4. Any arbitrary point is selected on the overlapping part of the two sheets (plots). This point need not be on the curves. It is often convenient to select a point on the type curve whose coordinates are a multiple of 10. Record \( W(u) \) and \( 1/u \) coordinates, and the corresponding \( s \) and \( t/r^2 \) coordinates, of this matching point.

5. The transmissivity is computed from the relation

\[
T = \frac{Q}{4\pi s} W(u)
\]

and the storage coefficient with

\[
S = 4T \frac{t}{r^2} u
\]

In the type curve, the maximum variation in \( W(u) \) takes place in the range of \( 1/u \) from \( 10^{-1} \) to \( 10^2 \) (type curve A in Figure 1) when from almost vertical the curve becomes almost horizontal. Gradually, the curve becomes more flat (horizontal). In many instances, the data curve might match curve A. The data curve should be visually compared with Figure 1 to decide which of the type curves might be the most appropriate.

Let us turn to the problem at hand. The data we need are computed in the following table.

<table>
<thead>
<tr>
<th>( t/r^2 )</th>
<th>0.000025</th>
<th>0.000038</th>
<th>0.000050</th>
<th>0.000063</th>
<th>0.000075</th>
<th>0.000100</th>
<th>0.000125</th>
<th>0.000200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>0.66</td>
<td>0.87</td>
<td>0.99</td>
<td>1.11</td>
<td>1.21</td>
<td>1.36</td>
<td>1.49</td>
<td>1.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t/r^2 )</th>
<th>0.000250</th>
<th>0.000350</th>
<th>0.000450</th>
<th>0.000600</th>
<th>0.000750</th>
<th>0.001000</th>
<th>0.001250</th>
<th>0.001500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>1.86</td>
<td>2.08</td>
<td>2.2</td>
<td>2.36</td>
<td>2.49</td>
<td>2.65</td>
<td>2.78</td>
<td>2.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t/r^2 )</th>
<th>0.002000</th>
<th>0.002500</th>
<th>0.003000</th>
<th>0.003750</th>
<th>0.004500</th>
<th>0.005250</th>
<th>0.006000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>3.04</td>
<td>3.16</td>
<td>3.28</td>
<td>3.42</td>
<td>3.51</td>
<td>3.61</td>
<td>3.67</td>
</tr>
</tbody>
</table>

The plot of \( s \) versus \( t/r^2 \) is shown in continuation.

The curve matches with type curve A. The match point corresponding to \( W(u) = 1 \) and \( 1/u = 10 \) on the data curve has coordinates \( s = 0.55 \) ft and \( t/r^2 = 5 \times 10^{-6} \) min/ft\(^2\). We can then determine the transmissivity of the aquifer,

\[
T = \frac{Q}{4\pi s} W(u) = \frac{1.2}{4\pi \times 0.55} \times 1 = 0.174 \ \text{ft}^2/\text{s} = 15,000 \ \text{ft}^3/\text{day}
\]

The storage coefficient, in turn, is given by

\[
S = \frac{t}{r^2} \times 4Tu = \left(5 \times 10^{-5}\right) \times 4 \times 0.174 \times 0.1 \times \left(\frac{60 \ \text{sec}}{1 \ \text{min}}\right) = 2.09 \times 10^{-2}
\]
Part B: Cooper and Jacob showed that when the well coefficient \( u = \frac{r^2S}{4Tt} \) is sufficiently small, steady-state conditions tend to develop in the cone of depression. Consequently, the well function can be approximated by only the first two terms of the infinite series typically used as its solution, so that the drawdowns becomes

\[
\frac{T}{s} = \frac{Q}{4\pi T} \left(-0.5772 - \ln u\right)
\]

or, manipulating this equation,

\[
\frac{T}{s} = \frac{Q}{4\pi T} \ln \frac{r^2S}{4Tt} \rightarrow \frac{T}{s} = \frac{Q}{4\pi T} \ln \frac{2.25Tt}{r^2S}
\]

and changing the logarithm base,

\[
\frac{T}{s} = \frac{2.3Q}{4\pi T} \log \left(\frac{2.25Tt}{r^2S}\right)
\]

On semilog paper, the equation above becomes a straight line, with the drawdowns in an ordinary scale on vertical coordinate and time \( t \) in a log scale on the horizontal coordinate. The line has a slope of \( \frac{2.3Q}{4\pi T} \); that is,

\[
m = \frac{2.3Q}{4\pi T}
\]

or

\[
m = \frac{2.3Q}{4\pi T} \rightarrow \frac{\Delta s}{\log(t_f/t_i)} = \frac{2.3Q}{4\pi T}
\]

If a change in the drawdown, \( \Delta s \), is considered for one log cycle, then \( \log(t_f/t_i) = 1 \) and the equation above reduces to

\[
\frac{\Delta s}{\log(t_f/t_i)} = \frac{2.3Q}{4\pi T} \rightarrow \Delta s = \frac{2.3Q}{4\pi T}
\]

or, solving for transmissivity,

\[
\frac{\Delta s}{\log(t_f/t_i)} = \frac{2.3Q}{4\pi T} \rightarrow T = \frac{2.3Q}{4\pi \Delta s}
\]

The graph in question is presented below.

It is seen that, for one cycle, \( \Delta s = 1.26 \) ft. Thus, the transmissivity is determined as

\[
T = \frac{2.3Q}{4\pi \Delta s} = \frac{2.3 \times 1.20}{4\pi \times 1.26} = 0.174 \text{ ft}^2/\text{s} = 15.028 \text{ ft}^2/\text{s}
\]

which is quite close to the result obtained in Part A. Note, now, that the drawdown is zero in the point where the line intersects the x-axis. The corresponding time is \( t_n \), as indicated in the graph. Mathematically, we have
\[
0 = \frac{2.25T_T}{r^2S} \log \frac{2.25T_T}{r^2S} \rightarrow \log \frac{2.25T_T}{r^2S} = 0
\]

which becomes

\[
\log \frac{2.25T_T}{r^2S} = 0 \rightarrow \frac{2.25T_T}{r^2S} = 1
\]

\[
\therefore S = \frac{2.25T_T}{r^2}
\]

We have thus obtained an equation for the storage coefficient of the aquifer. Substituting the pertaining data brings to

\[
S = \frac{2.25T_T}{r^2} = \frac{2.25 \times 15,028 \times 0.35}{200^2} \times \frac{1 \text{ day}}{24 \times 60 \text{ min}} = 2.05 \times 10^{-4}
\]

which is quite close to the value of \(2.09 \times 10^{-4}\) obtained earlier. Finally, it should be recognized that the derivations by Cooper-Jacob above are based on the assumption that the well coefficient \(u\) is quite small; that is,

\[
u \leq 0.05 \rightarrow \frac{r^2S}{4T_T} \leq 0.05
\]

\[
\therefore t \geq \frac{5r^2S}{T_T}
\]

If time is sufficiently long to satisfy the inequality above, the data points will begin to fall on a straight line. In the present case, we have

\[
t \geq \frac{5 \times 200^2 \times (2.05 \times 10^{-4})}{15,028} = 0.00273 \text{ day} = 4 \text{ min}
\]

That is to say, measurements after 4 minutes are valid.

**P.13 Solution**

Although the equation for groundwater flow through an unconfined aquifer may reduce to the same form as for confined flow, methods such as the Theis and Cooper-Jacob approaches are not readily applicable for three reasons, namely, (1) dewatering of the aquifer; (2) vertical flow near the well; and (3) delayed yield due to gravity drainage. However, these phenomena can be neglected under special circumstances: if the drawdown is small compared to the depth of the aquifer, the effect of dewatering can be neglected. Also, if pumping continues long enough, the effect of delayed yield becomes negligible. Furthermore, Hantush has established that the vertical effect is significant in the time period

\[
t < \frac{5bS_y}{K_z}
\]

where \(b\) is the thickness of the aquifer, \(S_y\) is the specific yield, and \(K_z\) is the hydraulic conductivity of the aquifer in the vertical \((z)\) direction.

Another investigator has established that the delayed yield is pronounced for the period

\[
t < 10S_y \frac{s}{K_z}
\]

where \(s\) is drawdown. If the test duration happens to be encompassed by one of the two preceding inequalities, a different analysis will be in order, and the type curves devised by Boulton and Neuman in the 1960s and 1970s should be used. These type curves comprise a set of curves between \(W(u)\) and \(1/u\) drawn for various values of \(K_z r^2/K_h b^2\), where \(K_h\) is the horizontal hydraulic conductivity, \(r\) is the distance from the pumping well, and \(b\) is the initial saturated thickness of the aquifer. The curves have two distinct segments. In the early part (time), the elastic storativity is responsible for the instantaneous release of water to the well similar to a confined aquifer. Later on, the specific yield is responsible for the delayed release of water to the well.
If, however, the pumping test can be extended to surpass the time values given in the two previous inequalities, the confined aquifer approach becomes valid and the late drawdown data follow the Theis curve. As for dewatering (lowering of the saturated thickness), the methods of a confined aquifer can be applied if the drawdown is less than 25% of the initial depth of saturation. The observed (measured) values of drawdown are corrected by the equation

\[ s' = s - \frac{s^2}{2b} \]

The value of specific yield determined from a confined aquifer method, in turn, is shifted in accordance with the equation

\[ S'_y = \frac{(b - \overline{s})S_y}{b} \]

where \( S_y \) is the computed specific yield and \( s \) is the drawdown at the end of pumping at a radius equal to the geometric mean of the radius of all observation wells.

Let us turn to the present problem. Below, the corrected values of drawdown are shown in the red row.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>200</th>
<th>400</th>
<th>800</th>
<th>1200</th>
<th>1700</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drawdown (m)</td>
<td>1.52</td>
<td>1.77</td>
<td>2.01</td>
<td>2.39</td>
<td>2.88</td>
<td>3.42</td>
<td>4.07</td>
<td>4.39</td>
<td>4.72</td>
<td>4.91</td>
<td>5.03</td>
</tr>
<tr>
<td>Corrected Drawdown (m)</td>
<td>1.50</td>
<td>1.74</td>
<td>1.97</td>
<td>2.33</td>
<td>2.80</td>
<td>3.30</td>
<td>3.90</td>
<td>4.20</td>
<td>4.50</td>
<td>4.67</td>
<td>4.78</td>
</tr>
</tbody>
</table>

For simplicity, we shall use the Cooper-Jacob method. A plot of drawdown and logarithm of time is shown in continuation.

It is seen that, for one log time cycle, \( \Delta s = 1.70 \) and \( t_0 = 2.75 \). The transmissivity is calculated as

\[ T = \frac{2.3Q}{4\pi \Delta s} = \frac{2.3 \times 0.8}{4 \pi \times 1.70} = 0.086 \text{ ft}^2/\text{s} = \frac{7448}{149} \text{ ft}^2/\text{day} \]

The value of \( S_y \), in turn, is

\[ S = \frac{2.25Tt_0}{r^2} = \frac{2.25 \times 7448 \times 2.75}{30^2} \times \left( \frac{1 \text{ day}}{24 \times 60 \text{ min}} \right) = 3.56 \times 10^{-2} \]

The adjusted specific yield is

\[ S'_y = \frac{(b - \overline{s})S_y}{b} = \frac{(50 - 4.78) \times 3.56 \times 10^{-3}}{50} = 3.22 \times 10^{-2} \]

We should also verify whether the pumping was carried out for a long enough period for vertical flow effects to be neglected. Given the vertical hydraulic conductivity \( K_z = \frac{7448}{50} = 149 \text{ ft/day} \), we have the inequality

\[ t > 5 \times 50 \times \left( \frac{3.22 \times 10^{-2}}{149} \right) = 0.054 \text{ day} = 78 \text{ min} \]

That is, the data over 78 min are free from vertical flow effects. Similarly, delayed yield effects can be neglected after the time period.
10 \times \left( \frac{3.22 \times 10^{-1}}{t} \right) \times 4.78 = 0.0103 \text{ day} = 15 \text{ min}

Thus, delayed yield effects can also be neglected for most of the test duration.

P.14 Solution

Water levels observed in wells under both confined and unconfined flow conditions are commonly affected by changes in atmospheric pressure, which is also known as barometric pressure. These changes sometimes become significant, especially during relatively long aquifer pumping tests. Typically, a pumping test period may range from a few hours to a couple of days, depending on the purpose of the test and the type of aquifer materials. If the barometric pressure changes significantly and causes sizable fluctuations during the test period, the drawdown data collected from an observation well distant from the pumping well will be inadequate for interpreting the test. As a result, to estimate the undisturbed hydraulic head, water-level fluctuations in observation wells resulting from the barometric pressure changes must be quantified, and the necessary changes must be made on the undisturbed drawdown data. The corrections can be performed with the use of barometric efficiency. Barometric efficiency is defined as the ratio of change in hydraulic head to the change in barometric pressure head,

$$BE = \frac{\Delta h}{\Delta \left( \frac{p_a}{\gamma} \right)}$$

This quantity is a particularly useful concept in that, besides being used to adjust hydraulic head values, it can also be used to estimate the storage coefficient of a confined aquifer, as revealed by Jacob in 1940, and the bulk elastic properties of the aquifer, as shown in papers published in the 1980s.

Let us turn to the example at hand. The calculated data are shown in the following table. Calculations include the variations $\Delta \left( \frac{p_a}{\gamma} \right)$ and $\Delta h$ of one time step to the next and the running total of these variations, which are shown in the blue (pressure head) and red (hydraulic head) columns.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>$\frac{p_a}{\gamma}$ (m)</th>
<th>$\Delta \left( \frac{p_a}{\gamma} \right)$ (m)</th>
<th>$\Delta h$ (m)</th>
<th>$\sum(\Delta h)$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00</td>
<td>10.256</td>
<td>0</td>
<td>0.000</td>
<td>1604.254</td>
</tr>
<tr>
<td>9:00</td>
<td>10.247</td>
<td>-0.009</td>
<td>-0.009</td>
<td>1604.257</td>
</tr>
<tr>
<td>10:00</td>
<td>10.244</td>
<td>-0.003</td>
<td>-0.012</td>
<td>1604.260</td>
</tr>
<tr>
<td>11:00</td>
<td>10.241</td>
<td>-0.003</td>
<td>-0.015</td>
<td>1604.260</td>
</tr>
<tr>
<td>12:00</td>
<td>10.232</td>
<td>-0.009</td>
<td>-0.024</td>
<td>1604.266</td>
</tr>
<tr>
<td>13:00</td>
<td>10.223</td>
<td>-0.009</td>
<td>-0.033</td>
<td>1604.269</td>
</tr>
<tr>
<td>14:00</td>
<td>10.217</td>
<td>-0.006</td>
<td>-0.039</td>
<td>1604.275</td>
</tr>
<tr>
<td>15:00</td>
<td>10.196</td>
<td>-0.021</td>
<td>-0.060</td>
<td>1604.284</td>
</tr>
<tr>
<td>16:00</td>
<td>10.208</td>
<td>0.012</td>
<td>-0.048</td>
<td>1604.278</td>
</tr>
<tr>
<td>17:00</td>
<td>10.214</td>
<td>0.006</td>
<td>-0.042</td>
<td>1604.375</td>
</tr>
<tr>
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<td>10.217</td>
<td>0.003</td>
<td>-0.039</td>
<td>1604.275</td>
</tr>
<tr>
<td>19:00</td>
<td>10.233</td>
<td>0.016</td>
<td>-0.023</td>
<td>1604.269</td>
</tr>
<tr>
<td>20:00</td>
<td>10.232</td>
<td>-0.001</td>
<td>-0.024</td>
<td>1604.260</td>
</tr>
</tbody>
</table>

A linear fit of the red column versus the absolute values of the blue column is shown in continuation.
The barometric efficiency is the ratio of change in hydraulic head, \( \Sigma (\Delta h) \), to the change in barometric pressure head, \( \Sigma (p_a/\gamma) \); that is, we are looking for the slope of the line above. Since the fit has the form \( y = 0.498x \), we conclude that the barometric efficiency is \( BE = 0.498 \). With this \( BE \) value, the hydraulic heads are corrected and listed in the next table. Negative \( \Delta(p_a/\gamma) \) values correspond to higher values. Hence, in the ensuing table, \( \Delta h \) values are subtracted from the observed head values.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>( h ) (m)</th>
<th>( \Delta(p_a/\gamma) )</th>
<th>( \Delta h = BE \Delta(p_a/\gamma) )</th>
<th>Corrected head ( h_c ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00</td>
<td>1604.254</td>
<td>0</td>
<td>0.0000</td>
<td>1604.2540</td>
</tr>
<tr>
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</tr>
<tr>
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<td>-0.003</td>
<td>-0.0015</td>
<td>1604.2585</td>
</tr>
<tr>
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<td>1604.260</td>
<td>-0.003</td>
<td>-0.0015</td>
<td>1604.2585</td>
</tr>
<tr>
<td>12:00</td>
<td>1604.266</td>
<td>-0.009</td>
<td>-0.0045</td>
<td>1604.2615</td>
</tr>
<tr>
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<td>-0.0045</td>
<td>1604.2645</td>
</tr>
<tr>
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<td>-0.0030</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.012</td>
<td>0.0060</td>
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</tr>
<tr>
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<tr>
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<td>0.0015</td>
<td>1604.2765</td>
</tr>
<tr>
<td>19:00</td>
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<td>0.0080</td>
<td>1604.2770</td>
</tr>
<tr>
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<td>1604.260</td>
<td>-0.001</td>
<td>-0.0005</td>
<td>1604.2595</td>
</tr>
</tbody>
</table>

Take, for instance, the row highlighted in bold. To correct this value, we first calculate the head adjustment \( \Delta h \),

\[
\Delta h = BE \times \Delta \left( \frac{p_a}{\gamma} \right) = 0.498 \times (-0.021) = -0.0105
\]

We then add apply this value to the observed head \( h \),

\[
h_c = h + \Delta h = 1604.284 - 0.0105 = 1604.2735
\]

**Answer Summary**

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 2</td>
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<tr>
<td></td>
<td>2B</td>
</tr>
<tr>
<td>Problem 3</td>
<td>B</td>
</tr>
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<td>Problem 4</td>
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<tr>
<td>Problem 5</td>
<td>5A</td>
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<td>5B</td>
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<tr>
<td>Problem 6</td>
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<tr>
<td>Problem 7</td>
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<td>8A</td>
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<td>8B</td>
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<tr>
<td>Problem 9</td>
<td>A</td>
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</table>
References


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