Montogue

Quiz SM208

DEFLECTIONS OF BEAMS

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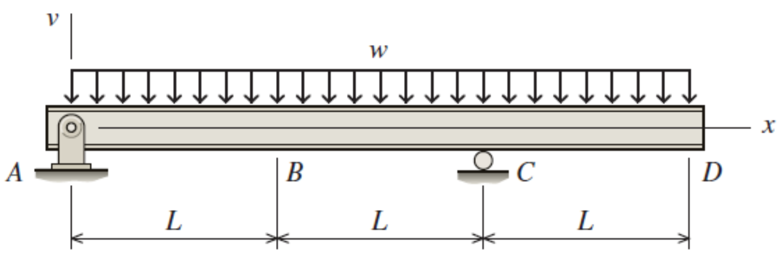
PROBLEMS

Problem 1 (Philpot, 2013, w/ permission)

For the beam and loading shown, use the double-integration method to calculate the deflection at point *B*. Assume that *EI* is constant for the beam.



A) $\delta_B = -\dfrac{wL^4}{24EI}$

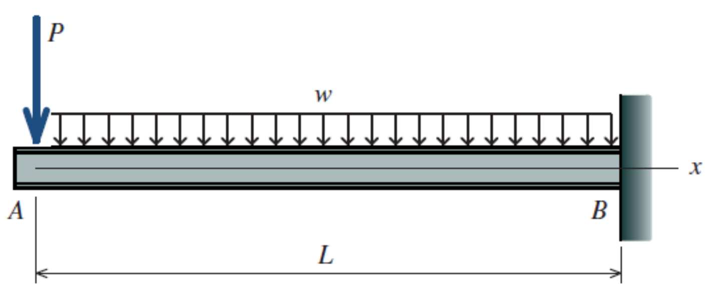
B) $\delta_B = -\dfrac{wL^4}{18EI}$

C) $\delta_B = -\dfrac{wL^4}{12EI}$

D) $\delta_B = -\dfrac{wL^4}{6EI}$

Problem 2 (Philpot, 2013, w/ permission)

For the cantilever steel beam (E = 200 GPa, I = 120×10^6 mm^4) shown, use the double-integration method to determine the deflection at point *A*. Assume that L = 2.5 m, P = 40 kN, and w = 30 kN/m.



A) $\delta_A = -8.71$ mm

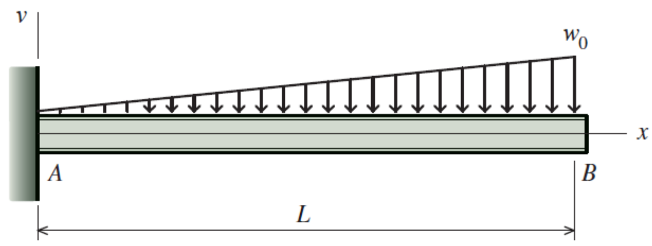
B) $\delta_A = -14.8$ mm

C) $\delta_A = -20.5$ mm

D) $\delta_A = -26.1$ mm

Problem 3 (Philpot, 2013, w/ permission)

For the beam and loading shown, use the double-integration method to calculate the deflection at point B. Assume that EI is constant for the beam.



A) $\delta_B = -\dfrac{w_0 L^4}{40EI}$

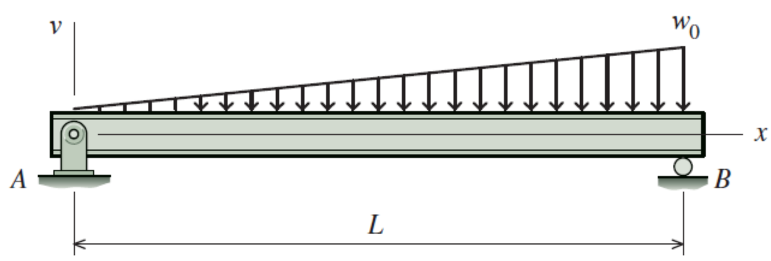
B) $\delta_B = -\dfrac{w_0 L^4}{24EI}$

C) $\delta_B = -\dfrac{7w_0 L^4}{120EI}$

D) $\delta_B = -\dfrac{11w_0 L^4}{120EI}$

Problem 4 (Philpot, 2013, w/ permission)

For the beam and loading shown, use the double-integration method to determine the maximum beam deflection. What is the maximum beam deflection if E = 200 GPa, $I = 120\times10^6$ mm^4, L = 5 m, and w_o = 50 kN/m?



A) $\delta_{max} = -8.49$ mm

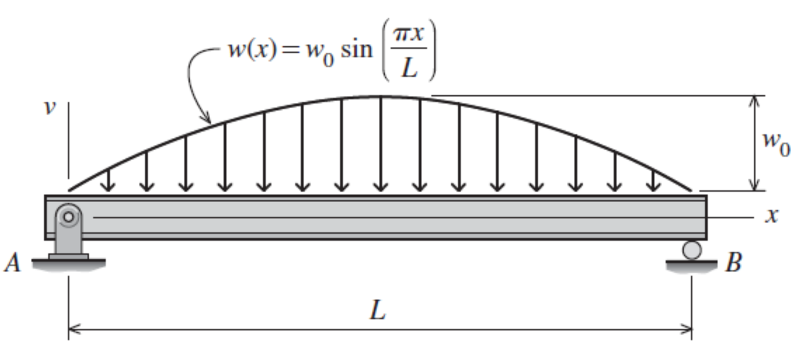
B) $\delta_{max} = -14.6$ mm

C) $\delta_{max} = -20.7$ mm

D) $\delta_{max} = -26.0$ mm

Problem 5 (Philpot, 2013, w/ permission)

For the beam and loading shown, determine the deflection midway between the supports. Assume that EI is constant for the beam.



A) $\delta_{L/2} = -\dfrac{w_0 L^4}{8\pi^4 EI}$

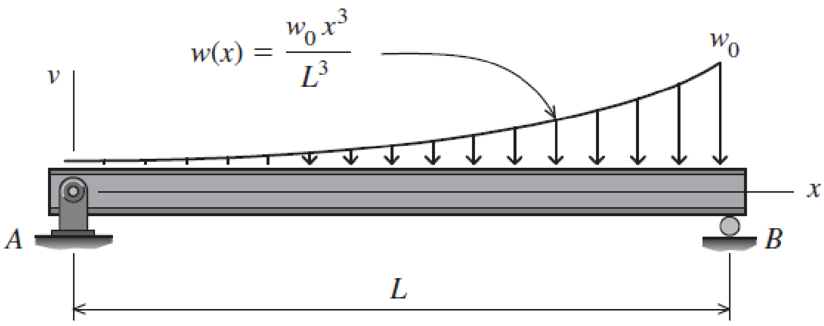
B) $\delta_{L/2} = -\dfrac{w_0 L^4}{4\pi^4 EI}$

C) $\delta_{L/2} = -\dfrac{w_0 L^4}{2\pi^4 EI}$

D) $\delta_{L/2} = -\dfrac{w_0 L^4}{\pi^4 EI}$

Problem 6 (Philpot, 2013, w/ permission)

For the beam and loading shown, determine the deflection midway between the supports. Assume that *EI* is constant for the beam.



A) $\delta_{L/2} = -\dfrac{11w_0L^4}{5120}$

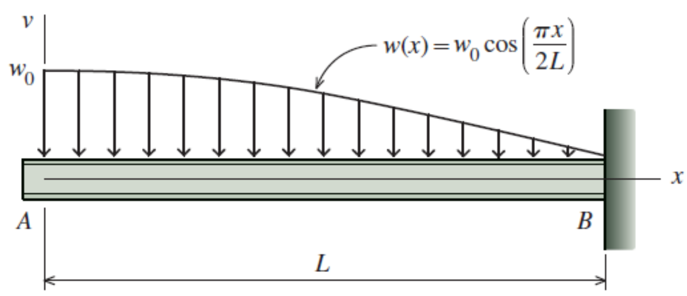
B) $\delta_{L/2} = -\dfrac{13w_0L^4}{5120EI}$

C) $\delta_{L/2} = -\dfrac{17w_0L^4}{5120EI}$

D) $\delta_{L/2} = -\dfrac{19w_0L^4}{5120EI}$

Problem 7 (Philpot, 2013, w/ permission)

For the beam and loading shown, determine the deflection at the left end of the beam. Assume that *EI* is constant for the beam.



A) $\delta_A = -0.109\dfrac{w_0L^4}{EI}$

B) $\delta_A = -0.405\dfrac{w_0L^4}{EI}$

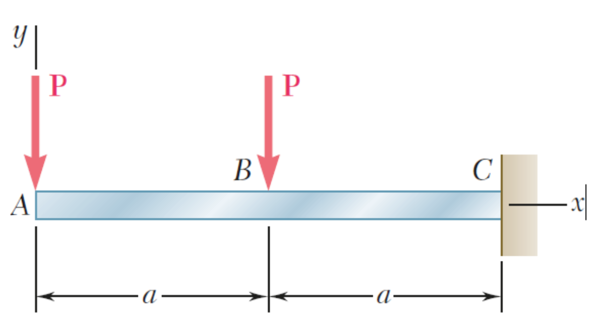
C) $\delta_A = -0.710\dfrac{w_0L^4}{EI}$

D) $\delta_A = -1.10\dfrac{w_0L^4}{EI}$

Problem 8 (Beer et al., 2012, w/ permission)

Use singularity functions to determine the elastic curve for the beam shown below. Consider the following statements.

Statement 1: The absolute value of the slope at the free end is greater than $3Pa^2/2EI$.
Statement 2: The absolute value of the deflection at the free end is greater than $4Pa^3/EI$.



A) Both statements are true.

B) Statement 1 is true and statement 2 is false.

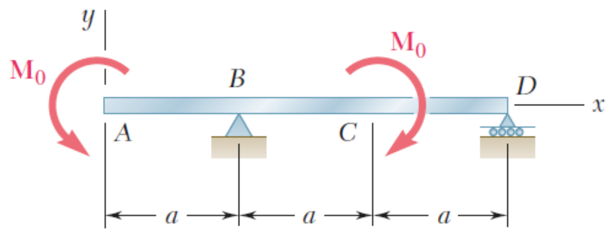
C) Statement 1 is false and statement 2 is true.

D) Both statements are false.

Problem 9 (Beer et al., 2012, w/ permission)

Use singularity functions to determine the elastic curve for the beam shown below. Consider the following statements.

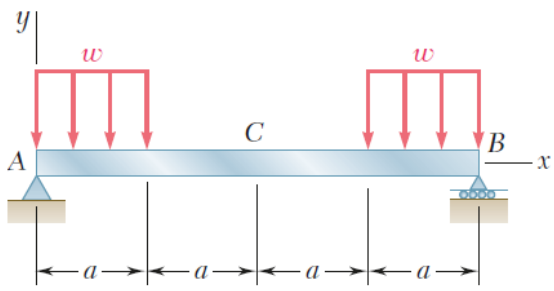
Statement 1: The absolute value of the slope at point *A* is greater than $M_0 a/2EI$.
Statement 2: The absolute value of the deflection at point *D* is greater than $M_0 a^2/EI$.



A) Both statements are true.
B) Statement 1 is true and statement 2 is false.
C) Statement 1 is false and statement 2 is true.
D) Both statements are false.

Problem 10 (Beer et al., 2012, w/ permission)

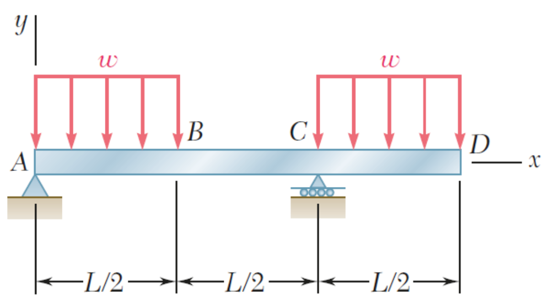
Use singularity functions to determine the elastic curve for the beam shown below, then determine the deflection at midpoint *C*.



A) $\delta_C = -\dfrac{13wa^4}{24EI}$
B) $\delta_C = -\dfrac{17wa^4}{24EI}$
C) $\delta_C = -\dfrac{19wa^4}{24EI}$
D) $\delta_C = -\dfrac{23wa^4}{24EI}$

Problem 11 (Beer et al., 2012, w/ permission)

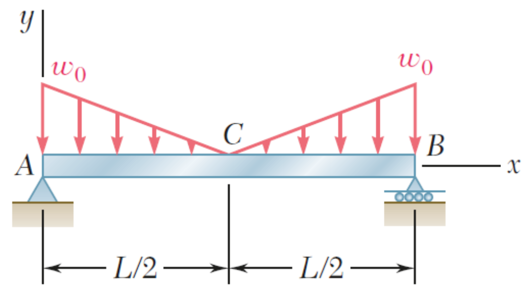
Use singularity functions to determine the elastic curve for the beam shown below, then determine the deflection at point *B*.



A) $\delta_B = -\dfrac{wL^4}{768EI}$
B) $\delta_B = -\dfrac{wL^4}{384EI}$
C) $\delta_B = +\dfrac{wL^4}{768EI}$
D) $\delta_B = +\dfrac{wL^4}{384EI}$

Problem 12 (Beer et al., 2012, w/ permission)

Use singularity functions to determine the elastic curve for the beam shown below, then determine the deflection at point *C*.



A) $\delta_C = -\dfrac{w_0 L^4}{640EI}$

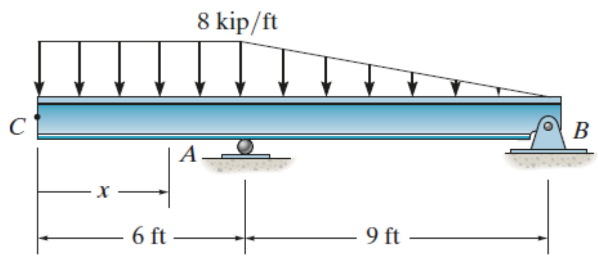
B) $\delta_C = -\dfrac{3w_0 L^4}{640EI}$

C) $\delta_C = -\dfrac{w_0 L^4}{128EI}$

D) $\delta_C = -\dfrac{7w_0 L^4}{640EI}$

Problem 13 (Hibbeler, 2014, w/ permission)

For the beam and loading shown, determine the displacement at point *C*. Assume *EI* to be constant for the beam. (The numerator in the right-hand side of each equation has units of kip-ft³.)



A) $\delta_C = -\dfrac{1040}{EI}$

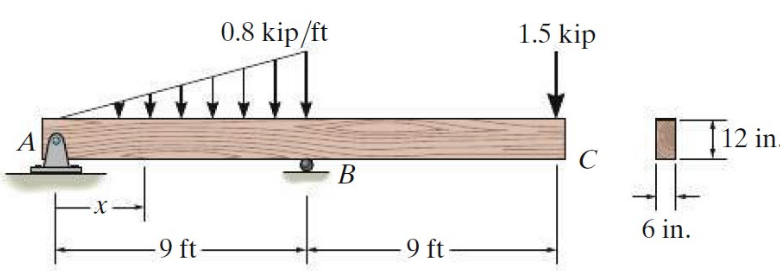
B) $\delta_C = -\dfrac{2100}{EI}$

C) $\delta_C = -\dfrac{3110}{EI}$

D) $\delta_C = -\dfrac{4090}{EI}$

Problem 14 (Hibbeler, 2014, w/ permission)

The wooden beam is subjected to the loading shown. Determine the equation of the elastic curve, then calculate the deflection at end *C*.



A) $\delta_C = -$ 0.103 in.

B) $\delta_C = -$ 0.326 in.

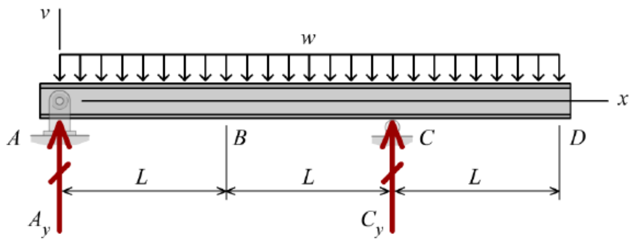
C) $\delta_C = -$ 0.544 in.

D) $\delta_C = -$ 0.767 in.

SOLUTIONS

P.1 ➡ *Solution*

Consider the free body diagram for the beam in question.



Taking moments about point *A*, we have

$$\Sigma M_A = 0 \rightarrow -w \times 3L \times \frac{3L}{2} + C_y \times 2L = 0$$

$$\therefore -\frac{9wL^2}{2} + 2LC_y = 0$$

$$\therefore C_y = \frac{9wL}{4}$$

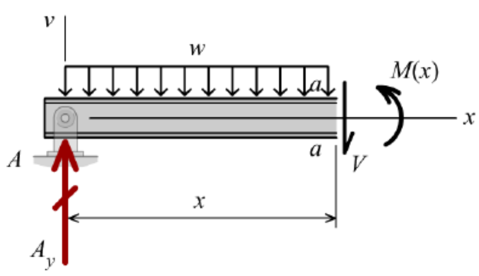
Summing forces in the *y*-direction, we have

$$\Sigma F_y = 0 \rightarrow A_y + C_y - w \times 3L = 0$$

$$\therefore A_y + \frac{9wL}{4} - 3wL = 0$$

$$\therefore A_y = \frac{3wL}{4}$$

Consider now a segment of the beam joining end *A* of the beam to a section *a-a* somewhere along its span, as shown.



Referring to the figure, the bending moment *M(x)* is determined as

$$\Sigma M_{a-a} = M(x) - A_y x + wx \times \left(\frac{x}{2}\right) = 0$$

$$\therefore M(x) - \frac{3wL}{4}x + \frac{w}{2}x^2 = 0$$

$$\therefore M(x) = -\frac{w}{2}x^2 + \frac{3wL}{4}x$$

We can then substitute this result into the moment equation,

$$EIv'' = M(x) \rightarrow EI\frac{d^2v}{dx^2} = -\frac{w}{2}x^2 + \frac{3wL}{4}x$$

Integrating once, we obtain

$$EI\frac{d^2v}{dx^2} = -\frac{w}{2}x^2 + \frac{3wL}{4}x \rightarrow EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{8}x^2 + C_1 \quad \text{(I)}$$

Integrating twice, we obtain

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{8}x^2 + C_1 \rightarrow EIv = -\frac{w}{24}x^4 + \frac{wL}{8}x^3 + C_1x + C_2 \quad \text{(II)}$$

The boundary conditions are $v(0) = 0$ (the deflection at support *A* is zero) and $v(2L) = 0$ (the deflection at support *C* is zero). Substituting the former into equation (II) gives

$$EI \times 0 = -\frac{w}{24} \times 0^4 + \frac{wL}{8} \times 0^3 + C_1 \times 0 + C_2$$
$$\therefore C_2 = 0$$

Substituting the remaining boundary condition into equation (II) yields

$$EI \times 2L = -\frac{w}{24} \times (2L)^4 + \frac{wL}{8} \times (2L)^3 + C_1 \times 2L + C_2 = 0$$
$$\therefore -\frac{2wL^4}{3} + wL^4 + 2LC_1 + 0 = 0$$
$$\therefore \frac{wL^3}{3} + 2C_1 = 0$$
$$\therefore C_1 = -\frac{wL^3}{6}$$

The elastic curve for the beam is then

$$EIv = -\frac{w}{24}x^4 + \frac{wL}{8}x^3 - \frac{wL^3}{6}x$$
$$\therefore v = -\frac{wx}{24EI}\left(x^3 - 3Lx^2 + 4L^3\right)$$

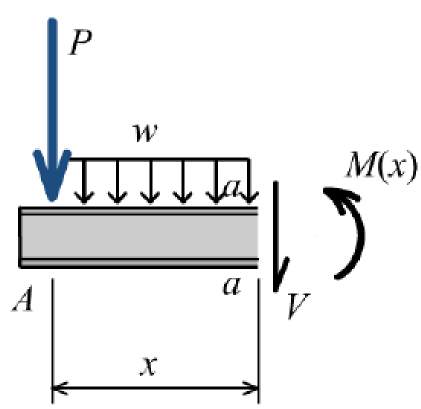
Substituting $x = L$, we can determine the deflection at *B*,

$$\delta_B = v(L) = -\frac{w \times L}{24EI}\left(L^3 - 3L \times L^2 + 4L^3\right) =$$
$$\therefore \delta_B = -\frac{wL}{24EI}\left(L^3 - 3L^3 + 4L^3\right)$$
$$\therefore \boxed{\delta_B = -\frac{wL^4}{12EI}}$$

The correct answer is **C**.

P.2 ➡ *Solution*

Consider a segment joining the left end *A* to a section *a-a* somewhere along the beam span.



Taking moments about section *a-a*, the bending moment *M(x)* is determined to be

$$\Sigma M_{a-a} = 0 \rightarrow M(x) + \frac{w}{2}x^2 + Px = 0$$
$$\therefore M(x) = -\frac{w}{2}x^2 - Px$$

Substituting into the bending moment equation, we have

$$EI\frac{d^2v}{dx^2} = M(x) = -\frac{w}{2}x^2 - Px$$

Integrating once, we get

$$EI\frac{d^2v}{dx^2} = -\frac{w}{2}x^2 - Px \rightarrow EI\frac{dv}{dx} = -\frac{w}{6}x^3 - \frac{P}{2}x^2 + C_1 \text{ (I)}$$

Integrating twice, we get

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 - \frac{P}{2}x^2 + C_1 \rightarrow EIv = -\frac{w}{24}x^4 - \frac{P}{6}x^3 + C_1x + C_2 \text{ (II)}$$

The available boundary conditions are $v'(L) = 0$ (the slope at support *B* is zero) and $v(L) = 0$ (the deflection at support *B* is zero). Substituting the former into equation (I) brings to

$$EI \times 0 = -\frac{w}{6} \times L^3 - \frac{P}{2} \times L^2 + C_1 = 0$$

$$\therefore C_1 = \frac{wL^3}{6} + \frac{PL^2}{2}$$

Substituting the remaining boundary condition into equation (II), we obtain

$$EI \times 0 = -\frac{w}{24} \times L^4 - \frac{P}{6}L^3 + \left(\frac{wL^3}{6} + \frac{PL^2}{2}\right) \times L + C_2 = 0$$

$$\therefore -\frac{wL^4}{24} - \frac{PL^3}{6} + \frac{wL^4}{6} + \frac{PL^3}{2} + C_2 = 0$$

$$\therefore \frac{wL^4}{8} + \frac{PL^3}{3} + C_2 = 0$$

$$\therefore C_2 = -\frac{wL^4}{8} - \frac{PL^3}{3}$$

Substituting the C_1 and C_2 into equation (II), the elastic curve is shown to be

$$EIv = -\frac{w}{24}x^4 - \frac{P}{6}x^3 + \left(\frac{wL^3}{6} + \frac{PL^2}{2}\right)x - \frac{wL^4}{8} - \frac{PL^3}{3}$$

$$\therefore EIv = -\frac{w}{24}x^4 + \frac{wL^3}{6}x - \frac{wL^4}{8} - \frac{P}{6}x^3 + \frac{PL^2}{2}x - \frac{PL^3}{3}$$

$$\therefore v = -\frac{w}{24EI}\left(x^4 - 4L^3x + 3L^4\right) - \frac{P}{6EI}\left(x^3 - 3L^2x + 2L^3\right)$$

To determine the deflection at the free end, we substitute $x = 0$ in the expression above,

$$v(0) = -\frac{w}{24EI}\left(0^4 - 4L^3 \times 0 + 3L^4\right) - \frac{P}{6EI}\left(0^3 - 3L^2 \times 0 + 2L^3\right)$$

$$\therefore v(0) = -\frac{wL^4}{8EI} - \frac{PL^3}{3EI}$$

Lastly, we can substitute the numerical data,

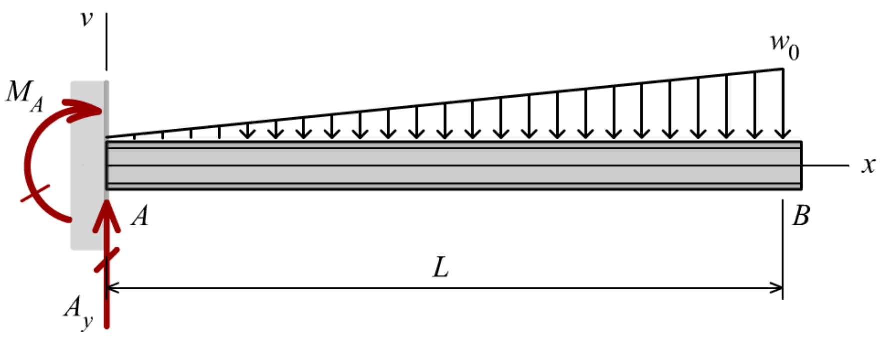
$$\delta_A = v(0) = -\frac{\left(30\times10^3\right)\times2.5^4}{8\times\left(200\times10^9\right)\times(120\times10^{-6})}\times1000 - \frac{\left(40\times10^3\right)\times2.5^3}{3\times\left(200\times10^9\right)\times\left(120\times10^{-6}\right)}\times1000$$

$$\therefore \boxed{\delta_A = -14.8 \text{ mm}}$$

The correct answer is **B**.

P.3 ➡ *Solution*

Consider the free body diagram for the beam.



Taking moments about point *A*, we have

$$\Sigma M_A = 0 \rightarrow -M_A - \frac{w_0 L}{2} \times \frac{2L}{3} = 0$$

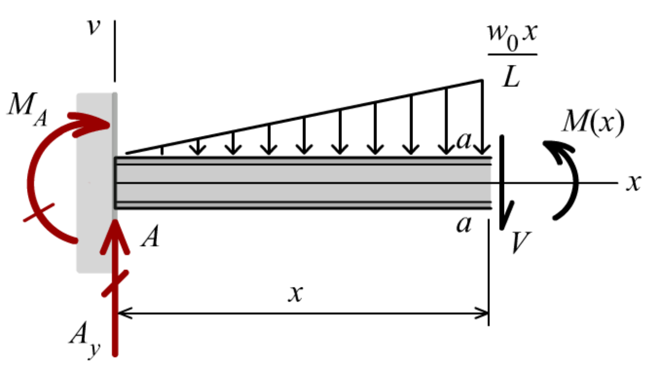
$$\therefore M_A = -\frac{w_0 L^2}{3}$$

Summing forces in the *y*-direction, we obtain

$$\Sigma F_y = 0 \rightarrow A_y - \frac{w_0 L}{2} = 0$$

$$\therefore A_y = \frac{w_0 L}{2}$$

Consider a segment joining the left end *A* to a section *a-a* somewhere along the beam span.



Taking moments about section *a-a*, the bending moment *M(x)* is determined to be

$$\Sigma M_{a-a} = 0 \rightarrow M(x) - M_A + \frac{w_0 x}{2L} \times x \times \frac{x}{3} - A_y \times x = 0$$

$$\therefore M(x) + \frac{w_0 L^2}{3} + \frac{w_0}{6L} x^3 - \frac{w_0 L}{2} x = 0$$

$$\therefore M(x) = -\frac{w_0}{6L} x^3 + \frac{w_0 L}{2} x - \frac{w_0 L^2}{3}$$

We can then set up the bending moment equation,

$$EI \frac{d^2 v}{dx^2} = M(x) = -\frac{w_0}{6L} x^3 + \frac{w_0 L}{2} x - \frac{w_0 L^2}{3}$$

Integrating once, we find that

$$EI \frac{d^2 v}{dx^2} = -\frac{w_0}{6L} x^3 + \frac{w_0 L}{2} x - \frac{w_0 L^2}{3} \rightarrow EI \frac{dv}{dx} = -\frac{w_0}{24L} x^4 + \frac{w_0 L}{4} x^2 - \frac{w_0 L^2}{3} x + C_1 \quad \text{(I)}$$

Integrating a second time, we find that

$$EI \frac{dv}{dx} = -\frac{w_0}{24L} x^4 + \frac{w_0 L}{4} x^2 - \frac{w_0 L^2}{3} x + C_1 \rightarrow EIv = -\frac{w_0}{120L} x^5 + \frac{w_0 L}{12} x^3 - \frac{w_0 L^2}{6} x^2 + C_1 x + C_2 \quad \text{(II)}$$

The available boundary conditions are $v'(0) = 0$ (the slope at the fixed end is zero) and $v(0) = 0$ (the deflection at the fixed end is zero). Substituting the former into equation (I), it is easy to see that $C_1 = 0$. Likewise, if we substitute the second boundary condition into equation (II), it follows that

$$EI \times 0 = -\frac{w_0}{120L} \times 0^5 + \frac{w_0 L}{12} \times 0^3 - \frac{w_0 L^2}{6} \times 0^2 + 0 \times 0 + C_2$$

$$\therefore C_2 = 0$$

That is to say, both integration constants are equal to zero. The elastic curve, then, is shown to be

$$EIv = -\frac{w_0}{120L} x^5 + \frac{w_0 L}{12} x^3 - \frac{w_0 L^2}{6} x^2$$

$$\therefore v = -\frac{w_0 x^2}{120LEI}\left(x^3 - 10L^2 x + 20L^3\right)$$

The deflection at the free end can be determined if we substitute $x = L$ in the relation above,

$$\delta_B = v(L) = -\frac{w_0 x^2}{120LEI}\left(L^3 - 10L^2 \times L + 20L^3\right)$$

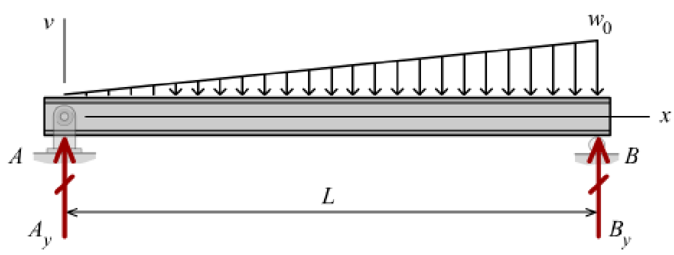
$$\therefore \delta_B = -\frac{w_0 L^2}{120LEI}\left(L^3 - 10L^3 + 20L^3\right)$$

$$\therefore \boxed{\delta_B = -\frac{11 w_0 L^4}{120EI}}$$

The correct answer is **D**.

P.4 ➡ *Solution*

Consider the free body diagram for the beam in question.



Summing moments about point A, we have

$$\Sigma M_A = 0 \rightarrow B_y L - \frac{w_0 L}{2} \times \frac{2L}{3} = 0$$

$$\therefore B_y L - \frac{w_0 L^2}{3} = 0$$

$$\therefore B_y = \frac{w_0 L}{3}$$

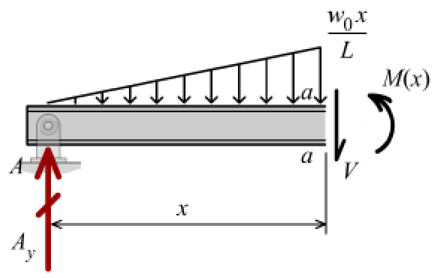
Summing forces in the y-direction, we find that

$$\Sigma F_y = 0 \rightarrow A_y + B_y - \frac{w_0 L}{2} = 0$$

$$\therefore A_y + \frac{w_0 L}{3} - \frac{w_0 L}{2} = 0$$

$$\therefore A_y = \frac{w_0 L}{6}$$

Consider a segment that goes from support A to a section *a-a* somewhere along the beam span, as shown.



Taking moments about section *a-a*, we can derive an expression for the bending moment *M(x)*,

$$\Sigma M_{a-a} = 0 \rightarrow M(x) + \frac{w_0 x^2}{2L} \times \frac{x}{3} - A_y \times x = 0$$

$$\therefore M(x) + \frac{w_0}{6L}x^3 - \frac{w_0 L}{6}x = 0$$

$$\therefore M(x) = -\frac{w_0}{6L}x^3 + \frac{w_0 L}{6}x$$

We can then set up and solve the bending moment equation,

$$EI\frac{d^2v}{dx^2} = M(x) = -\frac{w_0}{6L}x^3 + \frac{w_0 L}{6}x$$

Integrating a first time, it follows that

$$EI\frac{d^2v}{dx^2} = -\frac{w_0}{6L}x^3 + \frac{w_0 L}{6}x \rightarrow EI\frac{dv}{dx} = -\frac{w_0}{24L}x^4 + \frac{w_0 L}{12}x^2 + C_1 \ \text{(I)}$$

Integrating again, it follows that

$$EI\frac{dv}{dx} = -\frac{w_0}{24L}x^4 + \frac{w_0 L}{12}x^2 + C_1 \rightarrow EIv = -\frac{w_0}{120L}x^5 + \frac{w_0 L}{36}x^3 + C_1 x + C_2 \ \text{(II)}$$

The available boundary conditions are $v(0) = 0$ (the deflection at support *A* is zero) and $v(L) = 0$ (the deflection at support *B* is zero). Applying the former to equation (II), it is obvious that $C_2 = 0$. Applying the other boundary condition to equation (I), we obtain

$$EI \times 0 = -\frac{w_0}{120L} \times L^5 + \frac{w_0 L}{36} \times L^3 + C_1 \times L + 0 = 0$$

$$\therefore -\frac{w_0 L^4}{120} + \frac{w_0 L^4}{36} + C_1 L = 0$$

$$\therefore \frac{7w_0 L^4}{360} + C_1 L = 0$$

$$\therefore C_1 = -\frac{7w_0 L^3}{360}$$

The equation for the elastic curve is then

$$EIv = -\frac{w_0}{120L}x^5 + \frac{w_0 L}{36}x^3 - \frac{7w_0 L^3}{360}x$$

$$\therefore v = -\frac{w_0 x}{360LEI}\left(3x^4 - 10L^2x^2 + 7L^4\right)$$

It is not immediately clear where the maximum deflection occurs. We do know, however, that the maximum deflection occurs where the beam slope is zero. Accordingly, we can set equation (I) to zero and solve for *x*,

$$EI\frac{dv}{dx} = -\frac{\cancel{w_0}}{24L}x^4 + \frac{\cancel{w_0}L}{12}x^2 - \frac{7\cancel{w_0}L^3}{360} = 0$$

$$\therefore -\frac{0.0417}{L}x^4 + 0.0833Lx^2 - 0.0194L^3 = 0$$

There are two positive solutions to the equation above, namely, $x = 1.315L$, which is meaningless, and $x = 0.519L$, which is the one feasible result. Thus, the maximum deflection occurs slightly to the right of the middle of the beam. Substituting $x = 0.519L$ in the equation for the elastic curve, we obtain

$$\delta_{\max} = v(0.519L) = -\frac{w_0 \times 0.519L}{360LEI} \times \left[3\times(0.519L)^4 - 10L^2 \times (0.519L)^2 + 7\times L^4\right]$$

$$\therefore \delta_{\max} = -0.00652\frac{w_0 L^4}{EI}$$

Substituting the numerical data we were given, the maximum deflection follows as

$$\delta_{\max} = -0.00652\times\frac{(50\times10^3)\times5^4}{(200\times10^9)\times(120\times10^{-6})}\times1000 = \boxed{-8.49\text{ mm}}$$

The correct answer is **A**.

P.5 ➡ *Solution*

The load equation for this beam is

$$EI\frac{d^4v}{dx^4} = -w_0\sin\left(\frac{\pi x}{L}\right)$$

Integrating successively, we have

$$EI\frac{d^4v}{dx^4} = -w_0\sin\left(\frac{\pi x}{L}\right) \to EI\frac{d^3v}{dx^3} = \frac{w_0 L}{\pi}\cos\left(\frac{\pi x}{L}\right) + C_1 \quad \text{(I)}$$

$$EI\frac{d^3v}{dx^3} = \frac{w_0 L}{\pi}\cos\left(\frac{\pi x}{L}\right) + C_1 \to EI\frac{d^2v}{dx^2} = \frac{w_0 L^2}{\pi^2}\sin\left(\frac{\pi x}{L}\right) + C_1x + C_2 \quad \text{(II)}$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0 L^2}{\pi^2}\sin\left(\frac{\pi x}{L}\right) + C_1x + C_2 \to EI\frac{dv}{dx} = -\frac{w_0 L^3}{\pi^3}\cos\left(\frac{\pi x}{L}\right) + \frac{C_1x^2}{2} + C_2x + C_3 \quad \text{(III)}$$

$$EI\frac{dv}{dx} = -\frac{w_0 L^3}{\pi^3}\cos\left(\frac{\pi x}{L}\right) + \frac{C_1x^2}{2} + C_2x + C_3 \to EIv = -\frac{w_0 L^4}{\pi^4}\sin\left(\frac{\pi x}{L}\right) + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4 \quad \text{(IV)}$$

The available boundary/continuity/symmetry conditions are listed below.

Number	*Condition*	*Meaning*
(1)	at $x = 0$, $M = EI\frac{d^2v}{dx^2} = 0$	The bending moment at the left end is zero.
(2)	at $x = L$, $M = EI\frac{d^2v}{dx^2} = 0$	The bending moment at the right end is zero.
(3)	at $x = 0$, $v = 0$	The deflection at the left end is zero.
(4)	at $x = L$, $v = 0$	The deflection at the right end is zero.

Substituting boundary condition (1) into equation (II), we obtain

$$EI\frac{d^2v}{dx^2} = \frac{w_0 L^2}{\pi^2}\underbrace{\sin\left(\frac{\pi\times0}{L}\right)}_{=0} + C_1\times0 + C_2 = 0$$

$$\therefore 0 + 0 + C_2 = 0$$

$$\therefore C_2 = 0$$

Substituting boundary condition (2) into equation (II), we obtain

$$EI\frac{d^2v}{dx^2}=\frac{w_0L^2}{\pi^2}\underbrace{\sin\left(\frac{\pi\times L}{L}\right)}_{=0}+C_1\times L+\underbrace{C_2}_{=0}=0$$

$$\therefore 0+C_1L+0=0$$

$$\therefore C_1=0$$

Substituting boundary condition (3) into equation (IV), we obtain

$$EIv=-\frac{w_0L^4}{\pi^4}\underbrace{\sin\left(\frac{\pi\times 0}{L}\right)}_{=0}+\frac{C_1\times 0^3}{6}+\frac{C_2\times 0^2}{2}+C_3\times 0+C_4=0$$

$$\therefore C_4=0$$

Lastly, substituting boundary condition (4) into equation (IV), we also obtain $C_3 = 0$. The equation of the elastic curve is then

$$EIv=-\frac{w_0L^4}{\pi^4}\sin\left(\frac{\pi}{L}x\right)\rightarrow v=-\frac{w_0L^4}{\pi^4EI}\sin\left(\frac{\pi}{L}x\right)$$

Substituting $x = L/2$ in this equation, we can determine the deflection midway between the supports,

$$\delta_{L/2}=v\left(\frac{L}{2}\right)=-\frac{w_0L^4}{\pi^4EI}\sin\left(\frac{\pi}{\cancel{L}}\times\frac{\cancel{L}}{2}\right)$$

$$\therefore \boxed{\delta_{L/2}=-\frac{w_0L^4}{\pi^4EI}}$$

The correct answer is **D**.

P.6 ➡ *Solution*

The load equation for this beam is

$$EI\frac{d^4v}{dx^4}=-\frac{w_0}{L^3}x^3$$

Integrating four times successively brings to

$$EI\frac{d^4v}{dx^4}=-\frac{w_0}{L^3}x^3\rightarrow EI\frac{d^3v}{dx^3}=-\frac{w_0}{4L^3}x^4+C_1\ \text{(I)}$$

$$\therefore EI\frac{d^3v}{dx^3}=-\frac{w_0}{4L^3}x^4+C_1\rightarrow EI\frac{d^2v}{dx^2}=-\frac{w_0}{20L^3}x^5+C_1x+C_2\ \text{(II)}$$

$$\therefore EI\frac{d^2v}{dx^2}=-\frac{w_0}{20L^3}x^5+C_1x+C_2\rightarrow EI\frac{dv}{dx}=-\frac{w_0}{120L^3}x^6+\frac{C_1}{2}x^2+C_2x+C_3\ \text{(III)}$$

$$EI\frac{dv}{dx}=-\frac{w_0}{120L^3}x^6+\frac{C_1}{2}x^2+C_2x+C_3\rightarrow EIv=-\frac{w_0}{840L^3}x^7+\frac{C_1}{6}x^3+\frac{C_2}{2}x^2+C_3x+C_4\ \text{(IV)}$$

The available boundary/continuity/symmetry conditions are listed below.

Number	*Condition*	*Meaning*
(1)	at $x = 0$, $M=EI\frac{d^2v}{dx^2}=0$	The bending moment at the left end is zero.
(2)	at $x = L$, $M=EI\frac{d^2v}{dx^2}=0$	The bending moment at the right end is zero.
(3)	at $x = 0$, $v=0$	The deflection at the left end is zero.
(4)	at $x = L$, $v=0$	The deflection at the right end is zero.

Applying condition (1) to equation (II), it is clear that $C_2 = 0$. Applying condition (2) to equation (II), in turn, we have

$$EI\frac{d^2v}{dx^2} = -\frac{w_0}{20L^3}\times L^5 + C_1\times L + 0 = 0$$

$$\therefore C_1 = \frac{w_0 L}{20}$$

Applying condition (3) to equation (IV), we effortlessly obtain $C_4 = 0$. Applying condition (4) to equation (IV), in turn, we see that

$$EIv = -\frac{w_0}{840L^3}\times L^7 + \frac{w_0 L}{20}\times\frac{1}{6}\times L^3 + \frac{0}{2}\times L^2 + C_3\times L + 0 = 0$$

$$\therefore -\frac{w_0 L^4}{840} + \frac{w_0 L^4}{120} + C_3\times L = 0$$

$$\therefore C_3 = -\frac{w_0 L^3}{140}$$

Gleaning our results, the equation of the elastic curve is

$$EIv = -\frac{w_0}{840L^3}x^7 + \frac{1}{6}\times\frac{w_0 L}{20}x^3 + \frac{0}{2}\times x^2 - \frac{w_0 L^3}{140}\times x + 0$$

$$\therefore EIv = -\frac{w_0}{840L^3}x^7 + \frac{w_0 L}{120}x^3 - \frac{w_0 L^3}{140}x$$

$$\therefore v = -\frac{w_0}{840L^3 EI}\left(x^7 - 7L^4x^3 + 6L^6x\right)$$

Substituting $x = L/2$, the deflection midway between the supports is shown to be

$$\delta_{L/2} = v\left(\frac{L}{2}\right) = -\frac{w_0}{840L^3 EI}\left[\left(\frac{L}{2}\right)^7 - 7L^4\times\left(\frac{L}{2}\right)^3 + 6L^6\times\left(\frac{L}{2}\right)\right]$$

$$\therefore \delta_{L/2} = -\frac{w_0}{840L^3 EI}\left(\frac{L^7}{128} - \frac{7L^7}{8} + 3L^7\right)$$

$$\therefore \delta_{L/2} = -\frac{w_0}{840L^3 EI}\left(\frac{L^7}{128} - \frac{112L^7}{128} + \frac{384L^7}{128}\right)$$

$$\therefore \delta_{L/2} = -\frac{w_0}{840L^3 EI}\left(\frac{273L^7}{128}\right)$$

$$\therefore \boxed{\delta_{L/2} = -\frac{13w_0 L^4}{5120EI}}$$

The correct answer is **B**.

P.7 ➡ *Solution*

The load equation for this beam is

$$EI\frac{d^4v}{dx^4} = -w_0\cos\left(\frac{\pi x}{2L}\right)$$

Integrating four times successively, we obtain

$$EI\frac{d^4v}{dx^4} = -w_0\cos\left(\frac{\pi x}{2L}\right) \to EI\frac{d^3v}{dx^3} = -\frac{2w_0 L}{\pi}\sin\left(\frac{\pi x}{2L}\right) + C_1 \quad \text{(I)}$$

$$\therefore EI\frac{d^3v}{dx^3} = -\frac{2w_0 L}{\pi}\sin\left(\frac{\pi x}{2L}\right) + C_1 \to EI\frac{d^2v}{dx^2} = \frac{4w_0 L^2}{\pi^2}\cos\left(\frac{\pi x}{2L}\right) + C_1x + C_2 \quad \text{(II)}$$

$$\therefore EI\frac{d^2v}{dx^2} = \frac{4w_0 L^2}{\pi^2}\cos\left(\frac{\pi x}{2L}\right) + C_1x + C_2 \to EI\frac{dv}{dx} = \frac{8w_0 L^3}{\pi^3}\sin\left(\frac{\pi x}{2L}\right) + \frac{C_1}{2}x^2 + C_2x + C_3 \quad \text{(III)}$$

$$EI\frac{dv}{dx} = \frac{8w_0 L^3}{\pi^3}\sin\left(\frac{\pi x}{2L}\right) + \frac{C_1}{2}x^2 + C_2x + C_3 \to EIv = -\frac{16w_0 L^4}{\pi^4}\cos\left(\frac{\pi x}{2L}\right) + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4 \quad \text{(IV)}$$

The available boundary/symmetry/continuity equations are listed below.

Number	*Condition*	*Meaning*
(1)	at $x = 0$, $V = EI\dfrac{d^3v}{dx^3} = 0$	The shear force at the free end is zero.
(2)	at $x = 0$, $M = EI\dfrac{d^2v}{dx^2} = 0$	The bending moment at the free end is zero.
(3)	at $x = L$, $\dfrac{dv}{dx} = 0$	The slope at the fixed end is zero.
(4)	at $x = L$, $v = 0$	The deflection at the fixed end is zero.

Applying condition (1) to equation (I), it is clear that $C_1 = 0$. Applying condition (2) to equation (II), in turn, we see that

$$EI\frac{d^2v}{dx^2} = \frac{4w_0L^2}{\pi^2}\underbrace{\cos\left(\frac{\pi\times 0}{2L}\right)}_{=1} + 0\times 0 + C_2 = 0$$

$$\therefore \frac{4w_0L^2}{\pi^2} + C_2 = 0$$

$$\therefore C_2 = -\frac{4w_0L^2}{\pi^2}$$

Likewise, mapping condition (3) onto equation (III) yields

$$EI\frac{dv}{dx} = \frac{8w_0L^3}{\pi^3}\underbrace{\sin\left(\frac{\pi\times L}{2L}\right)}_{=1} + \frac{0}{2}\times L^2 - \frac{4w_0L^2}{\pi^2}\times L + C_3 = 0$$

$$\therefore \frac{8w_0L^3}{\pi^3} - \frac{4w_0L^3}{\pi^2} + C_3 = 0$$

$$\therefore C_3 = -\frac{4w_0L}{\pi^3}(2-\pi)$$

Finally, applying condition (4) to equation (IV) gives

$$-\frac{16w_0L^4}{\pi^4}\times\underbrace{\cos\left(\frac{\pi\times L}{2L}\right)}_{=0} - \frac{4w_0\times L\times L^2}{2\pi^2} - \frac{4w_0\times L^3\times L}{\pi^3}\times(2-\pi) + C_4$$

$$\therefore C_4 = \frac{2w_0L^4}{\pi^3}(4-\pi)$$

Gleaning our results, the equation of the elastic curve is determined to be

$$EIv = -\frac{16w_0L^4}{\pi^4}\cos\left(\frac{\pi x}{2L}\right) + \frac{0}{6}\times x^3 + \frac{1}{2}\left(-\frac{4w_0L^2}{\pi^2}\right)x^2 - \frac{4w_0L}{\pi^3}(2-\pi)x + \frac{2w_0L^4}{\pi^3}(4-\pi)$$

$$\therefore v = -\frac{w_0}{2\pi^4EI}\left[32L^4\cos\left(\frac{\pi x}{2L}\right) + 4\pi^2L^2x^2 + 8\pi L^3(2-\pi)x - 4\pi L^4(4-\pi)\right]$$

Substituting $x = 0$, we can establish the deflection at the left end of the beam,

$$\delta_A = v(0) = -\frac{w_0}{2\pi^4EI}\left[32L^4\underbrace{\cos\left(\frac{\pi\times 0}{2L}\right)}_{=1} + 4\pi^2L^2\times 0^2 + 8\pi L^3(2-\pi)\times 0 - 4\pi L^4(4-\pi)\right]$$

$$\therefore \delta_A = -\frac{w_0}{2\pi^4EI}\times\left[32L^4 + 0 + 0 - 4\pi L^4(4-\pi)\right]$$

$$\therefore \delta_A = -\frac{w_0L^4}{EI}\times\left[\frac{32-4\pi(4-\pi)}{2\pi^4}\right]$$

Evaluating the quantity in brackets with the help of a CAS such as Mathematica, we ultimately have

$$\boxed{\delta_A \approx -0.109\frac{w_0 L^4}{EI}}$$

The correct answer is **A**.

P.8 ➡ *Solution*

To begin, we write the shear force equation, with *x* measured from the free end *A*,

$$V(x) = -P - P\langle x-a\rangle^0$$

Integrating, we obtain

$$M(x) = -Px - P\langle x-a\rangle^1$$

We can then set up the bending moment equation,

$$EIv'' = M(x) \to EI\frac{d^2v}{dx^2} = -Px - P\langle x-a\rangle^1$$

Integrating once, we obtain

$$EI\frac{d^2v}{dx^2} = -Px - P\langle x-a\rangle^1 \to EI\frac{dv}{dx} = -\frac{P}{2}x^2 - \frac{P}{2}\langle x-a\rangle^2 + C_1 \;\; \text{(I)}$$

Integrating a second time gives

$$EI\frac{dv}{dx} = -\frac{P}{2}x^2 - \frac{P}{2}\langle x-a\rangle^2 + C_1 \to EIv = -\frac{P}{6}x^3 - \frac{P}{6}\langle x-a\rangle^3 + C_1x + C_2 \;\; \text{(II)}$$

The available boundary conditions are $v'(2a) = 0$ (the slope at support *C* is zero) and $v(2a) = 0$ (the deflection at support *C* is zero). Applying the former to equation (I), it follows that

$$EI\frac{dv}{dx} = -\frac{P}{2}\times(2a)^2 - \frac{P}{2}\times\langle 2a-a\rangle^2 + C_1 = 0$$

$$\therefore -\frac{P}{2}\times 4a^2 - \frac{P}{2}\times a^2 + C_1 = 0$$

$$\therefore -2Pa^2 - \frac{Pa^2}{2} + C_1 = 0$$

$$\therefore C_1 = \frac{5Pa^2}{2}$$

Applying the remaining boundary condition to equation (II), we have

$$EIv = -\frac{P}{6}\times(2a)^3 - \frac{P}{6}\langle 2a-a\rangle^3 + \frac{5Pa^2}{2}\times 2a + C_2 = 0$$

$$\therefore -\frac{4Pa^3}{3} - \frac{Pa^3}{6} + 5Pa^3 + C_2 = 0$$

$$\therefore C_2 = -\frac{7Pa^3}{2}$$

Substituting these variables into equation (I), the beam slope is given by

$$EIv' = -\frac{P}{2}x^2 - \frac{P}{2}\langle x-a\rangle^2 + \frac{5Pa^2}{2}$$

$$\therefore v' = \frac{P}{EI}\left[-\frac{1}{2}x^2 - \frac{1}{2}\langle x-a\rangle^2 + \frac{5a^2}{2}\right]$$

The slope at the free end is then

$$\theta_A = v'(0) = \frac{P}{EI}\left[-\frac{1}{2}\times 0^2 - \underbrace{\frac{1}{2}\langle 0-a\rangle^2}_{=0} + \frac{5a^2}{2}\right]$$

$$\therefore \theta_A = \frac{5Pa^2}{2EI}$$

Thus, statement 1 is true. In a similar manner, the elastic curve is established as

$$EIv = -\frac{P}{6}x^3 - \frac{P}{6}\langle x-a\rangle^3 + +\frac{5Pa^2}{2}x - \frac{7Pa^3}{2}$$

$$\therefore v = \frac{P}{EI}\left[-\frac{1}{6}x^3 - \frac{1}{6}\langle x-a\rangle^3 + \frac{5a^2}{2}x - \frac{7a^3}{2}\right]$$

so that, with $x = 0$, we have

$$\delta_A = v(0) = \frac{P}{EI}\left[-\frac{1}{6}\times 0^3 - \underbrace{\frac{1}{6}\langle 0-a\rangle^3}_{=0} + \frac{5a^2}{2}\times 0 - \frac{7a^3}{2}\right]$$

$$\therefore \delta_A = -\frac{7Pa^3}{2EI}$$

Thus, statement 2 is false.

The correct answer is **B**.

P.9 ➡ *Solution*

It is easily shown that the system is self-equilibrated, i.e., $R_B = R_D = 0$. We can then set up the bending moment equation,

$$EI\frac{d^2v}{dx^2} = M(x) = -M_0\langle x-a\rangle^0$$

Integrating once, we obtain

$$EIv'' = -M_0\langle x-a\rangle^0 \rightarrow EI\frac{dv}{dx} = -M_0\langle x-a\rangle^1 + C_1 \quad \text{(I)}$$

Integrating a second time, we obtain

$$EIv' = -M_0\langle x-a\rangle^1 + C_1 \rightarrow EIv = -\frac{M_0}{2}\langle x-a\rangle^2 + C_1x + C_2 \quad \text{(II)}$$

The boundary conditions are $v(0) = 0$ (the deflection at support *A* is zero) and $v(2a) = 0$ (the deflection at support *C* is zero). Substituting the former into equation (II), it is easy to see that $C_2 = 0$. Substituting the remaining condition into the same equation, in turn, we see that

$$EIv = -\frac{M_0}{2}\langle 2a-a\rangle^2 + C_1\times 2a + 0 = 0$$

$$\therefore -\frac{M_0a^2}{2} + 2a\times C_1 = 0$$

$$\therefore C_1 = \frac{M_0a}{4}$$

We are now in position to evaluate the slope at end *A* of the beam,

$$\theta_A = v'(0) = \frac{1}{EI}\left[-M_0\underbrace{\langle 0-a\rangle^1}_{=0} + \frac{M_0a}{4}\right] = \frac{1}{EI}\times\frac{M_0a}{4}$$

$$\therefore \theta_A = \frac{M_0a}{4EI}$$

Thus, statement 1 is false. Next, to determine the deflection at the right end of the beam, we substitute $x = 3a$ in equation (II),

$$\delta_D = v(3a) = \frac{M_0}{EI}\left[-\frac{1}{2}\langle 3a-a\rangle^2 + \frac{a}{4}\times 3a\right]$$

$$\therefore \delta_D = \frac{M_0}{EI}\left[-2a^2 + \frac{3a^2}{4}\right]$$

$$\therefore \delta_D = -\frac{5M_0a^2}{4EI}$$

Thus, statement 2 is true.

The correct answer is **C**.

P.10 ➡ *Solution*

From symmetry, is easy to see that reactions $A_Y = B_y = wa$. The distributed loads can be represented by the equation

$$w(x) = w - w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

However, $dV/dx = -w(x)$; that is,

$$\frac{dV}{dx} = -w + w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

Moreover, $dM/dx = V(x)$; that is,

$$\frac{dM}{dx} = A_y - wx + w\langle x-a\rangle^1 - w\langle x-3a\rangle^1$$

The bending moment *M(x)* is such that

$$M(x) = \underbrace{M_A}_{=0} + A_Y x - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a\rangle^2 - \frac{1}{2}w\langle x-3a\rangle^2$$

$$\therefore M(x) = wax - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a\rangle^2 - \frac{1}{2}w\langle x-3a\rangle^2$$

We can then set up and integrate the bending moment equation,

$$EI\frac{d^2v}{dx^2} = M(x) \rightarrow EI\frac{d^2v}{dx^2} = wax - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a\rangle^2 - \frac{1}{2}w\langle x-3a\rangle^2$$

$$\therefore EI\frac{dv}{dx} = \frac{wa}{2}x^2 - \frac{1}{6}wx^3 + \frac{1}{6}\langle x-a\rangle^3 - \frac{1}{6}w\langle x-3a\rangle^3 + C_1 \quad \text{(I)}$$

$$\therefore EIv = \frac{wa}{6}x^3 - \frac{1}{24}wx^4 + \frac{1}{24}\langle x-a\rangle^4 - \frac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2 \quad \text{(II)}$$

The available boundary conditions are $v(0) = 0$ (the deflection at support *A* is zero) and $v(4a) = 0$ (the deflection at support *B* is zero). Substituting the former into equation (I), we effortlessly obtain $C_2 = 0$. Substituting the second boundary condition into equation (II), we have

$$EI\times 0 = \frac{wa}{6}\times(4a)^3 - \frac{1}{24}w\times(4a)^4 + \frac{1}{24}\langle 4a-a\rangle^4 - \frac{1}{24}w\langle 4a-3a\rangle^4 + C_1\times 4a + 0 = 0$$

$$\therefore \frac{64w}{6}a^4 - \frac{256w}{24}a^4 + \frac{81w}{24}a^4 - \frac{1}{24}a^4 + C_1\times 4a = 0$$

$$\therefore C_1 = -\frac{5wa^3}{6}$$

Gleaning our results, the elastic curve is determined to be

$$EIv = \frac{wa}{6}x^3 - \frac{1}{24}wx^4 + \frac{1}{24}\langle x-a\rangle^4 - \frac{1}{24}w\langle x-3a\rangle^4 + \left(-\frac{5wa^3}{6}\right)x + 0$$

$$\therefore v = \frac{w}{EI}\left[\frac{a}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{24}\langle x-a\rangle^4 - \frac{1}{24}\langle x-3a\rangle^4 - \frac{5a^3}{6}x\right]$$

Substituting $x = 2a$, the deflection at midpoint C follows as

$$\delta_C = v(2a) = \frac{w}{EI}\left[\frac{a}{6}\times(2a)^3 - \frac{1}{24}\times(2a)^4 + \frac{1}{24}\langle 2a-a\rangle^4 - \frac{1}{24}\underbrace{\langle 2a-3a\rangle^4}_{=0} - \frac{5a^3}{6}\times 2a\right]$$

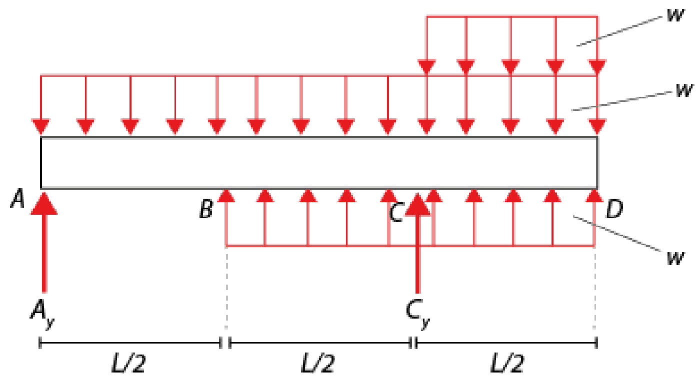
$$\therefore \delta_C = \frac{w}{EI}\left(\frac{8a^4}{6} - \frac{16a^4}{24} + \frac{a^4}{24} - 0 - \frac{10a^4}{6}\right)$$

$$\therefore \boxed{\delta_C = -\frac{23wa^4}{24EI}}$$

The correct answer is **D**.

P.11 ➡ *Solution*

Using statics, it is easy to see that vertical reactions A_y and C_y are such that $A_y = wL/4$ and $C_y = 3wL/4$. We can replace the loading configuration given with the following equivalent setup.



The shear force $V(x)$ is expressed as

$$V(x) = A_y - wx + w\left\langle x - \frac{L}{2}\right\rangle + C_y\langle x-L\rangle^0 + w\langle x-L\rangle$$

$$\therefore V(x) = \frac{wL}{4} - wx + w\left\langle x - \frac{L}{2}\right\rangle + \frac{3wL}{4}\langle x-L\rangle^0 + w\langle x-L\rangle$$

Integrating once, we obtain the bending moment $M(x)$,

$$V(x) = \frac{wL}{4} - wx + w\left\langle x - \frac{L}{2}\right\rangle + \frac{3wL}{4}\langle x-L\rangle^0 + w\langle x-L\rangle$$

$$\therefore M(x) = \frac{wL}{4}x - \frac{w}{2}x^2 + \frac{w}{2}\left\langle x - \frac{L}{2}\right\rangle^2 + \frac{3wL}{4}\langle x-L\rangle^1 + \frac{w}{2}\langle x-L\rangle^2$$

Since $EI(d^2v/dx^2) = M(x)$, the elastic curve can be obtained if we integrate the relation above twice more,

$$EI\frac{d^2v}{dx^2} = \frac{wL}{4}x - \frac{w}{2}x^2 + \frac{w}{2}\left\langle x - \frac{L}{2}\right\rangle^2 + \frac{3wL}{4}\langle x-L\rangle^1 + \frac{w}{2}\langle x-L\rangle^2$$

$$\therefore EI\frac{dv}{dx} = \frac{wL}{8}x^2 - \frac{w}{6}x^3 + \frac{w}{6}\left\langle x - \frac{L}{2}\right\rangle^3 + \frac{3wL}{8}\langle x-L\rangle^2 + \frac{w}{6}\langle x-L\rangle^3 + C_1 \quad \text{(I)}$$

$$\therefore EIv = \frac{wL}{24}x^3 - \frac{w}{24}x^4 + \frac{w}{24}\left\langle x - \frac{L}{2}\right\rangle^4 + \frac{3wL}{24}\langle x-L\rangle^3 + \frac{w}{24}\langle x-L\rangle^4 + C_1x + C_2 \quad \text{(II)}$$

The pertaining boundary conditions are $v(0) = 0$ (the deflection at support *A* is zero) and $v(L) = 0$ (the deflection at support *C* is zero). Applying the former to equation (II), we effortlessly have $C_2 = 0$. Resorting to the other boundary condition, equation (II) becomes

$$EIv = \frac{wL}{24}\times L^3 - \frac{w}{24}\times L^4 + \frac{w}{24}\left\langle L - \frac{L}{2}\right\rangle^4 + \frac{3wL}{24}\langle L-L\rangle^3 + \frac{w}{24}\langle L-L\rangle^4 + C_1\times L + \underbrace{C_2}_{=0} = 0$$

$$\therefore \cancel{\frac{wL^4}{24}} - \cancel{\frac{wL^4}{24}} + \frac{w}{24}\times\frac{L^4}{16} + 0 + 0 + C_1\times L = 0$$

$$\therefore C_1 = -\frac{wL^3}{384}$$

Therefore, the elastic curve is given by

$$EIv = \frac{wL}{24}x^3 - \frac{w}{24}x^4 + \frac{w}{24}\left\langle x - \frac{L}{2}\right\rangle^4 + \frac{3wL}{24}\langle x-L\rangle^3 + \frac{w}{24}\langle x-L\rangle^4 - \frac{wL^3}{384}x$$

$$v = \frac{w}{EI}\left[\frac{L}{24}x^3 - \frac{1}{24}x^4 + \frac{1}{24}\left\langle x - \frac{L}{2}\right\rangle^4 + \frac{3L}{24}\langle x-L\rangle^3 + \frac{1}{24}\langle x-L\rangle^4 - \frac{L^3}{384}x\right]$$

We can now determine the deflection at point *B*, which corresponds to $x = L/2$.

$$\delta_B = \frac{w}{EI}\left[\frac{L}{24}\times\left(\frac{L}{2}\right)^3 - \frac{1}{24}\times\left(\frac{L}{2}\right)^4 + \frac{1}{24}\underbrace{\left\langle\frac{L}{2} - \frac{L}{2}\right\rangle^4}_{=0} + \frac{3L}{24}\underbrace{\left\langle\frac{L}{2} - L\right\rangle^3}_{=0} + \frac{1}{24}\underbrace{\left\langle\frac{L}{2} - L\right\rangle^4}_{=0} - \frac{L^3}{384}\times\left(\frac{L}{2}\right)\right]$$

$$\therefore \delta_B = \frac{w}{EI}\left(\frac{L^4}{192} - \frac{L^4}{384} - \frac{L^4}{768}\right)$$

$$\therefore \delta_B = \frac{wL^4}{768EI}(4-2-1)$$

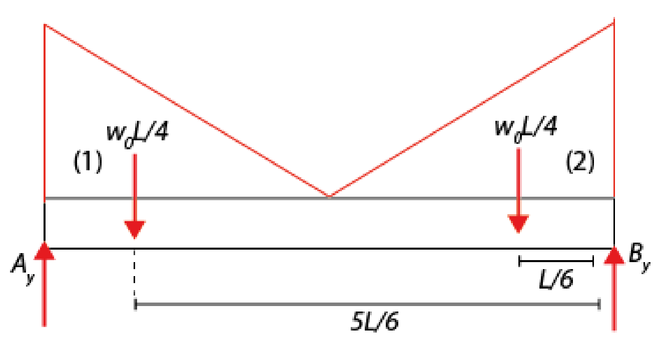
$$\therefore \boxed{\delta_B = +\frac{wL^4}{768EI}}$$

The positive sign indicates that the deflection of point *B* is upward.

The correct answer is **C**.

P.12 ➡ *Solution*

To begin, consider the free body diagram of the beam.



Taking moments about point *B*, we have

$$\Sigma M_B = 0 \rightarrow \frac{w_0L}{4}\times\frac{5L}{6} + \frac{w_0L}{4}\times\frac{L}{6} + A_y\times L = 0$$

$$\therefore \frac{5w_0L^2}{24} + \frac{w_0L^2}{24} + A_y\times L = 0$$

$$\therefore |A_y| = \frac{w_0L}{4}$$

Owing to symmetry, we have $B_y = A_y = w_0L/4$. The first distributed load, labeled (1) in the free body diagram, can be described with the relation

$$w_1(x) = w_0 - \frac{2w_0}{L}x$$

Likewise, the second distributed load, labeled (2) in the free body diagram, can be represented with the relation

$$w_2(x) = \frac{4w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

The distributed load imparted on the beam is then

$$w(x) = w_1(x) + w_2(x) = w_0 - \frac{2w_0}{L}x + \frac{4w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

Now, knowing that $dV/dx = -w(x)$, the shear force $V(x)$ can be obtained by integration,

$$\frac{dV}{dx} = -w(x) = -w_0 + \frac{2w_0}{L}x - \frac{4w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\therefore V(x) = -w_0x + \frac{w_0}{L}x^2 - \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^2$$

Since the variation in moment dM/dx is equal to the shear force $V(x)$, and

$$EI\frac{d^2v}{dx^2} = M(x)$$

it follows that

$$V(x) + A_y = \frac{w_0L}{4} - w_0x + \frac{w_0}{L}x^2 - \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^2$$

$$\therefore EI\frac{d^2v}{dx^2} = M(x) = \frac{w_0L}{4}x - \frac{w_0}{2}x^2 + \frac{w_0}{3L}x^3 - \frac{2w_0}{3L}\left\langle x - \frac{L}{2}\right\rangle^3$$

Integrating twice more gives

$$EI\frac{d^2v}{dx^2} = \frac{w_0L}{4}x - \frac{w_0}{2}x^2 + \frac{w_0}{3L}x^3 - \frac{2w_0}{3L}\left\langle x - \frac{L}{2}\right\rangle^3$$

$$\therefore EI\frac{dv}{dx} = \frac{w_0L}{8}x^2 - \frac{w_0}{6}x^3 + \frac{w_0}{12L}x^4 - \frac{w_0}{6L}\left\langle x - \frac{L}{2}\right\rangle^4 + C_1 \quad \text{(I)}$$

$$EIv = \frac{w_0L}{24}x^3 - \frac{w_0}{24}x^4 + \frac{w_0}{60L}x^5 - \frac{w_0}{30L}\left\langle x - \frac{L}{2}\right\rangle^5 + C_1x + C_2 \quad \text{(II)}$$

The available boundary conditions are $v(0) = 0$ (the deflection at support *A* is zero) and $v(L) = 0$ (the deflection at support *B* is zero). Applying the former to equation (II), it is clear that $C_2 = 0$. Applying the remaining boundary condition to equation (II), in turn, brings to

$$EIv = \cancel{\frac{w_0L}{24}\times L^3 - \frac{w_0}{24}\times L^4} + \frac{w_0}{60L}\times L^5 - \frac{w_0}{30L}\left\langle L - \frac{L}{2}\right\rangle^5 + C_1L + 0 = 0$$

$$\therefore \frac{w_0L^4}{60} - \frac{w_0}{30L}\times\frac{L^5}{32} + C_1L = 0$$

$$\therefore C_1 = -\frac{w_0L^3}{64}$$

The beam's elastic curve is determined to be

$$EIv = \frac{w_0 L}{24}x^3 - \frac{w_0}{24}x^4 + \frac{w_0}{60L}x^5 - \frac{w_0}{30L}\left\langle x - \frac{L}{2}\right\rangle^5 - \frac{w_0 L^3}{64}x$$

$$\therefore v = \frac{w_0}{EI}\left[\frac{L}{24}x^3 - \frac{1}{24}x^4 + \frac{1}{60L}x^5 - \frac{1}{30L}\left\langle x - \frac{L}{2}\right\rangle^5 - \frac{L^3}{64}x\right]$$

The deflection at the midpoint is then

$$\delta_C = v\left(\frac{L}{2}\right) = \frac{w_0}{EI}\left[\frac{L}{24}\times\left(\frac{L}{2}\right)^3 - \frac{1}{24}\times\left(\frac{L}{2}\right)^4 + \frac{1}{60L}\times\left(\frac{L}{2}\right)^5 - \frac{1}{30L}\underbrace{\left\langle\frac{L}{2} - \frac{L}{2}\right\rangle^5}_{=0} - \frac{L^3}{64}\times\left(\frac{L}{2}\right)\right]$$

$$\therefore \delta_C = \frac{w_0}{EI}\left(\frac{L^4}{192} - \frac{L^4}{384} + \frac{L^4}{1920} - 0 - \frac{L^4}{128}\right)$$

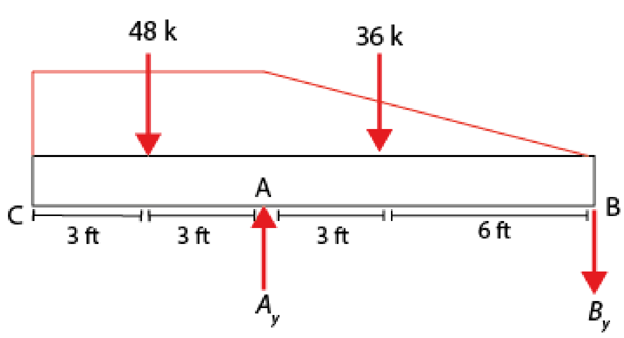
$$\therefore \delta_C = \frac{w_0 L^4}{1920EI}\times(10 - 5 + 1 - 15)$$

$$\therefore \boxed{\delta_C = -\frac{3w_0 L^4}{640EI}}$$

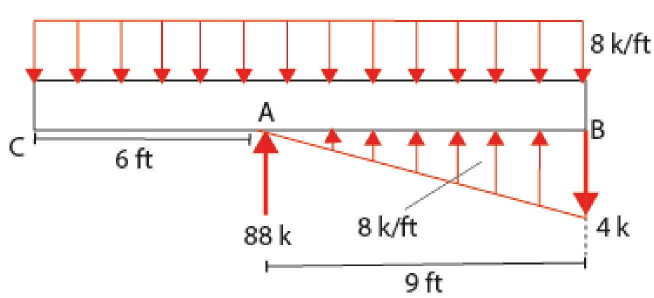
The correct answer is **B**.

P.13 ➡ *Solution*

The free body diagram for the beam is illustrated below.



From statics, we have $|A_y| = 88$ kip and $|B_y| = 4$ kip. We shall replace the system of loads on the beam with the following equivalent configuration.



Using discontinuity functions, the bending moment *M(x)* is written as

$$M(x) = -\frac{1}{2}\times 8\langle x - 0\rangle^2 - \frac{1}{6}\times\left(-\frac{8}{9}\right)\langle x - 6\rangle^3 - (-88)\langle x - 6\rangle$$

$$\therefore M(x) = -4x^2 + \frac{4}{27}\langle x - 6\rangle^3 + 88\langle x - 6\rangle$$

Knowing that $EI(d^2v/dx^2) = M(x)$, the beam deflection can be determined by integrating this relation twice,

$$EI\frac{d^2v}{dx^2} = -4x^2 + \frac{4}{27}\langle x - 6\rangle^3 + 88\langle x - 6\rangle$$

$$\therefore EI\frac{dv}{dx} = -\frac{4}{3}x^3 + \frac{1}{27}\langle x - 6\rangle^4 + 44\langle x - 6\rangle^2 + C_1 \quad \text{(I)}$$

$$\therefore EIv = -\frac{1}{3}x^4 + \frac{1}{135}\langle x - 6\rangle^5 + \frac{44}{3}\langle x - 6\rangle^3 + C_1 x + C_2 \quad \text{(II)}$$

The available boundary conditions are $v(6) = 0$ (the deflection at support *A* is zero) and $v(15) = 0$ (the deflection at support *B* is zero). Substituting the former into equation (II), we have

$$EIv = -\frac{1}{3}\times 6^4 + \frac{1}{135}\langle 6-6\rangle^5 + \frac{44}{3}\langle 6-6\rangle^3 + C_1\times 6 + C_2 = 0$$

$$\therefore -432 + 0 + 0 + 6C_1 + C_2 = 0$$

$$\therefore 6C_1 + C_2 = 432 \quad \text{(III)}$$

Substituting the remaining boundary condition into equation (II), we have

$$-\frac{1}{3}\times 15^4 + \frac{1}{135}\times\langle 15-6\rangle^5 + \frac{44}{3}\times\langle 15-6\rangle^3 + C_1\times 15 + C_2 = 0$$

$$\therefore -16{,}875 + 437.4 + 10{,}692 + 15C_1 + C_2 = 0$$

$$\therefore 15C_1 + C_2 = 5476 \quad \text{(IV)}$$

Equations (III) and (IV) can be solved simultaneously to yield $C_1 = 560.4$ and $C_2 = -3110$. Substituting these quantities into equation (II), the elastic curve is given by

$$EIv = -\frac{1}{3}x^4 + \frac{1}{135}\langle x-6\rangle^5 + \frac{44}{3}\langle x-6\rangle^3 + 560.4x - 3110$$

$$\therefore v = \frac{1}{EI}\left[-\frac{1}{3}x^4 + \frac{1}{135}\langle x-6\rangle^5 + \frac{44}{3}\langle x-6\rangle^3 + 560.4x - 3110\right]$$

The deflection at point *C* is then

$$\delta_C = v(0) = \frac{1}{EI}\left[-\frac{1}{3}\times 0^4 + \frac{1}{135}\times\langle 0-6\rangle^5 + \frac{44}{3}\langle 0-6\rangle^3 + 560.4\times 0 - 3110\right]$$

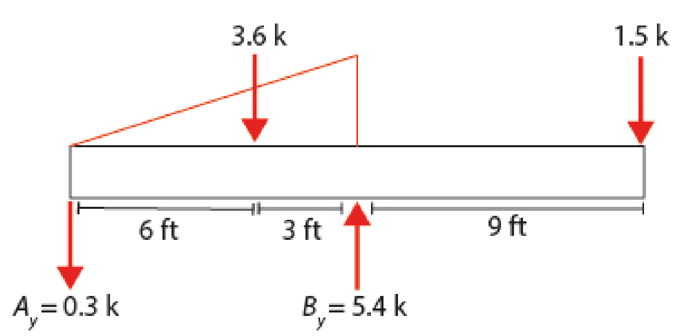
$$\therefore \delta_C = \frac{1}{EI}(-0+0+0+0-3110)$$

$$\therefore \boxed{\delta_C = -\frac{3110}{EI}}$$

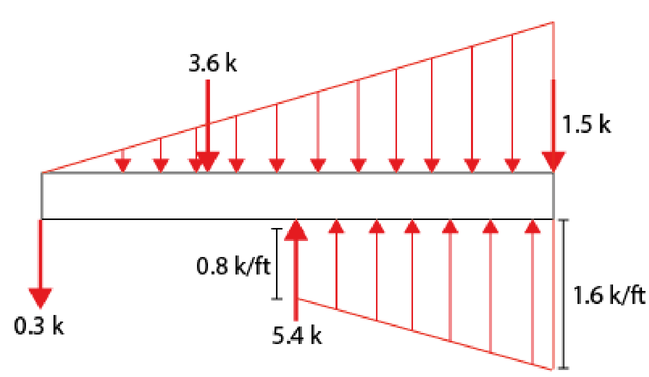
The correct answer is **C**.

P.14 ➡ *Solution*

The free body diagram for the beam is provided in continuation.



From statics, we have $|A_y| = 0.3$ k and $|B_y| = 5.4$ k. We shall replace the beam's loading configuration with the equivalent pattern shown below.



In this case, the bending moment *M(x)* is given by

$$M(x) = -0.3\langle x-0\rangle - \frac{1}{6}\times\frac{1.6}{18}\langle x-0\rangle^3 - (-5.4)\langle x-9\rangle - \left(-\frac{0.8}{2}\right)\langle x-9\rangle^2 - \frac{1}{6}\left(-\frac{0.8}{9}\right)\langle x-9\rangle^3$$

$$\therefore M(x) = -0.3x - 0.0148x^3 + 5.4\langle x-9\rangle + 0.4\langle x-9\rangle^2$$

Knowing that $EI(d^2v/dx^2) = M(x)$, the beam deflection can be determined by integrating the foregoing equation twice.

$$EI\frac{d^2v}{dx^2} = -0.3x - 0.0148x^3 + 5.4\langle x-9\rangle + 0.4\langle x-9\rangle^2 + 0.0148\langle x-9\rangle^3$$

$$\therefore EI\frac{dv}{dx} = -0.15x^2 - 0.0037x^4 + 2.7\langle x-9\rangle^2 + 0.13\langle x-9\rangle^3 + 0.0037\langle x-9\rangle^4 + C_1 \quad \text{(I)}$$

$$\therefore EIv = -0.05x^3 - 0.00074x^5 + 0.9\langle x-9\rangle^3 + 0.033\langle x-9\rangle^4 + 0.00074\langle x-9\rangle^5 + C_1x + C_2 \quad \text{(II)}$$

The available boundary conditions are $v(0) = 0$ (the deflection at support *A* is zero) and $v(9) = 0$ (the deflection at support *B* is zero). Substituting the first condition into equation (II), it is easily seen that $C_2 = 0$. Substituting the remaining condition into equation (II), in turn, we have

$$EIv = 0.05\times 9^3 - 0.00074\times 9^5 + 0.033\langle 9-9\rangle^4 + 0.00074\langle 9-9\rangle^5 + C_1\times 9 = 0$$

$$\therefore -36.5 - 43.7 + 0 + 0 + 9C_1 = 0$$

$$\therefore C_1 = 8.91$$

Consequently, the elastic curve is given by

$$v = \frac{1}{EI}\left[-0.05x^3 - 0.00074x^5 + 0.9\langle x-9\rangle^3 + 0.033\langle x-9\rangle^4 + 0.00074\langle x-9\rangle^5 + 8.91x\right] \text{kip-ft}^3$$

We are looking for the deflection at end *C*, for which $x = 18$ ft. Therefore,

$$\delta_C = v(18) = \frac{1}{EI}\begin{bmatrix} -0.05\times 18^3 - 0.00074\times 18^5 + 0.9(18-9)^3 \\ +0.033\times\langle 18-9\rangle^4 + 0.00074\langle 18-9\rangle^5 + 8.91\times 18 \end{bmatrix} \text{kip-ft}^3$$

$$\therefore \delta_C = -\frac{613.2}{EI} \text{ kip-ft}^3$$

Since $E = 1.6\times 10^3$ ksi and $I = 6\times 12^3/12 = 864$ in.^{4}, we ultimately have

$$\delta_C = -\frac{613.2}{(1.6\times 10^3)\times 864}\times 12^3 = \boxed{-0.767 \text{ in.}}$$

or approximately 19.5 mm.

The correct answer is **D**.

ANSWER SUMMARY

Problem 1	**C**
Problem 2	**B**
Problem 3	**D**
Problem 4	**A**
Problem 5	**D**
Problem 6	**B**
Problem 7	**A**
Problem 8	**B**
Problem 9	**C**
Problem 10	**D**
Problem 11	**C**
Problem 12	**B**
Problem 13	**C**
Problem 14	**D**

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