Quiz FM106
Differential Analysis of Fluid Flow
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PROBLEMS

Problem 1

The following is a list of statements related to fluid kinematics and differential fluid flow. True or false?

1. ( ) A velocity field in a plane flow is given by $V = 2yt\mathbf{i} + x\mathbf{j}$ m/s. The acceleration for this field at point (4,2) m at t = 3 s is given by the vector $\boldsymbol{a} = 28\mathbf{i} + 12\mathbf{j}$ m/s^2.

2. ( ) The angular velocity of the field given in the previous statement, evaluated at the same point and instant of time, is $\omega = -5$ rad/s.

3. ( ) The vorticity vector of the field given in statement 1, evaluated at the same point and instant of time, is $\boldsymbol{\zeta} = -10\mathbf{k}$ rad/s.

4. ( ) For a certain incompressible flow field, the velocity components are given by the equations

$$u = 2xy \;\; ; \;\; v = -x^2 y \;\; ; \;\; w = 0$$

We conclude that this is a physically possible flow field.

5. ( ) The velocity components of an incompressible, two-dimensional velocity field are given by the equations

$$\begin{cases} u = y^2 - x(1+x) \\ v = y(2x+1) \end{cases}$$

We conclude that this is a physically possible, irrotational flow field.

Problem 2

The following is a list of statements related to differential fluid flow. True or false?

1. ( ) The u velocity component of a steady, two-dimensional, incompressible flow field is $u = 3ax^2 - 2bxy$, where a and b are constants. The velocity component v is unknown. It can be shown that $v = -6axy + by^2 + f(x)$, where *f(x)* is an arbitrary function of x.

2. ( ) Two velocity components of a steady, incompressible flow field are given by $u = 2ax + bxy + cy^2$ and $v = axz - byz^2$, where a, b, and c are constants. Velocity component w is missing. It can be shown that $w = 2az + byz - bz^3/3 + f(x,y)$, where *f(x,y)* is an arbitrary function of x and y.

3. ( ) A flow field has velocity components $u = (x-y)$ and $v = -(x+y)$. It can be shown that the corresponding stream function is $\psi = xy - y^2/2 + x^2/2 + C$, where C is an arbitrary constant.

4. ( ) The velocity components for an incompressible, plane flow are $v_r = Ar^{-1} + Br^{-2}\cos\theta$ and $v_\theta = Br^{-2}\sin\theta$. It can be shown that the corresponding stream function is $\psi = -A\theta - Br^{-1}\sin\theta + C$, where C is an arbitrary constant.

Problem 3

The stream function for a given two-dimensional flow field is

$$\psi = 5x^2 y - (5/3)\, y^3$$

Determine the corresponding velocity potential. Consider C to be an arbitrary constant.

A) $\phi = (5/3)\, x^3 - 5xy^2 + C$

B) $\phi = (5/3)\, x^3 - 10xy^2 + C$

C) $\phi = 5x^3 - 5xy^2 + C$

D) $\phi = 5x^3 - 10xy^2 + C$

Problem 4A

Determine the stream function corresponding to the velocity potential

$$\phi = x^3 - 3xy^2$$

Consider C to be an arbitrary constant.

A) $\psi = 3x^2 y + y^3 + C$

B) $\psi = 3x^2 y - y^3 + C$

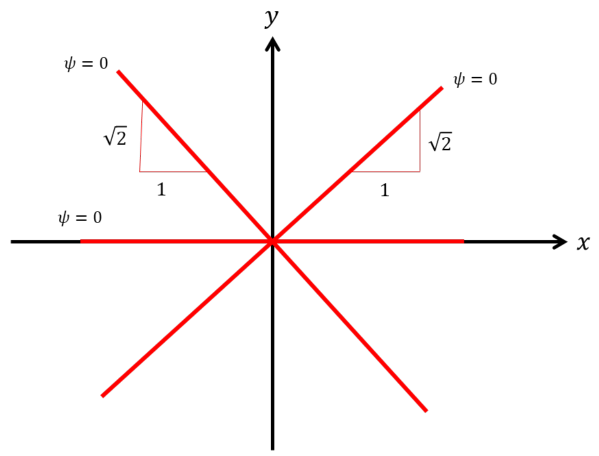
C) $\psi = 3x^2 y + 2y^3 + C$

D) $\psi = 3x^2 y - 2y^3 + C$

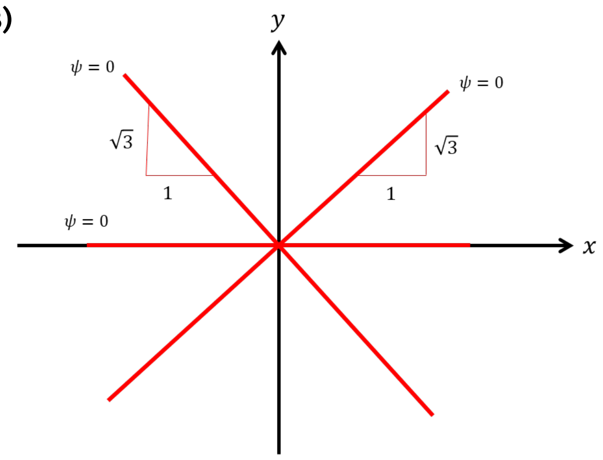
Problem 4B

Determine the streamlines $\psi = 0$, which pass through the origin. Which of the following graphs correctly illustrates these streamlines?

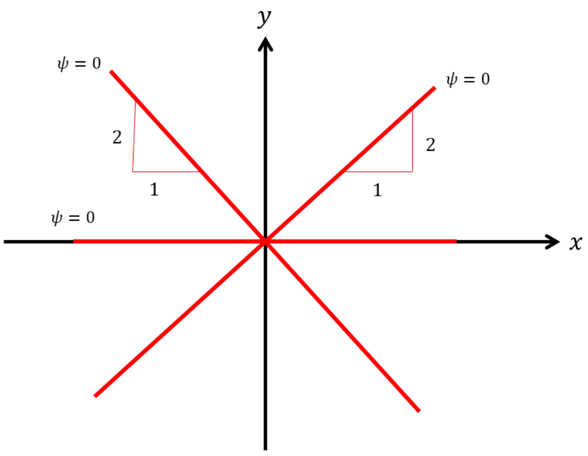
A)



B)



C)



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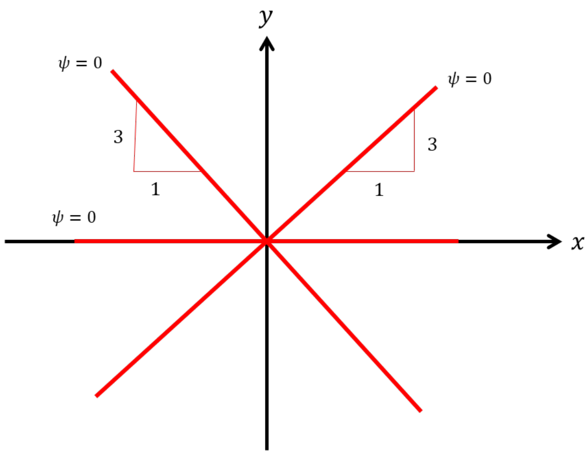
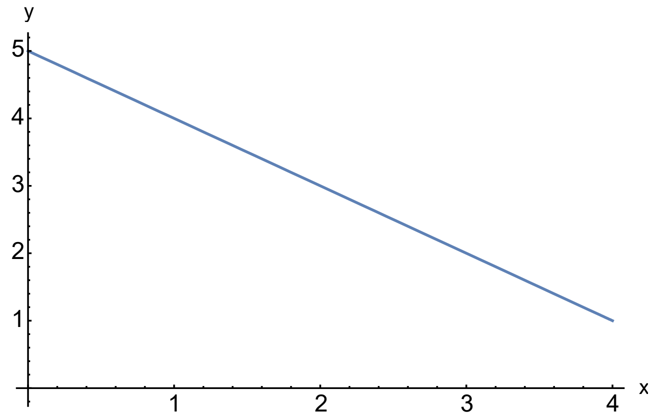


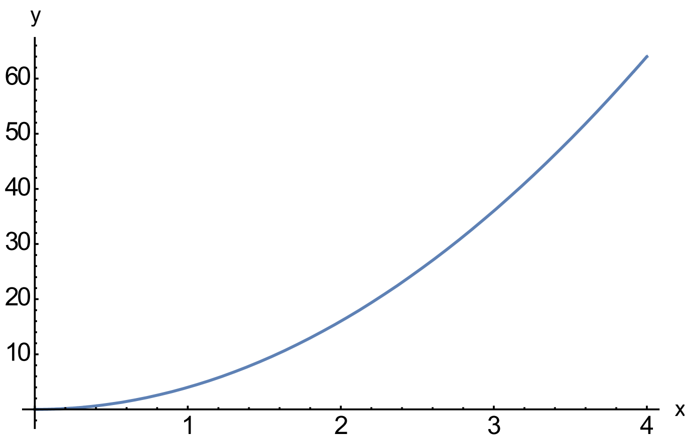
Problem 5

A two-dimensional flow field is defined by its components $u = 2x^2$ ft/s, and $v = (-4xy + x^2)$ ft/s, where x and y are in feet. Determine the streamline that passes through point (3 ft, 1 ft). Which of the following graphs corresponds to this streamline?

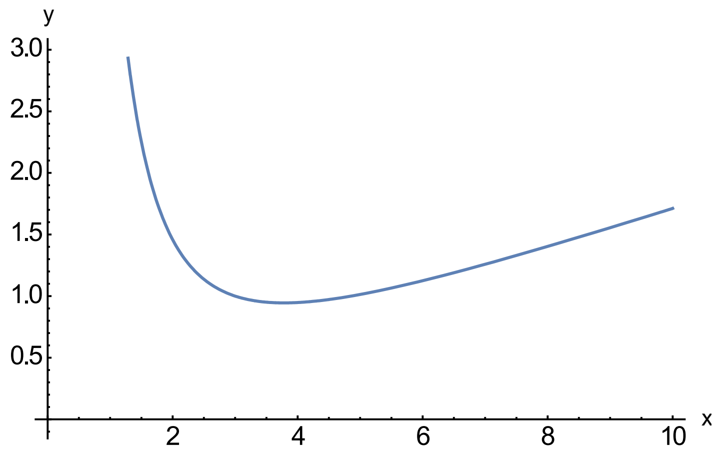
A)



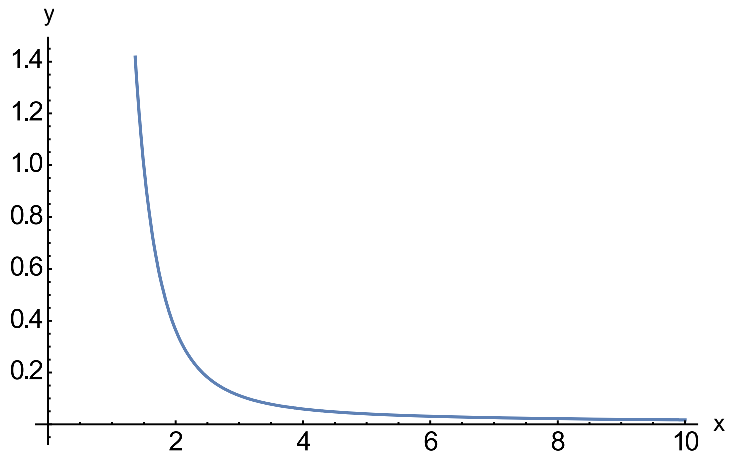
B)



C)



D)



Problem 6

A certain flow field is described by the stream function

$$\psi = A\theta + B\,r\sin\,\theta$$

where A and B are positive constants. Which of the following coordinates specify a stagnation point for this flow field?

A) $(r,\theta) = (-A/B\,,0)$

B) $(r,\theta) = (-A/B\,,\pi)$

C) $(r,\theta) = (A/B\,,0)$

D) $(r,\theta) = (A/B\,,\pi)$

Problem 7

The velocity potential for a certain inviscid flow field is

$$\phi = -\left(3x^2y - y^3\right)$$

where ϕ has units of ft²/s when x and y are in feet. Determine the pressure difference (in psi) between the points (1, 2) and (4, 4), where the coordinates are in feet. The fluid is water (γ = 62.4 lb/ft³) and elevation changes are negligible.

A) Δp = 31.5 psi

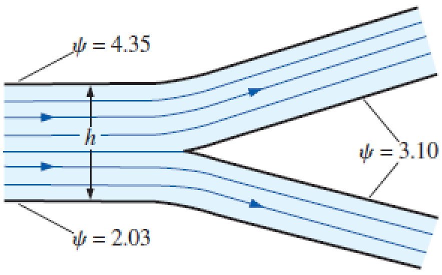
B) Δp = 40.5 psi

C) Δp = 51.5 psi

D) Δp = 60.5 psi

Problem 8A (Çengel & Cimbala, 2014, w/ permission)

A steady, incompressible, two-dimensional CFD calculation of flow through an asymmetric two-dimensional branching duct reveals the streamline pattern sketched in the figure below, where the values of ψ are in units of m^2/s, and b is the width of the duct into the page. The values of stream function ψ on the duct walls are shown. What percentage of flow goes through the *upper* branch of the duct?



A) $Q_{\text{upper}}/Q_{\text{main}} = 53.9\%$

B) $Q_{\text{upper}}/Q_{\text{main}} = 58.5\%$

C) $Q_{\text{upper}}/Q_{\text{main}} = 63.7\%$

D) $Q_{\text{upper}}/Q_{\text{main}} = 68.6\%$

Problem 8B

If the average velocity in the main branch of the duct of the previous problem is 13.4 m/s, calculate the duct height h in units of cm.

A) $h = 8.9$ cm

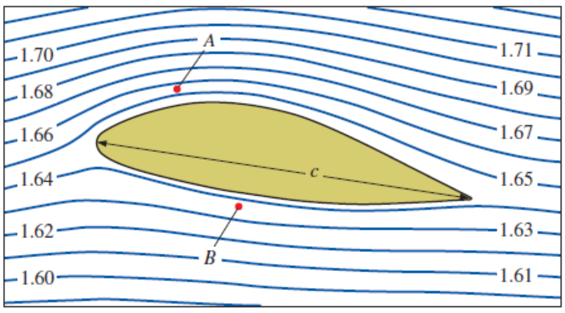
B) $h = 17.3$ cm

C) $h = 28.1$ cm

D) $h = 37.4$ cm

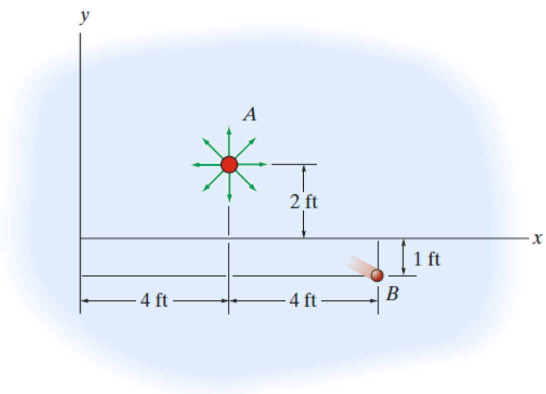
Problem 9 (Çengel & Cimbala, 2014, w/ permission)

Steady, incompressible, two-dimensional flow over a newly designed small hydrofoil is modeled with a commercial computational fluid dynamics (CFD) code. A close-up view of flow streamlines is shown in the following figure. Values of the stream function are in units of m^2/s. The fluid is water at room temperature. Draw an arrow on the plot to indicate the direction and relative magnitude of the velocity at point A. Repeat for point B. Discuss how your results can be used to explain how such a body creates lift.



Problem 10A (Hibbeler, 2015, w/ permission)

A source having a strength of q = 80 ft²/s is located at point A(4 ft, 2 ft). Determine the magnitude of the velocity at point B(8 ft, −1 ft).



A) $V = 2.03$ ft/s
B) $V = 2.55$ ft/s
C) $V = 3.10$ ft/s
D) $V = 3.58$ ft/s

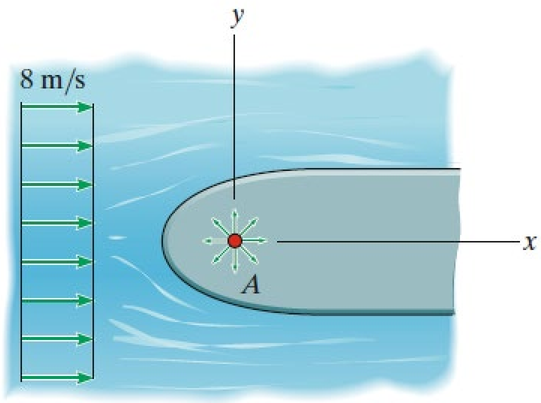
Problem 10B

Determine the acceleration of fluid particles at the same point B(8 ft, −1 ft).

A) $a = 1.3$ ft/s^2
B) $a = 1.9$ ft/s^2
C) $a = 2.5$ ft/s^2
D) $a = 3.1$ ft/s^2

Problem 11 (Hibbeler, 2015, w/ permission)

Determine the equation of the boundary of the half body formed by placing a source of 0.5 m^2/s in the uniform flow of 8 m/s. Express the result in Cartesian coordinates.



A) $y/x = \tan[\pi(1 - 16y)]$

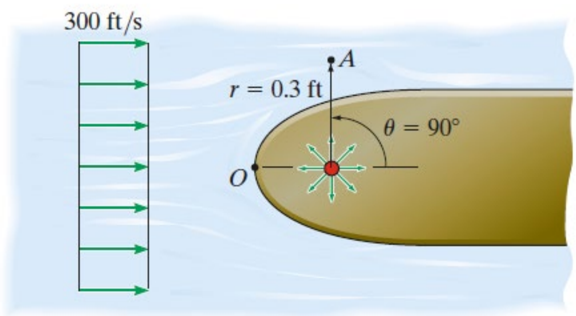
B) $y/x = \tan[\pi(1 - 32y)]$

C) $y/x = \tan[2\pi(1 - 16y)]$

D) $y/x = \tan[2\pi(1 - 32y)]$

Problem 12 (Çengel & Cimbala, 2014, w/ permission)

The leading edge of a wing is approximated by the half body illustrated below. It is formed from the superposition of the uniform air flow of 300 ft/s and a source having a strength of 100 ft^2/s. Determine the difference in pressure between the stagnation point O and point A, where r = 0.3 ft and θ = 90°. Take ρ = 2.35×10^{-3} slug/ft^3.



A) $\Delta p = 0.34$ psi
B) $\Delta p = 0.55$ psi
C) $\Delta p = 0.76$ psi
D) $\Delta p = 0.98$ psi

SOLUTIONS

P.1 ● Solution

1. True. The acceleration is given by the material derivative of velocity, $D\mathbf{V}/Dt$,

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\cancel{\frac{\partial \mathbf{V}}{\partial z}} = 2y\mathbf{i} + 2yt(\mathbf{j}) + x(2t\mathbf{i}) = 2(xt+y)\mathbf{i} + 2yt\mathbf{j}$$

Substituting x = 4 m, y = 2 m, and t = 3 s gives

$$\mathbf{a} = 2(4\times3+2)\mathbf{i} + 2\times2\times3\mathbf{j} = \boxed{28\mathbf{i} + 12\mathbf{j}\ \text{m/s}^2}$$

2. False. The angular velocity of a two-dimensional flow field such as the present one is given by

$$\boldsymbol{\omega}_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k} = \frac{1}{2}(1-2t)\mathbf{k}$$

Substituting t = 3 s gives

$$\omega_z = 0.5(1-2\times3) = 0.5(-5) = \boxed{-2.5\ \text{rad/s}}$$

3. False. The vorticity vector is twice the angular velocity vector; that is,

$$\boldsymbol{\zeta} = 2\boldsymbol{\omega} = (1-2t)\mathbf{k}$$

$$\therefore \boldsymbol{\zeta} = (1-2\times3)\mathbf{k}$$

$$\boxed{\boldsymbol{\zeta} = -5\mathbf{k}\ \text{rad/s}}$$

4. False. For the field to be a feasible flow field, it must satisfy the equation of conservation of mass. Mathematically,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

The necessary derivatives are

$$\frac{\partial}{\partial x}(2xy) = 2y\ \ ;\ \ \frac{\partial}{\partial y}(-x^2y) = -x^2\ \ ;\ \ \frac{\partial}{\partial z}(0) = 0$$

Substituting in the continuity equation yields

$$2y - x^2 + 0 = 2y - x^2 \neq 0$$

Because the sum is not zero, the field does not satisfy the equation of continuity and, consequently, does not constitute a physically possible flow field.

5. True. We can verify whether this is a physically possible flow field by writing the equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

For a two-dimensional flow field to be irrotational, the z-component of angular velocity (or the z-component of vorticity) must be zero. Mathematically,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left\{\frac{\partial}{\partial x}\left[y(2x+1)\right] - \frac{\partial}{\partial y}\left[y^2 - x(1+x)\right]\right\}$$

$$\therefore \omega_z = \frac{1}{2}(2y - 2y) = 0$$

The z-component of vorticity is indeed zero, and the flow field is irrotational.

P.2 ● Solution

1. True. For the flow to be indeed incompressible, it must satisfy the equation of continuity. Accordingly, we write

$$\frac{\partial v}{\partial y}=-\frac{\partial u}{\partial x}$$

The derivative on the right-hand side is $u_x = 6ax - 2by$. Substituting this result into the equation above, we have

$$\frac{\partial v}{\partial y}=-(6ax-2by)=-6ax+2by$$

$$\partial v=(-6ax+2by)\partial y$$

Integrating, we obtain

$$\int \partial v=\int (-6ax+2by)\partial y$$

$$\therefore \boxed{v=-6axy+by^2+f(x)}$$

where $f(x)$ is an unknown function of x.

2. False. Applying the steady incompressible continuity equation and integrating, it follows that

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\therefore \frac{\partial w}{\partial z}=-\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}=-(2a+by)-(-bz^2)$$

$$\therefore \frac{\partial w}{\partial z}=-2a-by+bz^2$$

$$\therefore \int \partial w=\int (-2a-by+bz^2)\partial z$$

$$\therefore \boxed{w=-2az-byz+\frac{bz^3}{3}+f(x,y)}$$

where $f(x,y)$ is an unknown function of x and y. It can be noted, however, that any function of x and y will work to satisfy the continuity equation, since there are no derivatives of w with respect to x or y in it.

3. True. We begin by writing the formula for the x-component of velocity,

$$u=\frac{\partial \psi}{\partial y}=x-y$$

$$\therefore \int \partial \psi=\int (x-y)\partial y$$

$$\therefore \psi=xy-\frac{y^2}{2}+f(x)$$

where $f(x)$ is an unknown function of x. We then proceed to write the formula for the y-component of velocity,

$$v=-\frac{\partial \psi}{\partial y}=-(x+y)$$

Substituting ψ as obtained earlier, we have

$$-\left(x+y\right)=-\frac{\partial}{\partial x}\left[xy-\frac{y^2}{2}+f\left(x\right)\right]$$

$$\therefore x+y=y+\frac{\partial}{\partial x}\left[f\left(x\right)\right]$$

$$\therefore x=\frac{\partial}{\partial x}\left[f\left(x\right)\right]$$

Integrating on both sides, we obtain

$$\int x\partial x=\int \partial\left[f\left(x\right)\right]$$

$$\therefore \frac{x^2}{2}+C=f\left(x\right)$$

We have obtained *f(x)*, the unknown function of *x* that we sought. Substituting in the equation for ψ, the result is

$$\psi=xy-\frac{y^2}{2}+f\left(x\right)=xy-\frac{y^2}{2}+\frac{x^2}{2}+C$$

Thus, we ultimately have

$$\boxed{\psi=xy-\frac{y^2}{2}+\frac{x^2}{2}+C}$$

4. False. From the definition of stream function in polar coordinates, we have

$$v_r=\frac{1}{r}\frac{\partial\psi}{\partial\theta}\;;\;v_\theta=-\frac{\partial\psi}{\partial r}$$

so that, for the velocity distribution given,

$$\begin{cases}\dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}=Ar^{-1}+Br^{-2}\cos\theta\rightarrow\dfrac{\partial\psi}{\partial\theta}=A+Br^{-1}\cos\theta\\ -\dfrac{\partial\psi}{\partial r}=Br^{-2}\sin\theta\rightarrow\dfrac{\partial\psi}{\partial r}=-Br^{-2}\sin\theta\end{cases}$$

Integrating the first equation with respect to θ gives

$$\int\partial\psi=\int\left(A+Br^{-1}\cos\theta\right)d\theta=A\theta+Br^{-1}\sin\theta+f_1\left(r\right)$$

Similarly, we integrate the second equation with respect to *r* and obtain

$$\int\partial\psi=-\int\left(Br^{-2}\sin\theta\right)dr=Br^{-1}\sin\theta+f_2\left(\theta\right)$$

For both preceding equations to be satisfied, ψ must have the form

$$\boxed{\psi=A\theta+Br^{-1}\sin\theta+C}$$

where *C* is an arbitrary constant.

P.3 ● Solution

The *x*-component of velocity is such that

$$u=\frac{\partial\psi}{\partial y}=\frac{\partial\phi}{\partial x}=5x^2-5y^2$$

Integrating, we have

$$\partial\phi=\left(5x^2-5y^2\right)\partial x$$

$$\therefore\int\partial\phi=\int\left(5x^2-5y^2\right)\partial x$$

$$\therefore\phi=\frac{5x^3}{3}-5xy^2+f\left(y\right)$$

where *f(y)* is an unknown function of *y*. Similarly, considering the *y*-component of velocity, we have

$$v=-\frac{\partial\psi}{\partial x}=\frac{\partial\phi}{\partial y}=-10xy$$

Integrating, we obtain

$$\int\partial\phi=\int(-10xy)\partial y$$

$$\therefore\phi=-5xy^2+f(x)$$

To satisfy both equations for ϕ, we must have

$$\boxed{\phi=\frac{5x^3}{3}-5xy^2+C}$$

where *C* is an arbitrary constant.

➡ The correct answer is **A**.

P.4 ● Solution

Part A: We begin by considering the *x*-component of velocity,

$$u=\frac{\partial\psi}{\partial y}=\frac{\partial\phi}{\partial x}=3x^2-3y^2$$

Integrating with respect to *y*, we obtain

$$\int\partial\psi=\int(3x^2-3y^2)\partial y$$

$$\therefore\psi=3x^2y-y^3+f_1(x)$$

Similarly, the *y*-component of velocity is determined as

$$v=-\frac{\partial\psi}{\partial x}=\frac{\partial\phi}{\partial y}=-6xy$$

$$\therefore-\int\partial\psi=\int(-6xy)\partial x$$

$$\therefore\psi=3x^2y+f(y)$$

Combining the two foregoing equations, we see that the stream function is, ultimately,

$$\boxed{\psi=3x^2y-y^3+C}$$

where *C* is an arbitrary constant.

➡ The correct answer is **B**.

Part B: Since the streamline $\psi = 0$ passes through the origin, constant *C* must become zero. The equation of the streamline is then

$$\psi=3x^2y-y^3=0$$

$$\therefore y(3x^2-y^2)=0$$

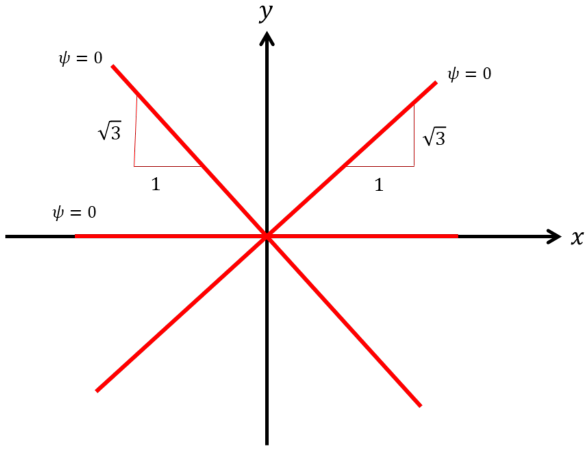
One of the factors above must be zero. $y = 0$ would be a trivial solution; for the other solution, we are left with

$$3x^2-y^2=0$$

$$\therefore y^2=3x^2$$

$$\therefore y=\pm\sqrt{3}x$$

Hence, the equations for the streamlines are three lines: one is the x-axis itself; the other two are lines that pass through the origin and have slopes of $\pm\sqrt{3}$, as shown.



➡ The correct answer is **B**.

P.5 ● Solution

To obtain the stream function, we appeal to the x-component of velocity,

$$u = \frac{\partial \psi}{\partial y}$$

$$\therefore \partial \psi = 2x^2 \partial y$$

$$\therefore \int \partial \psi = \int 2x^2 \partial y$$

$$\therefore \psi = 2x^2 y + f(x)$$

where $f(x)$ is an unknown function of x. Next, we consider the y-component of velocity,

$$v = -\frac{\partial \psi}{\partial x} = -\left(-4xy + x^2\right)$$

$$\therefore \int \partial \psi = \int \left(4xy - x^2\right) \partial x$$

$$\therefore \psi = 2x^2 y - \frac{x^3}{3} + f(y)$$

where $f(y)$ is an unknown function of y. Combining the two results, we conclude that the stream function has the form

$$\psi = 2x^2 y - \frac{x^3}{3} + C$$

where C is a constant. We take $C = 0$. The stream function is then

$$\psi = 2x^2 y - \frac{x^3}{3}$$

To find the streamline that passes through the point (3, 1) ft, we substitute these coordinates into the expression for ψ,

$$\psi(3,1) = 2 \times 3^2 \times 1 - \frac{3^3}{3} = 18 - 9 = 9$$

Hence, the equation for the stream function becomes

$$9 = 2x^2 y - \frac{x^3}{3}$$

$$\therefore 27 = 6x^2 y - x^3$$

$$\therefore 6x^2 y = x^3 + 27$$

$$\therefore y = \frac{x^3 + 27}{6x^2}$$

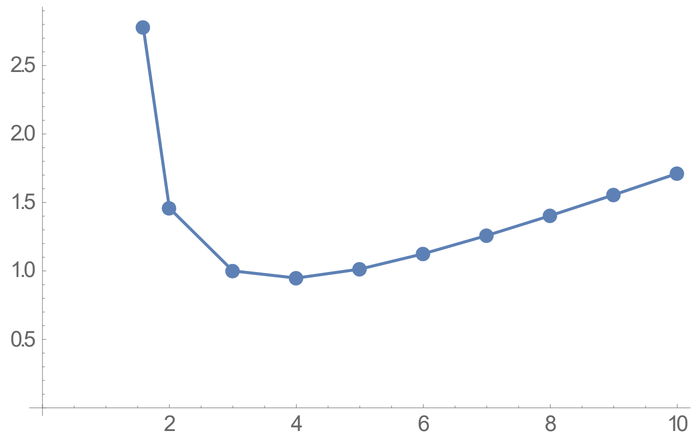
To deduce the general shape of this curve, we find the *y* coordinates for some values of *x*. In Mathematica, this can be done with the following syntax,

$$\text{list} = \text{Transpose@}\{\text{Table}[a,\{a,1,10\}], N[\text{Table}[\frac{a^3+27}{6a^2},\{a,1,10\}]]\}$$

Notice that we have taken integers ranging from 1 to 10, not zero to 10, because we can easily tell that the curve is not defined when *x* = 0. To plot these points, we use the *ListPlot* function,

$$\text{ListPlot[list, Joined} \to \text{True, PlotMarkers} \to \text{\{Automatic, Medium\}, AxesOrigin} \to \{0,0\}]$$

The following graph is obtained.



The graph that closely resembles this plot is the one in option C.

➡ The correct answer is **C**.

P.6 ● Solution

We begin by finding the potential function for this flow field. The *r*-component of velocity is

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r} = \frac{A}{r} + B\cos\theta$$

Integrating with respect to *r*, we have

$$\int \partial\phi = \phi = \int\left(\frac{A}{r} + B\cos\theta\right)\partial r = A\ln r + Br\cos\theta + f(\theta)$$

where $f(\theta)$ is an unknown function of θ. Similarly, the θ component of velocity is

$$v_\theta = -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -B\sin\theta$$

Integrating with respect to θ, we have

$$\int \partial\phi = \phi = -\int B\ r\ \sin\theta\ \partial\theta = Br\cos\theta + f(r)$$

where $f(r)$ is an unknown function of *r*. For both of the equations for the potential function to be satisfied, ϕ must have the form

$$\phi = A\ln r + Br\cos\theta + C$$

where *C* is an arbitrary constant. Now, stagnation points occur where $v_r = 0$ and $v_\theta = 0$. From the equation for v_θ, we see that this component of velocity becomes zero when $\sin\theta = 0$, that is, when $\theta = 0$ and $\theta = \pi$. Substituting $\theta = 0$ in the equation for v_r gives

$$v_r\big|_{\theta=0} = \frac{A}{r} + B$$

Accordingly, $v_r = 0$ if $r = -A/B$. However, since *A* and *B* are both positive constants, this result indicates a negative value for *r*, which is not defined. It remains to check the validity of $\theta = \pi$. Substituting this value into the equation for v_r, we obtain

$$v_r\Big|_{\theta=\pi} = \frac{A}{r} + B\underbrace{\cos\pi}_{=-1}$$

$$\therefore \frac{A}{r} - B = 0$$

$$\therefore r = \frac{A}{B}$$

which is a valid coordinate. Thus, a stagnation point occurs at $(r,\theta) = (A/B\,,\pi)$.

➡ The correct answer is **D**.

P.7 ● Solution

Since the flow field is described by a velocity potential, the flow is irrotational and the Bernoulli equation can be applied between any two points. The velocity components are

$$u = \frac{\partial\phi}{\partial x} = -6xy$$

and

$$v = \frac{\partial\phi}{\partial y} = -3x^2 + 3y^2$$

Then, at x = 1 ft, y = 2 ft, we have

$$u(1,2) = -6\times1\times2 = -12 \text{ ft/s}$$

$$v(1,2) = -3\times1^2 + 3\times2^2 = -3+12 = 9 \text{ ft/s}$$

and the square of the resultant velocity is

$$V_1^2 = \left[u(1,2)\right]^2 + \left[v(1,2)\right]^2 = (-12)^2 + (9)^2 = 225 \text{ (ft/s)}^2$$

At x = 4 ft, y = 4 ft, we have

$$u(4,4) = -6\times4\times4 = -96 \text{ ft/s}$$

$$v(4,4) = -3\times4^2 + 3\times4^2 = 0 \text{ ft/s}$$

$$\therefore V_2^2 = \left[u(4,4)\right]^2 + \left[v(4,4)\right]^2 = 96^2 + 0 = 9216 \text{ (ft/s)}^2$$

Solving for pressure difference in the Bernoulli equation, we obtain

$$p_1 - p_2 = \frac{\gamma}{2g}\left(V_2^2 - V_1^2\right)$$

$$\therefore p_1 - p_2 = \frac{62.4}{2\times32.2}(9216-225) = 8712 \text{ lb/ft}^2$$

That is,

$$\Delta p = 8712 \text{ lb}/\cancel{\text{ft}^2} \times \frac{\cancel{\text{ft}^2}}{144 \text{ in.}^2} = \boxed{60.5 \text{ psi}}$$

➡ The correct answer is **D**.

P.8 ● Solution

Part A: Recall that the difference in the value of the stream function between two streamlines is equal to the volume flow rate per unit width between the two streamlines. Thus, for the main branch we have

$$\frac{Q}{b}\bigg|_{\text{main}} = \psi_{\text{upper wall}} - \psi_{\text{lower wall}} = 4.35 - 2.03 = 2.32 \text{ ft}^2/\text{s}$$

Similarly, in the upper branch,

$$\left.\frac{Q}{b}\right|_{\text{upper}} = \psi_{\text{upper wall}} - \psi_{\text{branch wall}} = 4.35 - 3.10 = 1.25\ \text{ft}^2/\text{s}$$

On a percentage basis, the percentage of volume flow through the upper branch is calculated as

$$\frac{Q_{\text{upper}}}{Q_{\text{main}}} = \frac{1.25}{2.32} = \boxed{53.9\%}$$

➡ The correct answer is **A**.

Part B: One way to determine the height h is to assume uniform flow in the main branch, for which $\psi = V_{\text{avg}}y$. In order to find h, we take the difference between ψ at the top of the duct and ψ at the bottom of the duct,

$$\psi_{\text{upper wall}} - \psi_{\text{lower wall}} = V_{\text{avg}}y_{\text{upper wall}} - V_{\text{avg}}y_{\text{lower wall}} = V_{\text{avg}}\left(y_{\text{upper wall}} - y_{\text{lower wall}}\right) = V_{\text{avg}}h$$

Solving for h, we have

$$h = \frac{\psi_{\text{upper wall}} - \psi_{\text{lower wall}}}{V_{\text{avg}}} = \frac{4.35 - 2.03}{13.4} = 0.173 = \boxed{17.3\ \text{cm}}$$

An alternative way to solve for the height h is to consider that the volume flow rate through the main branch of the duct is equal to the average velocity times the cross-sectional area of the duct,

$$Q = V_{\text{avg}}bh \rightarrow h = \frac{1}{V_{\text{avg}}} \times \left(\frac{Q}{b}\right)_{\text{main}}$$

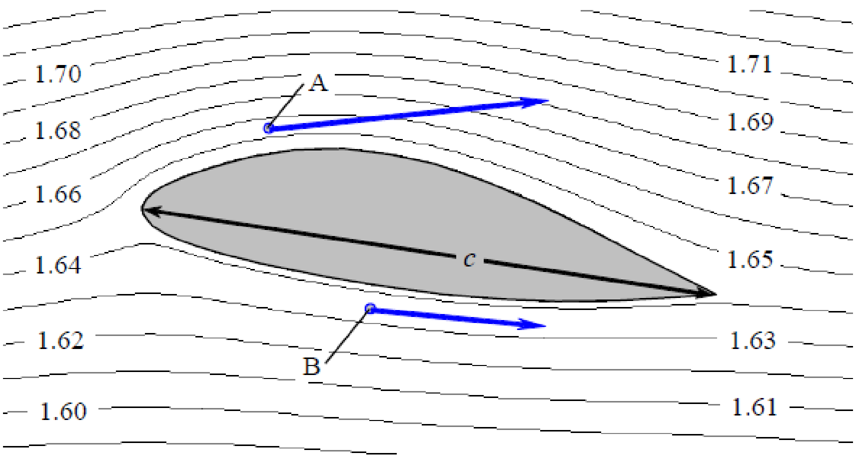
$$\therefore h = \frac{1}{13.4} \times 2.32 = 0.173 = 17.3\ \text{cm}$$

as expected.

➡ The correct answer is **B**.

P.9 ● Solution

We can tell the direction of the flow by whether ψ increases or decreases in the vertical direction (left side rule). Furthermore, since the streamlines near point B are somewhat further apart than those near point A (by a factor of about 1.6), the speed at point A is a factor of about 1.6 times greater than that at point B. The arrows are drawn below.



In terms of lift, it is obvious that the flow speeds near the upper surface of the hydrofoil are greater than those near the lower surface. From the Bernoulli equation we know that low speeds lead to (relatively) higher pressures; thus, the pressure on the lower half of the hydrofoil is greater than that on the upper half, leading to lift.

P.10 ● Solution

Part A: The radial and transverse components of velocity for a point source are

$$v_r = \frac{q}{2\pi r} \;;\; v_\theta = 0$$

The magnitude of the velocity easily follows,

$$V = v_r = \frac{q}{2\pi r}$$

Here, r is such that

$$r = \sqrt{(8-4)^2 + (-1-2)^2} = 5\text{ ft}$$

Therefore,

$$V = \frac{80}{2\pi \times 5} = \boxed{2.55\text{ ft/s}}$$

➡ The correct answer is **B**.

Part B: The x and y components of velocity are

$$u = v_r \cos\theta \;;\; v = v_r \sin\theta$$

Here, $\cos\theta = x/r$ and $\sin\theta = y/r$. Thus,

$$u = \frac{q}{2\pi r}\left(\frac{x}{r}\right) = \frac{q}{2\pi}\left(\frac{x}{r^2}\right) \;;\; v = \frac{q}{2\pi}\left(\frac{y}{r^2}\right)$$

However, $r^2 = x^2 + y^2$. Thus,

$$u = \frac{q}{2\pi}\left(\frac{x}{x^2+y^2}\right) \;;\; y = \frac{q}{2\pi}\left(\frac{y}{x^2+y^2}\right)$$

Now, the x-component of acceleration is

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

Substituting the pertaining terms in this expression, we get

$$a_x = 0 + \frac{q}{2\pi}\left(\frac{x}{x^2+y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{y^2-x^2}{\left(x^2+y^2\right)^2}\right]\right\} + \frac{q}{2\pi}\left(\frac{y}{x^2+y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{-2xy}{\left(x^2+y^2\right)^2}\right]\right\}$$

$$\therefore a_x = -\frac{q^2}{4\pi^2}\left[\frac{x^3+xy^2}{\left(x^2+y^2\right)^3}\right]$$

As for the y-component of acceleration, we have

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$\therefore a_y = 0 + \frac{q}{2\pi}\left(\frac{x}{x^2+y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{-2xy}{\left(x^2+y^2\right)^2}\right]\right\} + \frac{q}{2\pi}\left(\frac{y}{x^2+y^2}\right)\left\{\frac{q}{2\pi}\left[\frac{x^2-y^2}{\left(x^2+y^2\right)^2}\right]\right\}$$

$$\therefore a_y = -\frac{q^2}{4\pi^2}\left[\frac{y^3+x^2y}{\left(x^2+y^2\right)^3}\right]$$

With respect to point A, the coordinates of point B are B[(8 − 4) ft, (−1 − 2) ft] = B(4, −3 ft). Hence, a_x and a_y are such that

$$a_x = -\frac{80^2}{4\pi^2}\left\{\frac{4^3 + 4\times(-3)^2}{\left[4^2+(-3)^2\right]^3}\right\} = -1.04\ \text{ft/s}^2$$

$$a_y = -\frac{80^2}{4\pi^2}\left\{\frac{(-3)^3 + 4^2\times(-3)}{\left[4^2+(-3)^2\right]^3}\right\} = 0.78\ \text{ft/s}^2$$

The magnitude of acceleration is calculated to be

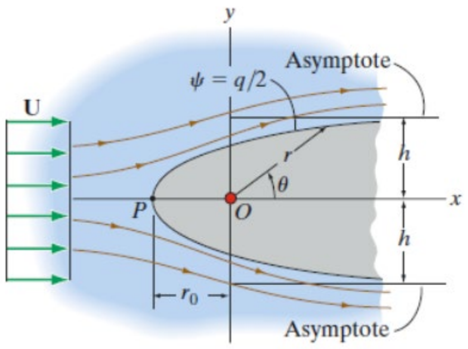
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-1.04)^2 + (0.78)^2} = \boxed{1.3\ \text{ft/s}^2}$$

➡ The correct answer is **A**.

P.11 ● Solution

We begin by computing r_o, the distance from the line source to stagnation point P,

$$r_o = \frac{q}{2\pi U} = \frac{0.5}{2\pi\times 8} = \frac{0.03125}{\pi}$$



Then, the equation of the boundary of a half body is given by

$$r = \frac{r_o(\pi-\theta)}{\sin\theta} = \frac{\frac{0.03125}{\pi}(\pi-\theta)}{\sin\theta}$$

$$\therefore r\sin\theta = \frac{0.03125}{\pi}(\pi-\theta)$$

Here, $y = r\sin\theta$ and $\theta = \tan^{-1} y/x$. Manipulating this relation, it follows that

$$\frac{\pi}{0.03125}y = 32\pi y = \pi - \tan^{-1}\frac{y}{x}$$

$$\therefore \tan^{-1}\frac{y}{x} = \pi - 32\pi y$$

$$\therefore \tan^{-1}\frac{y}{x} = \pi(1-32y)$$

$$\therefore \boxed{\frac{y}{x} = \tan\left[\pi(1-32y)\right]}$$

➡ The correct answer is **B**.

P.12 ● Solution

At the stagnation point, we obviously have $V_0 = 0$. The r and θ components of velocity at point A can be determined using the relations

$$v_r = \frac{q}{2\pi r} + U\cos\theta = \frac{100}{2\pi \times 0.3} + \underbrace{300\cos 90^\circ}_{=0} = 53.05\text{ ft/s}$$

$$v_\theta = -U\sin\theta = -300\sin 90^\circ = -300\text{ ft/s}$$

Thus, the magnitude of the velocity is

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{53.05^2 + (-300)^2} = 304.65\text{ ft/s}$$

This flow past a half body is irrotational. Accordingly, the Bernoulli equation for an ideal fluid is applicable from point O at A. Neglecting the elevation term, we have

$$\frac{p_o}{\rho} + \frac{V_o^2}{2} = \frac{p_A}{\rho} + \frac{V_A^2}{2}$$

Substituting the pertaining variables, we get

$$\frac{p_o}{2.35\times 10^{-3}} + 0 = \frac{p_A}{2.35\times 10^{-3}} + \frac{304.65^2}{2}$$

$$\therefore p_o - p_A = \Delta p = \left(2.35\times 10^{-3}\right)\times\frac{304.65^2}{2} = 109.05\text{ lb/ft}^2$$

$$\therefore \Delta p = 109.05\text{ lb/ft}^2\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 = \boxed{0.76\text{ psi}}$$

➡ The correct answer is **C**.

ANSWER SUMMARY

Problem 1		T/F
Problem 2		T/F
Problem 3		**A**
Problem 4	**4A**	**B**
	4B	**B**
Problem 5		**C**
Problem 6		**D**
Problem 7		**D**
Problem 8	**8A**	**A**
	8B	**B**
Problem 9		Open-ended qst.
Problem 10	**10A**	**B**
	10B	**A**
Problem 11		**B**
Problem 12		**C**

REFERENCES

- ÇENGEL, Y. and CIMBALA, J. (2014). *Fluid Mechanics: Fundamentals and Applications*. 3rd edition. New York: McGraw-Hill.
- HIBBELER, R. (2015). *Fluid Mechanics*. Upper Saddle River: Pearson.
- MUNSON, B., YOUNG, D., OKIISHI, T., and HUEBSCH, W. (2009). *Fundamentals of Fluid Mechanics*. 6th edition. Hoboken: John Wiley and Sons.