Quiz EL404
Digital Modulation Schemes

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PROBLEMS

Problem 1 (Sklar, 2001)

Find the number of bit errors made in one day by the following continuously operating coherent BPSK receiver. The data rate is 5000 bits/s. The input digital waveforms are $s_1(t) = A\cos(\omega_0 t)$ and $s_2(t) = -A\cos(\omega_0 t)$ where A = 1 mV and the single-sided noise power spectral density is $N_0 = 10^{-11}$ W/Hz. Assume that signal power and energy per bit are normalized relative to a 1-Ω resistive load.

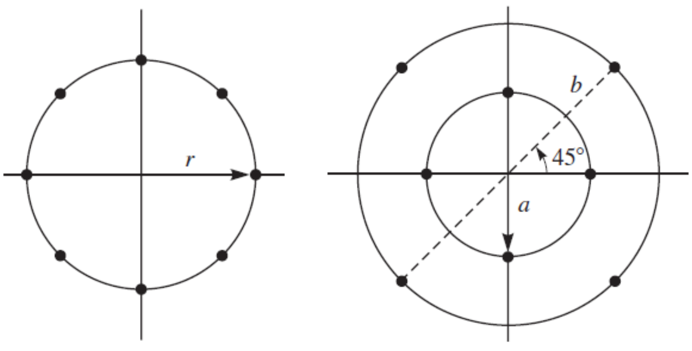
Problem 2 (Sklar, 2001)

A continuously operating coherent BPSK system makes errors at the average rate of 100 errors per day. The data rate is 1000 bits/s. The single-sided noise power spectral density is $N_0 = 10^{-10}$ W/Hz.

Problem 2.1: If the system is ergodic, what is the average bit error probability?

Problem 2.2: If the value of received average signal power is adjusted to be 10^{-6} W, will this received power be adequate to maintain the error probability found in part 1?

Problem 3 (Proakis and Salehi, 2008)



8-PSK 8-QAM

Problem 3.1: The nearest-neighbor signal points in the 8-QAM signal constellation are separated in distance by A units. Determine the radii a and b of the inner and outer circles, respectively.

Problem 3.2: The adjacent signal points in the 8-PSK are separated by a distance of A units. Determine the radius r of the circle.

Problem 3.3: Determine the average transmitter powers for the two signal constellations, and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume that all signal points are equiprobable.)

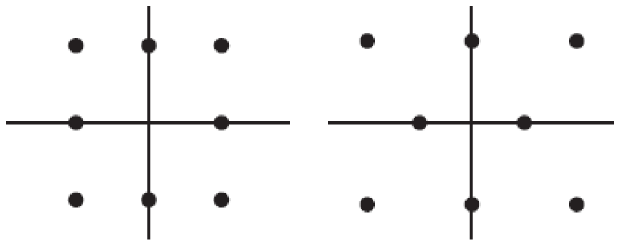
Problem 4 (Proakis and Salehi, 2008)

Problem 4.1: Reconsider the 8-point QAM signal constellation shown in the introduction to Problem 3. Is it possible to assign 3 data bits to each point of the signal constellation such that the nearest (adjacent) points differ in only 1 bit position?

Problem 4.2: Determine the symbol rate if the desired bit rate is 90 Mbits/s.

Problem 5 (Proakis and Salehi, 2008)

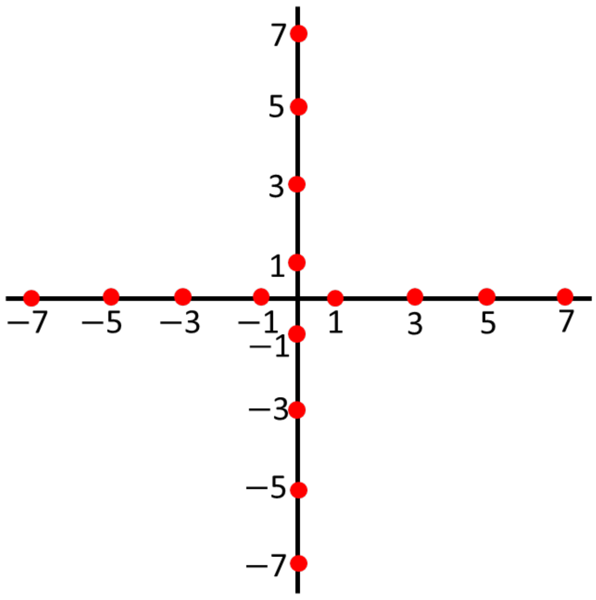
Consider the two 8-point QAM signal constellations illustrated in figures (a) and (b) below. The minimum distance between two adjacent points is $2A$. Determine the average transmitted power for each constellation, assuming that the signal points are equally probable. Which constellation is more power-efficient?



Problem 5.1: What is the maximum allowable time interval between sample values that will ensure perfect signal reproduction?
Problem 5.2: If we want to reproduce 1 hour of this waveform, how many sample values need to be stored?

Problem 6 (Proakis and Salehi, 2008)

Specify a Gray code for the 16-QAM signal constellation illustrated below.



Problem 7 (Proakis and Salehi, 2008)

In an MSK signal, the initial state for the phase is either 0 or π rad. Determine the terminal phase state for the following four input pairs of input data:
1. 00
2. 01
3. 10
4. 11

Problem 8 (Sklar, 2001)

If a system's main performance criterion is bit-error probability, which of the following two modulation schemes would be selected for an AWGN channel? Show your calculations.

Binary noncoherent orthogonal FSK with E_b/N_0 = 13 dB
Binary coherent PSK with E_b/N_0 = 8 dB

Problem 9 (Sklar, 2001)

A bit-error probability $P_B = 10^{-3}$ is required for a system with a data rate of 100 kbits/s to be transmitted over an AWGN channel using coherently detected MPSK modulation. The system bandwidth is 50 kHz. Assume that the system frequency transfer function is a raised cosine with a roll-off characteristic of $r = 1$ and that a Gray code is used for the symbol to bit assignment.
Problem 9.1: What symbol energy to noise power ratio, E_s/N_0, is required for the specified P_b?
Problem 9.2: What bit energy to noise power ratio, E_b/N_0, is required?

Problem 10

A differentially coherently MPSK system operates over an AWGN channel with a bit energy to noise density ratio, E_b/N_0, of 10 dB. What is the symbol error probability $M = 8$ and equally likely symbols?

Problem 11 (Sklar, 2001)

If a system's main performance criterion is bit-error probability, which of the following two modulation schemes would be selected for transmission over an AWGN channel?

Coherent 8-ary orthogonal FSK with E_b/N_0 = 8 dB
Coherent 8-ary PSK with E_b/N_0 = 13 dB

Problem 12 (Haykin, 2001)

Problem 12.1: In a coherent FSK system, the signals $s_1(t)$ and $s_2(t)$ representing symbols 1 and 0, respectively, are defined by

$$s_1(t), s_2(t) = A_c \cos\left[2\pi\left(f_c \pm \frac{\Delta f}{2}\right)t\right] \;;\; 0 \le t \le T_b$$

Assuming that $f_c \gg \Delta f$, show that the correlation coefficient of the signals $s_1(t)$ and $s_2(t)$ is approximately given by

$$\rho = \frac{\int_0^{T_b} s_1(t) s_2(t)\,dt}{\int_0^{T_b} s_1^2(t)dt} \approx \text{sinc}(2\Delta f T_b)$$

Problem 12.2: What is the minimum value of frequency shift Δf for which the signals $s_1(t)$ and $s_2(t)$ are orthogonal?
Problem 12.3: What is the value of Δf that minimizes the average probability of symbol error?
Problem 12.4: For the value of Δf obtained in part 3, determine the increase in E_b/N_0 required so that this coherent FSK system has the same noise performance as a coherent binary PSK system.

Problem 13 (Haykin, 2001)

A PSK signal is applied to a correlator supplied with a phase reference that lies within φ radians of the exact carrier phase. Determine the effect of the phase error φ on the average probability of error of the system.

Problem 14

There are two ways of detecting an MSK signal. One way is to use a coherent receiver to take full account of the phase information content of the MSK signal. Another way is to use a noncoherent receiver and disregard the phase information. The second method offers the advantage of simplicity of implementation, at the expense of a degraded noise performance. By how many decibels do we have to increase the bit energy-to-noise density ratio E_b/N_0 in the second case so as to realize an average probability of symbol error equal to 10^{-5} in both cases?

Problem 15 (Haykin, 2001)

Problem 15.1: Just as in an ordinary QPSK modulator, the output of a $\pi/4$-shifted DQPSK modulator may be expressed in terms of its in-phase and quadrature components as follows:

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

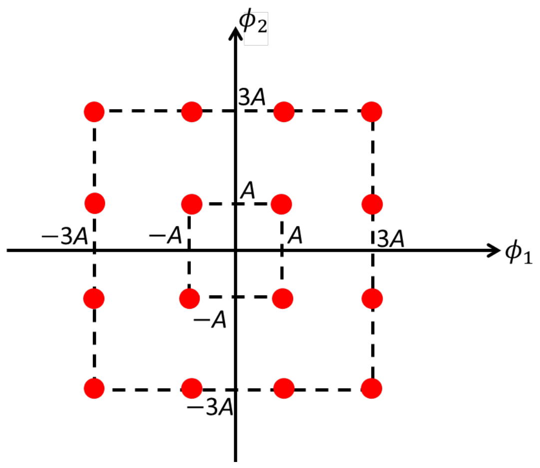
Formulate the in-phase component $s_I(t)$ and quadrature component $s_Q(t)$ of the $\pi/4$-shifted DQPSK signal. Hence, outline a scheme for the generation of $\pi/4$-shifted DQPSK signals.
Problem 15.2: An interesting property of $\pi/4$-shifted DQPSK signals is that they can be demodulated using an FM discriminator. Demonstrate the validity of this property.

Problem 16 (Lathi, 1998)

Problem 16.1: A binary source emits data at a rate of 400,000 bit/s. Multiamplitude shift keying (M-PAM) with M = 2, 16, and 32 is considered. In each case, determine the signal power required at the receiver input and the minimum transmission bandwidth required if $S_n(\omega) = 10^{-8}$ and the bit error rate P_b is required to be less than 10^{-6}.
Problem 16.2: Repeat the previous problem for M-ary PSK.

Problem 17 (Proakis and Salehi, 2008)

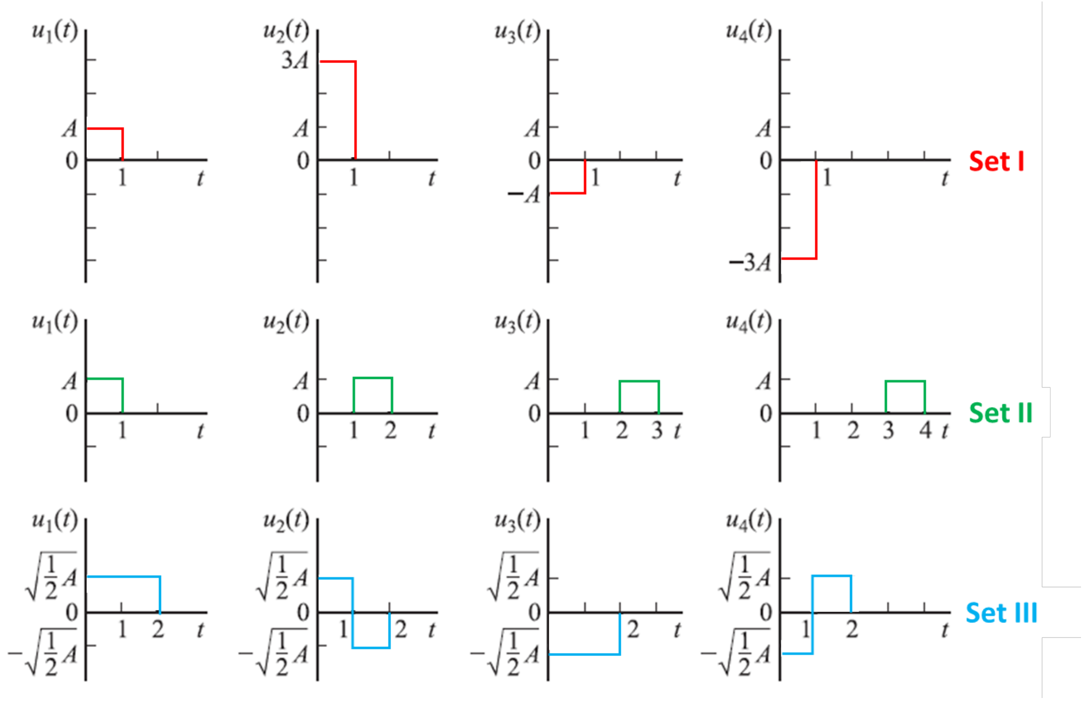
The signal constellation for a communication system with 16 equiprobable symbols is illustrated below. The channel is AWGN with noise power spectral density of $N_0/2$.



Problem 17.1: Using the union bound, find a bound in terms of A and N_0 on the error probability for this channel.
Problem 17.2: Determine the average signal to noise ratio, SNR, per bit for this channel.
Problem 17.3: Express the bound found in part 1 in terms of the average SNR per bit.
Problem 17.4: Compare the power efficiency of this system with a 16-level PAM system.

Problem 18 (Proakis and Salehi, 2008)

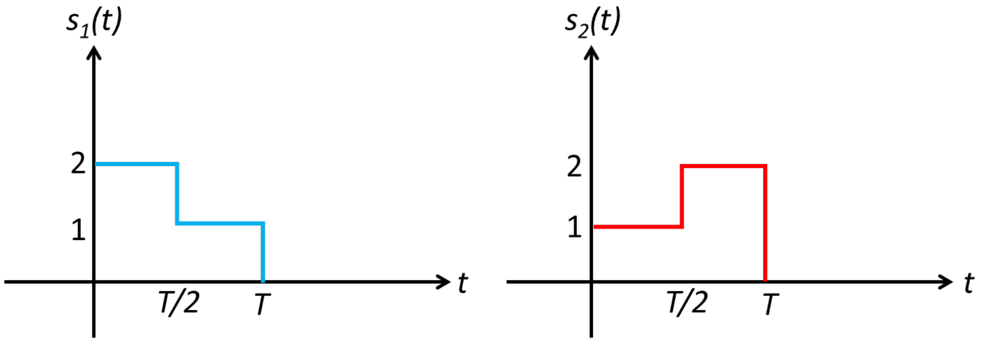
The equivalent lowpass waveforms for three signal sets are shown in the following figure. Each set may be used to transmit one of four equally probable messages over an additive white Gaussian noise channel. The equivalent lowpass noise $z(t)$ has zero-mean and autocorrelation function $R_z(\tau) = 2N_0\delta(\tau)$.



Problem 18.1: Classify the signal waveforms in sets I, II, and III. In other words, state the category or class to which each signal set belongs.
Problem 18.2: What is the *average* transmitted energy for each signal set?
Problem 18.3: For signal set I, specify the average probability of error if the signals are detected coherently.
Problem 18.4: For signal set II, give a union bound on the probability of a symbol error if the detection is performed (i) coherently and (ii) noncoherently.
Problem 18.5: Is it possible to use noncoherent detection on signal set III? Explain.
Problem 18.6: Is it possible to use noncoherent detection on signal set III? Explain.

Problem 19 (Proakis and Salehi, 2008)

A binary signaling scheme over an AWGN channel with noise power spectral density of $N_0/2$ uses the equiprobable messages illustrated below and is operating at a bit rate of R bits/sec.



Problem 19.1: What is E_b/N_0 for this system (in terms of N_0 and R)?
Problem 19.2: What is the error probability for this system (in terms of N_0 and R)?
Problem 19.3: By how many decibels does this system underperform a binary antipodal signaling system with the same E_b/N_0?
Problem 19.4: Now assume that this system is augmented with two more signals $s_3(t) = -s_1(t)$ and $s_4(t) = -s_2(t)$ to result in a 4-ary equiprobable system. What is the resulting transmission bit rate?
Problem 19.5: Using the union bound, find a bound on the error probability of the 4-ary system introduced in part 4.

ADDITIONAL INFORMATION

The following two pages show tabulated values of the Q function.

TABLE 8.2[3]
$Q(x)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0000	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.1000	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2000	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3000	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4000	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5000	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6000	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7000	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8000	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9000	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.000	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.100	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.200	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.9853E-01
1.300	.9680E-01	.9510E-01	.9342E-01	.9176E-01	.9012E-01	.8851E-01	.8691E-01	.8534E-01	.8379E-01	.8226E-01
1.400	.8076E-01	.7927E-01	.7780E-01	.7636E-01	.7493E-01	.7353E-01	.7215E-01	.7078E-01	.6944E-01	.6811E-01
1.500	.6681E-01	.6552E-01	.6426E-01	.6301E-01	.6178E-01	.6057E-01	.5938E-01	.5821E-01	.5705E-01	.5592E-01
1.600	.5480E-01	.5370E-01	.5262E-01	.5155E-01	.5050E-01	.4947E-01	.4846E-01	.4746E-01	.4648E-01	.4551E-01
1.700	.4457E-01	.4363E-01	.4272E-01	.4182E-01	.4093E-01	.4006E-01	.3920E-01	.3836E-01	.3754E-01	.3673E-01
1.800	.3593E-01	.3515E-01	.3438E-01	.3362E-01	.3288E-01	.3216E-01	.3144E-01	.3074E-01	.3005E-01	.2938E-01
1.900	.2872E-01	.2807E-01	.2743E-01	.2680E-01	.2619E-01	.2559E-01	.2500E-01	.2442E-01	.2385E-01	.2330E-01
2.000	.2275E-01	.2222E-01	.2169E-01	.2118E-01	.2068E-01	.2018E-01	.1970E-01	.1923E-01	.1876E-01	.1831E-01
2.100	.1786E-01	.1743E-01	.1700E-01	.1659E-01	.1618E-01	.1578E-01	.1539E-01	.1500E-01	.1463E-01	.1426E-01
2.200	.1390E-01	.1355E-01	.1321E-01	.1287E-01	.1255E-01	.1222E-01	.1191E-01	.1160E-01	.1130E-01	.1101E-01
2.300	.1072E-01	.1044E-01	.1017E-01	.9903E-02	.9642E-02	.9387E-02	.9137E-02	.8894E-02	.8656E-02	.8424E-02
2.400	.8198E-02	.7976E-02	.7760E-02	.7549E-02	.7344E-02	.7143E-02	.6947E-02	.6756E-02	.6569E-02	.6387E-02
2.500	.6210E-02	.6037E-02	.5868E-02	.5703E-02	.5543E-02	.5386E-02	.5234E-02	.5085E-02	.4940E-02	.4799E-02
2.600	.4661E-02	.4527E-02	.4396E-02	.4269E-02	.4145E-02	.4025E-02	.3907E-02	.3793E-02	.3681E-02	.3573E-02
2.700	.3467E-02	.3364E-02	.3264E-02	.3167E-02	.3072E-02	.2980E-02	.2890E-02	.2803E-02	.2718E-02	.2635E-02
2.800	.2555E-02	.2477E-02	.2401E-02	.2327E-02	.2256E-02	.2186E-02	.2118E-02	.2052E-02	.1988E-02	.1926E-02
2.900	.1866E-02	.1807E-02	.1750E-02	.1695E-02	.1641E-02	.1589E-02	.1538E-02	.1489E-02	.1441E-02	.1395E-02
3.000	.1350E-02	.1306E-02	.1264E-02	.1223E-02	.1183E-02	.1144E-02	.1107E-02	.1070E-02	.1035E-02	.1001E-02
3.100	.9676E-03	.9354E-03	.9043E-03	.8740E-03	.8447E-03	.8164E-03	.7888E-03	.7622E-03	.7364E-03	.7114E-03
3.200	.6871E-03	.6637E-03	.6410E-03	.6190E-03	.5976E-03	.5770E-03	.5571E-03	.5377E-03	.5190E-03	.5009E-03
3.300	.4834E-03	.4665E-03	.4501E-03	.4342E-03	.4189E-03	.4041E-03	.3897E-03	.3758E-03	.3624E-03	.3495E-03
3.400	.3369E-03	.3248E-03	.3131E-03	.3018E-03	.2909E-03	.2802E-03	.2701E-03	.2602E-03	.2507E-03	.2415E-03
3.500	.2326E-03	.2241E-03	.2158E-03	.2078E-03	.2001E-03	.1926E-03	.1854E-03	.1785E-03	.1718E-03	.1653E-03
3.600	.1591E-03	.1531E-03	.1473E-03	.1417E-03	.1363E-03	.1311E-03	.1261E-03	.1213E-03	.1166E-03	.1121E-03
3.700	.1078E-03	.1036E-03	.9961E-04	.9574E-04	.9201E-04	.8842E-04	.8496E-04	.8162E-04	.7841E-04	.7532E-04
3.800	.7235E-04	.6948E-04	.6673E-04	.6407E-04	.6152E-04	.5906E-04	.5669E-04	.5442E-04	.5223E-04	.5012E-04
3.900	.4810E-04	.4615E-04	.4427E-04	.4247E-04	.4074E-04	.3908E-04	.3747E-04	.3594E-04	.3446E-04	.3304E-04
4.000	.3167E-04	.3036E-04	.2910E-04	.2789E-04	.2673E-04	.2561E-04	.2454E-04	.2351E-04	.2252E-04	.2157E-04
4.100	.2066E-04	.1978E-04	.1894E-04	.1814E-04	.1737E-04	.1662E-04	.1591E-04	.1523E-04	.1458E-04	.1395E-04
4.200	.1335E-04	.1277E-04	.1222E-04	.1168E-04	.1118E-04	.1069E-04	.1022E-04	.9774E-05	.9345E-05	.8934E-05
4.300	.8540E-05	.8163E-05	.7801E-05	.7455E-05	.7124E-05	.8807E-05	.6503E-05	.6212E-05	.5934E-05	.5668E-05
4.400	.5413E-05	.5169E-05	.4935E-05	.4712E-05	.4498E-05	.4294E-05	.4098E-05	.3911E-05	.3732E-05	.3561E-05
4.500	.3398E-05	.3241E-05	.3092E-05	.2949E-05	.2813E-05	.2682E-05	.2558E-05	.2439E-05	.2325E-05	.2216E-05
4.600	.2112E-05	.2013E-05	.1919E-05	.1828E-05	.1742E-05	.1660E-05	.1581E-05	.1506E-05	.1434E-05	.1366E-05
4.700	.1301E-05	.1239E-05	.1179E-05	.1123E-05	.1069E-05	.1017E-05	.9680E-06	.9211E-06	.8765E-06	.8339E-06
4.800	.7933E-06	.7547E-06	.7178E-06	.6827E-06	.6492E-06	.6173E-06	.5869E-06	.5580E-06	.5304E-06	.5042E-06
4.900	.4792E-06	.4554E-06	.4327E-06	.4111E-06	.3906E-06	.3711E-06	.3525E-06	.3448E-06	.3179E-06	.3019E-06
5.000	.2867E-06	.2722E-06	.2584E-06	.2452E-06	.2328E-06	.2209E-06	.2096E-06	.1989E-06	.1887E-06	.1790E-06
5.100	.1698E-06	.1611E-06	.1528E-06	.1449E-06	.1374E-06	.1302E-06	.1235E-06	.1170E-06	.1109E-06	.1051E-06

(continued)

TABLE 8.2
Continued

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.200	.9964E-07	.9442E-07	.8946E-07	.8476E-07	.8029E-07	.7605E-07	.7203E-07	.6821E-07	.6459E-07	.6116E-07
5.300	.5790E-07	.5481E-07	.5188E-07	.4911E-07	.4647E-07	.4398E-07	.4161E-07	.3937E-07	.3724E-07	.3523E-07
5.400	.3332E-07	.3151E-07	.2980E-07	.2818E-07	.2664E-07	.2518E-07	.2381E-07	.2250E-07	.2127E-07	.2010E-07
5.500	.1899E-07	.1794E-07	.1695E-07	.1601E-07	.1512E-07	.1428E-07	.1349E-07	.1274E-07	.1203E-07	.1135E-07
5.600	.1072E-07	.1012E-07	.9548E-08	.9010E-08	.8503E-08	.8022E-08	.7569E-08	.7140E-08	.6735E-08	.6352E-08
5.700	.5990E-08	.5649E-08	.5326E-08	.5022E-08	.4734E-08	.4462E-08	.4206E-08	.3964E-08	.3735E-08	.3519E-08
5.800	.3316E-08	.3124E-08	.2942E-08	.2771E-08	.2610E-08	.2458E-08	.2314E-08	.2179E-08	.2051E-08	.1931E-08
5.900	.1818E-08	.1711E-08	.1610E-08	.1515E-08	.1425E-08	.1341E-08	.1261E-08	.1186E-08	.1116E-08	.1049E-08
6.000	.9866E-09	.9276E-09	.8721E-09	.8198E-09	.7706E-09	.7242E-09	.6806E-09	.6396E-09	.6009E-09	.5646E-09
6.100	.5303E-09	.4982E-09	.4679E-09	.4394E-09	.4126E-09	.3874E-09	.3637E-09	.3414E-09	.3205E-09	.3008E-09
6.200	.2823E-09	.2649E-09	.2486E-09	.2332E-09	.2188E-09	.2052E-09	.1925E-09	.1805E-09	.1692E-09	.1587E-09
6.300	.1488E-09	.1395E-09	.1308E-09	.1226E-09	.1149E-09	.1077E-09	.1009E-09	.9451E-10	.8854E-10	.8294E-10
6.400	.7769E-10	.7276E-10	.6814E-10	.6380E-10	.5974E-10	.5593E-10	.5235E-10	.4900E-10	.4586E-10	.4292E-10
6.500	.4016E-10	.3758E-10	.3515E-10	.3288E-10	.3077E-10	.2877E-10	.2690E-10	.2516E-10	.2352E-10	.2199E-10
6.600	.2056E-10	.1922E-10	.1796E-10	.1678E-10	.1568E-10	.1465E-10	.1369E-10	.1279E-10	.1195E-10	.1116E-10
6.700	.1042E-10	.9731E-11	.9086E-11	.8483E-11	.7919E-11	.7392E-11	.6900E-11	.6439E-11	.6009E-11	.5607E-11
6.800	.5231E-11	.4880E-11	.4552E-11	.4246E-11	.3960E-11	.3692E-11	.3443E-11	.3210E-11	.2993E-11	.2790E-11
6.900	.2600E-11	.2423E-11	.2258E-11	.2104E-11	.1960E-11	.1826E-11	.1701E-11	.1585E-11	.1476E-11	.1374E-11
7.000	.1280E-11	.1192E-11	.1109E-11	.1033E-11	.9612E-12	.8946E-12	.8325E-12	.7747E-12	.7208E-12	.6706E-12
7.100	.6238E-12	.5802E-12	.5396E-12	.5018E-12	.4667E-12	.4339E-12	.4034E-12	.3750E-12	.3486E-12	.3240E-12
7.200	.3011E-12	.2798E-12	.2599E-12	.2415E-12	.2243E-12	.2084E-12	.1935E-12	.1797E-12	.1669E-12	.1550E-12
7.300	.1439E-12	.1336E-12	.1240E-12	.1151E-12	.1068E-12	.9910E-13	.9196E-13	.8531E-13	.7914E-13	.7341E-13
7.400	.6809E-13	.6315E-13	.5856E-13	.5430E-13	.5034E-13	.4667E-13	.4326E-13	.4010E-13	.3716E-13	.3444E-13
7.500	.3191E-13	.2956E-13	.2739E-13	.2537E-13	.2350E-13	.2176E-13	.2015E-13	.1866E-13	.1728E-13	.1600E-13
7.600	.1481E-13	.1370E-13	.1268E-13	.1174E-13	.1086E-13	.1005E-13	.9297E-14	.8600E-14	.7954E-14	.7357E-14
7.700	.6803E-14	.6291E-14	.5816E-14	.5377E-14	.4971E-14	.4595E-14	.4246E-14	.3924E-14	.3626E-14	.3350E-14
7.800	.3095E-14	.2859E-14	.2641E-14	.2439E-14	.2253E-14	.2080E-14	.1921E-14	.1773E-14	.1637E-14	.1511E-14
7.900	.1395E-14	.1287E-14	.1188E-14	.1096E-14	.1011E-14	.9326E-15	.8602E-15	.7934E-15	.7317E-15	.6747E-15
8.000	.6221E-15	.5735E-15	.5287E-15	.4874E-15	.4492E-15	.4140E-15	.3815E-15	.3515E-15	.3238E-15	.2983E-15
8.100	.2748E-15	.2531E-15	.2331E-15	.2146E-15	.1976E-15	.1820E-15	.1675E-15	.1542E-15	.1419E-15	.1306E-15
8.200	.1202E-15	.1106E-15	.1018E-15	.9361E-16	.8611E-16	.7920E-16	.7284E-16	.6698E-16	.6159E-16	.5662E-16
8.300	.5206E-16	.4785E-16	.4398E-16	.4042E-16	.3715E-16	.3413E-16	.3136E-16	.2881E-16	.2646E-16	.2431E-16
8.400	.2232E-16	.2050E-16	.1882E-16	.1728E-16	.1587E-16	.1457E-16	.1337E-16	.1227E-16	.1126E-16	.1033E-16
8.500	.9480E-17	.8697E-17	.7978E-17	.7317E-17	.6711E-17	.6154E-17	.5643E-17	.5174E-17	.4744E-17	.4348E-17
8.600	.3986E-17	.3653E-17	.3348E-17	.3068E-17	.2811E-17	.2575E-17	.2359E-17	.2161E-17	.1979E-17	.1812E-17
8.700	.1659E-17	.1519E-17	.1391E-17	.1273E-17	.1166E-17	.1067E-17	.9763E-18	.8933E-18	.8174E-18	.7478E-18
8.800	.6841E-18	.6257E-18	.5723E-18	.5234E-18	.4786E-18	.4376E-18	.4001E-18	.3657E-18	.3343E-18	.3055E-18
8.900	.2792E-18	.2552E-18	.2331E-18	.2130E-18	.1946E-18	.1777E-18	.1623E-18	.1483E-18	.1354E-18	.1236E-18
9.000	.1129E-18	.1030E-18	.9404E-19	.8584E-19	.7834E-19	.7148E-19	.6523E-19	.5951E-19	.5429E-19	.4952E-19
9.100	.4517E-19	.4119E-19	.3756E-19	.3425E-19	.3123E-19	.2847E-19	.2595E-19	.2365E-19	.2155E-19	.1964E-19
9.200	.1790E-19	.1631E-19	.1486E-19	.1353E-19	.1232E-19	.1122E-19	.1022E-19	.9307E-20	.8474E-20	.7714E-20
9.300	.7022E-20	.6392E-20	.5817E-20	.5294E-20	.4817E-20	.4382E-20	.3987E-20	.3627E-20	.3299E-20	.3000E-20
9.400	.2728E-20	.2481E-20	.2255E-20	.2050E-20	.1864E-20	.1694E-20	.1540E-20	.1399E-20	.1271E-20	.1155E-20
9.500	.1049E-20	.9533E-21	.8659E-21	.7864E-21	.7142E-21	.6485E-21	.5888E-21	.5345E-21	.4852E-21	.4404E-21
9.600	.3997E-21	.3627E-21	.3292E-21	.2986E-21	.2709E-21	.2458E-21	.2229E-21	.2022E-21	.1834E-21	.1663E-21
9.700	.1507E-21	.1367E-21	.1239E-21	.1123E-21	.1018E-21	.9223E-22	.8358E-22	.7573E-22	.6861E-22	.6215E-22
9.800	.5629E-22	.5098E-22	.4617E-22	.4181E-22	.3786E-22	.3427E-22	.3102E-22	.2808E-22	.2542E-22	.2300E-22
9.900	.2081E-22	.1883E-22	.1704E-22	.1541E-22	.1394E-22	.1261E-22	.1140E-22	.1031E-22	.9323E-23	.8429E-23
10.00	.7620E-23	.6888E-23	.6225E-23	.5626E-23	.5084E-23	.4593E-23	.4150E-23	.3749E-23	.3386E-23	.3058E-23

Notes: (1) E-01 should be read as $\times 10^{-1}$; E-02 should be read as $\times 10^{-2}$, and so on.
(2) This table lists $Q(x)$ for x in the range of 0 to 10 in the increments of 0.01. To find $Q(5.36)$, for example, look up the row starting with $x = 5.3$. The sixth entry in this row (under 0.06) is the desired value 0.4161×10^{-7}.

SOLUTIONS

P.1 Solution

The bit-error probability can be restated as

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{N_0}}\right)$$

Here, amplitude $A = 10^{-3}$ V, period $T = 1/5000 = 2.0\times10^{-4}$ s, and noise power spectral density $N_0 = 10^{-11}$ W/Hz, giving

$$P_b = Q\left(\sqrt{\frac{\left(10^{-3}\right)^2\times\left(2.0\times10^{-4}\right)}{10^{-11}}}\right) = Q(4.47)$$

Referring to the Q function table, we see that $Q(4.47) = 0.3911\times10^{-5}$, or 3.91×10^{-6}. It follows that the average number of errors in one day amounts to

$$\text{No. of errors} = 5000\frac{\cancel{\text{bit}}}{\cancel{\text{sec}}}\times 86,400\frac{\cancel{\text{sec}}}{\text{day}}\times 3.91\times10^{-6}\frac{\text{errors}}{\cancel{\text{bit}}}$$

$$\therefore \boxed{\text{No. of errors} = 1690\ \text{errors/day}}$$

P.2 Solution

Problem 2.1: For a data rate of 1000 bits/s, the number of bits detected in one day is 1000 × 86,400 = 8.64×10^7 bits. The bit-error probability is then

$$P_b = \frac{100}{8.64\times10^7} = \boxed{1.16\times10^{-6}}$$

Problem 2.2: For the given data ($S = 10^{-6}$ W, $T = 1/1000 = 10^{-3}$ s, $N_0 = 10^{-10}$ W/Hz), we may write

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2ST}{N_0}}\right) = Q\left(\sqrt{\frac{2\times10^{-6}\times10^{-3}}{10^{-10}}}\right) = Q(4.47)$$

In Problem 1, $Q(\sqrt{4.47})$ was read to be 3.91×10^{-6}. Thus,

$$P_b' = 3.91\times10^{-6}$$

This is greater than the probability found in part 1; therefore, a received power of 10^{-6} W does not suffice to maintain the bit-error probability found in the previous part.

P.3 Solution

Problem 3.1: Using the Pythagorean theorem, we can find the radius of the inner circle as

$$a^2 + a^2 = A^2 \rightarrow \boxed{a = \frac{A}{\sqrt{2}}}$$

The radius of the outer circle can be found using the cosine rule. Since b is the third side of a triangle with sides a and A and angle between them equal to $\theta = 75°$, we may write

$$b^2 = a^2 + A^2 - 2aA\cos 75° \rightarrow \boxed{b = \frac{1+\sqrt{3}}{2}A}$$

Problem 3.2: If we denote by r the radius of the circle, then using the cosine theorem we obtain

$$A^2 = r^2 + r^2 - 2r^2\cos 45° \rightarrow \boxed{r = \frac{A}{\sqrt{2-\sqrt{2}}}}$$

Problem 3.3: The average transmitted power of the PSK constellation is

$$P_{PSK} = 8\times\frac{1}{8}\times\left(\frac{A}{\sqrt{2-\sqrt{2}}}\right)^2 = \boxed{\frac{A^2}{2-\sqrt{2}}}$$

The average transmitted power of the QAM constellation is, in turn,

$$P_{QAM} = \frac{1}{8}\times\left[4\frac{A^2}{2}+4\frac{\left(1+\sqrt{3}\right)^2}{4}A^2\right] \to \boxed{P_{QAM} = \left[\frac{2+\left(1+\sqrt{3}\right)^2}{8}\right]A^2}$$

The relative power advantage of the PSK constellation over the QAM constellation is

$$\text{Gain} = \frac{P_{PSK}}{P_{QAM}} = \frac{1/\left(2-\sqrt{2}\right)}{\left[2+\left(1+\sqrt{3}\right)^2\right]/8} = 1.443$$

or $10\log_{10}(1.443) = 1.593$ dB.

P.4 ➡ *Solution*

Problem 4.1: Although it is possible to assign three bits to each point of the 8-PSK signal constellation so that adjacent points differ in only one bit (e.g., going in a clockwise direction: 000, 001, 011, 010, 110, 111, 101, 100), this is not the case for the 8-QAM constellation of the given figure. To see this, consider an equilateral triangle with vertices *A*, *B* and *C*. If, without loss of generality, we assign the all zero sequence {0, 0, ..., 0} to point *A*, then points *B* and *C* should have the form

$$B = \{0,\ldots,0,1,0,\ldots,0\}\ ;\ C = \{0,\ldots,0,1,0,\ldots,0\}$$

where the position of the 1 in the sequences is not the same, otherwise $B = C$. Thus, the sequences of *B* and *C* differ in two bits.

Problem 4.2: Since each symbol conveys 3 bits of information, the resulting symbol rate is $90\times10^6/3$ = 30 million symbols per second.

P.5 ➡ *Solution*

The constellation in figure (*a*) has four points at a distance 2*A* from the origin (the points over one of the coordinate axes) and four points at a distance $2\sqrt{2}A$ (the ones on the vertices of the square formed by the eight points). Thus, the average transmitted power of the second constellation is

$$P_a = \frac{1}{8}\times\left[4\times(2A)^2+4\times\left(2\sqrt{2}A\right)^2\right] = \frac{1}{8}\times\left(16A^2+32A^2\right)$$

$$\therefore P_a = 6A^2$$

The second constellation has four points at a distance $\sqrt{7}A$ from the origin, two points at a distance $\sqrt{3}A$, and two points at a distance *A*. Thus, the average transmitted power of the second constellation is

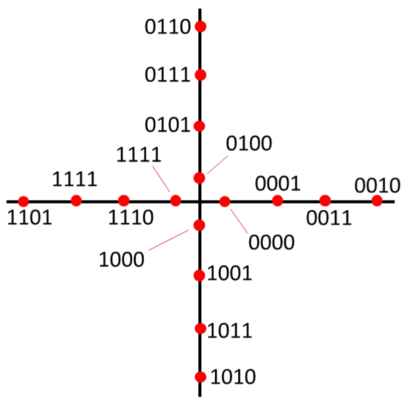
$$P_b = \frac{1}{8}\times\left[4\times\left(\sqrt{7}A\right)^2+2\times\left(\sqrt{3}A\right)^2+2A^2\right] = \frac{1}{8}\times\left(28A^2+6A^2+2A^2\right)$$

$$\therefore P_b = \frac{9}{2}A^2$$

Since $P_b < P_a$, the second constellation is more power efficient.

P.6 ➡ *Solution*

One way to label the points of the constellation at hand using a Gray code is depicted below.



P.7 ➡ Solution

We assume that the input bits 0, 1 are mapped to the symbols −1 and 1, respectively. The terminal phase of an MSK signal at time instant *n* is given by

$$\theta(n;\mathbf{a}) = \frac{\pi}{2}\sum_{k=0}^{k} a_k + \theta_0$$

where θ_0 is the initial phase and a_k is ±1 depending on the input bit at the time instant *k*. The following table shows $\theta(n;\mathbf{a})$ for two different values of $\theta_0(0,\pi)$ and the four input pairs of data: {00, 01, 10, 11}.

θ_0	b_0	b_1	a_0	a_1	$\theta(n;\mathbf{a})$
0	0	0	−1	−1	$-\frac{\pi}{2}-\frac{\pi}{2}+0=-\pi$
0	0	1	−1	1	$-\frac{\pi}{2}+\frac{\pi}{2}+0=0$
0	1	0	1	−1	$\frac{\pi}{2}-\frac{\pi}{2}+0=0$
0	1	1	1	1	$\frac{\pi}{2}+\frac{\pi}{2}+0=\pi$
π	0	0	−1	−1	$-\frac{\pi}{2}-\frac{\pi}{2}+\pi=0$
π	0	1	−1	1	$-\frac{\pi}{2}+\frac{\pi}{2}+\pi=\pi$
π	1	0	1	−1	$\frac{\pi}{2}-\frac{\pi}{2}+\pi=\pi$
π	1	1	1	1	$\frac{\pi}{2}+\frac{\pi}{2}+\pi=2\pi$

P.8 ➡ Solution

First note that 13 dB = $10^{1.3} \approx 20$. Evoking the expression for bit-error probability in noncoherent BFSK, we write

$$P_{b,1} = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) = \frac{1}{2}\times\exp\left(-\frac{20}{2}\right) = \underline{2.27\times10^{-5}}$$

Next, noting that 8 dB ≈ 6.31, we appeal to the expression for BER in coherent BPSK, giving

$$P_{b,2} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\times6.31}\right) = Q(3.55)$$

Referring to the *Q* function table, we read a probability of 0.1926×10^{-3}, or

$$P_{b,2} \approx 0.1926\times10^{-3} = \underline{1.93\times10^{-4}}$$

Since $P_{b,1} < P_{b,2}$, use of noncoherent BPSK with energy per bit to noise power equal to 13 dB should be selected in lieu of coherent BPSK with E_b/N_0 equal to 8 dB.

P.9 ➡ Solution

Problem 9.1: With roll-off $r = 1$, no ISI, and a bandwidth of 50 kHz, the symbol rate can be calculated as

$$W_{DSB} = (1+r)R_s \rightarrow R_s = \frac{W_{DSB}}{1+r}$$

$$\therefore R_s = \frac{50}{1+1} = 25\,\text{ksymbols/s}$$

We proceed to determine *M* for this MPSK modulation,

$$k = \log_2 M = \frac{R}{R_S} \rightarrow M = 2^{R/R_S}$$

$$\therefore M = 2^{100/25} = 16$$

Utilizing a Gray code assignment, the bit error and symbol error probability are associated by the simple relationship $P_B \approx P_E/\log_2 M$, so that

$$P_B \approx \frac{P_E}{\log_2 M} \rightarrow P_E = P_B\times\log_2 M$$

$$\therefore P_E = 10^{-3}\times\log_2 16 = 4.0\times10^{-3}$$

We aim for the symbol energy to noise density ratio, E_s/N_0, which can be found from the expression

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right) = 4.0\times10^{-3} \rightarrow Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right) = 2.0\times10^{-3}$$

Referring to the Q function table, we see that a value of 2.0×10^{-3} or, equivalently, 0.2×10^{-2}, corresponds to about 2.88 (the closest entry is for 0.1988×10^{-2}). It follows that

$$\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M} = 2.88 \rightarrow \sqrt{\frac{E_s}{N_0}} = \frac{2.88}{\sqrt{2}\sin(\pi/M)}$$

$$\therefore \frac{E_s}{N_0} = \left[\frac{2.88}{\sqrt{2}\sin(\pi/M)}\right]^2$$

$$\therefore \frac{E_s}{N_0} = \left[\frac{2.88}{\sqrt{2}\sin(\pi/16)}\right]^2 = 109$$

$$\therefore \boxed{\frac{E_s}{N_0} = 20.4\,\text{dB}}$$

Problem 9.2: The value of E_b/N_0 is

$$\frac{E_b}{N_0} = \frac{109}{k} = \frac{109}{4} = 27.3$$

or, equivalently, 14.4 dB.

P.10 ➡ *Solution*

For M = 8, we have $k = log_2 M = log_2 8 = 3$ and

$$\frac{E_s}{N_0} = k\frac{E_b}{N_0} = 3\times10 = 30 = 14.8\,\text{dB}$$

The error probability is then

$$P_E = 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{\sqrt{2}M}\right)\right] = 2Q\left[\sqrt{2\times30}\times\sin\left(\frac{\pi}{\sqrt{2}\times8}\right)\right] = 2Q(2.12)$$

Referring to the Q function table, we read $Q(2.12) = 0.1743\times10^{-1}$, giving

$$P_E = 2\times\left(0.174\times10^{-1}\right) = \boxed{3.48\times10^{-2}}$$

P.11 ➡ *Solution*

Noting that 8 dB = 6.31 and $k = log_2 M = 3$, we proceed to compute the symbol error probability

$$\frac{E_s}{N_0} = k\frac{E_b}{N_0} = 3\times6.31 = 18.9$$

so that

$$P_E(M) = (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right) = (8-1)\times Q\left(\sqrt{18.9}\right) = 7Q(4.35)$$

$$\therefore P_E(M) = 7Q(4.35) = 7\times\left(0.8807\times10^{-5}\right) = 6.16\times10^{-5}$$

The corresponding bit error probability is then

$$P_{b,1} = \frac{2^{k-1}}{2^k-1}P_E = \frac{2^{3-1}}{2^3-1}\times\left(6.16\times10^{-5}\right) = \underline{3.52\times10^{-5}}$$

Consider now a coherent 8-ary PSK scheme with E_b/N_0 = 13 dB = 20. The symbol error probability is

$$\frac{E_s}{N_0} = k\frac{E_b}{N_0} = 3\times20 = 60$$

so that

$$P_E(M) = 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right] = 2Q\left[\sqrt{2\times 60}\times\sin\left(\frac{\pi}{8}\right)\right] = 2Q(4.19)$$

$$\therefore P_E(M) = 2Q(4.19) = 2\times\left(0.2157\times 10^{-4}\right) = 4.31\times 10^{-5}$$

The corresponding bit error probability is then

$$P_{b,2} \approx \frac{P_E}{k} = \frac{4.31\times 10^{-5}}{3} = \underline{1.44\times 10^{-5}}$$

Since $P_{b,2} < P_{b,1}$, use of coherent 8-ary PSK with E_b/N_0 = 13 dB is a better choice than use of coherent 8-ary orthogonal FSK with E_b/N_0 = 8 dB.

P.12 ➡ *Solution*

Problem 12.1: Appealing to the definition of correlation coefficient for two signals $s_1(t)$ and $s_2(t)$, we write

$$\rho = \frac{\int_0^{T_b} s_0(t)s_1(t)\,dt}{\left[\int_0^{T_b} s_0^2(t)\,dt\right]^{1/2}\left[\int_0^{T_b} s_1^2(t)\,dt\right]^{1/2}}$$

$$\therefore \rho = \frac{A_c^2\int_0^{T_b}\cos\left[2\pi\left(f_c+\frac{1}{2}\Delta f\right)t\right]\cos\left[2\pi\left(f_c-\frac{1}{2}\Delta f\right)t\right]dt}{\left(\frac{1}{2}A_c^2T_b\right)^{1/2}\times\left(\frac{1}{2}A_c^2T_b\right)^{1/2}}$$

$$\therefore \rho = \frac{1}{T_b}\int_0^{T_b}\cos\left[2\pi\left(f_c+\frac{1}{2}\Delta f\right)t\right]\cos\left[2\pi\left(f_c-\frac{1}{2}\Delta f\right)t\right]dt$$

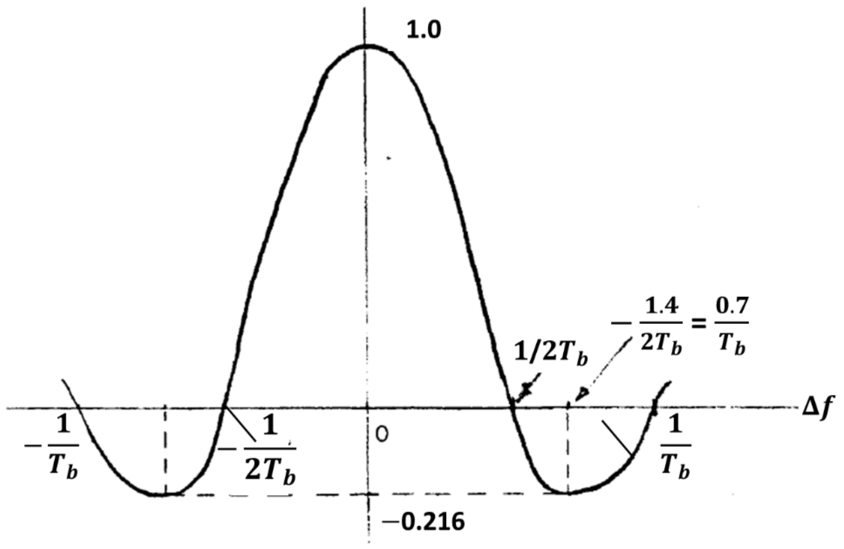
$$\therefore \rho = \frac{1}{T_b}\int_0^{T_b}\left[\cos(2\pi\Delta f t)+\cos(4\pi f_c t)\right]dt$$

$$\therefore \rho = \frac{1}{2\pi T_b}\left[\frac{\sin(2\pi\Delta f T_b)}{\Delta f}+\frac{\sin(4\pi f_c T_b)}{2f_c}\right]$$

Since $f_c \gg \Delta f$, we may ignore the term in red and obtain

$$\rho \approx \frac{1}{2\pi T_b}\times\frac{\sin(2\pi\Delta f T_b)}{\Delta f} = \text{sinc}(2\Delta f T_b)$$

Problem 12.2: The dependence of ρ on Δf is as shown below.



Signals $s_0(t)$ and $s_1(t)$ are orthogonal when $\rho = 0$. With reference to the graph above, we see that the minimum value of Δf for which the two signals are orthogonal is $1/2T_b$.

Problem 12.3: The average probability of error is given by

$$P_e = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{E_b(1-\rho)}{2N_0}}\right]$$

Referring to the graph in part 2, we see that the most negative value of ρ is −0.216, which occurs at Δf = 0.7/T_b. The minimum value of P_e is then

$$P_e = \frac{1}{2}\text{erfc}\left\{\sqrt{\frac{E_b\left[1-(0.216)\right]}{2N_0}}\right\} = \boxed{\frac{1}{2}\text{erfc}\left(\sqrt{\frac{0.608E_b}{N_0}}\right)}$$

Problem 12.4: For a coherent binary PSK system, the average probability of error is

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Therefore, the E_b/N_0 of this coherent binary FSK system must be increased by a factor of 1/0.608 = 1.64, or 2.15 dB, in order to realize the same average probability of error as a coherent binary PSK system.

P.13 ➡ *Solution*

The transmitted binary PSK signal is defined by

$$s(t) = \begin{cases} \sqrt{E_b}\phi(t), & 0 \le t \le T_b, \text{symbol } 1 \\ -\sqrt{E_b}\phi(t), & 0 \le t \le T_b, \text{symbol } 0 \end{cases}$$

where the basis function $\phi(t)$ is defined by

$$\phi(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$$

The locally generated basis function in the receiver is

$$\phi_{\text{rec}}(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t + \varphi)$$

$$\therefore \phi_{\text{rec}}(t) = \sqrt{\frac{2}{T_b}}\left[\cos(2\pi f_c t)\cos(\varphi) - \sin(2\pi f_c t)\sin(\varphi)\right]$$

where φ is phase error. The correlator output is given by

$$y = \int_0^{T_b} x(t)\varphi_{\text{rec}}(t)\,dt$$

where

$$x(t) = s_k(t) + w(t) \;\; ; \;\; k = 1,2$$

Assuming that f_c is an integer multiple of $1/T_b$, and recognizing that $\sin(2\pi f_c t)$ is orthogonal to $\cos(2\pi f_c t)$ over the interval $0 \le t \le T_b$, we get

$$y = \pm\sqrt{E_b}\cos(\varphi) + W$$

where the plus sign corresponds to symbol 1 and the minus sign corresponds to symbol 0, and W is a zero-mean Gaussian variable of variance $N_0/2$. Accordingly, the average probability of error of the binary PSK system with phase error φ is given by

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b\cos\varphi}{N_0}}\right)$$

When $\varphi = 0$, the formula above reduces to that for the standard PSK system equipped with perfect phase recovery. At the other extreme, when $\varphi = \pm 90^\circ$, P_e attains its worst value of unity.

P.14 ➡ *Solution*

For coherent MSK, the probability of error is

$$P_e = \text{erfc}\left(\sqrt{E_b/N_0}\right) \quad (\text{I})$$

while for noncoherent MSK (i.e., noncoherent binary FSK),

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) \quad (\text{II})$$

To maintain a probability $P_e = 10^{-5}$ for coherent MSK, we let u denote the ratio we aim for and appeal to Mathematica's *FindRoot* command,

```
In[133]:= FindRoot[10^-5 - Erfc[√u], {u, 1.}]

Out[133]= {u → 9.75571}
```

That is, u = 9.76, which amounts to 9.89 dB. Similarly, we set (II) to 10^{-5} and apply the same command a second time, giving

```
In[136]:= FindRoot[10^-5 - 0.5 * Exp[-u / 2], {u, 1.}]

Out[136]= {u → 21.6396}
```

That is, u = 21.6, which becomes 13.3 dB; there is an increase of 3.41 decibels from one situation to the next.

P.15 ➡ *Solution*

Problem 15.1: The output of a $\pi/4$-shifted QPSK modulator may be expressed in terms of its in-phase and quadrature components as

$$s(t)=\sqrt{\frac{2E}{T}}\cos(i\pi/4)\cos(2\pi f_c t)-\sqrt{\frac{2E}{T}}\sin(i\pi/4)\sin(2\pi f_c t)\ ;\ i=0,1,2,\ldots,7$$

The different values of integer i correspond to the eight possible phase states in which the modulator can reside. But, unlike the 8-PSK modulator, the phase states of the $\pi/4$-shifted QPSK modulator are divided into two QPSK groups that are shifted by $\pi/4$ relative to each other. Accordingly,

$$s_I(t)=\sqrt{\frac{2E}{T}}\cos(i\pi/4)$$

$$s_Q(t)=\sqrt{\frac{2E}{T}}\sin(i\pi/4)$$

The orthonormal-basis functions for $\pi/4$-shifted QPSK may be defined as

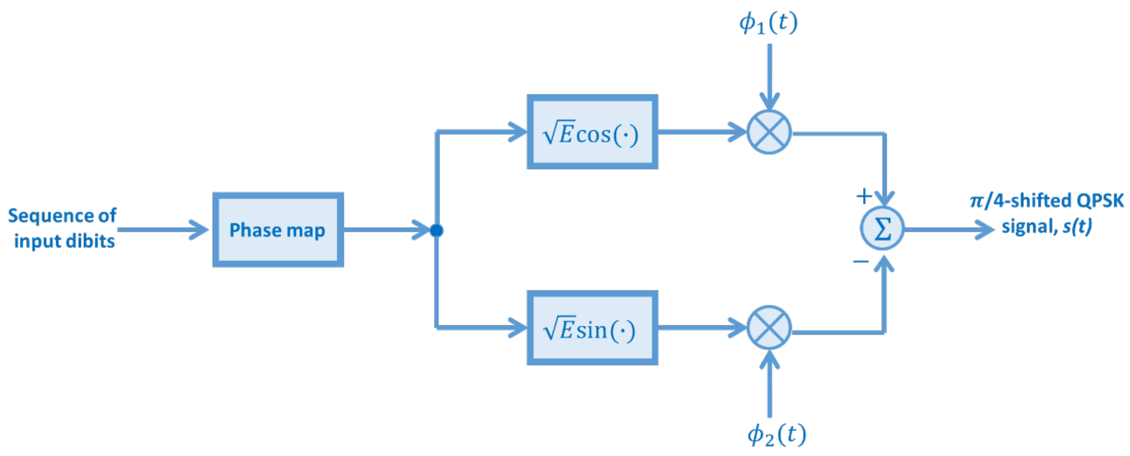
$$\phi_I(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$$

$$\phi_2(t)=\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

Then the $\pi/4$-shifted QPSK signal is defined in terms of these two basis functions as

$$s(t)=\sqrt{E}\cos(i\pi/4)\phi_1(t)-\sqrt{E}\sin(i\pi/4)\phi_2(t)$$

On the basis of this representation, Haykin proposes the following scheme for generating $\pi/4$-shifted QPSK signals:



Problem 15.2: A $\pi/4$-shifted DQPSK signal can be expressed as follows,

$$s(t)=\sqrt{\frac{2E}{T}}\cos(\phi_{k-1}+\Delta\phi_k)\cos(2\pi f_c t)-\sqrt{\frac{2E}{T}}\sin(\phi_{k-1}+\Delta\phi_k)\sin(2\pi f_c t)$$

$$\therefore s(t)=\sqrt{\frac{2E}{T}}\cos(2\pi f_c t+\phi_{k-1}+\Delta\phi_k)$$

where $\phi_{k-1} + \Delta\phi_k = \phi_k$, in which ϕ_{k-1} is the absolute angle of symbol k – 1, and $\Delta\phi_k$ is the differentially encoded phase change. In the demodulation process, the change in phase ϕ_k occurring over one symbol interval needs to be determined. If we demodulate the π/4-shifted DQPSK signal using an FM discriminator, the output of the discriminator is given by

$$v_{\text{out}}(t) = K\frac{d(2\pi f_c t + \phi_k)}{dt} \rightarrow v_{\text{out}}(t) = K\left(2\pi f_c + \frac{d\phi_k}{dt}\right)$$

$$\therefore v_{\text{out}}(t) = K(2\pi f_c + \Delta\phi_k)$$

where K is a constant. In a balanced FM discriminator, the DC offset $2\pi f_c K$ will not appear at the output. Hence, the output of the FM discriminator is $K\Delta\phi_k$.

P.16 ➡ *Solution*

Problem 16.1: First note that the bit error probability for M-ary PAM can be estimated as

$$P_b \approx \frac{1}{\log_2 M} P_{eM}$$

where, substituting P_{eM} with the corresponding expression for M-ary PAM,

$$P_b = 2\left(\frac{M-1}{M\log_2 M}\right)Q\left[\sqrt{\frac{6\log_2 M}{M^2-1}\left(\frac{E_b}{N_0}\right)}\right] \quad \text{(I)}$$

First, with M = 2,

$$P_b = 2\left(\frac{2-1}{2\log_2 2}\right)Q\left[\sqrt{\frac{6\log_2 2}{2^2-1}\left(\frac{E_b}{N_0}\right)}\right] = Q\left[\sqrt{2\left(\frac{E_b}{N_0}\right)}\right] < 10^{-6}$$

Referring to Table 8.2 in Lathi and Ding (2009), the Q function equals $0.968\times10^{-6} \approx 10^{-6}$ when its argument is 4.76. Accordingly,

$$\sqrt{2\left(\frac{E_b}{N_0}\right)} = 4.76 \rightarrow \frac{E_b}{N_0} = \frac{4.76^2}{2} = 11.3$$

The transmitted power is determined next,

$$S_T = R_b E_b = R_b\left(\frac{E_b}{N_0}\right)N_0 = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right]$$

$$\therefore S_T = 400{,}000\times11.3\times\left(2\times10^{-8}\right) = \boxed{90.4\,\text{mW}}$$

In M-ary PAM, each pulse carries information from $\log_2 M$ bits. It follows that the transmission bandwidth for M = 2 must be

$$B = \frac{R_b}{\log_2 M} = \frac{400{,}000}{\log_2 2} = \boxed{400\,\text{kHz}}$$

Consider now a PAM scheme with M = 16. Equation (I) is restated as

$$P_b = 2\left(\frac{16-1}{16\log_2 16}\right)Q\left[\sqrt{\frac{6\log_2 16}{16^2-1}\left(\frac{E_b}{N_0}\right)}\right] = 0.469Q\left[\sqrt{\frac{6\times4}{255}\left(\frac{E_b}{N_0}\right)}\right] < 10^{-6}$$

$$\therefore P_b = 0.469Q\left(0.307\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(0.307\sqrt{\frac{E_b}{N_0}}\right) < 2.13\times10^{-6}$$

Referring to the Q function table, we see that Q equals 0.211×10^{-5} if the argument is set to 4.60. Therefore,

$$0.307\sqrt{\frac{E_b}{N_0}} = 4.60 \rightarrow \frac{E_b}{N_0} = \frac{4.60^2}{0.307^2} = 225$$

leading to a power S_T such that

$$S_T = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right] = 400,000\times225\times\left(2\times10^{-8}\right) = \boxed{1.8\,\text{W}}$$

The associated bandwidth for M = 16 is

$$B = \frac{R_b}{\log_2 M} = \frac{400,000}{\log_2 16} = \boxed{100\,\text{kHz}}$$

Consider next a PAM scheme with M = 32. With recourse to equation (I), we have

$$P_b = 2\left(\frac{32-1}{32\log_2 32}\right)Q\left[\sqrt{\frac{6\log_2 32}{32^2-1}\left(\frac{E_b}{N_0}\right)}\right] = 0.388Q\left[\sqrt{\frac{6\times5}{1023}\left(\frac{E_b}{N_0}\right)}\right]$$

$$\therefore P_b = 0.388Q\left(0.171\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(0.171\sqrt{\frac{E_b}{N_0}}\right) < 2.58\times10^{-6}$$

Entering the rightmost value into the Q function table, we glean that the Q function equals 0.2558×10^{-5}, which is close to 2.58×10^{-6}, when the argument equals 4.56. Thus,

$$0.171\sqrt{\frac{E_b}{N_0}} = 4.56 \rightarrow \frac{E_b}{N_0} = \frac{4.56^2}{0.171^2} = 711$$

so that

$$S_T = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right] = 400,000\times711\times\left(2\times10^{-8}\right) = \boxed{2.84\,\text{W}}$$

The associated bandwidth for M = 32 is

$$B = \frac{R_b}{\log_2 M} = \frac{400,000}{\log_2 32} = \boxed{80\,\text{kHz}}$$

Problem 16.2: As before, the bit error probability is estimated as

$$P_b \approx \frac{1}{\log_2 M}P_{eM}$$

which, for M-ary PSK, becomes

$$P_b = \frac{2}{\log_2 M}Q\left[\sqrt{\frac{2\pi^2\log_2 M}{M^2}\left(\frac{E_b}{N_0}\right)}\right] \quad \text{(II)}$$

To begin, M = 2 and

$$P_b = \frac{2}{\log_2 2}Q\left[\sqrt{\frac{2\pi^2\times\log_2 2}{2^2}\left(\frac{E_b}{N_0}\right)}\right] = 2Q\left(2.22\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(2.22\sqrt{\frac{E_b}{N_0}}\right) < 0.5\times10^{-6}$$

Referring to Table 8.2 in Lathi and Ding (2009), the Q function attains a value close to 0.5×10^{-6} when its argument is 4.89. Accordingly,

$$2.22\sqrt{\frac{E_b}{N_0}} = 4.89 \rightarrow \frac{E_b}{N_0} = \frac{4.89^2}{2.22^2} = 4.85$$

The transmitted power is computed next,

$$S_T = R_bE_b = R_b\left(\frac{E_b}{N_0}\right)N_0 = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right]$$

$$\therefore S_T = 400,000\times4.85\times\left(2\times10^{-8}\right) = \boxed{38.8\,\text{mW}}$$

The bandwidth required is, in turn,

$$B = \frac{2R_b}{\log_2 M} = \frac{2\times 400,000}{\log_2 2} = \boxed{800\,\text{kHz}}$$

Next, with $M = 16$,

$$P_b = \frac{2}{\log_2 16} Q\left[\sqrt{\frac{2\pi^2 \times \log_2 16}{16^2}\left(\frac{E_b}{N_0}\right)}\right] = 0.5Q\left(0.555\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(0.555\sqrt{\frac{E_b}{N_0}}\right) < 2.0\times 10^{-6}$$

The Q function takes on a value close to 2.0×10^{-6}, or 0.2×10^{-5}, if the argument is set to 4.61. Therefore,

$$0.555\sqrt{\frac{E_b}{N_0}} = 4.61 \rightarrow \frac{E_b}{N_0} = \frac{4.61^2}{0.555^2} = 69.0$$

so that

$$S_T = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right] = 400,000\times 69.0\times\left(2\times10^{-8}\right) = \boxed{0.552\,\text{W}}$$

The bandwidth required is

$$B = \frac{2R_b}{\log_2 M} = \frac{2\times 400,000}{\log_2 16} = \boxed{200\,\text{kHz}}$$

Finally, with $M = 32$,

$$P_b = \frac{2}{\log_2 32} Q\left[\sqrt{\frac{2\pi^2 \times \log_2 32}{32^2}\left(\frac{E_b}{N_0}\right)}\right] = 0.4Q\left(0.310\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(0.310\sqrt{\frac{E_b}{N_0}}\right) < 2.5\times 10^{-6}$$

The Q function takes on a value close to 2.5×10^{-6}, or 0.25×10^{-5}, if the argument is set to 4.57. Therefore,

$$0.555\sqrt{\frac{E_b}{N_0}} = 4.61 \rightarrow \frac{E_b}{N_0} = \frac{4.61^2}{0.310^2} = 221$$

so that

$$S_T = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right] = 400,000\times 221\times\left(2\times10^{-8}\right) = \boxed{1.77\,\text{W}}$$

The bandwidth required is

$$B = \frac{2R_b}{\log_2 M} = \frac{2\times 400,000}{\log_2 32} = \boxed{160\,\text{kHz}}$$

As before, the bit error probability is estimated as

$$P_b \approx \frac{1}{\log_2 M} P_{eM}$$

which, for M-ary PSK, becomes

$$P_b = \frac{2}{\log_2 M} Q\left[\sqrt{\frac{2\pi^2 \log_2 M}{M^2}\left(\frac{E_b}{N_0}\right)}\right] \quad \text{(II)}$$

To begin, $M = 2$ and

$$P_b = \frac{2}{\log_2 2} Q\left[\sqrt{\frac{2\pi^2 \times \log_2 2}{2^2}\left(\frac{E_b}{N_0}\right)}\right] = 2Q\left(2.22\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(2.22\sqrt{\frac{E_b}{N_0}}\right) < 0.5\times10^{-6}$$

Referring to Table 8.2 in Lathi and Ding (2009), the Q function attains a value close to 0.5×10^{-6} when its argument is 4.89. Accordingly,

$$2.22\sqrt{\frac{E_b}{N_0}} = 4.89 \rightarrow \frac{E_b}{N_0} = \frac{4.89^2}{2.22^2} = 4.85$$

The transmitted power is computed next,

$$S_T = R_b E_b = R_b\left(\frac{E_b}{N_0}\right)N_0 = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right]$$

$$\therefore S_T = 400,000\times4.85\times\left(2\times10^{-8}\right) = \boxed{38.8\,\text{mW}}$$

The bandwidth required is, in turn,

$$B = \frac{2R_b}{\log_2 M} = \frac{2\times400,000}{\log_2 2} = \boxed{800\,\text{kHz}}$$

Next, with $M = 16$,

$$P_b = \frac{2}{\log_2 16}Q\left[\sqrt{\frac{2\pi^2\times\log_2 16}{16^2}\left(\frac{E_b}{N_0}\right)}\right] = 0.5Q\left(0.555\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(0.555\sqrt{\frac{E_b}{N_0}}\right) < 2.0\times10^{-6}$$

The Q function takes on a value close to 2.0×10^{-6}, or 0.2×10^{-5}, if the argument is set to 4.61. Therefore,

$$0.555\sqrt{\frac{E_b}{N_0}} = 4.61 \rightarrow \frac{E_b}{N_0} = \frac{4.61^2}{0.555^2} = 69.0$$

so that

$$S_T = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right] = 400,000\times69.0\times\left(2\times10^{-8}\right) = \boxed{0.552\,\text{W}}$$

The bandwidth required is

$$B = \frac{2R_b}{\log_2 M} = \frac{2\times400,000}{\log_2 16} = \boxed{200\,\text{kHz}}$$

Finally, with $M = 32$,

$$P_b = \frac{2}{\log_2 32}Q\left[\sqrt{\frac{2\pi^2\times\log_2 32}{32^2}\left(\frac{E_b}{N_0}\right)}\right] = 0.4Q\left(0.310\sqrt{\frac{E_b}{N_0}}\right) < 10^{-6}$$

$$\therefore P_b = Q\left(0.310\sqrt{\frac{E_b}{N_0}}\right) < 2.5\times10^{-6}$$

The Q function takes on a value close to 2.5×10^{-6}, or 0.25×10^{-5}, if the argument is set to 4.57. Therefore,

$$0.555\sqrt{\frac{E_b}{N_0}} = 4.61 \rightarrow \frac{E_b}{N_0} = \frac{4.61^2}{0.310^2} = 221$$

so that

$$S_T = R_b\left(\frac{E_b}{N_0}\right)\left[2S_n(\omega)\right] = 400,000\times221\times\left(2\times10^{-8}\right) = \boxed{1.77\,\text{W}}$$

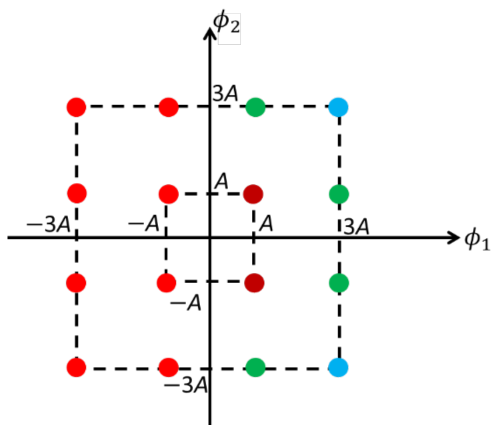
The bandwidth required is

$$B = \frac{2R_b}{\log_2 M} = \frac{2 \times 400,000}{\log_2 32} = \boxed{160\,\text{kHz}}$$

P.17 ➡ Solution

Problem 17.1: Since $d_{min} = 2A$, from the union bound we have $P_e \leq 15Q(\sqrt{d^2_{\min}/2N_0}) = 15Q(\sqrt{2A^2/N_0})$.

Problem 17.2: Referring to the illustration to the side, we see that three levels of energy are present, namely $E_1 = A^2 + A^2 = 2A^2$, $E_2 = A^2 + 9A^2 = 10A^2$, and $E_3 = 9A^2 + 9A^2 = 18A^2$. The average energy is



$$E_{\text{avg}} = \frac{1}{4}E_1 + \frac{1}{2}E_2 + \frac{1}{4}E_3$$

$$\therefore E_{\text{avg}} = \frac{1}{4} \times 2A^2 + \frac{1}{2} \times 10A^2 + \frac{1}{4} \times 18A^2$$

$$\therefore E_{\text{avg}} = 10A^2$$

The SNR per bit is then

$$E_{b,\text{avg}} = \frac{E_{\text{avg}}}{\log_2 16} = \boxed{\frac{5A^2}{2}}$$

Problem 17.3: Simply replace A^2 with $2E_{b,avg}/5$, giving

$$P_e \leq 15Q\left(\sqrt{\frac{2A^2}{N_0}}\right) = 15Q\left[\sqrt{\frac{2 \times \left(\frac{2E_{b,\text{avg}}}{5}\right)}{N_0}}\right] = \boxed{15Q\left(\sqrt{\frac{4E_{b,\text{avg}}}{5N_0}}\right)}$$

Problem 17.4: For a 16-level PAL system, the probability of error is

$$P_e \approx 2Q\left(\sqrt{\frac{6\log_2 M}{M^2-1}\frac{E_{b,\text{avg}}}{N_0}}\right) = 2Q\left(\sqrt{\frac{6\log_2 16}{16^2-1}\frac{E_{b,\text{avg}}}{N_0}}\right)$$

$$\therefore P_e = 2Q\left(\sqrt{\frac{24}{255}\frac{E_{b,\text{avg}}}{N_0}}\right)$$

The ratio of power efficiency of the two systems is then (4/5)/(24/255) = 8.5, which amounts to 9.29 dB.

P.18 ➡ Solution

Problem 18.1: Set I is a four-level PAM; Set II is a orthogonal set; Set III is a biorthogonal set.

Problem 18.2: For Set I, the transmitted waveforms have energy equal to $A^2/2$ in the case of $u_1(t)$ and $u_3(t)$ or $9A^2/2$ in the case of $u_2(t)$ and $u_4(t)$. The corresponding average energy is

$$E_1 = \frac{1}{4} \times \left[2 \times \left(\frac{A^2}{2}\right) + 2 \times \left(\frac{9A^2}{2}\right)\right] = \boxed{\frac{5A^2}{2}}$$

All waveforms in the second and third sets have the same energy $A^2/2$; therefore,

$$E_2 = E_3 = \boxed{\frac{A^2}{2}}$$

Problem 18.3: The average probability of a symbol error for M-PAM is, in general,

$$P_{M-PAM} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6E_{\text{avg}}}{(M^2-1)N_0}}\right)$$

so that, with $E_{avg} = 5A^2/2$ and $M = 4$,

$$P_{M-PAM} = \frac{2(4-1)}{4} Q\left(\sqrt{\frac{6\times 5A^2/2}{(4^2-1)N_0}}\right) = \boxed{\frac{3}{2}Q\left(\sqrt{\frac{A^2}{N_0}}\right)}$$

Problem 18.4: For coherent detection, a union bound can be given by

$$P_{4,\text{orth}} < (M-1)Q\left(\sqrt{E_s/N_0}\right) = \boxed{3Q\left(\sqrt{\frac{A^2}{2N_0}}\right)}$$

For non-coherent detection, in turn,

$$P_{4,\text{orth,nc}} \le (M-1)P_{2,\text{nc}} = (4-1)\times\frac{1}{2}e^{-E_s/(2N_0)} = \boxed{\frac{3}{2}e^{-A^2/(4N_0)}}$$

Problem 18.5: It is not possible to use non-coherent detection for a biorthogonal signal set; for example, we cannot distinguish between signals $u_1(t)$ and $u_3(t)$ without phase knowledge.

Problem 18.6: The bit rate to bandwidth ratio for M-PAM is given by $2\log_2 M$, which in the case of Set I becomes

$$\left(\frac{R}{W}\right)_{\text{Set 1}} = 2\log_2 M = 2\log_2 4 = 4$$

For Set II, note that the symbol interval used is 4 times larger than the one used in Set I, resulting in a bit rate 4 times smaller,

$$\left(\frac{R}{W}\right)_{\text{Set 2}} = \frac{2\log_2 M}{M} = \frac{2\log_2 4}{4} = 1$$

Finally, the biorthogonal Set III has double the bandwidth efficiency of the orthogonal set,

$$\left(\frac{R}{W}\right)_{\text{Set 3}} = 2$$

Hence, set I is the most bandwidth efficient (at the expense of larger average power), but set III also satisfies the requirement posed in the problem statement.

P.19 ➡ Solution

Problem 19.1: The signals have equal energy E such that

$$E = \int_0^T s^2(t)\,dt = \int_0^{T/2} 2^2\,dt + \int_{T/2}^T 1^2\,dt = \frac{5T}{2}$$

The bit energy is

$$E_b = \frac{E}{\underbrace{\log_2 M}_{=1}} = \frac{5T}{2}$$

and the bit rate is

$$R = \frac{\log_2 M}{T} = \frac{1}{T}$$

so that $T = 1/R$. It follows that $E_b = 5/(2R)$ and

$$\boxed{\frac{E_b}{N_0} = \frac{5}{2RN_0}}$$

Problem 19.2: Since the system is binary equiprobable, $P_e = Q(\sqrt{d^2/2N_0})$, where

$$d^2 = \int_0^T \left[s_1(t) - s_2(t)\right]^2 dt = \int_0^{T/2} (2-1)^2\,dt + \int_{T/2}^T (1-2)^2\,dt = T$$

so that

$$P_e = Q\left(\sqrt{\frac{\overbrace{d^2}^{=T}}{2N_0}}\right) = Q\left(\sqrt{\frac{T}{2N_0}}\right)$$

Lastly, with $T = 1/R$,

$$P_e = Q\left(\sqrt{\frac{T}{2N_0}}\right) = \boxed{Q\left(\sqrt{\frac{1}{2RN_0}}\right)}$$

Problem 19.3: Since $E_b = 5/(2R)$, we can restate the result at the end of the previous part as

$$P_e = Q\left(\sqrt{\frac{1}{\underbrace{2R}_{=5/E_b}\ N_0}}\right) = Q\left(\sqrt{\frac{E_b}{5N_0}}\right)$$

or

$$P_e = Q\left(\sqrt{\frac{1}{10}\times\frac{2E_b}{N_0}}\right)$$

Now, noting that for binary antipodal signaling,

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

it is easy to see, from the two foregoing equations, that the system at hand underperforms a binary antipodal scheme by a factor of 10.

Problem 19.4: The updated bit rate is

$$R_{\text{new}} = \frac{\log_2 4}{T} = \frac{2}{T} = \boxed{2R}$$

Problem 19.5: We need to find $d_{\min}^2$ in the new four-signal constellation. There is no obvious candidate for $d_{\min}^2$, so we must consider each distance individually. For starters, $d_{34}^2 = d_{12}^2 = T$. Also, $d_{13}^2 = d_{42}^2 = \int_0^T 4s_1^2(t)dt = 4E = 4(5T/2) = 10T$. The two remaining distances are d_{14}^2 and d_{23}^2, which are determined as

$$d_{14}^2 = d_{23}^2 = \int_0^T \left[s_1(t) + s_2(t)\right]^2 dt = \int_0^T 9dt = 9T$$

Clearly, $d_{\min}^2 = d_{12}^2 = T = 1/R$, leading to an union bound such that

$$P_e \le (M-1)e^{-d_{\min}^2/4N_0} = (4-1)e^{-T/4N_0}$$

$$\therefore \boxed{P_e \le 3e^{-1/4RN_0}}$$

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