Montogue

Quiz FM107
Dimensional Analysis and Similitude
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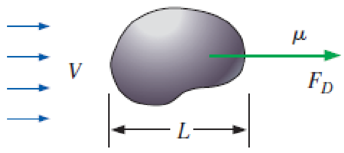
PROBLEMS

Problem 1 (Çengel & Cimbala, 2014, w/ permission)

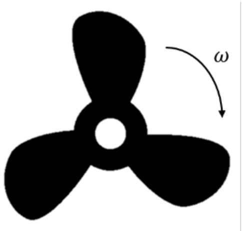
When small aerosol particles or microorganisms move through air or water, the Reynolds number is very small (*Re* << 1). Such flows are called *creeping flows*. The aerodynamic drag on an object is creeping flow is a function only of its speed V, some characteristic length scale L of the object, and fluid viscosity μ. Use dimensional analysis to generate a relationship for F_D as a function of the independent variables.



____ = *f* (__________)

Problem 2A (Çengel & Cimbala, 2014, w/ permission)

A propeller of diameter D rotates at angular velocity ω in a liquid of density ρ and viscosity μ. The required torque T is determined to be a function of D, ω, ρ, and μ. Using dimensional analysis, generate a dimensionless relationship. Identify any established nondimensional parameters that appear in the result.



____ = *f* (__________)

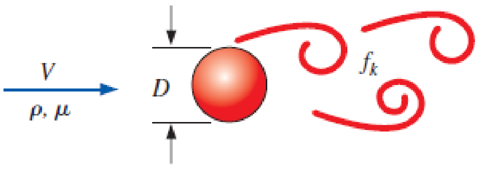
Problem 2B

Repeat the previous problem for the case in which the propeller operates in a compressible gas instead of a liquid.

____ = *f* (__________)

Problem 3 (Çengel & Cimbala, 2014, w/ permission)

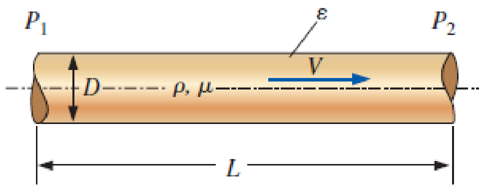
A "periodic Kármán vortex street" is formed when a uniform stream flows over a circular cylinder. Use the method of repeating variables to generate a dimensionless relationship for the Kármán vortex shedding frequency f_k as a function of free-stream speed V, fluid density ρ, fluid viscosity μ, and cylinder diameter D.



____ = f (__________)

Problem 4 (Çengel & Cimbala, 2014, w/ permission)

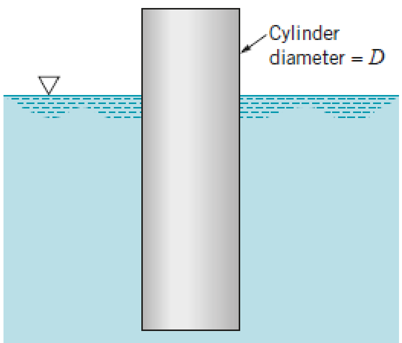
An incompressible fluid of density ρ and viscosity μ flows at average speed V through a long, horizontal section of round pipe of length L, inner diameter D, and inner wall roughness height ε. The pipe is long enough that the flow is fully developed, meaning that the velocity profile does not change down the pipe. Pressure decreases (linearly) down the pipe in order to "push" the fluid through the pipe to overcome friction. Using the method of repeating variables, develop a nondimensional relationship between pressure drop $\Delta p = p_1 - p_2$ and the other parameters in the problem. Modify your Π groups as necessary to achieve established nondimensional parameters, and name them.



____ = f (__________)

Problem 5 (Munson et al., 2009, w/ permission)

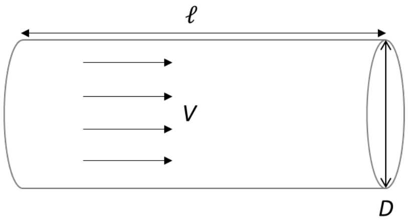
A cylinder with a diameter D floats upright in a liquid as shown. When the cylinder is displaced slightly along its vertical axis it will oscillate about its equilibrium position with a frequency, ω. Assume that the frequency is a function of the diameter D, the mass of the cylinder m, and the specific weight γ of the liquid. Determine, with the aid of dimensional analysis, how the frequency is related to these variables. If the mass of the cylinder were increased, would the frequency increase or decrease?



A) The frequency would increase.
B) The frequency would remain the same.
C) The frequency would decrease.
D) A relationship between frequency and cylinder mass cannot be established with dimensional analysis.

Problem 6 (Munson et al., 2009, w/ permission)

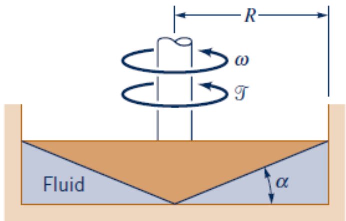
The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to infer how the pressure drop varies with pipe diameter.



A) $\Delta p \propto D$
B) $\Delta p \propto 1/D$
C) $\Delta p \propto 1/D^2$
D) $\Delta p \propto 1/D^3$

Problem 7 (Munson et al., 2009, w/ permission)

A cone-and-plate viscometer consists of a cone with a very small angle α that rotates above a flat surface as shown in the figure below. The torque T required to rotate the cone at an angular velocity ω is a function of the radius R, the cone angle α, and the fluid viscosity μ, in addition to ω. With the aid of dimensional analysis, determine how the torque will change if both the viscosity and angular velocity are doubled.

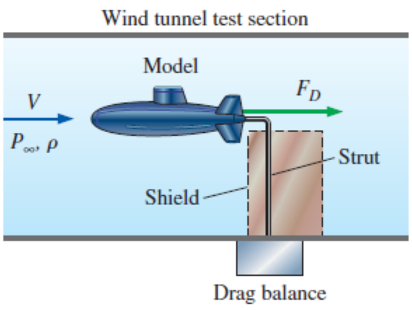


A) The torque will be reduced by a factor of 1/2.
B) The torque will be increased by a factor of 2.
C) The torque will be increased by a factor of 4.
D) The torque will be increased by a factor of 8.

Problem 8A (Çengel & Cimbala, 2014, w/ permission)

A student team is to design a human-powered submarine for a design competition. The overall length of the prototype submarine is 4.85 m, and its student designers hope that it can travel fully submerged through water at 0.44 m/s. The water is freshwater (a lake) at T = 15°C. The design team builds a one-fifth scale model to test in their university's wind tunnel. A shield surrounds the drag balance strut so that the aerodynamic drag of the strut itself does not influence the measured drag. The air in the wind tunnel is at 25°C and at one standard atmosphere pressure. At what air speed do they need to run the wind tunnel in order to achieve similarity? The data for the fluids involved are provided below.

	Water at 15°C	Air at 25°C
Density (ρ, kg/m^3)	999.1	1.184
Viscosity (μ, Pa·s)	1.138×10^{-3}	1.849×10^{-5}



A) V_m = 21.3 m/s
B) V_m = 30.2 m/s
C) V_m = 40.6 m/s
D) V_m = 51.5 m/s

● Problem 8B (Çengel & Cimbala, 2014, w/ permission)

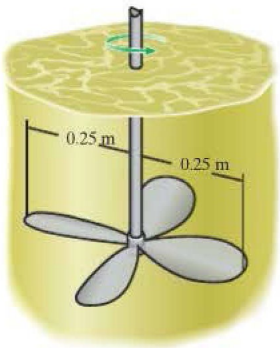
The students performing the tests in the model submarine are careful to run the wind tunnel at conditions that ensure similarity with the prototype submarine. Their measured drag force is 5.70 N. Estimate the drag force on the prototype submarine at the conditions given in the previous problem.

A) $F_{D,p}$ = 15.4 N
B) $F_{D,p}$ = 25.5 N
C) $F_{D,p}$ = 34.6 N
D) $F_{D,p}$ = 44.7 N

Problem 9 (Çengel & Cimbala, 2014, w/ permission)

The optimum performance of mixing blades 0.5 m in diameter is to be tested using a model one-fourth the size of the prototype. If the test of the model in water reveals the optimum speed to be 8 rad/s, determine the optimum angular speed of the prototype when it is used to mix ethyl alcohol. The data for the fluids involved are provided below.

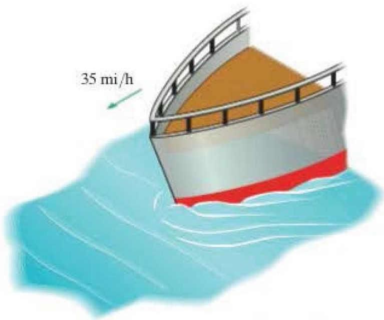
	Water at 20°C	Ethyl alcohol at 20°C
Kinematic viscosity (ν, m²/s)	1.00×10^{-6}	1.51×10^{-6}



A) ω_p = 0.175 rad/s
B) ω_p = 0.365 rad/s
C) ω_p = 0.545 rad/s
D) ω_p = 0.755 rad/s

Problem 10 (Hibbeler, 2015, w/ permission)

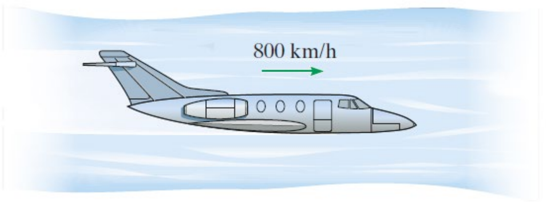
The resistance of waves on a 250-ft-long ship is tested in a channel using a model that is 15 ft long. If the ship travels at 35 mi/h, what should be the speed of the model to resist the waves?



A) V_m = 4.79 mi/h
B) V_m = 6.68 mi/h
C) V_m = 8.57 mi/h
D) V_m = 10.63 mi/h

Problem 11 (Hibbeler, 2015, w/ permission)

The flow around the airplane flying at an altitude of 10 km is to be studied using a wind tunnel and a model that is built to a 1/15 scale. If the plane has an air speed of 800 km/h, what should the speed of the air be inside the wind tunnel? Is the speed obtained reasonable? Explain.



	Air at 15°C	Air at 10 km altitude
Kinematic viscosity (ν, m²/s)	14.6×10^{-6}	35.25×10^{-6}

ADDITIONAL INFORMATION

Table 1 Common dimensionless numbers encountered in fluid mechanics and heat transfer

Parameter	Definition	Ratio of Significance
Drag coefficient	$C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 S}$	$\dfrac{\text{Drag force}}{\text{Dynamic Pressure} \times \text{Area}}$
Euler number	$\text{Eu} = \dfrac{\Delta p}{\rho V^2}$ (sometimes $\frac{1}{2}\rho V^2$)	$\dfrac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Froude number	$\text{Fr} = \dfrac{V}{\sqrt{gL}}$	$\dfrac{\text{Inertial force}}{\text{Gravitational force}}$
Mach number	$M = \dfrac{V}{c}$ (sometimes *Ma*)	$\dfrac{\text{Flow speed}}{\text{Speed of sound}}$
Reynolds number	$\text{Re} = \dfrac{\rho V L}{\mu} = \dfrac{VL}{\nu}$	$\dfrac{\text{Inertial force}}{\text{Viscous force}}$
Strouhal number	$\text{Sr} = \dfrac{fL}{V}$ (sometimes *S* or *St*)	$\dfrac{\text{Characteristic flow time}}{\text{Period of oscillation}}$
Torque coefficient	$C_T = \dfrac{T}{\rho \omega^2 D^5}$	$\dfrac{\text{Applied torque}}{\text{Reference torque}}$

SOLUTIONS

P.1 ● Solution

The primary dimensions of each variable involved are listed below.

F_D	$[F_D] = MLT^{-2}$
V	$[V] = LT^{-1}$
L	$[L] = L$
μ	$[\mu] = ML^{-1}T^{-1}$

The number of variables involved is n = 4. The number of primary dimensions represented in the problem is 3; hence, we take j = 3 as a first guess. The number of dimensionless parameters (Π's) shall be

$$k = n - j = 4 - 3 = 1$$

With j = 3, we are expected to choose three repeating variables. Since we cannot choose the dependent variable (F_D), we are left with V, L, and μ as obvious choices. We combine the repeating parameters into a product with the dependent variable F_D to generate the first (and only) dimensionless parameter Π_1; that is,

$$\Pi_1 = F_D V^a L^b \mu^c$$

Decomposing these variables to their primary dimensions, we obtain

$$M^0 L^0 T^0 = MLT^{-2} \times \left(LT^{-1}\right)^a \times L^b \times \left(ML^{-1}T^{-1}\right)^c$$

$$M^0 L^0 T^0 = M^{1+c} L^{1+a+b-c} T^{-2-a-c}$$

Equating the exponents in each side of the equation brings to the following system of linear equations,

$$\begin{cases} 1 + c = 0 \\ 1 + a + b - c = 0 \\ -2 - a - c = 0 \end{cases}$$

The solution to this system is a = -1, b = -1, and c = -1. We can then return to the equation for Π_1 and obtain the dimensionless parameter in question,

$$\Pi_1 = F_D V^{-1} L^{-1} \mu^{-1}$$

$$\therefore \Pi_1 = \frac{F_D}{\mu VL}$$

We now write the functional relationship between the dimensionless parameters. In the case at hand, there is only one Π, which is a function of nothing. This is possible only if the Π is constant. Thus, we state that

$$\Pi_1 = \frac{F_D}{\mu VL} = f\left(\text{nothing}\right) = k$$

where k is a constant. So,

$$\boxed{F_D = f(\text{nothing})}$$

or

$$F_D = k \times \mu VL$$

We have shown that, for creeping flow around an object, the aerodynamic drag force is simply a constant multiplied by μVL regardless of the shape of the object.

P.2 ● Solution

Part A: There are five parameters in the problem, n = 5. The functional relationship for torque is

$$T = f\left(\rho, \omega, D, \mu\right)$$

The primary dimensions of each variable are listed below.

T	$[T] = ML^2T^{-2}$
ρ	$[\rho] = ML^{-3}$
ω	$[\omega] = T^{-1}$
D	$[D] = L$
μ	$[\mu] = ML^{-1}T^{-1}$

We have 3 reference dimensions, so j = 3. The expected number of parameters is then

$$k = n - j = 5 - 3 = 2$$

The repeating variables shall be the fluid density ρ, the length scale D, and the angular velocity ω. To begin, we determine dependent parameter Π_1, which is found by combining the repeating variables with the torque T,

$$\Pi_1 = T\rho^a\omega^b D^c$$

Decomposing the variables to their primary dimensions, we obtain

$$M^0L^0T^0 = ML^2T^{-2}\times(ML^{-3})^a\times\left(T^{-1}\right)^b\times L^c$$

$$\therefore M^0L^0T^0 = ML^2T^{-2}\times M^aL^{-3a}\times T^{-b}\times L^c$$

$$\therefore M^0L^0T^0 = M^{1+a}L^{2-3a+c}T^{-2-b}$$

This leads to the following system of linear equations,

$$\begin{cases} 1+a=0 \\ 2-3a+c=0 \\ -2-b=0 \end{cases}$$

The solution to this system is a = -1, b = -2, and c = -5. The ensuing dimensionless parameter is

$$\Pi_1 = T\rho^{-1}\omega^{-2}D^{-5}$$

$$\therefore \Pi_1 = \frac{T}{\rho\omega^2D^5}$$

This is a kind of torque coefficient. Then, the second Π is obtained using viscosity and the three repeating variables,

$$\Pi_2 = \mu\rho^a\omega^b D^c$$

Decomposing the variables as usual, we have

$$M^0L^0T^0 = ML^{-1}T^{-1}\times\left(M^1L^{-3}\right)^a\times\left(T^{-1}\right)^b\times L^c$$

$$M^0L^0T^0 = M^{1+a}L^{-1-3a+c}T^{-1-b}$$

The ensuing system of equations is

$$\begin{cases} 1+a=0 \\ 1-3a+c=0 \\ -1-b=0 \end{cases}$$

of which the solution is a = -1, b = -1, and c = -2. Substituting these exponents into the equation for Π_2 gives

$$\Pi_2 = \mu\rho^{-1}\omega^{-1}D^{-2}$$

$$\therefore \Pi_2 = \frac{\mu}{\rho\omega D^2}$$

Observe that this is the inverse of a kind of Reynolds number. The corrected form of Π_2 is, accordingly,

$$\Pi_{2,\mathrm{corr}} = \frac{\rho\omega D^2}{\mu} = \mathrm{Re}$$

We write the final functional relationship as

$$\frac{T}{\rho\omega^2D^5} = f\left(\frac{\rho\omega D^2}{\mu}\right)$$

or, denoting the torque coefficient by C_T and the Reynolds number by Re,

$$\boxed{C_T = f\left(\mathrm{Re}\right)}$$

Part B: The updated functional relationship is

$$T = f\left(\rho,\omega,D,\mu,c\right)$$

There are now six variables in this problem, n = 6. The number of reference dimensions remains the same, j = 3. Therefore, the expected number of parameters is $k = n - j = 6 - 3 = 3$. As before, the repeating variables are ρ, D, and ω. The dependent Π is obtained by combining T with the repeating variables,

$$\Pi_1 = T\rho^a\omega^b D^c$$

Observe that this equation has the same form as the one used to obtain the torque coefficient in the previous problem. It is safe to conclude, then, that

$$\Pi_1 = \frac{T}{\rho\omega^2 D^5}$$

The second Π is obtained by combining viscosity with the repeating variables,

$$\Pi_2 = \mu\rho^a\omega^b D^c$$

Here as well we see that the equation should yield the same result as before, namely

$$\Pi_2 = \frac{\mu}{\rho\omega D^2}$$

This result is a kind of Reynolds number, which can be adjusted to yield

$$\Pi_{2,\text{corr}} = \frac{\rho\omega D^2}{\mu}$$

In similar fashion, we obtain the third dimensionless parameter by combining the speed of sound with the repeating variables,

$$\Pi_3 = c\rho^a\omega^b D^c$$

Decomposing the variables to their primary dimensions, we get

$$M^0L^0T^0 = LT^{-1}\times\left(ML^{-3}\right)^a\times\left(T^{-1}\right)^b\times L^c$$

$$\therefore M^0L^0T^0 = M^aL^{1-3a+c}T^{-1-b}$$

This leads to the system of linear equations

$$\begin{cases} a = 0 \\ 1-3a+c=0 \\ -1-b=0 \end{cases}$$

The solution is a = 0, b = -1, and c = -1. The ensuing parameter is

$$\Pi_3 = c\rho^0\omega^{-1}D^{-1}$$

$$\therefore \Pi_3 = \frac{c}{\omega D}$$

Since ωD is a speed (the tip speed of the propeller), this ratio of velocities is clearly a modified Mach number. The corrected Π_3 is

$$\Pi_{3,\text{corr}} = \frac{\omega D}{c}$$

Finally, we write the functional relationship

$$\frac{T}{\rho\omega^2D^5} = f\left(\frac{\rho\omega D^2}{\mu},\frac{\omega D}{c}\right)$$

Or, recalling that C_T is the torque coefficient, Re is the Reynolds number, and M is the Mach number,

$$\boxed{C_T = f\left(\text{Re},\text{M}\right)}$$

P.3 ● Solution

There are five parameters in this problem. Mathematically,

$$f_k = f\left(V, \rho, \mu, D\right)$$

The primary dimensions of each variable are listed below.

f_k	$[f_k] = T^{-1}$
V	$[V] = LT^{-1}$
ρ	$[\rho] = ML^{-3}$
μ	$[\mu] = ML^{-1}T^{-1}$
D	$[D] = L$

The number of variables, n, is 5. As a first guess, j is set equal to 3, which is the number of primary dimensions used. The expected number of dimensionless parameters is then

$$k = n - j = 5 - 3 = 2$$

Since $j = 3$, we need to choose 3 repeating parameters. A wise choice of repeating parameters for most fluid flow problems is a length, a velocity, and a mass or density. Thus, we take D, V, and ρ. Generating the dependent Π, it follows that

$$\Pi_1 = f_k V^a \rho^b D^c$$

$$\therefore M^0 L^0 T^0 = T^{-1} \times \left(LT^{-1}\right)^a \times \left(ML^{-3}\right)^b \times L^c$$

$$\therefore M^0 L^0 T^0 = M^b L^{a-3b+c} T^{-1-a}$$

This leads to the following system of linear equations,

$$\begin{cases} b = 0 \\ a - 3b + c = 0 \\ -1 - a = 0 \end{cases}$$

of which the solution is $a = -1$, $b = 0$, and $c = 1$. The dependent Π is then

$$\Pi_1 = f_k V^{-1} \rho^0 D^1 = \frac{f_k D}{V} = \mathrm{Sr}$$

where we have identified this parameter as the Strouhal number, the ratio of characteristic flow time to the period of oscillation. The second Π, which is the only independent parameter in this problem, is generated with the viscosity μ as the accompanying variable; that is,

$$\Pi_2 = \mu V^a \rho^b D^c$$

$$\therefore M^0 L^0 T^0 = \left(ML^{-1}T^{-1}\right) \times \left(LT^{-1}\right)^a \times \left(ML^{-3}\right)^b \times L^c$$

$$\therefore M^0 L^0 T^0 = M^{1+b} L^{-1+a-3b+c} T^{-1-a}$$

This leads to the following system of linear equations,

$$\begin{cases} 1 + b = 0 \\ -1 + a - 3b + c = 0 \\ -1 - a = 0 \end{cases}$$

The solution is $a = -1$, $b = -1$, and $c = -1$. The ensuing dimensionless parameter is then

$$\Pi_2 = \mu V^{-1} \rho^{-1} D^{-1} = \frac{\mu}{\rho V D} = \mathrm{Re}^{-1}$$

This parameter is easily recognized as a modified Reynolds number. Finally, we are able to write a functional relationship of the form

$$\boxed{\mathrm{Sr} = f(\mathrm{Re})}$$

Although we cannot tell the exact form of the functional relationship, experiments confirm that the Strouhal number is indeed a function of the Reynolds number.

P.4 ● Solution

The relevant parameters in the problem are listed below in functional form,

$$\Delta p = f(V, \varepsilon, \rho, \mu, D, L)$$

The primary dimensions of each variable are shown in continuation.

Δp	$[\Delta p] = ML^{-1}T^{-2}$
V	$[V] = LT^{-1}$
ε	$[\varepsilon] = L$
ρ	$[\rho] = ML^{-3}$
μ	$[\mu] = ML^{-1}T^{-1}$
D	$[D] = L$
L	$[L] = L$

As a first guess, j is set equal to 3, which is the number of primary dimensions represented in the problem. The expected number of dimensionless parameters is

$$k = n - j = 7 - 3 = 4$$

Since j = 3, we shall choose 3 repeating variables. Following the guidelines available in most fluid mechanics textbooks, we should not choose the dependent variable, Δp; we cannot choose any two of parameters ε, L, and D since their dimensions are identical; also, it is not desirable to have μ or ε appear in all the Π's. Thus, we take V, D, and ρ. We begin by generating the dependent parameter,

$$\Pi_1 = \Delta p V^a D^b \rho^c$$

Decomposing the variables to their primary dimensions, we get

$$M^0L^0T^0 = ML^{-1}T^{-2} \times \left(LT^{-1}\right)^a \times L^b \times \left(ML^{-3}\right)^c$$

$$\therefore M^0L^0T^0 = M^{1+c}L^{-1+a+b-3c}T^{-2-a}$$

We then proceed to solve the following system of linear equations,

$$\begin{cases} 1 + c = 0 \\ -1 + a + b - 3c = 0 \\ -2 - a = 0 \end{cases}$$

The solution is a = -2, b = 0, and c = -1. The dependent Π is then

$$\Pi_1 = \Delta p V^{-2} D^0 \rho^{-1}$$

$$\Pi_1 = \frac{\Delta p}{\rho V^2} = \mathrm{Eu}$$

The nondimensional parameter above is the Euler number, *Eu*, which represents the ratio of pressure difference to dynamic pressure. Now, to compute the second dimensionless parameter, we make use of the viscosity μ along with the repeating variables,

$$\Pi_2 = \mu V^a D^b \rho^c$$

Decomposing the variables as before yields

$$M^0L^0T^0 = ML^{-1}T^{-1} \times \left(LT^{-1}\right)^a \times L^b \times \left(ML^{-3}\right)^c$$

$$\therefore M^0L^0T^0 = M^{1+c}L^{-1+a+b-3c}T^{-1-a}$$

The solution is a = -1, b = -1, c = -1. It follows that Π_2 has the form

$$\Pi_2 = \mu V^{-1} D^{-1} \rho^{-1}$$

$$\therefore \Pi_2 = \frac{\mu}{\rho V D}$$

Clearly, this dimensionless quantity is a modified Reynolds number, which can be corrected to give

$$\Pi_{2,\text{corr}} = \frac{\rho V D}{\mu} = \text{Re}$$

We now proceed to form the third dimensionless parameter, this time using the roughness height ε along with the repeating variables,

$$\Pi_3 = \varepsilon V^a D^b \rho^c$$

Decomposing the variables gives

$$M^0 L^0 T^0 = L \times \left(LT^{-1}\right)^a \times L^b \times \left(ML^{-3}\right)^c$$

$$\therefore M^0 L^0 T^0 = M^c L^{1+a+b-3c} T^{-a}$$

The following system of equations ensues,

$$\begin{cases} c = 0 \\ 1 + a + b - 3c = 0 \\ -a = 0 \end{cases}$$

The solution is a = 0, b = -1, and c = 0. Parameter Π_3 then becomes

$$\Pi_3 = \varepsilon V^0 D^{-1} \rho^0$$

$$\therefore \Pi_3 = \frac{\varepsilon}{D}$$

This dimensionless quantity is the roughness ratio. It remains to compute the fourth dimensionless parameter, Π_4, in which case the length L accompanies the repeating variables,

$$\Pi_4 = L V^a D^b \rho^c$$

Because L has the same dimensions as the roughness height, we surmise that the solution to the system of equations obtained after decomposition will be the same as for Π_3. Thus, we have a = 0, b = -1, and c = 0. Parameter Π_4 then becomes

$$\Pi_4 = L V^0 D^{-1} \rho^0$$

$$\therefore \Pi_4 = \frac{L}{D}$$

This parameter is a length-to-diameter ratio or an aspect ratio. We are now able to write the final functional relationship as

$$\Pi_1 = f\left(\Pi_2, \Pi_3, \Pi_4\right) \rightarrow \frac{\Delta p}{\rho V^2} = f\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}, \frac{L}{D}\right)$$

Or, using the pertaining dimensionless parameters,

$$\boxed{\text{Eu} = f\left(\text{Re}, \frac{\varepsilon}{D}, \frac{L}{D}\right)}$$

P.5 ● Solution

The oscillation frequency of the cylinder follows the functional relationship

$$\omega = f\left(D, m, \gamma\right)$$

The primary dimensions of each variable are listed below.

ω	$[\omega] = T^{-1}$
D	$[D] = L$
m	$[m] = M$
γ	$[\gamma] = ML^{-2}T^{-2}$

Note that we have four variables (n = 4) and three reference dimensions (j = 3). The expected number of dimensionless parameters is

$$k = n - j = 4 - 3 = 1$$

Since j = 3, we need to select three repeating parameters. Let us choose D, m, and γ. We calculate the dependent Π by combining ω with the repeating variables,

$$\Pi_1 = \omega D^a m^b \gamma^c$$

Decomposing the variables to their primary dimensions, we obtain

$$M^0L^0T^0 = T^{-1} \times L^a \times M^b \times \left(ML^{-2}T^{-2}\right)^c$$

$$\therefore M^0L^0T^0 = T^{-1} \times L^a \times M^b \times M^c L^{-2c} T^{-2c}$$

$$\therefore M^0L^0T^0 = M^{b+c} L^{a-2c} T^{-1-2c}$$

This brings to the following system of linear equations,

$$\begin{cases} b + c = 0 \\ a - 2c = 0 \\ -1 - 2c = 0 \end{cases}$$

The solution is a = -1, b = 1/2, and c = -1/2. Substituting these into the equation for Π_1, we see that

$$\Pi_1 = \omega D^{-1} m^{1/2} \gamma^{-1/2}$$

$$\therefore \Pi_1 = \frac{\omega}{D}\sqrt{\frac{m}{\gamma}}$$

Since there is only one Π term, we can write

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} = k$$

where k is a constant. Solving for ω gives

$$\omega = kD\sqrt{\frac{\gamma}{m}}$$

From this result, we conclude that $\omega \propto m^{-1/2}$. Accordingly, an increase in mass will cause the frequency to decrease.

➡ The correct answer is **C**.

P.6 ● Solution

The pressure drop along the pipe obeys the functional relationship

$$\Delta p = f\left(D, \ell, V, \mu\right)$$

The primary dimensions of each variable are listed below.

Δp	$[\Delta p] = ML^{-1}T^{-2}$
D	$[D] = L$
ℓ	$[\ell] = L$

V	$[V] = LT^{-1}$
μ	$[\mu] = ML^{-1}T^{-1}$

Knowing that the number of variables n = 5 and that the number of reference dimensions j = 3, the number of required dimensionless parameters will be

$$k = n - j = 5 - 3 = 2$$

We take D, μ, and V as repeating variables. The first dimensionless parameter involves the dependent variable Δp,

$$\Pi_1 = \Delta p D^a \mu^b V^c$$

Decomposing the variables to their primary dimensions, we obtain

$$M^0L^0T^0 = ML^{-1}T^{-2} \times L^a \times \left(ML^{-1}T^{-1}\right)^b \times \left(LT^{-1}\right)^c$$

$$M^0L^0T^0 = ML^{-1}T^{-2} \times L^a \times M^bL^{-b}T^{-b} \times L^cT^{-c}$$

$$M^0L^0T^0 = M^{1+b}L^{-1+a-b+c}T^{-2-b-c}$$

This brings to the following system of linear equations,

$$\begin{cases} 1+b=0 \\ -1+a-b+c=0 \\ -2-b-c=0 \end{cases}$$

The solution is a = 1, b = -1, and c = -1. Substituting these exponents into the equation for Π_1 gives

$$\Pi_1 = \Delta p D^1 \mu^{-1} V^{-1}$$

$$\therefore \Pi_1 = \frac{\Delta p D}{\mu V}$$

Next, to obtain the second dimensionless parameter Π_2, let us combine the repeating variables with the independent variable ℓ,

$$\Pi_2 = \ell D^a \mu^b V^c$$

Decomposing the variables to their primary dimensions yields

$$M^0L^0T^0 = L \times L^a \times \left(ML^{-1}T^{-1}\right)^b \times \left(LT^{-1}\right)^c$$

$$M^0L^0T^0 = L \times L^a \times M^bL^{-b}T^{-b} \times L^cT^{-c}$$

$$M^0L^0T^0 = M^bL^{1+a-b+c}T^{-b-c}$$

This leads to the following system of linear equations,

$$\begin{cases} b=0 \\ 1+a-b+c=0 \\ -b-c=0 \end{cases}$$

The solution is a = -1, b = 0, and c = 0. The ensuing dimensionless parameter Π_2 is

$$\Pi_2 = \ell D^{-1} \mu^0 V^0$$

$$\therefore \Pi_2 = \frac{\ell}{D}$$

We can now write the functional relationship

$$\Pi_1 = f\left(\Pi_2\right) \to \frac{\Delta p D}{\mu V} = f\left(\frac{\ell}{D}\right)$$

From the statement of the problem, $\Delta p \propto \ell$, so that the foregoing equation must be of the form

$$\frac{\Delta p D}{\mu V} = k \times \frac{\ell}{D}$$

where k is some constant. Solving for Δp gives

$$\Delta p = k\frac{\mu V}{D^2}$$

That is, the pressure drop is proportional to the inverse square of the pipe diameter.

$$\boxed{\Delta p \propto \frac{1}{D^2}}$$

➡ The correct answer is **C**.

P.7 ● Solution

The torque T required to rotate the cone obeys the following functional relation,

$$T = f\left(R, \alpha, \mu, \omega\right)$$

The primary dimensions of each variable are listed below.

T	$[T] = ML^2T^{-2}$
R	$[R] = L$
α	$[\alpha] = M^0L^0T^0$
μ	$[\mu] = ML^{-1}T^{-1}$
ω	$[\omega] = L^{-1}$

We have five variables (n = 5) and 3 reference dimensions (j = 3). The number of dimensionless parameters is then

$$k = n - j = 5 - 3 = 2$$

Let R, ω, and μ be the repeating variables. We proceed to compute the dependent dimensionless parameter, Π_1,

$$\Pi_1 = TR^a\omega^b\mu^c$$

Decomposing these variables to their elementary dimensions, we obtain

$$M^0L^0T^0 = TR^a\omega^b\mu^c$$

$$\therefore M^0L^0T^0 = ML^2T^{-2} \times L^a \times \left(T^{-1}\right)^b \times \left(ML^{-1}T^{-1}\right)^c$$

$$\therefore M^0L^0T^0 = M^{1+c}L^{2+a-c}T^{-2-b-c}$$

This brings to the system of linear equations

$$\begin{cases} 1 + c = 0 \\ 2 + a - c = 0 \\ -2 - b - c = 0 \end{cases}$$

The solution is a = -3, b = -1, and c = -1. Substituting these into the equation for Π_1 gives

$$\Pi_1 = TR^{-3}\omega^{-1}\mu^{-1}$$

$$\therefore \Pi_1 = \frac{T}{\omega\mu R^3}$$

For the second dimensionless parameter, we combine the repeating variables with the cone angle α,

$$\Pi_2 = \alpha R^a\omega^b\mu^c$$

$$\therefore M^0L^0T^0 = M^0L^0T^0 \times L^a \times \left(T^{-1}\right)^b \times \left(ML^{-1}T^{-1}\right)^c$$

$$\therefore M^0L^0T^0 = L^a \times T^{-b} \times M^cL^{-c}T^{-c}$$

$$\therefore M^0L^0T^0 = M^cL^{a-c}T^{-b-c}$$

This leads to the following system of linear equations,

$$\begin{cases} c = 0 \\ a - c = 0 \\ -b - c = 0 \end{cases}$$

The solution is trivial: $a = 0, b = 0, c = 0$. Thus, the dimensionless parameter Π_2 is simply

$$\Pi_2 = \alpha R^0 \omega^0 \mu^0$$
$$\therefore \Pi_2 = \alpha$$

We are then in position to write the functional relationship between the two parameters,

$$\frac{T}{R^3 \omega \mu} = f(\alpha)$$

Solving for *T* gives

$$T = \omega \mu R^3 f(\alpha)$$

Suppose the viscosity and angular velocity were both doubled. The resulting torque *T** would be

$$T^* = (2\omega)(2\mu) R^3 f(\alpha)$$
$$\therefore T^* = 4\underbrace{\left[\omega \mu R^3 f(\alpha)\right]}_{=T}$$
$$\therefore \boxed{T^* = 4T}$$

Therefore, doubling the viscosity and angular velocity would increase the torque four-fold.

➡ The correct answer is **C**.

P.8 ● Solution

Part A: Similarity is achieved when the Reynolds number of the model is equal to that of the prototype. Mathematically,

$$\mathrm{Re}_m = \mathrm{Re}_p \rightarrow \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

Solving the equation above for V_m (the velocity of flow over the model), we obtain

$$V_m = \left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right) V_p = \left(\frac{1.849 \times 10^{-5}}{1.138 \times 10^{-3}}\right)\left(\frac{999.1}{1.184}\right)(5) \times 0.44 = \boxed{30.2\text{ m/s}}$$

At this air temperature, the speed of sound is around 346 m/s. Thus the Mach number in the wind tunnel is equal to 30.2/346 = 0.087. This is sufficiently low for an incompressible flow approximation to be reasonable.

➡ The correct answer is **B**.

Part B: Since the Reynolds numbers have been matched, the nondimensionalized drag coefficient of the model should equal that of the prototype,

$$\frac{F_{D,m}}{\rho_m V_m^2 L_m^2} = \frac{F_{D,p}}{\rho_p V_p^2 L_2^2}$$

Solving the equation above for $F_{D,p}$ (the drag force on the prototype) and substituting, we obtain

$$F_{D,p} = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2 \left(\frac{L_p}{L_m}\right)^2 F_{D,m} = \left(\frac{999.1}{1.184}\right)\left(\frac{0.44}{30.2}\right)^2 (5)^2 \times 5.70 = \boxed{25.5\text{ N}}$$

➡ The correct answer is **B**.

P.9 ● Solution

The Reynolds number is given by

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

The velocity of the mixing blade is $\omega D/2$. Thus,

$$\mathrm{Re} = \frac{\left(\frac{\omega D}{2}\right)D}{\nu} = \frac{\omega D^2}{2\nu}$$

Assuming that the Reynolds number is the same for both the prototype and the model, we can write

$$\mathrm{Re}_m = \mathrm{Re}_p \rightarrow \frac{\omega_m D_m^2}{2\nu_m} = \frac{\omega_p D_p^2}{2\nu_p}$$

$$\therefore \omega_p = \left(\frac{\nu_p}{\nu_m}\right)\left(\frac{D_m}{D_p}\right)^2 \omega_m$$

Substituting the pertaining variables brings to

$$\omega_p = \left(\frac{1.51\times10^{-6}}{1.00\times10^{-6}}\right)\times\left(\frac{1}{4}\right)^2 \times 8 = \boxed{0.755\ \mathrm{rad/s}}$$

The optimum angular speed of the prototype is 0.755 rad/s.

➡ The correct answer is **D**.

P.10 ● Solution

Given the fact that model and ship are similar, they should run at the same Froude number. Mathematically,

$$\mathrm{Fr}_m = \mathrm{Fr}_s \rightarrow \left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_p$$

Solving for the model velocity V_m gives

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_s}{\sqrt{gL_s}} \rightarrow V_m = \sqrt{\frac{L_m}{L_p}}V_p$$

Substituting the pertaining variables brings to

$$V_m = \sqrt{\frac{15}{250}}\times 35 = \boxed{8.57\ \mathrm{mi/h}}$$

➡ The correct answer is **C**.

P.11 ● Solution

As usual, we posit that the Reynolds number is the same for model and prototype, giving

$$\left(\frac{VL}{\nu}\right)_m = \left(\frac{VL}{\nu}\right)_p \rightarrow V_m = \left(\frac{\nu_m}{\nu_p}\right)\left(\frac{L_p}{L_m}\right)V_p$$

Substituting the pertaining variables, we get

$$V_m = \left(\frac{14.60\times10^{-6}}{35.25\times10^{-6}}\right)\times 15 \times 800 = \boxed{4970\ \mathrm{km/h}}$$

The speed of sound at room temperature is around 346 m/s, or 1246 km/h. The Mach number of flow in the wind tunnel for the model proposed should be around 4970/1246 ≈ 4. Thus, flow in the wind tunnel will be well into the supersonic range, which is unreasonable given the fact that the actual aircraft flies at a subsonic speed. The flow fields would have considerably different behavior. In addition, the speed V_m we have obtained for the wind tunnel may be exceedingly high for many facilities.

ANSWER SUMMARY

Problem 1		$F_D = f(\text{Nothing})$
Problem 2	**2A**	$C_T = f(\text{Re})$
	2B	$C_T = f(\text{Re}, \text{M})$
Problem 3		$\text{Sr} = f(\text{Re})$
Problem 4		$\text{Eu} = f\left(\text{Re}, \frac{\varepsilon}{D}, \frac{L}{D}\right)$
Problem 5		**C**
Problem 6		**C**
Problem 7		**C**
Problem 8	**8A**	**B**
	8B	**B**
Problem 9		**D**
Problem 10		**C**
Problem 11		Open-ended pb.

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