Montogue

Quiz EL202

Diodes

Lucas Monteiro Nogueira

PROBLEM DISTRIBUTION

Problems	Subject
1 - 11	*pn* junction diodes
12 - 18	Diode circuits
19 - 23	Half-wave and full-wave rectifiers

Refer to Table 3 for *pn* junction data. Note that values provided in individual problems override those of Table 3.

PROBLEMS

Problem 1

Consider an ideal *pn* junction diode at T = 300 K operating in the forward-bias region. Calculate the change in diode voltage that will cause a factor of 10 increase in current. Repeat for a factor of 100 increase in current.

Problem 2 (Neamen, 2003, w/ permission)

Consider a GaAs *pn* junction with doping concentrations $N_a = 5\times10^{16}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. The junction cross-sectional area is $A = 10^{-3}$ cm^2 and the applied forward-bias voltage is V_a = 1.10 V. Calculate:

Problem 2.1: The minority electron diffusion current at the edge of the space charge region.

Problem 2.2: The minority hole diffusion current at the edge of the space charge region.

Problem 2.3: The total current in the *pn* junction diode.

Problem 3 (Neamen, 2003, w/ permission)

An ideal germanium diode at T = 300 K has the following parameters: $N_a = 4\times10^{15}$ cm^{-3}, $N_d = 2\times10^{17}$ cm^{-3}, D_p = 48 cm^2/s, D_n = 90 cm^2/s, $\tau_{p0} = \tau_{n0} = 2\times10^{-6}$ s, and cross-sectional area $A = 10^{-4}$ cm^2. Determine the diode current for:

Problem 3.1: A forward-bias voltage of 0.25 V.

Problem 3.2: A reverse-biased voltage of 0.25 V.

Problem 4 (Neamen, 2003, w/ permission)

An n^+p silicon diode at T = 300 K has the following parameters: $N_d = 10^{18}$ cm^{-3}, $N_a = 10^{16}$ cm^{-3}, D_n = 25 cm^2/s, D_p = 10 cm^2/s, $\tau_{n0} = \tau_{p0}$ = 1 μs, and $A = 10^{-4}$ cm^2. Determine the diode current for:

Problem 4.1: A forward-bias voltage of 0.5 V.

Problem 4.2: A reverse-biased voltage of 0.5 V.

Problem 5 (Neamen, 2003, w/ permission)

A one-sided p^+n silicon diode has doping concentrations of $N_a = 5\times10^{17}$ cm^{-3} and $N_d = 8\times10^{15}$ cm^{-3}. The minority carrier lifetimes are $\tau_{n0} = 10^{-7}$ s and $\tau_{p0} = 8\times10^{-8}$ s. The cross-sectional area is $A = 2\times10^{-4}$ cm^2. Calculate:

Problem 5.1: The reverse-biased saturation current.

Problem 5.2: The forward-bias current at

→ V_a = 0.45 V;
→ V_a = 0.55 V;
→ V_a = 0.65 V;

Problem 6 (Neamen, 2003, w/ permission)

Consider an ideal silicon *pn* junction diode. What must be the ratio N_d/N_a so that 90 percent of the current in the depletion region is due to the flow of electrons?

Problem 7 (Neamen, 2003, w/ permission)

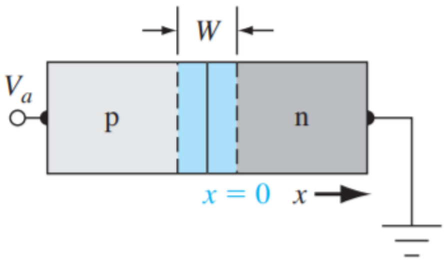
A silicon *pn* junction diode is to be designed to operate at T = 300 K such that the diode current is I = 10 mA at a diode voltage of V_D = 0.65 V. The ratio of electron current to total current is to be 0.10 and the maximum current density is to be no more than 20 A/cm^2. The semiconductor parameters are D_n = 25 cm^2/s, D_p = 10 cm^2/s, and $\tau_{n0} = \tau_{p0} = 5\times10^{-7}$ s. Determine the doping concentrations N_d and N_a.

Problem 8 (Neamen, 2003, w/ permission)

The cross-sectional area of a silicon *pn* junction is 10^{-3} cm^2. The temperature of the diode is T = 300 K, and the doping concentrations are $N_d = 10^{16}$ cm^{-3} and $N_a = 8\times10^{15}$ cm^{-3}. Assume minority carrier lifetimes of $\tau_{n0} = 10^{-6}$ s and $\tau_{p0} = 10^{-7}$ s. Calculate the total number of excess electrons in the *p* region and the total number of excess holes in the *n* region for voltages V_a = 0.3 V, V_a = 0.4 V, and V_a = 0.5 V.

Problem 9 (Neamen, 2003, w/ permission)

Consider the ideal long silicon *pn* junction illustrated to the side. The device temperature is T = 300 K. The *n* region is doped with 10^{16} donor atoms per cm^3 and the *p* region is doped with 5×10^{16} acceptor atoms per cm^3. The minority carrier lifetimes are τ_{n0} = 0.05 µs and τ_{p0} = 0.01 µs. The minority carrier diffusion coefficients are D_n = 23 cm^2/s and D_p = 8 cm^2/s. The forward-bias voltage is V_a = 0.610 V. Calculate:



Problem 9.1: The excess hole concentration as a function of x for $x \geq 0$.
Problem 9.2: The hole diffusion current density at $x = 3\times10^{-4}$ cm.
Problem 9.3: The electron current density at $x = 3\times10^{-4}$ cm.

Problem 10 (Neamen, 2003, w/ permission)

Problem 10.1: The reverse-biased saturation current is a function of temperature. Assuming that I_s varies with temperature only from the intrinsic carrier concentration, show that we can write $I_s = CT^3 exp(-E_g/kT)$, where C is a constant and a function only of the diode parameters, T is temperature, E_g is the bandgap energy, and k is Boltzmann's constant.
Problem 10.2: Determine the increase in I_s as the temperature increases from T = 300 K to T = 400 K for (*i*) a germanium diode and (*ii*) a silicon diode. Use E_g = 0.66 eV as the bandgap energy of germanium and E_g = 1.12 eV as the bandgap energy of silicon.

Problem 11 (Neamen, 2003, w/ permission)

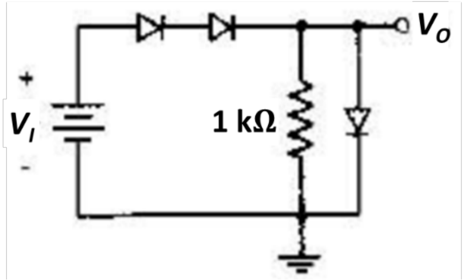
An ideal silicon *pn* junction diode has a cross-sectional area $A = 5\times10^{-4}$ cm^2. The doping concentrations are $N_a = 4\times10^{15}$ cm^{-3} and $N_d = 2\times10^{17}$ cm^{-3}. Assume that the bandgap energy E_g = 1.12 eV as well as the diffusion coefficients and lifetimes are independent of temperature. The ratio of the magnitude of forward- to reverse-biased currents is to be no less than 2×10^4 with forward- and reverse-biased voltages of 0.50 V, and the maximum reverse-biased current is to be limited to 1.2 µA. Determine the maximum temperature at which the diode will meet these specifications and state which specification is the limiting factor. Use $N_c = 2.8\times10^{19}$ cm^{-3} and $N_v = 1.04\times10^{19}$ cm^{-3} as the effective density of states functions in the conduction and valence bands of silicon, respectively.

Problem 12 (Neamen, 2000, w/ permission)

A pn junction diode is in series with a 10-MΩ resistor and a 1.5 V power supply. The reverse-saturation current of the diode is I_S = 30 nA. Determine the diode current and voltage if the diode is forward biased. Use V_T = 0.026 V as the thermal voltage.

Problem 13 (Neamen, 2000, w/ permission)

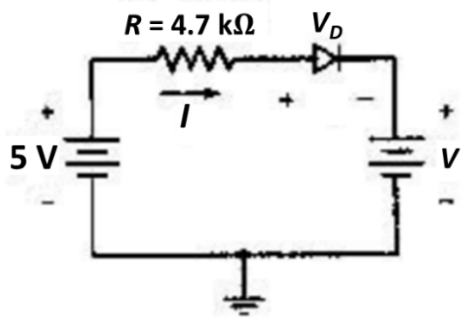
The reverse-saturation current of each diode in the circuit illustrated below if reverse-saturation current $I_S = 2\times10^{-13}$ A. Also determine the input voltage V_I required to produce an output voltage of $V_O = 0.60$ V.



Problem 14 (Neamen, 2000, w/ permission)

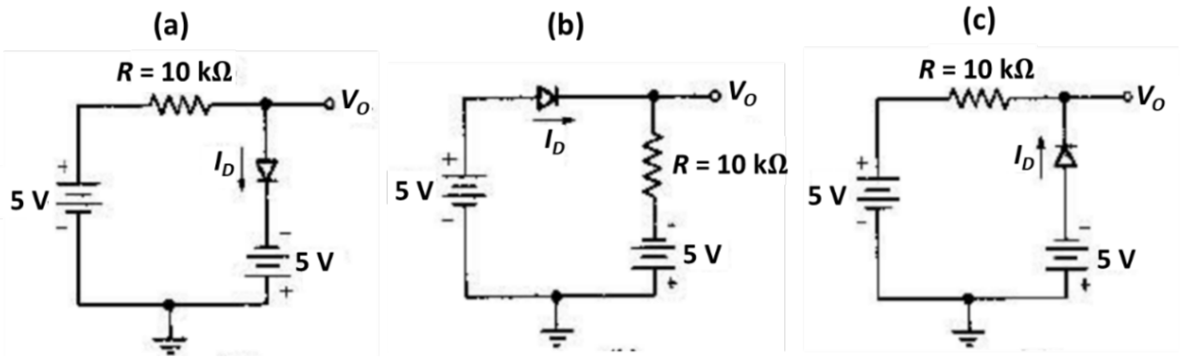
Problem 14.1: In the circuit illustrated below, find the diode voltage V_D and the supply voltage V such that the current in the loop is $I = 0.50$ mA. Assume the reverse-saturation current is $I_S = 5\times10^{-12}$ A. Also determine the power dissipated in the diode.

Problem 14.2: Reconsider the circuit introduced in Problem 14.1. If the voltage V is $V = 1.7$ V and the cut-in voltage of the diode is $V_\gamma = 0.65$ V, determine the new value of R required to limit the power dissipation in the diode to no more than 0.20 mW.



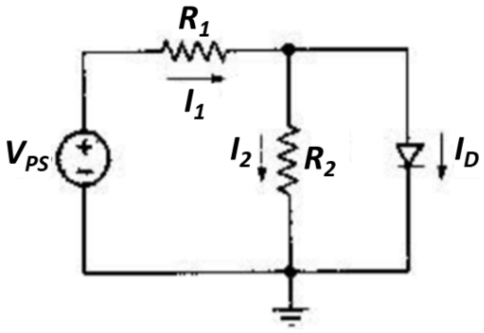
Problem 15 (Neamen, 2000, w/ permission)

The cut-in voltage for each voltage in the circuits shown below is $V_\gamma = 0.6$ V. For each circuit, determine the diode current I_D and the voltage V_O (measured with respect to ground potential).



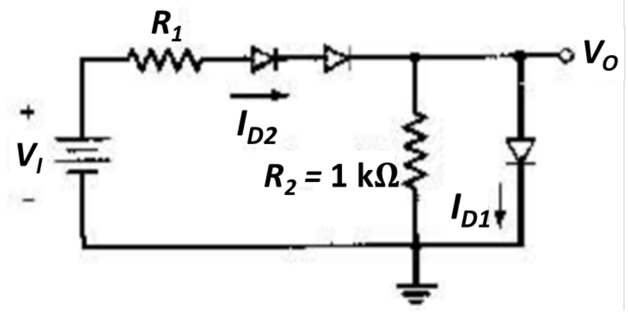
Problem 16 (Neamen, 2000, w/ permission)

The cut-in voltage of the diode in the circuit illustrated below is $V_\gamma = 0.7$ V. The diode is to remain biased "on" for a power supply voltage in the range $5 \le V_{PS} \le 10$ V. The minimum diode current is to be $I_{D,\min} = 2$ mA. The maximum power dissipated in the diode is to be no more than 10 mW. Determine the appropriate values of R_1 and R_2.



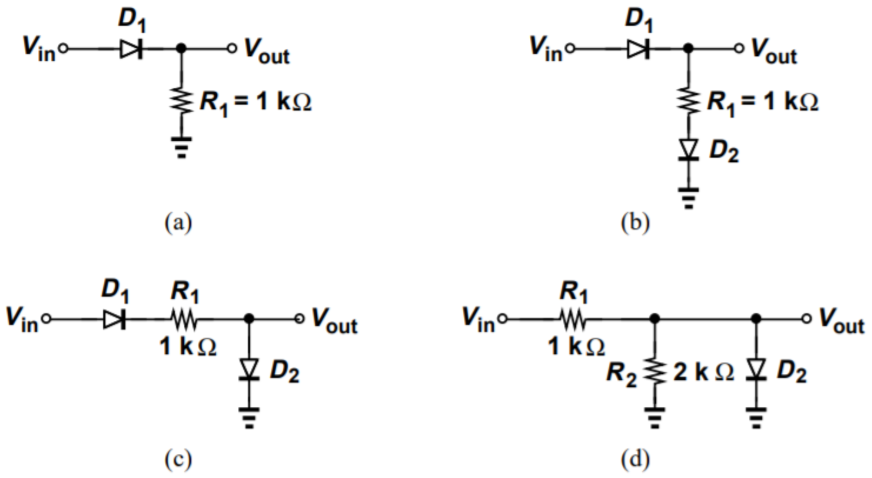
Problem 17 (Neamen, 2000, w/ permission)

Assume each diode in the circuit illustrated below has a cut-in voltage of V_γ = 0.65 V. The input voltage is V_I = 5 V. Determine the value of R_1 required such that current I_{D1} is one-half the value of I_{D2}. What are the values of I_{D1} and I_{D2}?



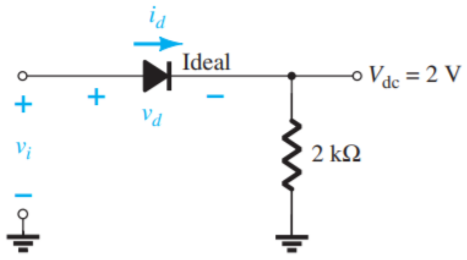
Problem 18

Beginning with a cut-in voltage $V_\gamma \approx$ 800 mV for each diode, determine the change in V_{out} if V_{in} changes from +2.4 V to +2.5 V for the circuits illustrated below. Use the small-signal model.



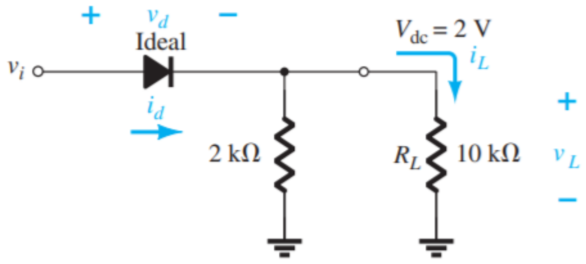
Problem 19 (Boylestad and Nashelsky, 2013, w/ permission)

Problem 19.1: Assuming an ideal diode, sketch the input voltage v_i, the diode voltage v_d, and the diode current i_d for the half-wave rectifier illustrated below if the input is a sinusoidal waveform with a frequency of 60 Hz.



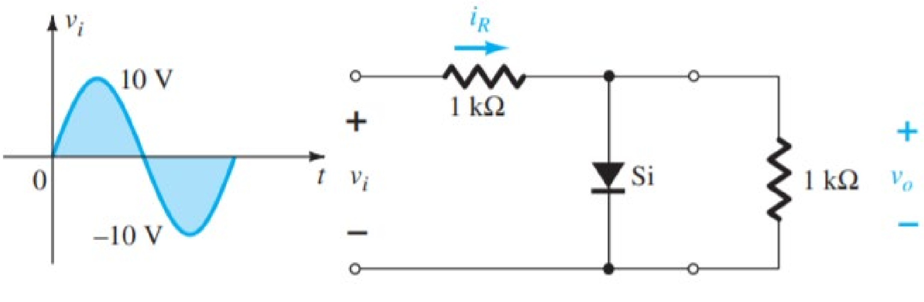
Problem 19.2: Repeat the previous problem if the ideal diode is replaced with a silicon diode, which has a "turn-on" voltage V_K = 0.7 V.

Problem 19.3: Repeat Problem 19.1 if a 10-kΩ load is applied, as shown below. Sketch the input voltage v_i, the diode current i_d, the load voltage v_L and the load current i_L.



Problem 20 (Boylestad and Nashelsky, 2013, w/ permission)

For the network illustrated below, sketch the voltage v_o and current i_R.

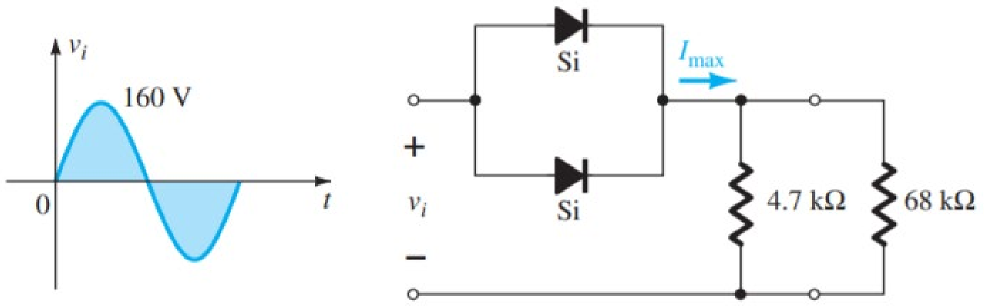


Problem 21 (Boylestad and Nashelsky, 2013, w/ permission)

Problem 21.1: Given the maximum power rating P_{max} = 14 mW for each diode in the following circuit, determine the maximum current rating of each diode (using the approximate equivalent model).

Problem 21.2: Determine current I_{max}.

Problem 21.3: Determine the current through each diode at maximum input voltage $V_{i,max}$ using the results of Problem 21.2. If only one diode were present, what would be the expected result?



Problem 22 (Boylestad and Nashelsky, 2013, w/ permission)

A full-wave bridge rectifier with a 120-V rms sinusoidal input has a load resistor of 1 kΩ.

Problem 22.1: If silicon diodes are employed, what is the dc voltage available at the load?

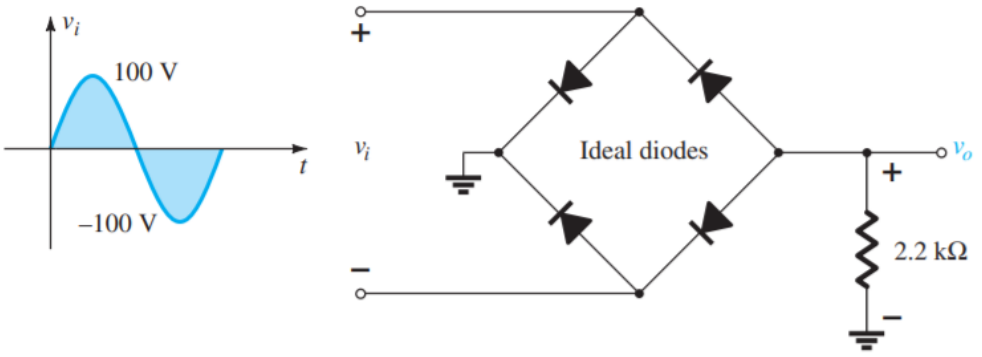
Problem 22.2: Determine the required peak inverse voltage (PIV) rating of each diode.

Problem 22.3: Find the maximum current through each diode during conduction.

Problem 22.4: What is the required power rating of each diode?

Problem 23 (Boylestad and Nashelsky, 2013, w/ permission)

Determine v_o and the required peak inverse voltage (PIV) rating of each diode for the full-wave rectifier illustrated below. In addition, determine the maximum current through each diode.



ADDITIONAL INFORMATION

Table 1 Concentration variables

Variable	Meaning
I_S	Reverse-saturation current in a diode
$V_T = kT/e$	Thermal voltage in a diode; (k is Boltzmann's constant, T is temperature, and e is elementary charge)
J	Current density
A	Cross-sectional area
D_n	Diffusion coefficient of electrons in a *pn* diode junction
D_p	Diffusion coefficient of holes in a *pn* diode junction
τ_{n0}	Lifetime of electrons in a *pn* diode junction
τ_{p0}	Lifetime of holes in a *pn* diode junction
$L_n = \sqrt{D_n \tau_{n0}}$	Diffusion length of electrons in a *pn* diode junction
$L_p = \sqrt{D_p \tau_{p0}}$	Diffusion length of holes in a *pn* diode junction

Table 2 More variables related to *pn* junction diodes

Variable	Meaning
N_a	Acceptor concentration in the *p* region of the *pn* junction
N_d	Donor concentration in the *n* region of the *pn* junction
n_i	Intrinsic carrier concentration
$n_{n0} = N_d$	Thermal-equilibrium majority carrier hole concentration in the *n* region
$p_{p0} = N_a$	Thermal-equilibrium majority carrier hole concentration in the *p* region
$n_{p0} = n_i^2/N_a$	Thermal-equilibrium minority carrier electron concentration in the *p* region
$p_{n0} = n_i^2/N_d$	Thermal-equilibrium minority carrier hole concentration in the *p* region
n_p	Total minority carrier electron concentration in the *p* region
p_n	Total minority carrier
$n_p(-x_p)$	Minority carrier electron concentration in the *p* region at the space charge edge
$p_n(x_n)$	Minority carrier hole concentration in the *n* region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the *p* region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the *n* region

Table 3 Silicon, and gallium arsenide *pn* junction properties* (T = 300 K)

Property	Si	GaAs
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}
Electron diffusion coefficient, D_n (cm^2/s)	25	205
Hole diffusion coefficient, D_p (cm^2/s)	10	9.8
Electron lifetime, τ_{n0} (sec)	5×10^{-7}	5×10^{-8}
Hole lifetime, τ_{p0} (sec)	10^{-7}	10^{-8}
*Values provided in individual problems override these values.		

Equations

1 → *Basic pn junction diode equation*

$$I_D = I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

where I_D is diode current, I_S is reverse-saturation current, *e* is elementary charge, *V* is the applied voltage. Note that factor *kT/e* can be replaced with the so-called *thermal voltage* V_T.

Some problems use the approximation

$$I_D \approx I_S \exp\left(\frac{eV}{kT}\right)$$

2 → *Electron current density in a pn junction diode*

$$J_n\left(-x_p\right) = \frac{eD_n n_{po}}{L_n}\exp\left(\frac{V_a}{V_T}\right)$$

where J_n is electron current density, x_p is the horizontal coordinate of one of the edges of the *pn* junction, *e* is elementary charge, L_n is electron diffusion length, V_a is applied voltage, and V_T is thermal voltage. Concentration variables are as defined in Table 1. The equation can be restated as

$$J_n\left(-x_p\right) = \frac{en_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}\exp\left(\frac{V_a}{V_T}\right)$$

where D_n is electron diffusion coefficient and τ_{n0} is the lifetime of electrons. Concentration variables are as defined in Table 1.

3 → *Hole current density in a pn junction diode*

$$J_p\left(x_n\right)=\frac{eD_p p_{no}}{L_p}\exp\left(\frac{V_a}{V_T}\right)$$

where J_p is hole current density, x_n is the horizontal coordinate of one of the edges of the *pn* junction, *e* is elementary charge, L_p is hole diffusion length, V_a is applied voltage, and V_T is thermal voltage. Concentration variables are as defined in Table 1. The equation can be restated as

$$J_p\left(x_n\right)=\frac{en_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\exp\left(\frac{V_a}{V_T}\right)$$

where D_p is hole diffusion coefficient and τ_{p0} is the lifetime of electrons. Concentration variables are as defined in Table 1.

4 → *Reverse-saturation current density*

$$J_s=\frac{eD_n n_{p0}}{L_n}+\frac{eD_p p_{n0}}{L_p}$$

where *e* is the elementary charge, D_n is the diffusion coefficient of electrons, D_P is the diffusion coefficient of holes, L_n is the electron diffusion length, and L_p is the hole diffusion length. Concentration variables are as defined in Table 1. Replacing the concentration variables and using the definition of diffusion length ($L_n^2 = D_n\tau_{n0}$, $L_p^2 = D_n\tau_{p0}$), we obtain the more convenient form

$$J_s=en_i^2\left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right)$$

where n_i is the intrinsic carrier concentration, τ_{n0} is the electron lifetime, and τ_{p0} is hole lifetime. Concentration variables are as defined in Table 1.

5 → *Distribution of excess electron concentration in a pn junction*

$$\delta n_p=n_p-n_{p0}=n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(\frac{-x}{L_n}\right)$$

where V_a is applied voltage, V_T is thermal voltage, *x* is distance from the center of the *pn* junction, and L_n is electron diffusion length. Concentration variables are as defined in Table 1.

6 → *Distribution of excess hole concentration in a pn junction*

$$\delta p_n=p_n-p_{n0}=p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(\frac{-x}{L_p}\right)$$

where V_a is applied voltage, V_T is thermal voltage, *x* is distance from the center of the *pn* junction, and L_p is hole diffusion length.

SOLUTIONS

P.1 ➡ *Solution*

In the forward bias region, the forward-bias current flowing through a *pn* junction diode is approximated as

$$I_f\approx I_S\exp\left(\frac{eV}{kT}\right)$$

where I_S is the saturation current, $e = 1.60\times10^{-19}$ C is the charge of an electron, *V* is diode voltage, $k = 1.38\times10^{-23}$ J/K is Boltzmann's constant, and *T* is the junction temperature. *k*, *e* and *T* can be condensed in the so-called thermal voltage V_T, which at 300 K amounts to

$$V_T=\frac{kT}{e}=\frac{\left(1.38\times10^{-23}\right)\times300}{1.60\times10^{-19}}=0.0259\text{ V}\approx0.026\text{ V}$$

This is the approximation used in future problems. Now, letting subscripts 1 and 2 denote variables under two different conditions, we have the ratio

$$\frac{I_{f,2}}{I_{f,1}}=\frac{I_S\exp\left(V_2/V_T\right)}{I_S\exp\left(V_1/V_T\right)}=\exp\left[\frac{1}{V_T}\left(V_2-V_1\right)\right]$$

or, solving for voltage difference,

$$\frac{I_{f,2}}{I_{f,1}} = \exp\left[\frac{1}{V_T}(V_2 - V_1)\right] \rightarrow \Delta V = V_T \ln\left(\frac{I_{f,2}}{I_{f,1}}\right)$$

For a 10-fold increase in current, we require a change in diode voltage such that

$$\Delta V = 0.026 \times \ln(10) = 59.9 \times 10^{-3}\text{ V} \approx \boxed{60\text{ mV}}$$

while for a 100-fold increase,

$$\Delta V = 0.026 \times \ln(100) = 120 \times 10^{-3}\text{ V} \approx \boxed{120\text{ mV}}$$

P.2 ➡ Solution

Problem 2.1: The electron diffusion current can be obtained by multiplying the corresponding current density $J_n(-x_p)$ by the cross-sectional area of the junction; that is,

$$I_n = AJ_n(-x_p) \text{ (I)}$$

where A is cross-sectional area. J_n is given by

$$J_n(-x_p) = \frac{eD_n n_{po}}{L_n}\exp\left(\frac{V_a}{V_T}\right)$$

which can be restated as

$$J_n(-x_p) = \frac{en_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}\exp\left(\frac{V_a}{V_T}\right)$$

$$\therefore J_n(-x_p) = \frac{(1.6\times10^{-19})\times(1.8\times10^{6})^2}{5\times10^{16}} \times \sqrt{\frac{205}{5\times10^{-8}}} \times \exp\left(\frac{1.10}{0.026}\right) = 1.57\text{ A/cm}^2$$

Substituting in (I) brings to

$$I_n = AJ_n(-x_p) = 10^{-3} \times 1.57 = \boxed{1.57\text{ mA}}$$

Problem 2.2: To find the minority hole diffusion current, we write

$$J_p(x_n) = \frac{eD_p p_{no}}{L_p}\exp\left(\frac{V_a}{V_T}\right)$$

which can be restated as

$$J_p(x_n) = \frac{en_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\exp\left(\frac{V_a}{V_T}\right)$$

$$\therefore J_p(x_n) = \frac{(1.6\times10^{-19})\times(1.8\times10^{6})^2}{10^{16}} \times \sqrt{\frac{9.80}{10^{-8}}} \times \exp\left(\frac{1.10}{0.026}\right) = 3.84\text{ A/cm}^2$$

so that

$$I_p = AJ_p(x_n) = 10^{-3} \times 3.84 = \boxed{3.84\text{ mA}}$$

Problem 2.3: To find the total current, simply add I_n and I_p,

$$I = I_n + I_p = 1.57 + 3.84 = \boxed{5.41\text{ mA}}$$

P.3 ➡ Solution

Problem 3.1: The current can be obtained by multiplying the ideal current density by the cross-sectional area of the junction,

$$I = AJ$$

Using the ideal-diode equation, the product above can be restated as

$$I = AJ_s \exp\left(\frac{V_a}{V_T}\right) \text{ (I)}$$

All but one of the variables in the equation above are given; the missing one is the ideal reverse-saturation current density J_s, which is given by equation 4,

$$J_s = \frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}$$

This can be restated as

$$J_s = en_i^2\left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right)$$

$$\therefore J_s = \left(1.6\times10^{-19}\right)\times\left(2.4\times10^{13}\right)^2\times\left[\frac{1}{4\times10^{15}}\sqrt{\frac{90}{2.0\times10^{-6}}}+\frac{1}{2\times10^{17}}\sqrt{\frac{48}{2.0\times10^{-6}}}\right]=1.57\times10^{-4}\ \text{A/cm}^2$$

so that, substituting in (I),

$$I = 10^{-4}\times\left(1.57\times10^{-4}\right)\times\exp\left(\frac{0.25}{0.026}\right)=2.35\times10^{-4}\ \text{A}=\boxed{0.235\,\text{mA}}$$

Problem 3.2: In this case, the diode would yield a current given by its reverse-bias saturation value, which, using J_s = 1.57×10^{-4} A/cm^2 determined above,

$$I \approx -I_s = -AJ_s = -10^{-4}\times\left(1.57\times10^{-4}\right)=-1.57\times10^{-8}\ \text{A}$$

$$\therefore \boxed{I=-15.7\,\text{nA}}$$

P.4 ➡ *Solution*

Problem 4.1: The current can be determined with equation (I) of the previous problem,

$$I = AJ_s\exp\left(\frac{V_a}{V_T}\right)$$

where J_s is given by, as before (equation 4)

$$J_s = \frac{eD_n n_{p0}}{L_n}+\frac{eD_p p_{n0}}{L_p}$$

Since this is an n^+p diode, however, the n-side is more heavily doped and the p-side contributes little to the saturation current; thus, we may write

$$J_s = \frac{eD_n n_{p0}}{L_n}+\cancel{\frac{eD_p p_{n0}}{L_p}}\approx\frac{eD_n n_{p0}}{L_n}$$

or, equivalently,

$$J_s = \frac{eD_n n_{p0}}{L_n}=en_i^2\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}$$

$$\therefore J_s = \left(1.6\times10^{-19}\right)\times\left(1.5\times10^{10}\right)^2\times\frac{1}{10^{16}}\sqrt{\frac{25}{10^{-6}}}=1.8\times10^{-11}\ \text{A/cm}^2$$

so that

$$I = AJ_s\exp\left(\frac{V_a}{V_T}\right)=10^{-4}\times\left(1.8\times10^{-11}\right)\times\exp\left(\frac{0.5}{0.026}\right)=4.05\times10^{-7}\ \text{A}$$

$$\therefore \boxed{I=0.405\ \mu\text{A}}$$

Problem 4.2: When reverse-biased, the diode will yield a current equal to its reverse-saturation value I_s,

$$I \approx -I_s = -AJ_s = -10^{-4}\times\left(1.8\times10^{-11}\right)=-1.8\times10^{-15}\ \text{A}$$

$$\therefore \boxed{I=-1.8\,\text{fA}}$$

P.5 ➡ *Solution*

Problem 5.1: To determine the reverse-biased saturation current, simply apply

$$I_s = AJ_s$$

where (equation 4)

$$J_s = en_i^2\left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right)$$

$$\therefore J_s = \left(1.60\times10^{-19}\right)\times\left(1.5\times10^{-10}\right)^2\times\left(\frac{1}{5.0\times10^{17}}\sqrt{\frac{25}{10^{-7}}}+\frac{1}{8.0\times10^{-15}}\sqrt{\frac{10}{8.0\times10^{-8}}}\right)=5.14\times10^{-11}\text{ A/cm}^2$$

so that

$$I_s = AJ_s = \left(2\times10^{-4}\right)\times\left(5.14\times10^{-11}\right)=1.03\times10^{-14}\text{ A}$$

$$\boxed{I_s = 10.3\,\text{fA}}$$

Problem 5.2: Using the ideal-diode equation, we have

$$I = I_s\exp\left(\frac{V_a}{V_T}\right)=\left(1.03\times10^{-14}\right)\times\exp\left(\frac{V_a}{0.026}\right)$$

Accordingly, with a bias voltage V_a = 0.45 V,

$$I=\left(1.03\times10^{-14}\right)\times\exp\left(\frac{0.45}{0.026}\right)=3.38\times10^{-7}\text{ A}$$

$$\boxed{I = 0.338\,\mu\text{A}}$$

With a bias voltage V_a = 0.55 V,

$$I=\left(1.03\times10^{-14}\right)\times\exp\left(\frac{0.55}{0.026}\right)=1.58\times10^{-5}\text{ A}$$

$$\boxed{I = 15.8\,\mu\text{A}}$$

Finally, with V_a = 0.65 V,

$$I=\left(1.03\times10^{-14}\right)\times\exp\left(\frac{0.65}{0.026}\right)=7.42\times10^{-4}\text{ A}$$

$$\boxed{I = 742\,\mu\text{A}}$$

P.6 ➡ *Solution*

If 90 percent of the current is to be due to the flow of electrons, we may write the ratio

$$\frac{J_n}{J_n+J_p}=0.9\rightarrow\frac{\dfrac{eD_nn_{p0}}{L_n}}{\dfrac{eD_nn_{p0}}{L_n}+\dfrac{eD_pp_{n0}}{L_p}}=0.9$$

Using the conversions we have been using in previous problems, namely

$$\frac{eD_nn_{p0}}{L_n}=\frac{en_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}\;;\;\frac{eD_pp_{n0}}{L_p}=\frac{en_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}$$

it follows that

$$\frac{J_n}{J_n+J_p}=\frac{\dfrac{\cancel{en_i^2}}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}}{\dfrac{\cancel{en_i^2}}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}+\dfrac{\cancel{en_i^2}}{N_d}\sqrt{\dfrac{D_p}{\tau_{p0}}}}=0.9$$

$$\therefore\frac{\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}}{\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}+\dfrac{1}{N_d}\sqrt{\dfrac{D_p}{\tau_{p0}}}}=0.9$$

$$\therefore\frac{1}{\dfrac{\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}}{\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}}+\dfrac{\dfrac{1}{N_d}\sqrt{\dfrac{D_p}{\tau_{p0}}}}{\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}}}=0.9$$

$$\therefore \frac{1}{1+\frac{N_a}{N_d}\sqrt{\frac{\tau_{n0}}{\tau_{p0}}\frac{D_p}{D_n}}} = 0.9$$

Denoting the ratio N_a/N_d by ζ and using the generic data for a silicon diode τ_{n0} = 5×10^{-7} s, τ_{p0} = 10^{-7} s, D_n = 25 cm^2/s, and D_p = 10 cm^2/s, we obtain

$$\therefore \frac{1}{1+\zeta\sqrt{\frac{5.0\times10^{-7}}{10^{-7}}\times\frac{10}{25}}} = 0.9$$

$$\therefore \frac{1}{1+\zeta\times1.41} = 0.9$$

$$\therefore 1 = 0.9\times(1+1.41\zeta)$$

$$\therefore 1 = 0.9+1.27\zeta$$

$$\therefore \zeta = \frac{1-0.9}{1.27} = 0.0787$$

Finally,

$$\frac{N_d}{N_a} = \zeta^{-1} = \frac{1}{0.0787} = \boxed{12.7}$$

Thus, 90 percent of the current in the depletion region will be due to the flow of electrons insasmuch as the donor concentration in the *n* region is about 12.7 times greater than the acceptor concentration in the *p* region.

P.7 ➡ *Solution*

If the ratio of electron current to total current must be 0.10, we may write

$$\frac{J_n}{J_n+J_p} = 0.1 \to \frac{\frac{eD_n n_{p0}}{L_n}}{\frac{eD_n n_{p0}}{L_n}+\frac{eD_p p_{n0}}{L_p}} = 0.1$$

$$\therefore \frac{\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}}{\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}} = 0.1$$

Performing the same manipulations employed in Problem 6, we get

$$\frac{\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}}{\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}} = 0.1 \to \frac{1}{1+\frac{N_a}{N_d}\sqrt{\frac{\tau_{n0}}{\tau_{p0}}\frac{D_p}{D_n}}} = 0.1$$

$$\therefore \frac{1}{1+\frac{N_a}{N_d}\sqrt{\frac{5.0\times10^{-7}}{5.0\times10^{-7}}\times\frac{10}{25}}} = 0.1$$

$$\therefore \frac{1}{1+\zeta\times0.632} = 0.1$$

$$\therefore 1 = 0.1+0.0632\zeta$$

$$\therefore \frac{N_a}{N_d} = \frac{1-0.1}{0.0632} = 14.2 \quad \text{(I)}$$

Observing that the current density J = 20 A/cm^2, we can determine the reverse-saturation CD as

$$J = J_s \exp\left(\frac{V_D}{V_T}\right) \to J_s = \frac{J}{\exp(V_D/V_T)}$$

$$\therefore J_s = \frac{20}{\exp(0.65/0.026)} = 2.78\times10^{-10}\ \text{A/cm}^2$$

Recalling another expression for J_s (equation 4), we obtain

$$J_s = en_i^2\left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right) = 2.78\times10^{-10}$$

$$\therefore \left(1.60\times10^{-19}\right)\times\left(1.5\times10^{10}\right)^2\times\left(\frac{1}{N_a}\sqrt{\frac{25}{5.0\times10^{-7}}} + \frac{1}{N_d}\sqrt{\frac{10}{5.0\times10^{-7}}}\right) = 2.78\times10^{-10}$$

$$\therefore \frac{1}{N_a}\sqrt{\frac{25}{5.0\times10^{-7}}} + \frac{1}{N_d}\sqrt{\frac{10}{5.0\times10^{-7}}} = \frac{2.78\times10^{-10}}{\left(1.60\times10^{-19}\right)\times\left(1.5\times10^{10}\right)^2}$$

$$\therefore \frac{7.07\times10^3}{N_a} + \frac{4.47\times10^3}{N_d} = 7.72\times10^{-12}$$

From (I), $N_a = 14.2N_d$; it follows that

$$\frac{7.07\times10^3}{14.2N_d} + \frac{4.47\times10^3}{N_d} = 7.72\times10^{-12} \to \frac{498}{N_d} + \frac{4.47\times10^3}{N_d} = 7.72\times10^{-12}$$

$$\therefore N_d = \frac{\left(498 + 4.47\times10^3\right)}{7.72\times10^{-12}} = \boxed{6.44\times10^{14}\ \text{cm}^{-3}}$$

and

$$N_a = 14.2\times\left(6.44\times10^{14}\right) = \boxed{9.15\times10^{15}\ \text{cm}^{-3}}$$

P.8 ➡ *Solution*

First, note that the excess electron concentration is given by equation 5,

$$\delta n_p = n_p - n_{p0} = n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]\exp\left(\frac{-x}{L_n}\right)$$

where x is the distance from the center of the pn junction. To establish the number of excess electrons, we integrate δn_p from $x = 0$ to $x \to \infty$ and multiply the result by the cross-sectional area A of the junction,

$$N_p = A\int_0^\infty \delta n_p\, dx$$

The improper integral in question is easy to evaluate,

$$\int_0^\infty \delta n_p\, dx = n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]\underbrace{\int_0^\infty \exp\left(\frac{-x}{L_n}\right)dx}_{=L_n} = n_{p0}L_n\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]$$

so that

$$N_p = An_{p0}L_n\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right] = \frac{An_i^2\sqrt{D_n\tau_{n0}}}{N_a}\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]$$

$$\therefore N_p = \underbrace{\frac{10^{-3}\times\left(1.5\times10^{10}\right)^2\times\sqrt{25\times\left(1.0\times10^{-6}\right)}}{8.0\times10^{15}}}_{=0.141}\times\left[\exp\left(\frac{0.3}{0.026}\right) - 1\right]$$

$$\therefore \boxed{N_p = 15{,}100\, e^-}$$

Repeating for a voltage of 0.4 V, the number of excess electrons is

$$N_p = 0.141\left[\exp\left(\frac{0.4}{0.026}\right) - 1\right] = \boxed{677{,}000\, e^-}$$

With V_a = 0.5 V, the number of excess electrons is

$$N_p = 0.141\left[\exp\left(\frac{0.5}{0.026}\right) - 1\right] = \boxed{31.7\ \text{million}\ e^-}$$

P.9 ➡ *Solution*

Problem 9.1: The excess hole concentration is expressed as (equation 6)

$$\delta p_n = p_n - p_{n0} = p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(-\frac{x}{L_p}\right)$$

Here, p_{n0}, the thermal-equilibrium minority carrier hole concentration in the *n* region, is given by

$$p_{n0} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$$

while the diffusion length L_p is determined as

$$L_p = \sqrt{D_p\tau_{p0}} = \sqrt{8\times\left(0.01\times10^{-6}\right)} = 2.83\times10^{-4}\ \text{cm}$$

so that

$$\delta p_n = \left(2.25\times10^4\right)\times\left[\exp\left(\frac{0.610}{0.026}\right)-1\right]\times\exp\left(-\frac{x}{2.83\times10^{-4}}\right)$$

$$\therefore \boxed{\delta p_n(x) = 3.48\times10^{14}\exp\left(\frac{-x}{2.83\times10^{-4}}\right)\left[\text{cm}^{-3}\right]}$$

Problem 9.2: The hole diffusion current density J_p at a distance *x* from the center of the device is

$$J_p(x) = -eD_p\frac{d(\delta p_n)}{dx} = -eD_p\frac{d}{dx}\left[3.48\times10^{14}\exp\left(\frac{-x}{2.83\times10^{-4}}\right)\right]$$

$$\therefore J_p(x) = -eD_p\times3.48\times10^{14}\times\left(-\frac{1}{2.83\times10^{-4}}\right)\times\exp\left(\frac{-x}{2.83\times10^{-4}}\right)$$

$$\therefore J_p(x) = 1.57\exp\left(\frac{-x}{2.83\times10^{-4}}\right)$$

so that, at $x = 3\times10^{-4}$ cm,

$$J_p\left(3\times10^{-4}\right) = 1.57\exp\left(\frac{-3\times10^{-4}}{2.83\times10^{-4}}\right) = \boxed{0.544\ \text{A/cm}^2}$$

Problem 9.3: For an ideal *pn* junction, the sum of electron and hole current densities at a distance *x* from the center of the junction must be constant. Accordingly, we may write

$$J_p(x) + J_n(x) = J_{p0} + J_{n0}$$

where $J_n(x)$ is the current density of electrons at a distance *x* from the center of the junction and J_{p0} and J_{n0} are the current densities at *x* = 0. Solving for $J_n(x)$,

$$J_n(x) = J_{p0} + J_{n0} - J_p(x) \quad \text{(I)}$$

We already have $J_p(x)$. J_{p0} is given by

$$J_{p0} = \frac{eD_p p_{n0}}{L_p}\exp\left(\frac{V_a}{V_T}\right)$$

$$J_{p0} = \frac{\left(1.6\times10^{-19}\right)\times8\times\left(2.25\times10^4\right)}{2.83\times10^{-4}}\exp\left(\frac{0.610}{0.026}\right) = 1.57\ \text{A/cm}^2$$

Likewise, the electron current density at the origin, J_{n0}, is stated as

$$J_{n0} = \frac{eD_n n_{p0}}{L_n}\exp\left(\frac{V_a}{V_T}\right)$$

where n_{p0}, the thermal-equilibrium minority carrier electron concentration in the *p* region, is given by

$$n_{p0} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^3\ \text{cm}^{-3}$$

while the diffusion length L_n is determined as

$$L_n = \sqrt{D_n \tau_{n0}} = \sqrt{23 \times \left(0.05 \times 10^{-6}\right)} = 1.07 \times 10^{-3} \text{ cm}$$

giving

$$J_{n0} = \frac{\left(1.6 \times 10^{-19}\right) \times 23 \times \left(4.5 \times 10^{3}\right)}{1.07 \times 10^{-3}} \exp\left(\frac{0.610}{0.026}\right) = 0.239 \text{ A/cm}^2$$

Substituting our results into (I), we have, with $x = 3\times10^{-4}$ cm,

$$J_n\left(3 \times 10^{-4}\right) = J_{p0} + J_{n0} - J_p\left(3 \times 10^{-4}\right)$$

$$\therefore J_n\left(3 \times 10^{-4}\right) = 1.57 + 0.239 - 0.544 = \boxed{1.27 \text{ A/cm}^2}$$

P.10 ➡ Solution

Problem 10.1: The reverse-saturation current is given by a modified form of equation 4,

$$I_s = Aen_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

Here, we may take the term in brackets and the *Ae* product as constants, but the intrinsic carrier concentration n_i varies sensibly with temperature. We thus restate I_s as

$$I_s = Aen_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right] = C_1 n_i^2$$

Recall that n_i is expressed as

$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

where N_c is the effective density of states function in the conduction band, N_v is the effective density of states function in the valence band, E_g is the bandgap energy, k is Boltzmann's constant, and T is temperature. N_c and N_v may be taken as functions of $T^{3/2}$, so their product must be a function of the cube of temperature; thus, writing N_{c0} and N_{v0} as the density of states functions at the reference temperature of 300 K, we have

$$I_s = C_1 n_i^2 = C_1 N_{c0} N_{v0} \left(\frac{T}{300}\right)^3 \exp\left(-\frac{E_g}{kT}\right) = C_1 \times \frac{N_{c0} N_{v0}}{300^3} \times T^3 \exp\left(-E_g/kT\right)$$

$$\therefore I_s = C_1 C_2 T^3 \exp\left(-E_g/kT\right)$$

$$\therefore \boxed{I_s = CT^3 \exp\left(-\frac{E_g}{kT}\right)}$$

Problem 10.2: Let $I_{s,1}$ denote the reverse-saturation current at the initial temperature T_1 = 300 K and $I_{s,2}$ denote the RSC at the final temperature T_2 = 400 K. Using the relation derived above, we have the ratio

$$\frac{I_{s,2}}{I_{s,1}} = \frac{\cancel{C} T_2^3 \exp\left(-\frac{E_g}{kT_2}\right)}{\cancel{C} T_1^3 \exp\left(-\frac{E_g}{kT_1}\right)} = \left(\frac{T_2}{T_1}\right)^3 \exp\left[-\frac{E_g}{kT_2} - \left(-\frac{E_g}{kT_1}\right)\right] = \left(\frac{T_2}{T_1}\right)^3 \exp\left[E_g\left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)\right]$$

Taking $k = 8.62\times10^{-5}$ eV/K as Boltzmann's constant, we have

$$\frac{1}{kT_1} = \frac{1}{\left(8.63 \times 10^{-5}\right) \times 300} = 38.6$$

$$\frac{1}{kT_2} = \frac{1}{\left(8.63 \times 10^{-5}\right) \times 400} = 29.0$$

so that, for germanium (E_g = 0.66 eV),

$$\frac{I_{s,2}}{I_{s,1}} = \left(\frac{400}{300}\right)^3 \exp\left[0.66 \times (38.6 - 29.0)\right] = \boxed{1340}$$

while for silicon (E_g = 1.12 eV),

$$\frac{I_{s,2}}{I_{s,1}} = \left(\frac{400}{300}\right)^3 \exp\left[1.12\times\left(38.6-29.0\right)\right] = \boxed{117,000}$$

P.11 ➡ *Solution*

Evoking the ideal-diode equation and noting that the ratio of I_f to I_s is to be no greater than 20,000, we see that the thermal voltage is limited to

$$I_f = I_s \exp\left(\frac{V_a}{V_T}\right) \rightarrow \left|\frac{I_f}{I_s}\right| = \exp\left(\frac{V_a}{V_T}\right)$$

$$\therefore V_T = \frac{V_a}{\ln\left|I_f/I_s\right|}$$

$$\therefore V_T = \frac{0.50}{\ln\left|20,000\right|} = 0.0505\text{ V}$$

If a thermal voltage of 0.026 V corresponds to a temperature of 300 K, a V_T of 0.0505 V pertains to a temperature T_1 such that

$$\frac{0.026}{300} = \frac{0.0505}{T_1} \rightarrow T_1 = \frac{300\times0.0505}{0.026} = 583\text{ K}$$

As an additional restriction, the reverse-biased current I_s is to be no greater than 1.2 μA. We first evoke the equation for I_s and solve for the squared intrinsic carrier concentration n_i^2,

$$I_s = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

$$\therefore 1.2\times10^{-6} = \left(5.0\times10^{-4}\right)\times\left(1.60\times10^{-19}\right)\times n_i^2\times\left[\frac{1}{4.0\times10^{15}}\sqrt{\frac{25}{5.0\times10^{-7}}} + \frac{1}{2.0\times10^{17}}\sqrt{\frac{10}{10^{-7}}}\right]$$

$$\therefore 1.2\times10^{-6} = 1.45\times10^{-34}\,n_i^2$$

$$\therefore n_i^2 = \frac{1.2\times10^{-6}}{1.45\times10^{-34}} = 8.28\times10^{27}$$

Now, using the definition of n_i^2 and recalling that the product of density of states functions varies with the cube of temperature, we may write

$$n_i^2 = N_cN_v\exp\left(-\frac{E_g}{kT}\right) = N_{c0}N_{v0}\left(\frac{T}{300}\right)^3\exp\left(-\frac{E_g}{kT}\right) = 8.28\times10^{27}$$

$$\therefore n_i^2 = \left(2.8\times10^{19}\right)\times\left(1.04\times10^{19}\right)\times\left(\frac{T}{300}\right)^3\exp\left[-\frac{1.12}{\left(8.62\times10^{-5}\right)\times T}\right] = 8.28\times10^{27}$$

$$\therefore n_i^2 = 1.08\times10^{31}T^3\exp\left(-1.30\times10^4T^{-1}\right) = 8.28\times10^{27}$$

The equation above is transcendental in nature and requires numerical methods to be solved. One way to go is to apply MATLAB's *fzero* function:

```
function y = intrinsic(T)
y = 1.08E31*T^3*exp(-1.3E4*T^-1)-8.28E27;

>> fun = @intrinsic;
x0 = 400;
z = fzero(fun,x0)
```

This returns $T_2 \approx$ 503 K. Gleaning our results, observe that the first specification limits the temperature of the diode to T_1 = 583 K, while the second specification limits the temperature of the device to T_2 = 503 K. The lower temperature governs, therefore the reverse-bias current limit of 1.2 μA is the limiting factor and the diode is to operate at a temperature no greater than 503 K.

P.12 ➡ *Solution*

The diode voltage V_D and current I_D are related by the ideal diode equation as

$$I_D = I_S\left[e^{\left(\frac{V_D}{V_T}\right)} - 1\right] \rightarrow I_D = \left(30\times10^{-9}\right)\times\left[e^{\left(\frac{V_D}{0.026}\right)} - 1\right] \quad \text{(I)}$$

If a dc voltage V_{PS} = 1.5 V is applied to the circuit with resistance R = 10 MΩ, we can write Kirchhoff's voltage law for the diode voltage V_D and current I_D,

$$V_{PS} = I_D R + V_D \rightarrow 1.5 = \left(10\times10^6\right)I_D + V_D \quad \text{(II)}$$

Equations (I) and (II) constitute a system of two nonlinear equations with two unknowns, I_D and V_D. Substituting (I) in (II) brings to

$$1.5 = \left(10\times10^6\right)\times\left\{\left(30\times10^{-9}\right)\times\left[e^{\left(\frac{V_D}{0.026}\right)} - 1\right]\right\} + V_D$$

This equation can be solved with MATLAB's *fzero* command,

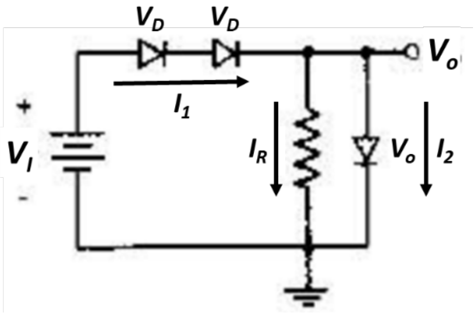
```
function y = diode(VD)
y = 1.5 - 10E6*30E-9*(exp(VD/0.026)-1) - VD;

>> fun = @diode;
x0 = 0.01;
z = fzero(fun,x0)
```

This returns V_D = 0.0459 V. Substituting in (I) yields the diode current I_D,

$$I_D = \left(30\times10^{-9}\right)\times\left[e^{(0.0459/0.026)} - 1\right] = 1.45\times10^{-7}\text{ A} = \boxed{0.145\,\mu\text{A}}$$

P.13 ➡ *Solution*

Refer to the figure below.



Noting that the diode below the output node is subjected to a potential difference V_o, which we prescribe as 0.6 V, current I_2 is calculated as

$$I_2 = I_S \exp\left(\frac{V_o}{V_T}\right) = \left(2\times10^{-13}\right)\times\exp\left(\frac{0.6}{0.026}\right) = 0.0021\text{ A} = 2.1\text{ mA}$$

The 1-kΩ resistor is subjected to the same voltage V_o and conducts a current I_R such that

$$I_R = \frac{V_o}{1\text{k}\Omega} = \frac{0.6}{10^3} = 0.6\text{ mA}$$

Using Kirchhoff's current law, the current I_1 flowing through the two in-series diodes is

$$I_1 = I_R + I_2 = 0.6 + 2.1 = 2.7\text{ mA}$$

The voltage V_D in either of these two diodes is then

$$V_D = V_T \ln\left(\frac{I_1}{I_S}\right) = 0.026\times\ln\left(\frac{2.7\times10^{-3}}{2\times10^{-13}}\right) = 0.606\text{ V}$$

Using Kirchhoff's voltage law, the input voltage V_I is calculated to be

$$V_I - V_D - V_D = V_o \rightarrow V_I = V_o + 2V_D$$

$$\therefore V_I = 0.6 + 2\times0.606 = \boxed{1.81\text{ V}}$$

P.14 ➡ Solution

Problem 14.1: If the current flowing through the diode is I = 0.5 mA, the diode voltage V_D is determined to be

$$V_D = V_T \ln\left(\frac{I}{I_S}\right) = 0.026 \times \ln\left(\frac{0.5\times10^{-3}}{5\times10^{-12}}\right) = \boxed{0.479 \text{ V}}$$

Applying Kirchhoff's voltage law, we can determine the supply voltage V,

$$5 - RI - V_D - V = 0 \rightarrow V = 5 - RI - V_D$$

$$\therefore V = 5 - \left(4.7\times10^{3}\right)\times\left(0.5\times10^{-3}\right) - 0.479 = \boxed{2.17 \text{ V}}$$

Finally, the power dissipated in the diode is

$$P_D = V_D I = 0.479 \times \left(0.5\times10^{-3}\right) = \boxed{0.240 \text{ mW}}$$

Problem 14.2: If the cut-in voltage of the diode is 0.65 V and the power dissipated is to be no greater than 0.20 mW, current I_D flowing through the loop is restricted to

$$P_D = V_\gamma I_D \rightarrow I_D = \frac{P_D}{V_\gamma} = \frac{0.20\times10^{-3}}{0.65} = 0.308 \text{ mA}$$

Applying Kirchhoff's voltage law to the loop yields

$$5 - RI - 0.65 - 1.7 = 0 \rightarrow R = \frac{5 - 0.65 - 1.7}{0.308\times10^{-3}} = 8600\ \Omega = \boxed{8.60 \text{ k}\Omega}$$

P.15 ➡ Solution

In circuit (*a*), the diode is conducting and contributes to the loop with an amount equal to the cut-in voltage V_γ. Applying KVL, we obtain

$$5 - 10{,}000 I_D - V_\gamma + 5 = 0 \rightarrow I_D = \frac{10 - V_\gamma}{10\text{k}}$$

$$\therefore I_D = \frac{10 - 0.7}{10\text{k}} = \boxed{0.94 \text{ mA}}$$

Voltage V_O is then

$$0 - 5 + V_\gamma = V_O \rightarrow V_O = -5 + 0.6 = \boxed{-4.4 \text{ V}}$$

Circuit (*b*) is simply a rearrangement of circuit (*a*). Current I_D continues to be 0.94 mA and, using KVL, voltage V_O is determined to be

$$0 + 5 - V_\gamma = V_O \rightarrow V_O = 5 - 0.6 = \boxed{+4.4 \text{ V}}$$

In circuit (*c*), notice the orientation of the power supplies relatively to the diode; the diode is reverse biased, hence I_D = 0 and V_D = −10 V.

P.16 ➡ Solution

In the mildest configuration for this circuit, a voltage equal to the cut-in value V_γ = 0.7 V appears in the diode and the diode conducts a current $I_{D,\text{min}}$ = 2 mA. The current flowing through resistance R_2, which is in parallel to the diode, is

$$I_2 = \frac{0.7}{R_2}$$

With the voltage supply set to its minimum value of 5 V, the current through resistor R_1 is determined as

$$I_1 = \frac{5 - 0.7}{R_1}$$

From Kirchhoff's current law,

$$I_1 = I_2 + I_D \rightarrow \frac{4.3}{R_1} = \frac{0.7}{R_2} + 2 \quad \text{(I)}$$

Now, when the circuit is set to its most extreme conditions, the voltage supply feeds 10 V to the circuit and the diode, which is to dissipate a power no greater than 10 mW, conducts a current $I_{D,\text{max}}$ such that

$$P_{max} = I_{D,max} V_D \rightarrow I_{D,max} = \frac{P_{max}}{V_D} = \frac{10}{0.7} = 14.3\,\text{mA}$$

Resistor R_2 conducts a current $I_2 = 0.7/R_2$, while R_1 receives $I_1 = (10 - 0.7)/R_1$. Applying KCL a second time and substituting from (I), we find that

$$I_1 = I_2 + I_D \rightarrow \frac{9.3}{R_1} = \frac{0.7}{R_2} + 14.3$$

$$\therefore \frac{9.3}{R_1} = \left(\frac{4.3}{R_1} - 2\right) + 14.3$$

$$\therefore \frac{5}{R_1} = 12.3$$

$$\therefore R_1 = \frac{5}{12.3} = 0.407\ \text{k}\Omega = \boxed{407\,\Omega}$$

Substituting in (I) and solving for R_2, we get

$$\frac{4.3}{R_1} = \frac{0.7}{R_2} + 2 \rightarrow \frac{4.3}{0.407} = \frac{0.7}{R_2} + 2$$

$$\therefore 10.6 - 2 = \frac{0.7}{R_2}$$

$$\therefore R_2 = \frac{0.7}{8.6} = 0.0814\,\text{k}\Omega = \boxed{81.4\,\Omega}$$

Resistances R_1 and R_2 have been specified.

P.17 ➡ *Solution*

Let I_R denote the current flowing through resistor R_2. Applying Kirchhoff's current law and noting that $I_{D1} = I_{D2}/2$ as prescribed, we get

$$I_{D2} = I_R + I_{D1} \rightarrow 2I_{D1} = I_R + I_{D1}$$

$$\therefore I_{D1} = I_R \quad \text{(I)}$$

Resistor R_2 is in series with diode D_1 and hence withstands a potential difference $V_\gamma = 0.65$ V. The corresponding current is

$$I_R = \frac{0.65}{R_2} = \frac{0.65}{10^3} = 0.65\ \text{mA}$$

so that, using (I),

$$I_{D1} = I_R = \boxed{0.65\,\text{mA}}$$

Then, recalling that $I_{D2} = 2I_{D1}$,

$$I_{D2} = 2 \times 0.65 = \boxed{1.3\,\text{mA}}$$

If 1.3 milliamperes flow through resistor R_1, the corresponding resistance must be, accounting for the voltages that appear in all three diodes,

$$R_1 = \frac{5 - 0.65 - 0.65 - 0.65}{1.3 \times 10^{-3}} = 2350\,\Omega = \boxed{2.35\,\text{k}\Omega}$$

P.18 ➡ *Solution*

Circuit (a): The current flowing through diode D_1 is initially given by

$$I_{D1} = \frac{V_{in} - V_\gamma}{R_1} = \frac{2.4 - 0.8}{10^3} = 1.6\,\text{mA}$$

Now, for calculation of voltage changes in the small-signal model, diode D_1 can be replaced with a linear resistor r_d given by the ratio of the device's thermal voltage V_T to the current flowing through it; mathematically,

$$r_d = \frac{V_T}{I_{D1}} = \frac{0.026}{1.6 \times 10^{-3}} = 16.3\,\Omega$$

Finally, using the voltage divider rule,

$$\Delta V_{out} = \frac{R_1}{r_d + R_1} \Delta V_{in} = \frac{1000}{16.3 + 1000} \times (2500 - 2400) = \boxed{98.4\,\text{mV}}$$

Circuit (b): Firstly, the current flowing through either diode is given by

$$I_{D1} = I_{D2} = \frac{V_{in} - 2V_\gamma}{R_1} = \frac{2.4 - 2\times 0.8}{10^3} = 0.8\,\text{mA}$$

For voltage change calculation purposes, the diodes can be replaced with linear resistors r_{d1} and r_{d2} such that

$$r_{d1} = r_{d2} = \frac{V_T}{I_{D1}} = \frac{V_T}{I_{D2}} = \frac{0.026}{0.8\times 10^{-3}} = 32.5\,\Omega$$

Lastly,

$$\Delta V_{out} = \frac{r_{d2} + R_1}{r_{d1} + r_{d2} + R_1}\Delta V_{in} = \frac{1032.5}{32.5 + 32.5 + 1000}\times(2500 - 2400) = \boxed{96.9\,\text{mV}}$$

Circuit (c): This is a slightly modified version of circuit (c). The current flowing through either diode continues to be

$$I_{D1} = I_{D2} = 0.8\,\text{mA}$$

while resistances r_{d1} and r_{d2} remain

$$r_{d1} = r_{d2} = 32.5\,\Omega$$

Now, noting that resistor R_1 has been displaced from the lower branch of the circuit to the segment that joins the V_{in} node to the V_{out} node, it no longer appears in the numerator of the voltage divider rule,

$$\Delta V_{out} = \frac{r_{d2}}{r_{d1} + r_{d2} + R_1}\Delta V_{in} = \frac{32.5}{32.5 + 32.5 + 1000}\times(2500 - 2400) = \boxed{3.05\,\text{mV}}$$

Circuit (d): In this case, the current flowing through diode D_2 equals the current stemming directly from the V_{in} node minus the amount deviated to resistor R_2,

$$I_{D2} = \frac{V_{in} - V_D}{R_1} - \frac{V_D}{R_2} = \frac{2.4 - 0.8}{10^3} - \frac{0.8}{2.0\times 10^3} = 1.2\,\text{mA}$$

The diode can be replaced with a resistance r_{d2} given by

$$r_{d2} = \frac{V_T}{I_{D2}} = \frac{0.026}{1.2\times 10^{-3}} = 21.7\,\Omega$$

Finally, we apply the voltage divider rule to obtain

$$\Delta V_{out} = \frac{R_2 \parallel r_{d2}}{R_1 + R_2 \parallel r_{d2}}\Delta V_{in} = \frac{\dfrac{2000\times 21.7}{2000 + 21.7}}{1000 + \dfrac{2000\times 21.7}{2000 + 21.7}}\times(2500 - 2400) = \boxed{2.10\,\text{mV}}$$

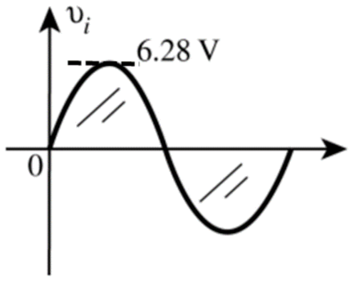
P.19 ➡ *Solution*

Problem 19.1: Per the problem statement, the input voltage is a sinusoidal waveform with 60-Hz frequency; from the circuit diagram, we see that the diode is supplied with a dc voltage V_{dc} = 2 V. It follows that the input sine wave will peak at a voltage V_m such that

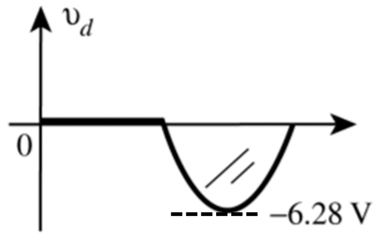
$$V_{dc} = 0.318V_m \rightarrow V_m = \frac{V_{dc}}{0.318}$$

$$\therefore V_m = \frac{2.0}{0.318} = 6.28\,\text{V}$$

The input v_i is sketched below.



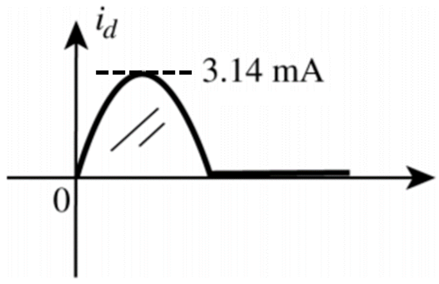
Now, as the diode is functioning as a half-wave rectifier, it will respond to v_i by suppressing the first half-period of the input sine wave and reproducing the second.



The average current crossing the diode is

$$I_m = \frac{V_m}{R} = \frac{6.28}{2000} = 3.14\,\text{mA}$$

The diode conducts for the positive half-cycle, leading to the following current profile,



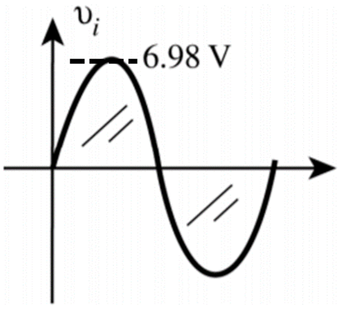
Problem 19.2: In this case, the input voltage will peak at a new value V_m' such that

$$V_{\text{dc}} \approx 0.318\left(V_m' - V_K\right) \rightarrow 2 = 0.318\left(V_m' - 0.7\right)$$

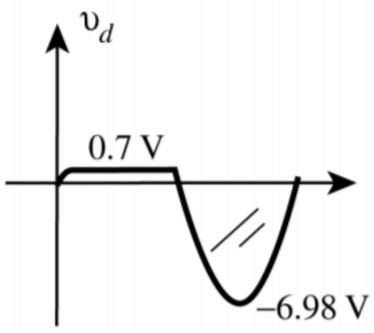
$$\therefore 2 = 0.318V_m' - 0.223$$

$$\therefore V_m' = \frac{2 + 0.223}{0.318} = 6.98\,\text{V}$$

The input voltage is sketched below.



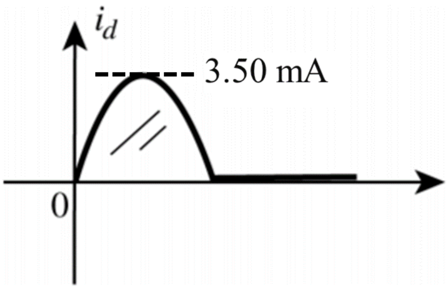
The voltage output profile is similar to the one obtained for the ideal diode, but differs from it in that the positive half-cycle will retain a positive plateau of V_K = 0.7 V that the rectifier cannot suppress.



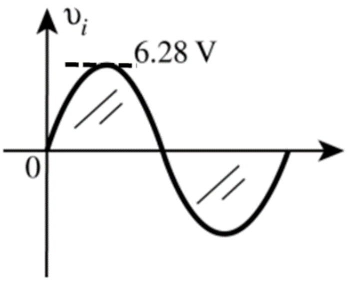
Lastly, the current profile will peak at an updated average value i_m' given by

$$i_m' = \frac{V_m'}{R} = \frac{6.99}{2000} = 3.50\,\text{mA}$$

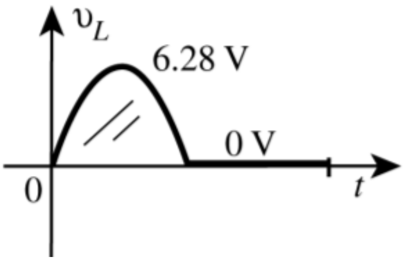
The waveform is sketched below.



Problem 19.3: The input waveform v_i remains unchanged relative to Problem 19.1.



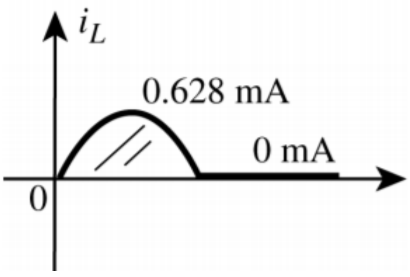
The load R_L sees a voltage waveform with a positive half-cycle only, as illustrated below.



The peak current flowing through R_L is

$$i_{L,\max} = \frac{6.28\,\text{V}}{10\,\text{k}\Omega} = 0.628\,\text{mA}$$

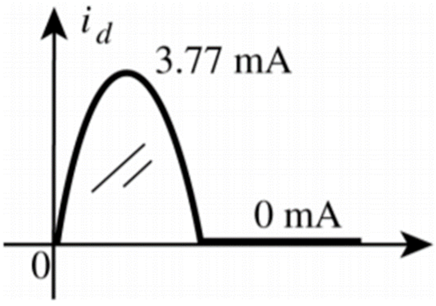
The i_L, waveform is shown in continuation.



Now, noting that the maximum current through the 2-kΩ resistor is $i_{2,max}$ = 6.28/2000 = 3.14 mA, the maximum current $i_{d,max}$ flowing in the diode follows as

$$i_{D,\max} = i_{L,\max} + i_{2,\max} = 0.628 + 3.14 = 3.77\,\text{mA}$$

The pertaining waveform is shown below.



P.20 ➡ *Solution*

A silicon diode will conduct when v_o = 0.7 V. Using the voltage divider rule and solving for input voltage v_i, we obtain

$$v_o = 0.7 = \frac{1.0 \times v_i}{1.0 + 1.0} \rightarrow v_i = 1.4\,\text{V}$$

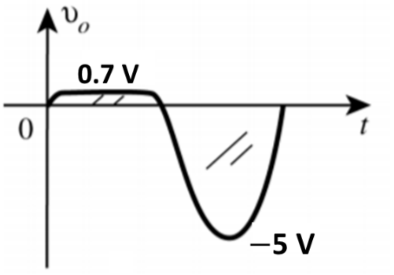
Accordingly, for $v_i \geq 1.4$ V the diode is “on” and v_o = 0.7 V; if $v_i \leq 1.4$ V, the diode is open and the level of v_o is determined by the voltage divider rule,

$$v_o = \frac{1.0 \times v_i}{1.0 + 1.0} = 0.5 v_i$$

With v_i = –10 V, the peak output voltage is then

$$v_o = 0.5 \times (-10) = -5.0\text{ V}$$

The v_o waveform is sketched below.



When v_o = 0.7 V, the maximum voltage across the resistor that receives current i_R is

$$v_{R_{\max}} = v_{i_{\max}} - 0.7 \rightarrow v_{R_{\max}} = 10 - 0.7 = 9.3\,\text{V}$$

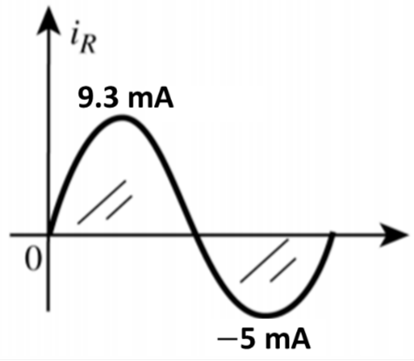
and corresponds to a peak current $i_{R_{\max}}$ such that

$$I_{R_{\max}} = \frac{9.3}{1\,\text{k}\Omega} = 9.3\,\text{mA}$$

The maximum reverse current is, in turn,

$$I_{\max}\left(\text{reverse}\right) = \frac{-10}{1\,\text{k}\Omega + 1\,\text{k}\Omega} = -5\,\text{mA}$$

The i_R waveform is sketched below.



P.21 ➡ *Solution*

Problem 21.1: Noting that "turn-on" voltage of a silicon diode is 0.7 V and the specified power rating is 14 mW, current I_D is determined as

$$I_D = \frac{14\times10^{-3}}{0.7} = \boxed{20\,\text{mA}}$$

Problem 21.2: If each of the two diodes is conducting a current of 20 mA, the maximum current $I_{\max}$ is found as

$$I_{\max} = 2I_D = 2\times20 = \boxed{40\,\text{mA}}$$

Problem 21.3: We aim to determine the largest current through each diode, which should occur when the input voltage waveform reaches its peak value of 160 V. The two resistances of 4.7 kΩ and 68 kΩ can be replaced with an equivalent resistance R given by

$$R = \left(4.7\times10^3\right) \| \left(68\times10^3\right) = 4.40\,\text{k}\Omega$$

The maximum voltage across this equivalent resistance equals 160 V minus the 0.7 V that appears in the two-diode arrangement; that is,

$$v_{\max} = 160 - 0.7 = 159.3\,\text{V}$$

The corresponding current is

$$I_{\max} = \frac{159.3}{4400} = 0.0362\,\text{A} = 36.2\,\text{mA}$$

Because the two diodes are identical and in parallel, each one receives a current I_d given by

$$I_d = \frac{36.2}{2} = \boxed{18.1\,\text{mA}}$$

Note that this current is lower than the 20-mA current rating obtained in Part 1, which indicates that the two diodes would operate safely even at the peak value of the input voltage waveform.

Operating the circuit with a single diode will cause one of the diodes to conduct a current of 36.2 mA at the peak of the input voltage waveform; since this current far exceeds the 20-mA current rating obtained in Part 1, the diode would be damaged.

P.22 ➡ *Solution*

Problem 22.1: Using the amplitude of the rms sinusoidal input, we obtain the average voltage

$$V_m = \sqrt{2}V\left(\text{rms}\right) = \sqrt{2}\times120 = 169.7\text{ V}$$

The voltage across the load, accounting for the silicon (V_K = 0.7 V) diodes, is

$$V_{L_m} = V_m - 2V_K = 169.7 - 2\times 0.7 = 168.3\text{ V}$$

Converting this result to a dc quantity,

$$V_{dc} \approx 0.636V_{L_m} = 0.636\times 168.3 = \boxed{107.04\text{ V}}$$

Problem 22.2: The peak inverse voltage is given by

$$\text{PIV} = V_{L_m} + V_K = 168.3 + 0.7 = \boxed{169.0\text{ V}}$$

Problem 22.3: To find the maximum current through the diodes, divide the load voltage V_{L_m} by the load resistance,

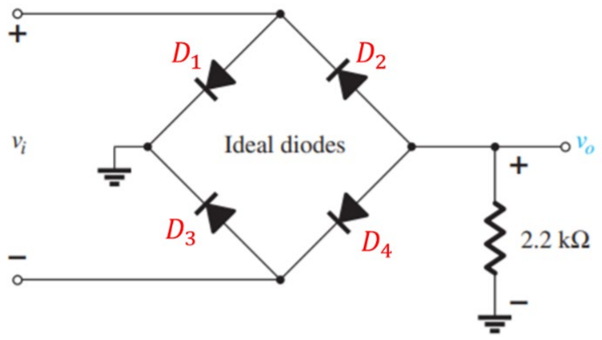
$$I_{D,\max} = \frac{V_{L_m}}{R_L} = \frac{168.3}{1000} = 0.168\text{ A} = \boxed{168\text{ mA}}$$

Problem 22.4: To find the power rating of each diode, use V_K = 0.7 V and the maximum current obtained just now,

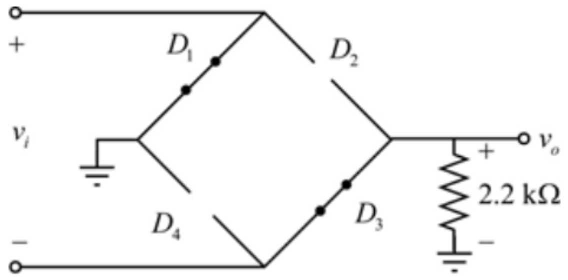
$$P_D = V_K I_{D,\max} = 0.7\times 0.168 = 0.118\text{ W} = \boxed{118\text{ mW}}$$

P.23 ➡ *Solution*

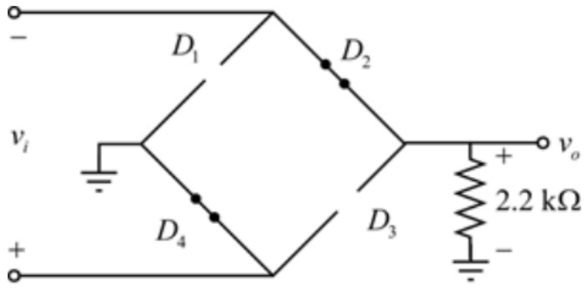
Let the four diodes be numbered as follows.



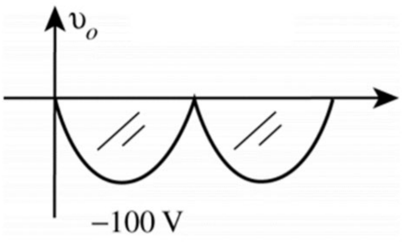
For the positive half-cycle of the input, diodes D_1 and D_3 are forward biased, whereas diodes D_2 and D_4 are reverse biased, as shown. It follows that, for the positive half-cycle, the output voltage is $v_o = -v_i$.



For the negative half-cycle of the input, diodes D_2 and D_4 are forward biased, whereas diodes D_1 and D_3 are reverse biased, as shown. It follows that, for the positive half-cycle, the output voltage is $v_o = v_i$.



With the two previous observations in mind, we can sketch the output voltage v_o:



The peak inverse voltage of the diodes is

$$\boxed{PIV = 100\text{ V}}$$

To establish the maximum current through each diode, apply Ohm's law with $|v_{o,\max}|$ = 100 V as the voltage,

$$i_{D,\max} = \frac{|v_{o,\max}|}{R_L} = \frac{100}{2200} = 0.0455\text{ A} = \boxed{45.5\text{ mA}}$$

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