Montogue

Quiz EL104

Economic Dispatch

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PROBLEMS

Problem 1 (Saadat, 1999, w/ permission)

Problem 1.1: The fuel-cost functions in \$/h for two 800 MW thermal plants are given by

$$\begin{cases} C_1 = 400 + 6.0P_1 + 0.004P_1^2 \\ C_2 = 500 + \beta P_2 + \gamma P_2^2 \end{cases}$$

The incremental cost of power λ is \$8/MWh when the total power demand is 550 MW. Neglecting losses, determine the optimal generation of each plant.

Problem 1.2: The incremental cost of power λ is \$10/MWh when the total power demand is 1300 MW. Neglecting losses, determine the optimal generation of each plant.

Problem 1.3: From the results of Parts 1 and 2, find the fuel-cost coefficients β and γ of the second plant.

Problem 2 (Saadat, 1999, w/ permission)

The fuel-cost functions in \$/h for three thermal plants are given by

$$\begin{cases} C_1 = 350 + 7.20P_1 + 0.0040P_1^2 \\ C_2 = 500 + 7.30P_2 + 0.0025P_2^2 \\ C_3 = 600 + 6.74P_3 + 0.0030P_3^2 \end{cases}$$

where P_1, P_2, and P_3 are in MW. The governors are set such that generators share the load equally. Neglecting line losses and generator limits, find the total cost in \$/h when the total load is

Problem 2.1: P_D = 450 MW.

Problem 2.2: P_D = 750 MW.

Problem 3 (Saadat, 1999, w/ permission)

Neglecting line losses and generator limits, determine the optimal scheduling of generation for each loading condition of Problem 2

Problem 3.1: using the analytical technique described in Chapter 7 of Saadat (1999).

Problem 3.2: using the iterative method described in Saadat's textbook.

Problem 4 (Saadat, 1999, w/ permission)

The fuel-cost function in \$/h of two thermal plants are

$$\begin{cases} C_1 = 320 + 6.2P_1 + 0.004P_1^2 \\ C_2 = 200 + 6.0P_2 + 0.003P_2^2 \end{cases}$$

where P_1 and P_2 are in MW. Plant outputs are subject to the following limits (in MW):

$$\begin{pmatrix} 50 \le P_1 \le 250\,\text{MW} \\ 50 \le P_2 \le 350\,\text{MW} \end{pmatrix}$$

The per-unit system real power loss with generation expressed in per unit on a 100-MVA base is given by

$$P_{L,(\text{pu})} = 0.0125P_{1,(\text{pu})}^2 + 0.00625P_{2,(\text{pu})}^2$$

The total load is 412.35 MW. Determine the optimal dispatch of generation. Start with an initial estimate of incremental cost of power λ = 7 \$/MWh.

Problem 5 (Grainger and Stevenson, Jr., 1994)

Problem 5.1: The incremental fuel costs in \$/MWh for four units of a plant are specified below.

$$\begin{cases} \lambda_1 = 0.012P_1 + 9.0 \\ \lambda_2 = 0.0096P_2 + 6.0 \\ \lambda_3 = 0.008P_3 + 8.0 \\ \lambda_4 = 0.0068P_4 + 10.0 \end{cases}$$

Assuming that all four units operate to meet the total plant load of 800 MW, find the incremental fuel cost λ of the plant and the required output of each unit for economic dispatch.

Problem 5.2: Assume that maximum loads on each of the four units described in Problem 5.1 are 200 MW, 400 MW, 270 MW, and 300 MW, respectively, and that minimum load on each unit is 50 MW, 100 MW, 80 MW, and 110 MW, respectively. With these maximum and minimum output limits, find the incremental fuel cost λ and MW output of each unit for economic dispatch.

Problem 5.3: Solve the previous problem if the minimum load on unit 4 is 50 MW rather than 110 MW.

Problem 6 (Grainger and Stevenson, Jr., 1994)

The incremental fuel costs for two units of a plant are

$$\begin{cases} \lambda_1 = 0.012P_1 + 8.0 \\ \lambda_2 = 0.008P_2 + 9.6 \end{cases}$$

where λ's are in dollars per hour and P's are in MW. If both units operate at all times and maximum and minimum loads on each unit are 550 MW and 100 MW, plot the incremental fuel cost λ of the plant in \$/MWh versus plant output in MW for economic dispatch as total load varies from 200 to 1100 MW.

SOLUTIONS

P.1 ➡ *Solution*

Problem 1.1: We begin by finding the fuel-cost coefficients β and γ of the second plant. Differentiating C_1 with respect to P_1 and C_2 with respect to P_2 brings to

$$\frac{dC_1}{dP_1} = 6.0 + 0.008P_1 = \lambda$$

$$\frac{dC_2}{dP_2} = \beta + 2\gamma P_2 = \lambda$$

With λ = 8 and P_D = 550 MW, we have

$$6.0 + 0.008P_1 = 8 \rightarrow P_1 = \frac{8 - 6.0}{0.008} = 250\,\text{MW}$$

and

$$P_2 = P_D - P_1 = 550 - 250 = 300\,\text{MW}$$

Problem 1.2: The same formulas used in the previous part apply here, namely

$$P_1 = \frac{10 - 6.0}{0.008} = 500\,\text{MW}$$

and

$$P_2 = P_D - P_1 = 1300 - 500 = 800\,\text{MW}$$

Problem 1.3: Using the two data points given in parts 1 and 2, we have, for the incremental cost of plant 2,

$$\beta + 2\gamma P_2 = \lambda \rightarrow \beta + 2\gamma \times 300 = 8$$

$$\therefore \beta + 600\gamma = 8 \quad \text{(I)}$$

$$\beta + 2\gamma P_2 = \lambda \rightarrow \beta + 2\gamma \times 800 = 10$$

$$\therefore \beta + 1600\gamma = 10 \quad \text{(II)}$$

Manipulating equation (II) brings to

$$\beta+1600\gamma=\underbrace{\beta+600\gamma}_{=8}+1000\gamma=10$$

$$\therefore 8+1000\gamma=10$$

$$\therefore \gamma=\frac{10-8}{1000}=\boxed{0.002}$$

Substituting γ in equation (I),

$$\beta+600\gamma=\beta+600\times 0.002=8$$

$$\therefore \beta+1.2=8$$

$$\therefore \boxed{\beta=6.8}$$

P.2 ➡ Solution

Problem 2.1: Since the governors are to share the load equally, $P_1=P_2=P_3=450/3=150$ MW. The total fuel cost is

$$C_t=\left(350+7.20\times 150+0.004\times 150^2\right)+\left(500+7.3\times 150+0.0025\times 150^2\right)$$
$$+\left(600+6.74\times 150+0.003\times 150^2\right)=\boxed{4{,}849.75\ \$/\text{h}}$$

Problem 2.2: With $P_D=750$ MW, we have $P_1=P_2=P_3=750/3=250$ MW. The total fuel cost is

$$C_t=\left(350+7.20\times 250+0.004\times 250^2\right)+\left(500+7.3\times 250+0.0025\times 250^2\right)$$
$$+\left(600+6.74\times 250+0.003\times 250^2\right)=\boxed{7{,}353.75\ \$/\text{h}}$$

P.3 ➡ Solution

Problem 3.1: The optimum incremental cost λ for n plants with fuel-cost functions of the form $C_i=c+\beta P_i+\gamma P_i^2$ is given by

$$\lambda=\frac{P_D+\sum_{i=1}^{n}\frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n}\frac{1}{2\gamma_i}}$$

$$\therefore \lambda=\frac{450+\frac{7.20}{2\times 0.004}+\frac{7.30}{2\times 0.0025}+\frac{6.74}{2\times 0.0030}}{\frac{1}{2\times 0.004}+\frac{1}{2\times 0.0025}+\frac{1}{2\times 0.0030}}=8\ \$/\text{MWh}$$

Substituting λ in the coordination equations gives the optimal dispatch

$$P_i=\frac{\lambda-\beta_i}{2\gamma_i}$$

$$P_1=\frac{8-7.20}{2\times 0.0040}=100\ \text{MW}$$

$$P_2=\frac{8-7.30}{2\times 0.0025}=140\ \text{MW}$$

$$P_3=\frac{8-6.74}{2\times 0.0030}=210\ \text{MW}$$

The optimized total cost is

$$C_t=\left(350+7.20\times 100+0.004\times 100^2\right)+\left(500+7.3\times 140+0.0025\times 140^2\right)$$
$$+\left(600+6.74\times 210+0.003\times 210^2\right)=\boxed{4{,}828.70\ \$/\text{h}}$$

which represents savings of 4,849.75 – 4,828.70 = $21.05 relatively to the cost implied when the load is shared equally between the three generators.

Repeating the procedure above with $P_D=750$ MW, the optimum incremental cost λ is

$$\lambda=\frac{750+\frac{7.20}{2\times 0.004}+\frac{7.30}{2\times 0.0025}+\frac{6.74}{2\times 0.0030}}{\frac{1}{2\times 0.004}+\frac{1}{2\times 0.0025}+\frac{1}{2\times 0.0030}}=8.61\ \$/\text{MWh}$$

so that

$$P_1 = \frac{8.61 - 7.20}{2 \times 0.0040} = 176.3\,\text{MW}$$

$$P_2 = \frac{8.61 - 7.30}{2 \times 0.0025} = 262\,\text{MW}$$

$$P_3 = \frac{8.61 - 6.74}{2 \times 0.0030} = 311.7\,\text{MW}$$

The optimized total cost is

$$C_t = \left(350 + 7.20 \times 176.3 + 0.004 \times 176.3^2\right) + \left(500 + 7.3 \times 262 + 0.0025 \times 262^2\right)$$
$$+\left(600 + 6.74 \times 311.7 + 0.003 \times 311.7^2\right) = \boxed{7{,}320.23\ \$/\text{h}}$$

which represents savings of $33.52 relatively to the cost implied when the load is shared equally between the three generators.

Problem 3.2: For the numerical solution using the gradient method, we assume an initial value $\lambda^{(1)} = 6$. From the coordination equations,

$$P_1^{(1)} = \frac{6 - 7.20}{2 \times 0.0040} = -150$$

$$P_2^{(1)} = \frac{6 - 7.30}{2 \times 0.0025} = -260$$

$$P_3^{(1)} = \frac{6 - 6.74}{2 \times 0.0030} = -123.3$$

The error ΔP, noting that $P_D = 450$ MW,

$$\Delta P^{(1)} = 450 - \left(-150 - 260 - 123.3\right) = 933.3$$

The value of $\Delta\lambda^{(1)}$ is

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\sum \frac{1}{2\gamma_i}} \rightarrow \Delta\lambda^{(1)} = \frac{\Delta P^{(1)}}{\sum \frac{1}{2\gamma_i}}$$

$$\therefore \Delta\lambda^{(1)} = \frac{933.3}{\frac{1}{2 \times 0.0040} + \frac{1}{2 \times 0.0025} + \frac{1}{2 \times 0.0030}} = 1.898$$

Thus, in the next iteration we shall use

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 6.0 + 1.898 = 7.898$$

Accordingly,

$$P_1^{(2)} = \frac{7.898 - 7.20}{2 \times 0.0040} = 87.25$$

$$P_2^{(2)} = \frac{7.898 - 7.30}{2 \times 0.0025} = 119.6$$

$$P_3^{(2)} = \frac{7.898 - 6.74}{2 \times 0.0030} = 193$$

The error ΔP is updated as

$$\Delta P^{(2)} = 450 - \left(87.25 + 119.6 + 193\right) = 50.15$$

The value of $\Delta\lambda^{(2)}$ is

$$\Delta\lambda^{(2)} = \frac{50.15}{\frac{1}{2 \times 0.0040} + \frac{1}{2 \times 0.0025} + \frac{1}{2 \times 0.0030}} = 0.102$$

The value of λ for the next iteration is then

$$\lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 7.898 + 0.102 = 8.000$$

Accordingly,

$$P_1^{(3)} = \frac{8.0 - 7.20}{2 \times 0.0040} = 100$$

$$P_2^{(3)} = \frac{8.0-7.30}{2\times 0.0025} = 140$$

$$P_3^{(3)} = \frac{8.0-6.74}{2\times 0.0030} = 210$$

The error ΔP is updated as

$$\Delta P^{(2)} = 450-\left(100+140+210\right) = 0$$

The error has been reduced to zero; convergence has been attained, and $P_1^{(3)}$ = 100, $P_2^{(3)}$ = 140, $P_3^{(3)}$ = 210 MW are the final optimized loads. Optimizing Problem 2.2 iteratively is no different from the previous situation. In the numerical analysis of Problem 2.1, we began with λ = 6.0, an <u>under</u>estimate relatively to the true value λ = 8.0. To make things a little different, let's start by <u>over</u>estimating λ relatively to the true value λ = 8.61; take $\lambda^{(1)}$ = 10.

$$P_1^{(1)} = \frac{10-7.20}{2\times 0.0040} = 350$$

$$P_2^{(1)} = \frac{10-7.30}{2\times 0.0025} = 540$$

$$P_3^{(1)} = \frac{10-6.74}{2\times 0.0030} = 543.333$$

With P_D = 750 W, error ΔP is calculated as

$$\Delta P^{(1)} = 750-\left(350+540+543.333\right) = -683.333$$

The value of $\Delta\lambda^{(1)}$ is

$$\Delta\lambda^{(1)} = \frac{-683.333}{\frac{1}{2\times 0.0040}+\frac{1}{2\times 0.0025}+\frac{1}{2\times 0.0030}} = -3.431$$

Thus, in the next iteration we shall use

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 10.0-3.431 = 6.569$$

Accordingly,

$$P_1^{(2)} = \frac{6.569-7.20}{2\times 0.0040} = -78.875$$

$$P_2^{(2)} = \frac{6.569-7.30}{2\times 0.0025} = -146.2$$

$$P_3^{(2)} = \frac{6.569-6.74}{2\times 0.0030} = -28.5$$

The error ΔP is updated as

$$\Delta P^{(2)} = 750-\left(-78.875-146.2-28.5\right) = 1003.58$$

The value of $\Delta\lambda^{(1)}$ is

$$\Delta\lambda^{(2)} = \frac{1003.58}{\frac{1}{2\times 0.0040}+\frac{1}{2\times 0.0025}+\frac{1}{2\times 0.0030}} = 2.041$$

For the next iteration,

$$\lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 6.569+2.041 = 8.610$$

Accordingly,

$$P_1^{(3)} = \frac{8.610-7.20}{2\times 0.0040} = 176.3$$

$$P_2^{(3)} = \frac{8.610-7.30}{2\times 0.0025} = 262$$

$$P_3^{(3)} = \frac{8.610-6.74}{2\times 0.0030} = 311.7$$

The error ΔP is calculated as

$$\Delta P^{(3)} = 750-\left(176.3+262+311.7\right) = 0$$

Convergence is reached, and $P_1^{(3)}$ = 176.3, $P_2^{(3)}$ = 262, $P_3^{(3)}$ = 311.7 MW are taken as the final optimized loads.

P.4 ➡ Solution

The coordination equations for a situation including losses are slightly more complicated than the ones used in the previous problem. The equations have general form

$$P_i^{(k)} = \frac{\lambda^{(k)} - \beta_i}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)}$$

so that

$$P_1^{(1)} = \frac{\lambda^{(1)} - \beta_1}{2\left(\gamma_1 + \lambda^{(1)} B_{11}\right)} = \frac{7 - 6.2}{2 \times \left(0.004 + 7 \times 0.000125\right)} = 82.0513\,\text{MW}$$

$$P_2^{(1)} = \frac{\lambda^{(1)} - \beta_2}{2\left(\gamma_2 + \lambda^{(1)} B_{22}\right)} = \frac{7 - 6.0}{2 \times \left(0.003 + 7 \times 0.0000625\right)} = 145.4545\,\text{MW}$$

Note that we have amped up the number of decimal places to four in order to better account for the effect of losses. The real power loss is

$$P_L^{(1)} = 0.000125\left[P_1^{(1)}\right]^2 + 0.0000625\left[P_2^{(1)}\right]^2 = 0.000125 \times 82.0513^2 + 0.0000625 \times 145.4545^2$$

$$\therefore P_L^{(1)} = 2.1638\,\text{MW}$$

Since P_D = 412.35 MW, the error $\Delta P^{(1)}$ is given by

$$\Delta P^{(k)} = P_D + P_L^{(k)} - \sum_{i=1}^{n} P_i^{(k)}$$

$$\therefore \Delta P^{(1)} = P_D + P_L^{(1)} - \sum_{i=1}^{2} P_i^{(1)} = 412.35 + 2.1638 - \left(82.0513 + 145.4545\right) = 187.0080\,\text{MW}$$

The change in incremental power cost for iteration k is given by

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\sum \left(\frac{\partial P_i}{\partial \lambda}\right)^{(k)}}$$

where

$$\sum_{i=1}^{n} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(k)} = \sum_{i=1}^{n} \frac{\gamma_i + B_{ii}\beta_i}{2\left(\gamma_i + \lambda^{(k)} B_{ii}\right)^2}$$

so that

$$\sum_{i=1}^{2} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)} = \frac{0.004 + 0.000125 \times 6.2}{2\left(0.0004 + 7.0 \times 0.000125\right)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2\left(0.003 + 7.0 \times 0.0000625\right)^2} = 243.2701$$

and

$$\Delta\lambda^{(1)} = \frac{\Delta P^{(1)}}{\sum \left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)}} = \frac{187.008}{243.2701} = 0.7687$$

Updating the value of λ, we find that

$$\lambda^{(2)} = \lambda^{(1)} + \Delta\lambda^{(1)} = 7.0 + 0.7687 = 7.7687$$

Moving on to the second iteration, we compute

$$P_1^{(2)} = \frac{\lambda^{(2)} - \beta_1}{2\left(\gamma_1 + \lambda^{(2)} B_{11}\right)} = \frac{7.7687 - 6.2}{2 \times \left(0.004 + 7.7687 \times 0.000125\right)} = 157.7824$$

$$P_2^{(2)} = \frac{\lambda^{(2)} - \beta_2}{2\left(\gamma_2 + \lambda^{(2)} B_{22}\right)} = \frac{7.7687 - 6.0}{2 \times \left(0.003 + 7.7687 \times 0.0000625\right)} = 253.7194$$

Updating the losses P_L,

$$P_L^{(2)} = 0.000125\left[P_1^{(2)}\right]^2 + 0.0000625\left[P_2^{(2)}\right]^2$$

$$= 0.000125 \times 157.7824^2 + 0.0000625 \times 253.7194^2 = 7.1353$$

and the error ΔP,

$$\Delta P^{(2)} = P_D + P_L^{(2)} - \sum_{i=1}^{2} P_i^{(2)} = 412.35 + 7.1353 - \left(157.7824 + 253.7194\right) = 7.9835$$

Also,

$$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)} = \frac{0.004 + 0.000125 \times 6.2}{2\left(0.004 + 7.7687 \times 0.000125\right)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2\left(0.003 + 7.7687 \times 0.0000625\right)^2} = 235.5143$$

so that

$$\Delta\lambda^{(2)} = \frac{\Delta P^{(2)}}{\sum\left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)}} = \frac{7.9835}{235.5143} = 0.03390$$

Updating the value of λ,

$$\lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 7.7687 + 0.0339 = 7.8026$$

The fact that $\Delta\lambda^{(2)}$ is quite small indicates that we are close to convergence. Proceeding with the third iteration, we have

$$P_1^{(3)} = \frac{\lambda^{(3)} - \beta_1}{2\left(\gamma_1 + \lambda^{(3)} B_{11}\right)} = \frac{7.8026 - 6.2}{2 \times \left(0.004 + 7.8026 \times 0.000125\right)} = 161.0548$$

$$P_2^{(3)} = \frac{\lambda^{(3)} - \beta_2}{2\left(\gamma_2 + \lambda^{(3)} B_{22}\right)} = \frac{7.8026 - 6.0}{2 \times \left(0.003 + 7.8026 \times 0.0000625\right)} = 258.4252$$

Updating the losses P_L,

$$P_L^{(3)} = 0.000125\left[P_1^{(3)}\right]^2 + 0.0000625\left[P_2^{(3)}\right]^2$$

$$= 0.000125 \times 161.0548^2 + 0.0000625 \times 258.4252^2 = 7.4163$$

and the error ΔP,

$$P^{(3)} = P_D + P_L^{(3)} - \sum_{i=1}^{2} P_i^{(3)} = 412.35 + 7.4163 - \left(161.0548 + 258.4252\right) = 0.2863$$

Further,

$$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)} = \frac{0.004 + 0.000125 \times 6.2}{2\left(0.004 + 7.8026 \times 0.000125\right)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2\left(0.003 + 7.8026 \times 0.0000625\right)^2} = 235.1810$$

giving

$$\Delta\lambda^{(3)} = \frac{\Delta P^{(3)}}{\sum\left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)}} = \frac{0.2863}{235.1810} = 0.0012$$

The new value of λ is

$$\lambda^{(4)} = \lambda^{(3)} + \Delta\lambda^{(3)} = 7.8026 + 0.0012 = 7.8038$$

Proceeding with the fourth iteration, we have

$$P_1^{(4)} = \frac{\lambda^{(4)} - \beta_1}{2\left(\gamma_1 + \lambda^{(4)} B_{11}\right)} = \frac{7.8038 - 6.2}{2 \times \left(0.004 + 7.8038 \times 0.000125\right)} = 161.1705$$

$$P_2^{(4)} = \frac{\lambda^{(4)} - \beta_2}{2\left(\gamma_2 + \lambda^{(4)} B_{22}\right)} = \frac{7.8038 - 6.0}{2 \times \left(0.003 + 7.8038 \times 0.0000625\right)} = 258.5917$$

Updating the losses P_L,

$$P_L^{(4)} = 0.000125\left[P_1^{(4)}\right]^2 + 0.0000625\left[P_2^{(4)}\right]^2$$

$$= 0.000125 \times 161.1705^2 + 0.0000625 \times 258.5917^2 = 7.4263$$

and the error ΔP,

$$\Delta P^{(4)} = P_D + P_L^{(4)} - \sum_{i=1}^{2} P_i^{(4)} = 412.35 + 7.4263 - \left(161.1705 + 258.5917\right) = 0.0141$$

Further,

$$\sum_{i=1}^{2}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(4)} = \frac{0.004 + 0.000125 \times 6.2}{2\left(0.004 + 7.8038 \times 0.000125\right)^2} + \frac{0.003 + 0.0000625 \times 6.0}{2\left(0.003 + 7.8038 \times 0.0000625\right)^2} = 235.1693$$

giving

$$\Delta\lambda^{(4)} = \frac{\Delta P^{(4)}}{\sum\left(\frac{\partial P_i}{\partial\lambda}\right)^{(4)}} = \frac{0.0141}{235.1693} = 0.0001$$

At this point, $\Delta\lambda$ is quite small and proceeding with a fifth iteration wouldn't improve the accuracy of the solution significantly. Thus, we take the fourth-iteration quantities as our final results:

$$\boxed{\begin{aligned} P_1 &= 161.1705\,\text{MW} \\ P_2 &= 258.5917\,\text{MW} \\ \lambda &= 7.8038\ \$/\text{MWh} \end{aligned}}$$

Note that P_1 and P_2 are within the specified limits. The power losses are P_L = 7.4263 MW. The optimal costs associated with plants 1 and 2 are, respectively,

$$C_1 = 320 + 6.2\times 161.1705 + 0.004\times 161.1705^2 = \boxed{1423.16\ \$/\text{h}}$$

$$C_2 = 200 + 6.0\times 258.5917 + 0.003\times 258.5917^2 = \boxed{1952.16\ \$/\text{h}}$$

P.5 ➡ *Solution*

Problem 5.1: The incremental fuel cost for all four units follows the general form $\lambda_i = a_i P_i + b_i$. Thus, the IFC of the plant is to be given by

$$\lambda = a_T P_T + b_T$$

where

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4}\right)^{-1} = \left(\frac{1}{0.012} + \frac{1}{0.0096} + \frac{1}{0.008} + \frac{1}{0.0068}\right)^{-1} = 0.002176$$

and

$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_3}{a_3} + \frac{b_4}{a_4}\right)^{-1} = 0.002176\times\left(\frac{9.0}{0.012} + \frac{6.0}{0.0096} + \frac{8.0}{0.008} + \frac{10.0}{0.0068}\right) = 8.368$$

with P_T = 800 MW, giving

$$\lambda = a_T P_T + b_T = 0.002176\times 800 + 8.368 = \boxed{10.109\,\$/\text{MWh}}$$

The loads assigned to each unit are

$$P_1 = \frac{\lambda - b_1}{a_1} = \frac{10.109 - 9.0}{0.012} = 92.42\,\text{MW}$$

$$P_2 = \frac{\lambda - b_2}{a_2} = \frac{10.109 - 6}{0.0096} = 428.02\,\text{MW}$$

$$P_3 = \frac{\lambda - b_3}{a_3} = \frac{10.109 - 8}{0.008} = 265.63\,\text{MW}$$

$$P_4 = \frac{\lambda - b_4}{a_4} = \frac{10.109 - 10}{0.0068} = 16.03\,\text{MW}$$

Problem 5.2: The solution to the previous problem shows that the loads attributed to each unit are P_1 = 92.42 MW, P_2 = 428.02 MW, P_3 = 265.63 MW, and P_4 = 16.03 MW. We see that P_2 is above the prescribed maximum of 400 MW, and P_3 violates the prescribed minimum of 110 MW; corrections for these two limits are in order. First, assume that unit 2 is operating at its upper limit of 400 MW. We recalculate the plant λ as follows,

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_3} + \frac{1}{a_4}\right)^{-1} = \left(\frac{1}{0.012} + \frac{1}{0.008} + \frac{1}{0.0068}\right)^{-1} = 0.00281379$$

$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_3}{a_3} + \frac{b_4}{a_4}\right) = 0.002814\times\left(\frac{9.0}{0.012} + \frac{8.0}{0.008} + \frac{10.0}{0.0068}\right) = 9.06273529$$

Since P_2 = 400 MW, the total output of units 1, 3 and 4 should be 400 MW. It follows that

$$\lambda = a_T P_T + b_T = 0.00281379\times 400 + 9.06273529 = 10.18825\,\$/\text{MWh}$$

Using this plant λ, the output in each unit is calculated to be

$$P_1 = \frac{\lambda - b_1}{a_1} = \frac{10.18825 - 9.0}{0.012} = 99.0208\,\text{MW}$$

$$P_3 = \frac{\lambda - b_3}{a_3} = \frac{10.18825 - 8}{0.008} = 273.5313\,\text{MW}$$

$$P_4 = \frac{\lambda - b_4}{a_4} = \frac{10.18825 - 10}{0.0068} = 27.6838\,\text{MW}$$

$$P_2 = 400\,\text{MW}$$

It is seen that the outputs of units 3 and 4 violate their respective upper and lower limits. Consequently, it is concluded that other units besides unit 2 must be operating at their limits if the output of unit 2 is specified to be 400 MW. This time assume that unit 4 is operating at its lower limit of 110 MW. Using units 1, 2 and 3 only, the incremental fuel cost is recalculated as follows.

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^{-1} = \left(\frac{1}{0.012} + \frac{1}{0.0096} + \frac{1}{0.008}\right)^{-1} = 0.00320$$

$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_3}{a_3}\right) = 0.00320 \times \left(\frac{9}{0.012} + \frac{6}{0.0096} + \frac{8}{0.008}\right) = 7.60$$

Since P_4 = 110 MW, the total output of units 1, 2 and 3 should be 690 MW. Thus,

$$\lambda = a_T P_T + b_T = 0.0032 \times 690 + 7.60 = 9.8080\,\$/\text{MWh}$$

and

$$P_1 = \frac{\lambda - b_1}{a_1} = \frac{9.8080 - 9.0}{0.012} = 67.3333\,\text{MW}$$

$$P_2 = \frac{\lambda - b_2}{a_2} = \frac{9.8080 - 6}{0.0096} = 396.6667\,\text{MW}$$

$$P_3 = \frac{\lambda - b_3}{a_3} = \frac{9.8080 - 8}{0.008} = 226.0\,\text{MW}$$

$$P_4 = 110\,\text{MW}$$

Notice that $P_1 \in [50, 200]$, $P_2 \in [100, 400]$, $P_3 \in [80, 270]$, and $P_4 \in [110, 300]$ MW. This is a valid configuration, and we conclude that economic dispatch requires that the output of unit 4 be set to its lower limit of 110 MW and the loads of units 1, 2 and 3 be set to the values above.

$$\boxed{\begin{aligned} P_1 &= 67.3\,\text{MW} \\ P_2 &= 396.7\,\text{MW} \\ P_3 &= 226.0\,\text{MW} \\ P_4 &= 110\,\text{MW} \end{aligned}}$$

Problem 5.3: It was shown in the previous problem that if the output of unit 2 is set to its maximum limit of 400 MW, some other units will also have to be operating at their limits. We now examine whether load limit constraints will be violated if unit 4 is set to its new lower limit of 50 MW. Using units 1, 2 and 3, the plant λ is calculated as follows,

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^{-1} = \left(\frac{1}{0.012} + \frac{1}{0.0096} + \frac{1}{0.008}\right)^{-1} = 0.0032$$

$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_3}{a_3}\right)^{-1} = 0.00320 \times \left(\frac{9}{0.012} + \frac{6}{0.0096} + \frac{8}{0.008}\right) = 7.6$$

$$\lambda = a_T P_T + b_T = 0.0032 \times (800 - 50) + 7.60 = 10\ \$/\text{MWh}$$

The unit outputs are calculated to be

$$P_1 = \frac{10-9.0}{0.012} = 83.333\,\text{MW}$$
$$P_2 = \frac{10-6}{0.0096} = 416.667\,\text{MW}$$
$$P_3 = \frac{10.0-8}{0.008} = 250.0\ \text{MW}$$
$$P_4 = 50\,\text{MW}$$

It is observed that the output of unit 2 exceeds the upper limit of 400 MW. Consequently, having unit 4 operate at its lower limit would lead to an unfeasible combination of loads; another configuration is in order. The above analysis suggests that units 2 and 4 should be operating at their upper and lower limits, respectively. Letting P_2 = 400 MW and P_4 = 50 MW, the plant λ is updated as

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_3}\right)^{-1} = \left(\frac{1}{0.012} + \frac{1}{0.008}\right)^{-1} = 0.0048$$
$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_3}{a_3}\right) = 0.0048 \times \left(\frac{9}{0.012} + \frac{8}{0.008}\right) = 8.40$$
$$\lambda = a_T P_T + b_T = 0.0048 \times (800 - 400 - 50) + 8.40 = 10.08\ \$/\text{MWh}$$

The unit outputs are determined to be

$$P_1 = \frac{10.08-9.0}{0.012} = 90.0\,\text{MW}$$
$$P_2 = 400\,\text{MW}$$
$$P_3 = \frac{10.08-8}{0.008} = 260.0\ \text{MW}$$
$$P_4 = 50\,\text{MW}$$

Notice that $P_1 \in [50, 200]$ and $P_3 \in [80, 270]$. This is a valid combination of powers, and we conclude that units 2 and 4 should function at upper and lower limit, respectively, while the other two units should supply the outputs calculated above.

$$\boxed{\begin{aligned} P_1 &= 90\,\text{MW} \\ P_2 &= 400\,\text{MW} \\ P_3 &= 260\,\text{MW} \\ P_4 &= 50\,\text{MW} \end{aligned}}$$

P.6 ➡ *Solution*

At their lower limit of 100 MW, the incremental costs of the units are

$$\lambda_1 = 0.012 \times 100 + 8.0 = 9.20$$
$$\lambda_2 = 0.008 \times 100 + 9.6 = 10.4$$

As the plant output exceeds 200 MW, initially the incremental fuel cost λ of the plant is determined by unit 1 alone and the additional power should come from unit 1. This will continue until the incremental fuel cost of unit 1 becomes 10.4 \$/MWh (i.e., $0.012P_1 + 8.0 = 10.4$), from which the value of P_1 = 200 MW. Therefore, for $200 \leq P_T \leq 300$,

$$\lambda = 0.012P_1 + 8.0 = 0.012(P_1 - 100) + 8.0 = 0.012P_1 + 6.80$$

For $P_T > 300$, both units will increase their outputs simultaneously. To determine which unit will reach its upper limit first, we calculate the incremental costs at the upper limit of 550 MW as follows,

$$\lambda_1\big|_{P_1=550} = 0.012P_1 + 8.0 = 0.012 \times 550 + 8.0 = 14.6$$
$$\lambda_2\big|_{P_2=550} = 0.008P_2 + 9.6 = 0.008 \times 550 + 9.6 = 14.0$$

The result shows that unit 2 will reach its maximum load limit earlier than unit 1. The value of P_1 for which the incremental cost becomes \$14.0/MWh is computed from $0.012P_1 + 8.0 = 14.0$, which yields P_1 = 500 MW.

For $300 \le P_T \le 1050$, the plant λ is calculated. Since the incremental fuel costs of units 1 and 2 should be the same, we may write

$$0.012P_1 + 8.0 = 0.008P_2 + 9.6$$
$$\therefore P_2 = 1.5P_1 - 200$$

Since $P_1 + P_2 = P_T$, P_1 can be represented in terms of P_T as $P_1 + (1.5P_1 - 200) = P_T$, from which $P_1 = 0.4P_T + 80$. The plant λ is then given by

$$\lambda = 0.012P_1 + 8.0 = 0.012(0.4P_T + 80) + 8.0 = 0.0048P_T + 8.96$$

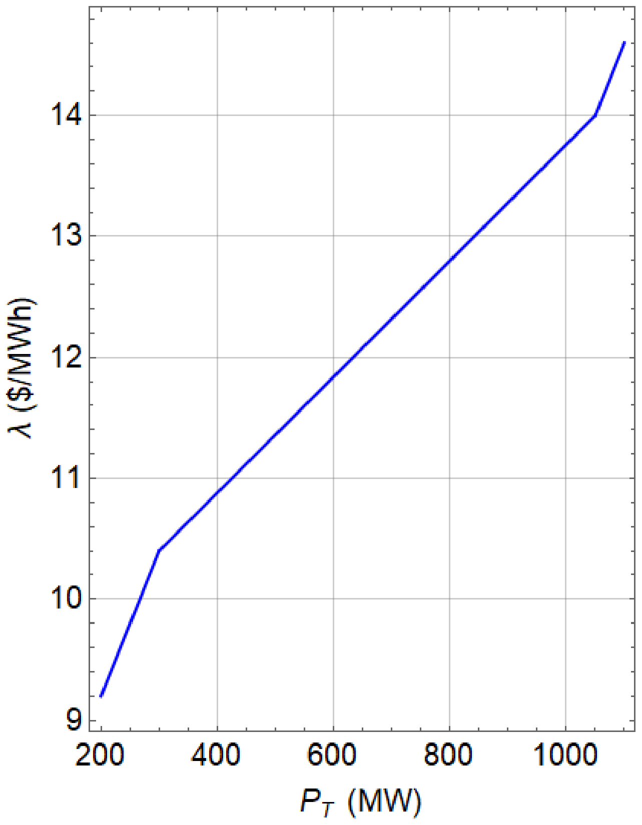
For $P_T > 1050$, only unit 1 will have an excess capacity, and the plant λ is determined by unit 1 alone as

$$\lambda = 0.012P_1 + 8.0 = 0.012(P_T - 550) + 8.0 = 0.012P_T + 1.40$$

The results are summarized as follows,

$$\text{For } 200 \le P_T \le 300,\ \ \lambda = 0.012P_T + 6.8$$
$$\text{For } 300 \le P_T \le 1050,\ \ \lambda = 0.0048P_T + 8.96$$
$$\text{For } 1050 \le P_T \le 1100,\ \ \lambda = 0.012P_T + 1.4$$

The ICF is plotted as a function of P_T below.



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