Montogue

Electromagnetics:

25 Practice Problems

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Here's a set of 25 problems on elementary electromagnetics. The problems were taken from a carefully researched assortment of textbooks and are solved step by step. Enjoy! ■

Problem Range	Subject
1 – 4	Electrostatics
5 – 8	Electric current, resistance and capacitance
9 - 13	Magnetostatics
14 – 16	Magnetic induction
17 - 21	Electromagnetic waves
22 – 25	Transmission lines

PROBLEMS

Problem 1. The plane $x + 2y = 5$ carries a charge density $\rho_S = 6$ nC/m^2. Determine electric field **E** at point $P(-1, 0, 1)$.

(A) $-112\mathbf{a_x} + 80\mathbf{a_y}$ V/m
(B) $-152\mathbf{a_x} + 304\mathbf{a_y}$ V/m
(C) $-152\mathbf{a_x} - 304\mathbf{a_y}$ V/m
(D) $-181\mathbf{a_x} - 362\mathbf{a_y}$ V/m
($\mathbf{a_x}$, $\mathbf{a_y}$ = Cartesian unit vectors in the x- and y-directions, respectively.)

Problems 2 and 3.
In a certain region in free space, the electric field is given by

$$\mathbf{D} = 2\rho(z+1)\cos(\phi)\mathbf{a}_\rho - \rho(z+1)\sin(\phi)\mathbf{a}_\phi + \rho^2\cos(\phi)\mathbf{a_z}\ \ [\mu\text{C/m}^2]$$

where ρ, ϕ, and z are cylindrical coordinates.

2. Find the charge density.
(A) $3\rho(z + 1)cos(\phi)$ μC/m^2
(B) $3(z + 1)cos(\phi)$ μC/m^2
(C) $4(z + 1)cos(\phi)$ μC/m^2
(D) $3(z + 1)sin(\phi)$ μC/m^2

3. Calculate the total charge enclosed by the volume $0 < \rho < 2$, $0 < \phi < \pi/2$, $0 < z < 4$.
(A) 36 μC
(B) 48 μC
(C) 60 μC
(D) 72 μC

Problem 4. Find the work done in carrying a 5-C charge from $P(1, 2, -4)$ to $R(3, -5, 6)$ in an electric field **E** such that

$$\mathbf{E} = \mathbf{a_x} + z^2\mathbf{a_y} + 2yz\mathbf{a_z}\ \ [\text{V/m}]$$

(A) 750 J
(B) 900 J
(C) 1050 J
(D) 1200 J

Problem 5. If a copper wire of cross-sectional area 1 mm^2 is carrying a current of 1 A, what is, most nearly, the average drift velocity of the conduction electrons in the wire? The density of conduction electrons in copper is n_c = 8.45×10^{28} elec/m^3.

(A) 7.40×10^{-5} m/s
(B) 1.82×10^{-4} m/s
(C) 7.40×10^{-4} m/s
(D) 2.23×10^{-3} m/s

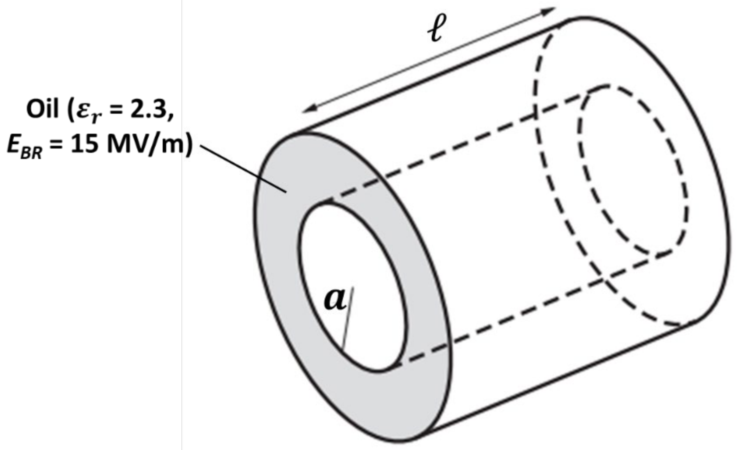
Problem 6. Copper has a conductivity of 5.8×10^5 S-$(cm)^{-1}$ at 20°C. Further, it is known that the temperature coefficient of copper is 0.0039 $(°C)^{-1}$. What is the resistance per unit length of a copper wire of 2 mm diameter at 60°C?

(A) 5.48×10^{-5} Ω-cm^{-1}
(B) 5.81×10^{-5} Ω-cm^{-1}
(C) 6.34×10^{-5} Ω-cm^{-1}
(D) 7.02×10^{-5} Ω-cm^{-1}

Problem 7. A thundercloud can be represented as a parallel-plate capacitor with horizontal plates of area S = 15 km^2 and vertical separation d = 1.13 km. Assume that the upper plate has a total charge Q = 300 C and the lower plate has an equal amount of negative charge. Find the electric field intensity in the cloud.

(A) 2.0 MV/m
(B) 2.26 MV/m
(C) 3.39 MV/m
(D) 4.0 MV/m

Problem 8. Consider the coaxial capacitor illustrated below. Given a = 9 mm, ℓ = 20 mm, and knowing that the voltage rating of the capacitor is 3 kV with a safety factor of 10, what is, most nearly, the maximum capacitance that can be designed using oil (relative permittivity ε_r = 2.3, breakdown field intensity E_{BR} = 15 MV/m)?



(A) 12 pF
(B) 24 pF
(C) 48 pF
(D) 96 pF

Problem 9. A long straight wire of radius R has a current distribution $j(r)$ = $j_0\cos(\pi r/2R)$, where $j_0 > 0$ and r is the radial distance from the centerline of the wire. The wire is in free space, at which the magnetic permeability is μ_0. Which of the following expressions describes the magnetic flux density magnitude |**B**| outside the wire?

(A) $|\mathbf{B}| = \dfrac{2R^2\mu_0 j_0}{\pi r}\left(1-\dfrac{2}{\pi}\right)$

(B) $|\mathbf{B}| = \dfrac{4R^2\mu_0 j_0}{\pi r}\left(1-\dfrac{2}{\pi}\right)$

(C) $|\mathbf{B}| = \dfrac{2R^2\mu_0 j_0}{\pi r}\left(1-\dfrac{1}{2\pi}\right)$

(D) $|\mathbf{B}| = \dfrac{2R^2\mu_0 j_0}{\pi r}\left(1-\dfrac{3}{\pi}\right)$

Problem 10. A Helmholtz coil affords remarkably near-uniform and smooth magnetic fields because some of the derivatives of central magnetic field B_z with respect to axial distance are zero. Specifically, for a Helmholtz coil with magnetic field B_z at the center and distance z along its central axis, we have:

(A) $\frac{\partial B_z}{\partial z} = 0, \frac{\partial^2 B_z}{\partial z^2} = 0, \frac{\partial^3 B_z}{\partial z^3} \neq 0, \frac{\partial^4 B_z}{\partial z^4} \neq 0$

(B) $\frac{\partial B_z}{\partial z} = 0, \frac{\partial^2 B_z}{\partial z^2} = 0, \frac{\partial^3 B_z}{\partial z^3} = 0, \frac{\partial^4 B_z}{\partial z^4} \neq 0$

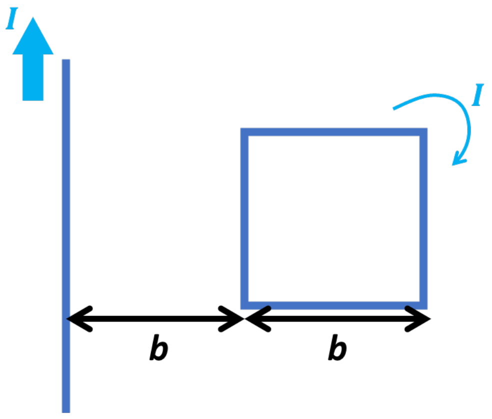
(C) $\frac{\partial B_z}{\partial z} = 0, \frac{\partial^2 B_z}{\partial z^2} = 0, \frac{\partial^3 B_z}{\partial z^3} \neq 0, \frac{\partial^4 B_z}{\partial z^4} = 0$

(D) $\frac{\partial B_z}{\partial z} = 0, \frac{\partial^2 B_z}{\partial z^2} \neq 0, \frac{\partial^3 B_z}{\partial z^3} = 0, \frac{\partial^4 B_z}{\partial z^4} \neq 0$

Problem 11. The earth's above-ground magnetic field density at a distance of r kilometers from the planet's surface is approximately 55 μT × $(r_e/r)^3$, where $r_e \approx$ 6400 km is the earth's radius. What is the order of magnitude of the total energy stored within our planet's above-ground magnetic field?

(A) 10^{15} J
(B) 10^{18} J
(C) 10^{20} J
(D) 10^{22} J

Problem 12. A current I flows in a very long straight wire, as illustrated below. Close to the wire is a square-shaped conductor of side b carrying a clockwise current I of the same magnitude as the wire. What is the magnitude of the force **F** that must be exerted on the square conductor to keep it at rest?



(A) $|\mathbf{F}| = \frac{\mu_0 I^2}{8\pi}$

(B) $|\mathbf{F}| = \frac{\mu_0 I^2}{4\pi}$

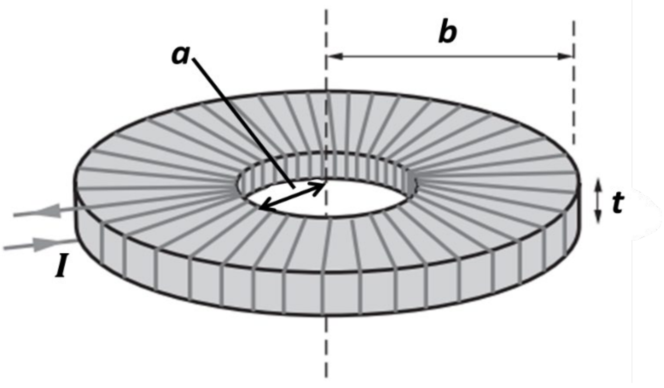
(C) $|\mathbf{F}| = \frac{3\mu_0 I^2}{8\pi}$

(D) $|\mathbf{F}| = \frac{4\mu_0 I^2}{9\pi}$

Problem 13. What magnetic flux density (in gauss) is required to bend electrons with kinetic energy 250 eV into a circular path with radius equal to 4 cm?

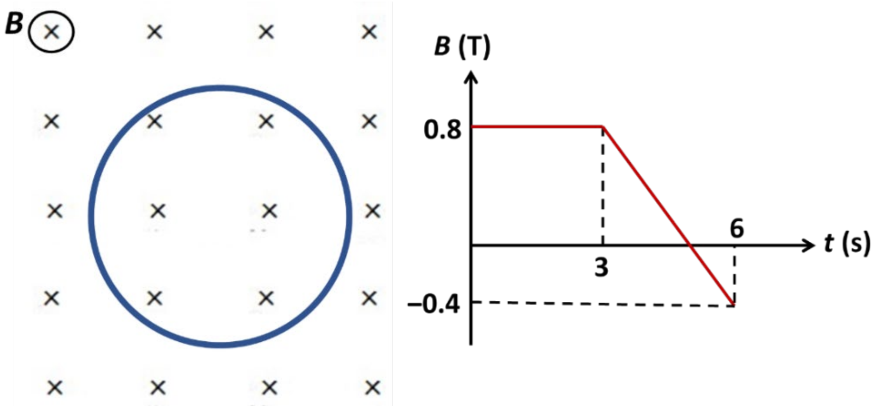
(A) 1500 Gs
(B) 2000 Gs
(C) 4000 Gs
(D) 6000 Gs

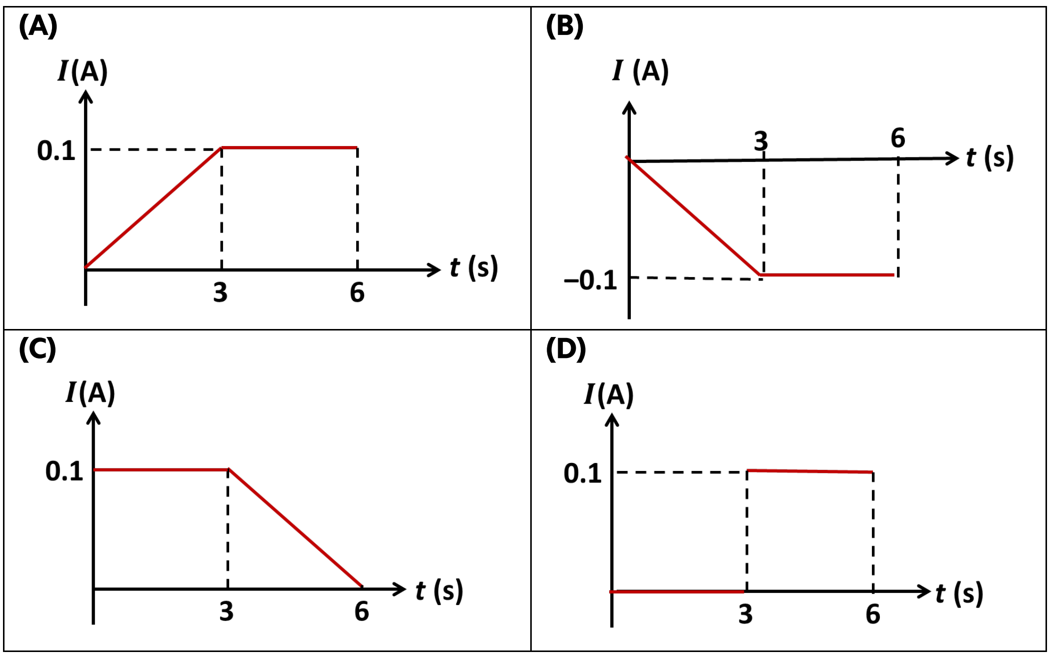
Problem 14. Consider an air-core toroidal coil of rectangular cross-section such as the one illustrated to the side. The coil has dimensions a = 4 mm, b = 7 mm and t = 5 mm. Estimate the total number of turns to be wound on this core so that the total inductance obtained is 1 mH.



(A) 975 turns
(B) 1340 turns
(C) 1800 turns
(D) 2110 turns

Problem 15. A circular loop of 2 Ω resistance and 0.5 m^2 area is on a plane located perpendicularly to a magnetic flux density field **B** whose intensity varies with time as shown on the following graph. Which of the following alternatives correctly shows the time history of the induced current I on the loop in the interval $t \in [0; 6]$ sec?





Problem 16. Consider a circular loop (radius a = 5 cm) of wire lying on the xy-plane with its center at the origin, in the presence of a z-directed magnetic flux density field

$$\mathbf{B}(r,t) = \mathbf{a_z} B_0 \left[1 - \left(10 \text{ m}^{-1}\right) r\right] \cos(2\pi f t)$$

where r is the radial coordinate in meters, B_0 = 10 mT, f is the field's frequency of oscillation in Hz, and t is time in seconds. The loop wire is made of copper (conductivity $\sigma = 5.8\times10^7$ S/m) and has a cross-sectional area of 1 mm^2. It is known that the wire will melt if the total current flowing through it exceeds 20 A. Most nearly, what is the maximum field frequency f_{max} for which the wire can operate in the presence of magnetic field **B**?

(A) f_{max} = 90.2 Hz
(B) f_{max} = 165 Hz
(C) f_{max} = 330 Hz
(D) f_{max} = 501 Hz

Problem 17. The electric field of a uniform plane wave in air is given by

$$\overline{\mathbf{E}}(z,t) = 4\cos\left(2\pi\times10^9 t - \beta z\right)\hat{\mathbf{x}} \;\; \text{V}\cdot\text{m}^{-1}$$

Find the phase constant β and the wavelength λ.

(A) β = 17.3 rad/m; λ = 0.301 m
(B) β = 17.3 rad/m; λ = 0.363 m
(C) β = 20.9 rad/m; λ = 0.301 m
(D) β = 20.9 rad/m; λ = 0.363 m

Problem 18. A 80-MHz uniform plane wave is normally incident from air into a layer of fat tissue. If the fat tissue has dielectric constant ε_r = 7.45 and conductivity σ = 50 mS/m, what is, most nearly, the percent power absorbed by the fat tissue as the wave strikes the boundary separating one medium from the other? Assume that the fat layer has semi-infinite extent.
(A) 32%
(B) 48%
(C) 62%
(D) 76%

Problem 19. A uniform plane wave operating at 100 MHz is normally incident from air onto an air-ceramic interface. It is known that ceramic is nonmagnetic (i.e., μ_r = 1) and that its complex relative dielectric constant at 100 MHz is ε_r = 4.5 – j2. Most nearly, what is the penetration depth of the wave in the ceramic? Assume that ceramic is a medium of semi-infinite extent.
(A) 10 cm
(B) 30 cm
(C) 60 cm
(D) 1 m

Problem 20. A sheet of aluminum foil of thickness 25 μm is used to shield an electronic instrument at 100 MHz. Most nearly, what is the dB attenuation of a plane wave that travels from one side to the other side of the aluminum foil? Neglect the effects from the boundaries. For aluminum, conductivity σ = 3.54×10^7 S/m and, for simplicity, take relative permittivity/permeability ε_r = μ_r = 1.
(A) 7.72 dB
(B) 10.1 dB
(C) 15.8 dB
(D) 25.6 dB

Problem 21. A uniform plane wave is normally incident on the interface between two lossless dielectric media. It is known that the magnitude of the reflection coefficient equals the magnitude of the transmission coefficient. Most nearly, what is the value of the standing wave ratio in decibels under these conditions?
(A) 9.54 dB
(B) 16.8 dB
(C) 21.1 dB
(D) 32.0 dB

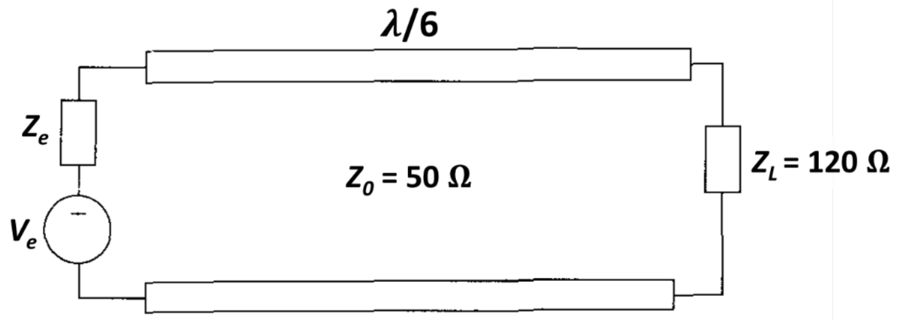
Problem 22. A lossless line operating at circular frequency ω = 5×10^8 rad/s has the following parameters:

Inductance, L = 0.3 μH/m
Capacitance, C = 45 pF/m
Shunt conductivity, G = 64 μS/m
Resistance, R = 25 Ω/m

Most nearly, what is the line's characteristic impedance?
(A) 48 – j6.7 Ω
(B) 48 – j13.4 Ω
(C) 82 – j6.7 Ω
(D) 82 – j13.4 Ω

Problem 23. A 500-Ω lossless line has load voltage V_L = $10e^{j25^\circ}$ V and load impedance Z_L = $50e^{j30^\circ}$ Ω. Find the current at $\lambda/8$ from the load (where λ = transmission line wavelength).
(A) 0.2*exp*(j35°) A
(B) 0.2*exp*(j40°) A
(C) 0.2*exp*(j50°) A
(D) 2.0*exp*(j40°) A

Problems 24 and 25.
Refer to the transmission line illustrated below. Z_0 and Z_L denote the characteristic and load impedances, respectively; λ denotes wavelength; Z_e and V_e are the generator impedance and voltage, respectively.



24. Most nearly, what is the reflection coefficient Γ_L at the load?
(A) 0.41
(B) 0.58
(C) 0.71
(D) 0.90

25. Most nearly, what is the input impedance Z_{in} at the generator?
(A) 26*exp*(–*j*41°) Ω
(B) 26*exp*(–*j*74°) Ω
(C) 35*exp*(–*j*41°) Ω
(D) 35*exp*(–*j*74°) Ω

ANSWER KEY

Problem	Answer	Problem	Answer
1	C	14	B
2	B	15	D
3	D	16	C
4	C	17	C
5	A	18	C
6	C	19	D
7	A	20	D
8	A	21	A
9	A	22	C
10	B	23	B
11	B	24	A
12	B	25	C
13	C		

SOLUTIONS

1 ➡ C

Let $f(x,y) = x + 2y - 5$, so that $\nabla f = \mathbf{a_x} + 2\mathbf{a_y}$. The normal unit vector is

$$\mathbf{a_n} = \pm\frac{\nabla f}{|\nabla f|} = \pm\frac{(\mathbf{a_x}+2\mathbf{a_y})}{\sqrt{1^2+2^2}} = \pm\frac{(\mathbf{a_x}+2\mathbf{a_y})}{\sqrt{5}}$$

Since point P is below the plane, normal vector $\mathbf{a_n}$ has a negative sign. Electric field $\mathbf{E}$ then becomes

$$\mathbf{E} = \frac{\rho_s}{2\varepsilon_0}\mathbf{a_n} = \frac{(6\times10^{-9})}{2\times(8.85\times10^{-12})}\left[-\frac{(\mathbf{a_x}+2\mathbf{a_y})}{\sqrt{5}}\right]$$

$$\therefore \boxed{\mathbf{E} = -152\mathbf{a_x} - 304\mathbf{a_y}\ \text{V/m}}$$

2 ➡ B

The charge density is given by the divergence of **D**, namely

$$\rho_V = \nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho D_\rho\right) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$\therefore \rho_V = \begin{Bmatrix} \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[2\rho^2(z+1)\cos\phi\right] + \frac{1}{\rho}\frac{\partial}{\partial \phi}\left[-\rho(z+1)\sin(\phi)\right] \\ +\frac{\partial}{\partial z}\left[\rho^2\cos(\phi)\right] \end{Bmatrix}$$

$$\therefore \rho_V = \begin{Bmatrix} \frac{1}{\rho}\times 4\rho(z+1)\cos\phi - (z+1)\cos(\phi) \\ +0 \end{Bmatrix}$$

$$\therefore \boxed{\rho_V = 3(z+1)\cos(\phi)\ \mu\text{C/m}^2}$$

3 ➡ D

The charge Q_{enc} enclosed by the volume in question is

$$Q_{\text{enc}} = \int \rho_V dv = \int_0^4 \int_0^{\pi/2} \int_0^2 3(z+1)\cos(\phi)\rho d\rho d\phi dz$$

$$\therefore Q_{\text{enc}} = 6\int_0^4 \int_0^{\pi/2} (z+1)\cos(\phi) d\phi dz$$

$$\therefore Q_{\text{enc}} = 6\int_0^4 (z+1) dz$$

$$\therefore Q_{\text{enc}} = 6\times\left(\frac{4^2}{2}+4\right) = \boxed{72\ \mu\text{C}}$$

4 ➡ C

The easiest way to proceed here is to break down the path outlined by the charge into incremental displacements of only one direction at a time:

$P(1, 2, -4) \rightarrow A(3, 2, -4) \rightarrow B(3, -5, -4) \rightarrow R(3, -5, 6)$

Integrating along each incremental displacement,

$$\left|\frac{W}{Q}\right| = \int \mathbf{E}\cdot d\mathbf{l} = \left(\int_P^A + \int_A^B + \int_B^P\right)\mathbf{E}\cdot d\mathbf{l}$$

$$\therefore \left|\frac{W}{Q}\right| = \int_{x=1}^3 dx + \int_{y=2}^{-5} z^2\Big|_{z=-4} dy + \int_{z=-4}^6 2yz\Big|_{y=-5} dz$$

Evaluating each integral separately, we find

$$\int_{x=1}^3 dx = 3-1 = 2$$

$$\int_{y=2}^{-5} z^2\Big|_{z=-4} dy = (-4)^2\times(-5-2) = -112$$

$$\int_{z=-4}^6 2yz\Big|_{y=-5} dz = 2\times(-5)\int_{-4}^6 zdz = -10\times\left(\frac{6^2}{2}-\frac{(-4)^2}{2}\right) = -100$$

so that

$$|W| = |(2-112-100)Q| = |(2-112-100)\times 5| = \boxed{1050\text{ J}}$$

5 ➡ A

The current density J may be expressed as

$$J = \frac{I}{A} = n_c|q_e|v_d$$

so that, solving for drift velocity v_d,

$$\frac{I}{A} = n_c|q_e|v_d \rightarrow v_d = \frac{I}{n_c|q_e|A}$$

$$\therefore v_d = \frac{1.0}{(8.45\times10^{28})\times(1.6\times10^{-19})\times(1.0\times10^{-6})} = \boxed{7.40\times10^{-5}\text{ m/s}}$$

6 ➡ C

The resistance of a copper wire of 1 cm length and 2 mm (= 0.2 cm) diameter at temperature T can be expressed as

$$R = \frac{\ell}{\sigma_T A} = \frac{1.0}{\sigma_T \times \left(\frac{\pi}{4} \times 0.2^2\right)} = \frac{31.8}{\sigma_T}$$

Here, σ_T for T = 60°C is determined as

$$\frac{1}{\sigma_T} \approx \frac{1}{\sigma_{20^\circ}}\left[1 + \alpha_\sigma (T - 20)\right] \rightarrow \sigma_T = \frac{\sigma_{20^\circ}}{1 + \alpha_\sigma (T - 20)}$$

$$\therefore \sigma_T = \frac{5.8 \times 10^5}{1 + 0.0039 \times (60 - 20)} = 5.017 \times 10^5 \text{ S} \cdot \text{cm}^{-1}$$

so that

$$\therefore R = \frac{31.8}{5.017 \times 10^5} = \boxed{6.34 \times 10^{-5} \ \Omega\text{-cm}^{-1}}$$

7 ➡ A

We first determine the capacitance of the hypothetical capacitor,

$$C = \frac{\varepsilon_0 S}{d} = \frac{\left(8.85 \times 10^{-12}\right) \times \left(15 \times 10^6\right)}{1000} = 1.33 \times 10^{-7} \text{ F} = 133 \text{ nF}$$

and then the voltage V,

$$V = \frac{Q}{C} = \frac{300}{1.33 \times 10^{-7}} = 2.26 \times 10^9 \text{ V} = 2.26 \text{ GV}$$

Finally, the electric field intensity is

$$E = \frac{V}{d} = \frac{2.26 \times 10^9}{1130} = 2.0 \times 10^6 \text{ V/m} = \boxed{2.0 \text{ MV/m}}$$

8 ➡ A

For starters, the electric field within a coaxial capacitor the region between the two concentric cylinders (i.e., $a < r < b$) is given by

$$E_r(r) = \frac{\rho_s a}{\varepsilon r}$$

The maximum field magnitude occurs at $r = a$ and is such that

$$E_{\max} = E_r(r = a) = \frac{\rho_s a}{\varepsilon a} = \frac{\rho_s}{\varepsilon}$$

Since FS = 10, $E_{\max}$ is not to exceed one-tenth of the breakdown field intensity,

$$E_{\max} = \frac{\rho_{s,\max}}{\varepsilon} = \frac{E_{BR}}{10} \quad \text{(I)}$$

In turn, the maximum electric potential difference (i.e., voltage rating) between the cylindrical conductors can be written as

$$\left|\Phi_{ba}\right|_{\max} = \frac{\rho_{s,\max} a}{\varepsilon} \ln\left(\frac{b}{a}\right) = 3000 \text{ V}$$

Using (I),

$$\frac{\rho_{s,\max} a}{\varepsilon} \ln\left(\frac{b}{a}\right) = 3000 \rightarrow aE_{\max} \ln\left(\frac{b}{a}\right) = 3000$$

Solving for $\ln(b/a)$,

$$aE_{\max} \ln\left(\frac{b}{a}\right) = 3000 \rightarrow \ln\left(\frac{b}{a}\right) = \frac{3000}{aE_{\max}} \quad \text{(II)}$$

Now, the capacitance of a coaxial capacitor is

$$C = \frac{2\pi\varepsilon\ell}{\ln(b/a)}$$

Substituting from (II),

$$C=\frac{2\pi\varepsilon\ell}{\ln(b/a)}=\frac{2\pi\varepsilon\ell}{\left(\dfrac{3000}{aE_{\max}}\right)}=\frac{2\pi\varepsilon\ell a}{3000}E_{\max}$$

Substituting from (I),

$$C=\frac{2\pi\varepsilon\ell a}{3000}E_{\max}=\frac{2\pi\varepsilon\ell a}{3000}\times\frac{E_{BR}}{10}$$

$$\therefore C=\frac{2\pi\varepsilon\ell a}{30{,}000}E_{BR}$$

For oil, with ε_r = 2.3 and E_{BR} = 15×10^6 V/m, we obtain

$$C_{\text{oil}}=\frac{2\pi\times\left[2.3\times\left(8.85\times10^{-12}\right)\right]\times\left(20\times10^{-3}\right)\times\left(9\times10^{-3}\right)}{30{,}000}\times\left(15\times10^{6}\right)$$

$$\therefore C_{\text{oil}}=1.15\times10^{-11}\text{ F}=\boxed{11.5\text{ pF}}$$

9 ➡ A

The total current is given by the integral

$$I=\int\mathbf{j}\cdot d\mathbf{S}=2\pi j_0\int_0^R\cos\left(\frac{\pi r}{2R}\right)rdr$$

Using integration by parts,

$$I=2\pi j_0\left\{\left[\frac{2rR}{\pi}\sin\left(\frac{\pi r}{2R}\right)\right]_{r=0}^{r=R}-\left(\frac{2R}{\pi}\right)\int_0^R\sin\left(\frac{\pi r}{2R}\right)dr\right\}$$

$$\therefore I=2\pi j_0\left\{\frac{2R^2}{\pi}-\left(\frac{2R}{\pi}\right)\left(\frac{2R}{\pi}\right)\left[-\cos\left(\frac{\pi}{2}\right)+\cos(0)\right]\right\}$$

$$\therefore I=2\pi j_0\left(\frac{2R^2}{\pi}-\frac{4R^2}{\pi^2}\right)$$

$$I=4R^2j_0\left(1-\frac{2}{\pi}\right)$$

The magnetic field outside the wire is given by Ampère's law,

$$B=\frac{\mu_0 I}{2\pi r}=\frac{\mu_0}{2\pi r}\times\left[4R^2j_0\left(1-\frac{2}{\pi}\right)\right]$$

$$\therefore\boxed{B=\frac{2R^2\mu_0 j_0}{\pi r}\left(1-\frac{2}{\pi}\right)}$$

10 ➡ B

In a Helmholtz coil, the first three derivatives of central magnetic field B_z with respect to axial distance z are equal to zero, which is one of the reasons why this type of device produces a remarkably smooth magnetic field.

11 ➡ B

The energy W_H associated with a **H** field is given by the volume integral

$$W_H=\frac{1}{2}\iiint_V\mu|\mathbf{H}|^2\,dv$$

Using |**H**| = |**B**|/μ and $dv=4\pi r^2dr$,

$$W_H=\frac{1}{2}\iiint_V\mu|\mathbf{H}|^2\,dv=\frac{1}{2}\iiint_V\mu\frac{|\mathbf{B}|^2}{\mu^2}dv=\frac{1}{2}\iiint_V\frac{|\mathbf{B}|^2}{\mu}dv=\frac{1}{2}\int_{r=r_e}^{r=\infty}\frac{\left[55\ \mu\text{T}\times\left(\dfrac{r_e}{r}\right)^3\right]^2\times4\pi r^2dr}{4\pi\times10^{-7}}$$

$$\therefore W_H=\frac{4\pi\times(55\ \mu\text{T})^2}{2\times\left(4\pi\times10^{-7}\right)}\int_{r=r_e}^{r=\infty}\left(\frac{r_e}{r}\right)^6\times r^2dr$$

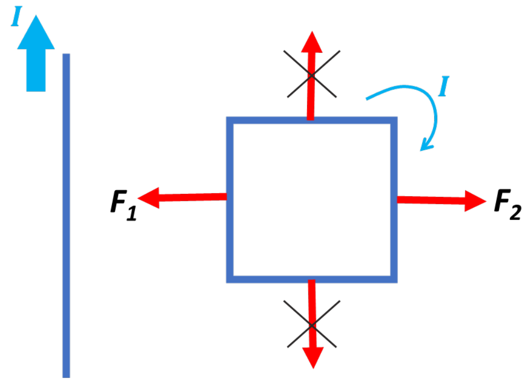
$$\therefore W_H=\frac{(55\ \mu\text{T})^2r_e^6}{2\times10^{-7}}\int_{r=r_e}^{r=\infty}r^{-4}dr$$

$$\therefore W_H = \frac{(55\ \mu\text{T})^2 r_e^6}{2\times10^{-7}} \times \left(-\frac{r^{-3}}{3}\right)\Bigg|_{r=r_e}^{r=\infty}$$

$$\therefore W_H = \frac{(55\ \mu\text{T})^2 r_e^3}{(2\times10^{-7})\times3} = \frac{(55\times10^{-6})^2\times(6400\times10^3)^3}{2\times10^{-7}\times3} = 1.32\times\boxed{10^{18}}\ \text{J}$$

12 ➡ B

The magnetic forces exerted on the sides of the square conductor that are perpendicular to the long wire cancel out, hence we need to worry only about the forces on the sides that are parallel to the long wire. In view of the specified current directions, we surmise that the side of the square conductor located closer to the wire is under the effect of a force $\mathbf{F_1}$ pointing to the left, whereas the side farther away from the wire is under the effect of a force $\mathbf{F_2}$ pointing to the right. Since $|\mathbf{F_1}| > |\mathbf{F_2}|$ because the side associated with $\mathbf{F_1}$ is closer to the wire than the side associated with $\mathbf{F_2}$, it follows that, in order to keep the conductor at rest, we must supply it with a rightward force **P** such that



$$|\mathbf{P}| = |\mathbf{F}_1| - |\mathbf{F}_2|$$

where

$$|\mathbf{F}_1| = B_1 Ib = \frac{\mu_0 I}{2\pi b}\times I\times b = \frac{\mu_0 I^2}{2\pi}$$

and

$$|\mathbf{F}_2| = B_2 Ib = \frac{\mu_0 I}{2\pi\times(2b)}\times I\times b = \frac{\mu_0 I^2}{4\pi}$$

Therefore,

$$|\mathbf{P}| = |\mathbf{F}_1| - |\mathbf{F}_2| = \frac{\mu_0 I^2}{2\pi} - \frac{\mu_0 I^2}{4\pi} = \boxed{\frac{\mu_0 I^2}{4\pi}}$$

13 ➡ C

The magnetic force on an electron moving with a velocity *v* is given by

$$F = -\frac{1}{c} e\mathbf{v}\times\mathbf{B}$$

This force will bend the electron in a circular path of radius *R* determined by equating the magnetic force to the centripetal force,

$$\frac{1}{c} evB = \frac{mv^2}{R}$$

Solving for *B* gives the required magnetic field as

$$B = \frac{mvc}{eR}$$

For electrons with kinetic energy equal to 250 eV in a circular path of radius 4 cm, the magnetic field is found as

$$B = \frac{mvc}{eR} = \frac{\sqrt{(mv^2)(mc^2)}}{eR} = \frac{\sqrt{2mc^2T}}{eR}$$

$$\therefore B = \frac{\sqrt{2\times(511\times10^3)\times250}}{4.0} \approx \boxed{4000\ \text{Gs}}$$

We took the *c* inside the square root to make mc^2 = 511 keV (the rest mass of an electron), we used $mv = \sqrt{2mT}$ where *T* is the kinetic energy, and we did not use the magnitude *e* of the electron charge because the energies were given in electron-volt units.

14 ➡ B

Here, we take the equation used to compute the inductance of a rectangular toroidal coil and solve for the number of turns *N*,

$$L = \frac{\mu_0 N^2 t}{2\pi} \ln\left(\frac{b}{a}\right) \quad \rightarrow \quad N = \sqrt{\frac{2\pi L}{\mu_0 t \ln(b/a)}}$$

$$\therefore N = \sqrt{\frac{2\pi \times \left(1.0 \times 10^{-3}\right)}{\left(4\pi \times 10^{-7}\right) \times \left(5 \times 10^{-3}\right) \times \ln(7/4)}} \approx \boxed{1340 \text{ turns}}$$

15 ➡ D

In the interval $t \in [0; 3]$ s, the magnetic field is constant and equal to 0.8 T; since there is no change of |**B**|, then, by Faraday's law, there is no induced voltage or current. In the interval $t \in [3; 6]$ sec, the intensity of **B** varies from 0.8 T at $t = 3$ s to −0.4 T at $t = 6$ s; as a result, there is an induced voltage such that

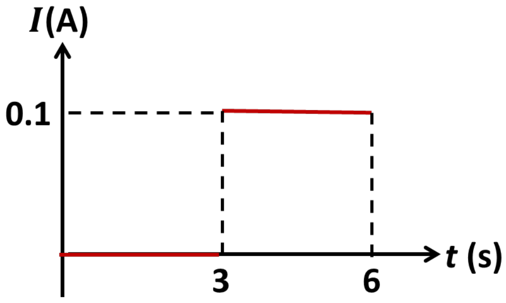
$$V_{\text{ind}} = -\frac{d\Psi}{dt} = -\frac{d}{dt}\left(B(t)A\right) = -A\frac{dB(t)}{dt}$$

$$\therefore V_{\text{ind}} = 0.5 \times \left(\frac{-0.4 - 0.8}{6 - 3}\right) = 0.2 \text{ V}$$

and the corresponding current is

$$I = \frac{V_{\text{ind}}}{R} = \frac{0.2}{2} = 0.1 \text{ A}$$

The time history of current I is shown below.



16 ➡ C

Firstly, the resistance of the circular loop is given by

$$R_{\text{loop}} = \frac{\ell}{\sigma A} = \frac{2\pi a}{\sigma A}$$

The current induced on the loop is

$$I = \frac{V_{\text{ind}}}{R_{\text{loop}}} = -\frac{1}{(2\pi a/\sigma A)}\frac{d\Psi}{dt} \quad \text{(I)}$$

To find the magnetic flux, we integrate the given **B** over the surface of the loop,

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_0^{2\pi}\int_0^a B(r,t)\, r\,dr\,d\phi$$

$$\therefore \Psi = B_0 \cos(2\pi f t)\int_0^{2\pi}\int_0^a (1 - 10r)\, r\,dr\,d\phi$$

$$\therefore \Psi = B_0 \cos(2\pi f t)\int_0^{2\pi}\int_0^a \left(r - 10r^2\right) dr\,d\phi$$

$$\therefore \Psi = B_0 \cos(2\pi f t)\int_0^{2\pi}\left(\frac{r^2}{2} - \frac{10r^3}{3}\right)\Bigg|_{r=0}^{r=a} d\phi$$

$$\therefore \Psi = B_0 \cos(2\pi f t)\left(\frac{a^2}{2} - \frac{10a^3}{3}\right)\int_0^{2\pi} d\phi$$

$$\therefore \Psi = 2\pi B_0 \cos(2\pi f t)\left(\frac{a^2}{2} - \frac{10a^3}{3}\right)$$

Differentiating with respect to time,

$$V_{\text{ind}} = -\frac{d\Psi}{dt} = -2\pi B_0\left(\frac{a^2}{2} - \frac{10a^3}{3}\right)\frac{d}{dt}\left[\cos(2\pi ft)\right]$$

$$\therefore V_{\text{ind}} = -2\pi B_0\left(\frac{a^2}{2} - \frac{10a^3}{3}\right)\times\left[-2\pi f\sin(2\pi ft)\right]$$

$$\therefore V_{\text{ind}} = 4\pi^2 fB_0\left(\frac{a^2}{2} - \frac{10a^3}{3}\right)\sin(2\pi ft)$$

The maximum current induced on the loop corresponds to the maximum induced voltage V_{ind}, which occurs when $\sin(2\pi ft) = 1$. Substituting in (I),

$$I_{\max} = \frac{V_{\text{ind}}}{R_{\text{loop}}} = \frac{\sigma A}{2\pi a}\times 4\pi^2 fB_0\left(\frac{a^2}{2} - \frac{10a^3}{3}\right) < 20$$

$$\therefore f < \frac{20}{\frac{\sigma A}{2\pi a}\times 4\pi^2 B_0\left(\frac{a^2}{2} - \frac{10a^3}{3}\right)}$$

$$\therefore f < \frac{20}{\frac{2\pi\sigma AB_0}{a}\left(\frac{a^2}{2} - \frac{10a^3}{3}\right)}$$

$$\therefore f < \frac{20}{\frac{2\pi\sigma AB_0 a^2}{a}\left(\frac{1}{2} - \frac{10a}{3}\right)}$$

$$\therefore f < \frac{20}{2\pi\sigma AB_0 a\left(\frac{1}{2} - \frac{10a}{3}\right)}$$

$$\therefore f < \frac{20}{2\pi\times\left(5.8\times10^7\right)\times\left(10^{-6}\right)\times\left(10\times10^{-3}\right)\times0.05\times\left(\frac{1}{2} - \frac{10\times0.05}{3}\right)}$$

$$\therefore \boxed{f < 329\text{ Hz}}$$

17 ➡ C

The phase constant β is given by

$$\beta = \frac{\omega}{c} = \frac{2\pi\times10^9}{3\times10^8} = \boxed{20.9\text{ rad}\cdot\text{m}^{-1}}$$

The wavelength λ is, in turn,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{20.9} = 0.301\text{ m} = \boxed{30.1\text{ cm}}$$

18 ➡ C

We first compute the loss tangent of fat tissue,

$$\tan\delta_c = \frac{\sigma}{\omega\varepsilon} = \frac{50\times10^{-3}}{\left[2\pi\times\left(80\times10^6\right)\right]\times\left[7.45\times\left(8.85\times10^{-12}\right)\right]} = 1.51$$

and the intrinsic impedance

$$\eta = \frac{\eta_0/\sqrt{\varepsilon_r}}{\left(1+\tan^2\delta_c\right)^{1/4}}e^{j(1/2)\tan^{-1}(\sigma/\omega\varepsilon)}$$

$$\therefore \eta = \frac{377/\sqrt{7.45}}{\left(1+1.51^2\right)^{1/4}}e^{j(1/2)\tan^{-1}(1.51)} = 103e^{j28.2^\circ}\ \Omega$$

Then, the reflection coefficient at the air-fat tissue boundary is

$$\Gamma = \rho e^{i\phi} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{103e^{j28.2^\circ} - 377}{103e^{j28.2^\circ} + 377} = 0.617e^{j2.87} = 0.617e^{j164^\circ}$$

The percentage of power absorbed by the fat tissue then becomes

$$\frac{\left|\left(\mathbf{S}_{\text{avg}}\right)_t\right|}{\left|\left(\mathbf{S}_{\text{avg}}\right)_i\right|}\times 100\% = \left(1-\rho^2\right)\times 100\% = \left(1-0.617^2\right)\times 100\% \approx \boxed{61.9\%}$$

19 ➡ D

The loss tangent of the ceramic at 100 MHz is $\tan\delta_c$ = 2/4.5 = 0.444. We proceed to compute the intrinsic impedance of the ceramic,

$$\eta_c = \frac{\eta_0/\sqrt{\varepsilon_r}}{\left(1+\tan^2\delta_c\right)^{1/4}}e^{j(1/2)\tan^{-1}(\sigma/\omega\varepsilon)}$$

$$\eta_d = \frac{377/\sqrt{4.5}}{\left(1+0.444^2\right)^{1/4}}e^{j(1/2)\tan^{-1}(0.444)} = 170e^{j12^\circ}\ \Omega$$

Then, the attenuation constant of the ceramic at 100 MHz is

$$\alpha_d \approx \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}}\left[\sqrt{1+\tan^2\delta_d}-1\right]^{1/2}$$

$$\therefore \alpha_d = \frac{2\pi\times 10^8}{3.0\times 10^8}\times\sqrt{\frac{1\times 4.5}{2}}\times\left[\sqrt{1+0.444^2}-1\right]^{1/2} = 0.964\ \text{Np/m}$$

The penetration depth d is simply the inverse of α_d,

$$d = \alpha_d^{-1} = 0.964^{-1} = \boxed{1.04\ \text{m}}$$

20 ➡ D

The loss tangent of aluminum is

$$\tan\delta_c = \frac{\sigma}{\omega\varepsilon} = \frac{3.54\times 10^7}{\left(2\pi\times 10^8\right)\times\left(8.85\times 10^{-12}\right)} = 6.37\times 10^9$$

Since $\tan(\delta_c) \gg 1$, aluminum is a good conductor and the attenuation constant can be approximated as

$$\alpha \approx \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{\left(2\pi\times 10^8\right)\times\left(4\pi\times 10^{-7}\right)\times\left(3.54\times 10^7\right)}{2}} = 118{,}000\ \text{Np/m}$$

The attenuation experienced by a plane wave at 100 MHz propagating in an aluminum foil of thickness d = 25 μm is

$$20\log_{10}e^{-\alpha d} = 20\log_{10}e^{-118{,}000\times\left(25\times 10^{-6}\right)} = \boxed{-25.6\ \text{dB}}$$

21 ➡ A

For normal incidence, $1+\Gamma=\tau$, where $|\Gamma|\le 1$. Let subscripts 1 and 2 denote the two lossless media. If the magnitude of the reflection coefficient, $|\Gamma|$, equals the magnitude of the transmission coefficient, $|\tau|$, we have $\Gamma < 0$ and may write

$$|\Gamma| = |\tau| \ \rightarrow\ \left|\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right| = \left|\frac{2\eta_2}{\eta_2+\eta_1}\right|$$

$$\therefore \frac{-(\eta_2-\eta_1)}{\cancel{\eta_2+\eta_1}} = \frac{2\eta_2}{\cancel{\eta_2+\eta_1}}$$

$$\therefore -\eta_2+\eta_1 = 2\eta_2$$

$$\therefore \eta_1 = 3\eta_2$$

so that

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\eta_2-3\eta_2}{\eta_2+3\eta_2} = \frac{-2\eta_2}{4\eta_2} = -\frac{1}{2}$$

$$\therefore |\Gamma| = \frac{1}{2}$$

The standing wave ratio s then becomes

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/2}{1-1/2} = 3.0$$

Finally,

$$s[\text{dB}] = 20\log_{10}(3.0) = \boxed{9.54 \text{ dB}}$$

22 ➡ C

The characteristic impedance Z_0 is

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{25+j\times(5\times10^8)\times(0.3\times10^{-6})}{(64\times10^{-6})+j\times(5\times10^8)\times(45\times10^{-12})}} = \boxed{81.9-j6.66\ \Omega}$$

23 ➡ B

The load current I_L is described by the phasor

$$I_L = \frac{V_L}{Z_L} = \frac{10e^{j25^\circ}}{50e^{j30^\circ}} = 0.2e^{-j5^\circ}$$

Noting that

$$\beta z = \frac{2\pi}{\lambda}\times\frac{\lambda}{8} = \frac{\pi}{4} = 45^\circ$$

we have

$$I\left(z=\frac{\lambda}{8}\right) = I_L e^{j\beta z} = 0.2e^{-j5^\circ}e^{j45^\circ} = \boxed{0.2e^{j40^\circ}}\ \text{A}$$

24 ➡ A

The reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{120-50}{120+50} = \boxed{0.412}$$

25 ➡ C

Firstly, we have the product

$$\beta\ell = \frac{2\pi}{\lambda}\times\frac{\lambda}{6} = \frac{\pi}{3} = 60^\circ$$

so that

$$Z_{\text{in}} = Z_0\left[\frac{Z_L\cos(\beta\ell)+jZ_0\sin(\beta\ell)}{Z_0\cos(\beta\ell)+jZ_L\sin(\beta\ell)}\right] = 50\times\left[\frac{120\cos(60^\circ)+j50\sin(60^\circ)}{50\cos(60^\circ)+j120\sin(60^\circ)}\right]$$

$$\therefore Z_{\text{in}} = 26.3 - j22.6\ \Omega = \boxed{34.6e^{j(-40.7^\circ)}\ \Omega}$$

REFERENCES

- CHENG, D.K. (1994). *Fundamentals of Engineering Electromagnetics – International Edition*. Pearson.
- HAYT, W.H., Jr. and BUCK, J.A. (2012). *Engineering Electromagnetics*. 8th edition. McGraw-Hill.
- INAN, U.S., INAN, A.S. and SAID, R.K. (2015). *Engineering Electromagnetics and Waves*. 2nd edition. Pearson. **Recommended book!**
- KAMBERAJ, H. (2022). *Electromagnetism – With Solved Problems*. Springer.
- NOTAROŠ, B.M. (2010). *Electromagnetics*. Prentice Hall.
- SADIKU, M.N.O. (2000). *Elements of Electromagnetics*. 3rd edition. Oxford University Press.