QUIZ MS201

Elementary Fracture Mechanics

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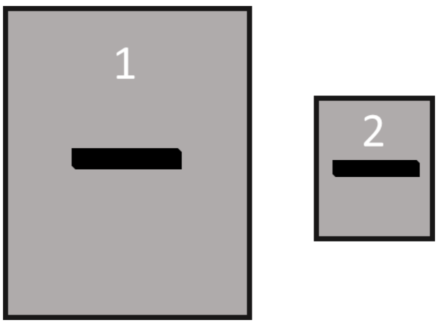
PROBLEMS

Problem 1 (Broek, 1988)

Given a toughness of K = 66 ksi-in.$^{1/2}$ and a collapse strength equal to the yield strength σ_Y = 72 ksi, determine the residual strength of a center cracked plate of 20-in. width with a crack of $2a$ = 8 in., labeled plate 1, and a center cracked plate of 2-in. width with a crack of $2a$ = 1 in, labeled plate 2. Will any of the plates collapse?



A) Neither plate will collapse.

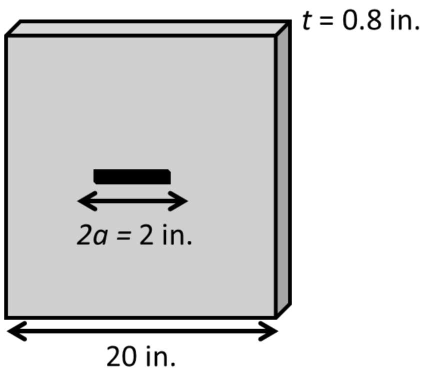
B) Plate 1 will collapse, whereas plate 2 will not collapse.

C) Plate 2 will collapse, whereas plate 1 will not collapse.

D) Both plates will collapse.

Problem 2 (Broek, 1988)

Calculate the fracture toughness of a material for which a plate test with a central crack with the following information: plate width = 20 in., plate thickness = 0.8 in., crack width $2a$ = 2 in., failure load P = 250 kip, yield strength σ_Y = 70 ksi. True or false?



1.( ) The stress intensity factor for the loading conditions is greater than 32 ksi-in.$^{1/2}$

2.( ) The plate is under plane strain conditions per the ASTM criterion.

3.( ) The size of the plastic zone is greater than 7.5×10^{-3} in.

Problem 3.1 (Broek, 1988)

The test record for a 150 ksi yield strength steel has been obtained from the compact tension specimens indicating a P_5 load of 48,200 lb and a maximum load of 50,200 lb as $P_{\max}$. Calculate the plane-strain fracture toughness K_{IC} if the specimen has width w = 5 in., thickness B = 1.8 in., and crack size a = 2 in. Given α = a/w, use the dimensionless geometric factor

$$f\left(\frac{a}{w}\right)_C = 29.6\alpha^{0.5} - 185.5\alpha^{1.5} + 655.7\alpha^{2.5} - 1017\alpha^{3.5} + 639\alpha^{4.5}$$

A) K_I = 23.2 ksi-in.$^{1/2}$

B) K_I = 45.4 ksi-in.$^{1/2}$

C) K_I = 60.9 ksi-in.$^{1/2}$

D) K_I = 87.8 ksi-in.$^{1/2}$

■ Problem 3.2

For the compact-tension (CT) test to yield a valid estimate of the fracture toughness of the specimen, the following conditions must be satisfied.

1. Plane strain conditions must hold, that is, $B \geq 2.5(K_{IC}/\sigma_Y)^2$;

2. The ratio of B to the size of the plastic zone r_y must be greater than about 50.

3. The ratio of a to the size of the plastic zone r_y must be greater than about 50.

4. The ratio of the maximum load to the P_5 load should be less than 1.10.

How many of these requirements are satisfied for the conditions introduced in Problem 3.1?

A) All four requirements are satisfied.

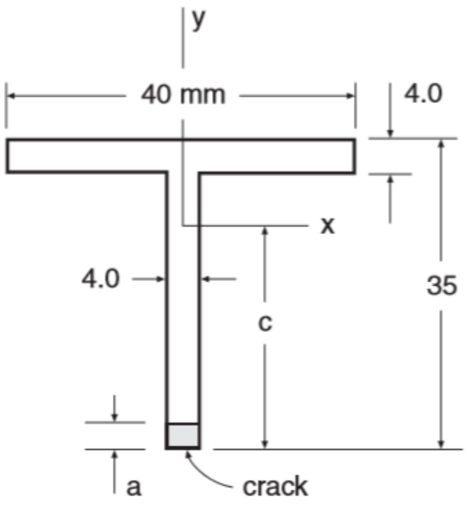
B) Only three requirements are satisfied.

C) Only two requirements are satisfied.

D) Only one requirement is satisfied.

■ Problem 4.1 (Dowling, 2013, w/permission)

A stiffener in aircraft structure is a T-section, as shown below, and is made of 7075-T561 aluminum. A crack of length a may be present on the bottom of the web as shown. A bending moment of 180 N·m is applied about the x-axis, such that the crack is subjected to tensile stresses. To enable stress calculations, locate the y-centroid of the T-section and its area moment of inertia about the centroidal x-axis. If the crack has length a = 1.5 mm, what is the safety factor against brittle fracture? The fracture toughness of 7075-T561 aluminum is 29 MPa-m$^{1/2}$.



A) FS_K = 1.44

B) FS_K = 2.59

C) FS_K = 3.79

D) FS_K = 4.56

■ Problem 4.2

What is the largest crack length that can be permitted if a safety factor against brittle fracture of 3.0 is considered adequate?

A) $a_{\max}$ = 1.11 mm

B) $a_{\max}$ = 2.34 mm

C) $a_{\max}$ = 3.09 mm

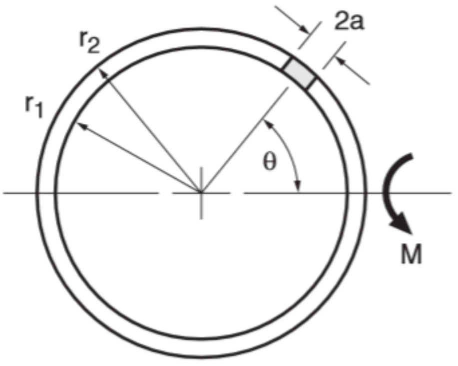
D) $a_{\max}$ = 4.06 mm

■ Problem 4.3

Consider the possibility of changing the material to the more expensive 7475-T7351 aluminum alloy, which has a fracture toughness of 52 MPa-m$^{1/2}$. What are the possible advantages and disadvantages of making this change? Support your comments with calculations where possible.

Problem 5 (Dowling, 2013, w/permission)

A tube having inner radius r_1 = 45 mm and outer radius r_2 = 50 mm is subjected to a bending moment of 8.0 kN·m. It is made of annealed titanium 6Al-4V, which has a fracture toughness of 66 MPa-m$^{1/2}$ and a yield strength of 925 MPa. As shown in the figure, the tube has a through-wall crack of width $2a$ = 10 mm, located at an angle θ = 50° relative to the bending axis. Estimate the safety factor for brittle failure and the safety factor for fully plastic yielding.



A) FS_K = 1.51 and FS_Y = 2.54
B) FS_K = 1.51 and FS_Y = 5.21
C) FS_K = 3.04 and FS_Y = 2.54
D) FS_K = 3.04 and FS_Y = 5.21

Problem 6 (Hertzberg et al., 2013, w/ permission)

A thin-walled pressure vessel 1.25-cm thick originally contained a small semicircular flaw or radius 0.25 cm located at the inner surface and oriented normal to the hoop stress direction. Repeated pressure cycling enabled the crack to grow larger. If the fracture toughness of the material is 88 MPa-m$^{1/2}$, the yield strength equal to 825 MPa, and the hoop stress equal to 275 MPa, would the vessel leak before it ruptured?

α) The vessel would leak before rupturing.
β) The vessel would rupture without leaking first.
γ) There is not enough information.

Problem 7.1 (Hertzberg et al., 2013, w/ permission)

An unreinforced polymeric pressure vessel is constructed with a diameter d = 0.44 m and a length L = 2 m. The vessel is designed to be capable of withstanding an internal pressure of p = 7 MPa at a nominal hoop stress of 70 MPa. However, in service the vessel bursts at an internal pressure of only 3.5 MPa, and a failure investigation reveals that the failure was initiated by a manufacturing-induced semicircular internal crack 2.5 mm in radius. Based on the original design criteria, compute the fracture toughness of the material used. You may assume that it is a thin-walled vessel for this calculation, even though this may not be true.

A) K_{IC} = 0.471 MPa-m$^{1/2}$
B) K_{IC} = 2.21 MPa-m$^{1/2}$
C) K_{IC} = 4.05 MPa-m$^{1/2}$
D) K_{IC} = 6.18 MPa-m$^{1/2}$

Problem 7.2

Given the following materials to choose among, is it possible for this pressure vessel to meet a leak-before-break criterion at the original design stress without reinforcing the polymer or changing the vessel dimensions?

Polymer	PMMA	PC	PVC	PET
K_{IC} **(MPa$\sqrt{m}$)**	1.65	3.2	3.8	5.0

A) All four polymers can be used to satisfy these criteria.
B) Only PC, PVC, and PET would satisfy these criteria.
C) Only PVC and PET would satisfy these criteria.
D) Only PET would satisfy these criteria.

Problem 8 (Hertzberg et al., 2013, w/ permission)

A 3 cm-diameter penny-shaped slag inclusion is found on the fracture surface of a very large component made of steel alloyed with Ni, Mo, and V. Could this defect have been responsible for the fracture if the stress acting on the component was 380 MPa? The only material data available are Charpy results in the transition temperature regime where impact energy values of 7 to 10 ft-lb were reported. The Young's modulus for the steel is 30×10^6 psi. Use the Barson-Rolfe correlation.

Problem 9.1 (Hertzberg et al., 2013, w/permission)

A rod of soda-lime-silica glass is rigidly constrained at 400 K and then cooled rapidly to 300 K. Assume that E = 70 GPa, $\alpha = 8 \times 10^{-6}$ K^{-1}, and K_{IC} = 0.8 MPa-m$^{1/2}$. The allowable stress is σ_a = 90 MPa. With no visible surface damage, would you expect the rod to survive this quench?

α) The rod would withstand the quench.

β) The rod would not withstand the quench.

γ) There is not enough information.

Problem 9.2

If the glass rod contained a 1-mm scratch that was oriented perpendicular to the axis of the rod, what would be your answer?

α) The rod would withstand the quench.

β) The rod would not withstand the quench.

γ) There is not enough information.

Problem 9.3

Would failure occur if the temperature drop and the crack size were each half the values given above?

α) The rod would withstand the quench.

β) The rod would not withstand the quench.

γ) There is not enough information.

Problem 10.1 (Anderson, 1995, w/permission)

A material exhibits the following crack growth resistance behavior.

$$R = 6.95\left(a - a_0\right)^{0.5}$$

where a_0 is the initial crack size. The R parameter has units of kJ/m^2 and the crack size is in millimeters. Consider a wide plate with a through crack such that $a \ll w$, where w is the width of the plate. The elastic modulus of the plate material is 207 GPa. If this plate fractures at 138 MPa, compute the crack size at failure.

A) a_c = 125 mm

B) a_c = 212 mm

C) a_c = 290 mm

D) a_c = 371 mm

Problem 10.2

If this plate has an initial crack length ($2a_0$) of 50.8 mm and the plate is loaded to failure, compute the stress at failure.

A) σ_c = 38.5 MPa

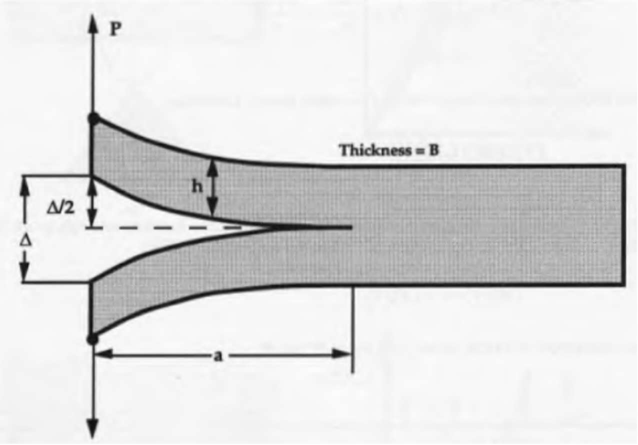
B) σ_c = 105 MPa

C) σ_c = 213 MPa

D) σ_c = 340 MPa

■ Problem 11.1 (Anderson, 1995, w/permission)

Suppose that a double cantilever beam specimen is fabricated from the same material considered in Problem 10. The dimensions of the beam are B = 25.4 mm and h = 12.7 mm (refer to figure below) and the initial crack size is a_0 = 152 mm. Calculate the load at failure.



A) P_c = 2.31 kN
B) P_c = 5.24 kN
C) P_c = 8.14 kN
D) P_c = 11.1 kN

■ Problem 11.2

The previous problem has shown that the energy release rate R of a double cantilever beam specimen increases with crack growth when the specimen is held at a constant load. Describe how you could alter the design of the DCB specimen so that a growing crack in load control would experience a constant G.

■ Problem 12.1 (Hertzberg et al., 2013, w/permission)

A particular pressure vessel is fabricated by bending a rolled aluminum alloy plate into a cylinder then welding on end caps. The alloy used for the cylinder has a distinct layered structure from the rolling process. The rolling direction is around the circumference of the cylinder. The plate thickness is 3 mm. The measured fracture toughness values for the different orientations of this material are provided below. The internal pressure leads to a hoop stress of 300 MPa. If two semicircular cracks are initiated on the inner surface of the cylinder such that one is growing along the cylinder length and other across the cylinder width, which one would be more likely to lead to fast fracture?

Direction	L-T	T-L	S-T
K_{IC} (MPa-m$^{1/2}$)	29.7	24.5	16.3

α) The L-T direction.
β) The T-L direction.
γ) The S-T direction.

■ Problem 12.2

If a circular embedded penny crack was created internally during fabrication so that the crack lies in the S-T orientation, will this fail before either of the cracks in the previous problem?

■ Problem 12.3

If only the longitudinal crack was present, would this design meet a leak-before-break criterion?

Problem 13

The records of a fracture toughness test are as follows.

Crack length a (mm)	Load (kN)	Load point displacement (mm)
20.1	100	0.291
21.1	100	0.298

The fracture load P_c = 120 kN for a = 20.6 mm. Given the thickness B = 25 mm, Young's modulus E = 70 GPa and Poisson's ratio ν = 0.3, determine G_{IC} and K_{IC}.

A) G_{IC} = 10.1 kJ/m² and K_{IC} = 19.7 MPa-m$^{1/2}$

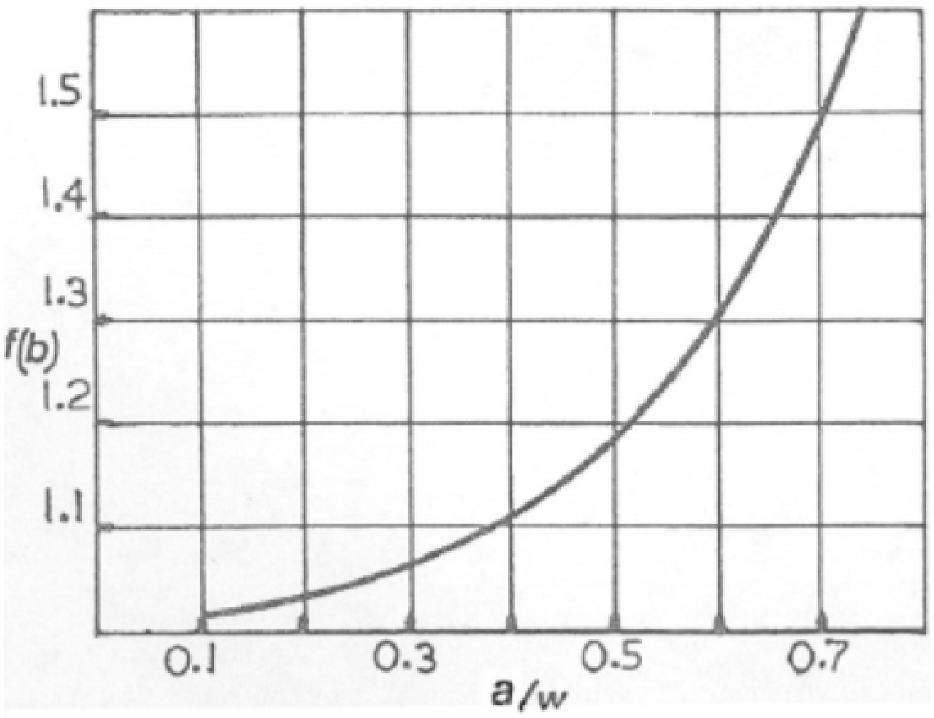
B) G_{IC} = 10.1 kJ/m² and K_{IC} = 39.4 MPa-m$^{1/2}$

C) G_{IC} = 20.2 kJ/m² and K_{IC} = 19.7 MPa-m$^{1/2}$

D) G_{IC} = 20.2 kJ/m² and K_{IC} = 39.4 MPa-m$^{1/2}$

ADDITIONAL INFORMATION

Figure 1 Secant correction factor for a central crack in a finite plate.



SOLUTIONS

P.1 ■ Solution

Since a/w = 4/10 = 0.4, we appeal to Figure 1 and read $f(b)$ = 1.1. The nominal stress in the first plate is then

$$\sigma_1 = \frac{K}{f(b)\sqrt{\pi a}} = \frac{66}{1.1\times\sqrt{\pi\times 4}} = 16.9\text{ ksi}$$

The corresponding nominal stress at collapse is given by

$$\sigma_{\text{col},1} = \frac{(w-a)\sigma_Y}{w} = \frac{(10-4)\times 72}{10} = 43.2\text{ ksi}$$

Since $\sigma_1 < \sigma_{\text{col},1}$, we conclude that plate 1 will not collapse. For the second plate, ratio a/w = 0.5/1.0 = 0.5 and $f(b)$ = 1.2. The nominal stress is

$$\sigma_2 = \frac{K}{f(b)\sqrt{\pi a}} = \frac{66}{1.2\times\sqrt{\pi\times 0.5}} = 43.9\text{ ksi}$$

The corresponding nominal stress at collapse is then

$$\sigma_{\text{col},2} = \frac{(w-a)\sigma_Y}{w} = \frac{(2-1)\times 72}{2} = 36\text{ ksi}$$

Since $\sigma_2 > \sigma_{\text{col},2}$, we surmise that plate 2 will collapse.

♦ The correct answer is **C**.

P.2 ■ Solution

1. False. The stress at fracture is σ = 250/(20 × 0.8) = 15.6 ksi. Given the ratio a/w = 1/(20/2) = 0.1, we read $f(b)$ = 1.0 from Figure 1. The stress intensity factor follows as

$$K = f(b) \times \sigma \times \sqrt{\pi a} = 15.6 \times 1.0 \times \sqrt{\pi \times 1.0} = 27.7 \text{ ksi}\sqrt{\text{in.}}$$

2. True. For plane strain to hold, the thickness B of the plate must be such that

$$B > 2.5\left(\frac{K}{\sigma_Y}\right)^2$$

In the case at hand, we have

$$B > 2.5\left(\frac{27.7}{70}\right)^2 = 0.391 \text{ in.}$$

Since B = 0.8 > 0.391 in., we surmise that the loading conditions are plane strain per the ASTM criterion.

3. True. The width of the plastic zone is

$$r_p = \frac{(K/\sigma_Y)^2}{6\pi} = \frac{(27.7/70)^2}{6\pi} = 8.31 \times 10^{-3} \text{ in.}$$

P.3 ■ Solution

Part 1: The fracture toughness for a compact-tension (CT) specimen can be estimated with the relation

$$K_I = \frac{P}{Ba^{1/2}} f\left(\frac{a}{w}\right)_C$$

where $f(a/w)_C$ follows from the regression equation

$$f\left(\frac{a}{w}\right)_C = 29.6\alpha^{0.5} - 185.5\alpha^{1.5} + 655.7\alpha^{2.5}$$
$$-1017\alpha^{3.5} + 639\alpha^{4.5}$$

In the present case, $\alpha = a/w$ = 2/5 = 0.4 and, accordingly,

$$f\left(\frac{a}{w}\right)_C = 29.6 \times 0.4^{0.5} - 185.5 \times 0.4^{1.5} + 655.7 \times 0.4^{2.5}$$
$$-1017 \times 0.4^{3.5} + 639 \times 0.4^{4.5} = 7.33$$

Inserting our data into the equation for K_I gives

$$K_I = \frac{48.2}{1.8 \times 5^{1/2}} \times 7.33 = \boxed{87.8 \text{ ksi}\sqrt{\text{in.}}}$$

♦ The correct answer is **D**.

Part 2: In order for plane strain to hold, we must have

$$B \geq 2.5\left(\frac{K_{IC}}{\sigma_Y}\right)^2$$

In the present case,

$$2.5\left(\frac{87.8}{150}\right)^2 = 0.857 < 1.8 \text{ (OK)}$$

This is also less than the crack size a = 1.5 in. Next, the ratio B/r_y of thickness to the size of the plastic zone should be greater than 50 or so, with r_y given by

$$r_y = \frac{1}{6\pi}\left(\frac{K_{CI}}{\sigma_Y}\right)^2 = \frac{1}{6\pi} \times \left(\frac{87.8}{150}\right)^2 = 0.0182$$

It follows that B/r_y = 1.8/0.0182 = 98.9. Likewise, a/r_y = 2.0/0.0182 = 110. Lastly, the ratio of maximum load P_{max} to the P_5 load should be no greater than 1.10. In the present case,

$$\frac{P_{\max}}{P_5}=\frac{50,200}{48,200}=1.04$$

which is lower than 1.1, as expected. The test satisfies all four constraints and, therefore, the value of K_{IC} afforded by the test is a valid estimate of the specimen's fracture toughness.

♦ The correct answer is **A.**

P.4 ■ Solution

Part 1: The first step is to establish the depth c of the section centroid, which is given by

$$c=\frac{\Sigma A y_c}{\Sigma A}=\frac{(40\times 35)\times 17.5-2\times(18\times 31)\times 15.5}{(40\times 35)-2\times(18\times 31)}=25.4\text{ mm}$$

We also require the moment of inertia with respect to the x-axis,

$$I_x'=\frac{40\times 35^3}{3}-2\times\frac{18\times 31^3}{3}=214,000\text{ mm}^4$$

The moment of inertia about the centroidal x-axis follows from the parallel axis theorem,

$$I_x'=I_x+(\Sigma A)c^2\rightarrow 214,000=I_x+284\times 25.4^2$$

$$\therefore I_x=30,800\text{ mm}^4$$

The nominal stress in the beam is then

$$\sigma=\frac{Mc}{I_x}=\frac{180\times 0.0254}{\left(30,800\times 10^{-12}\right)}=146\text{ MPa}$$

The corresponding stress intensity factor is

$$K=Y\sigma\sqrt{\pi a}=1.12\times 146\times\sqrt{\pi\times\left(1.5\times 10^{-3}\right)}=11.2\text{ MPa}\sqrt{\text{m}}$$

Given the fracture toughness of aluminum K_{IC} = 29 MPa-m$^{1/2}$, the safety factor against brittle fracture is calculated as

$$FS_K=\frac{K_{IC}}{K}=\frac{29}{11.2}=\boxed{2.59}$$

♦ The correct answer is **B.**

Part 2: The stress intensity factor that corresponds to a factor of safety of 3.0 is found as

$$FS_K=\frac{K_{IC}}{K_{\max}}\rightarrow K_{\max}=\frac{K_{IC}}{FS_K}$$

$$\therefore K_{\max}=\frac{29}{3.0}=9.67\text{ MPa}\sqrt{\text{m}}$$

The maximum permissible crack length is then

$$K_{\max}=Y\sigma\sqrt{\pi a_{\max}}\rightarrow a_{\max}=\frac{1}{\pi}\left(\frac{K_{\max}}{Y\sigma}\right)^2$$

$$\therefore a_{\max}=\frac{1}{\pi}\times\left(\frac{9.67}{1.12\times 146}\right)^2=0.00111=\boxed{1.11\text{ mm}}$$

♦ The correct answer is **A.**

Part 3: For this type of aluminum, the factor of safety against brittle fracture for a crack length of 1.5 mm is determined to be

$$FS'_K = \frac{K'_{IC}}{K} = \frac{52}{11.2} = 4.64$$

The maximum crack size for a factor of safety equal to 3 is, in turn,

$$K_{\max} = Y\sigma\sqrt{\pi a} \rightarrow \frac{52}{3} = 1.12 \times 146 \times \sqrt{\pi \times a_{\max}}$$

$$\therefore a_{\max} = 3.58 \text{ mm}$$

Use of this tougher aluminum will cause the factor of safety for a crack size of 1.5 mm to increase almost 80 percent; the allowable crack size for a safety factor of 3.0 against brittle fracture is increased by more than 220 percent.

P.5 ■ Solution

The average radius of the tube is r_m = (45 + 50)/2 = 47.5 mm and the vertical distance from the crack to the neutral axis is y = 47.5 × sin 50° = 36.4 mm. The moment of inertia about the neutral axis is estimated as $I = \pi r_m^3 t = \pi \times 47.5^3 \times 5 = 1.68 \times 10^6$ mm^4. The bending stress is determined next,

$$\sigma = \frac{My}{I} = \frac{8000 \times 0.0364}{\left(1.68 \times 10^{-6}\right)} = 173 \text{ MPa}$$

The stress intensity factor is

$$K = Y\sigma\sqrt{\pi a} = 1.0 \times 173 \times \sqrt{\pi \times \left(5 \times 10^{-3}\right)} = 21.7 \text{ MPa}\sqrt{\text{m}}$$

The factor of safety against brittle fracture is then

$$FS_K = \frac{66}{21.7} = \boxed{3.04}$$

Next, we assess the stability of the tube against fully plastic yielding. The bending moment at yield is

$$M_Y = 4r_m^2 t\sigma_Y = 4 \times 0.0475^2 \times 0.005 \times \left(925 \times 10^6\right) = 41.7 \text{ kN} \cdot \text{m}$$

and the corresponding factor of safety is calculated to be

$$FS_Y = \frac{41.7}{8} = \boxed{5.21}$$

♦ The correct answer is **D**.

P.6 ■ Solution

For leak-before-break conditions, the critical crack length a_c must be greater than the wall thickness t. The critical crack length is given by

$$a_c = \frac{1}{\pi}\left(\frac{K_C}{Y\sigma}\right)^2 = \frac{1}{\pi} \times \left[\frac{88}{\left(1.12 \times 2/\pi\right) \times 275}\right]^2 = 6.41 \text{ cm}$$

Since a_c > 1.25 cm, the crack will reach the outer surface of the wall before the crack length reaches the critical value. The vessel will therefore leak before it breaks.

♦ The correct answer is **α**.

P.7 ■ Solution

Part 1: Appealing to the equation for hoop stress in a thin-walled pressure vessel, we have

$$\sigma_h = \frac{pr}{t} \rightarrow t = \frac{pr}{\sigma_h}$$

$$\therefore t = \frac{7 \times 0.22}{70} = 0.022 \text{ m}$$

At fracture, the pressure was p = 3.5 MPa and the hoop stress must have been

$$\sigma_h = \frac{pr}{t} = \frac{3.5\times 0.22}{0.022} = 35 \text{ MPa}$$

The semicircular surface crack has a geometric factor Y = 1.12 × 2/π, so that

$$K_{IC} = Y\sigma\sqrt{\pi a} = \left(1.12\times\frac{2}{\pi}\right)\times 35\times\sqrt{\pi\times\left(2.5\times 10^{-3}\right)} = \boxed{2.21 \text{ MPa}\sqrt{\text{m}}}$$

♦ The correct answer is **B.**

Part 2: We want $a_c > t$ at the design stress of 70 MPa. An efficient approach is to set the crack length equal to the wall thickness in order to determine the fracture toughness at which $a_c = t$. This would be the minimum fracture toughness to establish a leak-before-break condition. Accordingly,

$$K_{IC,\min} = Y\sigma\sqrt{\pi a} = \left(1.12\times\frac{2}{\pi}\right)\times 70\times\sqrt{\pi\times 0.0022} = 4.15 \text{ MPa}\sqrt{\text{m}}$$

The only available polymer with a fracture toughness greater than 4.15 MPa-m$^{1/2}$ is polyethylene terephthalate (PET).

♦ The correct answer is **D.**

P.8 ▪ Solution

Most fracture toughness-Charpy V-notch correlations require the yield strength, which we do not have. Two exceptions are the Barson-Rolfe correlation,

$$\frac{K_{IC}^2}{E} = 2\left(CVN\right)^{1.5}$$

and the Sailors-Corten correlation,

$$K_{IC} = 15.5\left(CVN\right)^{0.5}$$

Since we were given the modulus of elasticity, it is reasonable to appeal to the former expression. Thus,

$$\frac{K_{IC}}{E} = 2\left(CVN\right)^{1.5} \rightarrow K_{IC} = \sqrt{2E\left(CVN\right)^{1.5}}$$

$$\therefore K_{IC,\min} = \sqrt{2\times\left(30\times 10^6\right)\times 7^{1.5}} = 33{,}300 = 33.3 \text{ ksi}\sqrt{\text{in.}}$$

Applying the conversion factor 1.099 MPa-in.$^{1/2}$/(ksi-in.$^{1/2}$) brings to

$$K_{IC,\min} = 33.3\times 1.099 = 36.6 \text{ MPa}\sqrt{\text{m}}$$

On the other extreme, with CVN = 10 ft-lb, we find that

$$K_{IC,\max} = \sqrt{2\times\left(30\times 10^6\right)\times 10^{3/2}} = 43.6 \text{ ksi}\sqrt{\text{in.}} = 47.9 \text{ MPa}\sqrt{\text{m}}$$

The critical stress intensity factor for the conditions in question is

$$K_{\text{crit}} = Y\sigma\sqrt{\pi a} = \frac{2}{\pi}\times 380\times\sqrt{\pi\times 0.015} = 52.5 \text{ MPa}\sqrt{\text{m}}$$

Since this is greater than the largest SIF obtained for the steel component ($K_{\text{IC,max}}$ = 47.9 MPa-m$^{1/2}$), we surmise that the slag inclusion was responsible for the failure of the steel component.

P.9 ▪ Solution

Part 1: The thermal stress due to the temperature change is

$$\sigma_\theta = \alpha E\Delta T = \left(8\times 10^{-6}\right)\times\left(70\times 10^9\right)\times\left(400-300\right) = 56 \text{ MPa}$$

Since σ_a = 90 MPa > σ_θ, the rod would likely withstand the quench.

♦ The correct answer is **α.**

Part 2: The stress intensity factor associated with the thermal stress is

$$K = \sigma_\theta \sqrt{\pi a} = 56 \times \sqrt{\pi \times \left(1 \times 10^{-3}\right)} = 3.14 \text{ MPa}\sqrt{\text{m}}$$

Since $K > K_{IC}$ = 0.8 MPa-m$^{1/2}$, we surmise that the rod would fail. A similar conclusion can be attained if we instead calculate the allowable stress for the given fracture toughness; that is,

$$K = \sigma\sqrt{\pi a} \to \sigma_{\text{allow}} = \frac{K_{IC}}{\sqrt{\pi a}}$$

$$\therefore \sigma_{\text{allow}} = \frac{0.8 \times 10^6}{\sqrt{\pi \times \left(1 \times 10^{-3}\right)}} = 14.3 \text{ MPa}$$

which is less than the applied thermal stress σ_θ, once again indicating that the rod will fail.

♦ The correct answer is **β**.

Part 3: Since the thermal stress is linearly dependent on the temperature drop, the corresponding stress with ΔT = 50 K is σ_θ = 56/2 = 28 MPa. The corresponding stress intensity factor is

$$K = \sigma_\theta \sqrt{\pi a} = 28 \times \sqrt{\pi \times \left(0.5 \times 10^{-3}\right)} = 1.11 \text{ MPa}\sqrt{\text{m}}$$

This is greater than K_{IC} = 0.8 MPa-m$^{1/2}$; although the conditions are less severe, the rod would still fail. This can be verified if we compute the allowable stress for this crack size,

$$\sigma_{\text{allow}} = \frac{K_{IC}}{\sqrt{\pi a}} = \frac{0.8}{\sqrt{\pi \times \left(0.5 \times 10^{-3}\right)}} = 20.2 \text{ MPa}$$

which is less than σ_θ, as expected.

♦ The correct answer is **β**.

P.10 ■ Solution

Part 1: At instability, the energy release rate equals the R parameter, that is, $G = R$. Accordingly,

$$\frac{\pi \sigma^2 a_c}{E} = 6.95\left(a_c - a_0\right)^{0.5} \quad \text{(I)}$$

In addition, we must have $dG/da = dR/dA$, so that

$$\frac{\pi \sigma^2}{E} = 3.48\left(a_c - a_0\right)^{-0.5} \quad \text{(II)}$$

Substituting the available data in equation (II) gives

$$\frac{\pi \times 138^2}{207{,}000} = 3.48\left(a_c - a_0\right)^{-0.5} \to \left(a_c - a_0\right) = 145 \text{ mm}$$

Backsubstituting in equation (I) gives

$$\frac{\pi \times 138^2 \times a_c}{207{,}000} = 6.95 \times 145^{0.5} \to \boxed{a_c = 290 \text{ mm}}$$

♦ The correct answer is **C**.

Part 2: Dividing equation (I) by equation (II) brings to

$$\frac{\pi\sigma^2 a_c / E}{\pi\sigma^2 / E} = \frac{6.95\left(a_c - a_0\right)^{0.5}}{3.48\left(a_c - a_0\right)^{-0.5}} \to a_c = 2\left(a_c - a_0\right)$$

Substituting the initial crack length a_0 = 25.4 mm and solving for a_c, we obtain

$$a_c = 2(a_c - a_0) \rightarrow a_c = 2(a_c - 25.4)$$

$$\therefore a_c = 50.8 \text{ mm}$$

We can then substitute the available data into equation (I) and solve for the critical stress σ_c,

$$\frac{\pi\sigma_c^2 a_c}{E} = 6.95(a_c - a_0)^{0.5} \rightarrow \frac{\pi \times \sigma_c^2 \times 0.0508}{207 \times 10^6} = 6.95 \times (0.0508 - 0.0254)^{0.5}$$

$$\therefore \sigma_c = 213{,}000 \text{ kPa} = \boxed{213 \text{ MPa}}$$

♦ The correct answer is **C**.

P.11 ■ Solution

Part 1: The first step is to obtain an expression for the energy release rate of a cantilever beam. From beam theory, the deflection Δ is found to be

$$\frac{\Delta}{2} = \frac{Pa^3}{3EI}$$

where moment of inertia $I = Bh^3/12$. The elastic compliance C, being the inverse of stiffness, is calculated as

$$C = \frac{\Delta}{P} = \frac{2a^3}{3EI}$$

At this point, we appeal to the equation for energy release rate, giving

$$G = \frac{P^2}{2B}\frac{dC}{da} = \frac{P^2}{2B} \times \frac{2a^2}{EI}$$

$$\therefore G = \frac{P^2 a^2}{BEI}$$

$$\therefore G = \frac{12P^2 a^2}{B^2 h^3 E}$$

At instability, $G = R$ and $dG/da = dR/da$. Hence, we can write

$$G_c = \frac{12P_c^2 a_c^2}{B^2 h^3} = 6.95(a_c - a_0)^{0.5}$$

and

$$\frac{2G_c}{a_c} = 3.48(a_c - a_0)^{-0.5}$$

Dividing one equation by the other yields

$$\frac{G_c}{2G_c/a_c} = \frac{6.95(a_c - a_0)^{0.5}}{3.48(a_c - a_0)^{-0.5}} \rightarrow \frac{a_c}{2} = 2(a_c - a_0)$$

With an initial crack length a_0 = 152 mm, we obtain

$$\frac{a_c}{2} = 2(a_c - a_0) \rightarrow \frac{a_c}{2} = 2(a_c - 152)$$

$$\therefore a_c = 203 \text{ mm}$$

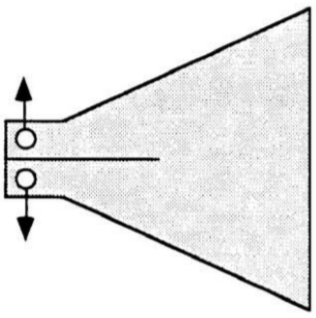
Backsubstituting in the equation for G_c, the load at failure is calculated to be

$$\frac{12P_c^2 a_c^2}{B^2 h^3 E} = 6.95(a_c - a_0)^{0.5} \rightarrow \frac{12 \times P_c^2 \times 0.203^2}{0.0254^2 \times 0.0127^3 \times 207 \times 10^6} = 6.95 \times (203 - 152)^{0.5}$$

$$\therefore \boxed{P_c = 5.24 \text{ kN}}$$

♦ The correct answer is **B**.

Part 2: In a conventional DCB specimen, compliance varies with a^3, while energy release is proportional to a^2 when load is fixed. In order for G to remain constant with crack growth, compliance must vary linearly with crack length. One way to accomplish this is to taper the specimen width, as illustrated in the following figure. Alternatively, the thickness can be tapered. The latter method is not as effective as the former because compliance is less sensitive to the thickness dimension (recall that the moment of inertia $I = Bh^3/12$. Specimens such as the one illustrated below have been successfully used in laboratory experiments.



P.12 ■ Solution

Part 1: The L-T orientation is slightly tougher than the T-L. However, the L-T crack lies along the cylinder axis and is therefore opened by the hoop stress. The stress is twice the longitudinal stress acting on the T-L orientation, and fracture toughness is directly proportional to stress. Thus, the L-T crack is favored to fail first.

♦ The correct answer is **α.**

Part 2: This crack would be opened by a radial tension stress. There is no radial stress in the radial direction of a pressure vessel, and the strain is compressive, so there is no chance that this crack will fail despite being in the least tough orientation.

Part 3: One possible approach is to find the critical crack length and compare it to the wall thickness. The appropriate fracture toughness in this case is the L-T value. Accordingly,

$$K_{IC} = Y\sigma\sqrt{\pi a} \rightarrow a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{Y\sigma}\right)^2$$

$$\therefore a_c = \frac{1}{\pi}\times\left[\frac{29.7}{(1.12\times 2/\pi)\times 300}\right]^2 = 6.14 \text{ mm}$$

The plate thickness is 3 mm, so it easily meets the leak-before-break criterion.

P.13 ■ Solution

The compliance at a = 20.1 mm is c_1 = 0.291 × $10^{-3}/10^5$ m/N, and that at a = 21.1 mm is c_2 = 0.298 × $10^{-3}/10^5$ m/N. The variation in compliance between the two data points is

$$\frac{\Delta c}{\Delta a} = \frac{(c_2 - c_1)}{\Delta a} = 7\times 10^{-8} \text{ N}^{-1}$$

noting that Δa = 1 mm. The energy release rate is determined to be

$$G_{IC} = \frac{P_c^2}{2B}\frac{\Delta c}{\Delta a} = \frac{\left(120\times 10^3\right)^2}{2\times\left(25\times 10^{-3}\right)}\times\left(7\times 10^{-8}\right) = \boxed{20.2 \text{ kJ/m}^2}$$

The energy release rate is associated to fracture toughness by the relation

$$G_{IC} = \frac{\left(1-\nu^2\right)K_{IC}^2}{E} \rightarrow K_{IC} = \sqrt{\frac{EG_{IC}}{1-\nu^2}}$$

so that

$$K_{IC} = \sqrt{\frac{\left(70\times 10^9\right)\times 20,200}{1-0.3^2}} = \boxed{39.4 \text{ MPa}\sqrt{\text{m}}}$$

♦ The correct answer is **D.**

ANSWER SUMMARY

Problem 1		C
Problem 2		T/F
Problem 3	3.1	D
	3.2	A
Problem 4	4.1	B
	4.2	A
	4.3	Open-ended pb.
Problem 5		D
Problem 6		α
Problem 7	7.1	B
	7.2	D
Problem 8		Open-ended pb.
Problem 9	9.1	α
	9.2	β
	9.3	β
Problem 10	10.1	C
	10.2	C
Problem 11	11.1	B
	11.2	Open-ended pb.
Problem 12	12.1	α
	12.2	Open-ended pb.
	12.3	Open-ended pb.
Problem 13		D

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