Quiz HT102

Extended Surfaces

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Problems

Problem 1 (Çengel & Ghajar, 2015, w/ permission)

Consider a very long rectangular aluminum fin attached to a flat surface such that the temperature at the end of the fin is essentially that of the surrounding air, i.e. 20°C; its width is 5.0 cm; the thickness is 1.0 mm; and the base temperature is 40°C. The heat transfer coefficient is 20 W/m²K. Estimate the fin temperature at a distance of 5.0 cm from the base and the rate of heat loss from the entire fin.

A) T_x = 29.8°C and $\dot{q}_x$ = 2.89 W
B) T_x = 29.8°C and $\dot{q}_x$ = 5.41 W
C) T_x = 36.5°C and $\dot{q}_x$ = 2.89 W
D) T_x = 36.5°C and $\dot{q}_x$ = 5.41 W

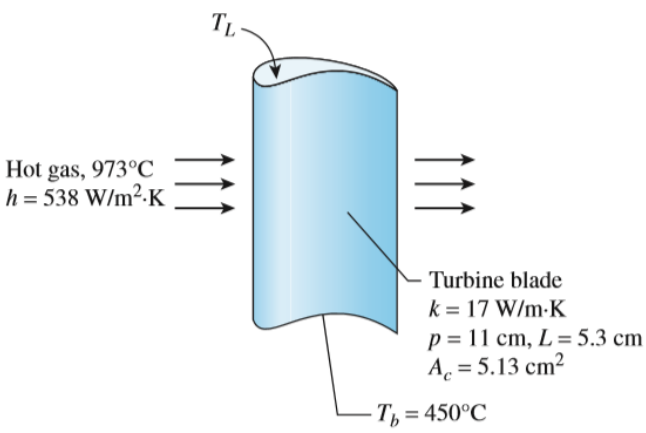
Problem 2 (Kreith et al., 2011, w/ permission)

The tip of a soldering iron consists of a 0.6-cm-diameter copper rod, 7.6 cm long. If the tip must be at 204°C, what are the required minimum temperature of the base and the heat flow into the base? Assume that the heat transfer coefficient h = 22.7 W/m²K and the air temperature T_{air} = 21°C.

A) T_b = 225°C and $\dot{q}_f$ = 3.16 W
B) T_b = 225°C and $\dot{q}_f$ = 6.32 W
C) T_b = 258°C and $\dot{q}_f$ = 3.16 W
D) T_b = 258°C and $\dot{q}_f$ = 6.32 W

Problem 3 (Çengel & Ghajar, 2015, w/ permission)

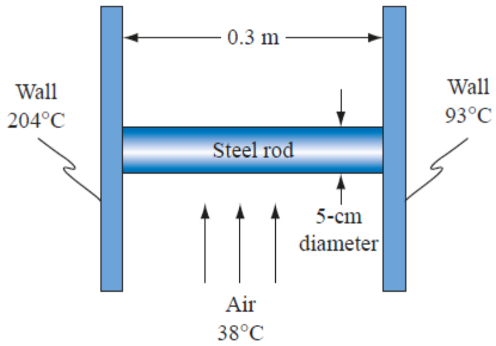
A turbine blade made of stainless steel has a length of 5.3 cm, a perimeter of 11 cm, and a cross-sectional area of 5.13 cm². The turbine blade is exposed to hot gas from the combustion chamber at 973°C with a convection heat transfer coefficient of 538 W/m²K. The base of the turbine blade maintains a constant temperature of 450°C and the tip is adiabatic. Determine the temperature at the tip of the blade and the heat transfer rate.



A) T_L = 912°C and $\dot{q}_f$ = 352.9 W
B) T_L = 912°C and $\dot{q}_f$ = 525.4 W
C) T_L = 963°C and $\dot{q}_f$ = 352.9 W
D) T_L = 963°C and $\dot{q}_f$ = 525.4 W

Problem 4 (Kreith et al., 2011, w/ permission)

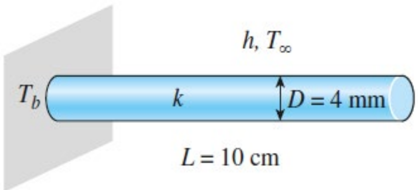
One end of a 0.3-m-long 1% carbon steel rod is connected to a wall at 204°C. The other end is connected to a wall which is maintained at 93°C. Air is blown across the rod so that a heat transfer coefficient of 17 W/m²K is maintained over the entire surface. If the diameter of the rod is 5 cm and the temperature of the air is 38°C, what is the net rate of heat loss to the air?



A) $\dot{q}_f$ = 54.9 W
B) $\dot{q}_f$ = 66.5 W
C) $\dot{q}_f$ = 74.4 W
D) $\dot{q}_f$ = 85.7 W

Problem 5 (Çengel & Ghajar, 2015, w/ permission)

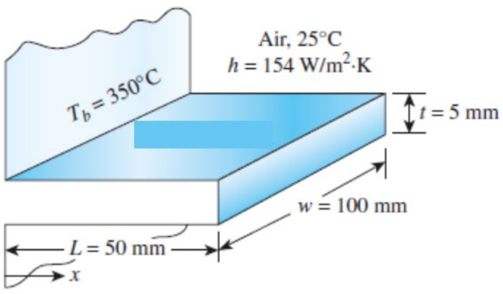
A 4-mm-diameter and 10-cm-long aluminum fin is attached to a surface. If the heat transfer coefficient is 12 W/m²K, determine the percent error in the rate of heat transfer from the fin when the infinitely long fin assumption is used instead of the adiabatic fin tip assumption.



A) $|\Delta\dot{q}|$= 28.2%
B) $|\Delta\dot{q}|$= 35.6%
C) $|\Delta\dot{q}|$= 41.5%
D) $|\Delta\dot{q}|$= 50.4%

Problem 6 (Çengel & Ghajar, 2015, w/ permission)

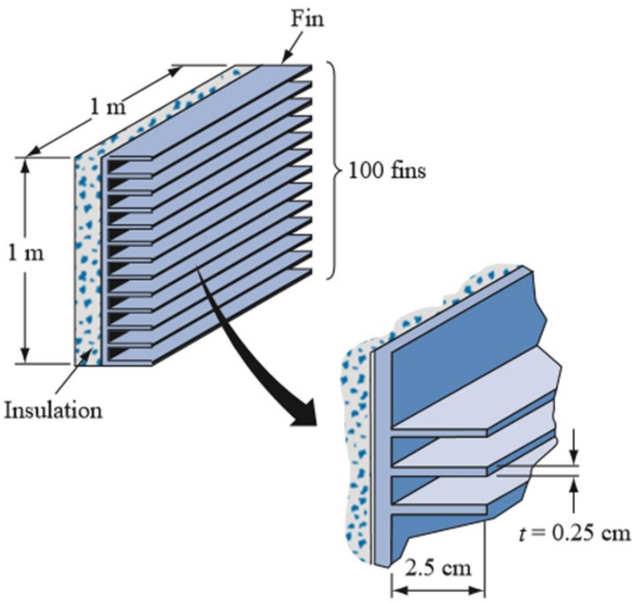
A plane wall with surface temperature of 350°C is attached with straight rectangular fins made of chromium. The fins are exposed to an ambient air condition of 25°C and the convection heat transfer coefficient is 154 W/m²K. Each fin has a length of 50 mm, a base of 5 mm thickness and a width of 100 mm. Determine the efficiency (η) and the effectiveness (ε) of each fin.



A) η_{fin} = 64.9% and ε_{fin} = 13.6
B) η_{fin} = 64.9% and ε_{fin} = 17.8
C) η_{fin} = 76.8% and ε_{fin} = 13.6
D) η_{fin} = 76.8% and ε_{fin} = 17.8

Problem 7 (Kreith et al., 2011, w/ permission)

The addition of aluminum fins has been suggested to increase the rate of heat dissipation from one side of an electronic device 1 m wide and 1 m tall. The fins are rectangular in cross-section, 2.5 cm long and 0.25 cm thick, as shown in the figure. There are to be 100 fins per meter. The heat transfer coefficient, both for the wall and the fins, is estimated to be 35 W/m^{2}K. With this information, determine the percent increase in the rate of heat transfer of the finned wall compared to the bare wall.



A) $|\Delta\dot{q}|$= 234.6%
B) $|\Delta\dot{q}|$= 317.5%
C) $|\Delta\dot{q}|$= 454.2%
D) $|\Delta\dot{q}|$= 510.6%

Problem 8A (Kreith et al., 2011, w/ permission)

Circular cooling fins of diameter D = 1 mm and length L = 25.4 mm made of copper are used to enhance heat transfer from a surface that is maintained at temperature T_{s1} = 132°C. Each rod has one end attached to this surface (x = 0) while the opposite end (x = L) is joined to a second surface, which is maintained at T_{s2} = 0°C. The air flowing between the surfaces and the rods is also at T_∞ = 0°C, and the heat transfer coefficient is h = 100 W/m^{2}K. Express the function $\theta(x) = T(x) - T_\infty$ along a fin, then answer: what is the temperature at x = $L/2$?

A) $T_{L/2}$ = 26.5°C
B) $T_{L/2}$ = 44.2°C
C) $T_{L/2}$ = 61.0°C
D) $T_{L/2}$ = 80.7°C

Problem 8B

Determine the rate of heat transferred from the hot surface through each fin and the fin effectiveness. Is the use of fins justified? Why?

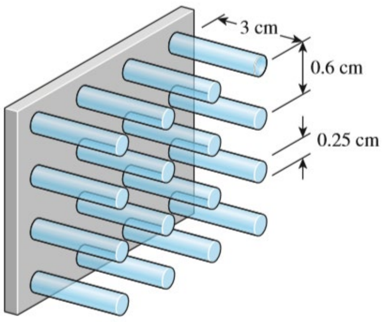
Problem 8C

What is the total rate of heat transfer from a 10-cm by 10-cm section of the wall, which has 625 uniformly distributed fins? Assume the same heat transfer coefficient for the fin and the unfinned surface.

A) $\dot{q}_{\text{total}}$ = 685.2 W
B) $\dot{q}_{\text{total}}$ = 981.4 W
C) $\dot{q}_{\text{total}}$ = 1357 W
D) $\dot{q}_{\text{total}}$ = 1725 W

Problem 9 (Çengel & Ghajar, 2015, w/ permission)

A hot surface at 100°C is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins to it, with center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m²K. Determine the rate of heat transfer from the surface for a 1-m × 1-m section of the plate. Also determine the overall effectiveness of the fins.



A) $\dot{q}_{\text{total}}$ = 17.27 kW and ε_{fin} = 3.45
B) $\dot{q}_{\text{total}}$ = 17.27 kW and ε_{fin} = 7.05
C) $\dot{q}_{\text{total}}$ = 34.54 kW and ε_{fin} = 3.45
D) $\dot{q}_{\text{total}}$ = 34.54 kW and ε_{fin} = 7.05

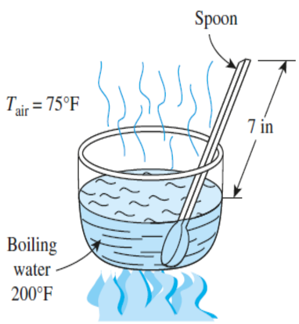
Problem 10 (Kreith et al., 2011, w/ permission)

To determine the thermal conductivity of a long, solid 2.5-cm-diameter rod, one half of the rod was inserted into a furnace while the other half is projected into air at 27°C. After steady state had been reached, the temperatures at two points 7.6 cm apart were measured and found to be 126°C and 91°C, respectively. The heat transfer coefficient over the surface of the rod exposed to the air was estimated to be 22.7 W/m²K. What is the thermal conductivity of the rod?

A) k = 50 W/m·K
B) k = 70 W/m·K
C) k = 90 W/m·K
D) k = 110 W/m·K

Problem 11 (Çengel & Ghajar, 2015, w/ permission)

Consider a stainless steel spoon partially immersed in boiling water at 200°F in a kitchen at 75°F. The handle of the spoon has a cross-section of 0.08 in. × 0.5 in. and extends 7 in. in the air from the free surface of the water. If the heat transfer coefficient at the exposed surfaces of the spoon handle is 1 Btu/h·ft²·°F, determine the temperature difference across the exposed surface of the spoon handle.



A) ΔT = 118.8°F
B) ΔT = 124.6°F
C) ΔT = 130.1°F
D) ΔT = 136.2°F

Problem 12A (Bergman et al., 2011, w/ permission)

A rectangular straight fin fabricated from aluminum has a base thickness of t = 3 mm and a length of L = 15 mm. Its base temperature is T_b = 100°C and it is exposed to a fluid for which T_∞ = 20°C and h = 50 W/m²K. For the foregoing conditions and a fin of unit width, determine the fin heat transfer rate.

A) $\dot{q}$ = 112.7 W
B) $\dot{q}$ = 130.1 W
C) $\dot{q}$ = 144.5 W
D) $\dot{q}$ = 178.0 W

Problem 12B

Determine the length of a triangular fin of unit width and base thickness t = 3 mm that will provide the same fin heat rate as the straight rectangular fin considered in the previous problem.

A) L_1 = 16.3 mm
B) L_1 = 21.4 mm
C) L_1 = 27.1 mm
D) L_1 = 32.2 mm

Problem 12C

Repeat the previous problem for a parabolic straight fin of unit width and base thickness t = 3 mm.

A) L_2 = 16.6 mm
B) L_1 = 21.8 mm
C) L_1 = 27.5 mm
D) L_1 = 32.4 mm

Additional Information

Table 1 Thermal conductivity of selected materials at room temperature

Material	k (W/mK)
1% carbon steel	43
Aluminum	205
Cast iron	52
Chromium	94
Copper	401
Stainless steel	15

Table 2 Temperature distribution and heat loss for fins of uniform cross-section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.80)	$M \tanh mL$ (3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.83)
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx} (3.84)	M (3.85)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$
$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\theta_b$

Table 3 Efficiency of common fin shapes

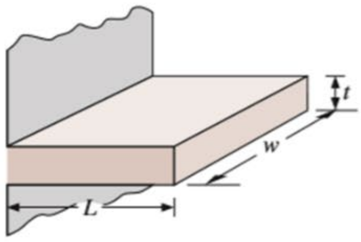
Straight Fins

Rectangular[a]

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$

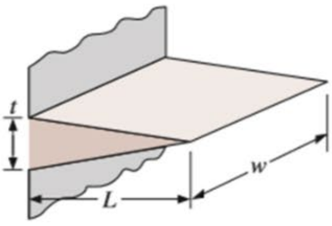


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

Triangular[a]

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$



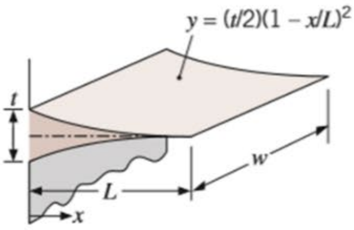
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

Parabolic[a]

$A_f = w[C_1 L + (L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

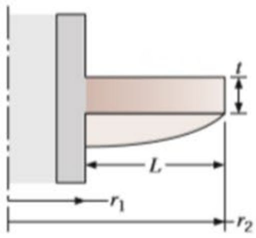
Circular Fin

Rectangular[a]

$A_f = 2\pi(r_{2c}^2 - r_1^2)$

$r_{2c} = r_2 + (t/2)$

$V = \pi(r_2^2 - r_1^2)t$



$$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})} \quad (3.96)$$

$$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$$

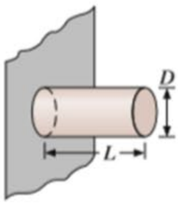
Pin Fins

Rectangular[b]

$A_f = \pi D L_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$

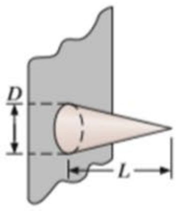


$$\eta_f = \frac{\tanh mL_c}{mL_c} \quad (3.100)$$

Triangular[b]

$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$

$V = (\pi/12)D^2 L$



$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} \quad (3.101)$$

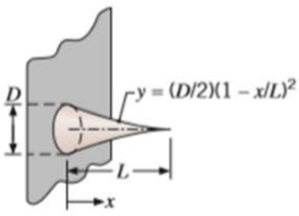
Parabolic[b]

$A_f = \frac{\pi L^3}{8D}\left\{C_3 C_4 - \frac{L}{2D}\ln[(2DC_4/L) + C_3]\right\}$

$C_3 = 1 + 2(D/L)^2$

$C_4 = [1 + (D/L)^2]^{1/2}$

$V = (\pi/20)D^2 L$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1} \quad (3.102)$$

[a] $m = (2h/kt)^{1/2}$.

[b] $m = (4h/kD)^{1/2}$.

Solutions

P.1 Solution

The temperature distribution for a very long rectangular fin is

$$\frac{T-T_\infty}{T_b-T_\infty}=e^{-mx}$$

where T is the temperature of the fin, T_b is the base temperature, T_∞ is the ambient temperature, and x is the distance from the base. Also, m is a coefficient given by

$$m=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{20\times\left[2\left(5\times10^{-2}+1\times10^{-3}\right)\right]}{205\times\left(5\times10^{-2}\times1\times10^{-3}\right)}}=14.28\ \text{m}^{-1}$$

Substituting the value of m and other variables in the initial expression, we can obtain the temperature distribution $T(x)$,

$$\frac{T-T_\infty}{T_b-T_\infty}=e^{-mx}\rightarrow\frac{T(x)-20}{40-20}=\frac{T(x)-20}{20}=e^{-14.28x}$$

The fin temperature at a distance of 5 cm from the base follows as

$$\frac{T(0.05)-20}{20}=e^{-14.28\times0.05}$$

$$\therefore T(0.05)=T_x=20e^{-14.28\times0.05}+20=\boxed{29.8^\circ\text{C}}$$

The rate of heat transfer, in turn, can be obtained with the following expression,

$$\dot{q}_x=\sqrt{hpkA_c}\,(T_b-T_\infty)$$

$$\therefore\dot{q}_x=\sqrt{20\times\left[2\times\left(5\times10^{-2}+1\times10^{-3}\right)\right]\times401\times\left(5\times10^{-2}\times1\times10^{-3}\right)}(40-20)=\boxed{2.89\ \text{W}}$$

♦ The correct answer is **A**.

P.2 Solution

The temperature distribution for a fin with a uniform cross-section and convection at the tip is given by

$$\frac{\theta}{\theta_b}=\frac{\cosh\left[m(L-x)\right]+(h/mk)\sinh\left[m(L-x)\right]}{\cosh mL+(h/mk)\sinh mL}$$

where factor m is calculated as

$$m=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{h\times\pi D}{k\times\pi D^2/4}}=\sqrt{\frac{4h}{kD}}$$

$$\therefore m=\sqrt{\frac{4\times22.7}{401\times\left(6\times10^{-3}\right)}}=6.14\,\text{m}^{-1}$$

Evaluating the temperature at x = L = 0.076 m and solving for the base temperature T_b, we obtain

$$\frac{\theta_L}{\theta_b}=\frac{T_L-T_\infty}{T_b-T_\infty}=\frac{\cosh 0+\left[22.7/(401\times6.14)\right]\sinh 0}{\cosh(6.14\times0.076)+\left[22.7/(401\times6.14)\right]\sinh(6.14\times0.076)}=0.897$$

$$\therefore T_b=T_\infty+\frac{T_L-T_\infty}{0.897}=21+\frac{204-21}{0.897}=\boxed{225^\circ\text{C}}$$

To maintain steady-state conditions, the rate of heat transfer into the rod must be equal to the rate of heat loss from the rod. The rate of heat loss is

$$\dot{q}_f = M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

Here, factor *M* is computed as

$$M = \sqrt{hpkA_c}\theta_b = \sqrt{\frac{\pi^2 hkD^3}{4}}\theta_b$$

$$\therefore M = \sqrt{\frac{\pi^2 \times 22.7 \times 401 \times \left(6\times 10^{-3}\right)^3}{4}} \times (226-21) = 14.28\ \text{W}$$

Accordingly,

$$\dot{q}_f = 14.28 \times \frac{\sinh(6.14\times 0.076) + \left[22.7/(401\times 6.14)\right]\cosh(6.14\times 0.076)}{\cosh(6.14\times 0.076) + \left[22.7/(401\times 6.14)\right]\sinh(6.14\times 0.076)} = \boxed{6.32\ \text{W}}$$

♦ The correct answer is **B**.

P.3 Solution

To begin, we compute factor *m*,

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{538\times 0.11}{15\times\left(5.13\times 10^{-4}\right)}} = 87.70\ \text{m}^{-1}$$

For an adiabatic fin tip, the temperature distribution is described by the equation

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

At the tip, $x = L$ and

$$\frac{T(L)-T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-L)}{\cosh mL} \rightarrow \frac{T_L - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh mL}$$

Substituting the pertaining variables and solving for T_L, we get

$$\frac{T_L - 973}{450-973} = \frac{1}{\cosh(87.7\times 0.053)} \rightarrow \boxed{T_L = 963^\circ\text{C}}$$

The heat transfer rate is given by

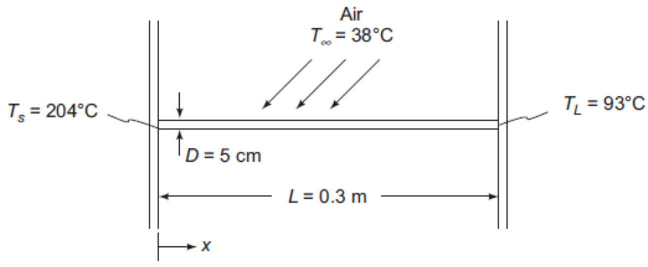
$$\dot{q}_f = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL$$

$$\therefore \dot{q}_f = \sqrt{538\times\left(11\times 10^{-2}\right)\times 15\times\left(5.13\times 10^{-4}\right)}\times(973-450)\times\tanh(87.70\times 0.053) = \boxed{352.9\ \text{W}}$$

♦ The correct answer is **C**.

P.4 Solution

The rod is idealized as a fin of uniform cross-section with fixed temperatures at both ends.



The rate of heat loss can be obtained with the expression

$$q_f = M\frac{\cosh mL - (\theta_L/\theta_s)}{\sinh mL}$$

where $\theta_L = T_L - T_\infty = 93 - 38 = 55$°C and $\theta_s = T_s - T_\infty = 204 - 38 = 166$°C. We proceed to compute factors *mL* and *M*,

$$mL = \sqrt{\frac{hp}{kA}}L = \sqrt{\frac{h\times\pi D}{k\times\frac{\pi D^2}{4}}}L = \sqrt{\frac{4h}{kD}}L = \sqrt{\frac{4\times 17}{43\times 0.05}}\times 0.3 = 1.69$$

$$M = \sqrt{hpkA}\theta_s = \frac{1}{2}\sqrt{h\pi^2 D^3 k}\theta_s = \frac{1}{2}\sqrt{17\times\pi^2\times 0.05^3\times 43}\times 166 = 78.82\text{ W}$$

Substituting these quantities into the equation for for q_f gives

$$\dot{q}_f = 78.82\times\frac{\cosh 1.69 - (55/166)}{\sinh 1.69} = \boxed{74.4\text{ W}}$$

♦ The correct answer is **C**.

P.5 Solution

First, suppose that the pin is of infinitely long length. Then, the rate of heat transfer is

$$\dot{q}_1 = \sqrt{hpkA}\left(T_b - T_\infty\right)$$

$$\therefore \dot{q}_1 = \sqrt{12\times\left(\pi\times 4\times 10^{-3}\right)\times 205\times\left[\pi\times\left(4\times 10^{-3}\right)^2/4\right]}\left(T_b - T_\infty\right) = 0.01971\left(T_b - T_\infty\right)$$

Next, suppose that the pin has an adiabatic tip. Then, the heat transfer rate becomes

$$\dot{q}_2 = \sqrt{hpkA}\left(T_b - T_\infty\right)\tanh\sqrt{\frac{hp}{kA}}L$$

$$\therefore \dot{q}_2 = \sqrt{12\times\left[\pi\times\left(4\times 10^{-3}\right)\right]\times 205\times\left[\pi\times\left(4\times 10^{-3}\right)^2/4\right]}\tanh\left\{\sqrt{\frac{12\times\left[\pi\times\left(4\times 10^{-3}\right)\right]}{205\times\left[\pi\times\left(4\times 10^{-3}\right)^2/4\right]}}\times 5\times 10^{-2}\right\}\left(T_b - T_\infty\right)$$

$$\therefore \dot{q}_2 = 0.01269\left(T_b - T_\infty\right)$$

The percentage error follows as

$$\left|\Delta\dot{q}\right| = \frac{\dot{q}_1 - \dot{q}_2}{\dot{q}_1}\times 100 = \frac{(0.01971 - 0.01269)\left(T_b - T_\infty\right)}{0.01269\left(T_b - T_\infty\right)} = \boxed{35.6\%}$$

Thus, assuming that the condition of an adiabatic tip is indeed applicable to the actual fin, we conclude that the infinite fin assumption overestimates the heat transfer rate by more than 30 percent.

♦ The correct answer is **B**.

P.6 Solution

The fin efficiency is the ratio of the fin heat transfer rate to the heat transfer rate if the entire fin were at the base temperature. From the relations available in the Additional Information section, we see that the efficiency for a straight rectangular fin is

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c}$$

where *m* and L_c are such that

$$m=\sqrt{\frac{2h}{kt}}=\sqrt{\frac{2\times 154}{94\times\left(5\times 10^{-3}\right)}}=25.6\text{ m}^{-1}$$

$$L_c=L+\frac{t}{2}=0.05+\frac{5\times 10^{-3}}{2}=0.0525\text{ m}$$

Substituting these quantities into the expression for η_{fin} gives

$$\eta_{\text{fin}}=\frac{\tanh\left(25.6\times 0.0525\right)}{25.6\times 0.0525}=\boxed{64.9\%}$$

Next, the effectiveness is the ratio of the fin heat transfer rate to the heat transfer rate if the surface had no fin. Mathematically,

$$\varepsilon_{\text{fin}}=\frac{\dot{q}_{\text{fin}}}{\dot{q}_{\text{without fin}}}=\frac{\dot{q}_{\text{fin}}}{hA_b\left(T_b-T_\infty\right)}$$

where A_b is the base area, $A_b = wt = 0.1 \times (5 \times 10^{-3}) = 5 \times 10^{-4}$ m^2. The rate of heat transfer with the fin, $\dot{q}_{\text{fin}}$, can be computed with the equation

$$\dot{q}_{\text{fin}}=\eta_{\text{fin}}hA_{\text{fin}}\left(T_b-T_\infty\right)$$

where A_{fin} is the fin surface area, $A_{fin} = 2 \times 0.1 \times 0.0525 = 0.105$ m^2. Substituting this quantity and other pertaining variables gives

$$\dot{q}_{\text{fin}}=\eta_{\text{fin}}hA_{\text{fin}}\left(T_b-T_\infty\right)=0.649\times 154\times 0.105\times\left(350-25\right)=341.1\text{ W}$$

Returning to the expression for fin effectiveness, we ultimately obtain

$$\varepsilon_{\text{fin}}=\frac{\dot{q}_{\text{fin}}}{hA_b\left(T_b-T_\infty\right)}=\frac{341.1}{154\times\left(5\times 10^{-4}\right)\times\left(350-25\right)}=\boxed{13.6}$$

♦ The correct answer is **A**.

P.7 Solution

The equation that models heat transfer in a fin with convection at the tip is

$$q_f=M\frac{\sinh mL+\left(h/mk\right)\cosh mL}{\cosh mL+\left(h/mk\right)\sinh mL}$$

For a width of 1 m, and noting that the thickness of each fin is t = 0.0025 m, factor M is calculated as

$$M=\sqrt{hpkA_c}\theta_b=\sqrt{h\times\left[2\left(t+w\right)\right]\times k\times\left(t\times w\right)}\theta_b$$

$$\therefore M=\sqrt{35\times\left(2\times 1.0025\right)\times 205\times\left(1.0\times 0.0025\right)}\theta_b$$

$$\therefore M=6.00\theta_b$$

Likewise, coefficient m is computed as

$$m=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{35\times\left(2\times 1.0025\right)}{205\times 0.0025}}=11.70\text{ m}^{-1}$$

The rate of heat transfer from one fin is

$$\dot{q}_f=6.00\theta_b\times\frac{\sinh\left(11.70\times 0.025\right)+\left[35/\left(11.70\times 205\right)\right]\cosh\left(11.70\times 0.025\right)}{\cosh\left(11.70\times 0.025\right)+\left[35/\left(11.70\times 205\right)\right]\sinh\left(11.70\times 0.025\right)}=1.787\theta_b$$

In 1 m^2 of wall area there are 100 fins covering $100 \times t \times w = 100 \times 0.0025 \times 1.0 = 0.25$ m^2 of wall area, which leaves 1.0 – 0.25 = 0.75 m^2 of bare wall. The total rate of heat transfer from the wall with fins is the sum of the heat transfer from the bare wall and the heat transfer from 100 fins. In mathematical terms,

$$\dot{q}_{\text{total}} = \dot{q}_{\text{bare wall}} + \dot{q}_{100\text{ fins}} = hA_{\text{bare}}\theta_b + 100\dot{q}_f$$

$$\therefore \dot{q}_{\text{total}} = 35\times1.0\times\theta_b + 100\times1.787\theta_b$$

$$\therefore \dot{q}_{\text{total}} = 213.7\theta_b$$

The rate of heat transfer from the wall without fins is

$$\dot{q}_{\text{bare}} = hA_c\theta_b = 35\times1.0\times\theta_b = 35\theta_b$$

The percent increase due to the addition of the fins follows as

$$|\Delta\dot{q}| = \frac{\dot{q}_{\text{total}} - \dot{q}_{\text{bare}}}{\dot{q}_{\text{bare}}} = \frac{213.7\theta_b - 35\theta_b}{35\theta_b} = \boxed{510.6\%}$$

That is, implementation of fins leads to a six-fold increase in the heat of heat transfer. The assumption that the convective heat transfer coefficient is the same for the fins and the wall is an oversimplification of the real situation, but does not affect the final results appreciably.

♦ The correct answer is **D**.

P.8 Solution

Part A: For a fin with prescribed tip temperature, we can write

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh mx + \sinh\left[m(L-x)\right]}{\sinh mL}$$

where $\theta_b = T_b - T_\infty = T_{s1}$ and $\theta_L = T_L - T_\infty = 0$. Substituting these quantities, the relation above simplifies to

$$\frac{\theta}{\theta_b} = \frac{\sinh\left[m(L-x)\right]}{\sinh mL}$$

We proceed to compute factor m,

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{100\times\left[\pi\times\left(1.0\times10^{-3}\right)\right]}{401\times\left[\pi\times\left(1.0\times10^{-3}\right)^2/4\right]}} = 31.58\ \text{m}^{-1}$$

Accordingly, the temperature at $x = L/2$ is determined to be

$$T_{L/2} = \theta_b\frac{\sinh(mL/2)}{\sinh mL} = T_{s1}\frac{\sinh(mL/2)}{\sinh mL}$$

$$\therefore T_{L/2} = 132\times\frac{\sinh\left[31.58\times\left(2.54\times10^{-2}\right)/2\right]}{\sinh\left[31.58\times\left(2.54\times10^{-2}\right)\right]} = \boxed{61.0^\circ\text{C}}$$

♦ The correct answer is **C**.

Part B: The rate of heat transfer for a single fin is

$$\dot{q}_{\text{one fin}} = \theta_b\sqrt{hpkA_c}\frac{\cosh mL}{\sinh mL}$$

$$\therefore \dot{q}_{\text{one fin}} == (132-0)\times\sqrt{100\times\left[\pi\times\left(1.0\times10^{-3}\right)\right]\times401\times\left[\pi\times\left(1.0\times10^{-3}\right)^2/4\right]}\times\frac{\cosh\left[31.58\times\left(2.54\times10^{-2}\right)\right]}{\sinh\left[31.58\times\left(2.54\times10^{-2}\right)\right]}$$

$$\therefore \dot{q}_{\text{one fin}} = 1.97\ \text{W}$$

If the effectiveness of the fin $\varepsilon > 2$, use of fins becomes a good decision. In the present case, we have

$$\varepsilon = \frac{\dot{q}_{\text{one fin}}}{A_c h\theta_b} = \frac{1.97}{\left[\pi\times\left(1.0\times10^{-3}\right)^2/4\right]\times100\times(132-0)} = 190.0$$

Since $\varepsilon > 2$, the use of fins is well justified.

Part C: The total rate of heat transfer is the sum of the heat transfer rates due to the fins and to the base. In mathematical terms,

$$\dot{q}_{\text{total}} = \dot{q}_{\text{fins}} + \dot{q}_{\text{base}}$$

$$\therefore \dot{q}_{\text{total}} = n_{\text{fin}}\dot{q}_{\text{one fin}} + \left(A_{\text{wall}} - n_{\text{fin}}A_c\right)h\theta_b$$

$$\therefore \dot{q}_{\text{total}} = 625\times1.97 + \left[(0.1\times0.1) - 625\times\pi\times\left(1.0\times10^{-3}\right)^2/4\right]\times100\times132$$

$$\therefore \boxed{\dot{q}_{\text{total}} = 1357\text{ W}}$$

♦ The correct answer is **C**.

P.9 Solution

The first step is to determine factor *m*,

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\times\pi D^2/4}} = \sqrt{\frac{4h}{kD}}$$

$$\therefore m = \sqrt{\frac{4\times35}{205\times\left(2.5\times10^{-3}\right)}} = 16.53\text{ m}^{-1}$$

The fin efficiency follows as

$$\eta_f = \frac{\tanh mL}{mL} = \frac{\tanh\left[16.53\times\left(3\times10^{-2}\right)\right]}{16.53\times\left(3\times10^{-2}\right)} = 92.54\%$$

The total number of fins is

$$n = \frac{1}{\left(6\times10^{-3}\right)\times\left(6\times10^{-3}\right)} = 27,778\text{ fins}$$

The total surface area of the fins is

$$A_{\text{fin}} = n\times\left(\pi DL + \frac{\pi D^2}{4}\right) = 27,778\times\left[\pi\times\left(2.5\times10^{-3}\right)\times\left(3\times10^{-2}\right) + \frac{\pi\times\left(2.5\times10^{-3}\right)^2}{4}\right] = 6.68\text{ m}^2$$

The unfinned surface area, in turn, is obtained by deducting A_{fin} from the area of the surface,

$$A_{\text{unfinned}} = 1.0 - 27,778\times\pi D^2/4 = 1.0 - 27,778\times\pi\times\left(2.5\times10^{-3}\right)^2/4 = 0.864\text{ m}^2$$

The heat transfer rate to the finned surface is calculated as

$$\dot{q}_{\text{finned}} = \eta_{\text{fin}}\dot{q}_{\text{fin,max}} = \eta_{\text{fin}}hA_{\text{fin}}\left(T_b - T_\infty\right)$$

$$\therefore \dot{q}_{\text{finned}} = 0.9254\times35\times6.68\times(100-30) = 15.15\text{ kW}$$

The heat transfer rate of the unfinned surface, in turn, is computed as

$$\dot{q}_{\text{unfinned}} = hA_{\text{unfinned}}\left(T_b - T_\infty\right)$$

$$\therefore \dot{q}_{\text{unfinned}} = 35\times0.864\times(100-30) = 2.12\text{ kW}$$

The total heat transfer rate from the finned plate becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{finned}} + \dot{q}_{\text{unfinned}} = 15.15 + 2.12 = \boxed{17.27\text{ kW}}$$

The heat transfer rate if there were no fins attached to the plate would be

$$\dot{q}_{\text{no fin}} = hA_{\text{no fin}}\left(T_b - T_\infty\right) = 35\times(1.0\times1.0)\times(100-30) = 2.45\text{ kW}$$

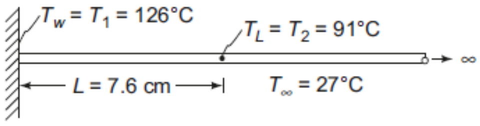
Accordingly, the fin efficiency becomes

$$\varepsilon_{\text{fin}} = \frac{\dot{q}_{\text{fin}}}{\dot{q}_{\text{no fin}}} = \frac{17.27}{2.45} = \boxed{7.05}$$

♦ The correct answer is **B**.

P.10 Solution

The system can be visualized as the pin fin illustrated below.



Assuming that the fin has infinite length, its temperature distribution is given by the expression

$$\frac{\theta}{\theta_s} = e^{-mx}$$

where coefficient *m* is given by

$$m = \sqrt{\frac{hp}{kA}} = \sqrt{\frac{h \times \pi D}{k \times \dfrac{\pi D^2}{4}}} = \sqrt{\frac{4h}{kD}}$$

We substitute this quantity into the initial equation and solve it for thermal conductivity, giving

$$\frac{\theta}{\theta_s} = e^{-mx} \rightarrow \frac{\theta}{\theta_s} = e^{-\sqrt{\frac{4h}{kD}}x} \therefore$$

$$\therefore k = \frac{4h}{D \times \left[\dfrac{\ln(\theta/\theta_s)}{x}\right]^2}$$

Now, at $x = L$, we have $\theta_L = T_L - T_\infty = 91 - 27 = 64°C$; also, $\theta_s = T_w - T_\infty = 126 - 27 = 99°C$. The ratio θ_L/θ_s = 64/99 = 0.646. Substituting in the previous equation gives

$$k = \frac{4 \times 22.7}{0.025 \times \left(\dfrac{\ln 0.646}{0.076}\right)^2} = \boxed{110 \text{ W/m} \cdot \text{K}}$$

♦ The correct answer is **D**.

P.11 Solution

Noting that the cross-sectional area of the spoon is constant and measuring *x* from the free surface of water, the variation of temperature along the spoon can be expressed as

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

Also, observing that 1 $\text{W/m} \cdot \text{K} = 1.73 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}$ and using the data available in the Additional Information section, we find that the thermal conductivity for stainless steel is k = 15/1.73 = 8.67 Btu/h·ft·°F. Given the perimeter p = 2(0.08 + 0.5) = 0.097 ft and the cross-sectional area A = (0.08/12) × (0.5/12) = 2.78 × 10^{-4} ft^2, we proceed to compute coefficient *m*,

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{1 \times 0.097}{8.67 \times (2.78 \times 10^{-4})}} = 6.34 \text{ ft}^{-1}$$

Noting that $x = L = 7/12 = 0.583$ ft at the tip and substituting, the tip temperature of the spoon is determined to be

$$T(x)=T_\infty+(T_b-T_\infty)\frac{\cosh m(L-x)}{\cosh mL}$$

$$\therefore T(L)=T(0.583)=75+(200-75)\frac{\cosh 6.34(0.583-0.583)}{\cosh 6.34(0.583)}=81.2^\circ\text{F}$$

Then, the temperature difference across the exposed surface of the spoon handle is

$$\Delta T=T_b-T(L)=200-81.2=\boxed{118.8^\circ\text{F}}$$

♦ The correct answer is **A**.

P.12 Solution

Part A: We begin by computing parameter *m*,

$$m=\sqrt{\frac{2h}{kt}}=\sqrt{\frac{2\times 50}{205\times\left(3\times 10^{-3}\right)}}=12.75\ \text{m}^{-1}$$

The surface area per unit width is

$$A_f=2L_c=2\left(L+\frac{t}{2}\right)=2\left(0.015+\frac{0.003}{2}\right)=0.033\,\text{m}^2$$

The efficiency of the fin for a rectangular geometry is given by

$$\eta_f=\frac{\tanh(mL_c)}{mL_c}=\frac{\tanh\left[12.75\times\left(0.015+\frac{0.003}{2}\right)\right]}{12.75\times\left(0.015+\frac{0.003}{2}\right)}=0.9855$$

We proceed to compute the fin heat transfer rate,

$$\dot{q}_f=\eta_f\left[hA_f(T_b-T_\infty)\right]=0.9855\times 50\times 0.033\times(100-20)=\boxed{130.1\ \text{W}}$$

♦ The correct answer is **B**.

Part B: The surface area of a triangular straight fin is given by

$$A_f=2w\sqrt{L_1^2+(t/2)^2}=2\times 1\times\sqrt{L_1^2+(0.003/2)^2}=2\sqrt{L_1^2+2.25\times 10^{-6}}$$

Now, the efficiency of the triangular straight fin is

$$\eta_f=\frac{\dot{q}_f}{hA_f(T_b-T_\infty)}=\frac{130.1}{50\times 2\sqrt{L_1^2+2.25\times 10^{-6}}\times 80}=\frac{0.016}{\sqrt{L_1^2+2.25\times 10^{-6}}}\quad\text{(I)}$$

However, tabulated values show that the efficiency must be

$$\eta_f=\frac{1}{mL}\times\frac{I_1(2mL)}{I_o(2mL)}$$

where I_0 and I_1 are modified Bessel functions of the first kind and orders zero and 1, respectively. Substituting variables *m* and *L* gives

$$\eta_f=\frac{1}{mL_1}\times\frac{I_1(2mL_1)}{I_o(2mL_1)}=\frac{1}{12.75L_1}\frac{I_1(2\times 12.75L_1)}{I_0(2\times 12.75L_1)}=\frac{1}{12.75L_1}\frac{I_1(25.5L_1)}{I_0(25.5L_1)}\quad\text{(II)}$$

Equations (I) and (II) should yield the same result; that is,

$$\frac{0.016}{\sqrt{L_1^2+2.25\times 10^{-6}}}=\frac{1}{12.75L}\frac{I_1(25.5L_1)}{I_0(25.5L_1)}$$

We now have a nonlinear equation in the variable that we seek, L_1, and a trial-and-error procedure is required. In Mathematica, one way to go is to use the *FindRoot* function,

$$\text{FindRoot}\left[\frac{0.016}{\sqrt{L_1^2+2.25*10^{-6}}}-\frac{1}{12.75L_1}*\frac{\text{BesselI}[1,25.5*L_1]}{\text{BesselI}[0,25.5*L_1]},\{L_1,0.01\}\right]$$

which produces the result L_1 = 16.3 mm.

♦ The correct answer is **A**.

Part C: Considering a parabolic straight fin, we first require the value of coefficient C_1, in which we substitute 0.003 m for the thickness t,

$$C_1=\sqrt{1+\left(\frac{t}{L}\right)^2}=\sqrt{1+\frac{0.003^2}{L^2}}=\sqrt{1+\frac{9\times10^{-6}}{L^2}}$$

Next, the surface area of the fin is given by

$$A_f=w\left[C_1L_2+\left(L_2^2/t\right)\ln\left(t/L_2+C_1\right)\right]$$

$$\therefore A_f=1\times\left[\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}L_2+\frac{L_2^2}{3\times10^{-3}}\times\ln\left(\frac{3\times10^{-3}}{L_2}+\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}\right)\right]$$

The efficiency of the parabolic straight fin follows as

$$\eta_f=\frac{\dot{q}_f}{hA_f\left(T_b-T_\infty\right)}$$

$$\therefore\eta_f=\frac{130.1}{50\times\left[\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}L_2+\frac{L_2^2}{3\times10^{-3}}\times\ln\left(\frac{3\times10^{-3}}{L_2}+\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}\right)\right]\times(100-20)}$$

$$\therefore\eta_f=\frac{0.0325}{\left[\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}L_2+\frac{L_2^2}{3\times10^{-3}}\times\ln\left(\frac{3\times10^{-3}}{L_2}+\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}\right)\right]}\quad\text{(III)}$$

From tabulated values, however, the efficiency is also

$$\eta_f=\frac{2}{\sqrt{4(mL)^2+1}+1}$$

$$\therefore\eta_f\frac{2}{\sqrt{4\times\left(12.75^2L_2^2\right)+1}+1}=\frac{2}{\sqrt{650.3L_2^2+1}+1}\quad\text{(IV)}$$

Since Equations (III) and (IV) should provide the same result, we write

$$\frac{0.0325}{\left[\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}L_2+\frac{L_2^2}{3\times10^{-3}}\times\ln\left(\frac{3\times10^{-3}}{L_2}+\sqrt{1+\frac{9\times10^{-6}}{L_2^2}}\right)\right]}=\frac{2}{\sqrt{650.3L_2{}^2+1}+1}$$

As before, we have a nonlinear equation for the length, L_2, and the *FindRoot* command can be used,

$$\text{FindRoot}[\frac{0.032}{\left(\sqrt{1+\frac{9*10^{-6}}{L_2^2}}L_2+\frac{L_2^2}{3*10^{-3}}*\text{Log}\left[\frac{3*10^{-3}}{L_2}+\sqrt{1+\frac{9*10^{-6}}{L_2^2}}\right]\right)}-\frac{2}{\sqrt{650.3L_2^2+1}+1},\{L_2,0.01\}]$$

which brings the result L_2 = 16.6 mm. In summary, the lengths of the three fins are all similar, which is a consequence of the fact that fin efficiencies are all close to unity, yielding fins of almost constant base temperature. Use of triangular and parabolic fins is appropriate when weight savings are important, such as in aerospace applications. However, reduction in cost due to reduction in the amount of raw material used is usually offset by higher manufacturing costs for the triangular and parabolic fins.

- The correct answer is **A**.

Answer Summary

Problem 1		A
Problem 2		B
Problem 3		C
Problem 4		C
Problem 5		B
Problem 6		A
Problem 7		D
Problem 8	8A	C
	8B	Open-ended pb.
	8C	C
Problem 9		B
Problem 10		D
Problem 11		A
Problem 12	12A	B
	12B	A
	12C	A

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