Quiz HT104
EXTERNAL CONVECTION

## Lucas Montogue

## Problems

Problem 1 (Bergman et al., 2011, w/ permission)
Engine oil at $100^{\circ} \mathrm{C}$ and a velocity of $0.1 \mathrm{~m} / \mathrm{s}$ flows over both surfaces of a 1 m long flat plate maintained at $20^{\circ} \mathrm{C}$. True or false? Use as properties $\rho=864 \mathrm{~kg} / \mathrm{m}^{3}$, $v=86.1 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}, k=0.140 \mathrm{~W} / \mathrm{mK}$, and $\operatorname{Pr}=1081$.


1. ( ) The thickness of the velocity boundary layer at the trailing edge is greater than 200 mm.
2.( ) The thickness of the thermal boundary layer at the trailing edge is greater than 10 mm .
2. ( ) The absolute value of the local heat flux at the trailing edge is greater than $1000 \mathrm{~W} / \mathrm{m}^{2}$.
3. ( ) The surface shear stress at the trailing edge is greater than $0.1 \mathrm{~N} / \mathrm{m}^{2}$.

Problem 2 (Çengel \& Ghajar, 2015, w/ permission)
The forming section of a plastics plant puts out a continuous sheet of plastic that is 1.2 m wide and 2 mm thick at a rate of $15 \mathrm{~m} / \mathrm{min}$. The temperature of the plastic sheet is $90^{\circ} \mathrm{C}$ when it is exposed to the surrounding air, and the sheet is subjected to flow of air at $30^{\circ} \mathrm{C}$ at a velocity of $3 \mathrm{~m} / \mathrm{s}$ on both sides along its surfaces normal to the direction of motion of the sheet. The width of the air cooling section is such that a fixed point on the plastic sheet passes through that section in 2 s . Determine the rate of heat transfer from the plastic sheet to the air. Use as properties $v=1.896 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}, k=0.0281 \mathrm{~W} / \mathrm{mK}$, and $\operatorname{Pr}=0.72$.

A) $\dot{q}=226 \mathrm{~W}$
B) $\dot{q}=437 \mathrm{~W}$
C) $\dot{q}=651 \mathrm{~W}$
D) $\dot{q}=814 \mathrm{~W}$

## Problem 3 (Çengel $\xi$ Ghajar, 2015, w/ permission)

An array of power transistors, dissipating 6 W of power each, is to be cooled by mounting them on a $25 \mathrm{~cm} \times 25 \mathrm{~cm}$ square aluminum plate and blowing air at $35^{\circ} \mathrm{C}$ over the plate with a fan at a velocity of $4 \mathrm{~m} / \mathrm{s}$. The average temperature of the plate is not to exceed $65^{\circ} \mathrm{C}$. Assuming the heat transfer from the back side of the plate to be negligible and disregarding radiation, determine the number of transistors that can be placed on this plate. Use as properties $v=1.798 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}, k$ $=0.0274 \mathrm{~W} / \mathrm{mK}$, and $\operatorname{Pr}=0.723$.

A) $n=3$
B) $n=4$
C) $n=5$
D) $n=6$

## Problem 4 (Bergman et al., 2011, w/ permission)

A flat plate of width 1 m is maintained at a uniform surface temperature of $T_{s}=150^{\circ} \mathrm{C}$ by using independently controlled, heat-generating rectangular modules of thickness $a=10 \mathrm{~mm}$ and length $b=50 \mathrm{~mm}$. Each module is insulated from its neighbors, as well as on its back side. Atmospheric air at $25^{\circ} \mathrm{C}$ flows over the plate at a velocity of $30 \mathrm{~m} / \mathrm{s}$. Find the required power generation, $\dot{q}\left(\mathrm{~W} / \mathrm{m}^{3}\right)$, in a module positioned at a distance 700 mm from the leading edge. Use as properties $v=$ $22.02 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}, k=0.0308 \mathrm{~W} / \mathrm{mK}$, and $\operatorname{Pr}=0.698$.

A) $\dot{q}=295.2 \mathrm{~kW} / \mathrm{m}^{3}$
B) $\dot{q}=469.2 \mathrm{~kW} / \mathrm{m}^{3}$
C) $\dot{q}=640.5 \mathrm{~kW} / \mathrm{m}^{3}$
D) $\dot{q}=871.1 \mathrm{~kW} / \mathrm{m}^{3}$

## Problem 5 (Bergman et al., 2011, w/ permission)

Explain under what conditions the total rate of heat transfer from an isothermal flat plate of dimensions $L \times 2 L$ would be the same, independent of whether parallel flow over the plate is directed along the side of length $L$ or $2 L$. With a critical Reynolds number of $5 \times 10^{5}$, for what values of $R e$ would the total heat transfer be independent of orientation?


## Problem 6A (Bergman et al., 2011, w/ permission)

In the production of sheet metals or plastics, it is customary to cool the material before it leaves the production process for storage or shipment to the customer. Typically, the process is continuous, with a sheet of thickness $\delta$ and width $W$ cooled as it transits the distance $L$ between two rollers at a velocity $V$. In this problem, we consider having an aluminum alloy (2024-T6) by an airstream moving at a velocity $u_{\infty}$ in counter flow over the top surface of the sheet. A turbulence promoter is used to provide turbulent boundary layer development over the entire surface. By applying conservation of energy to a differential control surface of length $d x$, which either moves with the sheet or is stationary and through which the sheet passes, derive a differential equation that governs the temperature distribution along the sheet. Because of the low emissivity of the aluminum, radiation effects may be neglected. Express your results in terms of the velocity, thickness, and properties of the sheet $\left(V, \delta, \rho, c_{p}\right)$, the local convection coefficient $h_{x}$ associated with the counter flow, and the air temperature. For a known temperature of the sheet ( $T_{i}$ ) at the onset of cooling and a negligible effect of the sheet velocity on boundary layer development, solve the equation to obtain an expression for the outlet temperature $T_{0}$.


## Problem 6B

For $\delta=2 \mathrm{~mm}, V=0.10 \mathrm{~m} / \mathrm{s}, L=5 \mathrm{~m}, W=1 \mathrm{~m}, u_{\infty}=20 \mathrm{~m} / \mathrm{s}, T_{\infty}=20^{\circ} \mathrm{C}$, and $T_{i}=$ $300^{\circ} \mathrm{C}$, what is the outlet temperature $T_{0}$ ? Use as properties $v=26.4 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}, k=$ 0.0338 W/mK, and $\operatorname{Pr}=0.690$.
A) $T_{o}=159.5^{\circ} \mathrm{C}$
B) $T_{o}=180.6^{\circ} \mathrm{C}$
C) $T_{o}=213.1^{\circ} \mathrm{C}$
D) $T_{o}=274.9^{\circ} \mathrm{C}$

Problem 7 (Bergman et al., 2011, w/ permission)
An array of electronic chips is mounted within a sealed rectangular enclosure, and cooling is implemented by attaching an aluminum heat sink ( $k=180$ $\mathrm{W} / \mathrm{mK})$. The base of the heat sink has dimensions of $w_{1}=w_{2}=100 \mathrm{~mm}$, while the 6 fins are of thickness $t=10 \mathrm{~mm}$ and pitch $S=18 \mathrm{~mm}$. The fin length is $L_{f}=50 \mathrm{~mm}$, and the base of the heat sink has a thickness of $L_{b}=10 \mathrm{~mm}$. If cooling is implemented by water flow through the heat sink, with $u_{\infty}=3 \mathrm{~m} / \mathrm{s}$, and $T_{\infty}=17^{\circ} \mathrm{C}$, what is the base temperature $T_{b}$ of the heat sink when power dissipation by the chips is $P_{\text {elec }}=1800 \mathrm{~W}$ ? The average convection coefficient for surfaces of the fins and the exposed base may be estimated by assuming parallel flow over a flat plate. Use as properties $k=0.62 \mathrm{~W} / \mathrm{mK}, \rho=995 \mathrm{~kg} / \mathrm{m}^{3}, c_{p}=4178 \mathrm{~J} / \mathrm{kgK}, v=7.73 \times 10^{-7} \mathrm{~m}^{2} / \mathrm{s}$, and $\operatorname{Pr}=$ 5.2.

A) $T_{b}=25.8^{\circ} \mathrm{C}$
B) $T_{b}=46.3^{\circ} \mathrm{C}$
C) $T_{b}=65.6^{\circ} \mathrm{C}$
D) $T_{b}=86.7^{\circ} \mathrm{C}$

## Problem 8 (Kreith et al., 2011, w/ permission)

A nuclear reactor fuel rod is a circular cylinder 6 cm in diameter. The rod is to be tested by cooling it with a flow of sodium at $205^{\circ} \mathrm{C}$ and a velocity of $5 \mathrm{~cm} / \mathrm{s}$ perpendicular to its axis. If the rod surface is not to exceed $300^{\circ} \mathrm{C}$, estimate the maximum allowed power dissipation in the rod. Use as properties $v=4.6 \times 10^{-7} \mathrm{~m}^{2} / \mathrm{s}$, $k=80.3 \mathrm{~W} / \mathrm{mK}$, and $\operatorname{Pr}=0.0072$.
A) $\dot{q}_{G}=46.8 \mathrm{MW} / \mathrm{m}^{3}$
B) $\dot{q}_{G}=77.5 \mathrm{MW} / \mathrm{m}^{3}$
C) $\dot{q}_{G}=109.7 \mathrm{MW} / \mathrm{m}^{3}$
D) $\dot{q}_{G}=140.2 \mathrm{MW} / \mathrm{m}^{3}$

## Problem 9 (Kreith et al., 2011, w/ permission)

An engineer is designing a heating system that will consist of multiple tubes placed in a duct carrying the air supply for a building. She decides to perform preliminary tests with a single copper tube of 2-cm OD carrying condensing steam at $100^{\circ} \mathrm{C}$. The air velocity in the duct is $5 \mathrm{~m} / \mathrm{s}$, and its temperature is $20^{\circ} \mathrm{C}$. The tube can be placed normal to the flow, but it may be advantageous to place the tube at an angle to the air flow and thus increase the heat transfer surface area, as illustrated below. If the duct width is 1 m , predict the outcome of the planned tests and estimate how the angle $\theta$ will affect the rate of heat transfer. Are there limits? Use as properties $v=1.57 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}, k=0.0251 \mathrm{~W} / \mathrm{mK}$, and $\operatorname{Pr}=0.71$.


Normal to flow


At an angle to flow

## Problem 10 (Çengel \& Ghajar, 2015, w/ permission)

A 6-mm diameter electrical transmission line carries an electrical current of 50 A and has a resistance of $0.02 \Omega$ per meter length. Determine the surface temperature of the wire during a windy day when the air temperature is $10^{\circ} \mathrm{C}$ and the wind is blowing across the transmission line at $40 \mathrm{~km} / \mathrm{h}$. Use as properties $\rho=$ $1.25 \mathrm{~kg} / \mathrm{m}^{3}, k=0.0244 \mathrm{~W} / \mathrm{mK}, v=1.43 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}$, and $\operatorname{Pr}=0.734$.

A) $T_{S}=11.8^{\circ} \mathrm{C}$
B) $T_{s}=16.9^{\circ} \mathrm{C}$
C) $T_{s}=22.7^{\circ} \mathrm{C}$
D) $T_{s}=27.5^{\circ} \mathrm{C}$

## Problem 11 (Çengel \& Ghajar, 2015, w/ permission)

In a geothermal power plant, the used geothermal water at $80^{\circ} \mathrm{C}$ enters a $15-\mathrm{cm}$ diameter and $400-\mathrm{m}$ long uninsulated pipe at a rate of $8.5 \mathrm{~kg} / \mathrm{s}$ and leaves at $70^{\circ} \mathrm{C}$ before being injected back to the ground. Windy air at $15^{\circ} \mathrm{C}$ flows normal to the pipe. Disregarding radiation, determine the average wind velocity in $\mathrm{km} / \mathrm{h}$. Use as properties $k=0.027 \mathrm{~W} / \mathrm{mK}, v=1.75 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$, and $\operatorname{Pr}=0.724$ for wind and $c_{p}=4193$ J/kgK for water.

A) $V_{\infty}=10.4 \mathrm{~km} / \mathrm{h}$
B) $V_{\infty}=20.5 \mathrm{~km} / \mathrm{h}$
C) $V_{\infty}=30.2 \mathrm{~km} / \mathrm{h}$
D) $V_{\infty}=40.1 \mathrm{~km} / \mathrm{h}$

## Problem 12 (çengel \& Ghajar, 2015, w/ permission)

Exhaust gases from a manufacturing plant are being discharged through a 10-m tall exhaust stack with outer diameter of 1 m . The exhaust gases are discharged at a rate of $1.2 \mathrm{~kg} / \mathrm{s}$, while the temperature drop between inlet and exit of the exhaust stack is $30^{\circ} \mathrm{C}$, and the constant pressure specific heat of the exhaust gases is $1600 \mathrm{~J} / \mathrm{kgK}$. On a particular day, wind at $27^{\circ} \mathrm{C}$ is blowing across the exhaust stack with a velocity of $10 \mathrm{~m} / \mathrm{s}$, while the outer surface of the exhaust stack experiences radiation with the surroundings at $27^{\circ} \mathrm{C}$. Solar irradiation is incident on the exhaust stack outer surface at a rate of $1400 \mathrm{~W} / \mathrm{m}^{2}$, and both the emissivity and solar absorptivity of the outer surface are 0.9 . Determine the exhaust stack outer surface temperature. Use as properties $v=2.10 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}, k=0.0295 \mathrm{~W} / \mathrm{mK}$, and $\operatorname{Pr}$ $=0.715$.

A) $T_{s}=48.8^{\circ} \mathrm{C}$
B) $T_{s}=88.5^{\circ} \mathrm{C}$
C) $T_{s}=133.2^{\circ} \mathrm{C}$
D) $T_{s}=171.8^{\circ} \mathrm{C}$

## Problem 13 (Bergman et al., 2011, w/ permission)

Pin fins are to be specified for use in an industrial cooling application. The fins will be subjected to a gas in cross-flow at $V=10 \mathrm{~m} / \mathrm{s}$. The cylindrical fin has a diameter of $D=15 \mathrm{~mm}$, and the cross-sectional area is the same for each configuration shown in the sketch. For fins of equal length and therefore equal mass, which fin has the largest heat transfer rate? Assume the pins can be treated as infinitely long. Assume the gas properties are those of air at $T=350 \mathrm{~K}$, namely, $\mathrm{k}=$ $0.030 \mathrm{~W} / \mathrm{mK}, v=2.09 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$, and $\mathrm{Pr}=0.700$.

A) Configuration $A$ has the largest heat transfer rate.
B) Configuration $B$ has the largest heat transfer rate.
C) Configuration C has the largest heat transfer rate.
D) The heat transfer rate is the same for all three configurations.

## Problem 14 (Bergman et al., 2011, w/ permission)

In a manufacturing process, long aluminum rods of square cross-section with $D=25 \mathrm{~mm}$ are cooled from an initial temperature of $T_{l}=400^{\circ} \mathrm{C}$. Which configuration in the sketch should be used to minimize the time needed for the rods to reach a safe-to-handle temperature of $60^{\circ} \mathrm{C}$ when exposed to air in cross-flow at $V$ $=8 \mathrm{~m} / \mathrm{s}, T_{\infty}=30^{\circ} \mathrm{C}$ ? What is the required cooling time for the preferred configuration? The emissivity of the rods is $\varepsilon=0.10$ and the temperature of the surroundings is $T_{\text {sur }}=20^{\circ} \mathrm{C}$. For air, use as properties $v=2.64 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}, k=0.0338$ $\mathrm{W} / \mathrm{mK}$, and $\operatorname{Pr}=0.690$. For aluminum, use $\rho=2702 \mathrm{~kg} / \mathrm{m}^{3}, c_{\rho}=991 \mathrm{~J} / \mathrm{kgK}$, and $k_{\mathrm{s}}=235$ W/mk.

A) Configuration A will cool faster, and the cooling time is about 10 minutes.
B) Configuration A will cool faster, and the cooling time is about 20 minutes.
C) Configuration B will cool faster, and the cooling time is about 10 minutes.
D) Configuration B will cool faster, and the cooling time is about 20 minutes.

## Additional Information

Table 1 Constants for use with the equation $N u=C \operatorname{Re}^{m} \operatorname{Pr}^{1 / 3}$ in circular cylinders in cross-flow of a gas

| $\boldsymbol{R} \boldsymbol{e}_{\boldsymbol{D}}$ | $\boldsymbol{C}$ | $\boldsymbol{m}$ |
| :---: | :---: | :---: |
| $0.4-4$ | 0.989 | 0.330 |
| $4-40$ | 0.911 | 0.385 |
| $40-4000$ | 0.683 | 0.466 |
| $4000-40,000$ | 0.193 | 0.618 |
| $40,000-400,000$ | 0.027 | 0.805 |

Table 2 Constants for use with the equation $N u=C \operatorname{Re}^{m} \mathrm{Pr}^{1 / 3}$ in noncircular cylinders in cross-flow of a gas

| Geometry |  |  | $\boldsymbol{R e} \boldsymbol{e}$ | C | $m$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Square |  |  |  |  |  |
| $V \rightarrow$ | T <br>  |  | 6000-60,000 | 0.304 | 0.59 |
| $V \rightarrow$ | $\begin{aligned} & \underline{\underline{T}} D \end{aligned}$ |  | 5000-60,000 | 0.158 | 0.66 |
| Hexagon |  |  |  |  |  |
| $v \rightarrow \square$ |  |  | 5200-20,400 | 0.164 | 0.638 |
|  |  |  | 20,400-105,000 | 0.039 | 0.78 |
| $v \rightarrow$ | T $D$ + + |  | 4500-90,700 | 0.150 | 0.638 |
| Thin plate perpendicular to flow |  |  |  |  |  |
| $V \rightarrow$ | $\stackrel{A}{\text { a }}$ | Front | 10,000-50,000 | 0.667 | 0.500 |
|  | $\pm$ | Back | 7000-80,000 | 0.191 | 0.667 |

## Solutions

## P. 1 Solution

1. False. Given $u_{\infty}=0.1 \mathrm{~m} / \mathrm{s}, L=1 \mathrm{~m}$, and $v=86.1 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}$, the Reynolds number is used to determine the nature of the flow,

$$
\operatorname{Re}=\frac{u_{\infty} L}{v}=\frac{0.1 \times 1}{\left(86.1 \times 10^{-6}\right)}=1161.4
$$

Since $R e<5 \times 10^{5}$, the flow is laminar. The equation to use for the velocity boundary layer thickness is then

$$
\delta=5 L \mathrm{Re}^{-1 / 2}=5 \times 1 \times 1161.4^{-0.5}=0.147 \mathrm{~m}=147 \mathrm{~mm}
$$

2. True. Noting that flow is laminar, the relation that defines the thickness of the thermal boundary layer is

$$
\delta_{t}=\delta \operatorname{Pr}^{-1 / 3}
$$

where $\delta=147 \mathrm{~mm}$ as determined just now and $\operatorname{Pr}=1081$. Therefore,

$$
\delta_{t}=\delta \operatorname{Pr}^{-1 / 3}=147 \times 1081^{-1 / 3}=14.3 \mathrm{~mm}
$$

3. True. We first require the local convection coefficient at the trailing edge of the plate, which is given by

$$
h=\frac{k \times N u}{L}=\frac{k}{L} \times 0.332 \operatorname{Re}^{1 / 2} \operatorname{Pr}^{1 / 3}
$$

where we have replaced the Nusselt number with the pertaining correlation. The result is

$$
h=\frac{0.140}{1} \times 0.332 \times 1161.4^{1 / 2} \times 1081^{1 / 3}=16.26 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

The heat flux follows from Newton's law of cooling,

$$
q^{\prime \prime}=h\left(T_{s}-T_{\infty}\right)=16.26 \times(20-100)=-1300.8 \mathrm{~W} / \mathrm{m}^{2}
$$

4. False. Recall that the friction coefficient for the flat plate in laminar flow is

$$
C_{f}=0.664 \mathrm{Re}^{-1 / 2}
$$

Or, with $R e=1161.4, C_{f}=0.0195$. From the definition of $C_{f}$, we have

$$
C_{f}=\frac{\tau_{s}}{\rho u_{\infty}^{2} / 2} \rightarrow \tau_{s}=C_{f} \times \frac{\rho u_{\infty}^{2}}{2}
$$

Substituting the proper variables, we obtain

$$
\tau_{s}=0.664 \times 1161.4^{-1 / 2} \times \frac{864 \times 0.1^{2}}{2}=0.0842 \mathrm{~N} / \mathrm{m}^{2}
$$

## P. 2 Solution

The width of the cooling section is given by $W=V \Delta t$, in which $V$ is the velocity at which the plant puts out a sheet and $\Delta t$ is the time of cooling, so that $W=$ $(15 / 60) \times 2=0.5 \mathrm{~m}$. The Reynolds number for air flow across the sheet is

$$
\mathrm{Re}_{L}=\frac{V L}{v}=\frac{3 \times 1.2}{\left(1.896 \times 10^{-5}\right)}=189,870
$$

Since $R e_{L}<5 \times 10^{5}$, the flow is laminar. Thus, the Nusselt number is calculated with the correlation

$$
N u=0.664 \mathrm{Re}_{L}^{0.5} \mathrm{Pr}^{1 / 3}=0.664 \times 189,870^{0.5} \times 0.72^{1 / 3}=259.3
$$

The convection coefficient easily follows,

$$
h=\frac{k \times N u}{L}=\frac{0.0281 \times 259.3}{1.2}=6.07 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

The heat transfer surface area, $A_{s}$, is $A_{s}=2 L W=2 \times 1.2 \times 0.5=1.2 \mathrm{~m}^{2}$. Finally, we calculate the rate of heat transfer from the plastic sheet to the air as

$$
\begin{gathered}
q_{\text {conv }}=h A_{s}\left(T_{s}-T_{\infty}\right) \\
\therefore q_{\text {conv }}=6.07 \times 1.2 \times(363-303)=437 \mathrm{~W}
\end{gathered}
$$

- The correct answer is B.


## P. 3 Solution

The Reynolds number for the plate is given by the usual expression,

$$
\operatorname{Re}=\frac{V L}{v}
$$

in which $V=4 \mathrm{~m} / \mathrm{s}, L=0.25 \mathrm{~m}$ (the length of the plate), and $v=1.798 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$. Thus,

$$
\operatorname{Re}=\frac{4 \times 0.25}{\left(1.798 \times 10^{-5}\right)}=55,620
$$

Clearly, the flow over the plate is laminar. We proceed to compute the Nusselt number for the plate using the correlation

$$
N u=0.664 \mathrm{Re}^{0.5} \mathrm{Pr}^{1 / 3}=0.664 \times 55,620^{0.5} \times 0.723^{1 / 3}=140.6
$$

and thence the heat transfer coefficient, which is given by

$$
h=\frac{k \times N u}{L}=\frac{0.0274 \times 140.6}{0.25}=15.41 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

Finally, we can determine the heat transfer over the plate with Newton's law of cooling,

$$
\begin{gathered}
q_{\mathrm{conv}}=h A_{s}\left(T_{s}-T_{\infty}\right) \\
\therefore \dot{q}_{\mathrm{conv}}=h A_{s}\left(T_{s}-T_{\infty}\right)=15.41 \times 0.25^{2} \times(338-308)=28.9 \mathrm{~W}
\end{gathered}
$$

Knowing that each transistor dissipates 6 W of power, the amount of transistors that can be placed on the plate, $n$, is such that

$$
n=\left\lfloor\frac{\dot{q}_{\text {conv }}}{6}\right\rfloor=\left\lfloor\frac{28.9}{6}\right\rfloor=4
$$

- The correct answer is B.


## P. 4 <br> Solution

The system in question is illustrated below.


The module power generation follows from an energy balance on the module surface,

$$
\begin{gathered}
q_{\mathrm{conv}}^{\prime \prime}=q_{\mathrm{gen}}^{\prime \prime} \rightarrow h\left(T_{s}-T_{\infty}\right)=\dot{q} a \\
\therefore \dot{q}=\frac{h\left(T_{s}-T_{\infty}\right)}{a}
\end{gathered}
$$

To select a convection correlation for estimating $h$, we first find the Reynolds number at $x=L$,

$$
\operatorname{Re}_{L}=\frac{u_{\infty} L}{v}=\frac{30 \times 0.70}{\left(22.02 \times 10^{-6}\right)}=953,680
$$

Since $R e_{L}>5 \times 10^{5}$, flow is turbulent over the plate. The approximation $\bar{h}=$ $h_{x}(L+b / 2)$ is appropriate, with the corresponding $R e$ being

$$
\operatorname{Re}_{L+b / 2}=\frac{30 \times(0.70+0.05 / 2)}{\left(22.02 \times 10^{-6}\right)}=987,740
$$

One turbulent flow correlation to use with $x=L+b / 2=0.725 \mathrm{~m}$ is the following,

$$
\begin{array}{r}
\qquad N u_{x}=0.0296 \mathrm{Re}_{x}^{4 / 5} \mathrm{Pr}^{1 / 3} \\
\text { Substituting } R e_{x}=R e_{L+b / 2} \text { and } \mathrm{Pr}=0.698 \text { gives }
\end{array}
$$

$$
N u_{x}=0.0296 \times 987,740^{4 / 5} \times 0.698^{1 / 3}=1640.4
$$

The average convection coefficient $h_{x}$ is then

$$
h_{x}=\frac{k \times N u_{x}}{L}=\frac{0.0308 \times 1640.4}{0.725}=69.69 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

and the required power generation $\dot{q}$ easily follows,

$$
\dot{q}=\frac{h_{x}\left(T_{s}-T_{\infty}\right)}{A}=\frac{69.69 \times(150-25)}{0.010}=871.1 \mathrm{~kW} / \mathrm{m}^{3}
$$

Another approach for estimating the average heat transfer coefficient for the module is to use the relation

$$
\begin{gathered}
q_{\text {module }}=q_{0 \rightarrow L+b}-q_{0 \rightarrow L} \\
\therefore h \times b=h_{L+b}(L+b)-h_{L} \times L
\end{gathered}
$$

or

$$
h=h_{L+b} \frac{(L+b)}{b}+h_{L} \frac{L}{b}
$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation would be, in such a case,

$$
N u_{x}=\left(0.037 \mathrm{Re}_{x}^{4 / 5}-871\right) \operatorname{Pr}^{1 / 3}
$$

with $x=L+b$ and $x=L$, we find

$$
h_{L+b}=54.79 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K} ; h_{L}=53.73 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

Substituting in the expression for $h$ gives

$$
h=\left[54.79 \times \frac{0.750}{0.05}-53.73 \times \frac{0.700}{0.05}\right]=69.63 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

which is in excellent agreement with the $h_{x}\left(=69.69 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}\right)$ computed with the previous approach.

- The correct answer is $\mathbf{D}$.


## P. 5 Solution

We are to find Reynolds numbers for which the total heat transfer rate is independent of orientation. Consider the following schematic.


The total heat transfer rate would be the same $\left(q_{L}=q_{2 L}\right)$ if the convection coefficients were equal, $h_{L}=h_{2 L}$. Conditions for which such an equality is possible may be inferred from a sketch of $h_{L}$ versus $R e_{L}$.


For laminar flow ( $R e_{L}<R e_{x, c}$ ), $h_{L} \propto L^{-1 / 2}$, and for mixed and turbulent flow $\left(R e_{L}>R e_{x, c}\right), h_{L}=C_{1} L^{-1 / 5}-C_{2} L^{-1}$. Hence $h_{L}$ varies with $R e_{L}$ as shown, and two possibilities are suggested.
$\checkmark$ Case A: Laminar flow exists on the shorter plate, while mixed flow conditions exist on the longer plate.
$\rightarrow$ Case B: Mixed boundary layer conditions exist on both plates.
In both cases, it is required that

$$
h_{L}=h_{2 L} \text { and } \operatorname{Re}_{2 L}=2 \operatorname{Re}_{L}
$$

Consider case A. From expressions for $h_{L}$ in laminar and mixed flow,

$$
\begin{gathered}
0.664 \frac{k}{L} \operatorname{Re}_{L}^{1 / 2} \operatorname{Pr}^{1 / 3}=\frac{k}{2 L}\left(0.037 \operatorname{Re}_{2 L}^{4 / 5}-871\right) \operatorname{Pr}^{1 / 3} \\
\therefore 0.664 \operatorname{Re}_{L}^{1 / 2}=0.032 \operatorname{Re}_{L}^{4 / 5}-435
\end{gathered}
$$

Since $R e_{L}<5 \times 10^{5}$ and $R e_{2 L}=2 R e_{L}>5 \times 10^{5}$, the required value of $R e_{L}$ may be narrowed to the range

$$
2.5 \times 10^{5}<\operatorname{Re}_{L}<5 \times 10^{5}
$$

From a trial-and-error solution of the foregoing equation, we obtain $R e=$ 319,410 , which is within this range.

Next, consider case B. For mixed flow on both plates, we apply the correlations

$$
\begin{gathered}
\frac{k}{L}\left(0.037 \mathrm{Re}_{L}^{4 / 5}-871\right) \operatorname{Pr}^{1 / 3}=\frac{k}{2 L}\left(0.037 \mathrm{Re}_{2 L}^{4 / 5}-871\right) \operatorname{Pr}^{1 / 3} \\
\therefore 0.037 \mathrm{Re}_{L}^{4 / 5}-871=0.032 \mathrm{Re}_{L}^{4 / 5}-435 \\
\therefore 0.005 \mathrm{Re}_{L}^{4 / 5}=436 \\
\therefore \operatorname{Re}_{L}=1,498,460
\end{gathered}
$$

Note that it is impossible to satisfy the requirement the $h_{L}=h_{2 L}$ if $R e_{L}<$ $0.25 \times 10^{5}$ (laminar flow for both plates). Furthermore, observe that the results are independent of the nature of the fluid (e.g., Pr cancels out in both sides in the analysis of case B).

## P. 6 Solution

Part A: Applying conservation of energy to a stationary control surface, through which the sheet moves, steady-state conditions exist and $\dot{E}_{\text {in }}-\dot{E}_{\text {out }}=0$. We have an inflow of energy due to advection and an outflow due to advection and convection. Mathematically,

$$
\begin{gathered}
\rho V A_{c} c_{p}(T+d T)-\rho V A_{c} c_{p} T-d q+\rho V \delta W c_{p} d T-h_{x}(d x \times W)\left(T-T_{\infty}\right)=0 \\
\therefore \frac{d T}{d x}=\frac{h_{x}}{\rho V \delta c_{p}}\left(T-T_{\infty}\right)
\end{gathered}
$$

Alternatively, if the control surface is fixed to the sheet, conditions are transient and the energy balance is of the form $-\dot{E}_{\text {out }}=\dot{E}_{\text {st }}$, or

$$
\begin{gathered}
-h_{x}(d x \times W)\left(T-T_{\infty}\right)=\rho(d x \times W \times \delta) c_{p} \frac{d T}{d t} \\
\therefore \frac{d T}{d t}=-\frac{h_{x}}{\rho \delta c_{p}}\left(T-T_{\infty}\right)
\end{gathered}
$$

Dividing the left- and right-hand sides of the foregoing equation by $d x / d t$ and $d x / d t=-V$, respectively, the result should be the first differential equation we proposed. This equation can be integrated on both sides from $x=0$ to $x=L$ to give

$$
\int_{T_{o}}^{T_{i}} \frac{d T}{T-T_{\infty}}=\frac{L}{\rho V \delta c_{p}}\left[\frac{1}{L} \int_{L}^{1} h_{x} d x\right]
$$

where $h_{x}=(k / x) 0.0296 \operatorname{Re}_{x}^{4 / 5} \operatorname{Pr}^{1 / 3}$ and the bracketed term on the right-hand side of the equation reduces to $h_{L}=(k / L) 0.037 \mathrm{Re}_{L}^{4 / 5} \mathrm{Pr}^{1 / 3}$. Hence,

$$
\ln \left(\frac{T_{i}-T_{\infty}}{T_{o}-T_{\infty}}\right)=\frac{L h_{L}}{\rho V \delta c_{p}} \rightarrow \frac{T_{o}-T_{\infty}}{T_{i}-T_{\infty}}=\exp \left(-\frac{L h_{L}}{\rho V \delta c_{p}}\right)
$$

This equation describes the evolution of the outlet temperature $T_{0}$.
Part B: For the prescribed conditions, we compute the Reynolds number as

$$
\operatorname{Re}_{L}=\frac{u_{\infty} L}{v}=\frac{20 \times 5}{\left(26.4 \times 10^{-6}\right)}=3.79 \times 10^{6}
$$

Then, we calculate the convection coefficient $h_{L}$ with the formula presented earlier,

$$
h_{L}=\left(\frac{k}{L}\right) 0.037 \mathrm{Re}_{L}^{4 / 5} \mathrm{Pr}^{1 / 3}=\left(\frac{0.0338}{5}\right) \times 0.037 \times\left(3.79 \times 10^{6}\right)^{4 / 5} \times 0.690^{1 / 3}=40.49 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

The relation we obtained in the previous part can be easily solved for the outlet temperature $T_{o}$,

$$
\frac{T_{o}-T_{\infty}}{T_{i}-T_{\infty}}=\exp \left(-\frac{L h_{L}}{\rho V \delta c_{p}}\right) \rightarrow T_{o}=T_{\infty}+\left(T_{i}-T_{\infty}\right) \exp \left(-\frac{L h_{L}}{\rho V \delta c_{p}}\right)
$$

Substituting the pertaining variables gives

$$
T_{o}=20+(300-20) \exp \left(-\frac{5 \times 40.49}{2770 \times 0.1 \times 0.002 \times 983}\right)=213.1^{\circ} \mathrm{C}
$$

The outlet temperature is about 90 degrees below the inlet temperature, $T_{i}$.

- The correct answer is $\mathbf{C}$.


## P. 7 Solution

The system in question is illustrated below.


The conduction resistance from the base of the fin is given by

$$
R_{b}=\frac{L_{b}}{k_{h s}\left(w_{1} \times w_{2}\right)}
$$

in which $L_{b}=0.001 \mathrm{~m}$ is the thickness of the base of the heat sink, $k_{h s}=180 \mathrm{~W} / \mathrm{Mk}$, and $w_{1}=w_{2}=0.1 \mathrm{~m}$. The resistance then becomes

$$
R_{b}=\frac{L_{b}}{k_{h s}\left(w_{1} \times w_{2}\right)}=\frac{0.01}{180 \times(0.1 \times 0.1)}=0.00556 \mathrm{~K} / \mathrm{W}
$$

The fin surface area is

$$
A_{f}=2 w_{2}\left(L_{f}+t / 2\right)=2 \times 0.1 \times\left(0.05+\frac{0.01}{2}\right)=0.011 \mathrm{~m}^{2}
$$

The total surface area of the fin array is obtained as

$$
A_{t}=N A_{f}+A_{b}=N A_{f}+(N-1)(S-t) w_{2}
$$

Here, $N=6$ is the number of fins, $A_{b}$ is the total area between the fins, and $S$ $=0.01 \mathrm{~m}$ is the pitch. Substituting the pertaining variables, we find

$$
A_{t}=N A_{f}+(N-1)(S-t) w_{2}=6 \times 0.011+(6-1)(0.018-0.01) \times 0.1=0.07 \mathrm{~m}^{2}
$$

The Reynolds number of the flow past the fins is determined as

$$
\operatorname{Re}=\frac{u_{\infty} w_{2}}{v}=\frac{3 \times 0.1}{\left(7.73 \times 10^{-7}\right)}=388,100
$$

We can then compute the heat transfer coefficient,
$h=\left(\frac{k}{w_{2}}\right) 0.664 \mathrm{Re}^{1 / 2} \operatorname{Pr}^{1 / 3}=\left(\frac{0.62}{0.01}\right) \times 0.664 \times(388,100)^{1 / 2} \times 5.2^{1 / 3}=4443.2 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$
Fin calculations require a characteristic length $L_{c}=L_{f}+t / 2=0.05+0.01 / 2=$ 0.055 m and a parameter $m$ such that

$$
m=\left(\frac{2 h}{k_{h s} t}\right)^{1 / 2}=\left(\frac{2 \times 4443.2}{180 \times 0.01}\right)^{1 / 2}=70.26 \mathrm{~m}^{-1}
$$

The efficiency of an individual fin, $\eta_{f}$, is

$$
\eta_{f}=\frac{\tanh m L_{c}}{m L_{c}}=\frac{\tanh (70.26 \times 0.055)}{(70.26 \times 0.055)}=0.259
$$

Then, the thermal resistance due to a fin is calculated as

$$
R_{t, o}=\left\{h A_{t}\left[1-\frac{N A_{f}}{A_{t}}\left(1-\eta_{f}\right)\right]\right\}^{-1}=\left\{4443.2 \times 0.07\left[1-\frac{6 \times 0.011}{0.07}(1-0.258)\right]\right\}^{-1}=0.0107 \mathrm{~K} / \mathrm{W}
$$

Finally, we can calculate the base temperature by means of the relation

$$
\begin{aligned}
& q=P_{\text {elec }}=\frac{T_{b}-T_{\infty}}{R_{b}+R_{t, o}} \rightarrow T_{b}=T_{\infty}+P_{\text {elec }}\left(R_{b}+R_{t, o}\right) \\
& \therefore T_{b}=17+1800(0.00556+0.0107)=46.3^{\circ} \mathrm{C}
\end{aligned}
$$

The temperature of the heat sink is about 30 degrees greater than the temperature of the water stream.

- The correct answer is $\mathbf{B}$.


## P. 8 Solution

The Reynolds number is

$$
\operatorname{Re}=\frac{U_{\infty} D}{v}=\frac{0.05 \times 0.06}{\left(4.6 \times 10^{-7}\right)}=6521.7
$$

The product of the Reynolds number just obtained and the Prandtl number $\operatorname{Pr}=0.0072$ is $\operatorname{Re} \times \operatorname{Pr}=6521.7 \times 0.0072=46.96$. Since this is within the range $1 \leq$ $\operatorname{RePr} \leq 100$, we can apply the Ishiguro correlation for a cylinder in cross-flow of liquid metals,

$$
N u=1.125(\operatorname{Re} \times \operatorname{Pr})^{0.413}=1.125 \times(6521.7 \times 0.0072)^{0.413}=5.52
$$

Given $k=80.3 \mathrm{~W} / \mathrm{mK}$ and the diameter $D=0.06 \mathrm{~m}$ of the rod, the corresponding heat transfer coefficient is

$$
N u=\frac{h \times D}{k} \rightarrow h=\frac{N u \times k}{D}=\frac{5.52 \times 80.3}{0.06}=7387.6 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

The rate of heat transfer at the maximum surface temperature is then

$$
\dot{q}=h A_{t}\left(T_{s}-T_{\infty}\right)
$$

so that

$$
\begin{aligned}
& \quad \dot{q}=h \times \pi D L \times\left(T_{s}-T_{\infty}\right) \rightarrow \frac{\dot{q}}{L}=h \times \pi D \times\left(T_{s}-T_{\infty}\right) \\
& \therefore \frac{\dot{q}}{L}=7381 \times \pi \times 0.06 \times(300-205)=132.2 \mathrm{~kW} / \mathrm{m}
\end{aligned}
$$

Denoting by $\dot{q}_{G}$ the maximum rate of heat generation per unit volume of the rod, we have

$$
\dot{q}_{G}=\frac{q}{\text { Volume }} \rightarrow \dot{q}_{G}=\frac{\dot{q}}{\frac{\pi}{4} \times D^{2} \times L}=\frac{4}{\pi D^{2}} \times\left(\frac{\dot{q}}{L}\right)
$$

We know the term in parentheses to be $132.2 \mathrm{~kW} / \mathrm{m}$. Accordingly,

$$
\dot{q}_{G}=\frac{4}{\pi D^{2}} \times\left(\frac{\dot{q}}{L}\right)=\frac{4}{\pi \times 0.06^{2}} \times 132.2=46.8 \mathrm{MW} / \mathrm{m}^{3}
$$

- The correct answer is A.


## P. 9 Solution

The system in question is illustrated below. The angle between the flow and the pipe is denoted as $\theta$.


Normal to flow


At an angle to flow

The Reynolds number based on the tube diameter is

$$
\operatorname{Re}=\frac{U_{\infty} D}{v}
$$

where $U_{\infty}=5 \mathrm{~m} / \mathrm{s}$ is the flow velocity, $D=0.02 \mathrm{~m}$ is the diameter of the tube, and $v=$ $1.57 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$ as given; that is,

$$
\operatorname{Re}=\frac{5 \times 0.02}{\left(1.57 \times 10^{-5}\right)}=6369.4
$$

The Nusselt number can be determined with the correlation

$$
N u=0.206 \operatorname{Pr}^{0.36} \operatorname{Re}^{0.63}
$$

In order to account for the inclination of the conduit, the Reynolds number is modified as

$$
\mathrm{Re}_{N}=\mathrm{Re} \times \sin \theta
$$

Note that $R e_{N}=R e$ when $\theta=90^{\circ}$, as one would expect. The Nusselt number is modified accordingly,

$$
N u=0.206 \times 0.71^{0.36} \times(6369.4 \times \sin \theta)^{0.63}=45.38(\sin \theta)^{0.63}
$$

The heat transfer coefficient is, in turn,
$N u=\frac{h \times D}{k} \rightarrow h=\frac{N u \times k}{D}=\frac{45.38(\sin \theta)^{0.63} \times 0.0251}{0.02}=56.95(\sin \theta)^{0.63}$
Now, the heat transfer rate can be determined with Newton's law of cooling,

$$
\dot{q}=h A_{s}\left(T_{s}-T_{\infty}\right)
$$

where the area term $A_{s}=\pi D L$, or, equivalently, $A_{s}=\pi D w / \sin \theta$, with $w=1 \mathrm{~m}$ being the width of the duct. Substituting this and other pertaining variables, we obtain

$$
\dot{q}=h \times \pi D L \times\left(T_{s}-T_{\infty}\right)=56.95(\sin \theta)^{0.63} \times\left(\pi \times 0.02 \times \frac{1}{\sin \theta}\right) \times(100-20)=\frac{286.3}{(\sin \theta)^{0.37}}
$$

The engineer will find that the rate of heat transfer will increase because the heat transfer coefficient decreases with $(\sin \theta)^{0.63}$, but the area increases with $1 / \sin \theta$. Therefore, the rate of heat transfer increases with $1 /(\sin \theta)^{0.37}$.

## P. 10 Solution

The Reynolds number is

$$
\operatorname{Re}=\frac{V D}{v}=\frac{(40 / 3.6) \times 0.006}{\left(1.43 \times 10^{-5}\right)}=4662
$$

The Nusselt number that corresponds to this Reynolds number can be determined with the correlation

$$
\begin{array}{r}
N u=0.3+\frac{0.62 \mathrm{Re}^{0.5} \operatorname{Pr}^{1 / 3}}{\left[1+(0.4 / \operatorname{Pr})^{2 / 3}\right]^{1 / 4}}\left[1+\left(\frac{\mathrm{Re}}{282,000}\right)^{5 / 8}\right]^{4 / 5} \\
\therefore N u=0.3+\frac{0.62 \times 4662^{0.5} \times 0.734^{1 / 3}}{\left[1+(0.4 / 0.734)^{2 / 3}\right]^{1 / 4}}\left[1+\left(\frac{4662}{282,000}\right)^{5 / 8}\right]^{4 / 5}=36.0
\end{array}
$$

Then, the convection coefficient is calculated as

$$
h=\frac{k \times N u}{D}=\frac{0.0244 \times 36.0}{0.006}=146.4 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

Given the current $I=50 \mathrm{~A}$ and the resistance $R=0.002 \Omega$, the rate of heat generated in the electrical transmission lines per meter length is

$$
\dot{q}=I^{2} R=50^{2} \times 0.002=5 \mathrm{~W} / \mathrm{m}
$$

The entire heat generated in the electrical transmission line has to be transferred to the ambient air. Given the surface area $A_{s}=\pi D L=\pi \times 0.006 \times 1=$ $0.0188 \mathrm{~m}^{2}$, the surface temperature of the wire can be established with Newton's law of cooling,

$$
\begin{gathered}
\dot{q}=h A_{s}\left(T_{s}-T_{\infty}\right) \rightarrow T_{s}=T_{\infty}+\frac{\dot{q}}{h A_{s}} \\
\therefore T_{s}=10+\frac{5}{146.4 \times 0.0188}=11.8^{\circ} \mathrm{C}
\end{gathered}
$$

The temperature of the wire is no more than two degrees above the temperature of the incoming air.

- The correct answer is A.

The rate of heat transfer from the pipe is the energy change of the water from inlet to exit of the pipe, and can be determined as

$$
\dot{q}=\dot{m} c_{p} \Delta T=8.5 \times 4193 \times(80-70)=356.4 \mathrm{~kW}
$$

The surface area of the tube is

$$
A=\pi D L=\pi \times 0.15 \times 400=188.5 \mathrm{~m}^{2}
$$

The heat transfer coefficient can be obtained with Newton's law of cooling,

$$
\begin{aligned}
\dot{q} & =h A\left(T_{s}-T_{\infty}\right) \rightarrow h=\frac{\dot{q}}{A\left(T_{s}-T_{\infty}\right)} \\
\therefore h & =\frac{356,400}{188.5 \times(75-15)}=31.51 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{aligned}
$$

The Nusselt number is

$$
N u=\frac{h \times D}{k}=\frac{31.51 \times 0.15}{0.027}=175.1
$$

However, recall that the Nusselt number for flow across a cylinder can be obtained with the following correlation,

$$
\begin{aligned}
N u & =0.3+\frac{0.62 \mathrm{Re}^{0.5} \mathrm{Pr}^{1 / 3}}{\left[1+(0.4 / \operatorname{Pr})^{2 / 3}\right]^{1 / 4}}\left[1+\left(\frac{\mathrm{Re}}{282,000}\right)^{5 / 8}\right]^{4 / 5} \\
\therefore 175.1 & =0.3+\frac{0.62 \mathrm{Re}^{0.5}(0.724)^{1 / 3}}{\left[1+(0.4 / 0.724)^{2 / 3}\right]^{1 / 4}}\left[1+\left(\frac{\mathrm{Re}}{282,000}\right)^{5 / 8}\right]^{4 / 5}
\end{aligned}
$$

where we have substituted $N u=175.1$ and $P r=0.724$. The ensuing expression is an implicit equation in the Reynolds number, $R e$, and can be solved by trial-and-error. Using the FindRoot function in Mathematica with an initial guess of 10,000, one appropriate code is

$$
\text { FindRoot }\left[175.1-0.3-\frac{0.62 R^{0.5}(0.724)^{1 / 3}}{\left(1+(0.4 / 0.724)^{2 / 3}\right)^{0.25}}\left(1+\left(\frac{R}{282000}\right)^{5 / 8}\right)^{0.8},\{R, 10000\}\right]
$$

The result is $R e=72,215$. Finally, the average wind velocity $V_{\infty}$ can be obtained from the definition of the Reynolds number, namely

$$
\begin{gathered}
\operatorname{Re}=\frac{V_{\infty} D}{v} \rightarrow V_{\infty}=\frac{\operatorname{Re} \times v}{D} \\
\therefore V_{\infty}=\frac{72,215 \times\left(1.75 \times 10^{-5}\right)}{0.15}=8.4 \mathrm{~m} / \mathrm{s}=30.2 \mathrm{~km} / \mathrm{h}
\end{gathered}
$$

- The correct answer is $\mathbf{C}$.


## P. 12 Solution

The Reynolds number for flow across the exhaust stack is

$$
\operatorname{Re}=\frac{V D}{v}=\frac{10 \times 1}{\left(2.10 \times 10^{-5}\right)}=476,190
$$

The Nusselt number for cross-flow over the exhaust, in turn, can be determined with the correlation

$$
N u=0.3+\frac{0.62(476,190)^{0.5}(0.715)^{1 / 3}}{\left[1+(0.4 / 0.715)^{2 / 3}\right]^{1 / 4}}\left[1+\left(\frac{476,190}{282,000}\right)^{5 / 8}\right]^{4 / 5}=674.5
$$

Using the definition of the Nusselt number, we compute the heat transfer coefficient as

$$
\begin{gathered}
N u=\frac{h \times D}{k} \rightarrow h=\frac{N u \times k}{D} \\
\therefore h=\frac{674.5 \times 0.0295}{1}=19.90 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{gathered}
$$

The surface area of the exhaust stack is $A_{s}=\pi D L=\pi \times 1 \times 10=31.42 \mathrm{~m}^{2}$. Given the mass flow $\dot{m}=1.2 \mathrm{~kg} / \mathrm{s}$, the specific heat $c_{p}=1600 \mathrm{~J} / \mathrm{kg} \cdot \mathrm{K}$, and the temperature $\operatorname{drop} \Delta T=30^{\circ} \mathrm{C}$, the rate of heat loss from the gases in the exhaust stack follows from the elementary equation

$$
\dot{q}_{\text {loss }}=\dot{m} c_{p} \Delta T=1.2 \times 1600 \times 30=57.6 \mathrm{~kW}
$$

The temperature of the outer surface of the exhaust stack is determined by applying an energy balance,

$$
\begin{gathered}
\alpha \times \dot{q}_{s}+\frac{\dot{q}_{\text {loss }}}{A_{s}}=\dot{q}_{\text {conv }}+\dot{q}_{\text {rad }} \\
\therefore \alpha \times \dot{q}_{s}+\frac{\dot{q}_{\text {loss }}}{A_{s}}=h\left(T_{s}-T_{\infty}\right)+\varepsilon \sigma\left(T_{s}^{4}-T_{\text {surr }}^{4}\right)
\end{gathered}
$$

In addition other known variables, here we have an emissivity/absorptivity $\alpha$ $=\varepsilon=0.9$, an intensity of solar radiation $\dot{q}_{s}=1400 \mathrm{~W} / \mathrm{m}^{2}$, the Stefan-Boltzmann constant $\sigma=5.67 \times 10^{-8} \mathrm{Wm}^{-2} \mathrm{~K}^{-4}$, the temperature of incoming wind $T_{\infty}=300 \mathrm{~K}$, and the temperature of the surroundings $T_{\text {surr }}=300 \mathrm{~K}$. Substituting these and other pertaining variables, we obtain

$$
\begin{gathered}
\alpha \times \dot{q}_{s}+\frac{\dot{q}_{\text {loss }}}{A_{s}}=h\left(T_{s}-T_{\infty}\right)+\varepsilon \sigma\left(T_{s}^{4}-T_{\text {surf }}^{4}\right) \\
\therefore 0.9 \times 1400+\frac{57,600}{31.42}=19.90\left(T_{s}-300\right)+0.9 \times\left(5.67 \times 10^{-8}\right) \times\left(T_{s}^{4}-300^{4}\right) \\
\therefore 3093.2=-6383.34+19.9 T_{s}+5.103 \times 10^{-8} T_{s}^{4} \\
\therefore 5.103 \times 10^{-8} T_{s}^{4}+19.9 T_{s}-9476.54=0
\end{gathered}
$$

The result is a fourth degree polynomial in the surface temperature $T_{s}$. This can be straightforwardly solved in Mathematica with the command Solve. There are two imaginary solutions, along with $T_{s}=-847.65$, and $T_{s}=406.32$. The latter is the solution we seek, and corresponds to a temperature $T_{s}=133.2^{\circ} \mathrm{C}$. The outer surface of the exhaust stack is at a temperature of about a hundred degrees greater than that of the surrounding air.

- The correct answer is C.


## P. 13 Solution

For purposes of comparison, suppose the fins have infinite length. For such infinitely long fins, the fin heat transfer rate is given by the equation

$$
\dot{q}_{f}=\sqrt{h P k A_{c}} \theta_{b}
$$

where $h$ is the heat transfer coefficient, $P$ is the perimeter of the cross-section, $k$ is the thermal conductivity, $A_{c}$ is the cross-sectional area, and $\theta_{b}=T_{b}-T_{\infty}$. In every case, the heat transfer coefficient is found from a correlation of the form

$$
N u=C \operatorname{Re}^{m} \operatorname{Pr}^{1 / 3}
$$

Thus,

$$
\dot{q}_{f}=\sqrt{\frac{k_{a}}{D} C \operatorname{Re}^{m} \operatorname{Pr}^{1 / 3} P k A_{c}} \theta_{b}
$$

where the subscript $a$ refers to air properties. Since each fin has the same crosssectional area, the parameters that vary from one configuration to another are $D, C$, $R e, m$, and $P$. Therefore, it suffices to examine the combination parameter $C \operatorname{Re}^{m} P / D$ to determine which fin has the largest heat transfer rate. Since the cross-sectional area of the cylinder is $A_{c}=\pi D^{2} / 4$ and the square has the same cross-sectional area, the dimension $L$ of the square follows as

$$
\begin{gathered}
L^{2}=\frac{\pi D^{2}}{4} \\
\therefore L_{s}=\frac{\pi^{1 / 2} D}{2}=\frac{\pi^{1 / 2} \times 15}{2}=13.3 \mathrm{~mm}
\end{gathered}
$$

The dimension of the diamond configuration is the same, but $D$ for the diamond configuration is defined differently. Referring to the first row of Table 2, the pertaining dimension of the diamond configuration is seen to be $L_{s}=\sqrt{2} L_{d}=18.8$ mm . The perimeters are $P_{c}=\pi D=47.1 \mathrm{~mm}$ for the circular cylinder and $P_{s}=P_{d}=4 /_{s}=$ 53.2 mm for both the square and diamond configurations. The Reynolds number can be calculated from $\operatorname{Re}=V D / v_{a}$, and the Nusselt number is then determined with the relation $N u=C \operatorname{Re}^{m} P r^{1 / 3}$. Constants $C$ and $m$ are taken from Tables 1 and 2 . The results are tabulated below for all three configurations. A line has been included to represent the heat transfer coefficient, which is proportional to $C \mathrm{Re}^{m} / D$; see the blue row below. The main variable we are concerned with, however, is the heat transfer rate, which is proportional to $C \operatorname{Re}^{m} P / D$ as discussed just now; check the red row below.

|  | Configuration A <br> (Circular) | Configuration B <br> (Square) | Configuration C <br> (Diamond) |
| :---: | :---: | :---: | :---: |
| Dimension (mm) | 15 | 13.3 | 18.8 |
| Perimeter $(\mathrm{mm})$ | 47.1 | 53.2 | 53.2 |
| Re | 7177 | 6364 | 8995 |
| C | 0.193 | 0.158 | 0.304 |
| $m$ | 0.618 | 0.66 | 0.59 |
| $\mathrm{CRe}^{\mathrm{m}} / D^{\sim} h$ | 3108 | 3848 | 3480 |
| $\mathrm{CRe}^{m} P / D^{\sim} q$ | 146 | 205 | 185 |

Clearly, the configuration that attains the highest product $C \operatorname{Re}^{m} P / D$, and consequently the highest heat transfer rate, is Configuration B. The square-shaped fin has the highest heat transfer rate.

- The correct answer is $\mathbf{B}$.


## P. 14 Solution

The Nusselt number can be calculated from the equation

$$
N u=C \operatorname{Re}^{m} \operatorname{Pr}^{1 / 3}
$$

where coefficients $C$ and $m$ are defined differently for the two configurations, as per Table 2. For the rod with a side perpendicular to the flow, we have $C=0.158, m=$ 0.66 , and the characteristic dimension $D=0.025 \mathrm{~m}$. The Reynolds number is

$$
\operatorname{Re}=\frac{V D}{v}=\frac{8 \times 0.025}{\left(2.64 \times 10^{-5}\right)}=7575.8
$$

and, accordingly,

$$
N u=0.158 \times 7575.8^{0.66} \times 0.690^{1 / 3}=50.74
$$

Then, the heat transfer coefficient is

$$
\begin{gathered}
N u=\frac{h \times D}{k} \rightarrow h=\frac{N u \times k}{D} \\
\therefore h=\frac{50.74 \times 0.0338}{0.025}=68.6 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
\end{gathered}
$$

When the rod is rotated so that it presents one edge to the flow, the coefficients to be used become $C=0.304$ and $m=0.59$ (Table 2). The characteristic length is shifted to $D=\sqrt{2} \times 0.025=0.0354 \mathrm{~m}$, and the Reynolds number is determined to be $R e=10,713$. The corresponding heat transfer coefficient follows as

$$
h=\frac{k}{D} \times 0.304 \mathrm{Re}^{0.59} \mathrm{Pr}^{1 / 3}=\frac{0.0338}{0.0354} \times 0.304 \times 10,713^{0.59} \times 0.690^{1 / 3}=61.2 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}
$$

Since radiation affects both rods the same way, the rod with the larger convection coefficient will cool faster. Because $h$ is greater for the rod with an edge perpendicular to flow, we conclude that it will have the smaller cooling time. The importance of radiation can be assessed from the Stefan-Boltzmann law,

$$
q_{\mathrm{rad}}^{\prime \prime}=\varepsilon \sigma\left(T_{s}^{4}-T_{\text {surr }}^{4}\right)=0.1 \times\left(5.67 \times 10^{-8}\right) \times\left(673^{4}-293^{4}\right)=1121.4 \mathrm{~W} / \mathrm{m}^{2}
$$

In contrast, the convection heat transfer flux is

$$
q_{\text {conv }}^{\prime \prime}=h\left(T_{s}-T_{\infty}\right)=68.6 \times(673-293)=26,068 \mathrm{~W} / \mathrm{m}^{2}
$$

Observe that radiation heat transfer is initially only a small fraction (= $1121.4 / 26,068 \approx 4.3 \%$ ) of the convection heat transfer and, in addition, it will only decrease in importance with time. Thus, radiation can be neglected in the calculation of the cooling time. The cooling process can be modeled using the lumped capacitance approximation, provided the Biot number is small. Using a characteristic length $L=V / A_{s}=D / 4=0.025 / 4=0.00625 \mathrm{~m}$, we have

$$
\mathrm{Bi}=\frac{h \times L}{k_{s}}=\frac{68.6 \times 0.00625}{235}=0.00182
$$

Consequently, the lumped capacitance approximation is valid and the cooling time can be determined with the relation

$$
t=\frac{\rho V c_{p}}{h A_{s}} \ln \left(\frac{\theta_{i}}{\theta}\right)=\frac{\rho D c_{p}}{4 h} \ln \left(\frac{T_{i}-T_{\infty}}{T-T_{\infty}}\right)
$$

where $\rho=2702 \mathrm{~kg} / \mathrm{m}^{3}$ is the density of aluminum, $D=0.025 \mathrm{~m}$ is the characteristic length, $c_{p}=991 \mathrm{~J} / \mathrm{kgK}$ is the specific heat of aluminum, $h=68.6 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$ is the heat transfer coefficient, $T_{i}=400^{\circ} \mathrm{C}$ is the initial temperature, $T_{\infty}=30^{\circ} \mathrm{C}$ is the temperature of incoming flow, and $T=60^{\circ} \mathrm{C}$ is the final, safe-to-handle temperature. Substituting the pertaining data, we obtain

$$
t=\frac{2702 \times 0.025 \times 991}{4 \times 68.6} \times \ln \left(\frac{400-30}{60-30}\right) \approx 613 \mathrm{~s}
$$

The time required for the aluminum rod to cool is close to 10 minutes.

- The correct answer is $\mathbf{A}$.

Answer Summary

| Problem 1 |  | T/F |
| :---: | :---: | :---: |
| Problem 2 |  | B |
| Problem 3 |  | B |
| Problem 4 |  | D |
| Problem 5 |  | Open-ended pb. |
| Problem 6 | 6A | Open-ended pb. |
|  | 6B | C |
| Problem 7 |  | B |
| Problem 8 |  | A |
| Problem 9 |  | Open-ended pb. |
| Problem 10 |  | A |
| Problem 11 |  | C |
| Problem 12 |  | C |
| Problem 13 |  | B |
| Problem 14 |  | A |

## References

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