Montogue

QUIZ MS202

Fatigue of Materials

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PROBLEMS

Problem 1 (Hertzberg et al., 2013, w/permission)

The fatigue life of a certain alloy at stress levels of σ_1, σ_2 and σ_3 is 10,000, 50,000, and 500,000 cycles, respectively. If a component of this material is subjected to 2500 cycles of σ_1 and 10,000 cycles of σ_2, estimate the remaining lifetime in association with cyclic stresses at a level of σ_3.

A) N = 168,000 cycles

B) N = 214,000 cycles

C) N = 275,000 cycles

D) N = 322,000 cycles

Problem 2 (Dowling, 2013, w/permission)

An unnotched member of AISI steel (A = 1640 MPa, B = −0.098 in equation 3) is subjected to uniaxial cyclic stressing at zero mean stress. The amplitude is at first σ_a = 640 MPa for 2000 cycles, followed by σ_a = 550 MPa for another 10,000 cycles. If the stress is then raised to 700 MPa, how many cycles can be applied at this third level before fatigue failure is expected?

A) N = 1150 cycles

B) N = 2360 cycles

C) N = 3090 cycles

D) N = 4280 cycles

Problem 3.1 (Dowling, 2013, w/permission)

Some values of stress amplitude and corresponding cycles to failure are given in the following table from tests on a AISI 4340 steel. The tests were carried out on unnotched, axially loaded specimens under zero mean stress. Plot these data on log-log coordinates and assess whether the trend seems to represent a straight line. Obtain rough estimates of constants A and B in a correlation of the form $\sigma = A \times N^B$ (equation 3).

σ (MPa)	N (cycles)
758	200
640	900
559	6000
490	14,000
430	44,000
418	130,000

Problem 3.2

For the AISI 4340 steel described in the previous problem, a stress amplitude of σ = 400 MPa will be applied in service for N = 2000 cycles. What are the safety factors in life and stress, respectively?

A) FS_N = 51 and FS_σ = 1.53

B) FS_N = 51 and FS_σ = 3.06

C) FS_N = 102 and FS_σ = 1.53

D) FS_N = 102 and FS_σ = 3.06

Problem 3.3

The AISI 4340 steel of the preceding problem is subjected to cyclic loading with a tensile mean stress of σ_m = 180 MPa. What life is expected if the stress amplitude is σ_a = 375 MPa?

A) N_f = 60,900 cycles

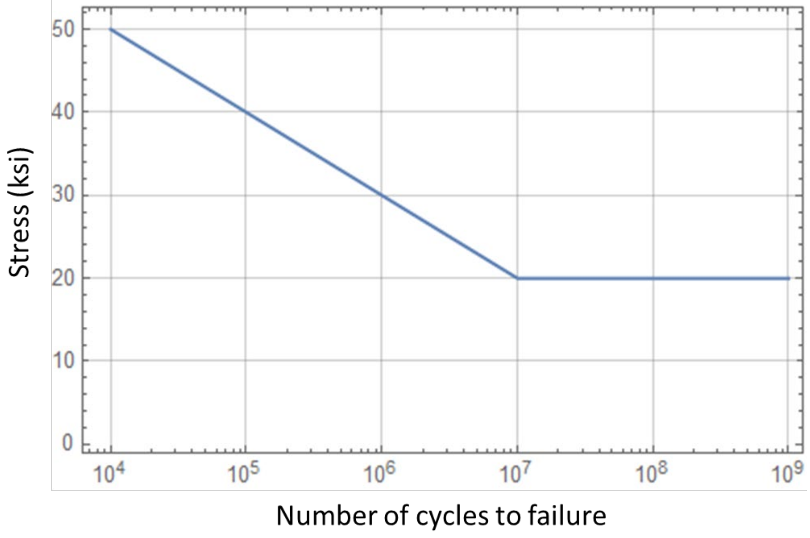
B) N_f = 81,100 cycles

C) N_f = 106,000 cycles

D) N_f = 136,000 cycles

Problem 4 (Shukla, 2005, w/permission)

A short-life bearing component in a rotating machinery undergoes cyclic dynamic loads due to synchronous rotor vibration. The rotor was operated at 50 rpm for 13 hrs and 20 min, then the speed was increased to 100 rpm and the rotor was run for an additional 8 hrs 20 min. Referring to the S-N curve and data shown below, estimate the allowable number of operational fatigue cycles and maximum run time at 200 rpm so that a cumulative damage factor of 0.75 is not exceeded. The maximum stresses that the component is subjected to at the abovementioned speeds are given below.



Frequency	Stress, σ
50 rpm	20 ksi
100 rpm	30 ksi
200 rpm	50 ksi

A) t = 17 min

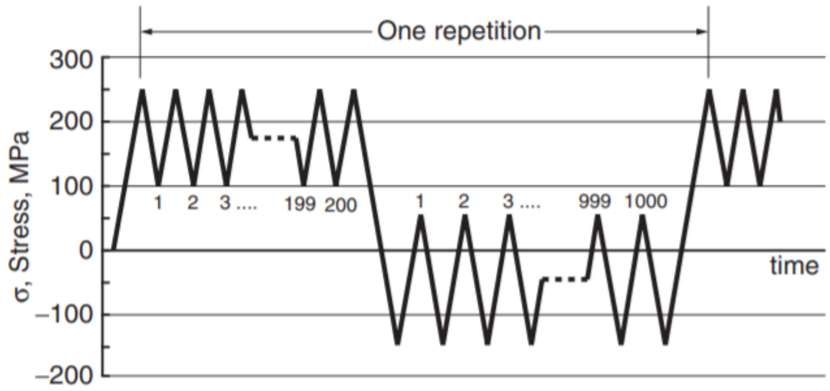
B) t = 35 min

C) t = 52 min

D) t = 70 min

Problem 5 (Dowling, 2013, w/permission)

At a location of interest in an engineering component made of 2024-T4 aluminum, the material is repeatedly subjected to the uniaxial stress history shown below. Estimate the number of repetitions necessary to cause failure. For the metal in question, σ_f = 900 MPa and b = −0.102 in equation 4. Use the SWT equation.



A) B= 83,200 cycles

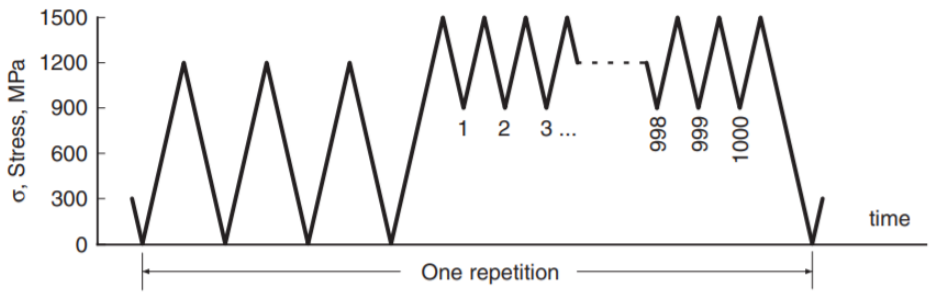
B) B= 111,000 cycles

C) B= 161,000 cycles

D) B= 210,000 cycles

Problem 6 (Dowling, 2013, w/permission)

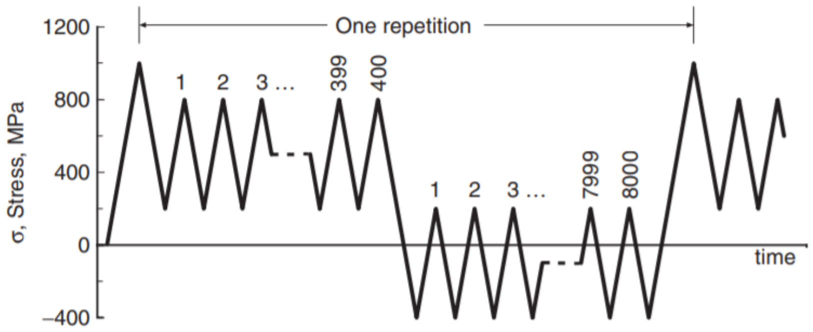
At a location of interest in an engineering component made of SAE 4142 steel, the material is repeatedly subjected to the uniaxial stress history shown below. Estimate the number of repetitions necessary to cause failure. For the metal in question, σ_f = 1940 MPa and b = −0.0762 in equation 4. Use the SWT equation.



A) B = 383 cycles
B) B = 850 cycles
C) B = 1250 cycles
D) B = 1690 cycles

Problem 7 (Dowling, 2013, w/permission)

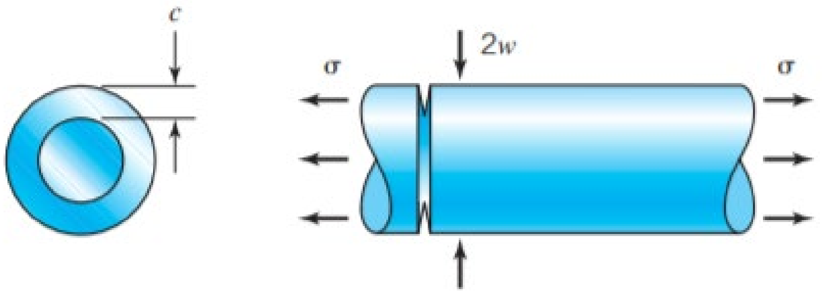
At a location of interest in an engineering component made of Ti-6Al-4V alloy, the material is repeatedly subjected to the uniaxial stress history shown below. Estimate the number of repetitions necessary to cause failure. For the metal in question, σ_f = 2030 MPa and b = −0.104 in equation 4. Use the SWT equation.



A) B = 742 cycles
B) B = 1660 cycles
C) B = 2540 cycles
D) B = 3100 cycles

Problem 8 (Juvinall & Marshek, 2012, w/permission)

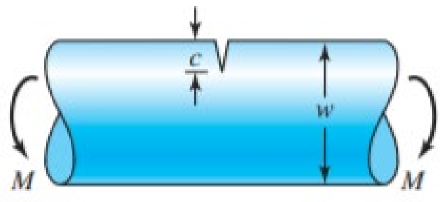
A 1.0-in. diameter aluminum bar is subjected to reversed axial loading of 1120 lb at 50 cycles per second. A circumferential crack, 0.004-in. deep, extends radially inward from the outside surface. The axial load is applied remote from the crack. Estimate the crack depth after 250 hours of operation, assuming a Paris exponent of 2.7 and a stress intensity range of 1.5 ksi-in.$^{1/2}$ corresponding to a growth rate of 0.036 in./10^6 cycles. The configuration factor Y may be approximated as $Y = [1.12 + \alpha(1.3\alpha - 0.88)]/(1 - 0.92\alpha)$, where $\alpha = a/w$, such that a is the crack width and w is the radius of the round bar.



A) a = 0.017 in.
B) a = 0.034 in.
C) a = 0.051 in.
D) a = 0.068 in.

Problem 9 (Juvinall & Marshek, 2012, w/permission)

A 2.0-in. diameter, 10-in. long aluminum shaft rotates at 3000 rpm and is subjected to a reverse bending moment of 1780 lb-in. A crack 0.004 in. deep extends radially inward from the external surface. The reverse bending moment is applied remote from the crack. Estimate the crack depth after 500 hours of operation assuming a Paris exponent of 2.6 and a stress intensity range of 1.1 ksi-in.$^{1/2}$ corresponding to a growth rate of 0.05 in./10^6 cycles. The configuration factor Y may be approximated as Y = [1.67 + α(451α – 80)]/(1 – 0.61α), where $\alpha = a/w$, such that a is the crack width and w is the radius of the round bar.



A) a = 0.0167 in.
B) a = 0.0221 in.
C) a = 0.0382 in.
D) a = 0.0501 in.

Problem 10.1 (Hertzberg et al., 2013, w/permission)

A material with a plane-strain fracture toughness of K_{IC} = 55 MPa-m$^{1/2}$ has a central crack in a very wide panel. If σ_Y = 1380 MPa and the design stress is limited to 50% of that value, compute the maximum allowable fatigue flaw size that can grow during cyclic loading.

A) a = 1.01 mm
B) a = 2.02 mm
C) a = 3.15 mm
D) a = 4.18 mm

Problem 10.2

If the initial crack had a total crack length of 2.5 mm, how many loading cycles (from zero to the design stress) could the panel endure? Assume that fatigue crack growth rates varied with the stress intensity factor range raised to the fourth power. The proportionality constant may be taken as 1.10×10^{-39}.

A) N_f = 25,400 cycles
B) N_f = 59,200 cycles
C) N_f = 85,100 cycles
D) N_f = 122,000 cycles

Problem 11 (Juvinall and Marshek, 2012, w/permission)

A 1-cm long through-thickness crack is discovered in a steel plate. If the plate experiences a stress of 50 MPa that is repeated at a frequency of 30 cpm, how long would it take to grow a crack, corresponding to a design limit where K_{limit} = K_{IC}/3. Assume that K_{IC} = 90 MPa-m$^{1/2}$ and the material possesses a growth rate relation where da/dN = 4×10^{-37}(ΔK)^4, with da/dN and ΔK given in units of m/cycle and Pa-m$^{1/2}$, respectively.

A) t = 15.8 days
B) t = 32.4 days
C) t = 53.1 days
D) t = 85.6 days

Problem 12 (Shukla, 2005, w/permission)

A wide plate made of A514 steel contains an initial edge crack a_i = 0.25 in. The plane-strain fracture toughness of the material is 125 ksi-in.$^{1/2}$. The plate is subjected to fluctuating tensile stresses of 25 ksi minimum intensity and 48 ksi maximum intensity. Calculate the number of cycles for the crack to reach the critical length. Use the Paris equation with C = 0.66×10^{-8} and n = 2.25.

A) N = 14,800 cycles
B) N = 32,100 cycles
C) N = 57,000 cycles
D) N = 76,100 cycles

■ Problem 13.1 (Dowling, 2013, w/permission)

A center-cracked plate of 7075-T6 aluminum was tested for evaluation of fatigue parameters. The specimen had dimensions of height h = 445 mm, width b = 152 mm, and thickness t = 2.29 mm. The force was cycled between a minimum value of $P_{\min}$ = 48.1 kN and a maximum value of $P_{\max}$ = 96.2 kN. The data obtained are listed below. Determine the da/dN and ΔK values from these data and make a da/dN versus ΔK plot of the results on log-log coordinates. Fit the data to the Paris equation to obtain values of C and n. Does the Paris equation represent the data well?

a (mm)	*N* (cycles)	*a* (mm)	*N* (cycles)
5.08	0	20.32	21,500
7.62	9500	22.86	22,300
10.16	14,300	25.4	22,900
12.7	17,100	30.48	23,500
15.24	19,100	35.56	24,000
17.78	20,500		

■ Problem 13.2

Employ the results of the previous problem for 7075-T6 aluminum tested at a stress ratio of R = 0.5 as follows: plot the da/dN versus ΔK data on log-log coordinates, then show the line corresponding to the Walker equation (equation 9), with C_0 = 2.71×10^{-8} mm/cycle/(MPa-m$^{-1/2}$)^{m}, m = 3.70, and γ = 0.641. Do the data and line agree?

Problem 14 (Dowling, 2013, w/permission)

A center-cracked plate made of 2024-T3 aluminum has width b = 50 mm, thickness t = 4 mm, a large height h, and an initial crack length of a_i = 2 mm. How many cycles between $P_{\min}$ = 18 kN and $P_{\max}$ = 60 kN are required to grow the crack to failure by either fully plastic yielding or brittle fracture? The properties of 2024-T4 aluminum are a plane-strain fracture toughness K_{IC} = 34 MPa-m$^{1/2}$, yield strength σ_Y = 353 MPa, C_0 = 1.42×10^{-11} m/cycle/(MPa-m$^{1/2}$)^{m}, m = 3.59, and γ = 0.680.

A) N_f = 18,800 cycles

B) N_f = 23,100 cycles

C) N_f = 39,000 cycles

D) N_f = 56,000 cycles

Problem 15 (Dowling, 2013, w/permission)

A bending member made of AISI 4340 steel has a rectangular cross-section with width b = 60 mm and thickness t = 9 mm. An initial edge crack of length a = 0.5 mm is present, and the member is subjected to cyclic bending between $M_{\min}$ = 1.2 kN·m and $M_{\max}$ = 3.0 kN·m. Estimate the number of cycles necessary to grow the cycle to failure. The properties of 2024-T4 aluminum are a plane-strain fracture toughness K_{IC} = 130 MPa-m$^{1/2}$, yield strength σ_Y = 1255 MPa, C_0 = 5.11×10^{-10} mm/cycle/(MPa-m$^{1/2}$)^{m}, m = 3.24, and γ = 0.420.

A) N_f = 8720 cycles

B) N_f = 25,600 cycles

C) N_f = 46,200 cycles

D) N_f = 85,000 cycles

■ Problem 16.1 (Dowling, 2013, w/permission)

A bending member made of 7075-T6 aluminum has a rectangular cross-section with width b = 40 mm and thickness t = 10 mm. Inspection can reliably find cracks only if they are larger than a = 0.25 mm, so it must be assumed that a through-thickness edge crack of this size may be present. A cyclic bending moment is applied with $M_{\min}$ = −90 N·m and $M_{\max}$ = 300 N·m. Estimate the number of cycles to grow the crack to failure. The properties of 2024-T4 aluminum are a plane-strain fracture toughness K_{IC} = 29 MPa-m$^{1/2}$, yield strength σ_Y = 523 MPa, C_0 = 2.71×10^{-8} mm/cycle/(MPa-m$^{1/2}$)^{m}, m = 3.70, and γ = 0 (because the stress ratio R < 0).

A) N_f = 40,400 cycles

B) N_f = 64,800 cycles

C) N_f = 80,100 cycles

D) N_f = 99,200 cycles

■ Problem 16.2

What is the factor of safety in life if the desired service life is 200,000 cycles? A factor of safety in life of 3 is required. What is the recommended interval of periodic inspection?

ADDITIONAL INFORMATION

Equations

1 → *Stress intensity factor equation*

$$K = F\sigma\sqrt{\pi a}$$

where K is the stress intensity factor, F (sometimes denoted as Y) is the dimensionless geometry function, σ is stress, and a is crack length.

2 → *Palmgren-Miner rule*

$$\Sigma\left(\frac{N_i}{N_{f,i}}\right) = 1$$

where N_i is the number of cycles for the i-th loading and $N_{f,i}$ is the number of cycles to failure for the i-th loading.

3 → *Equation for S-N curve*

$$\sigma_a = AN_f^B$$

where σ_a is the stress amplitude, N_f is the number of cycles, and A and B are constants.

4 → *Modified equation for S-N curve*

$$\sigma_a = \sigma'_f\left(2N_f\right)^b$$

where σ_a is the stress amplitude, N_f is the number of cycles, and σ'_f and b are constants.

5 → *Equation for completely reversed stress amplitude (σ_{ar})*

$$\sigma_{ar} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma_f}}$$

where σ_{ar} is the completely reversed stress amplitude, σ_a is the stress amplitude, σ_m is mean stress, and σ_f is the constant in the S-N curve equation $\sigma_{ar} = \sigma_f\left(2N_f\right)^b$ (eq. 4).

6 → *Smith, Watson and Tupper (SWT) equation*

$$\sigma_{ar} = \sqrt{\sigma_{\max}\sigma_a}$$

where σ_{ar} is the completely reversed stress amplitude, $\sigma_{\max}$ is the maximum stress (= $\sigma_m + \sigma_a$), and σ_a is the stress amplitude.

7 → *Paris equation*

$$\frac{da}{dN} = C\left(\Delta K_I\right)^m$$

where da/dN is the rate of increase of crack size per cycle, ΔK_I is the range of stress intensity factors, and C and n are material constants. Constant m is sometimes denoted as n.

8 → *Integrated Paris equation*

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\Delta\sigma\sqrt{\pi}\right)^m\left(1 - m/2\right)}$$

where N_f is the number of cycles to failure, a_f is the final crack length, a_i is the initial crack length, F is the geometric modification factor, $\Delta\sigma$ is the stress range, and C and m are the same material constants as in eq. 4.

9 → *Walker equation*

$$\frac{da}{dN} = C_0\left[\frac{\Delta K}{(1-R)^{(1-\gamma)}}\right]^m$$

where da/dN is the rate of increase of crack size per cycle, ΔK is the range of stress intensity factors, $R = \sigma_{\min}/\sigma_{\max}$ is the stress ratio, and C_0, γ and m are material constants.

10 → *Critical crack length for brittle fracture*

$$a_c = \frac{1}{\pi}\left(\frac{K_C}{F\sigma_{\max}}\right)^2$$

where a_c is the critical crack size for brittle fracture, K_C is the fracture toughness of the material, F is the modification factor (sometimes denoted as Y), and $\sigma_{\max}$ is the maximum stress.

11 → *Critical crack length for fully plastic yielding – maximum force*

$$a_Y = b\left(1-\frac{P_{\max}}{2bt\sigma_Y}\right)$$

where a_Y is the critical crack size for fully plastic yielding, $P_{\max}$ is the maximum loading, b is the member width, t is the member thickness, and σ_Y is the yield strength of the material.

12 → *Critical crack length for fully plastic yielding – maximum bending moment*

$$a_Y = b\left(1-\frac{2}{b}\sqrt{\frac{M_{\max}}{t\sigma_Y}}\right)$$

where $M_{\max}$ is the maximum bending moment; the other variables are the same as in eq. 9.

SOLUTIONS

P.1 ■ Solution

This is a straightforward application of the Miner rule,

$$\Sigma\frac{N}{N_f} = 1 \rightarrow \frac{2500}{10,000} + \frac{10,000}{50,000} + \frac{N}{500,000} = 1.0$$

$$\therefore \boxed{N = 275,000 \text{ cycles}}$$

♦ The correct answer is **C**.

P.2 ■ Solution

The S-N curve is $\sigma = 1640N^{-0.098}$. The number of cycles to failure at an amplitude stress of 640 MPa is

$$N_{f,1} = \left(\frac{\sigma_a}{A}\right)^{1/B} = \left(\frac{640}{1640}\right)^{-1/0.098} = 14,800 \text{ cycles}$$

while number of cycles to failure at an amplitude stress of 550 MPa is

$$N_{f,2} = \left(\frac{550}{1640}\right)^{-1/0.098} = 69,500 \text{ cycles}$$

and, for a stress of 700 MPa,

$$N_{f,3} = \left(\frac{700}{1640}\right)^{-1/0.098} = 5930 \text{ cycles}$$

The information we have are summarized in the following table.

No.	N (cycles)	σ_a (MPa)	N_f (cycles)
1	2000	640	14,800
2	10,000	550	69,500
3	N_3	700	5930

To find N_3, we apply the Miner rule,

$$\Sigma \frac{N}{N_f} = 1$$

Thus,

$$\Sigma \frac{N}{N_f} = 0 \rightarrow \frac{2000}{14,800} + \frac{10,000}{69,500} + \frac{N_3}{5930} = 1.0$$

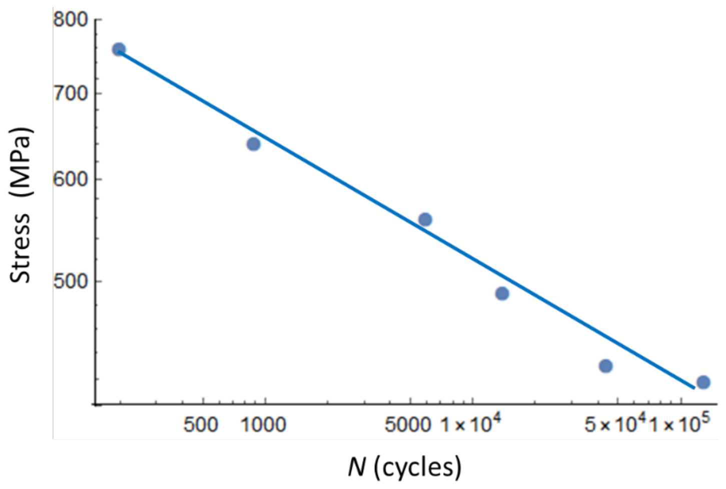
$$\therefore \boxed{N_3 = 4280 \text{ cycles}}$$

The member can withstand another 4280 cycles at 700 MPa before fatigue failure occurs.

♦ The correct answer is **D.**

P.3 ■ Solution

Part 1: The data are plotted on a log-log plot below.



The line is described by the relation $\sigma = A \times N^B$. For any two points we must have

$$\sigma_1 = A \times N_1^B \ ; \ \sigma_2 = A \times N_2^B$$

Dividing one equation by the other, taking logarithms and substituting, we obtain

$$B = \frac{\log \sigma_1 - \log \sigma_2}{\log N_1 - \log N_2} = \frac{\log 758 - \log 418}{\log 200 - \log 130,000} = -0.0919$$

where we have used points (200, 758) and (130,000, 418). Once B is known, coefficient A can be calculated from any data point; indeed,

$$A = \frac{\sigma}{N^B} = \frac{758}{200^{-0.0919}} = 1230 \text{ MPa}$$

The S-N curve is described by $\sigma = 1230 \times N^{-0.0919}$.

Part 2: The life that corresponds to a stress of 400 MPa is calculated as

$$\sigma = A \times N^B \rightarrow N_f = \left(\frac{\sigma}{A}\right)^{1/B}$$

$$\therefore N_f = \left(\frac{400}{1230}\right)^{1/-0.0919} = 203,000 \text{ cycles}$$

The factor of safety in life is then

$$FS_N = \frac{N_f}{\widehat{N}} = \frac{203,000}{200} = \boxed{102}$$

The safety factor in stress can be calculated if we first determine the stress amplitude corresponding to 2000 cycles, namely,

$$\sigma_f = 1230 \times 2000^{-0.0919} = 612 \text{ MPa}$$

Thus,

$$FS_\sigma = \frac{\sigma_f}{\hat{\sigma}} = \frac{612}{400} = \boxed{1.53}$$

♦ The correct answer is **C.**

Part 3: Another way to express the S-N curve obtained just now is to write

$$\sigma = \sigma_f'(2N)^B$$

Comparing this relation with the expression we have used heretofore gives

$$\sigma = \sigma_f'(2N)^B = A \times N^B \rightarrow \sigma_f'(2N)^{-0.0919} = 1230 \times N^{-0.919}$$

$$\therefore \sigma_f' = 1310 \text{ MPa}$$

Thus, another way to describe the S-N curve is to use $\sigma = 1310(2N)^{-0.0919}$, with $\sigma_f' = 1310$ MPa. We can now proceed to determine the completely reversed stress σ_{ar},

$$\sigma_{ar} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma_f'}} = \frac{375}{1 - \dfrac{180}{1310}} = 435 \text{ MPa}$$

The number of cycles the steel can withstand at this completely reversed stress is

$$\sigma_{ar} = \sigma_f'(2N)^B \rightarrow N_f = \frac{1}{2}\left(\frac{\sigma_{ar}}{\sigma_f'}\right)^{1/B}$$

$$\therefore N_f = \frac{1}{2}\left(\frac{435}{1310}\right)^{-1/0.0919} = \boxed{81{,}100 \text{ cycles}}$$

An alternative is to employ the SWT relationship,

$$\sqrt{\sigma_{\max}\sigma_a} = \sigma_f'(2N)^B$$

where $\sigma_{max} = \sigma_m + \sigma_a = 180 + 375 = 555$ MPa, so that

$$\sqrt{\sigma_{\max}\sigma_a} = \sigma_f'(2N)^B \rightarrow N = \frac{1}{2}\left(\frac{\sqrt{\sigma_{\max}\sigma_a}}{\sigma_f'}\right)^{1/B}$$

$$\therefore N = \frac{1}{2}\left(\frac{\sqrt{555 \times 375}}{1310}\right)^{-1/0.0919} = 48{,}300 \text{ cycles}$$

The value obtained is quite different from that of the preceding equation. Simple fatigue correlations such as those used here can be expected to agree only roughly.

♦ The correct answer is **B** .

P.4 ■ Solution

Referring to the S-N curve, we see that the number of cycles to failure at the abovementioned maximum stresses is $N = 10^7$ for $\sigma_{max} = 20$ ksi and $N = 10^6$ for $\sigma_{max} = 30$ ksi. The number of cycles the component was subjected to is 800 min × 50 rpm = 40,000 cycles at 20 ksi and 500 min × 100 rpm = 50,000 cycles at 30 ksi. In view of the Miner rule,

$$\Sigma\left(\frac{N_0}{N}\right) = 1$$

we have, in the present case,

$$\frac{N_0}{N}+\frac{40,000}{10,000,000}+\frac{50,000}{1,000,000}=0.75$$

$$\therefore \frac{N_0}{N}=0.696$$

Operation at 200 rpm implies a stress of σ_{max} = 50 ksi, which in turn corresponds to a number of cycles to failure of N = 10^4. Accordingly, the required number of operational cycles is

$$N_0=10^4\times 0.696=6960 \text{ cycles}$$

The maximum allowable service life at 200 rpm is then

$$t=\frac{6960 \text{ cycles}}{200 \ \frac{\text{cycles}}{\text{min}}}\approx \boxed{35 \text{ min}}$$

If the cumulative damage ratio is not to exceed 0.75, the component can operate at 200 rpm for little more than half an hour.

♦ The correct answer is **B.**

P.5 ■ Solution

The SWT equation can be used to estimate the number of cycles to failure,

$$\sqrt{\sigma_{max}\sigma_a}=\sigma_f'\left(2N_f\right)^b \rightarrow N_f=\frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma_f'}\right)^{1/b}$$

Here, the alternating stress is given by

$$\sigma_a=\frac{\sigma_{max}-\sigma_{min}}{2}$$

For the first 200 cycles in a given repetition, we have σ_{min} = 100 MPa and σ_{max} = 250 MPa, giving

$$\sigma_a=\frac{250-100}{2}=75 \text{ MPa}$$

and

$$N_f=\frac{1}{2}\times\left(\frac{\sqrt{250\times 75}}{900}\right)^{1/(-0.102)}=5.20\times 10^7 \text{ cycles}$$

Proceeding similarly with the other two loading steps that constitute a repetition, we obtain the following table.

Loading	σ_{max} (MPa)	σ_{min} (MPa)	σ_a (MPa)	N_f (cycles)	N (cycles)
1	250	100	75	5.20E+07	200
2	50	-150	100	3.39E+10	1000
3	250	-150	200	4.25E+05	1

The number of cycles to failure follows from the Miner rule,

$$B\left(\frac{N_1}{N_{f,1}}+\frac{N_2}{N_{f,2}}+\frac{N_3}{N_{f,3}}\right)=1\rightarrow B\left(\frac{200}{5.20\times 10^7}+\frac{50}{3.39\times 10^{10}}+\frac{1}{4.25\times 10^5}\right)=1.0$$

$$\therefore \boxed{B=161,000 \text{ cycles}}$$

♦ The correct answer is **C**.

P.6 ■ Solution

Applying the SWT equation, we see that the number of cycles to failure is

$$N_f=\frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma_f'}\right)^{1/b}$$

The alternating stress is given by

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

For the first three cycles in a given repetition, we have $\sigma_{\min}$ = 0 and $\sigma_{\max}$ = 1200 MPa, yielding

$$\sigma_a = \frac{1200-0}{2} = 600 \text{ MPa}$$

and

$$N_f = \frac{1}{2} \times \left(\frac{\sqrt{1200 \times 600}}{1940}\right)^{1/(-0.0762)} = 25{,}800 \text{ cycles}$$

Proceeding similarly with the other two loading steps that constitute a repetition, we obtain the following table.

Loading	$\sigma_{\max}$ (MPa)	$\sigma_{\min}$ (MPa)	σ_a (MPa)	N_f (cycles)	N (cycles)
1	1200	0	600	2.58E+04	3
2	1500	900	300	5.64E+05	1000
3	1500	0	750	1.38E+03	1

The number of cycles to failure follows from the Miner rule,

$$B\left(\frac{N_1}{N_{f,1}} + \frac{N_2}{N_{f,2}} + \frac{N_3}{N_{f,3}}\right) = 1 \rightarrow B\left(\frac{3}{2.58 \times 10^4} + \frac{1000}{5.64 \times 10^5} + \frac{1}{1.38 \times 10^3}\right) = 1.0$$

$$\therefore \boxed{B = 383 \text{ cycles}}$$

♦ The correct answer is **A**.

P.7 ■ Solution

Applying the SWT equation, we see that the number of cycles to failure is

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{\max}\sigma_a}}{\sigma_f'}\right)^{1/b}$$

The alternating stress is given by

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

For the first 400 cycles in a given repetition, we have $\sigma_{\min}$ = 200 MPa and $\sigma_{\max}$ = 800 MPa, with the result that

$$\sigma_a = \frac{800-200}{2} = 300 \text{ MPa}$$

and

$$N_f = \frac{1}{2} \times \left(\frac{\sqrt{800 \times 300}}{2030}\right)^{1/(-0.104)} = 432{,}000 \text{ cycles}$$

Proceeding similarly with the other steps that constitute a repetition, we obtain the following table.

Loading	$\sigma_{\max}$ (MPa)	$\sigma_{\min}$ (MPa)	σ_a (MPa)	N_f (cycles)	N (cycles)
1	800	200	300	4.32E+05	400
2	200	-400	300	3.39E+08	8000
3	1000	-400	700	2.51E+03	1

The number of cycles to failure follows from the Miner rule,

$$B\left(\frac{N_1}{N_{f,1}} + \frac{N_2}{N_{f,2}} + \frac{N_3}{N_{f,3}}\right) = 1 \rightarrow B\left(\frac{400}{4.32 \times 10^5} + \frac{8000}{3.39 \times 10^8} + \frac{1}{2.51 \times 10^3}\right) = 1.0$$

$$\therefore \boxed{B = 742 \text{ cycles}}$$

♦ The correct answer is **A**.

P.8 ■ Solution

The solution is started by computing the maximum stress in the aluminum bar,

$$\sigma_{\max} = \frac{P}{\frac{\pi}{4}d^2} = \frac{1120}{\frac{\pi}{4}\times 1.0^2} = 1.43 \text{ ksi}$$

In a similar manner, the minimum stress is $\sigma_{\min} = -1.43$ ksi. The range of stress is then $\Delta\sigma$ = 1.43 – (−1.43) = 2.86 ksi. Given the dimensionless parameter $\alpha = a/w$ = 0.004/0.5 = 0.008, the modification factor is calculated as

$$Y = \frac{1.12+\alpha(1.3\alpha-0.88)}{1-0.92\alpha} = \frac{1.12+0.008\times(1.3\times0.008-0.88)}{1-0.92\times0.008} = 1.12$$

The range of stress intensity factor is calculated next,

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} = 1.12\times2.86\times\sqrt{\pi\times0.004} = 0.359 \text{ ksi}\sqrt{\text{in.}}$$

Coefficient C for use with the Paris equation is

$$\frac{da}{dN} = C(\Delta K_I)^n \rightarrow C = \frac{da/dN}{(\Delta K_I)^n}$$

$$\therefore C = \frac{0.036/10^6}{1.5^{2.7}} = 1.20\times10^{-8} \text{ in.}/\text{ksi}\sqrt{\text{in.}}$$

The crack growth rate is then

$$\frac{da}{dN} = C\times(\Delta K)^n = (1.20\times10^{-8})\times0.359^{2.7} = 7.55\times10^{-10} \text{ in./cycle}$$

The number of cycles in a 250-hour period is

$$N = 50\times(250\times3600) = 4.5\times10^7 \text{ cycles}$$

The crack depth after this period of operation follows as

$$a = (7.55\times10^{-10})\times(4.5\times10^7) = \boxed{0.0340 \text{ in.}}$$

♦ The correct answer is **B.**

P.9 ■ Solution

To begin, we compute the section modulus of the bar,

$$z = \frac{\ell w^2}{6} = \frac{10\times(2.0/2)^2}{6} = 1.67 \text{ in.}^3$$

Given the bending moment $M_{\max}$ = 1780 lb-in., the maximum stress is calculated as

$$\sigma_{\max} = \frac{M_{\max}}{z} = \frac{1780}{1.67} = 1.07 \text{ ksi}$$

With a reverse moment $M_{\min}$ = −1780 lb-in., the minimum stress is easily seen to be $\sigma_{\min}$ = −1.07 ksi. The range of stress is then $\Delta\sigma$ = 1.07 – (−1.07) = 2.14 ksi. Given the dimensionless constant $\alpha = a/w$ = 0.004/1.0 = 0.004, the modification factor is

$$Y = \frac{1.67+\alpha(451\alpha-80)}{1-0.61\alpha} = \frac{1.67+0.004\times(451\times0.004-189)}{1-0.61\times0.004} = 1.36$$

The range of stress intensity factor is calculated next,

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} = 1.36\times2.14\times\sqrt{\pi\times0.004} = 0.326 \text{ ksi}\sqrt{\text{in.}}$$

Coefficient C for use with the Paris equation is determined as

$$\frac{da}{dN} = C\left(\Delta K_I\right)^n \rightarrow C = \frac{da/dN}{\left(\Delta K_I\right)^n}$$

$$\therefore C = \frac{0.05/10^6}{1.1^{2.6}} = 3.90\times10^{-8}\ \text{in.}/\text{ksi}\sqrt{\text{in.}}$$

The crack growth rate is then

$$\frac{da}{dN} = C\times\left(\Delta K\right)^n = \left(3.90\times10^{-8}\right)\times0.326^{2.6} = 2.12\times10^{-9}\ \text{in./cycle}$$

The number of cycles in a 100-hour period is

$$N = 3000\times\left(100\times60\right) = 1.8\times10^7\ \text{cycles}$$

The crack depth after this period of operation follows as

$$a = \left(2.12\times10^{-9}\right)\times\left(1.8\times10^7\right) = \boxed{0.0382\ \text{in.}}$$

♦ The correct answer is **C**.

P.10 ■ Solution

Part 1: The design stress is $\sigma_d = \sigma_Y/2$ = 1380/2 = 690 MPa. The maximum allowable fatigue flaw size is

$$K_{IC} = \sigma_d\sqrt{\pi a} \rightarrow a = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_d}\right)^2$$

$$\therefore a = \frac{1}{\pi}\times\left(\frac{55}{690}\right)^2 = \boxed{2.02\ \text{mm}}$$

♦ The correct answer is **B**.

Part 2: The Paris equation, in this case, is expressed as

$$\frac{da}{dN} = 1.1\times10^{-39}\left(\Delta K\right)^{4.0}$$

This can be restated as follows,

$$\frac{da}{dN} = 1.1\times10^{-39}\left(\Delta K\right)^{4.0} \rightarrow \frac{da}{dN} = 1.1\times10^{-39}\left(\Delta\sigma\right)^4\pi^2a^2$$

The number of cycles to failure is then

$$N_f = \frac{1}{C\times\left(\Delta\sigma\right)^4\times\pi^2}\left(\frac{1}{a_i}-\frac{1}{a_f}\right)$$

$$\therefore N_f = \frac{1}{\left(1.1\times10^{-39}\right)\times\left(690\times10^6\right)^4\times\pi^2}\left(\frac{1}{0.00125}=\frac{1}{0.002}\right) = \boxed{122{,}000\ \text{cycles}}$$

♦ The correct answer is **D**.

P.11 ■ Solution

The limiting K level is $K = K_{IC}/3$ = 90/3 = 30 MPa-m$^{1/2}$. The limiting crack size for an applied stress is

$$K = \sigma\sqrt{\pi a} \rightarrow a = \frac{1}{\pi}\left(\frac{K}{\sigma}\right)^2$$

$$\therefore a = \frac{1}{\pi}\times\left(\frac{30}{50}\right)^2 = 0.115\ \text{m}$$

The number of cycles to failure is then

$$N_f = \frac{1}{C\times\left(\Delta\sigma\right)^4\times\pi^2}\left(\frac{1}{a_i}-\frac{1}{a_f}\right)$$

$$\therefore \frac{1}{\left(4\times10^{-37}\right)\times\left(50\times10^{6}\right)^{4}\times\pi^{2}}\left(\frac{1}{0.01}-\frac{1}{0.115}\right)=3.70\times10^{6}\text{ cycles}$$

At 30 cpm, the remaining service lifetime is determined to be

$$t=\frac{3.7\times10^{6}}{30\times(60\times24)}=\boxed{85.6\text{ days}}$$

♦ The correct answer is **D**.

P.12 ■ Solution

The stress intensity factor is given by

$$K_I=1.12\sigma\sqrt{\pi a}\approx1.99\sigma\sqrt{a}$$

and the critical length is, accordingly,

$$K_I=1.99\sigma\sqrt{a}\rightarrow a_C=\left(\frac{1}{1.99}\right)^2\left(\frac{K_I}{\sigma_{\max}}\right)^2$$

$$\therefore a_C=\left(\frac{1}{1.99}\right)^2\left(\frac{125}{48}\right)^2=1.71\text{ in.}$$

The live-load stress range is, in turn,

$$\Delta\sigma=48-25=23\text{ ksi}$$

The stress intensity factor range is

$$\Delta K_I=1.99\Delta\sigma\sqrt{\bar{a}}=1.99\times23\sqrt{\bar{a}}=45.8\sqrt{\bar{a}}\quad(\text{I})$$

Let the increment of crack growth be Δa = 0.1 in. Appealing to the formula for fatigue crack growth per cycle, we write

$$\frac{\Delta a}{\Delta N}=0.66\times10^{-8}(\Delta K_I)^{2.25}\rightarrow\Delta N=\frac{\Delta a}{0.66\times10^{-8}(\Delta K_I)^{2.25}}$$

$$\therefore\Delta N=\frac{0.1}{0.66\times10^{-8}(\Delta K_I)^{2.25}}$$

$$\therefore\Delta N=\frac{1.52\times10^{7}}{(\Delta K_I)^{2.25}}\quad(\text{II})$$

Having set up the necessary equations, we proceed to perform the numerical integration; the calculations are summarized below.

Initial a (in.)	Final a (in.)	Avg. a (in.)	K_I (ksi-in.$^{1/2}$) (Eq. I)	N (cycles) (Eq. II)	ΣN (cycles)
0.25	0.35	0.3	25.1	10793	10793
0.35	0.45	0.4	29.0	7809	18602
0.45	0.55	0.5	32.4	6075	24677
0.55	0.65	0.6	35.5	4949	29625
0.65	0.75	0.7	38.3	4161	33786
0.75	0.85	0.8	41.0	3580	37366
0.85	0.95	0.9	43.4	3136	40502
0.95	1.05	1	45.8	2785	43288
1.05	1.15	1.1	48.0	2502	45790
1.15	1.25	1.2	50.2	2269	48059
1.25	1.35	1.3	52.2	2074	50132
1.35	1.45	1.4	54.2	1908	52040
1.45	1.55	1.5	56.1	1765	53805
1.55	1.65	1.6	57.9	1642	55447
1.65	1.71	1.68	59.4	1554	**57001**

Accordingly, the number of cycles required for the crack to reach the critical length is about 57,000. One rough estimate of N can be made as follows. The average crack length is

$$\bar{a}=\frac{0.25+1.71}{2}=0.98\text{ in.}$$

so that, referring to equation (I),

$$\Delta K_I = 45.8 \times \sqrt{0.98} = 45.3 \text{ ksi}\sqrt{\text{in.}}$$

and

$$\frac{\Delta a}{\Delta N} = 0.66 \times 10^{-8} \times 45.3^{2.25} = 3.51 \times 10^{-5} \text{ in./cycle}$$

Lastly, given Δa = 1.71 − 0.25 = 1.46 in., we obtain

$$\Delta N = \frac{1.46}{3.51 \times 10^{-5}} = 41,600 \text{ cycles}$$

This rudimentary approximation underestimates the number of cycles to failure by 27 percent.

♦ The correct answer is **C**.

P.13 ■ Solution

Part 1: To begin, we compute the load range ΔP, namely,

$$\Delta P = P_{\text{max}} - P_{\text{min}} = 96.2 - 48.1 = 48.1 \text{ kN}$$

The stress range is then

$$\Delta\sigma = \frac{\Delta P}{2bt} = \frac{48,100}{2 \times 0.152 \times 0.00229} = 69.1 \text{ MPa}$$

Ratio da/dN is, for the first two data points,

$$\frac{da}{dN} = \frac{7.62 - 5.08}{9500 - 0} = 2.67 \times 10^{-4}$$

The average crack length is $a_{\text{avg}} = (a_1 + a_2)/2$. For the first two data points, we have a_{avg} = (5.08 + 7.62)/2 = 6.35 mm. We also require ratio $\alpha = a_{\text{avg}}/b$, which for the first two data points is such that a_{avg} = 6.35/152 = 0.0418. This factor is used to determine the dimensionless geometry function F, which is given by

$$F = \frac{1 - 0.5\alpha_{\text{avg}} + 0.326\alpha_{\text{avg}}^2}{\sqrt{1 - \alpha_{\text{avg}}}}$$

For the data points in question, we have

$$F = \frac{1 - 0.5 \times 0.0418 + 0.326 \times 0.0418^2}{\sqrt{1 - 0.0418}} = 1.00$$

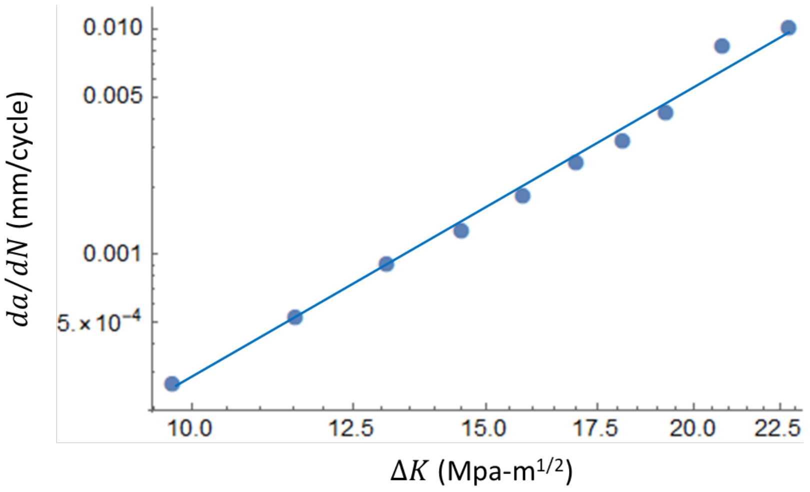
The stress intensity range follows as

$$\Delta K = F\Delta\sigma\sqrt{\pi a_{\text{avg}}} = 1.00 \times 69.1 \times \sqrt{\pi \times \left(6.35 \times 10^{-3}\right)} = 9.76 \text{ MPa}\sqrt{\text{m}}$$

The remaining calculations are summarized below.

a (mm)	N (cycles)	da/dN (mm/cycle)	a_{avg} (mm)	α_{avg}	F	ΔK (MPa-m$^{1/2}$)
5.08	0	-	-	-	-	-
7.62	9500	0.000267	6.35	0.042	1.0008	9.77
10.16	14300	0.000529	8.89	0.058	1.0016	11.57
12.7	17100	0.000907	11.43	0.075	1.0027	13.13
15.24	19100	0.001270	13.97	0.092	1.0041	14.53
17.78	20500	0.001814	16.51	0.109	1.0057	15.83
20.32	21500	0.002540	19.05	0.125	1.0077	17.03
22.86	22300	0.003175	21.59	0.142	1.0100	18.18
25.4	22900	0.004233	24.13	0.159	1.0127	19.27
30.48	23500	0.008467	27.94	0.184	1.0174	20.83
35.56	24000	0.010160	33.02	0.217	1.0249	22.81

The plot we are looking for is one of da/dN, the blue column, versus ΔK, the red column. Such a plot is shown below.



Clearly, the variables are related by a linear trend in the log-log-plane. To find the Paris equation coefficients, we can appeal to the *FindFit* command in Mathematica,

$$\text{DATA} = \{\{9.77,0.000267\},\{11.57,0.000529\},\{13.13,0.000907\},\{14.53,0.00127\},$$

$$\{15.83,0.001814\},\{17.03,0.00254\},\{18.18,0.003175\},\{19.27,0.004233\},$$

$$\{20.83,0.008467\},\{22.81,0.01016\}\}$$

$$\text{FindFit}[\text{DATA}, C * k^n, \{C,n\}, k]$$

This returns $C = 4.11 \times 10^{-9}$ and $n = 4.72$. Thus, the equation that describes crack growth in this aluminum plate is

$$\frac{da}{dN} = 4.11\times10^{-9}\times(\Delta K)^{4.72}$$

Part 2: The Walker equation models crack growth by an expression of the form

$$\frac{da}{dN} = C_0\left[\frac{\Delta K}{(1-R)^{(1-\gamma)}}\right]^m$$

Comparing this relation with the Paris equation,

$$\frac{da}{dN} = C(\Delta K)^m$$

we see that

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}}$$

We have $R = 0.5$, and all the other quantities in the right-hand side are given material properties, that is, $C_0 = 2.71\times10^{-8}$, $m = 3.70$, and $\gamma = 0.641$. Thus,

$$C = \frac{2.71\times10^{-8}}{(1-0.5)^{3.70\times(1-0.641)}} = 6.80\times10^{-8}$$

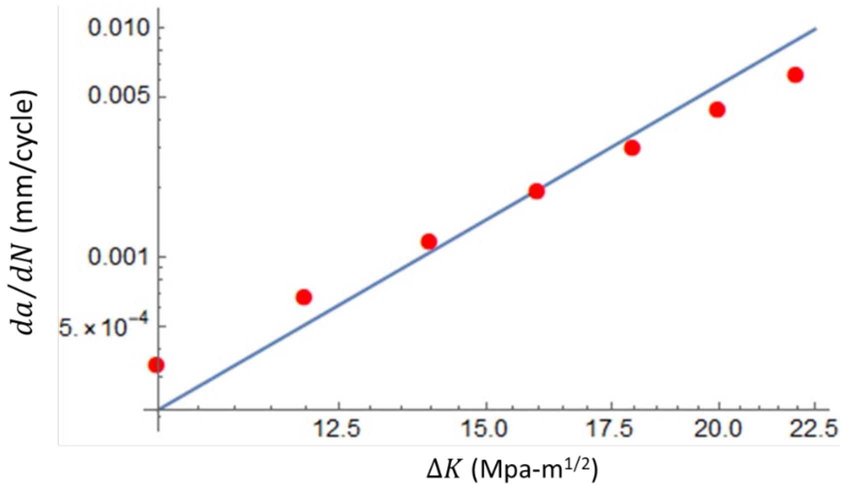
Hence, the line that corresponds to the Walker equation has the form

$$\frac{da}{dN} = C(\Delta K)^m \rightarrow \frac{da}{dN} = 6.80\times10^{-8}(\Delta K)^{3.70}$$

Some values of da/dN computed with this equation are listed below.

ΔK (MPa-m$^{1/2}$)	*da/dN* (mm/cycle)
8	0.00015
10	0.00034
12	0.00067
14	0.00118
16	0.00194
18	0.00300
20	0.00443
22	0.00630
24	0.00870

These values are then plotted along with the Paris equation obtained in the previous problem.



The line based on the Walker equation reasonably agrees with the Paris equation fit for intermediate values of ΔK, but the deviation becomes substantial at the lower and upper extremes.

P.14 ■ Solution

The stress ratio is $R = \sigma_{\min}/\sigma_{\max} = 18/60 = 0.3$. The maximum nominal stress is

$$\sigma_{\max} = \frac{P_{\max}}{2bt} = \frac{60{,}000}{2\times 0.05\times 0.004} = 150 \text{ MPa}$$

Let the dimension ratio $\alpha = 0.3$. The dimensionless geometry function follows as

$$F = \frac{1-0.5\alpha+0.326\alpha^2}{\sqrt{1-\alpha}} = \frac{1-0.5\times 0.3+0.326\times 0.3^2}{\sqrt{1-0.3}} = 1.05$$

The crack length for brittle fracture is determined as

$$a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{F\sigma_{\max}}\right)^2 = \frac{1}{\pi}\times\left(\frac{34}{1.05\times 150}\right)^2 = 14.8 \text{ mm}$$

The corresponding dimension ratio is

$$\alpha = \frac{a_c}{b} = \frac{14.8}{50} = 0.296$$

Since $a_c/b \approx 0.3$, there is no need for another iteration. In addition, $\alpha \le 0.4$ as it should be. The crack length for fully plastic yielding, in turn, is given by

$$a_Y = b\left(1-\frac{P_{\max}}{2bt\sigma_Y}\right) = 50\times\left[1-\frac{60{,}000}{2\times 0.05\times 0.004\times\left(353\times 10^6\right)}\right] = 28.8 \text{ mm}$$

The reference final crack length is the lesser of a_c and a_Y; thus, we take a_f = 14.8 mm. Constant C is calculated next,

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}} = \frac{1.42\times 10^{-11}}{(1-0.3)^{3.59\times(1-0.680)}} = 2.14\times 10^{-11} \frac{\text{m/cycle}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m}$$

The stress range is

$$\Delta\sigma = \sigma_{\max}(1-R) = 150\times(1-0.3) = 105 \text{ MPa}$$

We are now ready to evaluate the crack growth life,

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\Delta\sigma\sqrt{\pi}\right)^m(1-m/2)}$$

$$\therefore N_f = \frac{0.0148^{1-3.59/2} - 0.002^{1-3.59/2}}{\left(2.14\times 10^{-11}\right)\times\left(1.05\times 105\times\sqrt{\pi}\right)^{3.59}\times(1-3.59/2)} = \boxed{39{,}000 \text{ cycles}}$$

♦ The correct answer is **C**.

P.15 ■ Solution

From elementary fracture mechanics, the dimensionless geometry factor is F = 1.12. The stress ratio R = 1.2/3 = 0.4. The maximum nominal stress is

$$\sigma_{\max} = \frac{6M_{\max}}{b^2 t} = \frac{6 \times 3000}{0.06^2 \times 0.009} = 556 \text{ MPa}$$

The crack length for brittle fracture is determined as

$$a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{F\sigma_{\max}}\right)^2 = \frac{1}{\pi} \times \left(\frac{130}{1.12 \times 556}\right)^2 = 13.9 \text{ mm}$$

The corresponding dimension ratio is

$$\alpha = \frac{a_c}{b} = \frac{13.9}{60} = 0.232$$

Note that $\alpha \le 0.4$, as it should be; we can take F = 1.12. The crack length that corresponds to fully plastic yielding is, in turn,

$$a_Y = b\left(1 - \frac{2}{b}\sqrt{\frac{M_{\max}}{t\sigma_Y}}\right) = 60 \times \left(1 - \frac{2}{60}\sqrt{\frac{3000}{0.009 \times 1255}}\right) = 27.4 \text{ mm}$$

The reference final crack length is the lesser of a_c and a_Y; therefore, we take a_f = 13.9 mm. Constant C is calculated next,

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}} = \frac{5.11 \times 10^{-10}}{(1-0.4)^{3.24 \times (1-0.420)}} = 1.33 \times 10^{-9} \ \frac{\text{mm/cycle}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m}$$

The stress range is

$$\Delta\sigma = \sigma_{\max}(1-R) = 556 \times (1-0.4) = 334 \text{ MPa}$$

We are now ready to evaluate the crack growth life,

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\left(F\Delta\sigma\sqrt{\pi}\right)^m (1-m/2)}$$

$$\therefore N_f = \frac{0.0139^{1-3.24/2} - 0.0005^{1-3.24/2}}{\left[\left(1.33 \times 10^{-9}\right) \times 10^{-3}\right] \times \left(1.12 \times 556 \times \sqrt{\pi}\right)^{3.24} \times (1-3.24/2)} = \boxed{85{,}000 \text{ cycles}}$$

♦ The correct answer is **D.**

P.16 ■ Solution

Part 1: From elementary fracture mechanics, the dimensionless geometry factor is F = 1.12. The stress ratio is $R = M_{\min}/M_{\max}$ = −90/300 = −0.3. The maximum nominal stress is

$$\sigma_{\max} = \frac{6M_{\max}}{b^2 t} = \frac{6 \times 300}{0.04^2 \times 0.01} = 113 \text{ MPa}$$

The crack length for brittle fracture is determined to be

$$a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{F\sigma_{\max}}\right)^2 = \frac{1}{\pi} \times \left(\frac{29}{1.12 \times 113}\right)^2 = 16.7 \text{ mm}$$

The corresponding dimension ratio is

$$\alpha = \frac{a_c}{b} = \frac{16.7}{40} = 0.418$$

This dimension ratio is greater than 0.4, which is not acceptable. The crack length can be updated by trial-and-error. We shall assume different values of crack length, a_c, compute the dimension ratio α, the dimensionless geometry factor F, and the stress intensity factor K until the latter equals the plane-strain fracture toughness of 29 MPa-m$^{1/2}$. The equation to use for F is

$$F = \sqrt{\frac{1}{\beta}\tan\beta}\left[\frac{0.0923+0.199(1-\sin\beta)^4}{\cos\beta}\right]$$

where $\beta = \pi\,\alpha_c/2$. The calculations are summarized below.

a_c (mm)	α_c	β (deg)	β (rad)	F	K (MPa-m$^{1/2}$)
16.5	0.413	0.64793	0.01131	1.1132	28.639
16.6	0.415	0.65186	0.01138	1.1132	28.726
16.7	0.418	0.65579	0.01145	1.1131	28.810
16.8	0.420	0.65972	0.01151	1.1131	28.896
16.9	0.423	0.66364	0.01158	1.113	28.979
17	**0.425**	**0.66757**	**0.01165**	**1.113**	**29.065**
17.01	0.425	0.66796	0.01166	1.113	29.073
17.02	0.426	0.66835	0.01166	1.113	29.082

With reference to the table, we find that a_c = 17.0 mm, α = 0.425, F = 1.113, and of course K = 29.0 MPa-m$^{1/2}$. We proceed to compute the crack length that corresponds to fully plastic yielding,

$$a_Y = b\left(1-\frac{2}{b}\sqrt{\frac{M_{\max}}{t\sigma_Y}}\right) = 40\times\left(1-\frac{2}{40}\sqrt{\frac{300}{0.01\times 523}}\right) = 24.9 \text{ mm}$$

The reference final crack length is the lesser of a_c and a_Y; hence, we take a_f = 17.0 mm. Constant C is calculated next,

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}} = \frac{2.71\times10^{-8}}{\left[1-(-0.3)\right]^{3.70\times(1-0)}} = 1.03\times10^{-8}\ \frac{\text{mm/cycle}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m}$$

The stress range is

$$\Delta\sigma = \sigma_{\max}(1-R) = 113\times\left[1-(-0.3)\right] = 147 \text{ MPa}$$

It remains to evaluate the crack growth life,

$$N_f = \frac{a_f^{1-m/2}-a_i^{1-m/2}}{C\left(F\Delta\sigma\sqrt{\pi}\right)^m(1-m/2)}$$

$$\therefore N_f = \frac{0.017^{1-3.70/2}-0.00025^{1-3.70/2}}{\left[\left(1.03\times10^{-8}\right)\times10^{-3}\right]\times\left(1.113\times147\times\sqrt{\pi}\right)^{3.70}\times(1-3.70/2)} = \boxed{99{,}200 \text{ cycles}}$$

♦ The correct answer is **D**.

Part 2: The factor of safety in life with no periodic inspections is

$$FS_N = \frac{N_f}{\hat{N}} = \frac{99{,}200}{200{,}000} = 0.496$$

Since $FS_N <$ 1.0, periodic inspection is necessary. For a factor of safety in life of 3.0, the inspection interval is

$$FS_N = \frac{N_f}{\hat{N}} \rightarrow \hat{N} = \frac{N_f}{FS_N}$$

$$\therefore \hat{N} = \frac{99{,}200}{3} = 33{,}100 \text{ cycles}$$

ANSWER SUMMARY

Problem 1		C
Problem 2		D
Problem 3	3.1	Open-ended pb.
	3.2	C
	3.3	B
Problem 4		B
Problem 5		C
Problem 6		A
Problem 7		A
Problem 8		B
Problem 9		C
Problem 10	10.1	B
	10.2	D
Problem 11		D
Problem 12		C
Problem 13	13.1	Open-ended pb.
	13.2	Open-ended pb.
Problem 14		C
Problem 15		D
Problem 16	16.1	D
	16.2	Open-ended pb.

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