Montogue

Quiz FM109

Flow Over Immersed Surfaces

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PROBLEMS

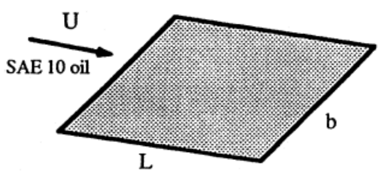
Problem 1 (Munson et al., 2009, w/ permission)

In two different experiments, air and water flow over an object that has the same shape in both experiments. It is found that the drag coefficient on each of the objects is the same for both the air and water flow. For the same velocity,

A) the drag force in the water flow is about equal to that for the airflow.
B) the drag force in the water flow is about 10 times that for the airflow.
C) the drag force in the water flow is about 100 times that for the airflow.
D) the drag force in the water flow is about 1000 times that for the airflow.

Problem 2 (White, 2003, w/ permission)

A thin flat plate 55 by 110 cm is immersed in a 6-m/s stream of SAE 10 oil at 20°C. Compute the total friction drag if the stream is parallel to the long side of the plate. Use ρ_{oil} = 891 kg/m³ and μ = 0.29 kg/m·s.



A) F = 181 N
B) F = 218 N
C) F = 250 N
D) F = 283 N

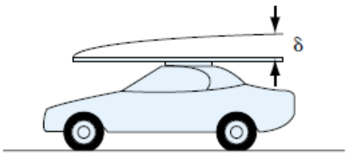
Problem 3 (White, 2003, w/ permission)

A smooth flat plate of length ℓ = 6 m and width b = 4 m is placed in water with an upstream velocity of U = 0.5 m/s. Determine the boundary layer thickness and the wall shear stress at the center of the plate. Use ρ_w = 1000 kg/m³ and μ = 1.12×10⁻³ kg/m·s.

A) δ = 1.3 cm and τ_w = 0.0716 N/m²
B) δ = 1.3 cm and τ_w = 0.1012 N/m²
C) δ = 2.1 cm and τ_w = 0.0716 N/m²
D) δ = 2.1 cm and τ_w = 0.1012 N/m²

Problem 4 (White, 2003, w/ permission)

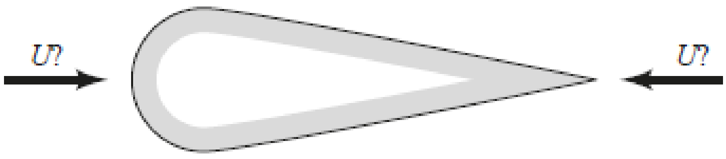
Suppose you buy a 2.44-m-long sheet of plywood and put it on your roof rack. You drive home at 67 mi/h. Assuming the board is perfectly aligned with the airflow, how thick is the boundary layer at the end of the board? Also, estimate the drag on the sheet of plywood if the boundary layer remains turbulent. Use ρ = 1.23 kg/m³ and μ = 1.8×10⁻⁵ kg/m·s.



A) δ = 18.6 mm and F_D = 11.3 N
B) δ = 18.6 mm and F_D = 22.5 N
C) δ = 43.1 mm and F_D = 11.3 N
D) δ = 43.1 mm and F_D = 22.5 N

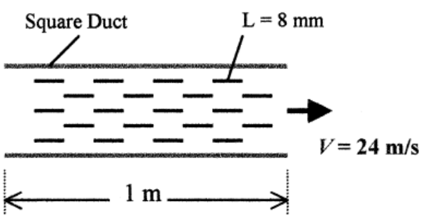
Problem 5 (Munson et al., 2009, w/ permission)

It is often assumed that "sharp objects can cut through the air better than blunt ones." Based on this assumption, the drag on the object shown below should be less when the wind blows from right to left than when it blows from left to right. Experiments show that the opposite is true. Explain.



Problem 6 (White, 2003, w/ permission)

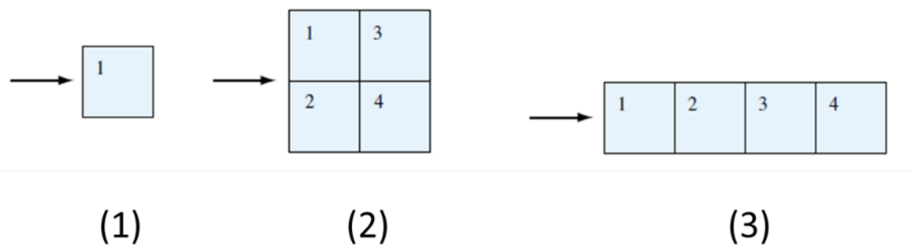
Consider the smooth square 10 cm by 10 cm duct illustrated below. The fluid is air at 20°C and 1 atm, flowing at V = 24 m/s. It is desired to increase the pressure drop over the 1-m length by adding sharp 8-mm long flat plates across the duct as shown. Estimate how many plates are needed to create a 100 Pa pressure drop. Use ρ = 1.23 kg/m³ and ν = 1.5×10⁻⁵ m²/s.



A) N_{plates} = 123
B) N_{plates} = 151
C) N_{plates} = 180
D) N_{plates} = 209

Problem 7 (White, 2003, w/ permission)

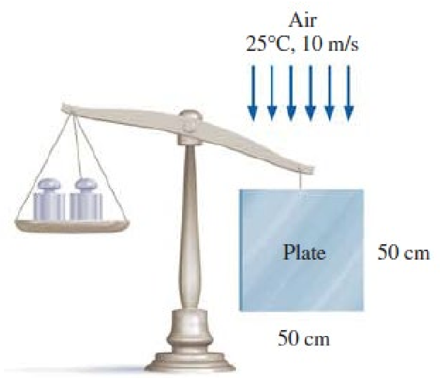
Consider laminar flow past the square-plate arrangements shown in the figure below. Comparing to the drag D_1 of a single plate, how much larger is the drag of the four plates D_2 and D_3 for the other two arrangements?



A) $D_2 = 1.73D_1$ and $D_3 = 2D_1$
B) $D_2 = 1.73D_1$ and $D_3 = 4D_1$
C) $D_2 = 2.83D_1$ and $D_3 = 2D_1$
D) $D_2 = 2.83D_1$ and $D_3 = 4D_1$

Problem 8 (Çengel & Cimbala, 2014, w/ permission)

The weight of a thin flat plate 50 cm × 50 cm in size is balanced by a counterweight that has a mass of 2 kg, as shown. Now a fan is turned on, and air at 1 atm and 25°C flows downward over both surfaces of the plate (front and back in the sketch) with a free-stream velocity of 10 m/s. Determine the mass of the counterweight that needs to be added in order to balance the plate. Use $\nu = 1.56\times10^{-5}$ m²/s.

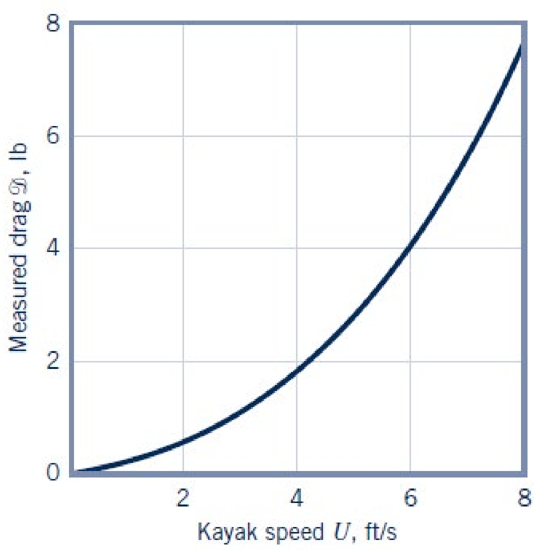


A) m = 7 g
B) m = 12 g
C) m = 17 g
D) m = 22 g

Problem 9 (Munson et al., 2009, w/ permission)

A kayak is a relatively streamlined object. As a first approximation in calculating the drag on a kayak, assume that the kayak acts as if it were a smooth, flat plate 17 ft long and 2 ft wide. Determine the drag as a function of speed and compare your results with the measured values given in the figure below. Comment on the reasons why the two sets of values may differ. Use ρ = 1.94 slugs/m³.

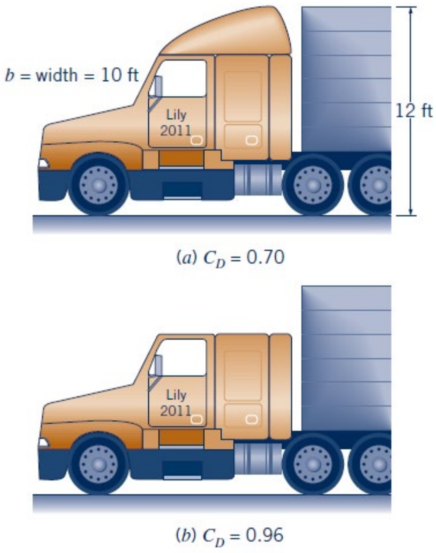
(*a*)



(*b*)

Problem 10 (Munson et al., 2009, w/ permission)

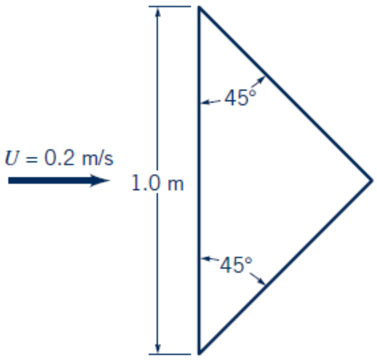
As shown in the figure below, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from $C_D = 0.96$ to $C_D = 0.70$ corresponds to a reduction in how many horsepower needed at a highway speed of 65 mph? Use $\rho_{air} = 2.38\times10^{-3}$ slugs/ft³.



A) ΔP = 10.4 hp
B) ΔP = 25.2 hp
C) ΔP = 43.1 hp
D) ΔP = 58.4 hp

Problem 11 (Munson et al., 2009, w/ permission)

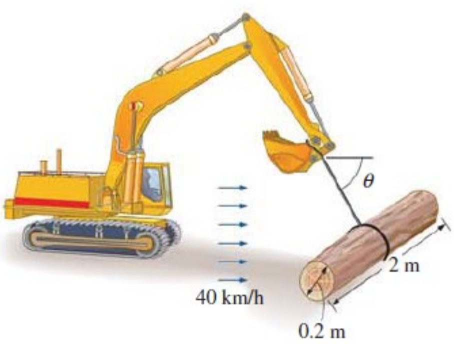
Water flows past a triangular flat plate oriented parallel to the free stream as shown. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow. Use ρ = 999 kg/m³ and $\mu = 1.12\times10^{-3}$ kg/m·s.



A) D = 0.0188 N
B) D = 0.0296 N
C) D = 0.0377 N
D) D = 0.0495 N

Problem 12 (Çengel & Cimbala, 2014, w/ permission)

A 2-m long, 0.2-m diameter pine log (density = 513 kg/m³) is suspended by a crane in the horizontal position. The log is subjected to normal winds at 40 km/h at 5°C and 88 kPa. Disregarding the weight of the cable and its drag, determine the angle θ the cable will make with the horizontal. Use ρ = 1.1 kg/m³ and $\mu = 1.75\times10^{-5}$ kg/m·s.



A) θ = 74°
B) θ = 79°
C) θ = 84°
D) θ = 89°

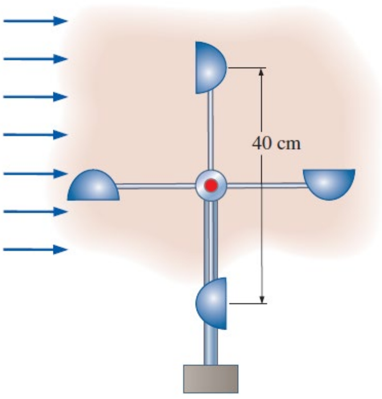
Problem 13 (Munson et al., 2009, w/ permission)

A hot-air balloon roughly spherical in shape has a volume of 70,000 ft^3 and a weight of 500 lb (including passengers, basket, balloon fabric, etc.). The outside temperature is 80°F and the temperature within the balloon is 165°F. Estimate the rate at which the balloon will rise under steady-state conditions if the atmospheric pressure is 14.7 psi. Use ρ_{air} = 2.286×10^{-3} slugs/ft^3 and R = 53.3 ft-lbf/lb·R.

A) U = 12.2 ft/s
B) U = 16.1 ft/s
C) U = 20.4 ft/s
D) U = 24.3 ft/s

Problem 14 (Çengel & Cimbala, 2014, w/ permission)

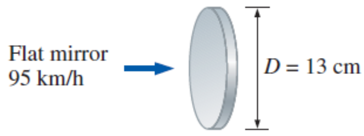
A wind turbine with two or four hollow hemispherical cups connected to a pivot is commonly used to measure wind speed. Consider a wind turbine with four 8-cm diameter cups with a center-to-center distance of 40 cm, as shown in the figure below. The pivot is stuck as a result of some malfunction, and the cups stop rotating. For a wind speed of 15 m/s and air density of 1.25 kg/m^3, determine the maximum torque this turbine applies on the pivot.



A) M_{max} = 0.087 N·m
B) M_{max} = 0.113 N·m
C) M_{max} = 0.145 N·m
D) M_{max} = 0.178 N·m

Problem 15A (Çengel & Cimbala, 2014, w/ permission)

To reduce the drag coefficient and thus to improve the fuel efficiency of cars, the design of side rearview mirrors has changed drastically in recent decades from a simple circular plate to a streamlined shape. Determine the amount money lost per year as a result of withstanding the drag due to a 13-cm-diameter flat mirror, as shown in the figure. Assume the car is driven 24,000 km a year at an average speed of 95 km/h. Take the density and price of gasoline to be 0.75 kg/L and $0.90/L, respectively; the heating value of gasoline to be 44,000 kJ/kg; and the overall efficiency of the engine to be 30 percent. Use ρ_{air} = 1.20 kg/m³.



A) Cost = $1.21/year

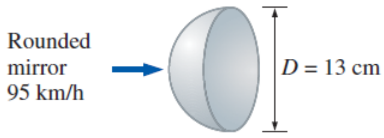
B) Cost = $6.88/year

C) Cost = $13.33/year

D) Cost = $18.46/year

• Problem 15B

Determine the amount of money saved per year as a result of replacing a 13-cm-diameter flat mirror by one with a hemispherical back, as shown in the figure.



A) Money saved = $2.43/year

B) Money saved = $4.67/year

C) Money saved = $6.75/year

D) Money saved = $8.48/year

ADDITIONAL INFORMATION

Figure 1 Friction drag coefficient for a flat plate parallel to the upstream flow.

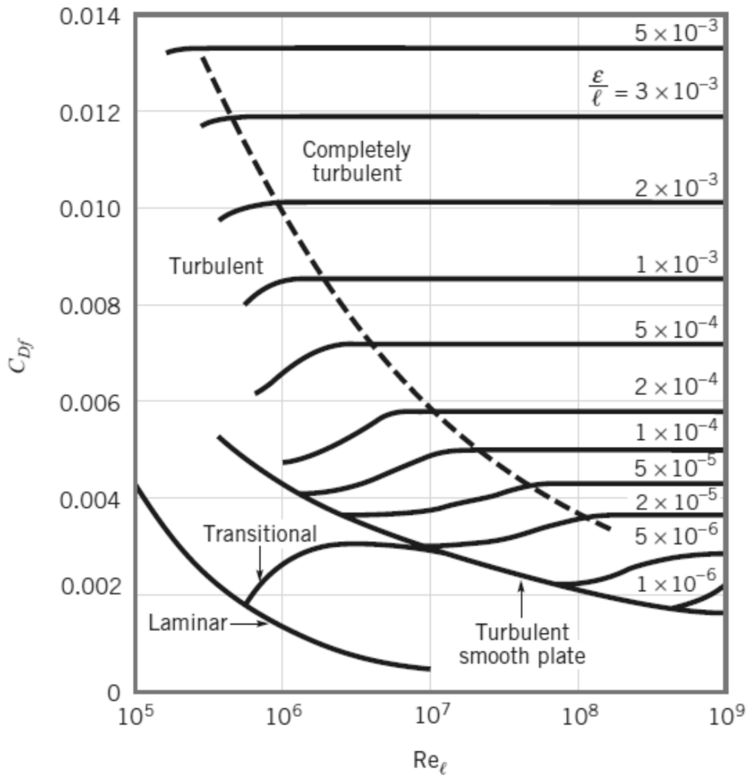


Figure 2 Drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere.

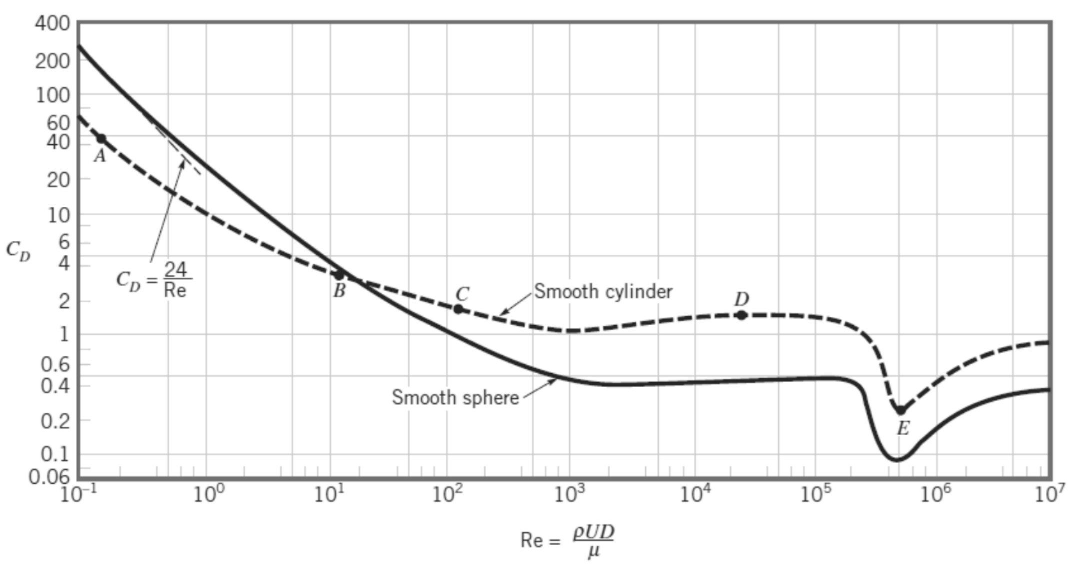
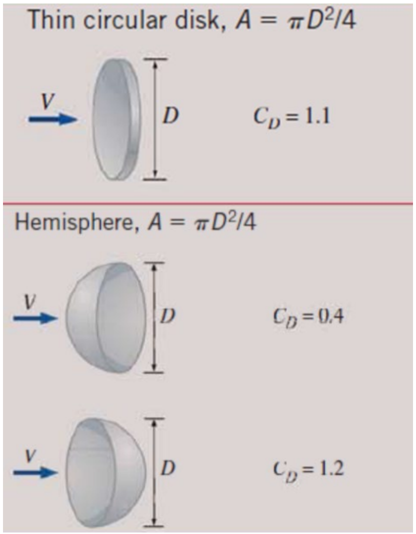


Figure 3 Drag coefficient for specific shapes.



SOLUTIONS

P.1 ● Solution

Recall that the drag force exerted on a surface immersed in a fluid is given by

$$F = \frac{1}{2}\rho V^2 A \times C_D$$

where ρ is the density of the fluid, V is the velocity of flow, A is the cross-sectional area, and C_D is the drag coefficient. Considering flow across the same body with the same velocity, we must have, in each case,

$$F_{\text{air}} = \frac{1}{2}\rho_{\text{air}} V^2 A \times C_{D,\text{air}} \rightarrow C_{D,\text{air}} = \frac{2F_{\text{air}}}{\rho_{\text{air}} V^2 A}$$

$$F_{\text{water}} = \frac{1}{2}\rho_{\text{water}} V^2 A \times C_{D,\text{water}} \rightarrow C_{D,\text{water}} = \frac{2F_{\text{water}}}{\rho_{\text{water}} V^2 A}$$

In order for the drag coefficient to be the same in both the air and water flow, we must have

$$C_{D,\text{air}} = C_{D,\text{water}} \rightarrow \frac{\cancel{2}F_{\text{air}}}{\rho_{\text{air}} \cancel{V^2 A}} = \frac{\cancel{2}F_{\text{water}}}{\rho_{\text{water}} \cancel{V^2 A}}$$

$$\therefore \frac{F_{\text{air}}}{\rho_{\text{air}}} = \frac{F_{\text{water}}}{\rho_{\text{water}}}$$

$$\therefore F_{\text{water}} = \frac{\rho_{\text{water}}}{\rho_{\text{air}}} F_{\text{air}}$$

Since $\rho_{\text{water}} \approx 1000$ kg/m³ and $\rho_{\text{air}} \approx 1.2$ kg/m³, we obtain

$$F_{\text{water}} = \frac{\rho_{\text{water}}}{\rho_{\text{air}}} F_{\text{air}} \rightarrow F_{\text{water}} = \frac{1000}{1.2} \times F_{\text{air}} = 833.3 F_{\text{air}}$$

That is, the drag force exerted by the water flow must be over 800 times, or nearly a thousand times, greater than the force of the air flow.

➡ The correct ans1wer is **D**.

P.2 ● Solution

The characteristic length of the flat plate is L = 110 cm. The Reynolds number for the flow in question is then

$$\text{Re} = \frac{\rho U L}{\mu} = \frac{891 \times 6 \times 1.1}{0.29} = 20,300$$

We conclude that the flow is laminar. The equation used to compute the drag coefficient is, in this case,

$$C_D = 1.328\,\mathrm{Re}^{-1/2} = \frac{1.328}{(20{,}300)^{0.5}} = 0.00932$$

Substituting this quantity and other pertaining variables in the equation for the friction drag, we obtain

$$F = C_D \times \frac{\rho U^2}{2} \times 2bL = 0.00932 \times \frac{891 \times 6^2}{2} \times 2 \times 0.55 \times 1.1 = \boxed{181\text{ N}}$$

➡ The correct answer is **A**.

P.3 ● Solution

The boundary layer thickness for laminar flow can be obtained with the equation

$$\delta = 5\sqrt{\frac{\nu x}{U}}$$

Substituting the appropriate variables, we express the boundary layer thickness as a function of the horizontal coordinate x,

$$\delta(x) = 5\sqrt{\frac{1.12 \times 10^{-6} \times x}{0.5}} = 0.00748x^{0.5}$$

Substituting x = 3 m gives

$$\delta(3) = 0.00748 \times 3^{0.5} = 0.013\,\text{m} = \boxed{1.3\text{ cm}}$$

The wall shear stress for laminar flow can be obtained with the equation

$$\tau_w = 0.332U^{3/2}\sqrt{\frac{\rho\mu}{x}}$$

Substituting the necessary variables, we can express the wall shear as a function of the horizontal coordinate x,

$$\tau_w(x) = 0.332 \times 0.5^{3/2} \times \sqrt{\frac{1000 \times 1.12 \times 10^{-3}}{x}} = \frac{0.124}{x^{0.5}}$$

Then, with x = 3 m, we find that

$$\tau_w(3) = \frac{0.124}{\sqrt{3}} = \boxed{0.0716\text{ N/m}^2}$$

➡ The correct answer is **A**.

P.4 ● Solution

We begin by converting U = 67 mi/h = 30 m/s. The Reynolds number is

$$\mathrm{Re} = \frac{\rho UL}{\mu} = \frac{1.23 \times 30 \times 2.44}{1.8 \times 10^{-5}} = 5 \times 10^6$$

Assuming turbulent flow, the thickness of the boundary layer is

$$\delta = \frac{0.16L}{\mathrm{Re}^{1/7}} = \frac{0.16 \times 2.44}{(5 \times 10^6)^{1/7}} = 0.0431\text{ m} = \boxed{43.1\text{ mm}}$$

It remains to evaluate the boundary layer drag for turbulent flow. This can be obtained with the expression

$$F_D = C_D \frac{\rho U^2}{2} A = \left[\frac{0.031}{(5 \times 10^6)^{1/7}}\right] \times \frac{1.23 \times 30^2}{2} \times (2.44 \times 1.22 \times 2) = \boxed{11.3\text{ N}}$$

➡ The correct answer is **C**.

P.5 ● Solution

A significant portion of the drag on an object can be from the relatively low pressure developed in the wake region behind the object. By making the object streamlined (i.e., flow from left to right, not right to left in the foregoing figure), boundary layer separation is avoided and a relatively thin wake with low drag is obtained. Whether the front of the object is "sharp" or "blunt" does not affect the contribution to drag from the front part of the body – at least not as much as the width of the wake affects the drag.

P.6 ● Solution

To estimate the plate-induced pressure drag, we first calculate the drag coefficient on one plate,

$$\mathrm{Re} = \frac{24\times 0.008}{1.5\times 10^{-5}} = 12{,}800 \rightarrow C_D = \frac{1.328}{\sqrt{12{,}800}} = 0.0117$$

The corresponding drag force is

$$F_D = C_D \times \frac{\rho V^2}{2}\times 2bL = 0.0117\times\frac{1.23\times 24^2}{2}\times 2\times 0.1\times 0.008 = 0.00663\,\mathrm{N}$$

Since the duct walls must support these plates, the effect is an additional pressure drop,

$$\Delta p_{\text{extra}} = 100 = \frac{FN_{\text{plates}}}{A_{\text{duct}}} \rightarrow N_{\text{plates}} = \frac{100\times 0.1^2}{0.00663} = \boxed{151}$$

That is to say, more than 150 plates are needed to create a pressure drop of 100 pascals.

➡ The correct answer is **B**.

P.7 ● Solution

The formula for drag coefficient in laminar flow is $C_D = 1.328/\mathrm{Re}^{1/2}$, whence we have the proportion $C_D \propto L^{-1/2}$. Thus, given an arbitrary constant k, it follows that

$$D_2 = \frac{k}{\sqrt{2L_1}}\times 4A_1 = \frac{4}{\sqrt{2}}\frac{kA_1}{\sqrt{L_1}} = 2\sqrt{2}\underbrace{\frac{kA_1}{\sqrt{L_1}}}_{=D_1} = \boxed{2.83D_1}$$

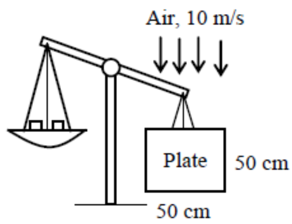
$$D_3 = \frac{k}{\sqrt{4L_1}}\times 4A_1 = 2\frac{kA_1}{\sqrt{L_1}} = \boxed{2D_1}$$

➡ The correct answer is **C**.

P.8 ● Solution

The Reynolds number for this flow field is

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{10\times 0.5}{1.56\times 10^{-5}} = 3.21\times 10^5$$



The average drag coefficient is

$$C_D = \frac{1.328}{\mathrm{Re}^{0.5}} = \frac{1.328}{\left(3.21\times 10^5\right)^{0.5}} = 0.002344$$

The drag force exerted on the plate is

$$F_D = C_D\times\frac{\rho V^2}{2}\times A = 0.00234\times\frac{1.18\times 10^2}{2}\times\left(2\times 0.5\times 0.5\right) = 0.069\ \mathrm{N}$$

The mass whose weight is 0.069 N is

$$m = \frac{F_D}{g} = \frac{0.069}{9.81} = 0.007\text{ kg} = \boxed{7\text{ g}}$$

The mass of the counterweight must be 7 grams to counteract the drag force acting on the plate.

➡ The correct answer is **A**.

P.9 ● Solution

The area of the flat plate that represents the kayak is A = 17×2 = 34 ft^2. Given the length L = 17 ft and the kinematic viscosity ν = 1.21×10^{-5} ft^2/s, the Reynolds number is determined as

$$\text{Re} = \frac{UL}{\nu} = \frac{17\times U}{\left(1.21\times10^{-5}\right)} = 1.405\times10^{6}U \quad \text{(I)}$$

In the range of interest, the speed varies from U = 1 ft/s to U = 8 ft/s, and the Reynolds number accordingly varies from Re = 1.405×10^6 to Re = 1.124×10^7. From Figure 1, we verify that this range of Reynolds numbers is in the transitional range. Consequently, the drag coefficient can be determined with the relation

$$C_{D,f} = \frac{0.455}{(\log \text{Re})^{2.58}} - \frac{1700}{\text{Re}}$$

or, replacing Re with equation (I),

$$C_{D,f} = \frac{0.455}{\left[\log\left(1.405\times10^{6}U\right)\right]^{2.58}} - \frac{1700}{1.405\times10^{6}U} \quad \text{(II)}$$

Since the drag force is given by

$$D = \frac{1}{2}\rho U^2 A \times C_{D,f}$$

we have, using equation (II) and substituting the adequate variables,

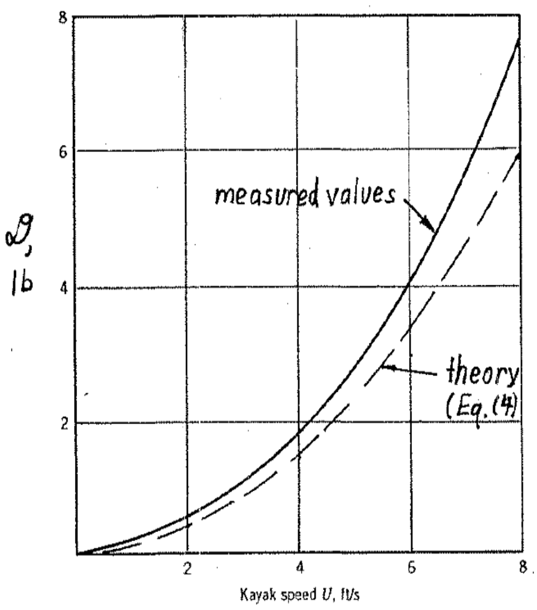
$$D = \frac{1}{2}\times1.94\times U^2\times34\times\left\{\frac{0.455}{\left[\log\left(1.405\times10^{6}U\right)\right]^{2.58}} - \frac{1700}{1.405\times10^{6}U}\right\}$$

$$\therefore D = 32.98\left\{\frac{0.455}{\left[\log\left(1.405\times10^{6}U\right)\right]^{2.58}} - \frac{1700}{1.405\times10^{6}U}\right\}$$

This expression provides the drag force as a function of the flow velocity U. The following table provides the drag force as a function of speed in the range of interest, $U \in [1, 8]$ ft/s.

U (ft/s)	D (lb)
1	0.0986
2	0.41
3	0.909
4	1.58
5	2.42
6	3.43
7	4.59
8	5.9

These values are then plotted along with the experimental data, as shown.



The actual kayak does not act as a smooth flat plate and friction plays a part. As a result, there is a mild shift from the obtained values to the measured values.

P.10 ● Solution

The velocity of the truck in ft/s is U = 65 × 5280/3600 = 95.33 ft/s. We compute the drag force D_1 acting on the truck without the air deflector,

$$D_1 = C_{D,1} \times \frac{1}{2}\rho U^2 A = 0.70 \times \frac{1}{2} \times \left(2.38 \times 10^{-3}\right) \times 95.33^2 \times (12 \times 10) = 908.4\text{ lb}$$

The horsepower at the initial state is

$$P_1 = D_1 U = 908.4 \times 95.33 \times \frac{1\text{ hp}}{550\text{ lb}\cdot\text{ft/s}} = 157.5\text{ hp}$$

Then, we calculate the drag force acting on the truck at the final state, D_2,

$$D_2 = C_{D,2} \times \frac{1}{2}\rho U^2 A = 0.96 \times \frac{1}{2} \times \left(2.38 \times 10^{-3}\right) \times 95.33^2 \times (12 \times 10) = 1245.8\text{ lb}$$

The corresponding horse power at the final state is

$$P_2 = D_2 U = 1245.8 \times 95.33 \times \frac{1\text{ hp}}{550\text{ lb}\cdot\text{ft/s}} = 215.9\text{ hp}$$

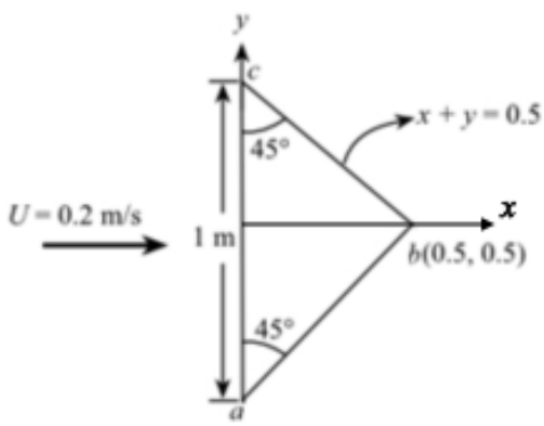
Accordingly, the reduction in horse power is

$$\Delta P = P_2 - P_1 = 215.9 - 157.5 = \boxed{58.4\text{ hp}}$$

➡ The correct answer is **D**.

P.11 ● Solution

Consider the following diagram.



To obtain the equation of line bc, we write

$$\frac{y-y_b}{y_c-y_b}=\frac{x-x_b}{x_c-x_b}$$

$$\therefore \frac{y-0}{0.5-0}=\frac{x-0.5}{0-0.5}$$

$$\therefore \frac{y}{0.5}=\frac{x-0.5}{-0.5}$$

$$\therefore -y=x-0.5$$

$$\therefore x+y=0.5$$

The friction drag can be obtained with the integral

$$D=\int \tau_w dA=\int 0.332U^{\frac{3}{2}}\sqrt{\frac{\rho\mu}{x}}dA$$

$$\therefore D=0.332U^{\frac{3}{2}}\sqrt{\rho\mu}\times 2\underbrace{\int_0^{x=0.5}\int_0^{y=0.5-x}\frac{1}{\sqrt{x}}dydx}_{=0.471}$$

$$\therefore D=0.332U^{\frac{3}{2}}\sqrt{\rho\mu}\times 2\times 0.471$$

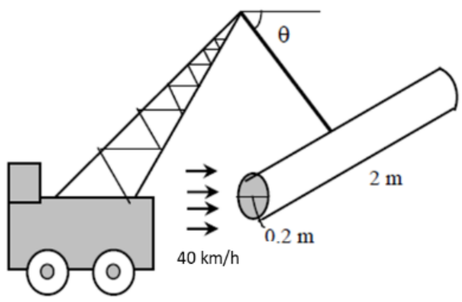
$$\therefore D=0.313U^{\frac{3}{2}}\sqrt{\rho\mu}=0.313\times 0.2^{\frac{3}{2}}\times\sqrt{999\times 1.12\times 10^{-3}}=\boxed{0.0296\text{ N}}$$

➡ The correct answer is **B**.

P.12 ● Solution

The solution is started by calculating the Reynolds number,

$$\text{Re}=\frac{\rho VD}{\mu}=\frac{1.1\times\left(40/3.6\right)\times 0.2}{1.75\times 10^{-5}}=139{,}700$$



Resorting to Figure 2, the drag coefficient that corresponds to this value of Re is $C_D = 1.2$. Noting that the frontal area for flow past the cylinder is $A = LD$, the drag force F_D imparted on the log is calculated as

$$F_D=C_D\times\frac{\rho V^2}{2}\times A=1.2\times\frac{1.1\times 11.11^2}{2}\times(2\times 0.2)=32.6\text{ N}$$

The weight of the log is

$$W=mg=\rho g\frac{\pi D^2L}{4}=513\times 9.81\times\frac{\pi\times 0.2^2\times 2}{4}=316.2\text{ N}$$

The angle that the resultant force (and, as a consequence, the cable) makes with the horizontal follows as

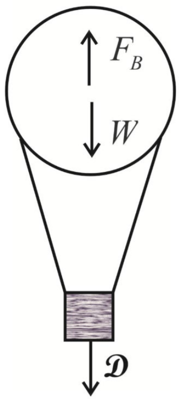
$$\tan\theta=\frac{W}{F_D}=\frac{316.2}{32.6}=9.699\rightarrow\boxed{\theta=84^{\circ}}$$

Drawing a free body diagram of the log and doing a force balance will show that the magnitude of the tension on the cable must be equal to the resultant force acting on the log. Therefore, the cable makes 84° with the horizontal.

➡ The correct answer is **C**.

P.13 ● Solution

The free body diagram for the balloon is shown in continuation.



The force due to buoyancy is

$$F_B = \rho_{\text{air}} g V = \left(2.286\times10^{-3}\right)\times32.2\times70{,}000 = 5152.6\text{ lb}$$

The density of the balloon can be obtained from the ideal gas law,

$$\rho_b = \frac{p}{RT} = \frac{\left(14.7\ \dfrac{\text{pound}}{\text{in.}^2}\times144\ \dfrac{\text{in.}^2}{\text{ft}^2}\right)}{53.3\text{ ft-lbf/lb}\cdot\text{R}\times\left(165+459.67\text{ R}\right)} = 0.0636\text{ lbm/ft}^3$$

$$\therefore \rho_b = 0.0636\ \frac{\text{lbm}}{\text{ft}^3}\times\frac{1\text{ slug}}{32.2\text{ lbm}} = 1.975\times10^{-3}\text{ slug/ft}^3$$

The corresponding specific weight is

$$\gamma_b = \rho_b g = \left(1.975\times10^{-3}\right)\times32.2 = 0.0636\text{ lb/ft}^3$$

The total weight of the balloon and its accompanying load is

$$W = 500 + \gamma_b V = 500 + 0.0636\times70{,}000 = 4952\text{ lb}$$

The radius of the balloon (considered a sphere) is

$$\frac{4}{3}\pi R^3 = 70{,}000 \rightarrow R = \left(\frac{3\times70{,}000}{4\pi}\right)^{\frac{1}{3}} = 25.57\text{ ft}$$

Now, the drag force is given by

$$D = \frac{1}{2}\rho U^2 C_D A$$

$$\therefore D = \frac{1}{2}\times\left(2.286\times10^{-3}\right)\times\left(\pi\times25.57^2\right)U^2 C_D$$

$$\therefore D = 2.348U^2 C_D$$

Applying an equilibrium of forces in the y-direction gives

$$\Sigma F_y = 0 \rightarrow F_B - W - D = 0$$

$$\therefore F_B = W + D$$

Substituting F_B = 5152.6 lb, W = 4952 lb and $D = 2.348U^2C_D$, we obtain

$$5156.6 = 4952 + 2.348U^2 C_D$$

$$\therefore 2.348U^2 C_D = 200.6$$

$$\therefore U^2 C_D = 85.43$$

$$\therefore U = \sqrt{\frac{85.43}{C_D}}\quad\text{(I)}$$

Next, we compute the Reynolds number,

$$\mathrm{Re} = \frac{UD}{\nu} = \frac{U\times 2R}{\nu} = \frac{U\times 2\times 25.57}{1.69\times 10^{-4}} = 3.026\times 10^{5} U \text{ (II)}$$

The velocity of the balloon can be computed by trial-and-error in equations (I) and (II), along with the data available in Figure 2. Assume the value of C_D is 0.5. Substituting in equation (I) yields

$$U = \sqrt{\frac{85.43}{(0.5)}} = 13.1\,\text{ft/s}$$

Substituting in equation (II), then, it follows that

$$\mathrm{Re} = 3.026\times 10^{5}\times(13.1) = 39.6\times 10^{5} = 3.96\times 10^{6}$$

Using Figure 2, we see that $C_D \approx 0.21$ when $\mathrm{Re} = 3.96\times 10^6$. However, the assumed value of C_D was 0.5; the values do not match, and another trial is in order. In a second trial, suppose that $C_D = 0.21$. Substituting in the equation for U gives

$$U = \sqrt{\frac{85.43}{(0.21)}} = 20.2\,\text{ft/s}$$

Substituting this quantity in the relation for *Re*, we obtain

$$\mathrm{Re} = 3.026\times 10^{5}\times(20.2) = 6.11\times 10^{6}$$

Using Figure 2 as before, we obtain $C_D \approx 0.33$. Again, the assumed value of C_D and the actual value do not match. In a third trial, suppose that $C_D = 0.33$. Substituting in equation (I) gives

$$U = \sqrt{\frac{85.43}{(0.33)}} = 16.1\,\text{ft/s}$$

Substituting this quantity into the relation for *Re*, we obtain

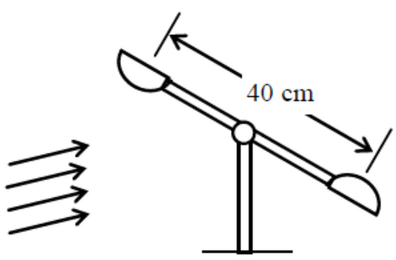
$$\mathrm{Re} = 3.026\times 10^{5}\times(16.1) = 48.72\times 10^{5} = 4.87\times 10^{6}$$

Using the same chart as before, we see that the drag coefficient is $C_D =$ 0.31. This is close enough to the assumed value of 0.33. We conclude that the velocity of the balloon is 16.1 ft/s.

➡ The correct answer is **B**.

P.14 ● Solution

The drag coefficient for a hemispherical cup is 0.4 and 1.2 when the hemispherical and plain surfaces are exposed to wind flow, respectively (see Figure 3).



The maximum torque occurs when the cups are normal to the wind since the length of the moment arm is maximum in this case. Noting that the frontal area is $\pi D^2/4$ for both cups, the drag force acting on each cup is determined to be

$$\text{Convex side: } F_{D,1} = C_{D,1}\times\frac{\rho V^2}{2}\times A = 0.4\times\frac{1.25\times 15^2}{2}\times\frac{\pi(0.08)^2}{4} = 0.283\,\text{N}$$

$$\text{Concave side: } F_{D,2} = C_{D,2}\times\frac{\rho V^2}{2}\times A = 1.2\times\frac{1.25\times 15^2}{2}\times\frac{\pi(0.08)^2}{4} = 0.848\,\text{N}$$

The moment arm for both forces is 20 cm. Taking moments about the pivot, the net torque applied on the pivot is calculated as

$$M_{max} = F_{D,2}L - F_{D,1}L = \left(F_{D,2} - F_{D,1}\right)L = (0.848 - 0.283) \times 0.20 = \boxed{0.113 \text{ N} \cdot \text{m}}$$

The torque varies from zero when both cups are aligned with the wind to the maximum value calculated above.

➡ The correct answer is **B**.

P.15 ● Solution

Part A: The drag coefficients are 1.1 for a circular disk and 0.40 for a hemispherical body (Figure 3). The velocity of the vehicle expressed in m/s is U = 95/3.6 = 26.4 m/s. The drag force acting on the flat mirror is

$$F_{D,\text{flat}} = C_{D,\text{flat}} \frac{\rho U^2}{2} A = 1.1 \times \frac{1.20 \times 26.4^2}{2} \times \frac{\pi (0.13)^2}{4} = 6.11 \text{ N}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 24,000 km are, respectively,

$$W_{\text{drag}} = F \times L = 6.11 \times 24{,}000 = 146{,}640 \text{ kJ/year}$$

$$E_{\text{in}} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{146{,}640}{0.3} = 488{,}800 \text{ kJ/year}$$

The amount and costs of the fuel that supplies this much energy are then

$$\text{Amount of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/HV}{\rho_{\text{fuel}}} = \frac{488{,}800 \ \frac{\text{kJ}}{\text{year}} / 44{,}000 \frac{\text{kJ}}{\text{kg}}}{0.75 \text{ kg/L}} = 14.81 \text{ L/year}$$

$$\text{Cost} = \text{Amount of fuel} \times \text{Unit cost} = 14.81 \frac{\text{L}}{\text{year}} \times \frac{\$0.90}{\text{L}} = \boxed{\$13.33 / \text{year}}$$

That is, the car uses 14.81 L of gasoline at a cost of $13.33 per year to overcome the drag generated by a flat mirror extending out from the side of the vehicle.

➡ The correct answer is **C**.

Part B: The drag force and the work done to overcome it are directly proportional to the drag coefficient. Then, the percent reduction in fuel consumption due to replacing the mirror is equal to the percent reduction in the drag coefficient, namely,

$$\text{Reduction ratio} = \frac{C_{D,\text{flat}} - C_{D,\text{hemisphere}}}{C_{D,\text{flat}}} = \frac{1.1 - 0.4}{1.1} = 0.636$$

$$\text{Amount reduction} = \text{Reduction ratio} \times \text{Amount} = 0.636 \times 14.79 \text{L/year} = 9.41 \text{ L/year}$$

$$\text{Cost reduction} = \text{Reduction ratio} \times \text{Cost} = 0.636 \times \$13.33 / \text{year} = \boxed{\$8.48 / \text{year}}$$

➡ The correct answer is **D**.

ANSWER SUMMARY

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