

Montague

## Quiz FM302

### Flow with Friction and Heat Addition

Lucas Monteiro Nogueira

#### PROBLEMS

##### Problem 1.1

Air ( $\gamma = 1.4$  and  $R = 287 \text{ J/kg}\cdot\text{K}$ ) flows into a constant-area insulated duct with a Mach number of 0.2. For a duct diameter of 1.5 cm and friction coefficient of 0.015, determine the duct length required to reach Mach 0.50. Then, determine the length required to attain Mach 1.

- A)** Length to attain Mach 0.50 = 8.10 m; length to attain Mach 1 = 10.9 m;
- B)** Length to attain Mach 0.50 = 8.10 m; length to attain Mach 1 = 12.9 m;
- C)** Length to attain Mach 0.50 = 10.1 m; length to attain Mach 1 = 10.9 m;
- D)** Length to attain Mach 0.50 = 10.1 m; length to attain Mach 1 = 12.9 m;

##### Problem 1.2

Suppose now that an additional 1 meter is added to the duct length needed to attain Mach 1. Assuming initial stagnation conditions are maintained, determine the reduction in flow rate that would occur.

##### Problem 2

Air ( $\gamma = 1.4$  and  $R = 287 \text{ J/kg}\cdot\text{K}$ ) enters a constant-area insulated duct with a Mach number of 0.32, a stagnation pressure of 95 kPa, and a stagnation temperature of 320 K. For a duct length of 60 cm, duct diameter of 1.2 cm, and friction coefficient of 0.02, determine the air force on the duct wall.

- A)**  $F = 0.615 \text{ N}$
- B)**  $F = 0.966 \text{ N}$
- C)**  $F = 1.33 \text{ N}$
- D)**  $F = 1.72 \text{ N}$

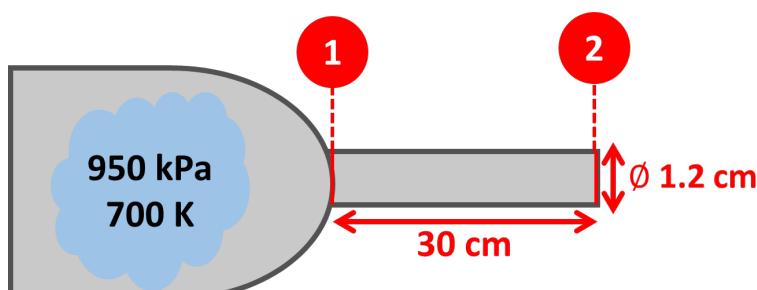
##### Problem 3

Hydrogen ( $\gamma = 1.4$ ,  $R = 4124 \text{ J/kg}\cdot\text{K}$ ) enters a constant-area with a velocity of 2735 m/s, static temperature of 400 K, and stagnation pressure of 525 kPa. The duct is 2 cm in diameter and 10 cm long. For a friction coefficient of 0.02, determine the change of static pressure and temperature in the duct and the exit velocity of the hydrogen. True or false?

- 1.** ( ) The exit velocity of the hydrogen is greater than 2200 m/s.
- 2.** ( ) The change in static pressure across the duct is greater than 32 kPa.
- 3.** ( ) The change in static temperature across the duct is greater than 80 K.

##### Problem 4

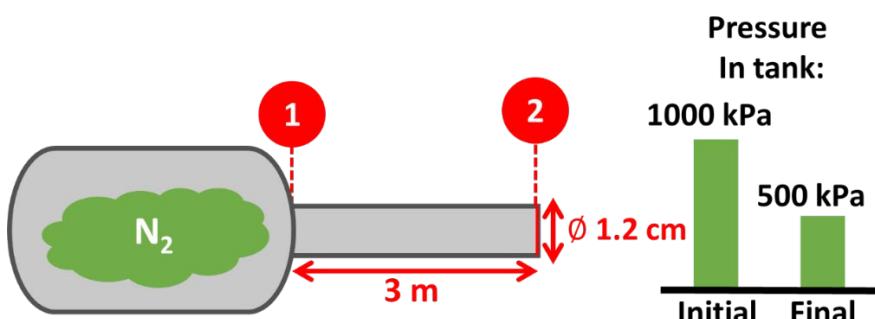
A constant-area duct, 30 cm in length by 1.2 cm in diameter, is connected to an air ( $\gamma = 1.4$  and  $R = 287 \text{ J/kg}$ ) reservoir through a converging nozzle, as shown below. For a constant reservoir pressure of 950 kPa and a constant reservoir temperature of 700 K, determine the flow rate through the duct for a back pressure of 101 kPa. Assume adiabatic flow in the tube with friction factor  $f = 0.022$ .



- A)**  $\dot{m} = 0.138 \text{ kg/s}$   
**B)**  $\dot{m} = 0.284 \text{ kg/s}$   
**C)**  $\dot{m} = 0.446 \text{ kg/s}$   
**D)**  $\dot{m} = 0.652 \text{ kg/s}$

### Problem 5

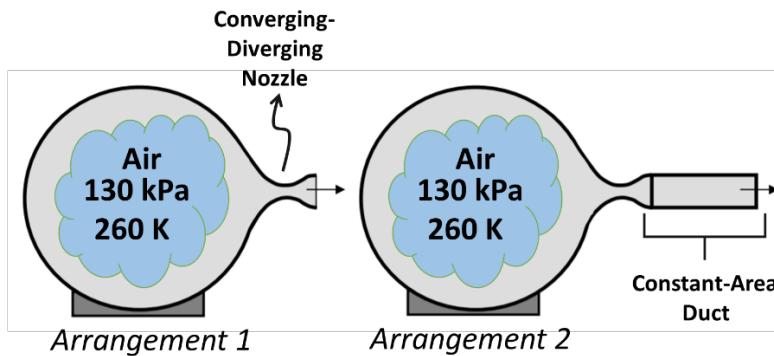
Find the time required for the pressure in the tank filled with nitrogen ( $\gamma = 1.4$ ,  $R = 296.8 \text{ J/kg}\cdot\text{K}$ ) illustrated below to drop from 1 MPa to 500 kPa. The tank volume is  $7.5 \text{ m}^3$  and the tank temperature is 350 K. Assume the tank temperature remains constant and flow in the 3-m long, 1.2-cm diameter connecting tube is adiabatic with  $f = 0.018$ . The back pressure is 101 kPa.



- A)**  $t = 105 \text{ sec}$   
**B)**  $t = 208 \text{ sec}$   
**C)**  $t = 292 \text{ sec}$   
**D)**  $t = 401 \text{ sec}$

### Problem 6

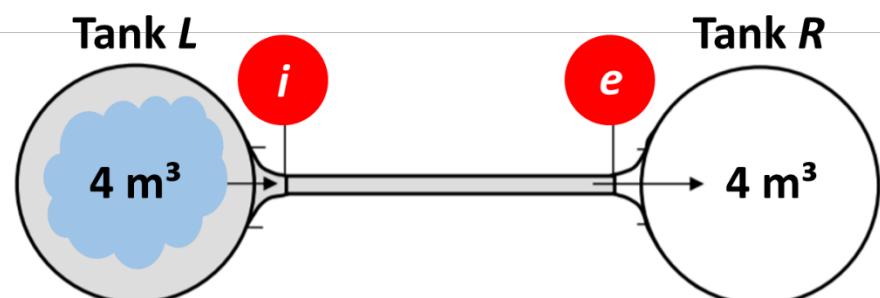
A converging-diverging nozzle has an area ratio of 3, i.e. the exit and therefore the duct area is 3 times the throat area, which is  $72 \text{ cm}^2$ . The nozzle is supplied from a tank containing air ( $\gamma = 1.4$ ,  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ ) at 130 kPa and 260 K. For arrangement 1 in the following figure, find the maximum mass flow possible through the nozzle and the range of back pressures over which the mass flow can be attained. Repeat for arrangement 2, in which a constant-area insulated duct of length 1.8 m and friction factor  $f = 0.022$  is added to the nozzle.



<b>Maximum mass flow</b>	_____ kg/s
<b>Range of back pressures for maximum mass flow in arrangement 1</b>	$0 \leq p_b \leq \text{___ kPa}$
<b>Range of back pressures for maximum mass flow in arrangement 2</b>	$0 \leq p_b \leq \text{___ kPa}$

### Problem 7

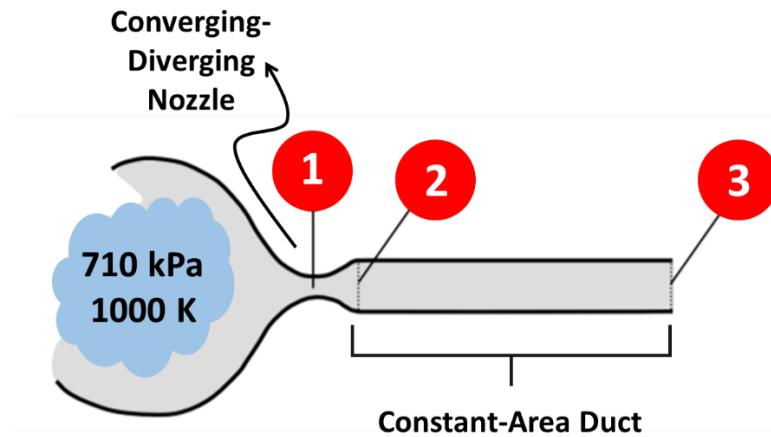
A  $4-\text{m}^3$  volume tank,  $R$ , is to be filled with air ( $\gamma = 1.4$ ), to a pressure of 180 kPa (initial pressure 0 kPa). The tank is connected to a reservoir tank,  $L$ , containing air at 3 MPa and 300 K, whose volume is also  $4 \text{ m}^3$ . A 30-m length of 2.5 cm-diameter tubing is used to connect the two vessels, as illustrated below. Determine the time required to fill the tank to 180 kPa. Assume Fanno flow with  $f = 0.02$ .



- A)**  $t = 3.51$  sec  
**B)**  $t = 9.03$  sec  
**C)**  $t = 14.1$  sec  
**D)**  $t = 17.2$  sec

### Problem 8.1

For the flow of air ( $\gamma = 1.4$ ,  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ ) from the reservoir at 710 kPa and 1000 K shown below, assume isentropic flow in the convergent-divergent nozzle and Fanno flow in the constant-area duct, which has a length of 24 cm and a diameter of 1.2 cm. The area ratio  $A_2/A_1$  of the convergent-divergent nozzle is 2.8. Take the friction factor to be 0.02. Find the mass flow rate for a back pressure of 0 kPa.



- A)**  $\dot{m} = 0.0183 \text{ kg/s}$   
**B)**  $\dot{m} = 0.0367 \text{ kg/s}$   
**C)**  $\dot{m} = 0.0734 \text{ kg/s}$   
**D)**  $\dot{m} = 0.105 \text{ kg/s}$

### Problem 8.2

Find the pressure at the exit plane of the duct.

- A)**  $p_e = 103 \text{ kPa}$   
**B)**  $p_e = 211 \text{ kPa}$   
**C)**  $p_e = 305 \text{ kPa}$   
**D)**  $p_e = 422 \text{ kPa}$

### Problem 8.3

Find the back pressure necessary for a normal shock to occur at the exit plane of nozzle (2).

- A)**  $p_e = 248 \text{ kPa}$   
**B)**  $p_e = 352 \text{ kPa}$   
**C)**  $p_e = 440 \text{ kPa}$   
**D)**  $p_e = 592 \text{ kPa}$

### Problem 8.4

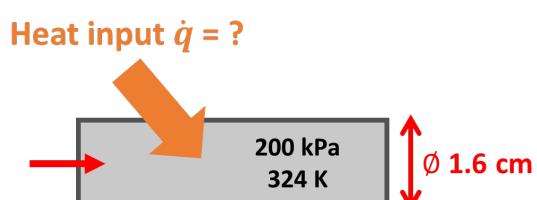
Find the back pressure necessary for a normal shock to appear just downstream of the nozzle throat (1).

- A)**  $p_e = 598 \text{ kPa}$   
**B)**  $p_e = 678 \text{ kPa}$   
**C)**  $p_e = 790 \text{ kPa}$   
**D)**  $p_e = 834 \text{ kPa}$

### Problem 9

Air ( $\gamma = 1.4$ ,  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ ,  $c_p = 1.004 \text{ kJ/kg}\cdot\text{K}$ ) is flowing in a constant-area duct of diameter 1.6 cm with a velocity of 120 m/s, static temperature of 324 K, and static pressure of 200 kPa. Determine the rate of heat input to the flow necessary to choke the duct. Assume Rayleigh line flow; express your answer in kilowatts. Assume air to behave as a perfect gas with constant specific heats.

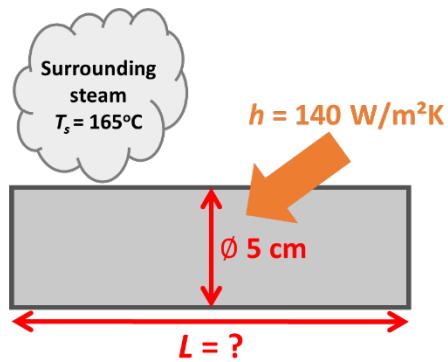
- A)**  $\dot{q} = 3.37 \text{ kW}$   
**B)**  $\dot{q} = 10.0 \text{ kW}$   
**C)**  $\dot{q} = 14.2 \text{ kW}$   
**D)**  $\dot{q} = 25.1 \text{ kW}$



## Problem 10.1

An airstream ( $\gamma = 1.4$ ,  $R = 0.287$  kJ/kg·K) passing through a 5-cm diameter, thin-walled tube is to be heated by high-pressure steam condensing on the outer surface of the tube at 165°C. The overall heat transfer coefficient between steam and air can be assumed to be 140 W/m<sup>2</sup>·K, with the air entering at 32 m/s, 72 kPa, and 8°C. The air is to be heated to 70°C. Determine the tube length required.

- A)**  $L = 1.30$  m
- B)**  $L = 2.60$  m
- C)**  $L = 3.91$  m
- D)**  $L = 5.08$  m



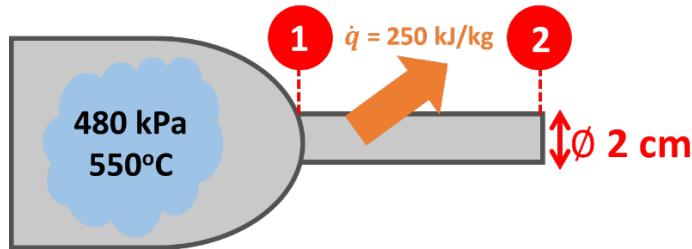
## Problem 10.2

Assuming Rayleigh line flow, calculate the static pressure change due to heat addition. Also, for the same inlet conditions, calculate the pressure drop due to friction, assuming Fanno flow in the duct with  $f = 0.025$ . To obtain an approximation to the overall pressure drop in this heat exchanger, add the two results.

- A)**  $|\Delta p_{\text{Rayleigh}}| + |\Delta p_{\text{Fanno}}| = 0.517$  kPa
- B)**  $|\Delta p_{\text{Rayleigh}}| + |\Delta p_{\text{Fanno}}| = 0.935$  kPa
- C)**  $|\Delta p_{\text{Rayleigh}}| + |\Delta p_{\text{Fanno}}| = 1.53$  kPa
- D)**  $|\Delta p_{\text{Rayleigh}}| + |\Delta p_{\text{Fanno}}| = 1.82$  kPa

## Problem 11.1

Air ( $\gamma = 1.4$ ,  $R = 0.287$  kJ/kg·K, and  $c_p = 1.004$  kJ/kg·K) flowing through a constant-area duct is connected to a reservoir at a temperature of 550°C and a pressure of 480 kPa by a converging nozzle, as shown below. Heat is lost at a rate of 250 kJ/kg. Determine the exit pressure, Mach number, and mass flow rate for a back pressure of 0 kPa. True or false?



1. ( ) The exit pressure  $p_2$  is greater than 65 kPa.
2. ( ) The exit Mach number  $M_2$  is greater than 2.2.
3. ( ) The mass flow rate  $\dot{m}$  is greater than 0.25 kg/s.

## Problem 11.2

Suppose a normal shock stands in the exit plane of the duct. In this case, the exit pressure and Mach number are, respectively:

- A)**  $p_e = 375$  kPa and  $M_e = 2.30$
- B)**  $p_e = 375$  kPa and  $M_e = 2.30$
- C)**  $p_e = 450$  kPa and  $M_e = 2.60$
- D)**  $p_e = 450$  kPa and  $M_e = 2.60$

## Problem 11.3

For the system introduced in Problem 11.1, determine the mass flow rate if 250 kJ/kg of heat energy is **added** to the flow in the duct. The duct diameter is 2 cm. Repeat for a back pressure of 100 kPa.

## Problem 12

Heat is added to airflow ( $\gamma = 1.4$ ,  $R = 0.287$  kJ/kg·K) in a horizontal constant-area duct at the rate of 28.9 kJ/m. If flow enters at Mach 0.20, temperature  $T_1 = 300$  K, and pressure  $p_1 = 100$  kPa, plot the variation of Mach number with respect to horizontal coordinate  $x$ . Also plot the variation of static pressure, temperature, and stagnation pressure.

## ADDITIONAL INFORMATION

### Tables for isentropic flow, normal shock, Fanno flow, and Rayleigh line flow

Pages 6 - 34 are isentropic flow, normal shock, and Fanno flow tables drawn from Rathakrishnan, E., 2019, *Applied Gas Dynamics*, 2nd edition. Tables were reproduced with permission from John Wiley and Sons, Inc., 111 River Street, Hoboken, New Jersey, USA.

Pages 35 – 37 are Rayleigh line flow tables drawn from John, J.E. and Keith, T., 2006, *Gas Dynamics*, 3rd edition. Tables were reproduced with permission from Pearson Education, Inc., 1 Lake Street, Upper Saddle River, New Jersey, USA.

## Appendix A

**Table A.1** Isentropic flow of perfect gas ( $\gamma = 1.4$ ).

M	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
0.00	1.0000	1.0000	1.0000	$\infty$	1.0000	0.0000		
0.01	0.9999	1.0000	1.0000	57.874	1.0000	0.0110		
0.02	0.9997	0.9999	0.9998	28.942	1.0000	0.0219		
0.03	0.9994	0.9998	0.9996	19.301	0.9999	0.0329		
0.04	0.9989	0.9997	0.9992	14.481	0.9998	0.0438		
0.05	0.9983	0.9995	0.9988	11.591	0.9998	0.0548		
0.06	0.9975	0.9993	0.9982	9.666	0.9996	0.0657		
0.07	0.9966	0.9990	0.9976	8.292	0.9995	0.0766		
0.08	0.9955	0.9987	0.9968	7.262	0.9994	0.0876		
0.09	0.9944	0.9984	0.9960	6.461	0.9992	0.0985		
0.10	0.9930	0.9980	0.9950	5.822	0.9990	0.1094		
0.11	0.9916	0.9976	0.9940	5.299	0.9988	0.1204		
0.12	0.9900	0.9971	0.9928	4.864	0.9986	0.1313		
0.13	0.9883	0.9966	0.9916	4.497	0.9983	0.1422		
0.14	0.9864	0.9961	0.9903	4.182	0.9980	0.1531		
0.15	0.9844	0.9955	0.9888	3.910	0.9978	0.1639		
0.16	0.9823	0.9949	0.9873	3.673	0.9974	0.1748		
0.17	0.9800	0.9943	0.9857	3.464	0.9971	0.1857		
0.18	0.9776	0.9936	0.9840	3.278	0.9968	0.1965		
0.19	0.9751	0.9928	0.9822	3.112	0.9964	0.2074		
0.20	0.9725	0.9921	0.9803	2.964	0.9960	0.2182		
0.21	0.9697	0.9913	0.9783	2.829	0.9956	0.2290		
0.22	0.9668	0.9904	0.9762	2.708	0.9952	0.2398		
0.23	0.9638	0.9895	0.9740	2.597	0.9948	0.2506		
0.24	0.9607	0.9886	0.9718	2.496	0.9943	0.2614		
0.25	0.9575	0.9877	0.9694	2.403	0.9938	0.2722		
0.26	0.9541	0.9867	0.9670	2.317	0.9933	0.2829		
0.27	0.9506	0.9856	0.9645	2.238	0.9928	0.2936		
0.28	0.9470	0.9846	0.9619	2.166	0.9923	0.3043		

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
0.29	0.9433	0.9835	0.9592	2.098	0.9917	0.3150		
0.30	0.9395	0.9823	0.9564	2.035	0.9911	0.3257		
0.31	0.9355	0.9811	0.9535	1.977	0.9905	0.3364		
0.32	0.9315	0.9799	0.9506	1.922	0.9899	0.3470		
0.33	0.9274	0.9787	0.9476	1.871	0.9893	0.3576		
0.34	0.9231	0.9774	0.9445	1.823	0.9886	0.3682		
0.35	0.9188	0.9761	0.9413	1.778	0.9880	0.3788		
0.36	0.9143	0.9747	0.9380	1.736	0.9873	0.3893		
0.37	0.9098	0.9733	0.9347	1.696	0.9866	0.3999		
0.38	0.9052	0.9719	0.9313	1.659	0.9859	0.4104		
0.39	0.9004	0.9705	0.9278	1.623	0.9851	0.4209		
0.40	0.8956	0.9690	0.9243	1.590	0.9844	0.4313		
0.41	0.8907	0.9675	0.9207	1.559	0.9836	0.4418		
0.42	0.8857	0.9659	0.9170	1.529	0.9828	0.4522		
0.43	0.8807	0.9643	0.9132	1.501	0.9820	0.4626		
0.44	0.8755	0.9627	0.9094	1.474	0.9812	0.4729		
0.45	0.8703	0.9611	0.9055	1.449	0.9803	0.4833		
0.46	0.8650	0.9594	0.9016	1.425	0.9795	0.4936		
0.47	0.8596	0.9577	0.8976	1.402	0.9786	0.5038		
0.48	0.8541	0.9559	0.8935	1.380	0.9777	0.5141		
0.49	0.8486	0.9542	0.8894	1.359	0.9768	0.5243		
0.50	0.8430	0.9524	0.8852	1.340	0.9759	0.5345		
0.51	0.8374	0.9506	0.8809	1.321	0.9750	0.5447		
0.52	0.8317	0.9487	0.8766	1.303	0.9740	0.5548		
0.53	0.8259	0.9468	0.8723	1.286	0.9730	0.5649		
0.54	0.8201	0.9449	0.8679	1.270	0.9721	0.5750		
0.55	0.8142	0.9430	0.8634	1.255	0.9711	0.5851		
0.56	0.8082	0.9410	0.8589	1.240	0.9700	0.5951		
0.57	0.8022	0.9390	0.8544	1.226	0.9690	0.6051		
0.58	0.7962	0.9370	0.8498	1.213	0.9680	0.6150		
0.59	0.7901	0.9349	0.8451	1.200	0.9669	0.6249		
0.60	0.7840	0.9328	0.8405	1.188	0.9658	0.6348		
0.61	0.7778	0.9307	0.8357	1.177	0.9647	0.6447		
0.62	0.7716	0.9286	0.8310	1.166	0.9636	0.6545		
0.63	0.7654	0.9265	0.8262	1.155	0.9625	0.6643		
0.64	0.7591	0.9243	0.8213	1.145	0.9614	0.6740		
0.65	0.7528	0.9221	0.8164	1.136	0.9603	0.6837		
0.66	0.7465	0.9199	0.8115	1.127	0.9591	0.6934		
0.67	0.7401	0.9176	0.8066	1.118	0.9579	0.7031		
0.68	0.7338	0.9153	0.8016	1.110	0.9567	0.7127		

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
0.69	0.7274	0.9131	0.7966	1.102	0.9555	0.7223		
0.70	0.7209	0.9107	0.7916	1.094	0.9543	0.7318		
0.71	0.7145	0.9084	0.7865	1.087	0.9531	0.7413		
0.72	0.7080	0.9061	0.7814	1.081	0.9519	0.7508		
0.73	0.7016	0.9037	0.7763	1.074	0.9506	0.7602		
0.74	0.6951	0.9013	0.7712	1.068	0.9494	0.7696		
0.75	0.6886	0.8989	0.7660	1.062	0.9481	0.7789		
0.76	0.6821	0.8964	0.7609	1.057	0.9468	0.7883		
0.77	0.6756	0.8940	0.7557	1.052	0.9455	0.7975		
0.78	0.6691	0.8915	0.7505	1.047	0.9442	0.8068		
0.79	0.6625	0.8890	0.7452	1.043	0.9429	0.8160		
0.80	0.6560	0.8865	0.7400	1.038	0.9416	0.8251		
0.81	0.6495	0.8840	0.7347	1.034	0.9402	0.8343		
0.82	0.6430	0.8815	0.7295	1.030	0.9389	0.8433		
0.83	0.6365	0.8789	0.7242	1.027	0.9375	0.8524		
0.84	0.6300	0.8763	0.7189	1.024	0.9361	0.8614		
0.85	0.6235	0.8737	0.7136	1.021	0.9347	0.8704		
0.86	0.6170	0.8711	0.7083	1.018	0.9333	0.8793		
0.87	0.6106	0.8685	0.7030	1.015	0.9319	0.8882		
0.88	0.6041	0.8659	0.6977	1.013	0.9305	0.8970		
0.89	0.5977	0.8632	0.6924	1.011	0.9291	0.9058		
0.90	0.5913	0.8606	0.6870	1.009	0.9277	0.9146		
0.91	0.5849	0.8579	0.6817	1.007	0.9262	0.9233		
0.92	0.5785	0.8552	0.6764	1.006	0.9248	0.9320		
0.93	0.5721	0.8525	0.6711	1.004	0.9233	0.9407		
0.94	0.5658	0.8498	0.6658	1.003	0.9219	0.9493		
0.95	0.5595	0.8471	0.6604	1.002	0.9204	0.9578		
0.96	0.5532	0.8444	0.6551	1.001	0.9189	0.9663		
0.97	0.5469	0.8416	0.6498	1.001	0.9174	0.9748		
0.98	0.5407	0.8389	0.6445	1.000	0.9159	0.9833		
0.99	0.5345	0.8361	0.6392	1.000	0.9144	0.9916		
1.00	0.5283	0.8333	0.6339	1.000	0.9129	1.0000	90.000	0.000
1.01	0.5221	0.8306	0.6287	1.000	0.9113	1.0083	81.931	0.045
1.02	0.5160	0.8278	0.6234	1.000	0.9098	1.0166	78.635	0.126
1.03	0.5099	0.8250	0.6181	1.001	0.9083	1.0248	76.138	0.229
1.04	0.5039	0.8222	0.6129	1.001	0.9067	1.0330	74.058	0.351
1.05	0.4979	0.8193	0.6077	1.002	0.9052	1.0411	72.247	0.487
1.06	0.4919	0.8165	0.6024	1.003	0.9036	1.0492	70.630	0.637
1.07	0.4860	0.8137	0.5972	1.004	0.9020	1.0573	69.160	0.797
1.08	0.4800	0.8108	0.5920	1.005	0.9005	1.0653	67.808	0.968

(continued)

**Table A.1** (Continued)

<i>M</i>	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
1.09	0.4742	0.8080	0.5869	1.006	0.8989	1.0733	66.553	1.148
1.10	0.4684	0.8052	0.5817	1.008	0.8973	1.0812	65.380	1.336
1.11	0.4626	0.8023	0.5766	1.010	0.8957	1.0891	64.277	1.532
1.12	0.4568	0.7994	0.5714	1.011	0.8941	1.0970	63.234	1.735
1.13	0.4511	0.7966	0.5663	1.013	0.8925	1.1048	62.246	1.944
1.14	0.4455	0.7937	0.5612	1.015	0.8909	1.1126	61.306	2.160
1.15	0.4398	0.7908	0.5562	1.017	0.8893	1.1203	60.408	2.381
1.16	0.4343	0.7879	0.5511	1.020	0.8877	1.1280	59.550	2.607
1.17	0.4287	0.7851	0.5461	1.022	0.8860	1.1356	58.727	2.839
1.18	0.4232	0.7822	0.5411	1.025	0.8844	1.1432	57.936	3.074
1.19	0.4178	0.7793	0.5361	1.028	0.8828	1.1508	57.176	3.314
1.20	0.4124	0.7764	0.5311	1.030	0.8811	1.1583	56.443	3.558
1.21	0.4070	0.7735	0.5262	1.033	0.8795	1.1658	55.735	3.806
1.22	0.4017	0.7706	0.5213	1.037	0.8778	1.1732	55.052	4.057
1.23	0.3964	0.7677	0.5164	1.040	0.8762	1.1806	54.391	4.312
1.24	0.3912	0.7648	0.5115	1.043	0.8745	1.1879	53.751	4.569
1.25	0.3861	0.7619	0.5067	1.047	0.8729	1.1952	53.130	4.830
1.26	0.3809	0.7590	0.5019	1.050	0.8712	1.2025	52.528	5.093
1.27	0.3759	0.7561	0.4971	1.054	0.8695	1.2097	51.943	5.359
1.28	0.3708	0.7532	0.4923	1.058	0.8679	1.2169	51.375	5.627
1.29	0.3658	0.7503	0.4876	1.062	0.8662	1.2240	50.823	5.898
1.30	0.3609	0.7474	0.4829	1.066	0.8645	1.2311	50.285	6.170
1.31	0.3560	0.7445	0.4782	1.071	0.8628	1.2382	49.761	6.445
1.32	0.3512	0.7416	0.4736	1.075	0.8611	1.2452	49.251	6.721
1.33	0.3464	0.7387	0.4690	1.080	0.8595	1.2522	48.753	7.000
1.34	0.3417	0.7358	0.4644	1.084	0.8578	1.2591	48.268	7.279
1.35	0.3370	0.7329	0.4598	1.089	0.8561	1.2660	47.795	7.561
1.36	0.3323	0.7300	0.4553	1.094	0.8544	1.2729	47.332	7.844
1.37	0.3277	0.7271	0.4508	1.099	0.8527	1.2797	46.880	8.128
1.38	0.3232	0.7242	0.4463	1.104	0.8510	1.2864	46.439	8.413
1.39	0.3187	0.7213	0.4418	1.109	0.8493	1.2932	46.007	8.699
1.40	0.3142	0.7184	0.4374	1.115	0.8476	1.2999	45.585	8.987
1.41	0.3098	0.7155	0.4330	1.120	0.8459	1.3065	45.171	9.276
1.42	0.3055	0.7126	0.4287	1.126	0.8442	1.3131	44.767	9.565
1.43	0.3012	0.7097	0.4244	1.132	0.8425	1.3197	44.371	9.855
1.44	0.2969	0.7069	0.4201	1.138	0.8407	1.3262	43.983	10.146
1.45	0.2927	0.7040	0.4158	1.144	0.8390	1.3327	43.603	10.438
1.46	0.2886	0.7011	0.4116	1.150	0.8373	1.3392	43.230	10.731
1.47	0.2845	0.6982	0.4074	1.156	0.8356	1.3456	42.865	11.023
1.48	0.2804	0.6954	0.4032	1.163	0.8339	1.3520	42.507	11.317
1.49	0.2764	0.6925	0.3991	1.169	0.8322	1.3583	42.155	11.611

(continued)

**Table A.1** (Continued)

<i>M</i>	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
1.50	0.2724	0.6897	0.3950	1.176	0.8305	1.3646	41.810	11.905
1.51	0.2685	0.6868	0.3909	1.183	0.8287	1.3708	41.472	12.200
1.52	0.2646	0.6840	0.3869	1.190	0.8270	1.3770	41.140	12.495
1.53	0.2608	0.6811	0.3829	1.197	0.8253	1.3832	40.813	12.790
1.54	0.2570	0.6783	0.3789	1.204	0.8236	1.3894	40.493	13.086
1.55	0.2533	0.6754	0.3750	1.212	0.8219	1.3955	40.178	13.381
1.56	0.2496	0.6726	0.3710	1.219	0.8201	1.4015	39.868	13.677
1.57	0.2459	0.6698	0.3672	1.227	0.8184	1.4075	39.564	13.973
1.58	0.2423	0.6670	0.3633	1.234	0.8167	1.4135	39.265	14.269
1.59	0.2388	0.6642	0.3595	1.242	0.8150	1.4195	38.971	14.565
1.60	0.2353	0.6614	0.3557	1.250	0.8133	1.4254	38.682	14.860
1.61	0.2318	0.6586	0.3520	1.258	0.8115	1.4313	38.398	15.156
1.62	0.2284	0.6558	0.3483	1.267	0.8098	1.4371	38.118	15.452
1.63	0.2250	0.6530	0.3446	1.275	0.8081	1.4429	37.843	15.747
1.64	0.2217	0.6502	0.3409	1.284	0.8064	1.4487	37.572	16.043
1.65	0.2184	0.6475	0.3373	1.292	0.8046	1.4544	37.305	16.338
1.66	0.2151	0.6447	0.3337	1.301	0.8029	1.4601	37.043	16.633
1.67	0.2119	0.6419	0.3302	1.310	0.8012	1.4657	36.784	16.928
1.68	0.2088	0.6392	0.3266	1.319	0.7995	1.4713	36.530	17.222
1.69	0.2057	0.6364	0.3232	1.328	0.7978	1.4769	36.279	17.516
1.70	0.2026	0.6337	0.3197	1.338	0.7961	1.4825	36.032	17.810
1.71	0.1996	0.6310	0.3163	1.347	0.7943	1.4880	35.789	18.103
1.72	0.1966	0.6283	0.3129	1.357	0.7926	1.4935	35.549	18.396
1.73	0.1936	0.6256	0.3095	1.367	0.7909	1.4989	35.312	18.689
1.74	0.1907	0.6229	0.3062	1.376	0.7892	1.5043	35.080	18.981
1.75	0.1878	0.6202	0.3029	1.386	0.7875	1.5097	34.850	19.273
1.76	0.1850	0.6175	0.2996	1.397	0.7858	1.5150	34.624	19.565
1.77	0.1822	0.6148	0.2964	1.407	0.7841	1.5203	34.400	19.855
1.78	0.1794	0.6121	0.2931	1.418	0.7824	1.5256	34.180	20.146
1.79	0.1767	0.6095	0.2900	1.428	0.7807	1.5308	33.963	20.436
1.80	0.1740	0.6068	0.2868	1.439	0.7790	1.5360	33.749	20.725
1.81	0.1714	0.6041	0.2837	1.450	0.7773	1.5411	33.538	21.014
1.82	0.1688	0.6015	0.2806	1.461	0.7756	1.5463	33.329	21.302
1.83	0.1662	0.5989	0.2776	1.472	0.7739	1.5514	33.124	21.590
1.84	0.1637	0.5963	0.2745	1.484	0.7722	1.5564	32.921	21.877
1.85	0.1612	0.5936	0.2715	1.495	0.7705	1.5614	32.720	22.163
1.86	0.1587	0.5910	0.2686	1.507	0.7688	1.5664	32.523	22.449
1.87	0.1563	0.5884	0.2656	1.519	0.7671	1.5714	32.328	22.734
1.88	0.1539	0.5859	0.2627	1.531	0.7654	1.5763	32.135	23.019
1.89	0.1516	0.5833	0.2598	1.543	0.7637	1.5812	31.945	23.303
1.90	0.1492	0.5807	0.2570	1.555	0.7620	1.5861	31.757	23.586

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
1.91	0.1470	0.5782	0.2542	1.568	0.7604	1.5909	31.571	23.869
1.92	0.1447	0.5756	0.2514	1.580	0.7587	1.5957	31.388	24.151
1.93	0.1425	0.5731	0.2486	1.593	0.7570	1.6005	31.207	24.432
1.94	0.1403	0.5705	0.2459	1.606	0.7553	1.6052	31.028	24.712
1.95	0.1381	0.5680	0.2432	1.619	0.7537	1.6099	30.852	24.992
1.96	0.1360	0.5655	0.2405	1.633	0.7520	1.6146	30.677	25.271
1.97	0.1339	0.5630	0.2378	1.646	0.7503	1.6192	30.505	25.549
1.98	0.1318	0.5605	0.2352	1.660	0.7487	1.6239	30.335	25.827
1.99	0.1298	0.5580	0.2326	1.674	0.7470	1.6284	30.166	26.104
2.00	0.1278	0.5556	0.2300	1.688	0.7454	1.6330	30.000	26.380
2.01	0.1258	0.5531	0.2275	1.702	0.7437	1.6375	29.836	26.655
2.02	0.1239	0.5506	0.2250	1.716	0.7420	1.6420	29.673	26.930
2.03	0.1220	0.5482	0.2225	1.730	0.7404	1.6465	29.512	27.203
2.04	0.1201	0.5458	0.2200	1.745	0.7388	1.6509	29.353	27.476
2.05	0.1182	0.5433	0.2176	1.760	0.7371	1.6553	29.196	27.748
2.06	0.1164	0.5409	0.2152	1.775	0.7355	1.6597	29.041	28.020
2.07	0.1146	0.5385	0.2128	1.790	0.7338	1.6640	28.888	28.290
2.08	0.1128	0.5361	0.2104	1.806	0.7322	1.6683	28.736	28.560
2.09	0.1111	0.5337	0.2081	1.821	0.7306	1.6726	28.585	28.829
2.10	0.1094	0.5313	0.2058	1.837	0.7289	1.6769	28.437	29.097
2.11	0.1077	0.5290	0.2035	1.853	0.7273	1.6811	28.290	29.364
2.12	0.1060	0.5266	0.2013	1.869	0.7257	1.6853	28.145	29.630
2.13	0.1043	0.5243	0.1990	1.885	0.7241	1.6895	28.001	29.896
2.14	0.1027	0.5219	0.1968	1.902	0.7225	1.6936	27.859	30.161
2.15	0.1011	0.5196	0.1946	1.919	0.7208	1.6977	27.718	30.425
2.16	0.0996	0.5173	0.1925	1.935	0.7192	1.7018	27.578	30.688
2.17	0.0980	0.5150	0.1903	1.953	0.7176	1.7059	27.441	30.951
2.18	0.0965	0.5127	0.1882	1.970	0.7160	1.7099	27.304	31.212
2.19	0.0950	0.5104	0.1861	1.987	0.7144	1.7139	27.169	31.473
2.20	0.0935	0.5081	0.1841	2.005	0.7128	1.7179	27.036	31.732
2.21	0.0921	0.5059	0.1820	2.023	0.7112	1.7219	26.903	31.991
2.22	0.0906	0.5036	0.1800	2.041	0.7097	1.7258	26.773	32.249
2.23	0.0892	0.5014	0.1780	2.059	0.7081	1.7297	26.643	32.507
2.24	0.0878	0.4991	0.1760	2.078	0.7065	1.7336	26.515	32.763
2.25	0.0865	0.4969	0.1740	2.096	0.7049	1.7374	26.388	33.018
2.26	0.0851	0.4947	0.1721	2.115	0.7033	1.7412	26.262	33.273
2.27	0.0838	0.4925	0.1702	2.134	0.7018	1.7450	26.138	33.527
2.28	0.0825	0.4903	0.1683	2.154	0.7002	1.7488	26.014	33.780
2.29	0.0812	0.4881	0.1664	2.173	0.6986	1.7526	25.892	34.032
2.30	0.0800	0.4859	0.1646	2.193	0.6971	1.7563	25.771	34.283
2.31	0.0787	0.4837	0.1628	2.213	0.6955	1.7600	25.652	34.533

(continued)

**Table A.1** (Continued)

<i>M</i>	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
2.32	0.0775	0.4816	0.1609	2.233	0.6940	1.7637	25.533	34.782
2.33	0.0763	0.4794	0.1592	2.254	0.6924	1.7673	25.416	35.031
2.34	0.0751	0.4773	0.1574	2.274	0.6909	1.7709	25.300	35.279
2.35	0.0740	0.4752	0.1556	2.295	0.6893	1.7745	25.184	35.526
2.36	0.0728	0.4731	0.1539	2.316	0.6878	1.7781	25.070	35.771
2.37	0.0717	0.4709	0.1522	2.338	0.6863	1.7817	24.957	36.017
2.38	0.0706	0.4688	0.1505	2.359	0.6847	1.7852	24.845	36.261
2.39	0.0695	0.4668	0.1488	2.381	0.6832	1.7887	24.734	36.504
2.40	0.0684	0.4647	0.1472	2.403	0.6817	1.7922	24.624	36.747
2.41	0.0673	0.4626	0.1456	2.425	0.6802	1.7956	24.515	36.988
2.42	0.0663	0.4606	0.1439	2.448	0.6786	1.7991	24.407	37.229
2.43	0.0653	0.4585	0.1424	2.471	0.6771	1.8025	24.300	37.469
2.44	0.0643	0.4565	0.1408	2.494	0.6756	1.8059	24.195	37.708
2.45	0.0633	0.4544	0.1392	2.517	0.6741	1.8092	24.090	37.946
2.46	0.0623	0.4524	0.1377	2.540	0.6726	1.8126	23.985	38.183
2.47	0.0613	0.4504	0.1362	2.564	0.6711	1.8159	23.882	38.420
2.48	0.0604	0.4484	0.1346	2.588	0.6696	1.8192	23.780	38.655
2.49	0.0594	0.4464	0.1332	2.612	0.6682	1.8225	23.679	38.890
2.50	0.0585	0.4444	0.1317	2.637	0.6667	1.8257	23.578	39.124
2.51	0.0576	0.4425	0.1302	2.661	0.6652	1.8290	23.479	39.357
2.52	0.0567	0.4405	0.1288	2.686	0.6637	1.8322	23.380	39.589
2.53	0.0559	0.4386	0.1274	2.712	0.6622	1.8354	23.282	39.820
2.54	0.0550	0.4366	0.1260	2.737	0.6608	1.8386	23.185	40.050
2.55	0.0542	0.4347	0.1246	2.763	0.6593	1.8417	23.089	40.280
2.56	0.0533	0.4328	0.1232	2.789	0.6578	1.8448	22.993	40.508
2.57	0.0525	0.4309	0.1218	2.815	0.6564	1.8479	22.899	40.736
2.58	0.0517	0.4289	0.1205	2.842	0.6549	1.8510	22.805	40.963
2.59	0.0509	0.4271	0.1192	2.869	0.6535	1.8541	22.712	41.189
2.60	0.0501	0.4252	0.1179	2.896	0.6521	1.8571	22.620	41.415
2.61	0.0493	0.4233	0.1166	2.923	0.6506	1.8602	22.528	41.639
2.62	0.0486	0.4214	0.1153	2.951	0.6492	1.8632	22.438	41.863
2.63	0.0478	0.4196	0.1140	2.979	0.6477	1.8662	22.348	42.086
2.64	0.0471	0.4177	0.1128	3.007	0.6463	1.8691	22.259	42.307
2.65	0.0464	0.4159	0.1115	3.036	0.6449	1.8721	22.170	42.529
2.66	0.0457	0.4141	0.1103	3.065	0.6435	1.8750	22.082	42.749
2.67	0.0450	0.4122	0.1091	3.094	0.6421	1.8779	21.995	42.968
2.68	0.0443	0.4104	0.1079	3.123	0.6406	1.8808	21.909	43.187
2.69	0.0436	0.4086	0.1067	3.153	0.6392	1.8837	21.823	43.405
2.70	0.0430	0.4068	0.1056	3.183	0.6378	1.8865	21.738	43.621
2.71	0.0423	0.4051	0.1044	3.213	0.6364	1.8894	21.654	43.838
2.72	0.0417	0.4033	0.1033	3.244	0.6350	1.8922	21.571	44.053

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
2.73	0.0410	0.4015	0.1022	3.275	0.6337	1.8950	21.488	44.267
2.74	0.0404	0.3998	0.1010	3.306	0.6323	1.8978	21.405	44.481
2.75	0.0398	0.3980	0.0999	3.338	0.6309	1.9005	21.324	44.694
2.76	0.0392	0.3963	0.0989	3.370	0.6295	1.9033	21.243	44.906
2.77	0.0386	0.3945	0.0978	3.402	0.6281	1.9060	21.162	45.117
2.78	0.0380	0.3928	0.0967	3.434	0.6268	1.9087	21.083	45.327
2.79	0.0374	0.3911	0.0957	3.467	0.6254	1.9114	21.003	45.537
2.80	0.0368	0.3894	0.0946	3.500	0.6240	1.9140	20.925	45.746
2.81	0.0363	0.3877	0.0936	3.534	0.6227	1.9167	20.847	45.954
2.82	0.0357	0.3860	0.0926	3.567	0.6213	1.9193	20.770	46.161
2.83	0.0352	0.3844	0.0916	3.601	0.6200	1.9219	20.693	46.368
2.84	0.0347	0.3827	0.0906	3.636	0.6186	1.9246	20.617	46.573
2.85	0.0341	0.3810	0.0896	3.671	0.6173	1.9271	20.541	46.778
2.86	0.0336	0.3794	0.0886	3.706	0.6159	1.9297	20.466	46.982
2.87	0.0331	0.3777	0.0877	3.741	0.6146	1.9323	20.391	47.185
2.88	0.0326	0.3761	0.0867	3.777	0.6133	1.9348	20.318	47.388
2.89	0.0321	0.3745	0.0858	3.813	0.6119	1.9373	20.244	47.589
2.90	0.0317	0.3729	0.0849	3.850	0.6106	1.9398	20.171	47.790
2.91	0.0312	0.3712	0.0840	3.887	0.6093	1.9423	20.099	47.990
2.92	0.0307	0.3696	0.0831	3.924	0.6080	1.9448	20.027	48.190
2.93	0.0302	0.3681	0.0822	3.961	0.6067	1.9472	19.956	48.388
2.94	0.0298	0.3665	0.0813	3.999	0.6054	1.9497	19.885	48.586
2.95	0.0293	0.3649	0.0804	4.038	0.6041	1.9521	19.815	48.783
2.96	0.0289	0.3633	0.0796	4.076	0.6028	1.9545	19.745	48.980
2.97	0.0285	0.3618	0.0787	4.115	0.6015	1.9569	19.676	49.175
2.98	0.0281	0.3602	0.0779	4.155	0.6002	1.9593	19.607	49.370
2.99	0.0276	0.3587	0.0770	4.194	0.5989	1.9616	19.539	49.564
3.00	0.0272	0.3571	0.0762	4.235	0.5976	1.9640	19.471	49.757
3.01	0.0268	0.3556	0.0754	4.275	0.5963	1.9663	19.404	49.950
3.02	0.0264	0.3541	0.0746	4.316	0.5951	1.9686	19.337	50.142
3.03	0.0260	0.3526	0.0738	4.357	0.5938	1.9709	19.271	50.333
3.04	0.0256	0.3511	0.0730	4.399	0.5925	1.9732	19.205	50.523
3.05	0.0253	0.3496	0.0723	4.441	0.5913	1.9755	19.139	50.713
3.06	0.0249	0.3481	0.0715	4.483	0.5900	1.9777	19.074	50.902
3.07	0.0245	0.3466	0.0707	4.526	0.5887	1.9800	19.010	51.090
3.08	0.0242	0.3452	0.0700	4.570	0.5875	1.9822	18.946	51.277
3.09	0.0238	0.3437	0.0692	4.613	0.5862	1.9844	18.882	51.464
3.10	0.0234	0.3422	0.0685	4.657	0.5850	1.9866	18.819	51.650
3.11	0.0231	0.3408	0.0678	4.702	0.5838	1.9888	18.756	51.835
3.12	0.0228	0.3393	0.0671	4.747	0.5825	1.9910	18.694	52.020
3.13	0.0224	0.3379	0.0664	4.792	0.5813	1.9931	18.632	52.203

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
3.14	0.0221	0.3365	0.0657	4.838	0.5801	1.9953	18.571	52.386
3.15	0.0218	0.3351	0.0650	4.884	0.5788	1.9974	18.509	52.569
3.16	0.0215	0.3337	0.0643	4.930	0.5776	1.9995	18.449	52.751
3.17	0.0211	0.3323	0.0636	4.977	0.5764	2.0016	18.388	52.932
3.18	0.0208	0.3309	0.0630	5.025	0.5752	2.0037	18.329	53.112
3.19	0.0205	0.3295	0.0623	5.073	0.5740	2.0058	18.269	53.291
3.20	0.0202	0.3281	0.0617	5.121	0.5728	2.0079	18.210	53.470
3.21	0.0199	0.3267	0.0610	5.170	0.5716	2.0099	18.151	53.649
3.22	0.0196	0.3253	0.0604	5.219	0.5704	2.0119	18.093	53.826
3.23	0.0194	0.3240	0.0597	5.268	0.5692	2.0140	18.035	54.003
3.24	0.0191	0.3226	0.0591	5.319	0.5680	2.0160	17.977	54.179
3.25	0.0188	0.3213	0.0585	5.369	0.5668	2.0180	17.920	54.355
3.26	0.0185	0.3199	0.0579	5.420	0.5656	2.0200	17.863	54.529
3.27	0.0183	0.3186	0.0573	5.472	0.5645	2.0220	17.807	54.704
3.28	0.0180	0.3173	0.0567	5.523	0.5633	2.0239	17.751	54.877
3.29	0.0177	0.3160	0.0561	5.576	0.5621	2.0259	17.695	55.050
3.30	0.0175	0.3147	0.0555	5.629	0.5609	2.0278	17.640	55.222
3.31	0.0172	0.3134	0.0550	5.682	0.5598	2.0297	17.585	55.393
3.32	0.0170	0.3121	0.0544	5.736	0.5586	2.0317	17.530	55.564
3.33	0.0167	0.3108	0.0538	5.790	0.5575	2.0336	17.476	55.734
3.34	0.0165	0.3095	0.0533	5.845	0.5563	2.0355	17.422	55.904
3.35	0.0163	0.3082	0.0527	5.900	0.5552	2.0373	17.368	56.073
3.36	0.0160	0.3069	0.0522	5.956	0.5540	2.0392	17.315	56.241
3.37	0.0158	0.3057	0.0517	6.012	0.5529	2.0411	17.262	56.409
3.38	0.0156	0.3044	0.0511	6.069	0.5517	2.0429	17.209	56.576
3.39	0.0153	0.3032	0.0506	6.126	0.5506	2.0447	17.157	56.742
3.40	0.0151	0.3019	0.0501	6.184	0.5495	2.0466	17.105	56.908
3.41	0.0149	0.3007	0.0496	6.242	0.5484	2.0484	17.053	57.073
3.42	0.0147	0.2995	0.0491	6.301	0.5472	2.0502	17.002	57.237
3.43	0.0145	0.2982	0.0486	6.360	0.5461	2.0520	16.950	57.401
3.44	0.0143	0.2970	0.0481	6.420	0.5450	2.0537	16.900	57.564
3.45	0.0141	0.2958	0.0476	6.480	0.5439	2.0555	16.849	57.726
3.46	0.0139	0.2946	0.0471	6.541	0.5428	2.0573	16.799	57.888
3.47	0.0137	0.2934	0.0466	6.602	0.5417	2.0590	16.749	58.050
3.48	0.0135	0.2922	0.0462	6.664	0.5406	2.0607	16.700	58.210
3.49	0.0133	0.2910	0.0457	6.727	0.5395	2.0625	16.651	58.370
3.50	0.0131	0.2899	0.0452	6.790	0.5384	2.0642	16.602	58.530
3.51	0.0129	0.2887	0.0448	6.853	0.5373	2.0659	16.553	58.689
3.52	0.0127	0.2875	0.0443	6.917	0.5362	2.0676	16.505	58.847
3.53	0.0126	0.2864	0.0439	6.982	0.5351	2.0693	16.456	59.005
3.54	0.0124	0.2852	0.0434	7.047	0.5340	2.0709	16.409	59.162

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
3.55	0.0122	0.2841	0.0430	7.113	0.5330	2.0726	16.361	59.318
3.56	0.0120	0.2829	0.0426	7.179	0.5319	2.0743	16.314	59.474
3.57	0.0119	0.2818	0.0421	7.246	0.5308	2.0759	16.267	59.629
3.58	0.0117	0.2806	0.0417	7.313	0.5298	2.0775	16.220	59.784
3.59	0.0115	0.2795	0.0413	7.381	0.5287	2.0792	16.174	59.938
3.60	0.0114	0.2784	0.0409	7.450	0.5276	2.0808	16.128	60.091
3.61	0.0112	0.2773	0.0405	7.519	0.5266	2.0824	16.082	60.244
3.62	0.0111	0.2762	0.0401	7.589	0.5255	2.0840	16.036	60.397
3.63	0.0109	0.2751	0.0397	7.659	0.5245	2.0856	15.991	60.549
3.64	0.0108	0.2740	0.0393	7.730	0.5234	2.0871	15.946	60.700
3.65	0.0106	0.2729	0.0389	7.802	0.5224	2.0887	15.901	60.850
3.66	0.0105	0.2718	0.0385	7.874	0.5213	2.0903	15.856	61.001
3.67	0.0103	0.2707	0.0381	7.947	0.5203	2.0918	15.812	61.150
3.68	0.0102	0.2697	0.0378	8.020	0.5193	2.0933	15.768	61.299
3.69	0.0100	0.2686	0.0374	8.094	0.5183	2.0949	15.724	61.447
3.70	0.0099	0.2675	0.0370	8.169	0.5172	2.0964	15.680	61.595
3.71	0.0098	0.2665	0.0367	8.244	0.5162	2.0979	15.637	61.743
3.72	0.0096	0.2654	0.0363	8.320	0.5152	2.0994	15.594	61.889
3.73	0.0095	0.2644	0.0359	8.397	0.5142	2.1009	15.551	62.036
3.74	0.0094	0.2633	0.0356	8.474	0.5132	2.1024	15.508	62.181
3.75	0.0092	0.2623	0.0352	8.552	0.5121	2.1039	15.466	62.326
3.76	0.0091	0.2613	0.0349	8.630	0.5111	2.1053	15.424	62.471
3.77	0.0090	0.2602	0.0345	8.709	0.5101	2.1068	15.382	62.615
3.78	0.0089	0.2592	0.0342	8.789	0.5091	2.1082	15.340	62.758
3.79	0.0087	0.2582	0.0339	8.869	0.5081	2.1097	15.299	62.901
3.80	0.0086	0.2572	0.0335	8.951	0.5072	2.1111	15.258	63.044
3.81	0.0085	0.2562	0.0332	9.032	0.5062	2.1125	15.217	63.186
3.82	0.0084	0.2552	0.0329	9.115	0.5052	2.1140	15.176	63.327
3.83	0.0083	0.2542	0.0326	9.198	0.5042	2.1154	15.135	63.468
3.84	0.0082	0.2532	0.0323	9.282	0.5032	2.1168	15.095	63.608
3.85	0.0081	0.2522	0.0320	9.366	0.5022	2.1182	15.055	63.748
3.86	0.0080	0.2513	0.0316	9.451	0.5013	2.1195	15.015	63.887
3.87	0.0078	0.2503	0.0313	9.537	0.5003	2.1209	14.975	64.026
3.88	0.0077	0.2493	0.0310	9.624	0.4993	2.1223	14.936	64.164
3.89	0.0076	0.2484	0.0307	9.711	0.4984	2.1236	14.896	64.302
3.90	0.0075	0.2474	0.0304	9.799	0.4974	2.1250	14.857	64.440
3.91	0.0074	0.2464	0.0302	9.888	0.4964	2.1263	14.818	64.576
3.92	0.0073	0.2455	0.0299	9.977	0.4955	2.1277	14.780	64.713
3.93	0.0072	0.2446	0.0296	10.067	0.4945	2.1290	14.741	64.848
3.94	0.0071	0.2436	0.0293	10.158	0.4936	2.1303	14.703	64.984
3.95	0.0070	0.2427	0.0290	10.250	0.4926	2.1316	14.665	65.118

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
3.96	0.0069	0.2418	0.0287	10.342	0.4917	2.1329	14.627	65.253
3.97	0.0069	0.2408	0.0285	10.435	0.4908	2.1342	14.589	65.386
3.98	0.0068	0.2399	0.0282	10.529	0.4898	2.1355	14.552	65.520
3.99	0.0067	0.2390	0.0279	10.623	0.4889	2.1368	14.515	65.652
4.00	0.0066	0.2381	0.0277	10.719	0.4880	2.1381	14.478	65.785
4.01	0.0065	0.2372	0.0274	10.815	0.4870	2.1394	14.441	65.917
4.02	0.0064	0.2363	0.0271	10.912	0.4861	2.1406	14.404	66.048
4.03	0.0063	0.2354	0.0269	11.009	0.4852	2.1419	14.367	66.179
4.04	0.0062	0.2345	0.0266	11.108	0.4843	2.1431	14.331	66.309
4.05	0.0062	0.2336	0.0264	11.207	0.4833	2.1444	14.295	66.439
4.06	0.0061	0.2327	0.0261	11.307	0.4824	2.1456	14.259	66.569
4.07	0.0060	0.2319	0.0259	11.408	0.4815	2.1468	14.223	66.698
4.08	0.0059	0.2310	0.0256	11.509	0.4806	2.1480	14.188	66.826
4.09	0.0058	0.2301	0.0254	11.611	0.4797	2.1493	14.152	66.954
4.10	0.0058	0.2293	0.0252	11.715	0.4788	2.1505	14.117	67.082
4.11	0.0057	0.2284	0.0249	11.819	0.4779	2.1517	14.082	67.209
4.12	0.0056	0.2275	0.0247	11.923	0.4770	2.1529	14.047	67.336
4.13	0.0055	0.2267	0.0245	12.029	0.4761	2.1540	14.012	67.462
4.14	0.0055	0.2258	0.0242	12.135	0.4752	2.1552	13.978	67.588
4.15	0.0054	0.2250	0.0240	12.243	0.4743	2.1564	13.943	67.713
4.16	0.0053	0.2242	0.0238	12.351	0.4735	2.1576	13.909	67.838
4.17	0.0053	0.2233	0.0236	12.460	0.4726	2.1587	13.875	67.963
4.18	0.0052	0.2225	0.0234	12.570	0.4717	2.1599	13.841	68.087
4.19	0.0051	0.2217	0.0231	12.680	0.4708	2.1610	13.808	68.210
4.20	0.0051	0.2208	0.0229	12.792	0.4699	2.1622	13.774	68.333
4.21	0.0050	0.2200	0.0227	12.904	0.4691	2.1633	13.741	68.456
4.22	0.0049	0.2192	0.0225	13.017	0.4682	2.1644	13.708	68.578
4.23	0.0049	0.2184	0.0223	13.131	0.4673	2.1655	13.675	68.700
4.24	0.0048	0.2176	0.0221	13.246	0.4665	2.1667	13.642	68.821
4.25	0.0047	0.2168	0.0219	13.362	0.4656	2.1678	13.609	68.942
4.26	0.0047	0.2160	0.0217	13.479	0.4648	2.1689	13.576	69.063
4.27	0.0046	0.2152	0.0215	13.597	0.4639	2.1700	13.544	69.183
4.28	0.0046	0.2144	0.0213	13.715	0.4631	2.1711	13.512	69.303
4.29	0.0045	0.2136	0.0211	13.835	0.4622	2.1721	13.480	69.422
4.30	0.0044	0.2129	0.0209	13.955	0.4614	2.1732	13.448	69.541
4.31	0.0044	0.2121	0.0207	14.076	0.4605	2.1743	13.416	69.659
4.32	0.0043	0.2113	0.0205	14.198	0.4597	2.1754	13.384	69.777
4.33	0.0043	0.2105	0.0203	14.322	0.4588	2.1764	13.353	69.895
4.34	0.0042	0.2098	0.0202	14.446	0.4580	2.1775	13.321	70.012
4.35	0.0042	0.2090	0.0200	14.571	0.4572	2.1785	13.290	70.129
4.36	0.0041	0.2083	0.0198	14.697	0.4563	2.1796	13.259	70.245

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
4.37	0.0041	0.2075	0.0196	14.823	0.4555	2.1806	13.228	70.361
4.38	0.0040	0.2067	0.0194	14.951	0.4547	2.1816	13.198	70.476
4.39	0.0040	0.2060	0.0193	15.080	0.4539	2.1827	13.167	70.591
4.40	0.0039	0.2053	0.0191	15.210	0.4531	2.1837	13.137	70.706
4.41	0.0039	0.2045	0.0189	15.341	0.4522	2.1847	13.106	70.820
4.42	0.0038	0.2038	0.0187	15.472	0.4514	2.1857	13.076	70.934
4.43	0.0038	0.2030	0.0186	15.605	0.4506	2.1867	13.046	71.048
4.44	0.0037	0.2023	0.0184	15.739	0.4498	2.1877	13.016	71.161
4.45	0.0037	0.2016	0.0182	15.873	0.4490	2.1887	12.986	71.274
4.46	0.0036	0.2009	0.0181	16.009	0.4482	2.1897	12.957	71.386
4.47	0.0036	0.2002	0.0179	16.146	0.4474	2.1907	12.927	71.498
4.48	0.0035	0.1994	0.0178	16.284	0.4466	2.1917	12.898	71.610
4.49	0.0035	0.1987	0.0176	16.422	0.4458	2.1926	12.869	71.721
4.50	0.0035	0.1980	0.0174	16.562	0.4450	2.1936	12.840	71.832
4.51	0.0034	0.1973	0.0173	16.703	0.4442	2.1946	12.811	71.942
4.52	0.0034	0.1966	0.0171	16.845	0.4434	2.1955	12.782	72.052
4.53	0.0033	0.1959	0.0170	16.988	0.4426	2.1965	12.753	72.162
4.54	0.0033	0.1952	0.0168	17.132	0.4418	2.1974	12.725	72.271
4.55	0.0032	0.1945	0.0167	17.277	0.4411	2.1984	12.696	72.380
4.56	0.0032	0.1938	0.0165	17.423	0.4403	2.1993	12.668	72.489
4.57	0.0032	0.1932	0.0164	17.570	0.4395	2.2002	12.640	72.597
4.58	0.0031	0.1925	0.0163	17.718	0.4387	2.2012	12.612	72.705
4.59	0.0031	0.1918	0.0161	17.867	0.4380	2.2021	12.584	72.812
4.60	0.0031	0.1911	0.0160	18.018	0.4372	2.2030	12.556	72.919
4.61	0.0030	0.1905	0.0158	18.169	0.4364	2.2039	12.528	73.026
4.62	0.0030	0.1898	0.0157	18.322	0.4357	2.2048	12.501	73.132
4.63	0.0029	0.1891	0.0156	18.476	0.4349	2.2057	12.473	73.238
4.64	0.0029	0.1885	0.0154	18.630	0.4341	2.2066	12.446	73.344
4.65	0.0029	0.1878	0.0153	18.786	0.4334	2.2075	12.419	73.449
4.66	0.0028	0.1872	0.0152	18.943	0.4326	2.2084	12.392	73.554
4.67	0.0028	0.1865	0.0150	19.101	0.4319	2.2093	12.365	73.659
4.68	0.0028	0.1859	0.0149	19.261	0.4311	2.2102	12.338	73.763
4.69	0.0027	0.1852	0.0148	19.421	0.4304	2.2110	12.311	73.867
4.70	0.0027	0.1846	0.0146	19.583	0.4296	2.2119	12.284	73.970
4.71	0.0027	0.1839	0.0145	19.746	0.4289	2.2128	12.258	74.073
4.72	0.0026	0.1833	0.0144	19.910	0.4281	2.2136	12.232	74.176
4.73	0.0026	0.1827	0.0143	20.075	0.4274	2.2145	12.205	74.279
4.74	0.0026	0.1820	0.0141	20.241	0.4267	2.2154	12.179	74.381
4.75	0.0025	0.1814	0.0140	20.408	0.4259	2.2162	12.153	74.482
4.76	0.0025	0.1808	0.0139	20.577	0.4252	2.2170	12.127	74.584
4.77	0.0025	0.1802	0.0138	20.747	0.4245	2.2179	12.101	74.685

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
4.78	0.0025	0.1795	0.0137	20.918	0.4237	2.2187	12.076	74.786
4.79	0.0024	0.1789	0.0135	21.090	0.4230	2.2196	12.050	74.886
4.80	0.0024	0.1783	0.0134	21.264	0.4223	2.2204	12.025	74.986
4.81	0.0024	0.1777	0.0133	21.438	0.4216	2.2212	11.999	75.086
4.82	0.0023	0.1771	0.0132	21.614	0.4208	2.2220	11.974	75.185
4.83	0.0023	0.1765	0.0131	21.792	0.4201	2.2228	11.949	75.285
4.84	0.0023	0.1759	0.0130	21.970	0.4194	2.2236	11.924	75.383
4.85	0.0023	0.1753	0.0129	22.150	0.4187	2.2245	11.899	75.482
4.86	0.0022	0.1747	0.0128	22.331	0.4180	2.2253	11.874	75.580
4.87	0.0022	0.1741	0.0126	22.513	0.4173	2.2261	11.849	75.678
4.88	0.0022	0.1735	0.0125	22.696	0.4166	2.2268	11.825	75.775
4.89	0.0022	0.1729	0.0124	22.881	0.4159	2.2276	11.800	75.872
4.90	0.0021	0.1724	0.0123	23.067	0.4152	2.2284	11.776	75.969
4.91	0.0021	0.1718	0.0122	23.254	0.4145	2.2292	11.751	76.066
4.92	0.0021	0.1712	0.0121	23.443	0.4138	2.2300	11.727	76.162
4.93	0.0021	0.1706	0.0120	23.633	0.4131	2.2308	11.703	76.258
4.94	0.0020	0.1700	0.0119	23.824	0.4124	2.2315	11.679	76.353
4.95	0.0020	0.1695	0.0118	24.017	0.4117	2.2323	11.655	76.449
4.96	0.0020	0.1689	0.0117	24.211	0.4110	2.2331	11.631	76.544
4.97	0.0020	0.1683	0.0116	24.406	0.4103	2.2338	11.608	76.638
4.98	0.0019	0.1678	0.0115	24.603	0.4096	2.2346	11.584	76.732
4.99	0.0019	0.1672	0.0114	24.801	0.4089	2.2353	11.560	76.826
5.00	0.0019	0.1667	0.0113	25.000	0.4082	2.2361	11.537	76.920

Note: In Table A.1  $\mu$  and  $\nu$  values are in degrees.

**Table A.2** Normal shock in perfect gas ( $\gamma = 1.4$ ).

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.01	0.9901	1.0234	1.0167	1.0066	1.0033	1.0000
1.02	0.9805	1.0471	1.0334	1.0132	1.0066	1.0000
1.03	0.9712	1.0710	1.0502	1.0198	1.0099	1.0000
1.04	0.9620	1.0952	1.0671	1.0263	1.0131	0.9999
1.05	0.9531	1.1196	1.0840	1.0328	1.0163	0.9999
1.06	0.9444	1.1442	1.1009	1.0393	1.0195	0.9998
1.07	0.9360	1.1690	1.1179	1.0458	1.0226	0.9996
1.08	0.9277	1.1941	1.1349	1.0522	1.0258	0.9994
1.09	0.9196	1.2194	1.1520	1.0586	1.0289	0.9992
1.10	0.9118	1.2450	1.1691	1.0649	1.0320	0.9989
1.11	0.9041	1.2708	1.1862	1.0713	1.0350	0.9986

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.12	0.8966	1.2968	1.2034	1.0776	1.0381	0.9982
1.13	0.8892	1.3230	1.2206	1.0840	1.0411	0.9978
1.14	0.8820	1.3495	1.2378	1.0903	1.0442	0.9973
1.15	0.8750	1.3762	1.2550	1.0966	1.0472	0.9967
1.16	0.8682	1.4032	1.2723	1.1029	1.0502	0.9961
1.17	0.8615	1.4304	1.2896	1.1092	1.0532	0.9953
1.18	0.8549	1.4578	1.3069	1.1154	1.0561	0.9946
1.19	0.8485	1.4854	1.3243	1.1217	1.0591	0.9937
1.20	0.8422	1.5133	1.3416	1.1280	1.0621	0.9928
1.21	0.8360	1.5414	1.3590	1.1343	1.0650	0.9918
1.22	0.8300	1.5698	1.3764	1.1405	1.0680	0.9907
1.23	0.8241	1.5984	1.3938	1.1468	1.0709	0.9896
1.24	0.8183	1.6272	1.4112	1.1531	1.0738	0.9884
1.25	0.8126	1.6562	1.4286	1.1594	1.0767	0.9871
1.26	0.8071	1.6855	1.4460	1.1657	1.0797	0.9857
1.27	0.8016	1.7150	1.4634	1.1720	1.0826	0.9842
1.28	0.7963	1.7448	1.4808	1.1783	1.0855	0.9827
1.29	0.7911	1.7748	1.4983	1.1846	1.0884	0.9811
1.30	0.7860	1.8050	1.5157	1.1909	1.0913	0.9794
1.31	0.7809	1.8354	1.5331	1.1972	1.0942	0.9776
1.32	0.7760	1.8661	1.5505	1.2035	1.0971	0.9758
1.33	0.7712	1.8970	1.5680	1.2099	1.0999	0.9738
1.34	0.7664	1.9282	1.5854	1.2162	1.1028	0.9718
1.35	0.7618	1.9596	1.6028	1.2226	1.1057	0.9697
1.36	0.7572	1.9912	1.6202	1.2290	1.1086	0.9676
1.37	0.7527	2.0230	1.6376	1.2354	1.1115	0.9653
1.38	0.7483	2.0551	1.6549	1.2418	1.1144	0.9630
1.39	0.7440	2.0874	1.6723	1.2482	1.1172	0.9607
1.40	0.7397	2.1200	1.6897	1.2547	1.1201	0.9582
1.41	0.7355	2.1528	1.7070	1.2612	1.1230	0.9557
1.42	0.7314	2.1858	1.7243	1.2676	1.1259	0.9531
1.43	0.7274	2.2190	1.7416	1.2741	1.1288	0.9504
1.44	0.7235	2.2525	1.7589	1.2807	1.1317	0.9476
1.45	0.7196	2.2862	1.7761	1.2872	1.1346	0.9448
1.46	0.7157	2.3202	1.7934	1.2938	1.1374	0.9420
1.47	0.7120	2.3544	1.8106	1.3003	1.1403	0.9390
1.48	0.7083	2.3888	1.8278	1.3069	1.1432	0.9360
1.49	0.7047	2.4234	1.8449	1.3136	1.1461	0.9329
1.50	0.7011	2.4583	1.8621	1.3202	1.1490	0.9298
1.51	0.6976	2.4934	1.8792	1.3269	1.1519	0.9266
1.52	0.6941	2.5288	1.8963	1.3336	1.1548	0.9233

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.53	0.6907	2.5644	1.9133	1.3403	1.1577	0.9200
1.54	0.6874	2.6002	1.9303	1.3470	1.1606	0.9166
1.55	0.6841	2.6362	1.9473	1.3538	1.1635	0.9132
1.56	0.6809	2.6725	1.9643	1.3606	1.1664	0.9097
1.57	0.6777	2.7090	1.9812	1.3674	1.1694	0.9062
1.58	0.6746	2.7458	1.9981	1.3742	1.1723	0.9026
1.59	0.6715	2.7828	2.0149	1.3811	1.1752	0.8989
1.60	0.6684	2.8200	2.0317	1.3880	1.1781	0.8952
1.61	0.6655	2.8574	2.0485	1.3949	1.1811	0.8915
1.62	0.6625	2.8951	2.0653	1.4018	1.1840	0.8877
1.63	0.6596	2.9330	2.0820	1.4088	1.1869	0.8838
1.64	0.6568	2.9712	2.0986	1.4158	1.1899	0.8799
1.65	0.6540	3.0096	2.1152	1.4228	1.1928	0.8760
1.66	0.6512	3.0482	2.1318	1.4299	1.1958	0.8720
1.67	0.6485	3.0870	2.1484	1.4369	1.1987	0.8680
1.68	0.6458	3.1261	2.1649	1.4440	1.2017	0.8639
1.69	0.6431	3.1654	2.1813	1.4512	1.2046	0.8599
1.70	0.6405	3.2050	2.1977	1.4583	1.2076	0.8557
1.71	0.6380	3.2448	2.2141	1.4655	1.2106	0.8516
1.72	0.6355	3.2848	2.2304	1.4727	1.2136	0.8474
1.73	0.6330	3.3250	2.2467	1.4800	1.2165	0.8431
1.74	0.6305	3.3655	2.2629	1.4873	1.2195	0.8389
1.75	0.6281	3.4062	2.2791	1.4946	1.2225	0.8346
1.76	0.6257	3.4472	2.2952	1.5019	1.2255	0.8302
1.77	0.6234	3.4884	2.3113	1.5093	1.2285	0.8259
1.78	0.6210	3.5298	2.3273	1.5167	1.2315	0.8215
1.79	0.6188	3.5714	2.3433	1.5241	1.2346	0.8171
1.80	0.6165	3.6133	2.3592	1.5316	1.2376	0.8127
1.81	0.6143	3.6554	2.3751	1.5391	1.2406	0.8082
1.82	0.6121	3.6978	2.3909	1.5466	1.2436	0.8038
1.83	0.6099	3.7404	2.4067	1.5541	1.2467	0.7993
1.84	0.6078	3.7832	2.4224	1.5617	1.2497	0.7948
1.85	0.6057	3.8262	2.4381	1.5693	1.2527	0.7902
1.86	0.6036	3.8695	2.4537	1.5770	1.2558	0.7857
1.87	0.6016	3.9130	2.4693	1.5847	1.2588	0.7811
1.88	0.5996	3.9568	2.4848	1.5924	1.2619	0.7765
1.89	0.5976	4.0008	2.5003	1.6001	1.2650	0.7720
1.90	0.5956	4.0450	2.5157	1.6079	1.2680	0.7674
1.91	0.5937	4.0894	2.5310	1.6157	1.2711	0.7627
1.92	0.5918	4.1341	2.5463	1.6236	1.2742	0.7581
1.93	0.5899	4.1791	2.5616	1.6314	1.2773	0.7535

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.94	0.5880	4.2242	2.5767	1.6394	1.2804	0.7488
1.95	0.5862	4.2696	2.5919	1.6473	1.2835	0.7442
1.96	0.5844	4.3152	2.6069	1.6553	1.2866	0.7395
1.97	0.5826	4.3610	2.6220	1.6633	1.2897	0.7349
1.98	0.5808	4.4071	2.6369	1.6713	1.2928	0.7302
1.99	0.5791	4.4534	2.6518	1.6794	1.2959	0.7255
2.00	0.5774	4.5000	2.6667	1.6875	1.2990	0.7209
2.01	0.5757	4.5468	2.6815	1.6956	1.3022	0.7162
2.02	0.5740	4.5938	2.6962	1.7038	1.3053	0.7115
2.03	0.5723	4.6410	2.7109	1.7120	1.3084	0.7069
2.04	0.5707	4.6885	2.7255	1.7203	1.3116	0.7022
2.05	0.5691	4.7362	2.7400	1.7285	1.3147	0.6975
2.06	0.5675	4.7842	2.7545	1.7369	1.3179	0.6928
2.07	0.5659	4.8324	2.7689	1.7452	1.3211	0.6882
2.08	0.5643	4.8808	2.7833	1.7536	1.3242	0.6835
2.09	0.5628	4.9294	2.7976	1.7620	1.3274	0.6789
2.10	0.5613	4.9783	2.8119	1.7704	1.3306	0.6742
2.11	0.5598	5.0274	2.8261	1.7789	1.3338	0.6696
2.12	0.5583	5.0768	2.8402	1.7875	1.3370	0.6649
2.13	0.5568	5.1264	2.8543	1.7960	1.3402	0.6603
2.14	0.5554	5.1762	2.8683	1.8046	1.3434	0.6557
2.15	0.5540	5.2262	2.8823	1.8132	1.3466	0.6511
2.16	0.5525	5.2765	2.8962	1.8219	1.3498	0.6464
2.17	0.5511	5.3270	2.9101	1.8306	1.3530	0.6419
2.18	0.5498	5.3778	2.9238	1.8393	1.3562	0.6373
2.19	0.5484	5.4288	2.9376	1.8481	1.3594	0.6327
2.20	0.5471	5.4800	2.9512	1.8569	1.3627	0.6281
2.21	0.5457	5.5314	2.9648	1.8657	1.3659	0.6236
2.22	0.5444	5.5831	2.9784	1.8746	1.3691	0.6191
2.23	0.5431	5.6350	2.9918	1.8835	1.3724	0.6145
2.24	0.5418	5.6872	3.0053	1.8924	1.3756	0.6100
2.25	0.5406	5.7396	3.0186	1.9014	1.3789	0.6055
2.26	0.5393	5.7922	3.0319	1.9104	1.3822	0.6011
2.27	0.5381	5.8450	3.0452	1.9194	1.3854	0.5966
2.28	0.5368	5.8981	3.0584	1.9285	1.3887	0.5921
2.29	0.5356	5.9514	3.0715	1.9376	1.3920	0.5877
2.30	0.5344	6.0050	3.0845	1.9468	1.3953	0.5833
2.31	0.5332	6.0588	3.0976	1.9560	1.3986	0.5789
2.32	0.5321	6.1128	3.1105	1.9652	1.4019	0.5745
2.33	0.5309	6.1670	3.1234	1.9745	1.4052	0.5702
2.34	0.5297	6.2215	3.1362	1.9838	1.4085	0.5658

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
2.35	0.5286	6.2762	3.1490	1.9931	1.4118	0.5615
2.36	0.5275	6.3312	3.1617	2.0025	1.4151	0.5572
2.37	0.5264	6.3864	3.1743	2.0119	1.4184	0.5529
2.38	0.5253	6.4418	3.1869	2.0213	1.4217	0.5486
2.39	0.5242	6.4974	3.1994	2.0308	1.4251	0.5444
2.40	0.5231	6.5533	3.2119	2.0403	1.4284	0.5401
2.41	0.5221	6.6094	3.2243	2.0499	1.4317	0.5359
2.42	0.5210	6.6658	3.2367	2.0595	1.4351	0.5317
2.43	0.5200	6.7224	3.2489	2.0691	1.4384	0.5276
2.44	0.5189	6.7792	3.2612	2.0788	1.4418	0.5234
2.45	0.5179	6.8362	3.2733	2.0885	1.4452	0.5193
2.46	0.5169	6.8935	3.2855	2.0982	1.4485	0.5152
2.47	0.5159	6.9510	3.2975	2.1080	1.4519	0.5111
2.48	0.5149	7.0088	3.3095	2.1178	1.4553	0.5071
2.49	0.5140	7.0668	3.3215	2.1276	1.4586	0.5030
2.50	0.5130	7.1250	3.3333	2.1375	1.4620	0.4990
2.51	0.5120	7.1834	3.3452	2.1474	1.4654	0.4950
2.52	0.5111	7.2421	3.3569	2.1574	1.4688	0.4911
2.53	0.5102	7.3010	3.3686	2.1674	1.4722	0.4871
2.54	0.5092	7.3602	3.3803	2.1774	1.4756	0.4832
2.55	0.5083	7.4196	3.3919	2.1875	1.4790	0.4793
2.56	0.5074	7.4792	3.4034	2.1976	1.4824	0.4754
2.57	0.5065	7.5390	3.4149	2.2077	1.4858	0.4715
2.58	0.5056	7.5991	3.4263	2.2179	1.4893	0.4677
2.59	0.5047	7.6594	3.4377	2.2281	1.4927	0.4639
2.60	0.5039	7.7200	3.4490	2.2383	1.4961	0.4601
2.61	0.5030	7.7808	3.4602	2.2486	1.4995	0.4564
2.62	0.5022	7.8418	3.4714	2.2590	1.5030	0.4526
2.63	0.5013	7.9030	3.4826	2.2693	1.5064	0.4489
2.64	0.5005	7.9645	3.4937	2.2797	1.5099	0.4452
2.65	0.4996	8.0262	3.5047	2.2902	1.5133	0.4416
2.66	0.4988	8.0882	3.5157	2.3006	1.5168	0.4379
2.67	0.4980	8.1504	3.5266	2.3111	1.5202	0.4343
2.68	0.4972	8.2128	3.5374	2.3217	1.5237	0.4307
2.69	0.4964	8.2754	3.5482	2.3323	1.5272	0.4271
2.70	0.4956	8.3383	3.5590	2.3429	1.5307	0.4236
2.71	0.4949	8.4014	3.5697	2.3536	1.5341	0.4201
2.72	0.4941	8.4648	3.5803	2.3642	1.5376	0.4166
2.73	0.4933	8.5284	3.5909	2.3750	1.5411	0.4131
2.74	0.4926	8.5922	3.6015	2.3858	1.5446	0.4097
2.75	0.4918	8.6562	3.6119	2.3966	1.5481	0.4062

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
2.76	0.4911	8.7205	3.6224	2.4074	1.5516	0.4028
2.77	0.4903	8.7850	3.6327	2.4183	1.5551	0.3994
2.78	0.4896	8.8498	3.6431	2.4292	1.5586	0.3961
2.79	0.4889	8.9148	3.6533	2.4402	1.5621	0.3928
2.80	0.4882	8.9800	3.6636	2.4512	1.5656	0.3895
2.81	0.4875	9.0454	3.6737	2.4622	1.5691	0.3862
2.82	0.4868	9.1111	3.6838	2.4733	1.5727	0.3829
2.83	0.4861	9.1770	3.6939	2.4844	1.5762	0.3797
2.84	0.4854	9.2432	3.7039	2.4955	1.5797	0.3765
2.85	0.4847	9.3096	3.7139	2.5067	1.5833	0.3733
2.86	0.4840	9.3762	3.7238	2.5179	1.5868	0.3701
2.87	0.4833	9.4430	3.7336	2.5292	1.5903	0.3670
2.88	0.4827	9.5101	3.7434	2.5405	1.5939	0.3639
2.89	0.4820	9.5774	3.7532	2.5518	1.5974	0.3608
2.90	0.4814	9.6450	3.7629	2.5632	1.6010	0.3577
2.91	0.4807	9.7128	3.7725	2.5746	1.6046	0.3547
2.92	0.4801	9.7808	3.7821	2.5861	1.6081	0.3517
2.93	0.4795	9.8490	3.7917	2.5976	1.6117	0.3487
2.94	0.4788	9.9175	3.8012	2.6091	1.6153	0.3457
2.95	0.4782	9.9862	3.8106	2.6206	1.6188	0.3428
2.96	0.4776	10.0552	3.8200	2.6322	1.6224	0.3398
2.97	0.4770	10.1244	3.8294	2.6439	1.6260	0.3369
2.98	0.4764	10.1938	3.8387	2.6555	1.6296	0.3340
2.99	0.4758	10.2634	3.8479	2.6673	1.6332	0.3312
3.00	0.4752	10.3333	3.8571	2.6790	1.6368	0.3283
3.01	0.4746	10.4034	3.8663	2.6908	1.6404	0.3255
3.02	0.4740	10.4738	3.8754	2.7026	1.6440	0.3227
3.03	0.4734	10.5444	3.8845	2.7145	1.6476	0.3200
3.04	0.4729	10.6152	3.8935	2.7264	1.6512	0.3172
3.05	0.4723	10.6862	3.9025	2.7383	1.6548	0.3145
3.06	0.4717	10.7575	3.9114	2.7503	1.6584	0.3118
3.07	0.4712	10.8290	3.9203	2.7623	1.6620	0.3091
3.08	0.4706	10.9008	3.9291	2.7744	1.6656	0.3065
3.09	0.4701	10.9728	3.9379	2.7865	1.6693	0.3038
3.10	0.4695	11.0450	3.9466	2.7986	1.6729	0.3012
3.11	0.4690	11.1174	3.9553	2.8108	1.6765	0.2986
3.12	0.4685	11.1901	3.9639	2.8230	1.6802	0.2960
3.13	0.4679	11.2630	3.9725	2.8352	1.6838	0.2935
3.14	0.4674	11.3362	3.9811	2.8475	1.6875	0.2910
3.15	0.4669	11.4096	3.9896	2.8598	1.6911	0.2885
3.16	0.4664	11.4832	3.9981	2.8722	1.6948	0.2860

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
3.17	0.4659	11.5570	4.0065	2.8846	1.6984	0.2835
3.18	0.4654	11.6311	4.0149	2.8970	1.7021	0.2811
3.19	0.4648	11.7054	4.0232	2.9095	1.7057	0.2786
3.20	0.4643	11.7800	4.0315	2.9220	1.7094	0.2762
3.21	0.4639	11.8548	4.0397	2.9345	1.7131	0.2738
3.22	0.4634	11.9298	4.0479	2.9471	1.7167	0.2715
3.23	0.4629	12.0050	4.0561	2.9598	1.7204	0.2691
3.24	0.4624	12.0805	4.0642	2.9724	1.7241	0.2668
3.25	0.4619	12.1562	4.0723	2.9851	1.7277	0.2645
3.26	0.4614	12.2322	4.0803	2.9979	1.7314	0.2622
3.27	0.4610	12.3084	4.0883	3.0106	1.7351	0.2600
3.28	0.4605	12.3848	4.0963	3.0234	1.7388	0.2577
3.29	0.4600	12.4614	4.1042	3.0363	1.7425	0.2555
3.30	0.4596	12.5383	4.1120	3.0492	1.7462	0.2533
3.31	0.4591	12.6154	4.1198	3.0621	1.7499	0.2511
3.32	0.4587	12.6928	4.1276	3.0751	1.7536	0.2489
3.33	0.4582	12.7704	4.1354	3.0881	1.7573	0.2468
3.34	0.4578	12.8482	4.1431	3.1011	1.7610	0.2446
3.35	0.4573	12.9262	4.1507	3.1142	1.7647	0.2425
3.36	0.4569	13.0045	4.1583	3.1273	1.7684	0.2404
3.37	0.4565	13.0830	4.1659	3.1405	1.7721	0.2383
3.38	0.4560	13.1618	4.1734	3.1537	1.7759	0.2363
3.39	0.4556	13.2408	4.1809	3.1669	1.7796	0.2342
3.40	0.4552	13.3200	4.1884	3.1802	1.7833	0.2322
3.41	0.4548	13.3994	4.1958	3.1935	1.7870	0.2302
3.42	0.4544	13.4791	4.2032	3.2069	1.7908	0.2282
3.43	0.4540	13.5590	4.2105	3.2203	1.7945	0.2263
3.44	0.4535	13.6392	4.2179	3.2337	1.7982	0.2243
3.45	0.4531	13.7196	4.2251	3.2472	1.8020	0.2224
3.46	0.4527	13.8002	4.2323	3.2607	1.8057	0.2205
3.47	0.4523	13.8810	4.2395	3.2742	1.8095	0.2186
3.48	0.4519	13.9621	4.2467	3.2878	1.8132	0.2167
3.49	0.4515	14.0434	4.2538	3.3014	1.8170	0.2148
3.50	0.4512	14.1250	4.2609	3.3150	1.8207	0.2129
3.51	0.4508	14.2068	4.2679	3.3287	1.8245	0.2111
3.52	0.4504	14.2888	4.2749	3.3425	1.8282	0.2093
3.53	0.4500	14.3710	4.2819	3.3562	1.8320	0.2075
3.54	0.4496	14.4535	4.2888	3.3701	1.8358	0.2057
3.55	0.4492	14.5362	4.2957	3.3839	1.8395	0.2039
3.56	0.4489	14.6192	4.3026	3.3978	1.8433	0.2022
3.57	0.4485	14.7024	4.3094	3.4117	1.8471	0.2004

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
3.58	0.4481	14.7858	4.3162	3.4257	1.8509	0.1987
3.59	0.4478	14.8694	4.3229	3.4397	1.8546	0.1970
3.60	0.4474	14.9533	4.3296	3.4537	1.8584	0.1953
3.61	0.4471	15.0374	4.3363	3.4678	1.8622	0.1936
3.62	0.4467	15.1218	4.3429	3.4819	1.8660	0.1920
3.63	0.4463	15.2064	4.3496	3.4961	1.8698	0.1903
3.64	0.4460	15.2912	4.3561	3.5103	1.8736	0.1887
3.65	0.4456	15.3762	4.3627	3.5245	1.8774	0.1871
3.66	0.4453	15.4615	4.3692	3.5388	1.8812	0.1855
3.67	0.4450	15.5470	4.3756	3.5531	1.8850	0.1839
3.68	0.4446	15.6328	4.3821	3.5674	1.8888	0.1823
3.69	0.4443	15.7188	4.3885	3.5818	1.8926	0.1807
3.70	0.4439	15.8050	4.3949	3.5962	1.8964	0.1792
3.71	0.4436	15.8914	4.4012	3.6107	1.9002	0.1777
3.72	0.4433	15.9781	4.4075	3.6252	1.9040	0.1761
3.73	0.4430	16.0650	4.4138	3.6397	1.9078	0.1746
3.74	0.4426	16.1522	4.4200	3.6543	1.9116	0.1731
3.75	0.4423	16.2396	4.4262	3.6689	1.9154	0.1717
3.76	0.4420	16.3272	4.4324	3.6836	1.9193	0.1702
3.77	0.4417	16.4150	4.4385	3.6983	1.9231	0.1687
3.78	0.4414	16.5031	4.4447	3.7130	1.9269	0.1673
3.79	0.4410	16.5914	4.4507	3.7278	1.9307	0.1659
3.80	0.4407	16.6800	4.4568	3.7426	1.9346	0.1645
3.81	0.4404	16.7688	4.4628	3.7575	1.9384	0.1631
3.82	0.4401	16.8578	4.4688	3.7723	1.9423	0.1617
3.83	0.4398	16.9470	4.4747	3.7873	1.9461	0.1603
3.84	0.4395	17.0365	4.4807	3.8022	1.9499	0.1589
3.85	0.4392	17.1262	4.4866	3.8172	1.9538	0.1576
3.86	0.4389	17.2162	4.4924	3.8323	1.9576	0.1563
3.87	0.4386	17.3064	4.4983	3.8473	1.9615	0.1549
3.88	0.4383	17.3968	4.5041	3.8625	1.9653	0.1536
3.89	0.4380	17.4874	4.5098	3.8776	1.9692	0.1523
3.90	0.4377	17.5783	4.5156	3.8928	1.9730	0.1510
3.91	0.4375	17.6694	4.5213	3.9080	1.9769	0.1497
3.92	0.4372	17.7608	4.5270	3.9233	1.9807	0.1485
3.93	0.4369	17.8524	4.5326	3.9386	1.9846	0.1472
3.94	0.4366	17.9442	4.5383	3.9540	1.9885	0.1460
3.95	0.4363	18.0362	4.5439	3.9694	1.9923	0.1448
3.96	0.4360	18.1285	4.5494	3.9848	1.9962	0.1435
3.97	0.4358	18.2210	4.5550	4.0003	2.0001	0.1423

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
3.98	0.4355	18.3138	4.5605	4.0157	2.0039	0.1411
3.99	0.4352	18.4068	4.5660	4.0313	2.0078	0.1399
4.00	0.4350	18.5000	4.5714	4.0469	2.0117	0.1388
4.01	0.4347	18.5934	4.5769	4.0625	2.0156	0.1376
4.02	0.4344	18.6871	4.5823	4.0781	2.0194	0.1364
4.03	0.4342	18.7810	4.5876	4.0938	2.0233	0.1353
4.04	0.4339	18.8752	4.5930	4.1096	2.0272	0.1342
4.05	0.4336	18.9696	4.5983	4.1253	2.0311	0.1330
4.06	0.4334	19.0642	4.6036	4.1412	2.0350	0.1319
4.07	0.4331	19.1590	4.6089	4.1570	2.0389	0.1308
4.08	0.4329	19.2541	4.6141	4.1729	2.0428	0.1297
4.09	0.4326	19.3494	4.6193	4.1888	2.0467	0.1286
4.10	0.4324	19.4450	4.6245	4.2048	2.0506	0.1276
4.11	0.4321	19.5408	4.6296	4.2208	2.0545	0.1265
4.12	0.4319	19.6368	4.6348	4.2368	2.0584	0.1254
4.13	0.4316	19.7330	4.6399	4.2529	2.0623	0.1244
4.14	0.4314	19.8295	4.6450	4.2690	2.0662	0.1234
4.15	0.4311	19.9262	4.6500	4.2852	2.0701	0.1223
4.16	0.4309	20.0232	4.6550	4.3014	2.0740	0.1213
4.17	0.4306	20.1204	4.6601	4.3176	2.0779	0.1203
4.18	0.4304	20.2178	4.6650	4.3339	2.0818	0.1193
4.19	0.4302	20.3155	4.6700	4.3502	2.0857	0.1183
4.20	0.4299	20.4133	4.6749	4.3666	2.0896	0.1173
4.21	0.4297	20.5115	4.6798	4.3830	2.0936	0.1164
4.22	0.4295	20.6098	4.6847	4.3994	2.0975	0.1154
4.23	0.4292	20.7084	4.6896	4.4159	2.1014	0.1144
4.24	0.4290	20.8072	4.6944	4.4324	2.1053	0.1135
4.25	0.4288	20.9063	4.6992	4.4489	2.1092	0.1126
4.26	0.4286	21.0056	4.7040	4.4655	2.1132	0.1116
4.27	0.4283	21.1051	4.7087	4.4821	2.1171	0.1107
4.28	0.4281	21.2048	4.7135	4.4988	2.1210	0.1098
4.29	0.4279	21.3048	4.7182	4.5155	2.1250	0.1089
4.30	0.4277	21.4050	4.7229	4.5322	2.1289	0.1080
4.31	0.4275	21.5055	4.7275	4.5490	2.1328	0.1071
4.32	0.4272	21.6062	4.7322	4.5658	2.1368	0.1062
4.33	0.4270	21.7071	4.7368	4.5827	2.1407	0.1054
4.34	0.4268	21.8083	4.7414	4.5995	2.1447	0.1045
4.35	0.4266	21.9096	4.7460	4.6165	2.1486	0.1036
4.36	0.4264	22.0113	4.7505	4.6335	2.1525	0.1028
4.37	0.4262	22.1131	4.7550	4.6505	2.1565	0.1020
4.38	0.4260	22.2152	4.7595	4.6675	2.1604	0.1011

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
4.39	0.4258	22.3175	4.7640	4.6846	2.1644	0.1003
4.40	0.4255	22.4201	4.7685	4.7017	2.1683	0.0995
4.41	0.4253	22.5229	4.7729	4.7189	2.1723	0.0987
4.42	0.4251	22.6259	4.7773	4.7361	2.1763	0.0979
4.43	0.4249	22.7291	4.7817	4.7533	2.1802	0.0971
4.44	0.4247	22.8326	4.7861	4.7706	2.1842	0.0963
4.45	0.4245	22.9363	4.7904	4.7879	2.1881	0.0955
4.46	0.4243	23.0403	4.7948	4.8053	2.1921	0.0947
4.47	0.4241	23.1445	4.7991	4.8227	2.1961	0.0940
4.48	0.4239	23.2489	4.8034	4.8401	2.2000	0.0932
4.49	0.4237	23.3535	4.8076	4.8576	2.2040	0.0924
4.50	0.4236	23.4584	4.8119	4.8751	2.2080	0.0917
4.51	0.4234	23.5635	4.8161	4.8926	2.2119	0.0910
4.52	0.4232	23.6689	4.8203	4.9102	2.2159	0.0902
4.53	0.4230	23.7745	4.8245	4.9279	2.2199	0.0895
4.54	0.4228	23.8803	4.8287	4.9455	2.2239	0.0888
4.55	0.4226	23.9864	4.8328	4.9632	2.2278	0.0881
4.56	0.4224	24.0926	4.8369	4.9810	2.2318	0.0874
4.57	0.4222	24.1992	4.8410	4.9988	2.2358	0.0867
4.58	0.4220	24.3059	4.8451	5.0166	2.2398	0.0860
4.59	0.4219	24.4129	4.8492	5.0344	2.2438	0.0853
4.60	0.4217	24.5201	4.8532	5.0523	2.2477	0.0846
4.61	0.4215	24.6276	4.8572	5.0703	2.2517	0.0839
4.62	0.4213	24.7353	4.8612	5.0883	2.2557	0.0832
4.63	0.4211	24.8432	4.8652	5.1063	2.2597	0.0826
4.64	0.4210	24.9513	4.8692	5.1243	2.2637	0.0819
4.65	0.4208	25.0597	4.8731	5.1424	2.2677	0.0813
4.66	0.4206	25.1683	4.8771	5.1605	2.2717	0.0806
4.67	0.4204	25.2772	4.8810	5.1787	2.2757	0.0800
4.68	0.4203	25.3863	4.8849	5.1969	2.2797	0.0793
4.69	0.4201	25.4956	4.8887	5.2152	2.2837	0.0787
4.70	0.4199	25.6051	4.8926	5.2335	2.2877	0.0781
4.71	0.4197	25.7149	4.8964	5.2518	2.2917	0.0775
4.72	0.4196	25.8249	4.9002	5.2701	2.2957	0.0769
4.73	0.4194	25.9352	4.9040	5.2885	2.2997	0.0762
4.74	0.4192	26.0457	4.9078	5.3070	2.3037	0.0756
4.75	0.4191	26.1564	4.9116	5.3255	2.3077	0.0750
4.76	0.4189	26.2673	4.9153	5.3440	2.3117	0.0745
4.77	0.4187	26.3785	4.9190	5.3625	2.3157	0.0739
4.78	0.4186	26.4900	4.9227	5.3811	2.3197	0.0733
4.79	0.4184	26.6016	4.9264	5.3998	2.3237	0.0727

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
4.80	0.4183	26.7135	4.9301	5.4184	2.3277	0.0721
4.81	0.4181	26.8256	4.9338	5.4372	2.3318	0.0716
4.82	0.4179	26.9380	4.9374	5.4559	2.3358	0.0710
4.83	0.4178	27.0505	4.9410	5.4747	2.3398	0.0705
4.84	0.4176	27.1634	4.9446	5.4935	2.3438	0.0699
4.85	0.4175	27.2764	4.9482	5.5124	2.3478	0.0694
4.86	0.4173	27.3897	4.9518	5.5313	2.3519	0.0688
4.87	0.4172	27.5032	4.9553	5.5502	2.3559	0.0683
4.88	0.4170	27.6170	4.9589	5.5692	2.3599	0.0677
4.89	0.4169	27.7310	4.9624	5.5882	2.3639	0.0672
4.90	0.4167	27.8452	4.9659	5.6073	2.3680	0.0667
4.91	0.4165	27.9596	4.9694	5.6264	2.3720	0.0662
4.92	0.4164	28.0743	4.9728	5.6455	2.3760	0.0657
4.93	0.4162	28.1893	4.9763	5.6647	2.3801	0.0652
4.94	0.4161	28.3044	4.9797	5.6839	2.3841	0.0647
4.95	0.4160	28.4198	4.9831	5.7032	2.3881	0.0642
4.96	0.4158	28.5354	4.9865	5.7225	2.3922	0.0637
4.97	0.4157	28.6513	4.9899	5.7418	2.3962	0.0632
4.98	0.4155	28.7673	4.9933	5.7612	2.4002	0.0627
4.99	0.4154	28.8837	4.9967	5.7806	2.4043	0.0622
5.00	0.4152	29.0002	5.0000	5.8000	2.4083	0.0617

**Table A.3** Oblique shock in perfect gas ( $\gamma = 1.4$ ).

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.05	0	72.07	0.998	1.052	89.66	1.120	0.953
1.10	0	65.28	0.998	1.101	89.83	1.245	0.912
1.10	1	69.80	1.077	1.039	83.57	1.227	0.925
1.15	0	60.34	0.998	1.151	89.89	1.376	0.875
1.15	1	63.16	1.062	1.102	85.98	1.369	0.880
1.15	2	67.00	1.141	1.043	81.17	1.340	0.901
1.20	0	56.39	0.998	1.201	89.92	1.513	0.842
1.20	1	58.55	1.056	1.158	87.04	1.509	0.845
1.20	2	61.05	1.120	1.111	83.86	1.494	0.855
1.20	3	64.34	1.198	1.056	80.03	1.463	0.876
1.25	0	53.08	0.999	1.251	89.94	1.656	0.813
1.25	1	54.88	1.053	1.211	87.65	1.653	0.815
1.25	2	56.85	1.111	1.170	85.21	1.644	0.821

(continued)

**Table A.4** One-dimensional flow with friction ( $\gamma = 1.4$ ).

$M$	$T/T^*$	$p/p^*$	$p_0/p_0^*$	$\rho^*/\rho$	$F/F^*$	$4 f L_{\max}/D$
0.02	1.19990	54.77007	28.94214	0.02191	22.83364	1778.4498
0.04	1.19962	27.38175	14.48149	0.04381	11.43462	440.3522
0.06	1.19914	18.25085	9.66591	0.06570	7.64285	193.0311
0.08	1.19847	13.68431	7.26161	0.08758	5.75288	106.7182
0.10	1.19760	10.94351	5.82183	0.10944	4.62363	66.9216
0.12	1.19655	9.11559	4.86432	0.13126	3.87473	45.4080
0.14	1.19531	7.80932	4.18240	0.15306	3.34317	32.5113
0.16	1.19389	6.82907	3.67274	0.17482	2.94743	24.1978
0.18	1.19227	6.06618	3.27793	0.19654	2.64223	18.5427
0.20	1.19048	5.45545	2.96352	0.21822	2.40040	14.5333
0.22	1.18850	4.95537	2.70760	0.23984	2.20464	11.5961
0.24	1.18633	4.53829	2.49556	0.26141	2.04344	9.3865
0.26	1.18399	4.18506	2.31729	0.28291	1.90880	7.6876
0.28	1.18147	3.88199	2.16555	0.30435	1.79503	6.3572
0.30	1.17878	3.61906	2.03506	0.32572	1.69794	5.2993
0.32	1.17592	3.38874	1.92185	0.34701	1.61440	4.4467
0.34	1.17288	3.18529	1.82288	0.36822	1.54200	3.7520
0.36	1.16968	3.00422	1.73578	0.38935	1.47888	3.1801
0.38	1.16632	2.84200	1.65870	0.41039	1.42356	2.7054
0.40	1.16279	2.69582	1.59014	0.43133	1.37487	2.3085
0.42	1.15911	2.56338	1.52890	0.45218	1.33184	1.9744
0.44	1.15527	2.44280	1.47400	0.47293	1.29371	1.6915
0.46	1.15128	2.33256	1.42463	0.49357	1.25981	1.4509
0.48	1.14714	2.23135	1.38010	0.51410	1.22962	1.2453
0.50	1.14286	2.13809	1.33984	0.53452	1.20268	1.0691
0.52	1.13843	2.05187	1.30339	0.55483	1.17860	0.9174
0.54	1.13387	1.97192	1.27032	0.57501	1.15705	0.7866
0.56	1.12918	1.89755	1.24029	0.59507	1.13777	0.6736
0.58	1.12435	1.82820	1.21301	0.61501	1.12050	0.5757
0.60	1.11940	1.76336	1.18820	0.63481	1.10504	0.4908
0.62	1.11433	1.70261	1.16565	0.65448	1.09120	0.4172
0.64	1.10914	1.64556	1.14515	0.67402	1.07883	0.3533
0.66	1.10383	1.59187	1.12653	0.69342	1.06777	0.2979
0.68	1.09842	1.54126	1.10965	0.71268	1.05792	0.2498
0.70	1.09290	1.49345	1.09437	0.73179	1.04915	0.2081
0.72	1.08727	1.44823	1.08057	0.75076	1.04137	0.1721
0.74	1.08155	1.40537	1.06814	0.76958	1.03449	0.1411
0.76	1.07573	1.36470	1.05700	0.78825	1.02844	0.1145
0.78	1.06982	1.32606	1.04705	0.80677	1.02314	0.0917
0.80	1.06383	1.28928	1.03823	0.82514	1.01853	0.0723
0.82	1.05775	1.25423	1.03046	0.84335	1.01455	0.0559
0.84	1.05160	1.22080	1.02370	0.86140	1.01115	0.0423

(continued)

**Table A.4** (Continued)

<i>M</i>	<i>T/T*</i>	<i>p/p*</i>	<i>p<sub>0</sub>/p<sub>0</sub>*</i>	<i>p*/ρ</i>	<i>F/F*</i>	<i>4 fL<sub>max</sub>/D</i>
0.86	1.04537	1.18888	1.01787	0.87929	1.00829	0.0310
0.88	1.03907	1.15835	1.01294	0.89703	1.00591	0.0218
0.90	1.03270	1.12913	1.00886	0.91460	1.00399	0.0145
0.92	1.02627	1.10114	1.00560	0.93201	1.00248	0.0089
0.94	1.01978	1.07430	1.00311	0.94925	1.00136	0.0048
0.96	1.01324	1.04854	1.00136	0.96633	1.00059	0.0026
0.98	1.00664	1.02379	1.00034	0.98325	1.00014	0.0005
1.00	1.00000	1.00000	1.00000	1.00000	1.00000	0.0000
1.02	0.99331	0.97711	1.00033	1.01658	1.00014	0.0005
1.04	0.98658	0.95507	1.00130	1.03300	1.00053	0.0018
1.06	0.97982	0.93383	1.00291	1.04925	1.00116	0.0038
1.08	0.97302	0.91335	1.00512	1.06533	1.00200	0.0066
1.10	0.96618	0.89359	1.00792	1.08124	1.00305	0.0099
1.12	0.95932	0.87451	1.01131	1.09698	1.00429	0.0138
1.14	0.95244	0.85608	1.01527	1.11256	1.00569	0.0182
1.16	0.94554	0.83827	1.01978	1.12797	1.00726	0.0230
1.18	0.93861	0.82104	1.02484	1.14321	1.00897	0.0281
1.20	0.93168	0.80436	1.03044	1.15828	1.01081	0.0336
1.22	0.92473	0.78822	1.03657	1.17319	1.01278	0.0394
1.24	0.91777	0.77258	1.04323	1.18792	1.01486	0.0455
1.26	0.91080	0.75743	1.05041	1.20249	1.01705	0.0517
1.28	0.90383	0.74274	1.05810	1.21690	1.01933	0.0582
1.30	0.89686	0.72848	1.06630	1.23114	1.02170	0.0648
1.32	0.88989	0.71465	1.07502	1.24521	1.02414	0.0716
1.34	0.88292	0.70122	1.08424	1.25912	1.02666	0.0785
1.36	0.87596	0.68818	1.09396	1.27286	1.02925	0.0855
1.38	0.86901	0.67551	1.10419	1.28645	1.03189	0.0926
1.40	0.86207	0.66320	1.11493	1.29987	1.03459	0.0997
1.42	0.85514	0.65122	1.12616	1.31313	1.03733	0.1069
1.44	0.84822	0.63958	1.13790	1.32623	1.04012	0.1142
1.46	0.84133	0.62825	1.15015	1.33917	1.04295	0.1215
1.48	0.83445	0.61722	1.16290	1.35195	1.04581	0.1288
1.50	0.82759	0.60648	1.17617	1.36458	1.04870	0.1360
1.52	0.82075	0.59602	1.18994	1.37705	1.05162	0.1433
1.54	0.81393	0.58583	1.20423	1.38936	1.05456	0.1506
1.56	0.80715	0.57591	1.21904	1.40152	1.05752	0.1579
1.58	0.80038	0.56623	1.23438	1.41353	1.06049	0.1651
1.60	0.79365	0.55679	1.25023	1.42539	1.06348	0.1724
1.62	0.78695	0.54759	1.26662	1.43710	1.06647	0.1795
1.64	0.78028	0.53862	1.28355	1.44866	1.06948	0.1867
1.66	0.77363	0.52986	1.30102	1.46008	1.07249	0.1938

(continued)

**Table A.4** (Continued)

<i>M</i>	<i>T/T*</i>	<i>p/p*</i>	<i>p<sub>0</sub>/p<sub>0</sub>*</i>	<i>p*/p</i>	<i>F/F*</i>	<i>4 fL<sub>max</sub>/D</i>
1.68	0.76703	0.52131	1.31904	1.47135	1.07550	0.2008
1.70	0.76046	0.51297	1.33761	1.48247	1.07851	0.2078
1.72	0.75392	0.50482	1.35673	1.49345	1.08152	0.2147
1.74	0.74742	0.49686	1.37643	1.50429	1.08453	0.2216
1.76	0.74096	0.48909	1.39670	1.51499	1.08753	0.2284
1.78	0.73454	0.48149	1.41754	1.52555	1.09053	0.2352
1.80	0.72816	0.47407	1.43898	1.53598	1.09351	0.2419
1.82	0.72181	0.46681	1.46101	1.54626	1.09649	0.2485
1.84	0.71551	0.45972	1.48365	1.55642	1.09946	0.2551
1.86	0.70925	0.45278	1.50689	1.56644	1.10241	0.2616
1.88	0.70304	0.44600	1.53076	1.57633	1.10536	0.2680
1.90	0.69686	0.43936	1.55525	1.58609	1.10829	0.2743
1.92	0.69074	0.43287	1.58039	1.59572	1.11120	0.2806
1.94	0.68465	0.42651	1.60617	1.60523	1.11410	0.2868
1.96	0.67861	0.42030	1.63261	1.61460	1.11698	0.2929
1.98	0.67262	0.41421	1.65972	1.62386	1.11984	0.2990
2.00	0.66667	0.40825	1.68750	1.63299	1.12268	0.3050
2.02	0.66076	0.40241	1.71597	1.64200	1.12551	0.3109
2.04	0.65491	0.39670	1.74514	1.65090	1.12831	0.3168
2.06	0.64910	0.39110	1.77501	1.65967	1.13110	0.3225
2.08	0.64334	0.38562	1.80561	1.66833	1.13387	0.3282
2.10	0.63762	0.38024	1.83694	1.67687	1.13661	0.3339
2.12	0.63195	0.37498	1.86902	1.68530	1.13933	0.3394
2.14	0.62633	0.36982	1.90184	1.69362	1.14204	0.3449
2.16	0.62076	0.36476	1.93543	1.70182	1.14471	0.3503
2.18	0.61523	0.35980	1.96981	1.70992	1.14737	0.3556
2.20	0.60976	0.35494	2.00497	1.71791	1.15001	0.3609
2.22	0.60433	0.35017	2.04094	1.72579	1.15262	0.3661
2.24	0.59895	0.34550	2.07773	1.73357	1.15521	0.3712
2.26	0.59361	0.34091	2.11535	1.74125	1.15777	0.3763
2.28	0.58833	0.33641	2.15381	1.74882	1.16032	0.3813
2.30	0.58309	0.33200	2.19313	1.75629	1.16284	0.3862
2.32	0.57790	0.32767	2.23332	1.76366	1.16533	0.3911
2.34	0.57276	0.32342	2.27440	1.77093	1.16780	0.3959
2.36	0.56767	0.31925	2.31638	1.77811	1.17025	0.4006
2.38	0.56262	0.31516	2.35927	1.78519	1.17268	0.4053
2.40	0.55762	0.31114	2.40310	1.79218	1.17508	0.4099
2.42	0.55267	0.30720	2.44787	1.79907	1.17746	0.4144
2.44	0.54777	0.30332	2.49360	1.80587	1.17981	0.4189
2.46	0.54291	0.29952	2.54031	1.81258	1.18214	0.4233
2.48	0.53810	0.29579	2.58801	1.81921	1.18445	0.4277

(continued)

**Table A.4** (Continued)

<i>M</i>	<i>T/T*</i>	<i>p/p*</i>	<i>p<sub>0</sub>/p<sub>0</sub>*</i>	<i>ρ*/ρ</i>	<i>F/F*</i>	<i>4fL<sub>max</sub>/D</i>
2.50	0.53333	0.29212	2.63671	1.82574	1.18673	0.4320
2.52	0.52862	0.28852	2.68645	1.83219	1.18899	0.4362
2.54	0.52394	0.28498	2.73722	1.83855	1.19123	0.4404
2.56	0.51932	0.28150	2.78906	1.84483	1.19344	0.4445
2.58	0.51474	0.27808	2.84197	1.85103	1.19563	0.4486
2.60	0.51020	0.27473	2.89597	1.85714	1.19780	0.4526
2.62	0.50572	0.27143	2.95108	1.86318	1.19995	0.4565
2.64	0.50127	0.26818	3.00733	1.86913	1.20207	0.4604
2.66	0.49687	0.26500	3.06471	1.87501	1.20417	0.4643
2.68	0.49251	0.26186	3.12327	1.88081	1.20625	0.4681
2.70	0.48820	0.25878	3.18300	1.88653	1.20830	0.4718
2.72	0.48393	0.25576	3.24394	1.89218	1.21033	0.4755
2.74	0.47971	0.25278	3.30611	1.89775	1.21235	0.4791
2.76	0.47553	0.24985	3.36951	1.90325	1.21433	0.4827
2.78	0.47139	0.24697	3.43417	1.90868	1.21630	0.4863
2.80	0.46729	0.24414	3.50012	1.91404	1.21825	0.4898
2.82	0.46324	0.24135	3.56736	1.91933	1.22017	0.4932
2.84	0.45922	0.23861	3.63593	1.92455	1.22208	0.4966
2.86	0.45525	0.23592	3.70584	1.92970	1.22396	0.5000
2.88	0.45132	0.23326	3.77711	1.93479	1.22582	0.5033
2.90	0.44743	0.23066	3.84976	1.93981	1.22766	0.5065
2.92	0.44358	0.22809	3.92382	1.94477	1.22948	0.5097
2.94	0.43977	0.22556	3.99931	1.94966	1.23128	0.5129
2.96	0.43600	0.22307	4.07625	1.95449	1.23307	0.5160
2.98	0.43226	0.22063	4.15465	1.95925	1.23483	0.5191
3.00	0.42857	0.21822	4.23456	1.96396	1.23657	0.5222
3.02	0.42492	0.21585	4.31598	1.96861	1.23829	0.5252
3.04	0.42130	0.21351	4.39894	1.97319	1.23999	0.5281
3.06	0.41772	0.21121	4.48347	1.97772	1.24168	0.5310
3.08	0.41418	0.20895	4.56958	1.98219	1.24334	0.5339
3.10	0.41068	0.20672	4.65730	1.98661	1.24499	0.5368
3.12	0.40721	0.20453	4.74666	1.99097	1.24662	0.5396
3.14	0.40378	0.20237	4.83768	1.99527	1.24823	0.5424
3.16	0.40038	0.20024	4.93038	1.99952	1.24982	0.5451
3.18	0.39703	0.19814	5.02480	2.00371	1.25139	0.5478
3.20	0.39370	0.19608	5.12095	2.00786	1.25295	0.5504
3.22	0.39041	0.19405	5.21886	2.01195	1.25449	0.5531
3.24	0.38716	0.19204	5.31855	2.01599	1.25601	0.5557
3.26	0.38394	0.19007	5.42006	2.01998	1.25752	0.5582
3.28	0.38075	0.18812	5.52342	2.02392	1.25901	0.5607
3.30	0.37760	0.18621	5.62863	2.02781	1.26048	0.5632

(continued)

**Table A.4** (Continued)

<i>M</i>	<i>T/T*</i>	<i>p/p*</i>	<i>p<sub>0</sub>/p<sub>0</sub>*</i>	<i>ρ*/ρ</i>	<i>F/F*</i>	<i>4 fL<sub>max</sub>/D</i>
3.32	0.37448	0.18432	5.73574	2.03165	1.26193	0.5657
3.34	0.37139	0.18246	5.84478	2.03545	1.26337	0.5681
3.36	0.36833	0.18063	5.95576	2.03920	1.26479	0.5705
3.38	0.36531	0.17882	6.06872	2.04290	1.26620	0.5729
3.40	0.36232	0.17704	6.18368	2.04656	1.26759	0.5752
3.42	0.35936	0.17528	6.30068	2.05017	1.26897	0.5775
3.44	0.35643	0.17355	6.41974	2.05374	1.27033	0.5798
3.46	0.35353	0.17185	6.54090	2.05727	1.27167	0.5820
3.48	0.35066	0.17016	6.66418	2.06075	1.27300	0.5842
3.50	0.34783	0.16851	6.78961	2.06419	1.27432	0.5864
3.52	0.34502	0.16687	6.91721	2.06759	1.27562	0.5886
3.54	0.34224	0.16526	7.04704	2.07094	1.27691	0.5907
3.56	0.33949	0.16367	7.17911	2.07426	1.27818	0.5928
3.58	0.33677	0.16210	7.31345	2.07754	1.27944	0.5949
3.60	0.33408	0.16055	7.45010	2.08077	1.28068	0.5970
3.62	0.33141	0.15903	7.58908	2.08397	1.28191	0.5990
3.64	0.32877	0.15752	7.73043	2.08713	1.28313	0.6010
3.66	0.32616	0.15604	7.87419	2.09026	1.28433	0.6030
3.68	0.32358	0.15458	8.02038	2.09334	1.28552	0.6049
3.70	0.32103	0.15313	8.16904	2.09639	1.28670	0.6068
3.72	0.31850	0.15171	8.32021	2.09941	1.28787	0.6087
3.74	0.31600	0.15030	8.47391	2.10238	1.28902	0.6106
3.76	0.31352	0.14892	8.63018	2.10533	1.29016	0.6125
3.78	0.31107	0.14755	8.78905	2.10824	1.29128	0.6143
3.80	0.30864	0.14620	8.95057	2.11111	1.29240	0.6161
3.82	0.30624	0.14487	9.11475	2.11395	1.29350	0.6179
3.84	0.30387	0.14355	9.28164	2.11676	1.29459	0.6197
3.86	0.30151	0.14225	9.45128	2.11954	1.29567	0.6214
3.88	0.29919	0.14097	9.62371	2.12228	1.29674	0.6231
3.90	0.29688	0.13971	9.79895	2.12499	1.29779	0.6248
3.92	0.29460	0.13846	9.97704	2.12767	1.29883	0.6265
3.94	0.29235	0.13723	10.15803	2.13032	1.29987	0.6282
3.96	0.29011	0.13602	10.34194	2.13294	1.30089	0.6298
3.98	0.28790	0.13482	10.52883	2.13553	1.30190	0.6315
4.00	0.28571	0.13363	10.71872	2.13809	1.30290	0.6331
4.02	0.28355	0.13246	10.91166	2.14062	1.30389	0.6346
4.04	0.28140	0.13131	11.10768	2.14312	1.30487	0.6362
4.06	0.27928	0.13017	11.30681	2.14560	1.30583	0.6378
4.08	0.27718	0.12904	11.50912	2.14804	1.30679	0.6393
4.10	0.27510	0.12793	11.71463	2.15046	1.30774	0.6408
4.12	0.27304	0.12683	11.92337	2.15285	1.30868	0.6423
4.14	0.27101	0.12574	12.13540	2.15522	1.30960	0.6438

(continued)

**Table A.4** (Continued)

<i>M</i>	<i>T/T*</i>	<i>p/p*</i>	<i>p<sub>0</sub>/p<sub>0</sub>*</i>	<i>p*/p</i>	<i>F/F*</i>	<i>4 fL<sub>max</sub>/D</i>
4.16	0.26899	0.12467	12.35076	2.15756	1.31052	0.6452
4.18	0.26699	0.12362	12.56947	2.15987	1.31143	0.6467
4.20	0.26502	0.12257	12.79160	2.16215	1.31233	0.6481
4.22	0.26306	0.12154	13.01719	2.16442	1.31322	0.6495
4.24	0.26112	0.12052	13.24626	2.16665	1.31410	0.6509
4.26	0.25921	0.11951	13.47888	2.16886	1.31497	0.6523
4.28	0.25731	0.11852	13.71505	2.17105	1.31583	0.6536
4.30	0.25543	0.11753	13.95487	2.17321	1.31668	0.6550
4.32	0.25357	0.11656	14.19835	2.17535	1.31752	0.6563
4.34	0.25172	0.11560	14.44554	2.17747	1.31836	0.6576
4.36	0.24990	0.11466	14.69648	2.17956	1.31919	0.6589
4.38	0.24809	0.11372	14.95123	2.18163	1.32000	0.6602
4.40	0.24631	0.11279	15.20983	2.18368	1.32081	0.6615
4.42	0.24453	0.11188	15.47233	2.18571	1.32161	0.6627
4.44	0.24278	0.11097	15.73875	2.18771	1.32241	0.6640
4.46	0.24105	0.11008	16.00916	2.18970	1.32319	0.6652
4.48	0.23933	0.10920	16.28361	2.19166	1.32397	0.6664
4.50	0.23762	0.10833	16.56215	2.19360	1.32474	0.6676
4.52	0.23594	0.10746	16.84483	2.19552	1.32550	0.6688
4.54	0.23427	0.10661	17.13165	2.19742	1.32625	0.6700
4.56	0.23262	0.10577	17.42273	2.19930	1.32700	0.6712
4.58	0.23098	0.10494	17.71807	2.20116	1.32773	0.6723
4.60	0.22936	0.10411	18.01775	2.20300	1.32846	0.6734
4.62	0.22775	0.10330	18.32179	2.20482	1.32919	0.6746
4.64	0.22616	0.10249	18.63027	2.20662	1.32990	0.6757
4.66	0.22459	0.10170	18.94323	2.20841	1.33061	0.6768
4.68	0.22303	0.10091	19.26071	2.21017	1.33131	0.6779
4.70	0.22148	0.10013	19.58277	2.21192	1.33201	0.6790
4.72	0.21995	0.09936	19.90947	2.21365	1.33269	0.6800
4.74	0.21844	0.09860	20.24085	2.21536	1.33338	0.6811
4.76	0.21694	0.09785	20.57698	2.21705	1.33405	0.6821
4.78	0.21545	0.09711	20.91790	2.21872	1.33472	0.6831
4.80	0.21398	0.09637	21.26365	2.22038	1.33538	0.6842
4.82	0.21252	0.09564	21.61431	2.22202	1.33603	0.6852
4.84	0.21108	0.09492	21.96992	2.22365	1.33668	0.6862
4.86	0.20965	0.09421	22.33055	2.22526	1.33732	0.6872
4.88	0.20823	0.09351	22.69624	2.22685	1.33796	0.6881
4.90	0.20683	0.09281	23.06705	2.22842	1.33859	0.6891
4.92	0.20543	0.09212	23.44304	2.22998	1.33921	0.6901
4.94	0.20406	0.09144	23.82427	2.23153	1.33983	0.6910
4.96	0.20269	0.09077	24.21077	2.23306	1.34044	0.6920
4.98	0.20134	0.09010	24.60265	2.23457	1.34104	0.6929
5.00	0.20000	0.08944	24.99994	2.23607	1.34164	0.6938

TABLE G.1 Rayleigh Line Flow ( $\gamma = 1.4$ )

<i>M</i>	$T_o/T_o^*$	$T/T^*$	$p_o/p_o^*$	$p/p^*$	$V/V^*$
0.00	0.0000	0.0000	1.2679	2.4000	0.0000
0.02	0.0019	0.0023	1.2675	2.3987	0.0010
0.04	0.0076	0.0092	1.2665	2.3946	0.0038
0.06	0.0171	0.0205	1.2647	2.3880	0.0086
0.08	0.0302	0.0362	1.2623	2.3787	0.0152
0.10	0.0468	0.0560	1.2591	2.3669	0.0237
0.12	0.0666	0.0797	1.2554	2.3526	0.0339
0.14	0.0895	0.1069	1.2510	2.3359	0.0458
0.16	0.1151	0.1374	1.2461	2.3170	0.0593
0.18	0.1432	0.1708	1.2406	2.2959	0.0744
0.20	0.1736	0.2066	1.2346	2.2727	0.0909
0.22	0.2057	0.2445	1.2281	2.2477	0.1088
0.24	0.2395	0.2841	1.2213	2.2209	0.1279
0.26	0.2745	0.3250	1.2140	2.1925	0.1482
0.28	0.3104	0.3667	1.2064	2.1626	0.1696
0.30	0.3469	0.4089	1.1985	2.1314	0.1918
0.32	0.3837	0.4512	1.1904	2.0991	0.2149
0.34	0.4206	0.4933	1.1822	2.0657	0.2388
0.36	0.4572	0.5348	1.1737	2.0314	0.2633
0.38	0.4935	0.5755	1.1652	1.9964	0.2883
0.40	0.5290	0.6151	1.1566	1.9608	0.3137
0.42	0.5638	0.6535	1.1480	1.9247	0.3395
0.44	0.5975	0.6903	1.1394	1.8882	0.3656
0.46	0.6301	0.7254	1.1308	1.8515	0.3918
0.48	0.6614	0.7587	1.1224	1.8147	0.4181
0.50	0.6914	0.7901	1.1141	1.7778	0.4444
0.52	0.7199	0.8196	1.1059	1.7409	0.4708
0.54	0.7470	0.8469	1.0979	1.7043	0.4970
0.56	0.7725	0.8723	1.0901	1.6678	0.5230
0.58	0.7965	0.8955	1.0826	1.6316	0.5489
0.60	0.8189	0.9167	1.0753	1.5957	0.5745
0.62	0.8398	0.9358	1.0682	1.5603	0.5998
0.64	0.8592	0.9530	1.0615	1.5253	0.6248
0.66	0.8771	0.9682	1.0550	1.4908	0.6494
0.68	0.8935	0.9814	1.0489	1.4569	0.6737
0.70	0.9085	0.9929	1.0431	1.4235	0.6975
0.72	0.9221	1.0026	1.0376	1.3907	0.7209
0.74	0.9344	1.0106	1.0325	1.3585	0.7439
0.76	0.9455	1.0171	1.0278	1.3270	0.7665
0.78	0.9553	1.0220	1.0234	1.2961	0.7885
0.80	0.9639	1.0255	1.0193	1.2658	0.8101
0.82	0.9715	1.0276	1.0157	1.2362	0.8313
0.84	0.9781	1.0285	1.0124	1.2073	0.8519
0.86	0.9836	1.0283	1.0095	1.1791	0.8721
0.88	0.9883	1.0269	1.0070	1.1515	0.8918
0.90	0.9921	1.0245	1.0049	1.1246	0.9110

TABLE G.1 (Continued)

<i>M</i>	$T_o/T_o^*$	$T/T^*$	$p_o/p_o^*$	$p/p^*$	$V/V^*$
0.92	0.9951	1.0212	1.0031	1.0984	0.9297
0.94	0.9973	1.0170	1.0017	1.0728	0.9480
0.96	0.9988	1.0121	1.0008	1.0479	0.9658
0.98	0.9997	1.0064	1.0002	1.0236	0.9831
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.02	0.9997	0.9930	1.0002	0.9770	1.0164
1.04	0.9989	0.9855	1.0008	0.9546	1.0325
1.06	0.9977	0.9776	1.0017	0.9327	1.0480
1.08	0.9960	0.9691	1.0031	0.9115	1.0632
1.10	0.9939	0.9603	1.0049	0.8909	1.0780
1.12	0.9915	0.9512	1.0070	0.8708	1.0923
1.14	0.9887	0.9417	1.0095	0.8512	1.1063
1.16	0.9856	0.9320	1.0124	0.8322	1.1198
1.18	0.9823	0.9220	1.0157	0.8137	1.1330
1.20	0.9787	0.9118	1.0194	0.7958	1.1459
1.22	0.9749	0.9015	1.0235	0.7783	1.1584
1.24	0.9709	0.8911	1.0279	0.7613	1.1705
1.26	0.9668	0.8805	1.0328	0.7447	1.1823
1.28	0.9624	0.8699	1.0380	0.7287	1.1938
1.30	0.9580	0.8592	1.0437	0.7130	1.2050
1.32	0.9534	0.8484	1.0497	0.6978	1.2159
1.34	0.9487	0.8377	1.0561	0.6830	1.2264
1.36	0.9440	0.8269	1.0629	0.6686	1.2367
1.38	0.9391	0.8161	1.0701	0.6546	1.2467
1.40	0.9343	0.8054	1.0777	0.6410	1.2564
1.42	0.9293	0.7947	1.0856	0.6278	1.2659
1.44	0.9243	0.7840	1.0940	0.6149	1.2751
1.46	0.9193	0.7735	1.1028	0.6024	1.2840
1.48	0.9143	0.7629	1.1120	0.5902	1.2927
1.50	0.9093	0.7525	1.1215	0.5783	1.3012
1.52	0.9042	0.7422	1.1315	0.5668	1.3095
1.54	0.8992	0.7319	1.1419	0.5555	1.3175
1.56	0.8942	0.7217	1.1527	0.5446	1.3253
1.58	0.8892	0.7117	1.1640	0.5339	1.3329
1.60	0.8842	0.7017	1.1756	0.5236	1.3403
1.62	0.8792	0.6919	1.1877	0.5135	1.3475
1.64	0.8743	0.6822	1.2002	0.5036	1.3546
1.66	0.8694	0.6726	1.2131	0.4940	1.3614
1.68	0.8645	0.6631	1.2264	0.4847	1.3681
1.70	0.8597	0.6538	1.2402	0.4756	1.3746
1.72	0.8549	0.6445	1.2545	0.4668	1.3809
1.74	0.8502	0.6355	1.2692	0.4581	1.3870
1.76	0.8455	0.6265	1.2843	0.4497	1.3931
1.78	0.8409	0.6176	1.2999	0.4415	1.3989
1.80	0.8363	0.6089	1.3159	0.4335	1.4046
1.82	0.8317	0.6004	1.3324	0.4257	1.4102
1.84	0.8273	0.5919	1.3494	0.4181	1.4156
1.86	0.8228	0.5836	1.3669	0.4107	1.4209
1.88	0.8185	0.5754	1.3849	0.4035	1.4261
1.90	0.8141	0.5673	1.4033	0.3964	1.4311
1.92	0.8099	0.5594	1.4222	0.3895	1.4360
1.94	0.8057	0.5516	1.4417	0.3828	1.4408
1.96	0.8015	0.5439	1.4616	0.3763	1.4455
1.98	0.7974	0.5364	1.4821	0.3699	1.4501
2.00	0.7934	0.5289	1.5031	0.3636	1.4545
2.02	0.7894	0.5216	1.5246	0.3575	1.4589
2.04	0.7855	0.5144	1.5467	0.3516	1.4632
2.06	0.7816	0.5074	1.5693	0.3458	1.4673
2.08	0.7778	0.5004	1.5924	0.3401	1.4714

TABLE G.1 (Continued)

<i>M</i>	$T_o/T_o^*$	$T/T^*$	$p_o/p_o^*$	$p/p^*$	$V/V^*$
2.10	0.7741	0.4936	1.6162	0.3345	1.4753
2.12	0.7704	0.4868	1.6404	0.3291	1.4792
2.14	0.7667	0.4802	1.6653	0.3238	1.4830
2.16	0.7631	0.4737	1.6908	0.3186	1.4867
2.18	0.7596	0.4673	1.7168	0.3136	1.4903
2.20	0.7561	0.4611	1.7434	0.3086	1.4938
2.22	0.7527	0.4549	1.7707	0.3038	1.4973
2.24	0.7493	0.4488	1.7986	0.2991	1.5007
2.26	0.7460	0.4428	1.8271	0.2945	1.5040
2.28	0.7428	0.4370	1.8562	0.2899	1.5072
2.30	0.7395	0.4312	1.8860	0.2855	1.5103
2.32	0.7364	0.4256	1.9165	0.2812	1.5134
2.34	0.7333	0.4200	1.9476	0.2769	1.5165
2.36	0.7302	0.4145	1.9794	0.2728	1.5194
2.38	0.7272	0.4091	2.0119	0.2688	1.5223
2.40	0.7242	0.4038	2.0451	0.2648	1.5252
2.42	0.7213	0.3986	2.0789	0.2609	1.5279
2.44	0.7184	0.3935	2.1136	0.2571	1.5306
2.46	0.7156	0.3885	2.1489	0.2534	1.5333
2.48	0.7128	0.3836	2.1850	0.2497	1.5359
2.50	0.7101	0.3787	2.2218	0.2462	1.5385
2.52	0.7074	0.3739	2.2594	0.2427	1.5410
2.54	0.7047	0.3692	2.2978	0.2392	1.5434
2.56	0.7021	0.3646	2.3370	0.2359	1.5458
2.58	0.6995	0.3601	2.3770	0.2326	1.5482
2.60	0.6970	0.3556	2.4177	0.2294	1.5505
2.62	0.6945	0.3512	2.4593	0.2262	1.5527
2.64	0.6921	0.3469	2.5018	0.2231	1.5549
2.66	0.6896	0.3427	2.5451	0.2201	1.5571
2.68	0.6873	0.3385	2.5892	0.2171	1.5592
2.70	0.6849	0.3344	2.6343	0.2142	1.5613
2.72	0.6826	0.3304	2.6802	0.2113	1.5634
2.74	0.6804	0.3264	2.7270	0.2085	1.5654
2.76	0.6781	0.3225	2.7748	0.2058	1.5673
2.78	0.6760	0.3186	2.8235	0.2030	1.5693
2.80	0.6738	0.3149	2.8731	0.2004	1.5711
2.82	0.6717	0.3111	2.9237	0.1978	1.5730
2.84	0.6696	0.3075	2.9752	0.1953	1.5748
2.86	0.6675	0.3039	3.0278	0.1927	1.5766
2.88	0.6655	0.3004	3.0813	0.1903	1.5784
2.90	0.6635	0.2969	3.1359	0.1879	1.5801
2.92	0.6615	0.2934	3.1914	0.1855	1.5818
2.94	0.6596	0.2901	3.2481	0.1832	1.5834
2.96	0.6577	0.2868	3.3058	0.1809	1.5851
2.98	0.6558	0.2835	3.3646	0.1787	1.5867
3.00	0.6540	0.2803	3.4245	0.1765	1.5882
3.50	0.6158	0.2142	5.3280	0.1322	1.6198
4.00	0.5891	0.1683	8.2268	0.1026	1.6410
4.50	0.5698	0.1354	12.5023	0.0818	1.6559
5.00	0.5556	0.1111	18.6339	0.0667	1.6667
6.00	0.5363	0.0785	38.9459	0.0467	1.6809
7.00	0.5244	0.0583	75.4138	0.0345	1.6897
8.00	0.5165	0.0449	136.6235	0.0265	1.6954
9.00	0.5110	0.0356	233.8840	0.0210	1.6993
10.00	0.5070	0.0290	381.6149	0.0170	1.7021
$\infty$	0.4898	0.0000	$\infty$	0.0000	1.7143

## SOLUTIONS

### P.1 ■ Solution

**Part 1:** Entering  $M_1 = 0.2$  and  $\gamma = 1.4$  into the Fanno flow table, we read

$$\left( \frac{fL_{\max}}{D} \right)_1 = 14.533$$

From the isentropic table,

$$\frac{p}{p_0} = 0.9725 ; \frac{T}{T_0} = 0.9921$$

At the downstream location, with  $M_2 = 0.50$ ,

$$\left( \frac{fL_{\max}}{D} \right)_2 = 1.0691$$

so that

$$\left( \frac{fL_{\max}}{D} \right)_1 - \left( \frac{fL_{\max}}{D} \right)_2 = 14.533 - 1.0691 = 13.4639$$

Given  $f = 0.02$  and  $D = 0.015$  m, the length required to attain Mach 0.50 is calculated to be

$$\frac{0.02 \times L}{0.015} = 13.4639 \rightarrow L = \frac{13.4639 \times 0.015}{0.02} = \boxed{10.1 \text{ m}}$$

To reach Mach 1 at the exit, in turn, we set

$$\left( \frac{fL_{\max}}{D} \right)_2 = 0$$

so that

$$\left( \frac{fL_{\max}}{D} \right)_1 - \left( \frac{fL_{\max}}{D} \right)_2 = 14.533 - 0 = 14.533$$

giving

$$L = \frac{14.533 \times 0.015}{0.02} = \boxed{10.9 \text{ m}}$$

★ The correct answer is C.

**Part 2:** If 1 meter is added to the duct length, the flow rate will be reduced because  $M_1$  will be reduced. To determine the reduced Mach number  $M_{1R}$ , we first compute

$$\left( \frac{fL_{\max}}{D} \right)_{1R} = \frac{0.02 \times (10.9 + 1.0)}{0.015} = 15.9$$

The tabulated values of  $(fL_{\max}/D)$  are a bit too coarse for low Mach numbers, so we could instead compute the ratio directly from its definition,

$$\frac{fL_{\max}}{D} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma} \ln \left[ \frac{(\gamma + 1)M^2}{2 \left( 1 + \frac{\gamma - 1}{2} M^2 \right)} \right] = 15.9$$

One way to solve this transcendental equation is to apply Mathematica's *FindRoot* command with an initial guess of, say, 0.1,

$$\text{In[95]:= } \text{FindRoot}\left[\frac{1-m^2}{1.4*m^2} + \frac{1.4+1}{2*1.4} * \text{Log}\left[\frac{2.4*m^2}{2*(1+\frac{1.4-1}{2}*m^2)}\right] - 15.9, \{m, 0.1\}\right]$$

Out[95]= {m → 0.192431}

The program returns  $M_{1R} \approx 0.1924$ . We proceed to compute the isentropic flow ratios

$$\frac{p_{1R}}{p_{01}} = \left(1 + \frac{\gamma-1}{2} M_{1R}^2\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{1.4-1}{2} \times 0.192^2\right)^{1.4/(1.4-1)} = 0.9746$$

$$\frac{T_{1R}}{T_{01}} = \left(1 + \frac{\gamma-1}{2} M_{1R}^2\right)^{-1} = \left(1 + \frac{1.4-1}{2} \times 0.192^2\right)^{-1} = 0.9927$$

Now, the original mass flow rate and the reduced flow rate may be written as

$$\dot{m}_1 = \rho_1 A V_1 = \left(\frac{p_1}{RT_1}\right) A M_1 \sqrt{\gamma RT_1}$$

$$\dot{m}_{1R} = \rho_{1R} A V_{1R} = \left(\frac{p_{1R}}{RT_{1R}}\right) A M_{1R} \sqrt{\gamma RT_{1R}}$$

Since the stagnation conditions are maintained, we may write the ratio of mass flow rates as

$$\frac{\dot{m}_{1R}}{\dot{m}} = \frac{\left(\frac{p_{1R}}{p_{01}}\right) \left(\frac{T_1}{T_{01}}\right)}{\left(\frac{p_1}{p_{01}}\right) \left(\frac{T_{1R}}{T_{01}}\right)} \left(\frac{M_{1R}}{M_1}\right) \sqrt{\frac{\left(\frac{T_{1R}}{T_{01}}\right)}{\left(\frac{T_1}{T_{01}}\right)}} = \frac{\left(\frac{p_{1R}}{p_{01}}\right)}{\left(\frac{p_1}{p_{01}}\right)} \left(\frac{M_{1R}}{M_1}\right) \sqrt{\frac{\left(\frac{T_1}{T_{01}}\right)}{\left(\frac{T_{1R}}{T_{01}}\right)}}$$

$$\therefore \frac{\dot{m}_{1R}}{\dot{m}} = \frac{0.9746}{0.9725} \times \frac{0.1924}{0.2} \times \sqrt{\frac{0.9921}{0.9927}} = 0.9638$$

Thus, the percentage reduction in flow rate due to the 1-m increase in duct length equals

$$\Delta = \frac{1.0 - 0.9638}{1.0} \times 100\% = \boxed{3.62\%}$$

## P.2 ■ Solution

A force-momentum balance on a control volume within the duct reveals that

$$p_1 A_1 - p_2 A_2 - F = \dot{m}(V_2 - V_1)$$

where  $F$  is the force we aim to determine. To determine  $F$ , we require the entry and exit values of the static pressure and velocity, as well as the flow rate. Entering  $M_1 = 0.32$  into the Fanno flow table, we read

$$\left(\frac{fL_{\max}}{D}\right)_1 = 4.4467 ; \frac{p_1}{p^*} = 3.3887 ; \frac{\rho_1}{\rho^*} = \frac{V^*}{V_1} = \frac{1}{0.34701} = 2.8818$$

From the isentropic table,

$$\frac{p_1}{p_{01}} = 0.9315 ; \frac{T_1}{T_{01}} = 0.9799$$

In station 2, we have

$$\left(\frac{fL_{\max}}{D}\right)_2 = \left(\frac{fL_{\max}}{D}\right)_1 - \frac{fL}{D} = 4.4467 - \frac{0.02 \times 60}{1} = 3.2467$$

Entering this value of  $fL_{\max}/D$  into the Fanno flow table, we read  $M_2 = 0.3577$ . Also from the Fanno flow table, we find that

$$\frac{p_2}{p^*} = 3.0042 ; \frac{\rho_2}{\rho^*} = \frac{V^*}{V_2} = \frac{1}{0.38935} = 2.5784$$

From the isentropic table,

$$\frac{T_2}{T_{02}} = 0.9747$$

Noting that  $p_{01} = 95$  kPa, pressure  $p_1$  is determined as

$$p_1 = \left( \frac{p_1}{p_{01}} \right) p_{01} = 0.9315 \times 95 = 88.5 \text{ kPa}$$

Pressure  $p_2$  is given by

$$p_2 = \left( \frac{p_2}{p^*} \right) \left( \frac{p^*}{p_1} \right) p_1 = 3.00422 \times \frac{1}{3.38874} \times 88.5 = 78.5 \text{ kPa}$$

Because the flow is adiabatic,  $T_{01} = T_{02} = 320$  K. We proceed to determine a few more variables,

$$T_1 = \left( \frac{T_1}{T_{01}} \right) T_{01} = 0.9799 \times 320 = 314 \text{ K}$$

$$V_1 = M_1 a_1 = 0.32 \times \sqrt{1.4 \times 287 \times 314} = 114 \text{ m/s}$$

$$V_2 = \frac{V_2}{V^*} \frac{V^*}{V_1} V_1 = \left( \frac{\rho^*}{\rho_2} \right) \left( \frac{\rho_1}{\rho^*} \right) V_1 = \frac{1}{2.5784} \times 2.8818 \times 114 = 127 \text{ m/s}$$

$$\dot{m} = \rho_1 A V_1 = \frac{p_1}{R T_1} \times A \times M_1 \sqrt{\gamma R T_1} = \frac{88,500}{287 \times 314} \times \left( \frac{\pi}{4} \times 0.012^2 \right) \times 114 = 0.0127 \text{ kg/s}$$

We now have all the information needed to compute force  $F$ , namely

$$F = p_1 A_1 - p_2 A_2 - \dot{m} (V_2 - V_1)$$

$$\therefore F = (88,500 - 78,500) \times \left( \frac{\pi}{4} \times 0.012^2 \right) - 0.0127 \times (127 - 114) = \boxed{0.966 \text{ N}}$$

★ The correct answer is **B**.

### P.3 ■ Solution

The Mach number at the entrance of the duct is given by

$$M_1 = \frac{V_1}{a_1} = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{2735}{\sqrt{1.4 \times 4124 \times 400}} = 1.80$$

Entering this Mach number into the Fanno flow table, we read

$$\left( \frac{fL_{\max}}{D} \right)_1 = 0.2419 ; \frac{p_1}{p^*} = 0.4741 ; \frac{\rho_1}{\rho^*} = \frac{V^*}{V_1} = \frac{1}{1.53598} = 0.6511$$

$$\frac{T_1}{T^*} = 0.7282$$

From the isentropic table,

$$\frac{p_1}{p_{01}} = 0.1740$$

Now,

$$\left( \frac{fL_{\max}}{D} \right)_2 = \left( \frac{fL_{\max}}{D} \right)_1 - \frac{fL}{D} = 0.2419 - \frac{0.02 \times 10}{2} = 0.1419$$

This value of  $fL_{\max}/D$  corresponds to a Mach number  $M_2 = 1.5160$ . Appealing to the Fanno flow table a second time, we have

$$\frac{p_2}{p^*} = 0.5981 ; \frac{\rho_2}{\rho^*} = \frac{V^*}{V_2} = \frac{1}{1.3746} = 0.7275$$

$$\frac{T_2}{T^*} = 0.8221$$

Pressure ratio  $p_2/p_1$  is determined as

$$\frac{p_2}{p_1} = \frac{p_2}{p^*} \frac{p^*}{p_1} = 0.5981 \times \frac{1}{0.4741} = 1.2616$$

Temperature ratio  $T_2/T_1$  is

$$\frac{T_2}{T_1} = \frac{T_2}{T^*} \frac{T^*}{T_1} = 0.8221 \times \frac{1}{0.7282} = 1.1290$$

We were asked to determine velocity  $V_2$ ; here goes,

$$V_2 = \left( \frac{V_2}{V^*} \right) \left( \frac{V^*}{V_1} \right) V_1 = \left( \frac{\rho^*}{\rho_2} \right) \left( \frac{\rho_1}{\rho^*} \right) V_1 = \frac{1}{0.7275} \times 0.6511 \times 2735 = \boxed{2450 \text{ m/s}}$$

Pressure change  $p_2 - p_1$  is, in turn,

$$\Delta p = p_2 - p_1 = p_1 \left( \frac{p_2}{p_1} - 1 \right) = \left( \frac{p_1}{p_{01}} \right) p_{01} \left( \frac{p_2}{p_1} - 1 \right)$$

$$\therefore \Delta p = 0.1740 \times 525 \times (1.2616 - 1) = \boxed{23.9 \text{ kPa}}$$

Finally, temperature change  $T_2 - T_1$  is given by

$$\Delta T = T_2 - T_1 = T_1 \left( \frac{T_2}{T_1} - 1 \right) = 400 \times (1.1290 - 1) = \boxed{51.6 \text{ K}}$$

★ Statement **A** is true, whereas statements **B** and **C** are false.

#### P.4 ■ Solution

We must first determine the exit pressure. Assuming the duct is choked, we have

$$\frac{fL}{D} = \frac{0.02 \times 30}{1.2} = 0.5 = \left( \frac{fL_{\max}}{D} \right)_1 - \underbrace{\left( \frac{fL_{\max}}{D} \right)_2}_{=0} = \left( \frac{fL_{\max}}{D} \right)_1$$

Entering this  $fL_{\max}/D$  value into the Fanno flow table, we find  $M_1 = 0.5977$ . At this Mach number, using the isentropic and Fanno flow tables, we may write

$$p_e = p_2 = p^* = \left( \frac{p^*}{p_1} \right) \left( \frac{p_1}{p_{01}} \right) p_{01} = \frac{1}{1.7634} \times 0.7840 \times 950 = 422 \text{ kPa}$$

Since the back pressure is substantially below this value, the assumption that the duct is choked is correct and we may proceed to determine the flow rate. Now, at  $M_1 = 0.5977$ , referring to the isentropic flow table,

$$T_1 = \left( \frac{T_1}{T_{01}} \right) T_{01} = 0.9328 \times 700 = 653 \text{ K}$$

$$p_1 = \left( \frac{p_1}{p_{01}} \right) p_{01} = 0.7840 \times 950 = 745 \text{ kPa}$$

Finally, the mass flow rate is calculated to be

$$\dot{m} = \rho_1 A V_1 = \left( \frac{p_1}{R T_1} \right) A M_1 \sqrt{\gamma R T_1}$$

$$\therefore \dot{m} = \left( \frac{745,000}{287 \times 653} \right) \times \left( \frac{\pi}{4} \times 0.012^2 \right) \times 0.5977 \times \sqrt{1.4 \times 287 \times 653} = [0.138 \text{ kg/s}]$$

★ The correct answer is **A.**

## P.5 ■ Solution

We first determine the exit pressure assuming the duct is choked,

$$\frac{fL}{D} = \frac{0.018 \times 3}{0.012} = 4.5 = \left( \frac{fL_{\max}}{D} \right)_1 - \left( \frac{fL_{\max}}{D} \right)_2 = \left( \frac{fL_{\max}}{D} \right)_1 - 0 = \left( \frac{fL_{\max}}{D} \right)_1$$

From this value we can establish Mach number  $M_1 = 0.3186$ . At this Mach number, using the isentropic and Fanno flow pressure relations, we may write that

$$p_2 = p^* = \left( \frac{p^*}{p_1} \right) \left( \frac{p_1}{p_{01}} \right) \left( \frac{p_{01}}{p_R} \right) p_R = \frac{1}{3.4048} \times 0.9321 \times 1.0 \times p_R(t)$$

$$\therefore p_2 = 0.2738 p_R(t)$$

Here,  $p_R(t)$  denotes the pressure in the reservoir in kPa. The lowest value  $p^*$  is  $0.2738 \times 500 = 136.9$  kPa, which is greater than the back pressure  $p_b = 101$  kPa. Accordingly, the duct is indeed choked in the entire process. Using the isentropic table with  $M_1 = 0.3186$ , we proceed to determine temperature  $T_1$  and pressure  $p_1$ ,

$$T_1 = \left( \frac{T_1}{T_{01}} \right) T_{01} = 0.9801 \times 350 = 343 \text{ K}$$

$$p_1 = \left( \frac{p_1}{p_{01}} \right) \underbrace{p_{01}}_{=p_R} = 0.9321 p_R$$

Now, the mass flow rate is given by

$$\dot{m} = \rho_1 A V_1 = \left( \frac{p_1}{R T_1} \right) A M_1 \sqrt{\gamma R T_1}$$

$$\therefore \dot{m} = \left( \frac{0.9321 p_R}{0.2968 \times 343} \right) \times \left( \frac{\pi}{4} \times 0.012^2 \right) \times 0.3186 \times \sqrt{1.4 \times 296.8 \times 343} = 1.2455 \times 10^{-4} p_R$$

The mass of gas in the reservoir equals the product of density  $\rho_R$  and volume  $V_R$ ,

$$\rho_R V_R = m_R$$

Substituting  $\rho_R$  from the ideal gas law,

$$\rho_R V_R = m_R \rightarrow p_R V_R = m_R R T$$

Deriving with respect to time,

$$\frac{dp_R}{dt} = \left( \frac{RT}{V_R} \right) \frac{dm_R}{dt} = \left( \frac{0.2968 \times 350}{7.5} \right) \frac{dm_R}{dt}$$

$$\therefore \frac{dp_R}{dt} = 13.9 \frac{dm_R}{dt}$$

From a mass balance on the reservoir,

$$\frac{dm_R}{dt} = -\dot{m}$$

so that

$$\frac{dp_R}{dt} = 13.9 \times (-1.2455 \times 10^{-4} p_R) = -1.73 \times 10^{-3} p_R$$

Separating variables and integrating,

$$\frac{dp_R}{p_R} = -1.73 \times 10^{-3} dt \rightarrow \ln \left[ \frac{(p_R)_f}{(p_R)_i} \right] = -1.73 \times 10^{-3} t$$

$$\therefore t = -\frac{1}{1.73 \times 10^{-3}} \ln \left( \frac{500}{1000} \right) = \boxed{401 \text{ s}}$$

★ The correct answer is D.

## P.6 ■ Solution

Consider arrangement 1. The maximum flow rate will occur when the throat Mach number is 1. At this Mach number, we have

$$\frac{p_{\text{th}}}{p_0} = 0.5283 ; \frac{T_{\text{th}}}{T_0} = 0.8333$$

The mass flow rate follows as

$$\dot{m}_{\text{max}} = \rho_{\text{th}} A_{\text{th}} V_{\text{th}} = \left( \frac{p_{\text{th}}}{R T_{\text{th}}} \right) A M_{\text{th}} \sqrt{\gamma R T_{\text{th}}}$$

$$\therefore \dot{m}_{\text{max}} = \frac{\left( \frac{p_{\text{th}}}{p_0} \right) p_0}{R \left( \frac{T_{\text{th}}}{T_0} \right) T_0} A M_{\text{th}} \sqrt{\gamma R \left( \frac{T_{\text{th}}}{T_0} \right) T_0}$$

$$\therefore \dot{m}_{\text{max}} = \frac{0.5283 \times 130,000}{287 \times 0.8333 \times 260} \times (72 \times 10^{-4}) \times 1.0 \times \sqrt{1.4 \times 287 \times 0.8333 \times 260} = \boxed{2.35 \text{ kg/s}}$$

Entering  $A/A^* = 3$  into the isentropic table, we read a Mach number of 0.1977. At this value the exit static to total pressure ratio is 0.9731. Thus, the maximum flow rate will occur for

$$0 \leq p_b \leq 0.9731 \times 130$$

$$\therefore \boxed{0 \leq p_b \leq 126.5 \text{ kPa}}$$

Consider now arrangement 2. Here, too, the maximum flow rate will occur when the throat Mach number is 1. At this Mach number, the throat static to total pressure and temperature ratios are 0.5283 and 0.8333, and the maximum mass flow rate, as determined above, is 2.35 kg/s. Now, for subsonic flow at the nozzle exit, and the duct inlet, we have, for  $M_1 = 0.1977$ ,

$$\left( \frac{fL_{\text{max}}}{D} \right)_1 = 14.9315$$

The diameter of the duct is

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times (72 \times 3)}{\pi}} = 16.6 \text{ cm}$$

Thus,

$$\frac{fL}{D} = \frac{0.022 \times 1.8}{0.166} = 0.239$$

so that

$$\left( \frac{fL_{\text{max}}}{D} \right)_2 = \left( \frac{fL_{\text{max}}}{D} \right)_1 - \frac{fL}{D} = 14.9315 - 0.239 = 14.693$$

which corresponds to an exit Mach number of 0.199. The exit pressure, which is equal to the back pressure, is determined as

$$p_2 = \left( \frac{p_2}{p^*} \right) \left( \frac{p^*}{p_1} \right) \left( \frac{p_1}{p_{01}} \right) p_0 = 5.4555 \times \frac{1}{5.4860} \times 0.9725 \times 130 = 125.7 \text{ kPa}$$

Thus, the maximum flow rate will occur for a back pressure interval

$$0 \leq p_b \leq 125.7 \text{ kPa}$$

Maximum mass flow	2.35 kg/s
Range of back pressures for maximum mass flow in arrangement 1	$0 \leq p_b \leq 126.5 \text{ kPa}$
Range of back pressures for maximum mass flow in arrangement 2	$0 \leq p_b \leq 125.7 \text{ kPa}$

## P.7 ■ Solution

Because  $R$ , the tank on the right, is evacuated, it may be safely assumed that  $M_e = 1$ . Accordingly,

$$\begin{aligned} \frac{fL}{D} &= \frac{0.02 \times 30}{0.025} = 24 = \left( \frac{fL_{\max}}{D} \right)_i - \underbrace{\left( \frac{fL_{\max}}{D} \right)_e}_{=0} \\ \therefore \left( \frac{fL_{\max}}{D} \right)_i &= 24.0 \end{aligned}$$

This ratio corresponds to  $M_i = 0.161$ . Entering this Mach number into the isentropic table, we read

$$\left( \frac{p}{p_0} \right)_i = 0.9821 ; \left( \frac{T}{T_0} \right)_i = 0.9948$$

Then, substituting in the definition of mass flow,

$$\begin{aligned} \dot{m} &= \rho_i A_i V_i = \left( \frac{p_i}{RT_i} \right) A_i M_i \sqrt{\gamma RT_i} \\ \therefore \dot{m} &= \left[ \frac{0.9948 p_{0R}}{0.287 \times (0.9948 \times 300)} \right] \times \left( \frac{\pi}{4} \times 0.025^2 \right) \times 0.161 \times \sqrt{1.4 \times 287 \times 300} = 3.19 \times 10^{-4} p_{0R} \end{aligned}$$

As we did in Problem 5, note that mass, pressure, and density of gas in container  $L$  can be related as

$$\rho \forall = m \rightarrow p_{0L} \forall = m_L RT_0$$

Here,  $T_0$  must be constant for there to be Fanno flow. We differentiate with respect to time to obtain

$$\begin{aligned} p_{0L} \forall &= m_L RT_0 \rightarrow \forall \frac{dp_{0L}}{dt} = RT_0 \frac{dm_L}{dt} \\ \therefore \frac{dm_L}{dt} &= \left( \frac{\forall}{RT_0} \right) \frac{dp_{0L}}{dt} \\ \therefore \frac{dm_L}{dt} &= -\dot{m}_i = -3.19 \times 10^{-4} p_{0L} = \left( \frac{4}{0.287 \times 300} \right) \frac{dp_{0L}}{dt} = 0.0465 \frac{dp_{0L}}{dt} \quad (\text{I}) \end{aligned}$$

Similarly, for tank  $R$ ,

$$p_{0R} \forall = m_R RT_0 \rightarrow \frac{dm_R}{dt} = \left( \frac{\forall}{RT_0} \right) \frac{dp_{0R}}{dt} = \dot{m}_e$$

However,  $\dot{m}_e = \dot{m}_i$ , that is,

$$\left( \frac{\forall}{RT_0} \right) \frac{dp_{0R}}{dt} = \dot{m}_e = \dot{m}_i = -\left( \frac{\forall}{RT_0} \right) \frac{dp_{0L}}{dt}$$

Cancelling the terms in parentheses, we find that

$$\frac{dp_{0R}}{dt} = -\frac{dp_{0L}}{dt}$$

so that, integrating,

$$p_{0R}|_0^{180} = 180 \text{ kPa} = p_{0L2} - p_{0L1} \quad (\text{II})$$

From (I), we find that

$$\frac{dp_{0L}}{dt} = -\frac{3.19 \times 10^{-4}}{0.0465} p_{0L} = -0.00686 p_{0L}$$

Finally, we integrate, noting that integration bounds can be inferred from (II),

$$\Delta t = - \int_{3 \text{ MPa}}^{2.82 \text{ MPa}} \frac{1}{0.00686} \frac{dp_{0L}}{p_{0L}} = 146 \ln\left(\frac{3}{2.82}\right) = \boxed{9.03 \text{ s}}$$

The tank will be filled to 180 kPa within approximately nine seconds.

★ The correct answer is **B**.

## P.8 ■ Solution

**Part 1:** For  $A_2/A_1 = A_2/A^* = 2.8$ , the isentropic table reads  $M_2 \approx 2.5642$ . Entering this Mach number into the Fanno flow table, we find that

$$\left(\frac{fL_{\max}}{D}\right)_2 = 0.4454$$

Now,

$$\frac{fL}{D} = \frac{0.02 \times 24}{1.2} = 0.4$$

Since  $L < L_{\max}$ , the flow cannot reach  $M_e = 1$ . To compute the exit Mach number, we have

$$\left(\frac{fL_{\max}}{D}\right)_e = \left(\frac{fL_{\max}}{D}\right)_2 - \frac{fL}{D} = 0.4454 - 0.4 = 0.0454$$

from which we find  $M_e = 1.24$ . Now, at the nozzle throat,  $M = 1$ , so

$$p_{\text{th}} = \left(\frac{p_{\text{th}}}{p_{0\text{th}}}\right) p_{0\text{th}} = 0.5283 \times 710 = 375 \text{ kPa}$$

$$T_{\text{th}} = \left(\frac{p_{\text{th}}}{p_{0\text{th}}}\right) T_{0\text{th}} = 0.8333 \times 1000 = 833 \text{ K}$$

We also need the cross-sectional area at the throat,

$$A_{\text{th}} = \frac{\frac{\pi}{4} \times 0.012^2}{2.8} = 4.04 \times 10^{-5} \text{ m}^2$$

Finally, mass flow rate  $\dot{m}$  is calculated to be

$$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{p_{\text{th}}}{RT_{\text{th}}}\right) A_{\text{th}} M_{\text{th}} \sqrt{\gamma RT_{\text{th}}}$$

$$\therefore \dot{m}_1 = \left(\frac{375,000}{287 \times 833}\right) \times (4.04 \times 10^{-5}) \times 1.0 \times \sqrt{1.4 \times 287 \times 833} = \boxed{0.0367 \text{ kg/s}}$$

★ The correct answer is **B**.

**Part 2:** To determine the pressure at the exit plane of the duct, we first enter  $M_2 = 2.5642$  into the isentropic table to read  $p_2/p_{02} = 0.0530$ . Pressure  $p_2$  easily follows,

$$p_2 = \left( \frac{p_2}{p_{02}} \right) p_{02} = 0.0530 \times 710 = 37.6 \text{ kPa}$$

Reading two further pressure ratios from the Fanno flow table,

$$\frac{p_e}{p^*} = 0.7726 ; \frac{p^*}{p_2} = \frac{1}{0.2808}$$

we ultimately find that

$$p_e = \left( \frac{p_e}{p^*} \right) \left( \frac{p^*}{p_2} \right) p_2 = 0.7726 \times \frac{1}{0.2808} \times 37.6 = \boxed{103 \text{ kPa}}$$

★ The correct answer is A.

**Part 3:** A shock stands at the nozzle exit, which is station 2. We will call the duct inlet (on the downstream side of the shock), station 3. Entering  $M_2 = 2.5642$  into the normal shock relations, we find that  $M_3 = 0.5070$  and  $p_3/p_2 = 7.5043$ . Pressure  $p_3$  is determined next,

$$p_3 = \left( \frac{p_3}{p_2} \right) p_2 = 7.5043 \times 37.6 = 282 \text{ kPa}$$

From the Fanno flow table at  $M_3 = 0.5070$ ,  $(fL_{\max}/D)_3 = 1.0134$ . Since  $fL/D = 0.4$ , it follows that

$$\left( \frac{fL_{\max}}{D} \right)_e = \left( \frac{fL_{\max}}{D} \right)_3 - \frac{fL}{D} = 1.0134 - 0.4 = 0.6134$$

whence we find the corresponding Mach number  $M_e = 0.572$ . Reading two additional pressure ratios from the Fanno flow table, exit pressure  $p_e$  is calculated to be

$$p_e = \left( \frac{p_e}{p^*} \right) \left( \frac{p^*}{p_3} \right) p_3 = 1.8560 \times \frac{1}{2.1080} \times 282 = \boxed{248 \text{ kPa}}$$

★ The correct answer is A.

**Part 4:** In this case a shock appears just downstream of the nozzle throat. Consequently, subsonic flow exits the nozzle. For  $A_2/A_1 = A_2/A^* = 2.8$ ,  $M_2 = 0.212$ . The corresponding  $(fL_{\max}/D)$  is 12.7. Since  $fL/D = 0.4$ , then

$$\left( \frac{fL_{\max}}{D} \right)_e = 12.7 - 0.4 = 12.3$$

from which we find  $M_e = 0.215$ . The back pressure we're looking for follows as

$$p_e = \left( \frac{p_e}{p^*} \right) \left( \frac{p^*}{p_2} \right) \left( \frac{p_2}{p_{02}} \right) \left( \frac{p_{02}}{p_{0t}} \right) p_{0t} = 5.0803 \times \frac{1}{5.1552} \times 0.9697 \times 710 = \boxed{678 \text{ kPa}}$$

★ The correct answer is B.

## P.9 ■ Solution

The Mach number at the initial station is

$$M_1 = \frac{V_1}{a_1} = \frac{V_1}{\sqrt{\gamma RT_1}} = \frac{120}{\sqrt{1.4 \times 287 \times 324}} = 0.333$$

At this Mach number, we refer to the isentropic table and read  $(T/T_0)_1 = 0.9787$ . Thus,

$$T_{01} = \frac{T_1}{0.9787} = \frac{324}{0.9787} = 331\text{K}$$

Entering the initial Mach number into the Rayleigh flow table, we find that

$$\frac{T_{01}}{T_0^*} = 0.4077 \rightarrow T_0^* = \frac{331}{0.4077} = 812\text{K}$$

The mass flow rate is given by

$$\dot{m} = \rho_1 A V_1 = \left( \frac{p_1}{RT_1} \right) A V_1 = \frac{200,000}{287 \times 324} \times \left( \frac{\pi}{4} \times 0.016^2 \right) \times 120 = 0.0519\text{kg/s}$$

Lastly, the heat transfer rate for choked flow is

$$\dot{q} = \dot{m} c_p (T_0^* - T_{01}) = 0.0519 \times 1.004 \times (812 - 331) = \boxed{25.1\text{kW}}$$

★ The correct answer is D.

## P.10 ■ Solution

**Part 1:** Because the wall of the pipe is thin, assume the wall has a single temperature, i.e. it is radially lumped. Also, since the steam is condensing on the outside of the pipe, the pipe temperature may be assumed to be equal to the  $T_s$ . The air is treated as a perfect gas, so

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 287}{1.4 - 1} = 1004\text{J/kg}\cdot\text{K}$$

$$a_1 = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 281} = 336\text{m/s}$$

$$M_1 = \frac{V_1}{a_1} = \frac{32}{336} = 0.0952$$

At this Mach number, we find from the Rayleigh flow and isentropic tables,

$$\left( \frac{T}{T^*} \right)_1 = 0.0512 ; \left( \frac{p}{p^*} \right)_1 = 2.3699 ; \left( \frac{T}{T_0} \right)_1 = 0.9982$$

Now,

$$\frac{T_2}{T^*} = \left( \frac{T_2}{T_1} \right) \left( \frac{T_1}{T^*} \right)_1 = \frac{343}{281} \times 0.0512 = 0.0625$$

With this temperature ratio, we may refer to the Rayleigh flow table and read  $M_2 = 0.106$ . From the isentropic and Rayleigh flow tables, we read, respectively,

$$\left( \frac{T}{T_0} \right)_2 = 0.9978 ; \left( \frac{p}{p^*} \right)_2 = 2.3628$$

The mass flow rate in the pipe is determined next,

$$\dot{m} = \rho_1 A_1 V_1 = \left( \frac{p_1}{RT_1} \right) \left( \frac{\pi}{4} D^2 \right) V_1$$

$$\therefore \dot{m} = \left( \frac{72,000}{287 \times 281} \right) \times \left( \frac{\pi}{4} \times 0.05^2 \right) \times 32 = 0.0561\text{kg/s}$$

Applying an energy balance to a differential control volume, we may write

$$\dot{m} c_p dT_0 = \delta q = \bar{h} dA (T_w - T_0) = \bar{h} \times \pi D dx \times (T_s - T_0)$$

$$\therefore \dot{m}c_p dT_0 = \bar{h}\pi D(T_s - T_0)dx$$

Rearranging,

$$-\frac{dT_0}{T_0 - T_s} = \frac{\bar{h}\pi D}{\dot{m}c_p} dx$$

Integrating along the length of the pipe yields

$$\ln\left(\frac{T_{01} - T_s}{T_{02} - T_s}\right) = \frac{\pi D \bar{h}}{\dot{m}c_p} L$$

Temperatures  $T_{01}$  and  $T_{02}$  can be computed from the given static temperatures and the temperature ratios determined above,

$$T_{01} = \frac{T_{01}}{T_1} T_1 = \frac{1}{0.9982} \times 281 = 282 \text{ K}$$

$$T_{02} = \frac{T_{02}}{T_2} T_2 = \frac{1}{0.9978} \times 343 = 344 \text{ K}$$

Solving for pipe length and inserting the various parameters, we find that

$$L = \frac{\dot{m}c_p}{\pi D \bar{h}} \ln\left(\frac{T_{01} - T_s}{T_{02} - T_s}\right) = \frac{0.0561 \times 1004}{\pi \times 0.05 \times 140} \times \ln\left(\frac{282 - 438}{344 - 438}\right) = \boxed{1.30 \text{ m}}$$

★ The correct answer is A.

**Part 2:** To determine the pressure drop from Rayleigh flow, we make use of the pressure ratios determined above and write

$$p_2 = \left(\frac{p_2}{p^*}\right) \left(\frac{p^*}{p_1}\right) p_1 = 2.3628 \times \frac{1}{2.3699} \times 72 = 71.8 \text{ kPa}$$

Accordingly, the pressure drop is  $\Delta p \approx 71.8 - 72 = -0.2 \text{ kPa}$ . Expanding the number of decimal digits in the last calculation would yield  $p_2 = 71.7843 \text{ kPa}$  and  $\Delta p = -0.2157 \text{ kPa}$ .

We now proceed to determine the pressure drop associated with Fanno flow. At  $M_1 = 0.0952$ ,

$$\left(\frac{p}{p^*}\right)_1 = 11.496 ; \left(\frac{fL_{\max}}{D}\right)_1 = 74.222$$

(Since the tabulated pressure ratios and values of  $fL_{\max}/D$  are a bit too coarse at low Mach numbers, I used the working relations below directly.)

$$\frac{p}{p^*} = \frac{1}{M} \left[ \frac{\gamma + 1}{2 \left( 1 + \frac{\gamma - 1}{2} M^2 \right)} \right]^{1/2}$$

$$\frac{4fL_{\max}}{D} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma} \ln \left[ \frac{(\gamma + 1)M^2}{2 \left( 1 + \frac{\gamma - 1}{2} M^2 \right)} \right]$$

Inserting parameters into  $fL/D$  gives

$$\frac{fL}{D} = \frac{0.025 \times 1.30}{0.05} = 0.650$$

Therefore,

$$\left(\frac{fL_{\max}}{D}\right)_2 = \left(\frac{fL_{\max}}{D}\right)_1 - \frac{fL}{D} = 74.222 - 0.650 = 73.572$$

which corresponds to a Mach number  $M_2 = 0.0956$ . Thus,

$$p_2 = \left(\frac{p_2}{p^*}\right) \left(\frac{p^*}{p_1}\right) p_1 = 11.448 \times \frac{1}{11.496} \times 72 = 71.7 \text{ kPa}$$

Accordingly, the pressure drop is  $\Delta p \approx 71.7 - 72 = -0.3 \text{ kPa}$ . Expanding the number of decimal digits in the last calculation would yield  $p_2 = 71.699 \text{ kPa}$  and  $\Delta p = -0.3010 \text{ kPa}$ . Lastly, the combined pressure drops if added together amount to

$$|\Delta p| = |(\Delta p)_{\text{Rayleigh}}| + |(\Delta p)_{\text{Fanno}}| = 0.2157 + 0.3010 = \boxed{0.5167 \text{ kPa}}$$

★ The correct answer is **A**.

## P.11 ■ Solution

**Part 1:** Because the back pressure is 0 kPa and because heat is removed from air, flow in the duct will be supersonic and accelerating. This will only occur if  $M_1 = 1.0$ . It follows that  $T_{01} = T_0^* = 823 \text{ K}$ . From the isentropic table,

$$\left(\frac{T}{T_0}\right)_1 = 0.8333 ; \left(\frac{p}{p_0}\right)_1 = 0.5283$$

Accordingly,

$$T_1 = 0.8333 \times 823 = 686 \text{ K}$$

$$p_1 = p^* = 0.5283 \times 480 = 254 \text{ kPa}$$

Now, from an energy balance on the duct,

$$T_{02} = T_{01} + \frac{q}{c_p} = 823 - \frac{250}{1.004} = 574 \text{ K}$$

and

$$\frac{T_{02}}{T_{01}} = \frac{574}{823} = 0.697$$

From this ratio, we refer to the Rayleigh flow table and read a Mach number  $M_2 = 2.60$  and a pressure ratio  $p_2/p^* = 0.2294$ . Therefore,

$$p_2 = p_e = \left(\frac{p_2}{p^*}\right) p^* = 0.2294 \times 254 = \boxed{58.3 \text{ kPa}}$$

Expansion waves occur outside the duct to allow the pressure to reach the 0-kPa back pressure. The mass flow rate is determined next,

$$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{p_1}{RT_1}\right) \left(\frac{\pi}{4} D^2\right) M_1 \sqrt{\gamma RT_1}$$

$$\therefore \dot{m} = \left(\frac{254,000}{287 \times 686}\right) \times \left(\frac{\pi}{4} \times 0.02^2\right) \times 1.0 \times \sqrt{1.4 \times 287 \times 686} = \boxed{0.213 \text{ kg/s}}$$

★ Statement **2** is true, whereas statements **1** and **3** are false.

**Part 2:** As determined in the previous part, the Mach number just upstream of the shock is 2.60. Appealing to the normal shock table, we

read a downstream Mach number of 0.5039 and a downstream/upstream pressure ratio of 7.7200, giving an exit pressure  $p_e = 7.7200 \times 58.3 = 450$  kPa.

★ The correct answer is **D**.

**Part 3:** We first assume that  $M_2 = 1$ , so that the duct is choked. Accordingly,  $T_{02} = T_0^*$ . Applying an energy balance to the system, we find that

$$\frac{q}{c_p} = T_{02} - T_{01} = T_0^* - T_r$$

$$\therefore T_0^* = T_r + \frac{q}{c_p} = 823 + \frac{250}{1.004} = 1070 \text{ K}$$

Thus,

$$\frac{T_{01}}{T_0^*} = \frac{823}{1070} = 0.769$$

Entering this stagnation temperature ratio into the Rayleigh flow table, we find a corresponding Mach number  $M_1 \approx 0.5573$  and a pressure ratio  $p_1/p^* = 1.6727$ . In turn, we map these values onto the isentropic table to read

$$\frac{p_1}{p_{01}} = 0.8098 ; \quad \frac{T_1}{T_{01}} = 0.9415$$

Since  $p_{01} = p_r = 480$  kPa and  $T_{01} = T_r = 823$  K, we can use the ratios above to compute

$$p_1 = 0.8098 \times 480 = 389 \text{ kPa}$$

$$T_1 = 0.9415 \times 823 = 775 \text{ K}$$

Furthermore,

$$p^* = p_e = p_b = \left( \frac{p^*}{p_1} \right) p_1 = \frac{1}{1.6727} \times 389 = 233 \text{ kPa}$$

Consequently, for any back pressure  $p_b \leq 233$  kPa the duct will be choked due to the heat addition. It follows that for  $p_b = 0$  and 100 kPa the maximum flow rate will be realized, and can be calculated to be

$$\dot{m} = \rho_1 A_1 V_1 = \left( \frac{p_1}{R T_1} \right) \left( \frac{\pi}{4} D^2 \right) M_1 \sqrt{\gamma R T_1}$$

$$\therefore \dot{m} = \left( \frac{389,000}{287 \times 775} \right) \times \left( \frac{\pi}{4} \times 0.02^2 \right) \times 0.5573 \times \sqrt{1.4 \times 287 \times 775} = \boxed{0.171 \text{ kg/s}}$$

## P.12 ■ Solution

Entering  $M_1 = 0.20$  into the isentropic table, we read  $T_1/T_{01} = 0.9921$ , so that

$$T_{01} = \left( \frac{T_{01}}{T_1} \right) T_1 = \frac{1}{0.9921} \times 300 = 302 \text{ K}$$

From the Rayleigh flow table at  $M_1 = 0.2$ ,  $T_{01}/T_0^* = 0.1736$ , giving

$$T_0^* = \left( \frac{T_0^*}{T_{01}} \right) T_{01} = \frac{1}{0.1736} \times 302 = 1740 \text{ K}$$

The maximum length that the pipe may have without affecting the mass flow rate is obtained when the stagnation temperature at the exit,  $T_{0e}$ , equals  $T_0^*$ . Accordingly,

$$q_{\max} = q'L_{\max} = c_p (T_0^* - T_{01})$$

Solving for maximum length,

$$L_{\max} = \frac{c_p (T_0^* - T_{01})}{q'} = \frac{\gamma R (T_0^* - T_{01})}{(\gamma - 1) q'}$$

$$\therefore L_{\max} = \frac{1.4 \times 287 \times (1740 - 302)}{(1.4 - 1) \times 28,900} = 50 \text{ m}$$

Stagnation temperature evolves linearly with horizontal coordinate  $x$ ; that is,

$$T_0(x) = T_{01} + \frac{x}{L_{\max}} (T_0^* - T_{01}) = 302 + \frac{x}{50} \times (1740 - 302)$$

$$\therefore T_0(x) = T_{01} + \frac{x}{L_{\max}} (T_0^* - T_{01}) = 302 + 28.76x$$

Dividing by  $T_0^* = 1740 \text{ K}$ , we get

$$\frac{T_0(x)}{T_0^*} = \frac{302 + 28.76x}{1740} = 0.174 + 0.0165x \quad (\text{I})$$

Now, noting that  $T_0/T_0^*$  is related to Mach number by the expression

$$\frac{T_0}{T_0^*} = \frac{(1+\gamma)M^2 [2 + (\gamma-1)M^2]}{(1+\gamma M^2)^2}$$

which is a second-degree equation in  $M^2$ , we may write

$$M^2 = \frac{\alpha - \sqrt{\beta^2 - \alpha \times (T_0/T_0^*)}}{\alpha} \quad (\text{II})$$

Here,

$$\alpha = 1 + \gamma^2 \left[ \left( \frac{T_0}{T_0^*} \right) - 1 \right]; \quad \beta = 1 - \gamma \left[ \left( \frac{T_0}{T_0^*} \right) - 1 \right] \quad (\text{III, IV})$$

In equation (II), a negative sign is used in front the radical to yield a subsonic Mach number. Equipped with Mach number, the static pressure distribution can be determined from

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \quad (\text{V})$$

and the temperature distribution from

$$\frac{T_2}{T_1} = \frac{M_2^2}{M_1^2} \left( \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^2 \quad (\text{VI})$$

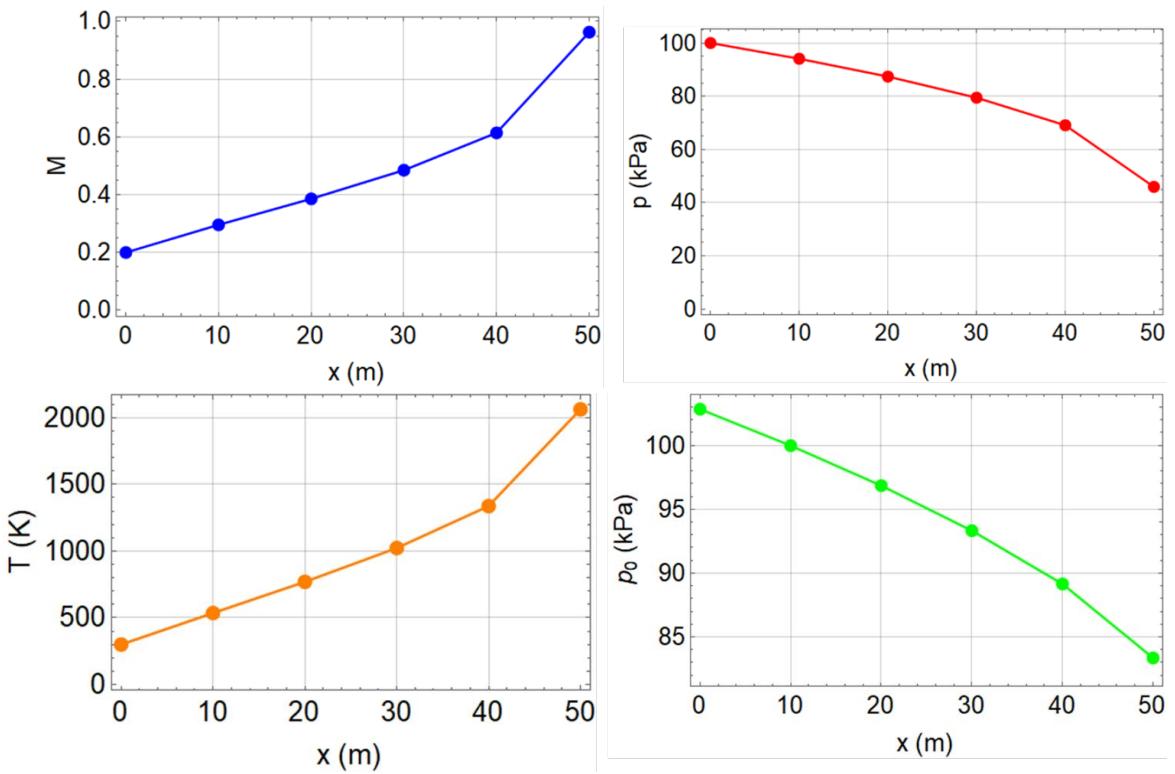
Lastly, the stagnation pressure can be determined from

$$p_0(x) = \frac{p_0(x)}{p(x)} p(x) = p(x) \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/(\gamma-1)} \quad (\text{VII})$$

The calculations are tabulated below.

$X$ (m)	$T_0/T_0^*$ (Eq. I)	$\alpha$ (Eq. III)	$\beta$ (Eq. IV)	$M(x)$ (Eq. II)	$p(x)$ (Eq. V)	$T(x)$ (Eq. VI)	$p_0(x)$ (Eq. VII)
0	0.174	-0.619	2.1564	0.2003	100.0000	300.0000	102.8363
5	0.2565	-0.4573	2.0409	0.2498	97.1302	419.9826	101.4397
10	0.339	-0.2956	1.9254	0.2957	94.0962	536.2411	99.9831
15	0.4215	-0.1339	1.8099	0.3405	90.8662	651.8966	98.4573
20	0.504	0.02784	1.6944	0.3859	87.3964	769.5820	96.8504
25	0.5865	0.18954	1.5789	0.4334	83.6239	892.0920	95.1461
30	0.669	0.35124	1.4634	0.4850	79.4521	1023.0299	93.3215
35	0.7515	0.51294	1.3479	0.5435	74.7188	1167.9479	91.3421
40	0.834	0.67464	1.2324	0.6143	69.1082	1337.2819	89.1496
45	0.9165	0.83634	1.1169	0.7115	61.8095	1557.7681	86.6269
50	0.999	0.99804	1.0014	0.9630	45.9549	2061.9982	83.3570

Finally, we pick six evenly spaced data points and plot curves for Mach number (blue), static pressure (red), temperature (orange), and stagnation pressure (green).



## ANSWER SUMMARY

Problem 1	1.1	C
	1.2	Open-ended
Problem 2		B
Problem 3		T/F
Problem 4		A
Problem 5		D
Problem 6		Open-ended
Problem 7		B
Problem 8	8.1	B
	8.2	A
	8.3	A
	8.4	B
Problem 9		D
Problem 10	10.1	A
	10.2	A
Problem 11	11.1	T/F
	11.2	D
	11.3	Open-ended
Problem 12		Open-ended

## REFERENCE

- JOHN, J. and KEITH, T. (2006). *Gas Dynamics*. 3rd edition. Upper Saddle River: Pearson Prentice Hall.



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