Montogue

Quiz FM102
Forces on Submerged Surfaces
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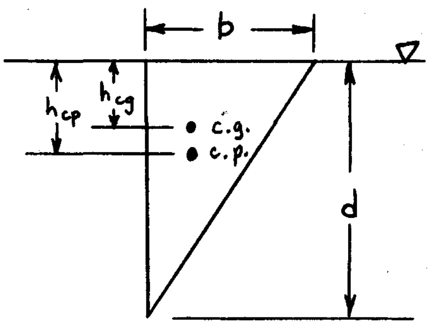
PROBLEMS

Problem 1 (Evett & Liu, 1989)

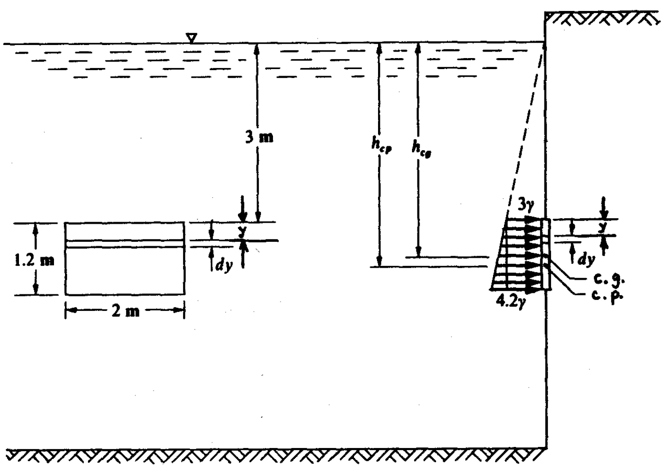
If a triangle of height d and base b is vertical and submerged in liquid with its base at the liquid surface, as illustrated below, what is the depth of its center of pressure?



A) $h_{cp} = d/3$
B) $h_{cp} = d/2$
C) $h_{cp} = 3d/5$
D) $h_{cp} = 2d/3$

Problem 2 (Evett & Liu, 1989)

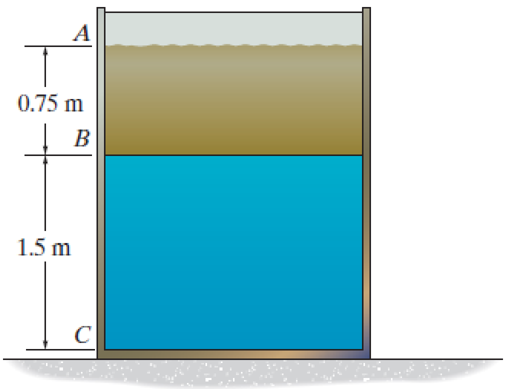
A vertical, rectangular plate with dimensions 1.2 m × 2 m and water on one side is shown below. The upper edge of the gate has is at a depth of 3 m relative to the water surface. What are the total resultant force acting on the gate and the depth of its center of pressure as measured from the surface?



A) F = 63.2 kN and h_{cp} = 3.31 m
B) F = 63.2 kN and h_{cp} = 3.63 m
C) F = 84.6 kN and h_{cp} = 3.31 m
D) F = 84.6 kN and h_{cp} = 3.63 m

Problem 3 (Hibbeler, 2017, w/ permission)

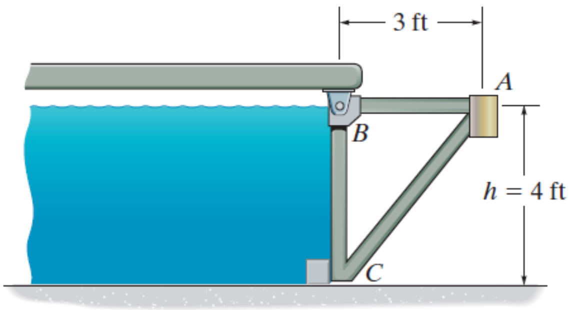
A storage tank contains oil and water acting at the depths shown. Determine the resultant force that both of these liquids exert on the side ABC of the tank if the side has a width of b = 1.25 m. Also, determine the location of this resultant, measured from the top of the tank. Consider the density of the oil to be ρ_o = 900 kg/m^3.



A) F = 29.3 kN and the force acts at a depth of 1.15 m.
B) F = 29.3 kN and the force acts at a depth of 1.51 m.
C) F = 37.5 kN and the force acts at a depth of 1.15 m.
D) F = 37.5 kN and the force acts at a depth of 1.51 m.

Problem 4 (Hibbeler, 2017, w/ permission)

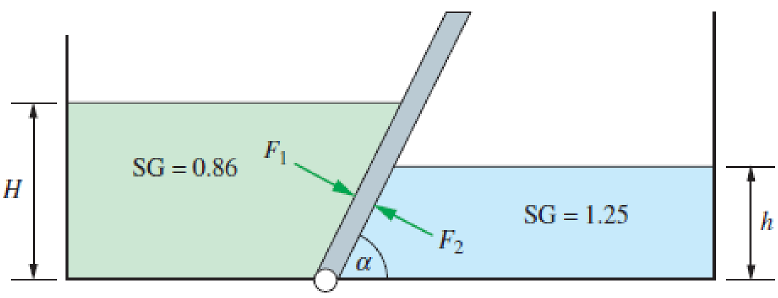
Determine the weight of block A so that the 2-ft radius circular gate BC begins to open when the water level reaches the top of the channel, h = 4 ft. There is a smooth stop block at C.



A) W_A = 1.31 kip
B) W_A = 1.62 kip
C) W_A = 1.93 kip
D) W_A = 2.24 kip

Problem 5 (Çengel & Cimbala, 2014, w/ permission)

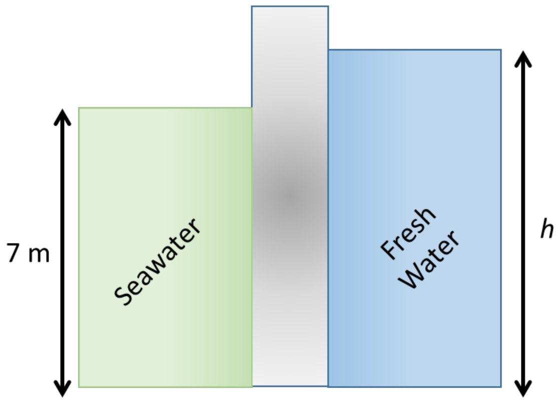
The weight of the plate separating the two fluids is such that the system shown in the figure below is in static equilibrium. If it is known that F_1/F_2 = 1.70, determine the value of the ratio H/h.



A) H/h = 1.25
B) H/h = 1.57
C) H/h = 2.22
D) H/h = 3.14

Problem 6 (Munson et al., 2009, w/ permission)

A long vertical wall separates seawater (γ = 10.1 kN/m^3) from freshwater. If the seawater stands at a depth of 7 m, what depth of freshwater is required to give a zero resultant force on the wall? When the resultant force is zero, will the sum of moments due to the fluid forces be zero?



A) The depth of freshwater required to give a zero resultant force on the wall is 7.10 m. When the resultant force is zero, the sum of moments due to the fluid forces will *not* be zero.

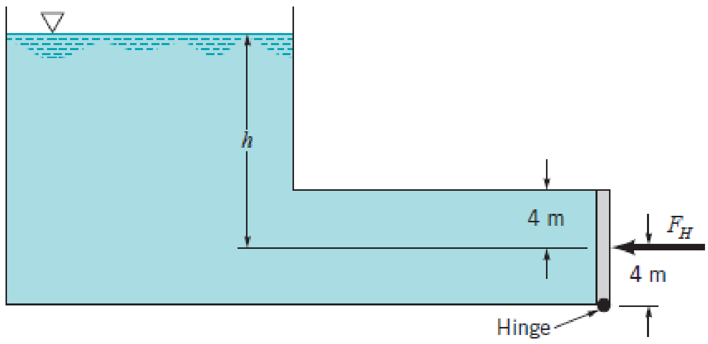
B) The depth of freshwater required to give a zero resultant force on the wall is 7.10 m. When the resultant force is zero, the sum of moments due to the fluid forces will be zero.

C) The depth of freshwater required to give a zero resultant force on the wall is 7.65 m. When the resultant force is zero, the sum of moments due to the fluid forces will *not* be zero.

D) The depth of freshwater required to give a zero resultant force on the wall is 7.65 m. When the resultant force is zero, the sum of moments due to the fluid forces will be zero.

Problem 7 (Munson et al., 2009, w/ permission)

A 3-m wide, 8-m high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in the figure below. The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum acceptable value of F_H is 3500 kN. Determine the maximum water depth, h, above the center of the gate that can exist without the gate opening.



A) h = 10.1 m

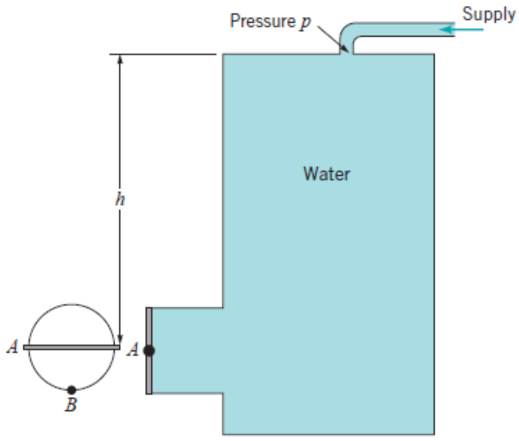
B) h = 12.5 m

C) h = 14.3 m

D) h = 16.2 m

Problem 8 (Munson et al., 2009, w/ permission)

A pump supplies water under pressure to a large tank as shown in the figure below. The circular-plate valve fitted in the short discharge pipe on the tank pivots about its diameter A-A and is held shut against the water pressure by a latch B. Which of the following statements is true?



A) The force on the latch is dependent on both the supply pressure, p, and the height of the tank, h.

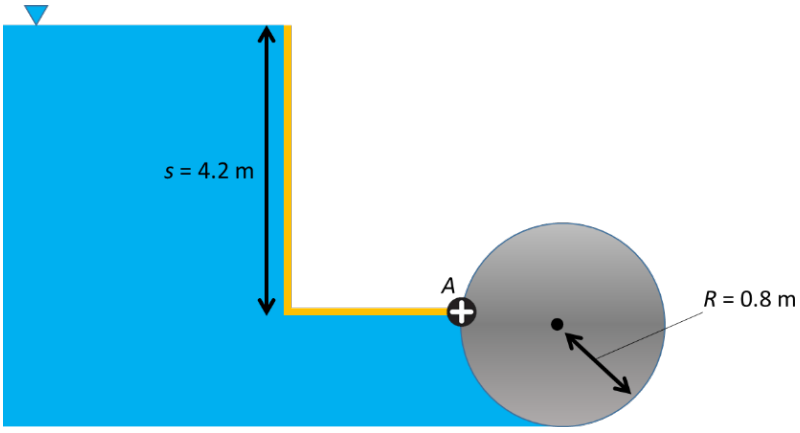
B) The force on the latch is dependent on the supply pressure, p, but not on the height of the tank, h.

C) The force on the latch is dependent on the height of the tank, h, but not on the supply pressure, p.

D) The force on the latch is independent of both the height of the tank, h, and the supply pressure, p.

Problem 9A (Munson et al., 2009, w/ permission)

A long solid cylinder of radius 0.8 m hinged at point A is used as an automatic gate, as shown in the figure below. When the water level reaches 5 m, the gate opens automatically by turning about the hinge at point A. Determine the hydrostatic force acting on the cylinder and its line of action when the gate opens.



A) The hydrostatic resultant on the cylinder has 52.3 kN intensity and makes an angle of 35.5° with the horizontal.

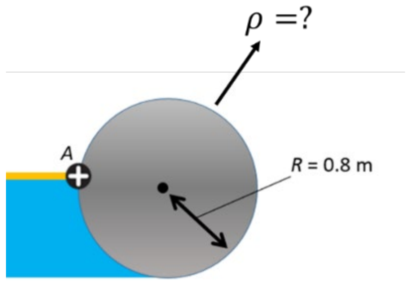
B) The hydrostatic resultant on the cylinder has 52.3 kN intensity and makes an angle of 46.4° with the horizontal.

C) The hydrostatic resultant on the cylinder has 67.7 kN intensity and makes an angle of 35.5° with the horizontal.

D) The hydrostatic resultant on the cylinder has 67.7 kN intensity and makes an angle of 46.4° with the horizontal.

Problem 9B

Considering the system in the previous problem, what is the density of the material that constitutes the cylinder?



A) ρ = 1534 kg/m³

B) ρ = 1765 kg/m³

C) ρ = 1921 kg/m³

D) ρ = 2103 kg/m³

Problem 10 (Munson et al., 2009, w/ permission)

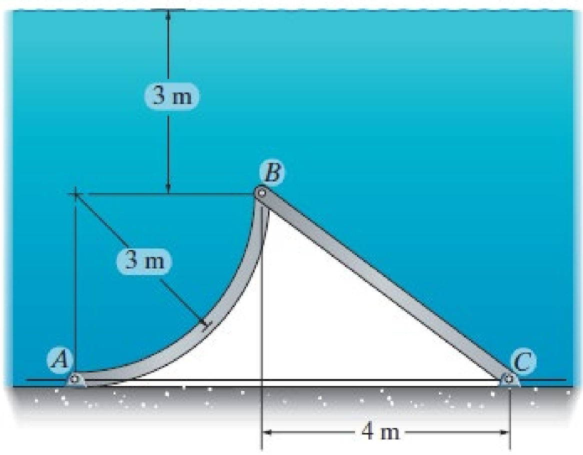
An open rectangular settling tank contains a liquid suspension that at a given time has a specific weight that varies approximately with depth according to the following data.

h (m)	γ (N/m^3)
0	10
0.4	10.1
0.8	10.2
1.2	10.6
1.6	11.3
2	12.3
2.4	12.7
2.8	12.9
3.2	13
3.6	13.1

The depth h = 0 corresponds to the free surface. Determine, using numerical integration, the magnitude and location of the resultant force that the liquid suspension exerts on a vertical wall of a tank that is 6 m wide. The depth of fluid in the tank is 3.6 m.

Problem 11 (Hibbeler, 2017, w/ permission)

The curved and flat plates are pin connected at A, B, and C. They are submerged in water at the depth shown. Determine the horizontal and vertical components at pin B. The plates have a width of 4 m.



A) B_x = 475.6 kN and B_y = 37.1 kN

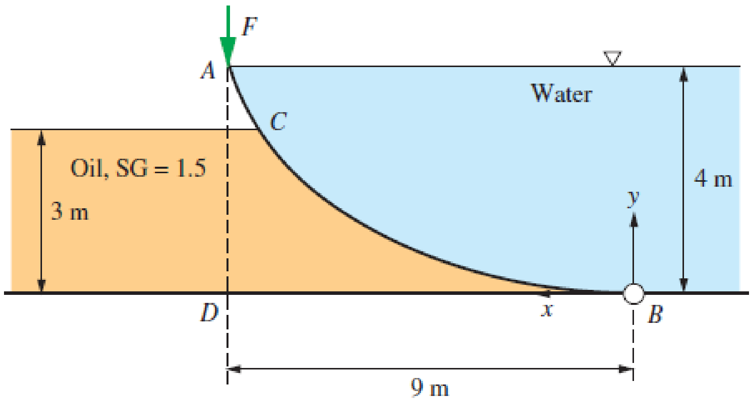
B) B_x = 475.6 kN and B_y = 53.1 kN

C) B_x = 582.9 kN and B_y = 37.1 kN

D) B_x = 582.9 kN and B_y = 53.1 kN

Problem 12 (Çengel & Cimbala, 2014, w/ permission)

The parabolic shaped gate with a width of 2 m shown below is hinged at point B. Determine the force F required to keep the gate stationary.



A) $|F|$ = 17.7 kN

B) $|F|$ = 24.6 kN

C) $|F|$ = 36.5 kN

D) $|F|$ = 43.0 kN

SOLUTIONS

P.1 Solution

All we have to do is apply the equation for the depth h_{cp} of the center of pressure, recalling that the moment of inertia relative to an axis is, in this case, $I_{cg} = bd^3/36$; that is,

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{d}{3} + \frac{bd^3}{36}\frac{1}{\left(\frac{d}{3}\right)\left(\frac{bd}{2}\right)} = \frac{d}{3} + \frac{bd^3}{36}\left(\frac{6}{bd^2}\right) = \frac{d}{3} + \frac{d}{6} = \boxed{\frac{d}{2}}$$

➡ The correct answer is **B**.

P.2 Solution

The force imparted on the gate is determined as

$$F = \gamma h_{cg} A = (9.79)\left(3 + \frac{1.2}{2}\right)(1.2 \times 2) = \boxed{84.6 \text{ kN}}$$

The center of pressure, in turn, can be obtained with the usual formula

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(3 + \frac{1.2}{2}\right) + \frac{(2 \times 1.2^3/12)}{\left(3 + \frac{1.2}{2}\right)(2 \times 1.2)} = \boxed{3.63 \text{ m}}$$

Alternatively, the force could have been obtained by integration,

$$F = \int_A \gamma h dA = \int_0^{1.2} 9.79(3+y)(2dy) = 19.58\left(3y + \frac{y^2}{2}\right)\Big|_0^{1.2} = 84.6 \text{ kN}$$

So could the depth of the center of pressure,

$$h_{cp} = \frac{\int_A \gamma h^2 dA}{F} = \frac{\int_0^{1.2} 9.79(3+y)^2(2dy)}{84.59} = \frac{19.58\left(9y + 3y^2 + \frac{y^3}{3}\right)\Big|_0^{1.2}}{84.59} = 3.63 \text{ m}$$

As expected, the results are the same.

➡ The correct answer is **D**.

P.3 Solution

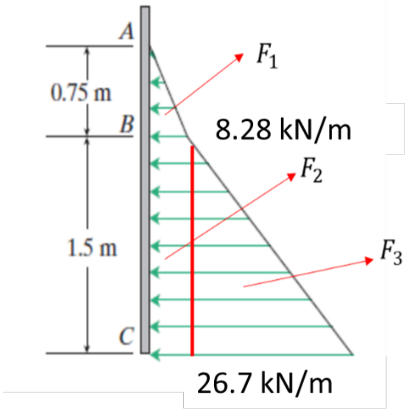
Since the side of the tank has a constant width, the intensities of the distributed loading at B and C are, respectively,

$$w_B = \rho_0 g h_{AB} b = 900 \times 9.81 \times 0.75 \times 1.25 = 8.28 \text{ kN}$$

and

$$w_C = w_B + \rho_w g h_{BC} b = 8.28 + 1000 \times 9.81 \times 1.5 \times 1.25 = 26.7 \text{ kN}$$

The resultant force can be determined by adding the shaded triangular and rectangular areas outlined in the figure below.



Thus, the intensities of forces F_1, F_2, and F_3 are, respectively,

$$F_1 = \frac{1}{2}(0.75)(8.28) = 3.11 \text{ kN}$$

$$F_2 = 1.5(8.28) = 12.4 \text{ kN}$$

$$F_3 = \frac{1}{2}(1.5)(26.67 - 8.28) = 13.8 \text{ kN}$$

and the resultant force is

$$F_R = F_1 + F_2 + F_3 = 3.11 + 12.4 + 13.8 = \boxed{29.3 \text{ kN}}$$

Each of these forces acts through the centroid of its respective area. Distance y_1 pertains to the triangle from which we obtained force F_1,

$$y_1 = \frac{2}{3}(0.75) = 0.50 \text{ m}$$

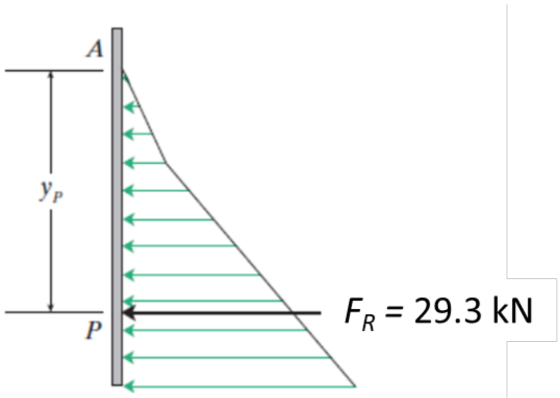
Length y_2, in turn, is the distance to the centroid of the rectangle from which we computed force F_2,

$$y_2 = 0.75 + \frac{1}{2}(1.5) = 1.50 \text{ m}$$

Finally, distance y_3 is associated with the triangle from which we calculated force F_3,

$$y_3 = 0.75 + \frac{2}{3}(1.5) = 1.75 \text{ m}$$

The location of the resultant force is determined by equating the moment of the resultant above A to the moments of the component forces about this point.



Accordingly, $\overline{y_p}$ is such that

$$\overline{y_p} F_R = \Sigma y F \rightarrow \overline{y_p}(29.3) = 0.5(3.11) + 1.5(12.4) + 1.75(13.8)$$

$$\therefore \boxed{\overline{y_p} = 1.51 \text{ m}}$$

The resultant force has an intensity of 29.3 kN and acts at a vertical distance of 1.51 m from point A.

➡ The correct answer is **B**.

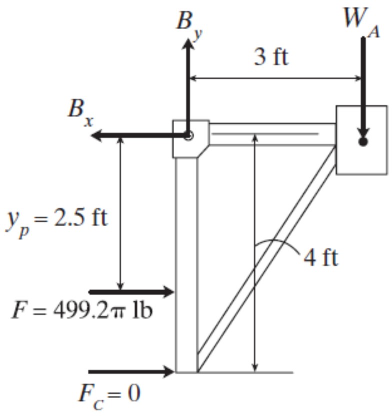
P.4 ● Solution

The resultant on the gate is calculated as

$$F_R = \gamma_w \bar{h} A = 62.4 \times 2 \times \pi(2)^2 = 1568.3 \text{ lb}$$

The location of the center of pressure is given by

$$y_p = \frac{I_x}{\bar{y}A} + \bar{y} = \frac{\left(\frac{\pi \times 2^4}{4}\right)}{2 \times \pi \times 2^2} + 2 = 2.5 \text{ ft}$$



Referring to the free-body diagram shown above, we apply the second condition of equilibrium to moment center B, giving

$$\Sigma M_B = 0 \rightarrow 1568.3(2.5) - W_A(3) = 0$$

$$\therefore W_A = \boxed{1.31 \text{ kip}}$$

➡ The correct answer is **A**.

P.5 ● Solution

Force F_1 is given by

$$F_1 = \gamma_1 h_{cg} A = \gamma_1 \frac{H}{2} \frac{H}{\sin\alpha} b = \frac{\gamma_1 H^2 b}{2 \sin\alpha}$$

while force F_2 is such that

$$F_2 = \gamma_2 h_{cg} A = \gamma_2 \frac{h}{2} \frac{h}{\sin\alpha} b = \frac{\gamma_2 h^2 b}{2 \sin\alpha}$$

Ratio F_1/F_2 is then

$$\frac{F_1}{F_2} = \frac{\gamma_1}{\gamma_2}\left(\frac{H}{h}\right)^2 \therefore \frac{H}{h} = \sqrt{\frac{F_1}{F_2} \times \frac{\gamma_2}{\gamma_1}} = \sqrt{1.70 \times \frac{1.25}{0.86}} = \boxed{1.57}$$

➡ The correct answer is **B**.

P.6 ● Solution

For a zero resultant force, we must have

$$F_{R,S} = F_{R,T}$$

which implies that

$$\gamma_S h_{CS} A_S = \gamma_F h_{CF} A_F$$

Thus, for a unit length of wall, we have

$$10.1 \times \frac{7}{2} \times 7(1) = 9.81 \times \frac{h}{2} \times h(1)$$

which can be solved for h to yield h = 7.10 m. In order for the moment to be zero, forces $F_{R,S}$ and $F_{R,T}$ must be collinear. For $F_{R,S}$, we have

$$y_R = \frac{I_x}{\bar{y}A} + \bar{y} = \frac{1 \times 7^3/_{12}}{\frac{7}{2} \times 7 \times 1} + \frac{7}{2} = 4.67 \text{ m}$$

Similarly for $F_{R,T}$,

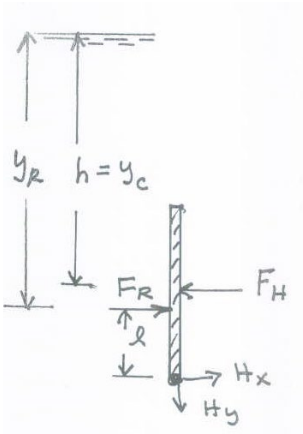
$$y_R = \frac{1\times 7.10^3/_{12}}{\frac{7.10}{2}\times 7.10\times 1} + \frac{7.10}{2} = 4.73\text{ m}$$

Thus, the distance to $F_{R,S}$ from the bottom is 7 – 4.67 = 2.33 m, and for $F_{R,T}$ this distance is 7.10 – 4.73 = 2.37 m. The forces are not collinear. Accordingly, when the resultant force is zero, the moment due to the forces will *not* be zero.

➡ The correct answer is **A**.

P.7 ● Solution

The free body diagram of the gate is shown in continuation.



For equilibrium to exist, the sum of moments relative to point H, where the hinge is located, must equal zero. In view of the figure above, we have

$$\Sigma M_H = 0 \rightarrow 4F_H - \ell F_R = 0 \;\; \text{(I)}$$

and

$$F_R = \gamma h_C A = 9.81\times h\times 3(8) = 235.4h$$

The force due to the water is located at a depth y_R, which is calculated as

$$y_R = \frac{I_x}{y_C A} + y_C = \frac{(3\times 8^3)/_{12}}{h(3\times 8)} + h = \frac{5.33}{h} + h$$

The length ℓ is such that $\ell = h + 4 - y_R$. Substituting, we obtain

$$\ell = h + 4 - y_R = h + 4 - \frac{5.33}{h} - h = 4 - \frac{5.33}{h}$$

Inserting this result into Equation (I), we have

$$4\times 3500 - \left(4 - \frac{5.33}{h}\right)\times 235.4h = 0$$

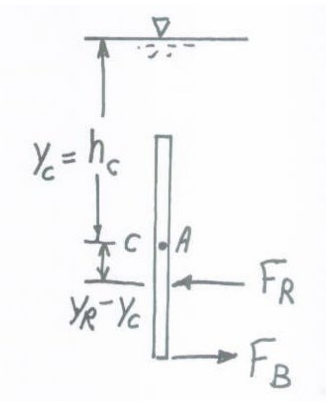
which, when solved for h, yields h = 16.2 m.

➡ The correct answer is **D**.

P.8 ● Solution

The pressure on the gate is the same as it would be for an open tank with a depth h_c given by

$$h_C = \frac{p + \gamma h}{\gamma}$$



The sum of moments relative to point A must equal zero. With reference to the figure above, we have

$$\Sigma M_A = 0 \rightarrow (y_R - y_C)F_R = RF_B \quad \text{(I)}$$

where $F_R = p_C A = \gamma h_C(\pi R^2) = (p + \gamma h)(\pi R^2)$. In addition,

$$y_R - y_C = \frac{I_x}{y_C A} = \frac{\frac{\pi R^4}{4}}{\left(\frac{p+\gamma h}{\gamma}\right)\pi R^2} = \frac{R^2}{4\left(\frac{p}{\gamma} + h\right)} \quad \text{(II)}$$

Combining Equations (I) and (II) yields

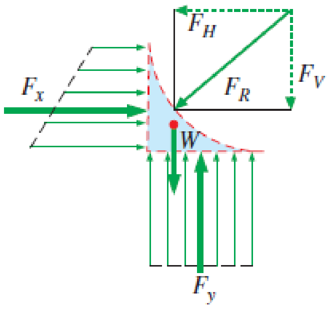
$$F_B = \frac{(y_R - y_C)}{R} F_R = \frac{R}{4\left(\frac{p}{\gamma} + h\right)}(p + \gamma h)\pi R^2 = \frac{\pi}{4}\gamma R^3$$

We conclude that the force on the latch is independent of both the supply pressure, p, and the height of the tank, h.

➡ The correct answer is **D**.

P.9 ● Solution

Part A: We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces, in addition to the weight of the liquid block, are determined as follows.



The horizontal force on the vertical surface is

$$F_H = F_x = p_{\text{avg}}A = \rho g h_C A = \rho g(s + R/2)A = 1000 \times 9.81 \times (4.2 + 0.8/2) \times 0.8(1)$$
$$= 36.1 \text{ kN}$$

The vertical force on the horizontal surface, directed upward, is

$$F_y = p_{\text{avg}}A = \rho g h_C A = \rho g h_{\text{bottom}} A = 1000 \times 9.81 \times 5 \times 0.8(1) = 39.2 \text{ kN}$$

The weight (downward) of the fluid block for 1-m width into the page is

$$W = mg = \rho g \forall = \rho g(R^2 - \pi R^2/4)(1) = 1000 \times 9.81 \times (0.8^2)(1 - \pi/4) \times 1$$
$$= 1.35 \text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.35 = 37.9 \text{ kN}$$

Then, the magnitude of the resultant hydrostatic force, F_R, acting on the cylindrical surface is

$$F_R = \sqrt{F_H^2 + F_V^2} = \boxed{52.3 \text{ kN}}$$

and its corresponding inclination relative to the horizontal is

$$\tan\theta = \frac{F_V}{F_H} = \frac{37.9}{36.1} \rightarrow \boxed{\theta = 46.4^\circ}$$

The magnitude of the hydrostatic force acting on the cylinder is 52.3 kN per meter length of the cylinder, and its line of action passes through the center of the cylinder and makes an angle of 46.4° with the horizontal.

➡ The correct answer is **B**.

Part B: When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. The forces acting on the cylinder, other than those at the hinge, are its weight, acting through the center, and the hydrostatic force exerted by the water. Taking moments about point A at the location of the hinge gives

$$F_R R \sin\theta - W_{\text{cyl}} R = 0 \rightarrow W_{\text{cyl}} = F_R \sin\theta = 52.3 \sin 46.4^{\text{o}} = 37.9 \text{ kN}$$

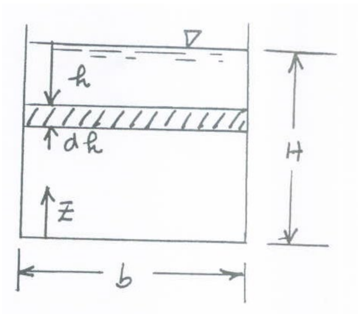
Hence, the weight of the cylinder per meter length is determined to be 37.9 kN. This corresponds to a mass of 37,870/9.81 = 3863 kg/m length and to a density of $3863/\pi(0.8)^2$ = 1921 kg/m^3 for the material that constitutes the cylinder.

➡ The correct answer is **C**.

P.10 ● Solution

The magnitude of the fluid force, F_R, can be found by summing the differential forces acting on the horizontal strip shown in the figure; that is,

$$F_R = \int_0^H dF_R = b\int_0^H p dh$$



where p is the pressure at depth h. To find p, we consider the relation

$$\frac{dp}{dz} = -\gamma$$

Since $dz = -dh$, we can write

$$p(h) = \int_o^h \gamma dh$$

This equation can be integrated numerically using the trapezoidal rule, i.e.,

$$I = \frac{1}{2}\sum_{i=1}^{n-1}(y_i + y_{i+1})(x_{i+1} - x_i)$$

Here, $y \sim \gamma, x \sim h$, and n is the number of data points. The pressure distribution is given in the following table.

h (m)	γ (kN/m^3)	Pressure (kPa)
0	10	0
0.4	10.1	4.02
0.8	10.2	8.08
1.2	10.6	12.24
1.6	11.3	16.62
2	12.3	21.34
2.4	12.7	26.34
2.8	12.9	31.46
3.2	13	36.64
3.6	13.1	41.86

The previous equation can then be integrated numerically using the trapezoidal rule, yielding an approximate value of 71.07 kN/m. Thus, with

$$F_R = \int_0^H pdh = 71.07 \text{ kN/m}$$

the resultant force is

$$F_R = 6 \times 71.07 = 426.4 \text{ kN}$$

To locate F_R, we sum moments about the axis formed by intersection of the vertical wall and the fluid surface; that is,

$$F_R h_R = b \int_0^H hpdh$$

The integrand, $h \times p$, is tabulated below.

h (m)	Pressure (kPa)	$h \times p$ (kN/m)
0	0	0.00
0.4	4.02	1.61
0.8	8.08	6.46
1.2	12.24	14.69
1.6	16.62	26.59
2	21.34	42.68
2.4	26.34	63.22
2.8	31.46	88.09
3.2	36.64	117.25
3.6	41.86	150.70

Using the trapezoidal rule with $y \sim h \times p$ and $x \sim h$, the approximate value of the integral is determined as 174.4 kN. Thus, with

$$\int_0^H hpdh = 174.4 \text{ kN}$$

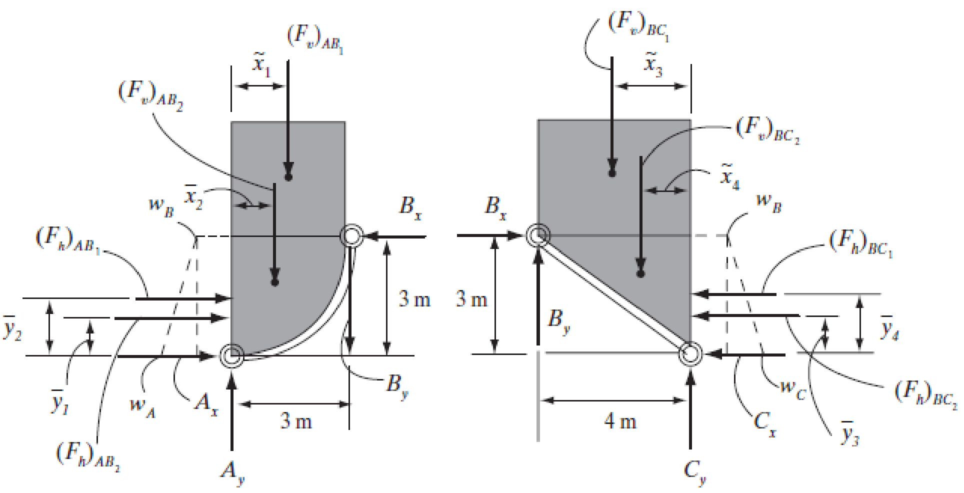
it follows that

$$h_R = \frac{b \int_0^H hpdh}{F_R} = \frac{6 \times 174.4}{426} = 2.46 \text{ m}$$

That is to say, the resultant force acts 2.46 m below the fluid surface.

P.11 ● Solution

The horizontal component of the resultant force is equal to the pressure force on the vertically projected area of the plate. Refer to the figure below.



Load w_B is such that

$$w_B = \rho_w g h_B b = 1000 \times 9.81 \times 3 \times 4 = 117.72 \text{ kN/m}$$

while w_A and w_C both equal

$$w_A = w_C = \rho_w g h_C b = 1000 \times 9.81 \times 6 \times 4 = 235.44 \text{ kN/m}$$

Thus,

$$(F_H)_{AB1} = (F_H)_{BC1} = 117.72 \times 3 = 353.16 \text{ kN}$$

$$(F_H)_{AB2} = (F_H)_{BC2} = 0.5 \times (235.44 - 117.72) \times 3 = 176.58 \text{ Kn}$$

These forces act at depths

$$\bar{y}_2 = \bar{y}_4 = \frac{1}{2}(3) = 1.5 \text{ m} \,; \bar{y}_1 = \bar{y}_3 = \frac{1}{3}(3) = 1.0 \text{ m}$$

The vertical component of the resultant force is equal to the weight of the column of water above the plates, shown shaded in the previous figures. Mathematically,

$$(F_V)_{AB1} = \rho_w g \forall = 1000 \times 9.81 \times [3(3)(4)] = 353.16 \text{ kN}$$

$$(F_V)_{AB2} = \rho_w g \forall = 1000 \times 9.81 \times \left[\frac{\pi}{4}(3)^2(4)\right] = 277.37 \text{ kN}$$

$$(F_V)_{BC1} = \rho_w g \forall = 1000 \times 9.81 \times [3(4)(4)] = 470.88 \text{ kN}$$

$$(F_V)_{BC2} = \rho_w g \forall = 1000 \times 9.81 \times [0.5(3)(4)(4)] = 235.44 \text{ kN}$$

These forces act at $\overline{x_1} = (1/2)(3) = 1.5$ m, $\overline{x_2} = (4)(3)/3\pi = 1.27$ m, $\overline{x_3} = (1/2)(4) = 2$ m, and $\overline{x_4} = (1/3)(4) = 1.33$ m. Then, we refer to the previous figure and write the moment equations for equilibrium at points A and C, namely,

$$\Sigma M_A = 0 \rightarrow B_x(3) - B_y(3) - 353.16(1.5) - 88.29\pi\left(\frac{4}{\pi}\right) - 353.16(1.5) - 176.58(1)$$
$$= 0$$

$$\therefore B_x - B_y = 529.74 \quad \text{(I)}$$

and

$$\Sigma M_C = 0 \rightarrow 470.88(2) + 235.44\left(\frac{4}{3}\right) + 353.16(1.5) + 176.58(1) - 3B_x - 4B_y = 0$$

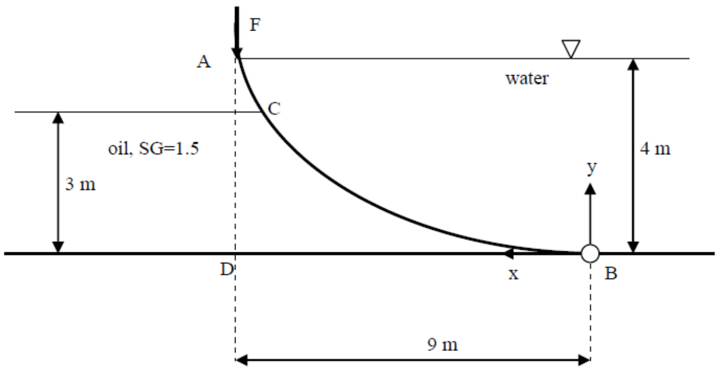
$$\therefore 3B_x + 4B_y = 1961.22 \quad \text{(II)}$$

Solving equations (I) and (II) simultaneously yields $B_x = 582.88 \approx 582.9$ kN and $B_y = 53.14 \approx 53.1$ kN.

➡ The correct answer is **D**.

P.12 ● Solution

Consider the hypothetical coordinate system shown below.



Generally, a parabolic shape is defined by the expression $y(x) = C_1x^2 + C_2x + C_3$. Since our parabola passes through the origin we have $C_2 = C_3 = 0$. Knowing that (x = 9, y = 4) is a point on the parabola, we write

$$y = C_1x^2 \rightarrow 4 = C_1 \times 9^2 \rightarrow C_1 = \frac{4}{81}$$

Thus, the parabola is described by the equation $y = (4/81)x^2$. Now, the force applied by the oil can be obtained via integration,

$$F_{H_O} = \int_{y_1}^{y_2} pbdy = \int_{y_1}^{y_2} (\gamma h)bdy = b\gamma\int_{y_1}^{y_2} hdy$$

Since $h + y = 3$ m, or $h = 3 - y$, the equation above takes the form

$$F_{H_O} = b\gamma\int_{y_1}^{y_2}(3-y)dy = 2 \times 1.5(9810) \times \int_0^3 (3-y)dy$$
$$= 2 \times 1.5(9810) \times \left(3y - \frac{y^2}{2}\Big|_0^3\right) = 2 \times 1.5(9810) \times 4.5 = 132.44 \text{ kN}$$

To locate F_{H_O}, we write

$$F_{H_0}y_{c-o} = \left(b\gamma\int_{y_1}^{y_2}(3-y)dy\right)y = 2\times 1.5(9810)\times\int_0^3(3y-y^2)dy$$
$$= 2\times 1.5(9810)\times\left(\frac{3}{2}y^2-\frac{1}{3}y^3\Big|_0^3\right) = 2\times 1.5(9810)\times 4.5$$
$$= 132.44\ \text{kN}\cdot\text{m}$$

Distance y_{c-o} is then

$$y_{c-o} = \frac{132.44}{132.44} = 1\ \text{m}$$

Alternatively, we could find y_{cp} from $y_{c-o} = (1/3)\times 3 = 1$ m.

Next, let us obtain the vertical component of the force. The incremental force $dF_{V_0} = pddA_x$, where $dA_x = bdx$. Thus,

$$F_{V_0} = \int_{x_1}^{x_2} pbdx = \int_{x_1}^{x_2}(\gamma h)bdx = b\gamma\int_{x_1}^{x_2} hdx$$

Since $h = 3 - y$ and $y = (4/81)x^2$, we get $h = 3-(4/81)x^2$. Force F_{V_0} is calculated as

$$F_{V_0} = b\gamma\int_{x_1}^{x_2}\left(3-\frac{4}{81}x^2\right)dx = 2\times 1.5(9810)\times\int_0^{7.79}\left(3-\frac{4}{81}x^2\right)dx$$
$$= 2\times 1.5(9810)\times\left(3x-\frac{4}{243}x^3\Big|_0^{7.79}\right)$$
$$= 2\times 1.5(9810)\times(15.59) = 458.81\ \text{kN}$$

To locate F_{V_0}, we take moments about the origin of the coordinate system,

$$F_{V_0}x_{c-o} = \left(b\gamma\int_{x_1}^{x_2}\left(3-\frac{4}{81}x^2\right)dx\right)x = b\gamma\int_0^{7.79}\left(3x-\frac{4}{81}x^3\right)dx$$
$$= 2\times 1.5(9810)\times\left(\frac{3}{2}x^2-\frac{1}{81}x^4\Big|_0^{7.79}\right)$$
$$= 2\times 1.5(9810)\times 45.56 = 1340.83\ \text{kN}$$

Finally,

$$x_{c-o} = \frac{1340.83}{458.81} = 2.92\ \text{m}$$

Next, we consider the force applied by the water. We shall use an alternative method. First, the horizontal component follows from

$$F_{H_w} = \gamma\left(h_{\text{cg}}A\right)_{\text{projected}} = 9810\times\frac{4}{2}\times 4\times 2 = 156.96\ \text{kN}$$

The force is concentrated at

$$y_{c-w} = \frac{1}{3}\times 4 = 1.33\ \text{m}$$

The vertical component, in turn, is

$$F_{V_w} = b\gamma\int_{x_1}^{x_2}\left(4-\frac{4}{81}x^2\right)dx = 2\times 9810\times\int_0^9\left(4-\frac{4}{81}x^2\right)dx$$
$$= 2\times 9810\times\left(4x-\frac{4}{243}x^3\Big|_0^9\right)$$
$$= 2\times 9810\times 24 = 470.88\ \text{kN}$$

Now, the location $x_{c\text{-}w}$ of the vertical component is determined as

$$F_{V_w}x_{c-w} = \left[b\gamma\int_{x_1}^{x_2}\left(4-\frac{4}{81}x^2\right)dx\right]x$$
$$= b\gamma\int_0^9\left(4x-\frac{4}{81}x^3\right)dx$$

$$= 2 \times 9810 \times \left(2x^2 - \frac{1}{81}x^4\Big|_0^9\right)$$

$$= 2 \times 9810 \times 81 = 1589.22 \text{ kN}$$

so that $x_{c\text{-}w}$ becomes

$$x_{\text{c-w}} = \frac{1589.22}{470.88} = 3.38 \text{ m}$$

Finally, a sum of moments about the hinge gives

$$F \times BD - F_{H_0} \times y_{c-o} - F_{V_0} \times x_{c-o} + F_{H_w} \times y_{c-w} + F_{V_w} \times x_{c-w} = 0$$

Solving for *F*, it follows that

$$F = \frac{132.44 \times 1 + 458.81 \times 2.92 - 156.96 \times 1.33 - 470.88 \times 3.38}{9} = -36.5 \text{ kN}$$

$$\therefore \boxed{|F| = 36.5 \text{ kN}}$$

Therefore, force *F* must be directed upward and have an intensity of about 36.5 kN.

➡ The correct answer is **C**.

ANSWER SUMMARY

Problem 1		**B**
Problem 2		**D**
Problem 3		**B**
Problem 4		**A**
Problem 5		**B**
Problem 6		**A**
Problem 7		**D**
Problem 8		**D**
Problem 9	**9A**	**B**
	9B	**C**
Problem 10		Open-ended pb.
Problem 11		**D**
Problem 12		**C**

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