Quiz HY201

Frequency Analysis

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Problems

Problem 1A

A highway is built near Montogue creek. The stream channel can carry 200 m^3/s, which is the peak flow of the 10-year storm of the watershed in which the road is located. Find the probability that the road will flood next year.

A) P_1 = 10%

B) P_1 = 20%

C) P_1 = 30%

D) P_1 = 40%

Problem 1B

Find the probability that the road will not flood at all in the next 5 years.

A) P_5 = 37.5%

B) P_5 = 59.0%

C) P_5 = 71.4%

D) P_5 = 88.5%

Problem 1C

Find the probability that the road will flood at least once in the next 20 years.

A) P_{20} = 34.8%

B) P_{20} = 57.9%

C) P_{20} = 69.5%

D) P_{20} = 87.8%

Problem 1D

Find the probability that the road will flood in the third and fourth years exactly in the next 5 years.

A) P_5 = 1.9%

B) P_5 = 7.3%

C) P_5 = 12.1%

D) P_5 = 17.0%

Problem 1E

Find the probability that the road will flood exactly 3 times in the next 10 years.

A) P_{10} = 5.7%

B) P_{10} = 12.3%

C) P_{10} = 17.5%

D) P_{10} = 22.4%

Problem 2

Given a mean and standard deviation of 1000 and 570 m^3/s, respectively, find the 2- and 10-year peak floods for a normal distribution.

A) Q_2 = 1000 m^3/s and Q_{10} = 1730 m^3/s

B) Q_2 = 1000 m^3/s and Q_{10} = 2450 m^3/s

C) Q_2 = 1320 m^3/s and Q_{10} = 1730 m^3/s

D) Q_2 = 1320 m^3/s and Q_{10} = 2450 m^3/s

Problem 3

Assuming a normal distribution with a mean and standard deviation of 1200 m^3/s and 830 m^3/s, find the return period for flood magnitudes of 1500 m^3/s and 2500 m^3/s.

A) T_{1500} = 1.2 years and T_{2500} = 17.2 years

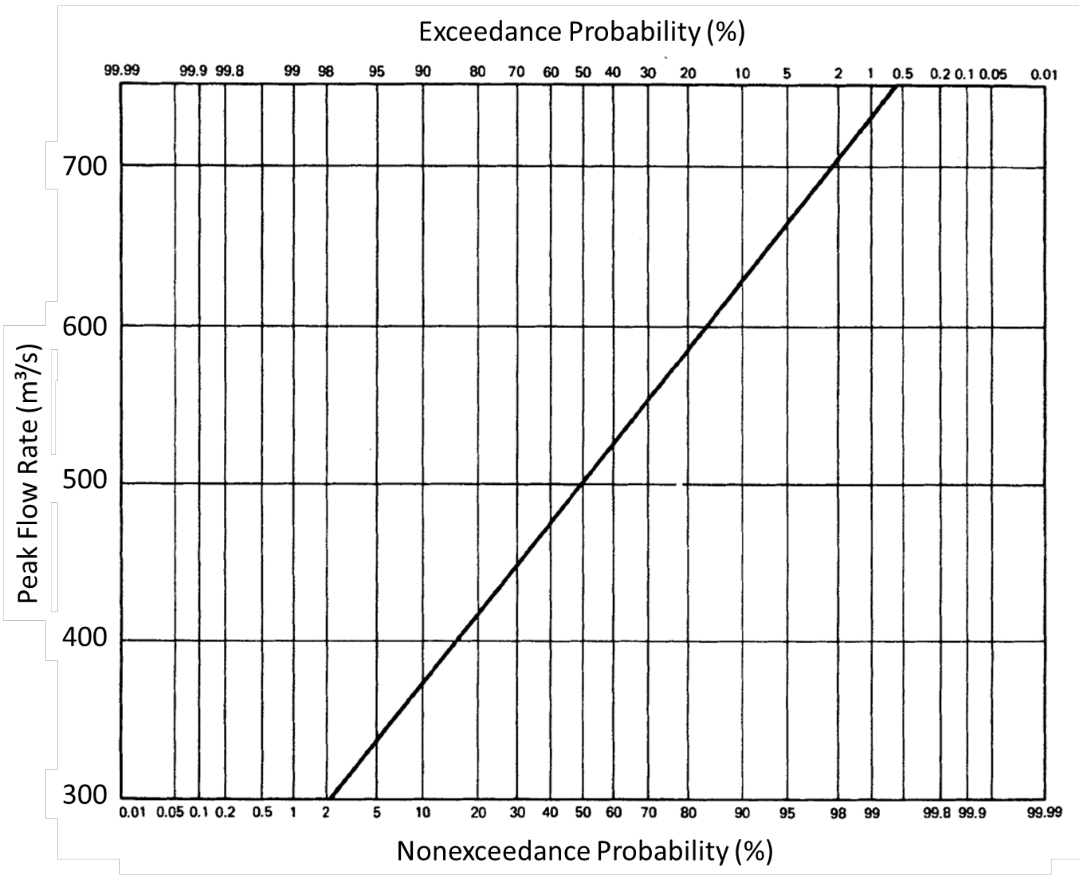
B) T_{1500} = 1.2 years and T_{2500} = 23.5 years

C) T_{1500} = 2.8 years and T_{2500} = 17.2 years

D) T_{1500} = 2.8 years and T_{2500} = 23.5 years

Problem 4

The following graph is the frequency curve from a normally distributed 20-acre urban watershed. Find the 25- and 100-year floods for this watershed.



A) Q_{25} = 584 m^3/s and Q_{100} = 733 m^3/s

B) Q_{25} = 584 m^3/s and Q_{100} = 855 m^3/s

C) Q_{25} = 675 m^3/s and Q_{100} = 733 m^3/s

D) Q_{25} = 675 m^3/s and Q_{100} = 855 m^3/s

Problem 5A

The peak discharge at a particular river gauging station is found to have a mean of 130 m^3/s and a standard deviation of 30 m^3/s. Considering the peak discharge to follow a lognormal distribution, evaluate the probability of the peak discharge being greater than 180 m^3/s.

A) P_{180} = 2.4%

B) P_{180} = 6.2%

C) P_{180} = 12.3%

D) P_{180} = 20.6%

Problem 5B

Determine the probability that the peak discharge will lie between 120 m^3/s and 150 m^3/s.

A) $P_{120-150}$ = 3.4%

B) $P_{120-150}$ = 15.4%

C) $P_{120-150}$ = 35.8%

D) $P_{120-150}$ = 48.5%

Problem 6A

The following data are the peak flows of the Drexciya river in the 39-year continuous period from 1991 to 2030. Assuming the data are normally distributed, find the peak flow of the 50-year flood. (The Excel spreadsheet can be found here.)

Year	Peak Flow (m^3/s)	Year	Peak Flow (m^3/s)
1991	338	2011	510
1992	635	2012	640
1993	306	2013	859
1994	453	2014	700
1995	823	2015	793
1996	288	2016	535
1997	741	2017	481
1998	1050	2018	974
1999	610	2019	280
2000	389	2020	676
2001	624	2021	604
2002	984	2022	531
2003	619	2023	698
2004	687	2024	482
2005	512	2025	598
2006	589	2026	359
2007	1160	2027	221
2008	673	2028	444
2009	666	2029	476
2010	401	2030	253

A) Q_{50} = 340 m^3/s

B) Q_{50} = 550 m^3/s

C) Q_{50} = 780 m^3/s

D) Q_{50} = 1050 m^3/s

Problem 6B

Maintaining the assumption that the data are normally distributed, find the return period of a flood with peak flow rate of 850 m^3/s.

A) T = 2.18 years

B) T = 4.42 years

C) T = 6.12 years

D) T = 8.26 years

Problem 6C

Suppose now that the Drexciya river data are lognormally distributed. Find the peak flow of the 100-year flood.

A) Q_{100} = 785 m^3/s

B) Q_{100} = 990 m^3/s

C) Q_{100} = 1380 m^3/s

D) Q_{100} = 1660 m^3/s

Problem 6D

Maintaining the assumption that the data are lognormally distributed, find the return period for a flood with peak flow rate of 825 m^3/s.

A) T = 0.778 years

B) T = 2.25 years

C) T = 4.33 years

D) T = 6.49 years

Problem 7A

The following data are the peak flows of the Efdemin river in the 25-year continuous period from 1998 to 2022. Perform a frequency analysis of this data adopting the log-Pearson type III distribution. The generalized skew from the regional map is −1.7. What is the exceedance probability of a flow of 1200 m^3/s?

Year	Peak Flow (m^3/s)	Year	Peak Flow (m^3/s)
1998	885	2011	1050
1999	1320	2012	1010
2000	847	2013	442
2001	1060	2014	366
2002	517	2015	1260
2003	649	2016	1050
2004	1020	2017	1550
2005	840	2018	1180
2006	716	2019	1080
2007	190	2020	793
2008	507	2021	1150
2009	890	2022	1010
2010	989		

A) P_{1200} = 10.1%

B) P_{1200} = 20.0%

C) P_{1200} = 30.6%

D) P_{1200} = 40.4%

Problem 7B

What is the magnitude of flow for a return period of 100 years?

A) Q_{100} = 1110 m^3/s

B) Q_{100} = 1435 m^3/s

C) Q_{100} = 1780 m^3/s

D) Q_{100} = 2050 m^3/s

Problem 8A (Bedient et al., 2013, w/ permission)

The random variable x represents the depth of rainfall in June, July, and August in Houston. The whole PDF is *symmetric* and is shaped as an isosceles triangle, with base 0 − 60 in. Between values of x = 0 and x = 30, the probability density function has the equation

$$f(x) = \frac{x}{900} \ ; \ (0 \le x \le 30)$$

Sketch the complete PDF.

Problem 8B

Find the probability that next summer's rainfall will not exceed 20 in.

A) P_{20} = 10.3%

B) P_{20} = 22.2%

C) P_{20} = 34.2%

D) P_{20} = 46.1%

Problem 8C

Find the probability that summer rainfall will equal or exceed 30 in. for the next three consecutive summers.

A) P_{30} = 13.3%

B) P_{30} = 20.8%

C) P_{30} = 27.6%

D) P_{30} = 34.4%

Problem 9A

Suppose that, in a year, there are approximately 90 storm events in the city of Kyoto, Japan, each with an average duration of 2.5 hours. Find the average interevent time ignoring seasonal variations and use it to fit an exponential distribution. What is the probability that the separation between two storms will be less than or equal to 12 hours?

A) P_{12} = 11.8%

B) P_{12} = 20.3%

C) P_{12} = 30.5%

D) P_{12} = 40.7%

Problem 9B

What is the probability that the separation between two storms will be *exactly* 12 hours?

A) P_{12} = 10.5%

B) P_{12} = 23.5%

C) P_{12} = 28.8%

D) None of the above.

Problem 9C

What is the probability that at least 3 days will elapse between storms?

A) P_{72} = 10.5%

B) P_{72} = 23.5%

C) P_{72} = 28.8%

D) P_{72} = 47.0%

Problem 10A

The following data give flood-data statistics for rivers Drexciya and Aisatsana. Estimate the 100-year flood for river Drexciya by using Gumbel's extreme value distribution.

River	Length of Records (Years)	Mean Annual Flood (m^3/s)	Sample SD (m^3/s)
Drexciya	92	5600	3450
Aisatsana	54	3000	1850

A) Q_{100} = 8950 m^3/s

B) Q_{100} = 13,000 m^3/s

C) Q_{100} = 17,200 m^3/s

D) Q_{100} = 21,100 m^3/s

Problem 10B

Estimate the 200-year flood for river Aisatsana by using Gumbel's extreme value distribution.

A) Q_{200} = 7200 m^3/s

B) Q_{200} = 10,500 m^3/s

C) Q_{200} = 13,400 m^3/s

D) Q_{200} = 16,500 m^3/s

Problem 10C

What is the 95% confidence interval for the value calculated in Part B?

Problem 11

For a data of maximum-recorded annual floods of a river the mean and the standard deviation are 4200 m^3/s and 1705 m^3/s respectively. Using Gumbel's extreme value distribution, estimate the return period of a design flood of 9500 m^3/s. Assume an infinite sample size.

A) T = 45 years

B) T = 97 years

C) T = 142 years

D) T = 191 years

Problem 12

For a certain river, the estimated flood peaks for two return periods as obtained from Gumbel's method are tabulated below. What flood discharge in this river will have a return period of 500 years?

Return Period (years)	Peak Flood (m^3/s)
50	3900
100	4450

A) Q_{500} = 5720 m^3/s

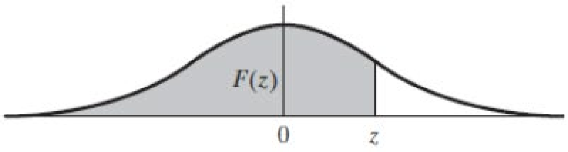
B) Q_{500} = 6580 m^3/s

C) Q_{500} = 7770 m^3/s

D) Q_{500} = 8590 m^3/s

Additional Information

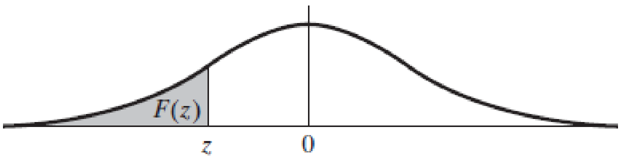
Table 1 Cumulative normal distribution



$$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table 2 Percentiles of the normal distribution



$$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

F(z)	z	F(z)	z
.0001	−3.719	.500	.000
.0005	−3.291	.550	.126
.001	−3.090	.600	.253
.002	−2.878	.650	.385
.005	−2.576	.700	.524
.010	−2.326	.750	.674
.020	−2.054	.800	.842
.025	−1.960	.850	1.036
.040	−1.751	.900	1.282
.050	−1.645	.950	1.645
.100	−1.282	.960	1.751
.150	−1.036	.975	1.960
.200	−.842	.980	2.054
.250	−.674	.990	2.326
.300	−.524	.995	2.576
.350	−.385	.998	2.878
.400	−.253	.999	3.090
.450	−.126	.9995	3.291
.500	.000	.9999	3.719

Table 3 Frequency factors K for log-Pearson type III distribution

Skew Coefficient, *g*	Probability							
	0.99	0.80	0.50	0.20	0.10	0.04	0.02	0.01
	Return Period							
	1.0101	1.2500	2	5	10	25	50	100
3.0	−0.667	−0.636	−0.396	0.420	1.180	2.278	3.152	4.051
2.8	−0.714	−0.666	−0.384	0.460	1.210	2.275	3.114	3.973
2.6	−0.769	−0.696	−0.368	0.499	1.238	2.267	3.071	3.889
2.4	−0.832	−0.725	−0.351	0.537	1.262	2.256	3.023	3.800
2.2	−0.905	−0.752	−0.330	0.574	1.284	2.240	2.970	3.705
2.0	−0.990	−0.777	−0.307	0.609	1.302	2.219	2.912	3.605
1.8	−1.087	−0.799	−0.282	0.643	1.318	2.193	2.848	3.499
1.6	−1.197	−0.817	−0.254	0.675	1.329	2.163	2.780	3.388
1.4	−1.318	−0.832	−0.225	0.705	1.337	2.128	2.706	3.271
1.2	−1.449	−0.844	−0.195	0.732	1.340	2.087	2.626	3.149
1.0	−1.588	−0.852	−0.164	0.758	1.340	2.043	2.542	3.022
0.8	−1.733	−0.856	−0.132	0.780	1.336	1.993	2.453	2.891
0.6	−1.880	−0.857	−0.099	0.800	1.328	1.939	2.359	2.755
0.4	−2.029	−0.855	−0.066	0.816	1.317	1.880	2.261	2.615
0.2	−2.178	−0.850	−0.033	0.830	1.301	1.818	2.159	2.472
0	−2.326	−0.842	0	0.842	1.282	1.751	2.054	2.326
−0.2	−2.472	−0.830	0.033	0.850	1.258	1.680	1.945	2.178
−0.4	−2.615	−0.816	0.066	0.855	1.231	1.606	1.834	2.029
−0.6	−2.755	−0.800	0.099	0.857	1.200	1.528	1.720	1.880
−0.8	−2.891	−0.780	0.132	0.856	1.166	1.448	1.606	1.733
−1.0	−3.022	−0.758	0.164	0.852	1.128	1.366	1.492	1.588
−1.2	−3.149	−0.732	0.195	0.844	1.086	1.282	1.379	1.449
−1.4	−3.271	−0.705	0.225	0.832	1.041	1.198	1.270	1.318
−1.6	−3.388	−0.675	0.254	0.817	0.994	1.116	1.166	1.197
−1.8	−3.499	−0.643	0.282	0.799	0.945	1.035	1.069	1.087
−2.0	−3.605	−0.609	0.307	0.777	0.895	0.959	0.980	0.990
−2.2	−3.705	−0.574	0.330	0.752	0.844	0.888	0.900	0.905
−2.4	−3.800	−0.537	0.351	0.725	0.795	0.823	0.830	0.832
−2.6	−3.889	−0.499	0.368	0.696	0.747	0.764	0.768	0.769
−2.8	−3.973	−0.460	0.384	0.666	0.702	0.712	0.714	0.714
−3.0	−4.051	−0.420	0.396	0.636	0.660	0.666	0.666	0.667

Table 4 Reduced mean $\bar{y}_n$ in Gumbel's extreme value distribution

N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.5070	0.5100	0.5128	0.5157	0.5181	0.5202	0.5220
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.5320	0.5332	0.5343	0.5353
30	0.5362	0.5371	0.5380	0.5388	0.5396	0.5402	0.5410	0.5418	0.5424	0.5430
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.5530	0.5533	0.5535	0.5538	0.5540	0.5543	0.5545
70	0.5548	0.5550	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.5570	0.5572	0.5574	0.5576	0.5578	0.5580	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.5600									

Table 5 Reduced standard deviation S_n in Gumbel's extreme value distribution

N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	1.1004	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.1480	1.1499	1.1519	1.1538	1.1557	1.1574	1.1590
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.1770	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.1890	1.1898	1.1906	1.1915	1.1923	1.1930
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.1980	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.2020	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.2060
100	1.2065									

Table 6 $f(c)$ coefficient as a function of confidence probability

c in percent	50	68	80	90	95	99
f(c)	0.674	1	1.282	1.645	1.96	2.58

Solutions

P.1 ■ Solution

Part A: Given the return period T = 10 years, the probability that the road will flood within the next year is

$$P_1 = \frac{1}{T} = \frac{1}{10} = \boxed{10\%}$$

★ The correct answer is **A**.

Part B: The probability that the road will not flood in any one year is 0.1. Accordingly, the probability that the road will *not* flood is 0.9. If the park is not to flood in the next five years, the corresponding probability must be $P_5 = 0.9^5 = 59\%$.

★ The correct answer is **B**.

Part C: The probability in question can be obtained with the risk formula

$$P_{20} = 1 - \left(1 - \frac{1}{T}\right)^n = 1 - \left(1 - \frac{1}{10}\right)^{20} = \boxed{87.8\%}$$

★ The correct answer is **D**.

Part D: The probability is given by the product

$$P_5 = 0.90 \times 0.90 \times 0.10 \times 0.10 \times 0.90 = \boxed{7.3\%}$$

★ The correct answer is **B**.

Part E: The desired probability can be determined with the binomial equation

$$P_{10} = \binom{n}{k} p^k (1-p)^{n-k}$$

Here, n = 10 is the total number of events, k = 3 is the number of favorable events, and p = 0.1 is the probability that the park will flood in any one year. Inserting these data in the relation above gives

$$P_{10} = \binom{10}{3} 0.1^3 (1-0.1)^7 = \boxed{5.7\%}$$

★ The correct answer is **A**.

P.2 ■ Solution

The exceedance probability for a 2-year flood is

$$p_2 = \frac{1}{T} = \frac{1}{2} = 0.5$$

The corresponding standard normal probability is

$$P_2 = 1 - p_2 = 1 - 0.50 = 0.50$$

Entering a percentile $F(z)$ of 0.50 into Table 2, we read a reduced variate z = 0. Consequently, the 2-year flood follows as

$$Q_2 = \bar{Q} + z \times \sigma = 1000 + 0 \times 570 = \boxed{1000 \text{ m}^3/\text{s}}$$

The exceedance probability for a 10-year flood is

$$p_{10} = \frac{1}{10} = 0.1$$

The corresponding standard normal probability is

$$P_{10} = 1 - p_{10} = 1 - 0.1 = 0.90$$

Mapping a percentile $F(z)$ of 0.90 onto Table 2, we extract a reduced variate z = 1.282. Accordingly, the 10-year flood follows as

$$Q_{10} = 1000 + 1.282 \times 570 = \boxed{1730 \text{ m}^3/\text{s}}$$

★ The correct answer is **A**.

P.3 ■ Solution

To begin, we calculate the frequency factor z for a flood magnitude of 1500 m^3/s,

$$z = \frac{Q - \bar{Q}}{\sigma} = \frac{1500 - 1200}{830} = 0.36$$

Referring to Table 1, we see that the standard normal probability is 0.6406. The corresponding exceedance probability is

$$p = 1 - 0.6406 = 0.359$$

or 35.9%. The return period for this flood magnitude is then

$$T_{1500} = \frac{1}{0.359} = \boxed{2.8 \text{ years}}$$

Consider now a flow of 2400 m^3/s. The value of z is now

$$z = \frac{2500-1200}{830} = 1.57$$

Referring to Table 1, we obtain a standard normal probability of 0.9418. The corresponding exceedance probability is

$$p = 1 - 0.9418 = 0.0582$$

or 5.82%. The pertaining return period follows as

$$T_{2500} = \frac{1}{0.0582} = \boxed{17.2 \text{ years}}$$

★ The correct answer is **C**.

P.4 ■ Solution

We know that the frequency curve given by a normal distribution will pass through the point defined by the mean and a probability of 0.5. Inspecting the graph, it is easy to see that $\bar{Q}$ = 500 m^3/s. Further, a normal probability plot should also pass through a point with ordinate $\bar{Q} - \sigma$ and exceedance probability of 0.8413 (or, equivalently, an ordinate $\bar{Q} + \sigma$ and an exceedance probability of 0.1587). Accordingly, we obtain σ = 100 m^3/s. Turning to the task of obtaining the desired floods, recall that the exceedance probability of a 25-year flood is 1/25 = 0.04, and hence the reduced variate is z = 1.751 (Table 2). The desired flood follows as

$$Q_{25} = \bar{Q} + z \times \sigma = 500 + 1.751 \times 100 = \boxed{675 \text{ m}^3/\text{s}}$$

The exceedance probability for a 100-year storm is 1/100 = 0.01, and the corresponding reduced variate, again appealing to Table 2, is z = 2.326. The sought flood is then

$$Q_{100} = \bar{Q} + z \times \sigma = 500 + 2.326 \times 100 = \boxed{733 \text{ m}^3/\text{s}}$$

★ The correct answer is **C**.

P.5 ■ Solution

Part A: As the peak discharge is described by a lognormal distribution, the parameters ($\bar{y}$ and σ_y) can be evaluated from the sample statistics ($\bar{x}$ and σ_x) as follows. The coefficient of variation C_v is given by

$$C_v = \frac{\sigma_x}{\bar{x}} = \frac{30}{130} = 0.231$$

Mean $\bar{y}$ follows as

$$\bar{y} = \frac{1}{2}\ln\left(\frac{\bar{x}^2}{C_v^2+1}\right) = 0.5 \times \ln\left(\frac{130^2}{0.231^2+1}\right) = 4.842$$

The standard deviation σ_y is calculated as

$$\sigma_y = \sqrt{\ln\left(C_v^2+1\right)} = \sqrt{\ln\left(0.231^2+1\right)} = 0.228$$

For x = 180, the reduced variate is

$$z = \frac{y-\bar{y}}{\sigma_y} = \frac{\ln x - \bar{y}}{\sigma_y} = \frac{\ln 180 - 4.842}{0.228} = 1.54$$

Referring to Table 1, we read $F(z)$ = 0.9382. the probability that the peak discharge will be greater than 180 m^3/s is evaluated as

$$P = P\left(Y > \ln 180\right) = 1 - P\left(Z < 1.54\right) = 1 - 0.9382 = \boxed{6.2\%}$$

★ The correct answer is **B**.

Part B: For x = 120, the corresponding reduced variate is

$$z_1 = \frac{y-\bar{y}}{\sigma_y} = \frac{\ln 120 - 4.841}{0.231} = -0.23$$

while for x = 150, we have

$$z_2 = \frac{\ln 150 - 4.841}{0.231} = 0.73$$

Referring to Table 1 as before, the probability we aim for is evaluated as

$$P(\ln 120 < Y < \ln 150) = P_{120-150} = P(-0.23 < z < 0.73)$$

$$\therefore P_{120-150} = P(z < 0.73) - P(z < -0.23)$$

$$\therefore P_{120-150} = P(z < 0.73) - P(z > 0.23)$$

$$\therefore P_{120-150} = P(z < 0.73) - [1 - P(z < 0.23)]$$

$$\therefore P_{120-150} = P(z < 0.73) + P(z < 0.23) - 1$$

$$\therefore P_{120-150} = 0.7673 + 0.5910 - 1 = \boxed{35.8\%}$$

★ The correct answer is **C**.

P.6 ■ Solution

Part A: The mean and standard deviation of the data are $\bar{Q}$ = 592 m^3/s and σ = 221 m^3/s, respectively. For a 50-year flood, we have $F = 1 - 1/T = 0.98$. With reference to Table 2, we read z = 2.054. The peak flow for the flood in question follows as

$$Q_{50} = \bar{Q} + z \times \sigma = 592 + 2.054 \times 221 = \boxed{1050 \text{ m}^3/\text{s}}$$

★ The correct answer is **D**.

Part B: The reduced variate z is

$$z = \frac{Q - \bar{Q}}{\sigma} = \frac{850 - 592}{221} = 1.17$$

Entering this value into Table 1, the corresponding probability is read as 87.9%. The return period is given by

$$1 - \frac{1}{T} = 0.879 \rightarrow \boxed{T = 8.26 \text{ years}}$$

★ The correct answer is **D**.

Part C: Transforming the data points to base-10 logarithms, summing these logarithms, and dividing by the number of data points, one obtains $\overline{\log Q}$ = 2.741. The standard deviation is $\sigma_{\log Q}$ = 0.171. With reference to Table 2, we take z = 2.326. The corresponding flow is then

$$\log Q = \overline{\log Q} + z \times \sigma_{\log Q} \rightarrow \log Q = 2.741 + 2.326 \times 0.171 = 3.139$$

$$\therefore Q_{100} = 10^{3.139} = \boxed{1380 \text{ m}^3/\text{s}}$$

★ The correct answer is **C**.

Part D: The logarithm of the flow is $\log Q$ = 2.916, and the reduced variate is calculated as

$$z = \frac{\log Q - \overline{\log Q}}{\sigma_{\log Q}} = \frac{2.916 - 2.741}{0.171} = 1.02$$

Entering this value into Table 1, the corresponding probability is read as 84.6%. The return period follows as

$$1 - \frac{1}{T} = 0.846 \rightarrow \boxed{T = 6.49 \text{ years}}$$

★ The correct answer is **D**.

P.7 ■ Solution

Part A: The base-10 logarithms of the flows are listed below.

Year	log(Peak Flow) [log(m³/s)]	Year	log(Peak Flow) [log (m³/s)]
1998	2.947	2011	3.021
1999	3.121	2012	3.004
2000	2.298	2013	2.645
2001	3.025	2014	2.563
2002	2.713	2015	3.1
2003	2.812	2016	3.021
2004	3.009	2017	3.19
2005	2.924	2018	3.072
2006	2.855	2019	3.033
2007	2.279	2020	2.899
2008	2.705	2021	3.061
2009	2.949	2022	3.004
2010	2.995		

The mean, standard deviation, and skew coefficient of the data are $\bar{x}$ = 2.915, σ = 0.202, and $g_s = -1.5$. The mean square error of the sample skew coefficient, $V(g_s)$, is calculated with the relation

$$V\left(g_s\right)=10^{A-B\log\left(N/10\right)}$$

Here coefficient A, for $|g_s| > 1.5$, is given by

$$A=-0.52+0.30\left|g_s\right|=-0.52+0.30\times1.5=-0.07$$

while coefficient B, with $|g_s| \leq 1.5$, is computed as

$$B=0.94-0.26\left|g_s\right|=0.94-0.26\times1.5=0.550$$

so that

$$V\left(g_s\right)=10^{-0.07-0.550\times\log\left(25/10\right)}=0.514$$

Knowing that $V(g_m) = 0.3025$ as always, the weighting factor W follows as

$$W=\frac{V\left(g_m\right)}{V\left(g_m\right)+V\left(g_s\right)}=\frac{0.3025}{0.3025+0.514}=0.370$$

With $g_m = -1.7$, we are now able to calculate the generalized skew coefficient g,

$$g=Wg_s+\left(1-W\right)g_m=0.370\times\left(-1.5\right)+\left(1-0.370\right)\times\left(-1.7\right)=-1.626$$

or simply $g = -1.6$. Mapping this quantity onto Table 3, we can extract a series of frequency factors K and thence compute the logarithm of the flows, $X = \bar{X} + K \times S$. Finally, the flows themselves are determined as $Q = 10^X$. The results of this procedure are shown in the following table.

Percent Probability	K	$X = X_{avg} + KS$ [log(m³/s)]	$Q = \log^{-1}X$ (m³/s)
1	1.197	3.157	1435
2	1.166	3.151	1414
4	1.116	3.140	1382
10	0.994	3.116	1306
20	0.817	3.080	1202
50	0.254	2.966	925
80	-0.675	2.779	601
99	-3.388	2.231	170

If need be, these values can be plotted on probability paper and the flood frequency curve thusly prepared. Reading the table above, we observe that the exceedance probability of a flow of 1200 m³/s is close to 20.0%.

★ The correct answer is **B**.

Part B: The exceedance probability for a return period of 100 years is 1/100 = 1%. Inspecting the table above, the flow that corresponds to this probability is 1435 m³/s.

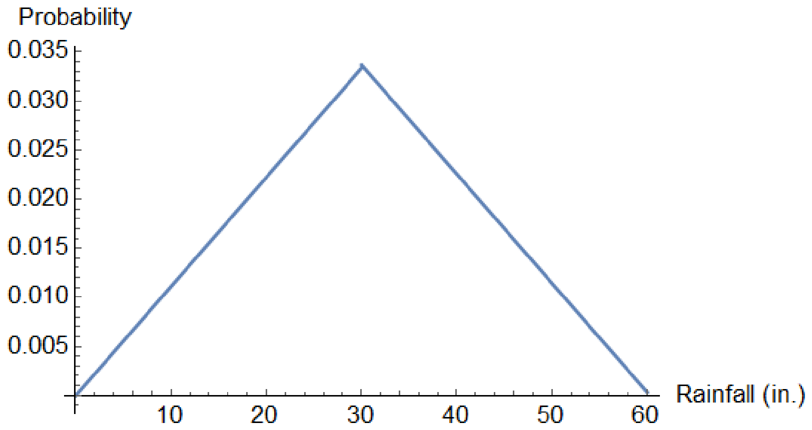
★ The correct answer is **B**.

P.8 ■ Solution

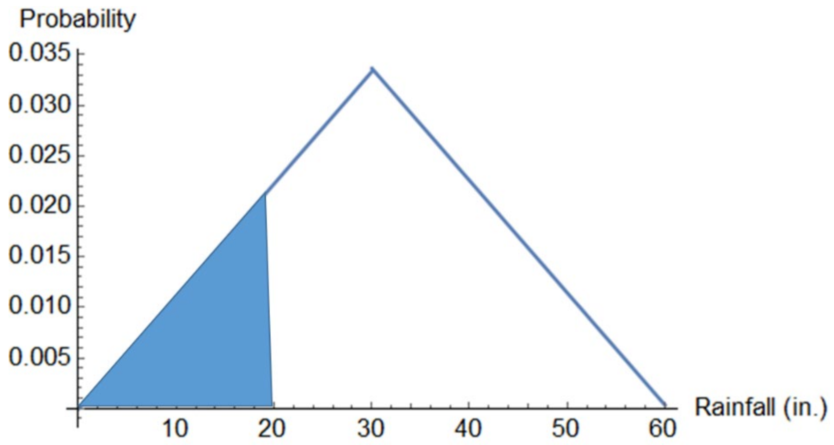
Part A: It is easy to see that, while the left half of the PDF follows the function $f(x) = x/900$, the right half is given by

$$f(x) = 0.067 - \frac{x}{300} \;\; ; \;\; (30 \le x \le 60)$$

A plot of the PDF is shown below.



Part B: This probability can be established by computing the area below the PDF over the interval of interest, namely $0 \le x \le 20$. Refer to the figure below.



Clearly, the probability in question is given by the area of the triangle highlighted in blue. Its height is 20/900 = 0.0222, and its area, accordingly, is such that

$$P_{20} = P(x < 20) = 0.5 \times 0.0222 \times 20 = \boxed{22.2\%}$$

Of course, the probability can also be obtained analytically,

$$P(x \le 20) = \int_0^{20} f(x)\,dx = \left.\frac{x^2}{1800}\right|_0^{20} = 22.2\%$$

★ The correct answer is **B**.

Part C: In this case, the probability we aim for is $[P(x \ge 30)]^3$. In analytical terms, we have

$$P_{30} = \left[P(x \ge 30)\right]^3 = \left[\int_{30}^{60}\left(0.067 - \frac{x}{900}\right)dx\right]^3$$

$$\therefore P_{30} = \left[\left.\left(0.067x - \frac{x^2}{1800}\right)\right|_{30}^{60}\right]^3$$

$$\therefore P_{30} = (2.02 - 1.51)^3 = \boxed{13.3\%}$$

The solution could just as well have been established geometrically, noting that $[P(x \ge 30)]^3$ is simply the area of the right half of the triangle taken to third power. Indeed,

$$P_{30} = \left[0.5 \times (60 - 30) \times 0.033\right]^3 = 0.50^3 = 12.5\%$$

★ The correct answer is **A**.

P.9 ■ Solution

Part A: Ignoring seasonal variations, the average interevent time is determined to be

$$\bar{t} = \frac{24\times 365 - 90\times 2.5}{90} = 94.8 \text{ h}$$

The rate parameter is then $\lambda = 1/\bar{t} = 1/94.8 = 0.0105 \text{ h}^{-1}$. The probability that the separation between two storms will be less than or equal to 12 hours is easily obtained with the cumulative distribution function,

$$P_{12} = P(t \le 12) = F(12) = 1 - e^{-0.0105\times 12} = \boxed{11.8\%}$$

★ The correct answer is **A**.

Part B: In this case, we simply have

$$P(t = 12) = 0$$

because the probability that a continuous random variable exactly equals a specific value is zero.

★ The correct answer is **D**.

Part C: The probability we are looking for is $P(t > 72)$, which can be easily obtained by recalling that

$$P(t > 72) = 1 - P(t \le 72) \rightarrow P(t > 72) = 1 - F(72)$$

$$\therefore P(t > 72) = 1 - \left(1 - e^{-0.0105\times 72}\right)$$

$$\therefore \boxed{P_{72} = 47.0\%}$$

★ The correct answer is **D**.

P.10 ■ Solution

Part A: Referring to Tables 4 and 5 with a sample size N = 92, we extract coefficients $\bar{y}_n$ = 0.5589 and S_n = 1.2020. Reduced variate y_T is determined as

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right] = -\ln\left[\ln\left(\frac{100}{99}\right)\right] = 4.60$$

Frequency factor K follows as

$$K = \frac{y_T - \bar{y}_n}{S_n} = \frac{4.60 - 0.5589}{1.2020} = 3.362$$

The 100-year flood for the river in question is then

$$Q_{100} = \bar{Q} + K\times\sigma = 5600 + 3.362\times 3450 = \boxed{17{,}200 \text{ m}^3/\text{s}}$$

★ The correct answer is **C**.

Part B: Entering a sample size N =54 into Tables 4 and 5, we read coefficients $\bar{y}_n$ = 0.5501 and S_n = 1.1667. Reduced variate y_T is calculated as

$$y_T = -\ln\left[\ln\left(\frac{200}{199}\right)\right] = 5.296$$

The value of K is

$$K = \frac{5.296 - 0.5501}{1.1667} = 4.068$$

Thus, the 200-year flood for the river being considered is

$$Q_{200} = 3000 + 4.068\times 1850 = \boxed{10{,}500 \text{ m}^3/\text{s}}$$

★ The correct answer is **B**.

Part C: For a confidence probability c, the confidence interval of the flow rate variate Q is bounded by values Q_1 and Q_2 given by

$$Q_{1,2} = Q \pm f(c) S_e$$

where $f(c)$ is a constant that varies with the confidence probability c in accordance with Table 6. For a 90% confidence, $f(90)$ = 1.645. In turn, S_e is the probable error, which is described by the relation

$$S_e = \sqrt{1+1.3K+1.1K^2}\,\frac{\sigma}{\sqrt{N}} = \sqrt{1+1.3\times4.068+1.1\times4.068^2}\times\frac{1850}{\sqrt{54}} = 1246$$

so that

$$Q_{1,2} = 10,500 \pm 1.645\times1246 = 8450 \text{ and } 12{,}550 \text{ m}^3/\text{s}$$

There is a 90% probability that the estimated discharge of 10,500 m^3/s will lie between 8450 and 12,550 m^3/s.

P.11 ■ Solution

The design flood Q = 9500 m^3/s. Appealing to the equation for Q, we have

$$Q = \bar{Q} + K\sigma \rightarrow 9500 = 4200 + K\times1705$$
$$\therefore K = 3.11$$

Using the relation for coefficient K, in turn, we can calculate the reduced variate y_T,

$$K = \frac{y_T - 0.577}{1.2825} = 3.11 \rightarrow y_T = 4.57$$

(Coefficients −0.577 and 1.2825 occur because the sample size is infinite.) Since y_T is a function of return period only, we are now able to determine T,

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right] = 4.57 \rightarrow \boxed{T = 97 \text{ years}}$$

★ The correct answer is **B**.

P.12 ■ Solution

The peak floods are described by the relation $Q = \bar{Q} + K\times\sigma$, so that, in the present case,

$$3900 = \bar{Q} + K_{50}\times\sigma$$
$$4450 = \bar{Q} + K_{100}\times\sigma$$

Subtracting one equation from the other gives

$$(4450-3900) = \bar{Q} + (K_{100} - K_{50})\times\sigma$$
$$\therefore 550 = \bar{Q} + (K_{100} - K_{50})\times\sigma \quad \text{(I)}$$

The reduced variates are

$$y_{50} = -\ln\left[\ln\left(\frac{50}{49}\right)\right] = 3.902\ ;\ y_{100} = -\ln\left[\ln\left(\frac{100}{99}\right)\right] = 4.60$$

and $y_{100} - y_{50}$ = 0.698. Observing that $K = (y_T - \bar{y}_n)/S_n$ and referring to equation (I), we have

$$0.698\times\frac{\sigma}{S_n} = 550 \rightarrow \frac{\sigma}{S_n} = 788$$

For a return period of 500 years, we have y_{500} = 6.214 and K_{500} = (6.214 − $\bar{y}_n$)/S_n. Subtracting K_{100} from K_{500} yields

$$K_{500} - K_{100} = \frac{6.214-4.60}{S_n} = \frac{1.614}{S_n}$$

so that the discharge difference $Q_{500} - Q_{100}$ becomes

$$Q_{500} - 4450 = \frac{1.614}{S_n}\sigma$$

Since σ/S_n = 788, we ultimately obtain

$$Q_{500} - 4450 = 1.614 \times 788 = 1272$$

$$\therefore \boxed{Q_{500} = 5720 \text{ m}^3/\text{s}}$$

★ The correct answer is **A**.

Answer Summary

Problem 1	**1A**	A
	1B	B
	1C	D
	1D	B
	1E	A
Problem 2		A
Problem 3		C
Problem 4		C
Problem 5	**5A**	B
	5B	C
Problem 6	**6A**	D
	6B	D
	6C	C
	6D	D
Problem 7	**7A**	B
	7B	B
Problem 8	**8A**	Open-ended pb.
	8B	B
	8C	A
Problem 9	**9A**	A
	9B	D
	9C	D
Problem 10	**10A**	C
	10B	B
	10C	Open-ended pb.
Problem 11		B
Problem 12		A

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