Montogue

Quiz EL205
Frequency Response

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PROBLEM DISTRIBUTION

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PROBLEMS

Problem 1 (Sedra and Smith, 2015, w/ permission)

A CS amplifier has $C_{C1} = C_S = C_{C2} = 1$ μF, $R_G = 10$ MΩ, $R_{\text{sig}} = 100$ kΩ, $g_m = 2$ mA/V, $R_D = R_L = R_S = 10$ kΩ. Find the midband gain A_M, pole frequencies f_{P1}, f_{P2} and f_{P3}, the lower 3-dB frequency f_L, and the zero frequency f_Z.

Theory: See Topic 1.

Problem 2 (Sedra and Smith, 2015, w/ permission)

The amplifier in Figure 1 (see Additional Information section) is biased to operate at $g_m = 5$ mA/V and $R_S = 1.8$ kΩ. Find the value of C_S that places its associated pole at 100 Hz or lower. What are the actual frequencies of the pole and zero realized?

Theory: See Topic 1.

Problem 3 (Sedra and Smith, 2015, w/ permission)

A common-emitter amplifier has $C_{C1} = C_E = C_{C2} = 1$ μF, $R_B = 100$ kΩ, $R_{\text{sig}} = 5$ kΩ, $g_m = 40$ mA/V, $r_\pi = 2.5$ kΩ, $R_E = 5$ kΩ, $R_C = 8$ kΩ, and $R_L = 5$ kΩ. Find the value of the time constant associated with each capacitor, and hence estimate the value of the lower 3-dB frequency f_L. Also compute the frequency of the transmission zero introduced by C_E and comment on its effect on f_L. Take $\beta = 100$ as the BJT current gain parameter.

Theory: See Topic 2.

Problem 4 (Sedra and Smith, 2015, w/ permission)

Problem 4.1: Consider the common-emitter amplifier of Figure 2 under the following conditions: $R_{\text{sig}} = 5$ kΩ, $R_{B1} = 33$ kΩ, $R_{B2} = 22$ kΩ, $R_E = 3.9$ kΩ, $R_C = 4.7$ kΩ, $R_L = 5.6$ kΩ, and $V_{CC} = 5$ V. The dc collector current can be shown to be $I_C \approx 0.3$ mA, at which $\beta = 120$. Find the input resistance R_{in} and the midband gain A_M. If $C_{C1} = C_{C2} = 1$ μF and $C_E = 20$ μF, find the three short-circuit time constants and an estimate for the lower 3-dB frequency f_L.

Problem 4.2: For the amplifier described in Problem 4.1, design the coupling and bypass capacitors for a lower 3-dB frequency of 50 Hz. Design so that each of C_{C1} and C_{C2} to determining f_L is only 10%.

Theory: See Topic 2.

Problem 5 (Sedra and Smith, 2015, w/ permission)

Problem 5.1: Consider a n-channel MOSFET with oxide thickness t_{ox} = 10 nm, length L = 1.0 µm, width W = 10 µm, overlap length L_{ov} = 0.05 µm, source-body capacitance at zero body-source bias C_{sb0} = 10 fF, drain-body capacitance at zero reverse-bias voltage C_{db0} = 10 fF, junction built-in voltage V_0 = 0.6 V, reverse-bias voltage V_{SB} = 1 V, and drain-source voltage V_{DS} = 2 V. The device is assumed to operate at 100 µA and transconductance parameter k_n' = 160 µA/V². Determine the oxide capacitance C_{ox}, the overlap capacitance C_{ov}, and the four internal capacitances (gate-to-source C_{gs}, gate-to-drain C_{gd}, source-body C_{sb}, and drain-body C_{db}). The permittivity of silicon oxide is taken as ε_{ox} = 3.45×10⁻¹¹ F/m.

Problem 5.2: Follow up on your results and determine the unity-gain frequency f_T.

Theory: See Topic 3.

Problem 6 (Sedra and Smith, 2015, w/ permission)

Starting from the expression for f_T for a MOSFET (equation 18),

$$f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)}$$

and making the approximation that $C_{gs} \gg C_{gd}$ (i.e., the gate-source capacitance is much greater than the gate-drain capacitance) and that the overlap component of C_{gs} is negligibly small, show that

$$f_T \approx \frac{1.5}{\pi L}\sqrt{\frac{\mu_n I_D}{2C_{ox}WL}}$$

where μ_n is electron mobility, I_D is drain current, C_{ox} is oxide capacitance, W is effective width, and L is effective length. Thus note that to obtain a high f_T from a given device, it must be operated at a high current. Also note that faster operation is obtained from smaller devices.

Theory: See Topic 3.

Problem 7 (Sedra and Smith, 2015, w/ permission)

Find the unity-gain bandwidth f_T for a MOSFET operating at bias current I_D = 200 µA and overdrive voltage V_{OV} = 0.3 V. The MOSFET has gate-source capacitance C_{gs} = 25 fF and gate-drain capacitance C_{gd} = 5 fF.

Theory: See Topic 3.

Problem 8 (Sedra and Smith, 2015, w/ permission)

A bipolar junction transistor operates at a dc collector current I_C = 1 mA and a collector-base junction reverse bias of 2 V. The device has forward base-transit time τ_F = 20 ps, base-emitter junction capacitance at zero emitter-base junction voltage C_{je0} = 20 fF, depletion capacitance at zero voltage $C_{\mu 0}$ = 20 fF, base-emitter junction built-in voltage V_{0e} = 0.9 V, collector-base junction built-in voltage V_{0c} = 0.5 V, and collector-base junction grading coefficient m_{CBJ} = 0.33. Determine the small-signal diffusion capacitance C_{de}, the base-emitter junction capacitance C_{je}, the emitter-base capacitance C_π, the depletion capacitance C_μ, and the unity-gain bandwidth f_T.

Theory: See Topic 4.

Problem 9 (Sedra and Smith, 2015, w/ permission)

Problem 9.1: Find the unity-gain bandwidth f_T and the beta cut-off frequency $f_\beta = f_T/\beta$ for a BJT operating at bias current I_C = 0.5 mA if the device has collector-base capacitance C_μ = 1 pF, emitter-base capacitance C_π = 8 pF, and current gain parameter β = 100.

Problem 9.2: For the bipolar transistor described in Problem 9.1, emitter-base capacitance C_π includes a relatively constant depletion-layer capacitance of 2 pF. If the device is operated at a bias current I_C = 0.25 mA, what does the unity-gain bandwidth f_T become?

Theory: See Topic 4.

Problem 10 (Sedra and Smith, 2015, w/ permission)

Problem 10.1: Find the midband gain A_M and the upper 3-dB frequency f_H of a CS amplifier fed with a signal source having an internal resistance R_{sig} = 120 kΩ. The amplifier has R_G = 3.2 MΩ, $R_D = R_L$ = 16 kΩ, transconductance g_m = 1 mA/V², r_o = 165 kΩ, gate-source capacitance C_{gs} = 1 pF, and gate-drain capacitance C_{gd} = 0.4 pF. Also, find the frequency of the transmission zero f_Z.

Problem 10.2: If it is possible to replace the MOSFET used in the amplifier in Problem 10.1 with another having the same gate-source capacitance C_{gs} but a smaller gate-drain capacitance C_{gd}, what is the maximum value that its C_{gd} can be in order to obtain an upper 3-dB frequency f_H of at least 1 MHz?

Theory: See Topic 5.

Problem 11 (Sedra and Smith, 2015, w/ permission)

In a particular common-source amplifier for which the midband voltage gain between gate and drain (i.e., $-g_m R_L'$) is −39 V/V, the NMOS transistor has gate-source capacitance C_{gs} = 1.0 pF and gate-drain capacitance C_{gd} = 0.1 pF. What input capacitance C_{in} would you expect? For what range of signal-source resistances can you expect the 3-dB frequency to exceed 1 MHz? Neglect the effect of R_G.

Theory: See Topic 5.

Problem 12 (Sedra and Smith, 2015, w/ permission)

It is required to find the midband gain and the upper 3-dB frequency of the CE amplifier illustrated in Figure 2. Use as data emitter current I_E = 1 mA, $R_B = R_{B1}||R_{B2}$ = 120 kΩ, collector resistance R_C = 7.5 kΩ, signal source resistance R_{sig} = 6 kΩ, load resistance R_L = 4.8 kΩ, β_0 = 100, Early voltage V_A = 100 V, emitter-base capacitance C_μ = 1 pF, transfer frequency f_T = 825 MHz, and r_x = 50 Ω; the circuit is biased at $I_C \approx$ 1 mA. Also determine the frequency of transmission zero.

Theory: See Topic 6.

Problem 13 (Sedra and Smith, 2015, w/ permission)

Some of the frequency response relations given in the Additional Information section can be somewhat simplified under the right circumstances. Consider the high-frequency response of a CE amplifier fed by a relatively large source resistance R_{sig}. Refer to the amplifier in Figure 2 and to its high-frequency, equivalent-circuit model and the analysis shown in Figure 5. Let $R_B \gg R_{\text{sig}}$, $r_x \ll R_{\text{sig}}$, $R_{\text{sig}} \gg r_\pi$, $g_m R_L' \gg 1$, and $g_m R_L' C_\mu \gg C_\pi$. Under these conditions, show that:

1. The midband gain $A_M \approx -\beta R_L'/R_{\text{sig}}$.

2. The upper 3-dB frequency $f_H \approx 1/(2\pi C_\mu \beta R_L')$.

3. The gain-bandwidth product $|A_M| f_H \approx 1/(2\pi C_\mu R_{\text{sig}})$.

4. Use the approximation to gain-bandwidth for the case R_{sig} = 25 kΩ and C_μ = 1 pF.

Theory: See Topic 6.

Problem 14 (Sedra and Smith, 2015, w/ permission)

A direct-coupled amplifier has a dc gain of 1000 V/V and an upper 3-dB frequency of 100 kHz. Find the transfer function and the gain-bandwidth product in hertz.

Problem 15 (Sedra and Smith, 2015, w/ permission)

Problem 15.1: The high-frequency response of an amplifier is characterized by two zeros at $s \to \infty$ and two poles at ω_{P1} and ω_{P2}. For $\omega_{P2} = k\omega_{P1}$, find the value of k that results in the exact value of 3-dB frequency ω_H being $0.9\omega_{P1}$.

Problem 15.2: For the amplifier described in Problem 15.1, find the exact and approximate values of the 3-dB frequency ω_H if the proportionality constant k equals 1.0. Repeat for k = 4.

Theory: See Topic 7.

Problem 16 (Sedra and Smith, 2015, w/ permission)

A direct-coupled amplifier has a low-frequency gain of 40 dB, poles at 2 MHz and 20 MHz, a zero on the negative real axis at 200 MHz, and another zero at infinite frequency. Write the amplifier gain function and compute the 3-dB frequency.

Theory: See Topic 7.

Problem 17 (Sedra and Smith, 2015, w/ permission)

An amplifier with a dc gain of 60 dB has a single-pole, high-frequency response with a 3-dB frequency of 100 kHz.

Problem 17.1: Give an expression for the gain function *A(s)*.

Problem 17.2: Sketch Bode plots for gain magnitude and phase.

Problem 17.3: If a change in the amplifier circuit causes its transfer function to acquire another pole at 1 MHz, sketch the resulting gain magnitude and specify the unity-gain frequency.

Problem 18 (Sedra and Smith, 2015, w/ permission)

An integrated-circuit CS amplifier has transconductance g_m = 1.4 mA/V, gate-source capacitance C_{gs} = 25 fF, gate-drain capacitance C_{gd} = 6 fF, output node-to-ground capacitance C_L = 27 fF, Thévenin resistance of signal generator R'_{sig} = 12 kΩ, and output-node-to-ground resistance R'_L = 12 kΩ.

Problem 18.1: Determine the 3-dB frequency f_H using open-circuit time constants. Also determine the frequency of the transmission zero f_Z caused by gate-to-drain capacitance C_{gd}. Then, determine the gain-bandwidth product.

Problem 18.2: Repeat the calculation of f_H for the CS amplifier of Problem 18.2, this time using the Miller effect method. By what percentage does this estimate differ from that obtained in Problem 18.1 using the method of open-circuit time constants? Which of the two estimates is more realistic, and why?

Problem 18.3: For the CS amplifier introduced above, using the value of f_H determined by the method of open-circuit time constants in Problem 18.1, find the gain-bandwidth product.

Theory: See Topic 8.

Problem 19 (Sedra and Smith, 2015, w/ permission)

A CS amplifier that can be represented by the equivalent circuit of Figure 6 has gate-source capacitance C_{gs} = 2 pF, gate-drain capacitance C_{gd} = 0.1 pF, output-node-to-ground capacitance C_L = 2 pF, transconductance g_m = 4 mA/V, and signal-source resistance $R'_{\mathrm{sig}} = R'_L$ = 20 kΩ. Find the midband gain $|A_M|$, the input capacitance using the Miller approximation, and hence an estimate of the 3-dB frequency f_H. Also, obtain another estimate of f_H using open-circuit time constants. Which of the two estimates is more accurate and why?

Theory: See Topic 8.

Problem 20 (Sedra and Smith, 2015, w/ permission)

For a common source amplifier with g_m = 5 mA/V, C_{gs} = 5 pF, C_{gd} = 1 pF, C_L = 5 pF, R'_{sig} = 10 kΩ, R'_L = 10 kΩ, find time constant τ_H using Miller's approximation and use your result to estimate the upper 3-dB frequency f_H. What is the percentage of τ_H that is caused by the interaction of R'_{sig} with the input capacitance? To what value must R'_{sig} be lowered in order to double f_H?

Theory: See Topic 8.

Problem 21 (Sedra and Smith, 2015, w/ permission)

Consider a bipolar active-loaded common-emitter amplifier having the load current source implemented with a *pnp* transistor. Let the circuit be operating at a 1-mA bias current. The transistors are specified as follows: current gain $\beta(npn)$ = 200, Early voltages V_{An} = 130 V, $|V_{Ap}|$ = 50 V, emitter-base capacitance C_π = 16 pF, collector-base capacitance C_μ = 0.3 pF, drain-to-ground capacitance C_L = 5 pF, and resistance r_x = 200 Ω. The amplifier is fed with a signal source having a resistance of 36 kΩ. Determine

1. The midband voltage gain A_M;

2. The input capacitance C_{in} and the 3-dB frequency f_H using the Miller effect approach;

3. The 3-dB frequency using open-circuit time constants;

4. The frequency of transmission zero f_Z;

5. The gain-bandwidth product.

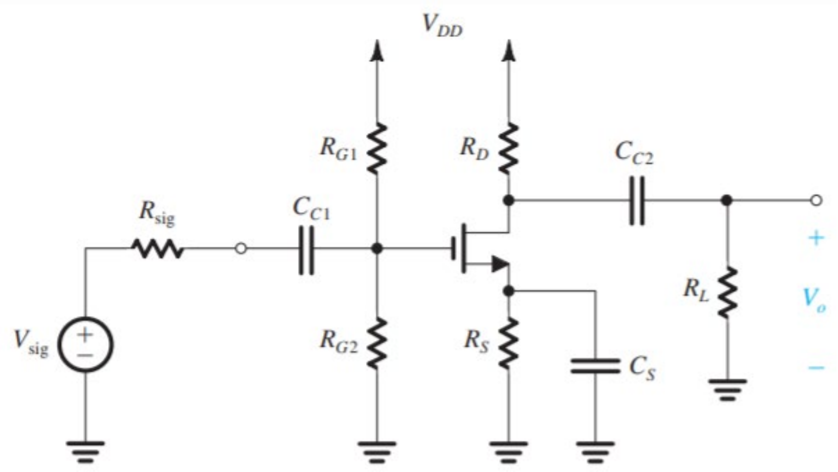
Theory: See Topic 9.

ADDITIONAL INFORMATION

Topic 1: Low-frequency response of a common-source amplifier

To analyze the low-frequency response of a CS amplifier, we refer to the amplifier equivalent circuit illustrated in Figure 1; this arrangement is obtained by short-circuit supply voltage V_{DD} and replacing the MOSFET with its T model.

Figure 1. Capacitatively coupled common-source amplifier



The midband gain A_M of the amplifier is given by

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m \left(R_D \parallel R_L\right) \quad (1)$$

where $R_G = R_{G1}||R_{G2}$ is the equivalent gate resistance, g_m is the MOSFET's transconductance, R_D is drain resistance, and R_L is load resistance.

→ Coupling capacitance C_{C1} introduces a pole with frequency ω_{P1} given by

$$\omega_{P1} = \frac{1}{C_{C1}\left(R_{sig} + R_G\right)} \quad (2)$$

where R_{sig} is the signal-source resistance and $R_G = R_{G1}||R_{G2}$ is the equivalent gate resistance.

→ Bypass capacitance C_S introduces a pole with frequency ω_{P2} given by

$$\omega_{P2} = \frac{g_m + 1/R_S}{C_S} \quad (3)$$

where g_m is transconductance and R_S is source resistance.

→ Coupling capacitance C_{C2} introduces a pole with frequency ω_{P3} given by

$$\omega_{P3} = \frac{1}{C_{C2}\left(R_D + R_L\right)} \quad (4)$$

where R_D is drain resistance and R_L is load resistance.

→ The zero frequency is given by

$$f_Z = \frac{1}{2\pi C_S R_S} \quad (5)$$

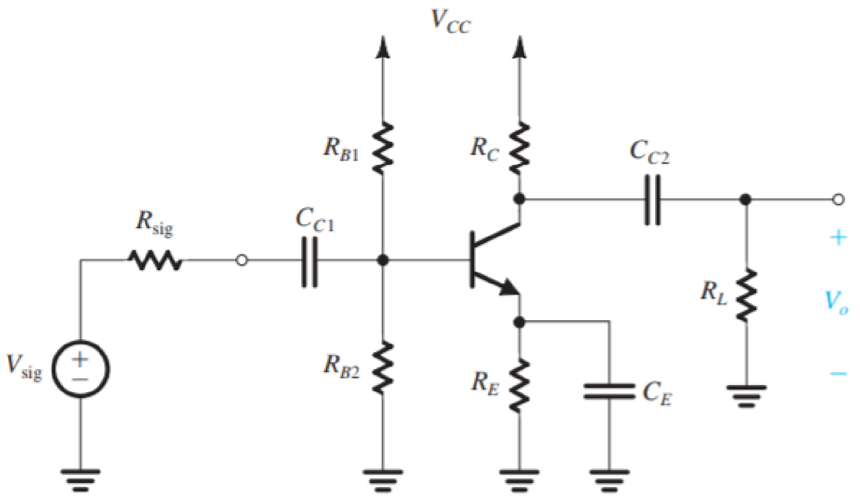
→ One crucial aspect of the low-frequency response of CS and CE amplifiers is the determination of 3-dB frequency f_L. A quick way to estimate this quantity is possible if the highest-frequency pole (here, assumed to be f_{P2}) is separated from the nearest pole or zero (here, f_{P1}) by at least a factor of 4 (two octaves). In such a case, f_L is approximately equal to the highest of the pole frequencies,

$$f_L \approx f_{P2}$$

Topic 2: Low-frequency response of the CE amplifier with short-circuit time constants

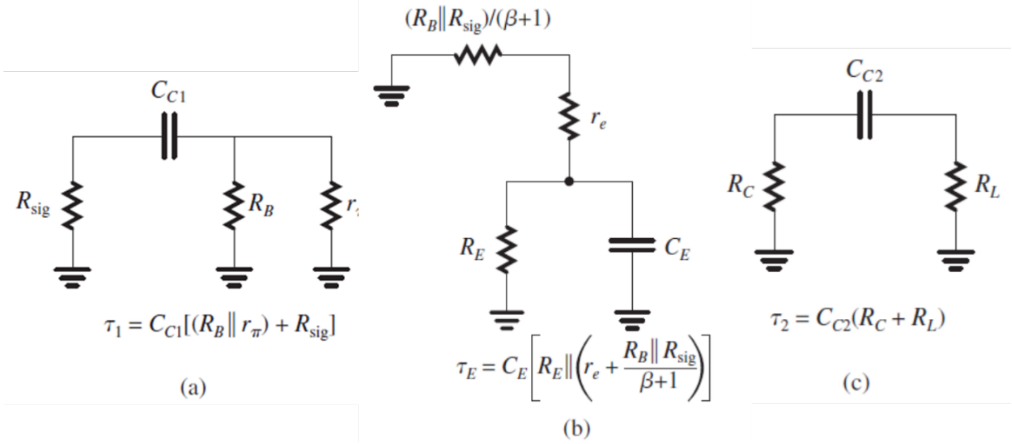
To analyze the low-frequency response of a CE amplifier, we utilize the equivalent circuit shown in Figure 2 below; this arrangement is obtained by short-circuiting voltage supply V_{CC} and replacing the BJT with its T model. In contrast to the low-freq model of the CS amplifier, here the finite input current in the base of the BJT causes coupling capacitors C_{C1} and C_{C2} to interact. That is, unlike the case of the CS amplifier, here each of the two poles caused by C_{C1} and C_{C2} will depend on both capacitor values in a complicated fashion that hinders design insight. Accordingly, Sedra and Smith chose not to determine individual pole frequencies directly, instead using the so-called method of short-circuit time constants to obtain an estimate of the 3-dB frequency f_L directly.

Figure 2. Capacitatively coupled common-emitter amplifier



Considering each capacitor in Figure 2, one at a time, while short-circuiting the other two results in the three circuits illustrated in Figure 3.

Figure 3. Capacitor circuits for modelling the low-freq. response of a CE amp.



→ With reference to figure (*a*) above, the resistance seen by coupling capacitor C_{C1} is

$$R_{C1} = (R_B \,\|\, r_\pi) + R_{\text{sig}} \quad (6)$$

where R_B is base resistance, r_π is input resistance at the base when C_E is short-circuited, and R_{sig} is signal source resistance. Multiplying R_{C1} by coupling capacitance C_{C1} gives time constant τ_{C1}.

→ With reference to figure (*b*) above, the resistance seen by bypass capacitor C_E is

$$R_{CE} = R_E \,\|\, \left(r_e + \frac{R_B \,\|\, R_{\text{sig}}}{\beta + 1} \right) \quad (7)$$

where R_E is the added emitter lead resistance, r_e is transistor emitter resistance, R_B is base resistance, R_{sig} is signal source resistance, and β is the BJT's current gain parameter. Multiplying R_E by bypass capacitance C_E gives time constant τ_E.

→ With reference to figure (c) above, the resistance seen by coupling capacitor C_{C2} is

$$R_{C2} = R_C + R_L \quad (8)$$

where R_C is collector resistance and R_L is load resistance. Multiplying R_{C2} by coupling capacitance C_{C2} gives time constant τ_{C2}.

→ With the three time constants in hand, the 3-dB frequency can be determined as

$$f_L = \frac{1}{2\pi}\left(\frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}} \right) \quad (9)$$

→ The zero frequency is given by

$$f_Z = \frac{1}{2\pi C_E R_E} \quad (10)$$

→ For the midband voltage gain, use the expression

$$A_M \text{ or } G_v = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \,\|\, R_L) \quad (11)$$

where $R_{in} = R_B \| r_\pi$.

Topic 3: Internal capacitive effects of the MOSFET

→ Problem 5 addresses the partition of internal capacitive components in high-frequency MOSFET operation. In a typical such separation, the capacitance of the MOSFET is made up of four contributions:

→ First, we have the gate-to-source capacitance C_{gs},

$$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov} \quad (12)$$

where W is channel width, L is channel length, C_{ox} is oxide (or gate) capacitance, and C_{ov} is overlap capacitance. The oxide capacitance C_{ox} should be familiar to most students and equals the ratio of the permittivity ε_{ox} of silicon oxide [ε_{ox} = 3.9 × (8.85×10^{-12}) F/m] to the thickness t_{ox} of the oxide layer,

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \quad (13)$$

The overlap capacitance C_{ov}, in turn, is a contribution to gate capacitance that arises from the overlap of the gate with the source region and the drain region. Each of these overlaps has a length L_{ov}, so that the C_{ov} can be expressed as

$$C_{ov} = WL_{ov}C_{ox} \quad (14)$$

Typically, L_{ov} is 0.05 to 0.1 times the channel length L.

→ Second, we have the gate-to-drain capacitance C_{gd}, which for most purposes equals the overlap capacitance C_{ov},

$$C_{gd} = C_{ov} = WL_{ov}C_{ox} \quad (15)$$

→ Third, we have the source-body capacitance C_{sb}, one of two depletion-layer capacitances in the MOSFET structure. C_{sb} can be determined as

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1+\frac{V_{SB}}{V_0}}} \quad (16)$$

Here, C_{sb0} is the value of C_{sb} at zero body-source bias, V_{SB} is the magnitude of this reverse-bias voltage, and V_0 is the junction built-in voltage (0.6 to 0.8 V).

→ Fourth, we have the drain-body capacitance C_{db}, which is the second depletion-layer capacitance. C_{db} can be determined as

$$C_{db} = \frac{C_{db0}}{\sqrt{1+\frac{V_{DB}}{V_0}}} \quad (17)$$

Here, C_{db0} is the value of C_{db} at zero drain-body bias, V_{DB} is the magnitude of this reverse-bias voltage, and V_0 is the junction built-in voltage.

→ A figure of merit for the high-frequency operation of the MOSFET is the so-called unity gain frequency f_T, also known as transfer frequency. f_T is defined as the frequency at which the short-circuit current gain of the common-source configuration becomes unity. To determine f_T, we use

$$f_T = \frac{g_m}{2\pi\left(C_{gs}+C_{gd}\right)} \quad (18)$$

where C_{gs} is the gate-source capacitance and C_{gd} is the gate-drain capacitance.

Topic 4: Internal capacitive effects of the BJT

→ Like the MOSFET, the bipolar junction transistor has internal capacitive effects of its own.

→ For small signals, we can define the small-signal diffusion capacitance C_{de},.

$$C_{de} = \tau_F g_m \quad (19)$$

where τ_F is a device constant known as the forward base-transit time and g_m is transconductance. As the name implies, τ_F has dimensions of time and is typically in the range of 10 ps to 100 ps. One simple interpretation of C_{de} is that when the base-emitter voltage changes by a certain amount ΔV_{BE}, the charge stored in the base changes by an amount $C_{de}\,\Delta V_{BE} = \tau_F g_m\,\Delta V_{BE}$.

→ A change in V_{BE} changes not only the charge stored in the base region but also the charge stored in the base-emitter depletion layer. This distinct charge-stored effect is represented by the emitter-base junction depletion-layer capacitance, C_{je}, which can be estimated with the simple relationship

$$C_{je} \approx 2C_{je0} \quad (20)$$

where C_{je0} is the value of C_{je} at zero EBJ voltage.
→ The emitter-base capacitance C_π is obtained by adding C_{de} to C_{je},

$$C_\pi = C_{de} + C_{je} \quad (21)$$

→ The junction or depletion capacitance C_μ can be estimated with the relationship

$$C_\mu = \frac{C_{\mu 0}}{\left(1+\frac{V_{CB}}{V_{0c}}\right)^m} \quad (22)$$

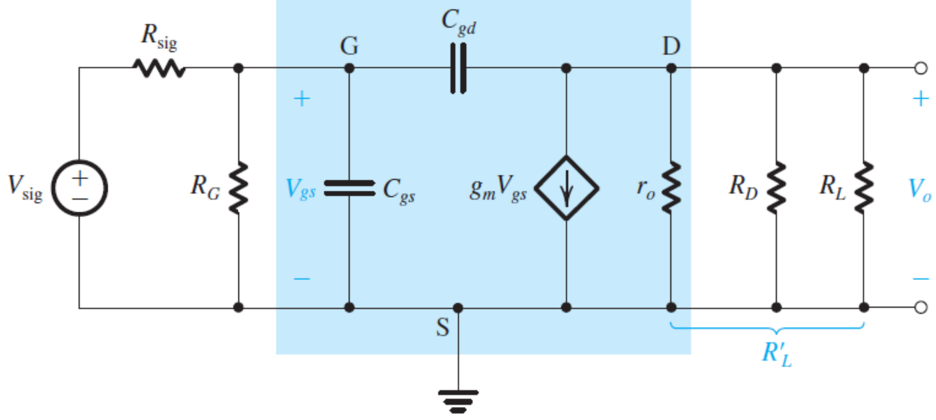
where $C_{\mu 0}$ is the value of C_μ at zero voltage; V_{CB} is the magnitude of the CBJ reverse-bias voltage, V_{0c} is the CBJ built-in voltage (typically, 0.75 V), and m is the grading coefficient (typically, 0.2 – 0.5).
→ The unity-gain bandwidth or transition frequency of a BJT is

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)} \quad (23)$$

Topic 5: High-frequency response of a CS amplifier

Figure 4 shows the circuit used to model the high-frequency response of a CS amplifier.

Figure 4. Equivalent circuit for high-frequency response of a CS amplifier.



→ The midband gain A_M can be estimated as

$$A_M = -\frac{R_G}{R_G + R_{sig}}\left(g_m R'_L\right) \quad (24)$$

where R_G is the gate resistance, R_{sig} is the signal source resistance, g_m is transconductance, and R'_L is the compound load resistance, which in the figure above consists of three generic components: transistor output resistance r_o, drain resistance R_D, and load resistance R_L; that is,

$$R'_L = \left(r_o \parallel R_D \parallel R_L\right) \quad (25)$$

→ The upper 3-dB frequency of the CS amplifier configuration is given by

$$f_H = \frac{1}{2\pi C_{in}\left(R_{sig} \parallel R_G\right)} \quad (26)$$

Here, input capacitance C_{in} is given by

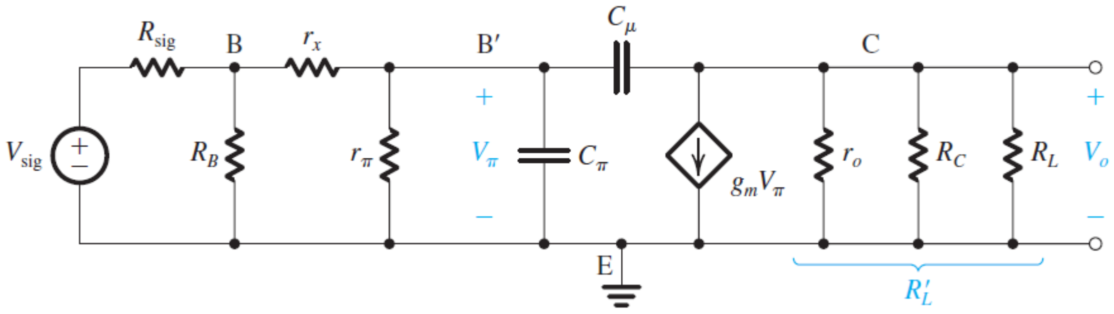
$$C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}\left(1 + g_m R'_L\right) \quad (27)$$

where C_{gs} and C_{gd} are capacitances shown in the foregoing circuit; the equivalent capacitance C_{eq} is found by multiplying the gate-drain capacitance C_{gd} by the so-called Miller multiplier $(1 + g_m R'_L)$.

Topic 6: High-frequency response of a CE amplifier

Figure 5 shows circuit used to model the high-frequency response of a CE amplifier.

Figure 5. Equivalent circuit for high-frequency response of a CE amplifier.



→ The midband gain A_M can be estimated as

$$A_M = -\frac{R_B}{R_B + R_{sig}}\frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)}(g_m R_L') \quad (28)$$

where R_B is base resistance, R_{sig} is signal source resistance, r_π is input resistance at the base when bypass capacitor C_E is short-circuited, r_x is the added base resistance, g_m is transconductance, and R_L' is the compound load resistance, which consists of three generic components: the transistor output resistance r_o, the collector resistance R_C, and the load resistance R_L; that is,

$$R_L' = (r_o \parallel R_C \parallel R_L) \quad (29)$$

→ The upper 3-dB frequency of the common-emitter configuration is

$$f_H = \frac{1}{2\pi C_{in} R_{sig}'} \quad (30)$$

Here, input capacitance C_{in} is expressed as

$$C_{in} = C_\pi + C_\mu (1 + g_m R_L') \quad (31)$$

and

$$R_{sig}' = r_\pi \parallel [r_x + (R_B \parallel R_{sig})] \quad (32)$$

where r_π is input resistance at the base when bypass capacitor C_E is short-circuited, r_x is the added base resistance, R_B is base resistance, and R_{sig} is signal source resistance.

Topic 7: Frequency analysis

→ The 3-dB frequency ω_H can be estimated with the general relationship

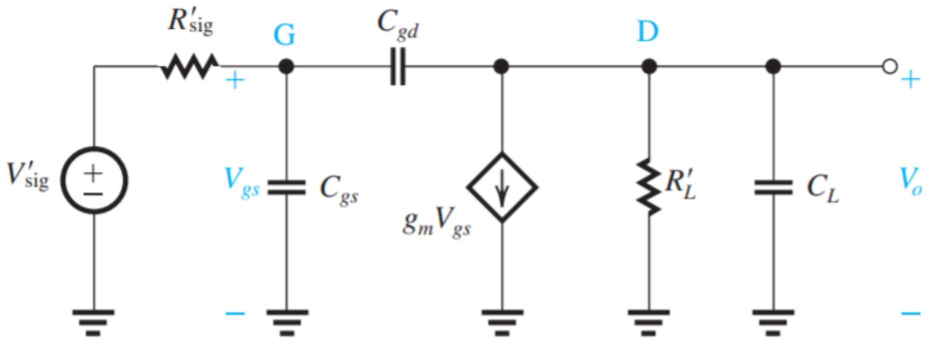
$$\omega_H \approx \frac{1}{\sqrt{\left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} + \ldots + \frac{1}{\omega_{Pn}^2}\right) - 2\left(\frac{1}{\omega_{Z1}^2} + \frac{1}{\omega_{Z2}^2} + \ldots + \frac{1}{\omega_{Zn}^2}\right)}} \quad (33)$$

where $\omega_{P1}, \omega_{P2}, \ldots$ are the pole frequencies and $\omega_{Z1}, \omega_{Z2}, \ldots$ are the zero frequencies.

Topic 8: High frequency response of a CS amplifier with open-circuit time constants

Figure 6 shows the generalized high-frequency equivalent circuit for the CS amplifier.

Figure 6. Generalized high-frequency equivalent circuit for a CS amp.



→ To estimate the 3-dB frequency of a CS amplifier with the open-circuit time constants method, we apply the formula

$$f_H \approx \frac{1}{2\pi\tau_H} \quad (34)$$

where τ_H, the effective high-frequency time constant, is made up of three contributions,

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L} \quad (35)$$

→ First, τ_{gs} is given by

$$\tau_{gs} = C_{gs} R_{gs} \quad (36)$$

where C_{gs} is the internal gate-source capacitance and R_{gs} is the resistance seen by C_{gs} when other capacitances are set to zero; in this case,

$$R_{gs} = R'_{\text{sig}} = r_\pi \,||\, \left[r_x + \left(R_B \,||\, R_{\text{sig}} \right) \right] \quad (37)$$

→ Second, τ_{gd} is given by

$$\tau_{gd} = C_{gd} R_{gd} \quad (38)$$

where C_{gd} is the internal gate-drain capacitance and R_{gd} is the resistance seen by C_{gd} when other capacitances are set to zero; in this case,

$$R_{gd} = R'_{\text{sig}} \left(1 + g_m R'_L \right) + R'_L \quad (39)$$

→ Third, τ_{C_L} is given by

$$\tau_{C_L} = C_L R_{C_L} \quad (40)$$

Where C_{C_L} represents the total capacitance between the drain node and ground and R_{C_L} is the resistance seen by C_{C_L} when other capacitances are set to zero; in this case,

$$R_{C_L} = R'_L = \left(r_o \,||\, R_D \,||\, R_L \right) \quad (41)$$

→ Expanding equation 35 with the respective resistance components, the effective time constant becomes

$$\tau_H = C_{gs} R'_{\text{sig}} + C_{gd} \left[R'_{\text{sig}} \left(1 + g_m R'_L \right) + R'_L \right] + C_L R'_L \quad (42)$$

Topic 9: High frequency response of a CE amplifier with open-circuit time constants

→ The formulas presented above for the CS case can be easily adapted to the case of the CE amplifier. Analogously to equation 35, the effective time constant for the CE amp is expressed as

$$\tau_H = \tau_\pi + \tau_\mu + \tau_{C_L} \quad (43)$$

→ Refer to Figure 5. First, τ_π is given by

$$\tau_\pi = C_\pi R_\pi \quad (44)$$

where

$$R_\pi = R'_{\text{sig}} = r_\pi \,||\, \left[r_x + \left(R_B \,||\, R_{\text{sig}} \right) \right] \quad (45)$$

→ Second, τ_μ is given by

$$\tau_\mu = C_\mu R_\mu \quad (46)$$

where

$$R_\mu = R'_{\text{sig}} \left(1 + g_m R'_L \right) + R'_L \quad (47)$$

→ Third, τ_{C_L} is given by

$$\tau_{C_L} = C_L R_{C_L} \quad (48)$$

where

$$R_{C_L} = R'_L = r_o \,||\, R_C \,||\, R_L \quad (49)$$

→ Expanding equation 43 with the respective resistance components, the effective time constant becomes

$$\tau_H = C_\pi R'_{\text{sig}} + C_\mu \left[R'_{\text{sig}} \left(1 + g_m R'_L \right) + R'_L \right] + C_L R'_L \quad (50)$$

SOLUTIONS

P.1 ➡ *Solution*

The midband gain is given by equation 1,

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} g_m \left(R_D \,||\, R_L \right) = -\frac{10}{10 + 0.1} \times \left(2.0 \times 10^{-3} \right) \times \left(\frac{10 \times 10}{10 + 10} \times 10^3 \right) = \boxed{-9.90\text{ V/V}}$$

Frequency f_{P1} is determined with equation 2,

$$\omega_{P1} = \frac{1}{C_{C1} \left(R_{\text{sig}} + R_G \right)} \rightarrow f_{P1} = \frac{1}{2\pi C_{C1} \left(R_{\text{sig}} + R_G \right)}$$

$$\therefore f_{P1} = \frac{1}{2\pi \times 10^{-6} \times \left[\left(0.1 + 10 \right) \times 10^6 \right]} = \boxed{0.0158\text{ Hz}}$$

The bypass capacitance C_S is chosen so that the highest frequency pole f_{P2} equals the lower 3-dB frequency f_L, that is (equation 3),

$$f_{P2} = \frac{g_m + R_S^{-1}}{2\pi C_S} \rightarrow f_{P2} = \frac{g_m + R_S^{-1}}{2\pi C_S}$$

$$\therefore f_{P2} = \frac{2.0\times10^{-3} + \left(10\times10^3\right)^{-1}}{2\pi\times10^{-6}} = \boxed{334\,\text{Hz}} = f_L$$

The third pole frequency we need is (equation 4)

$$f_{P3} = \frac{1}{2\pi C_{C2}\left(R_D + R_L\right)} = \frac{1}{2\pi\times10^{-6}\times\left[\left(10+10\right)\times10^3\right]} = \boxed{7.96\,\text{Hz}}$$

Lastly, the zero frequency is (equation 5)

$$f_Z = \frac{1}{2\pi C_S R_S} = \frac{1}{2\pi\times10^{-6}\times\left(10\times10^3\right)} = \boxed{15.92\,\text{Hz}}$$

P.2 ➡ *Solution*

Capacitance C_S is found by rearranging equation 3,

$$\omega_{P2} = \frac{g_m + R_S^{-1}}{C_S} \rightarrow C_S = \frac{g_m + R_S^{-1}}{2\pi f_{P2}}$$

Since $f_{P2} \propto C_S^{-1}$, the minimum capacitance required to maintain the corresponding pole frequency at 100 Hz or lower is given by

$$C_S = \frac{5.0\times10^{-3} + \dfrac{1}{1.8\times10^3}}{2\pi\times100} = 8.84\times10^{-6}\text{ F} = \boxed{8.84\,\mu\text{F}}$$

The corresponding zero frequency is (equation 5)

$$f_Z = \frac{1}{2\pi C_S R_S} = \frac{1}{2\pi\times\left(8.84\times10^{-6}\right)\times\left(1.8\times10^3\right)} = \boxed{10.0\,\text{Hz}}$$

P.3 ➡ *Solution*

Time constant τ_{C1} is given by equation 6,

$$\tau_{C1} = C_{C1}R_{C1} = C_{C1}\left[\left(R_B \parallel r_\pi\right) + R_{\text{sig}}\right] = 10^{-6}\times\left[\left(\frac{100\times2.5}{100+2.5}\times10^3\right) + \left(5.0\times10^3\right)\right] = \boxed{7.44\,\text{ms}}$$

Time constant τ_{C2} is, in turn (equation 8),

$$\tau_{C2} = C_{C2}R_{C2} = C_{C2}\left(R_C + R_L\right) = 10^{-6}\times\left[\left(8+5\right)\times10^3\right] = \boxed{13\,\text{ms}}$$

Lastly, determining the resistance seen by the bypass capacitance C_{CE} requires more algebra (equation 7),

$$R_{CE} = R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta+1}\right) = 5.0 \parallel \left[0.025 + \frac{\frac{100\times5}{100+5}}{100+1}\right] = 0.0711\,\text{k}\Omega$$

so that

$$\tau_{CE} = C_E R_{CE} = 10^{-6}\times\left(0.0711\times10^3\right) = \boxed{0.0711\,\text{ms}}$$

Gleaning the previous results, the lower 3-dB frequency is calculated to be (equation 9)

$$f_L = \frac{1}{2\pi}\left(\frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}}\right) = \frac{1}{2\pi}\times\left(\frac{1}{7.44} + \frac{1}{0.0711} + \frac{1}{13}\right)\times10^3 = 2270\,\text{Hz} = \boxed{2.27\text{ kHz}}$$

Noting that the zero frequency (equation 10)

$$f_Z = \frac{1}{2\pi C_E R_E} = \frac{1}{2\pi \times 10^{-6} \times \left(5.0 \times 10^3\right)} = 31.8\,\text{Hz}$$

is much lower than f_L, we surmise that it has negligible effect on the low-frequency response of the amplifier.

P.4 ➡ *Solution*

Problem 4.1: Firstly, note that resistances R_{B1} and R_{B2} are in parallel, so that

$$R_B = R_{B1} \,||\, R_{B2} = \frac{33 \times 22}{33 + 22} \times 10^3 = 13.2\,\text{k}\Omega$$

Taking 25 mV as the thermal voltage, transconductance g_m is

$$g_m = \frac{I_C}{V_T} = \frac{0.3 \times 10^{-3}}{25 \times 10^{-3}} = 0.012\,\text{A/V} = 12\,\text{mA/V}$$

Resistance r_π is

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12 \times 10^{-3}} = 10\,\text{k}\Omega$$

Input resistance R_{in} is

$$R_{\text{in}} = R_B \,||\, r_\pi = 13.2 \,||\, 10 = 5.69\,\text{k}\Omega$$

Emitter resistance r_e is

$$r_e = \frac{1}{g_m} = \frac{1}{12 \times 10^{-3}} = 83.3\,\Omega$$

Gleaning our results, the midband gain is calculated to be (equation 11)

$$A_M = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m \left(R_C \,||\, R_L\right) = -\frac{5.69}{5.69 + 5} \times \left(12 \times 10^{-3}\right) \times \left(\frac{4.7 \times 5.6}{4.7 + 5.6} \times 10^3\right) = \boxed{-16.3\,\text{V/V}}$$

To determine the first pole frequency, we write (resistance components taken from equation 6)

$$f_{P1} = \frac{1}{2\pi C_{C1}\left(R_B \,||\, r_\pi + R_{\text{sig}}\right)} = \frac{1}{2\pi \times 10^{-6} \times \left[(5.69 + 5) \times 10^3\right]} = 14.9\text{ Hz}$$

This result corresponds to a time constant τ_{C1} such that

$$\tau_{C1} = \frac{1}{f_{P1}} = \frac{1}{14.9} = 0.0671\,\text{s} = \boxed{67.1\,\text{ms}}$$

Next, we establish resistance R_{CE} (equation 7)

$$R_{CE} = R_E \,||\, \left(r_e + \frac{R_B \,||\, R_{\text{sig}}}{\beta + 1}\right) = 3.9 \,||\, \left(0.0833 + \frac{\frac{13.2 \times 5}{13.2 + 5}}{120 + 1}\right) = 0.110\,\text{k}\Omega$$

which corresponds to a time constant τ_{CE} such that

$$\tau_{CE} = C_E R_{CE} = \left(20 \times 10^{-6}\right) \times \left(0.110 \times 10^3\right) = 0.00220\,\text{s} = \boxed{2.2\,\text{ms}}$$

and to a pole frequency f_{P2} given by

$$f_{P2} = \frac{1}{2\pi C_E R_{CE}} = \frac{1}{2\pi \times \left(20 \times 10^{-6}\right) \times \left(0.110 \times 10^3\right)} = 72.3\,\text{Hz}$$

The third pole frequency is given by (resistance components taken from equation 8)

$$f_{P3} = \frac{1}{2\pi C_{C2}\left(R_C + R_L\right)} = \frac{1}{2\pi \times 10^{-6} \times \left[(4.7 + 5.6) \times 10^3\right]} = 15.5\text{ Hz}$$

and corresponds to a time constant τ_{C2} such that

$$\tau_{C2} = \frac{1}{f_{P3}} = \frac{1}{15.5} = 0.0645\,\text{s} = \boxed{64.5\,\text{ms}}$$

Gleaning the three pole frequencies determined just now, we can estimate the lower 3-dB frequency,

$$f_L = f_{P1} + f_{P2} + f_{P3} = 14.9 + 72.3 + 15.5 = \boxed{103\,\text{Hz}}$$

Problem 4.2: If coupling capacitors C_{C1} and C_{C2} each contribute 10% to the determination of f_L, the bypass capacitance C_E will contribute the remaining 80%. With a lower 3-dB frequency of 50 Hz and resistance R_{CE} determined in the previous problem as 0.11 kΩ, we have

$$C_E = \frac{1}{2\pi \underbrace{f_{P2}}_{=0.8 f_L} R_{CE}} = \frac{1}{2\pi \times (0.8\times 50)\times(0.110\times 10^3)} = 3.62\times 10^{-5}\text{ F} = \boxed{36.2\,\mu\text{F}}$$

Next, noting that coupling capacitor C_{C1} is to contribute 10% to the determination of f_L, we may write

$$C_{C1} = \frac{1}{2\pi \underbrace{f_{P1}}_{=0.1 f_L} (R_{\text{in}} + R_{\text{sig}})} = \frac{1}{2\pi \times (0.1\times 50)\times\left[(5.69+5)\times 10^3\right]} = \boxed{2.98\,\mu\text{F}}$$

Likewise for C_{C2},

$$C_{C2} = \frac{1}{2\pi f_{P3} (R_C + R_L)} = \frac{1}{2\pi \times (0.1\times 50)\times\left[(4.7+5.6)\times 10^3\right]} = \boxed{3.09\,\mu\text{F}}$$

P.5 ➡ *Solution*

Problem 5.1: To find the oxide capacitance, we divide the permittivity ε_{ox} of silicon oxide by the thickness t_{ox} (equation 13)

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times 10^{-11}}{10\times 10^{-9}} = 3.45\times 10^{-3}\text{ F/m}^2$$

Noting that 1 fF = 10^{-15} F and 1 m^2 = 10^{12} µm^2, the result above can be restated as

$$C_{ox} = 3.45\times 10^{-3}\,\frac{10^{15}\text{ fF}}{10^{12}\ \mu\text{m}^2} = \boxed{3.45\text{ fF}/\mu\text{m}^2}$$

To determine the overlap capacitance C_{ov}, simply multiply C_{ox} by the product of width W = 10 µm and overlap length L_{ov} = 0.05 µm (equation 14),

$$C_{ov} = WL_{ov}C_{ox} = 10\times 0.05\times 3.45 = \boxed{1.73\text{ fF}}$$

To determine the gate-to-source capacitance C_{gs}, use equation 12,

$$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov} = \frac{2}{3}\times 10\times 1.0\times 3.45 + 1.73 = \boxed{24.73\text{ fF}}$$

The gate-to-drain capacitance C_{gs} is given by equation 15,

$$C_{gd} = WL_{ov}C_{ox} = 10\times 0.05\times 3.45 = \boxed{1.73\text{ fF}} = C_{ov}$$

which of course happens to be the same result as the overlap capacitance C_{ov}. Next, the source-body capacitance is expressed as (equation 16)

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1+\frac{V_{SB}}{V_0}}} = \frac{10}{\sqrt{1+\frac{1.0}{0.6}}} = \boxed{6.12\text{ fF}}$$

Similarly, the drain-body capacitance is given by equation 17,

$$C_{db} = \frac{C_{db0}}{\sqrt{1+\frac{\overbrace{V_{DB}}^{=V_{SB}+V_{DS}}}{V_0}}} = \frac{10}{\sqrt{1+\frac{1.0+2.0}{0.6}}} = \boxed{4.08\text{ fF}}$$

Problem 5.2: The unity-gain frequency in Hz is given by equation 18,

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})}$$

We already have C_{gs} = 24.73 fF and C_{gd} = 1.73 fF, and the transconductance g_m can be determined from the operational characteristics of the device,

$$g_m = \sqrt{2k_n' \frac{W}{L} I_D} = \sqrt{2\times\left(160\times10^{-6}\right)\times\frac{10}{1.0}\times\left(100\times10^{-6}\right)} = 5.66\times10^{-4}\text{ A/V}$$

so that

$$f_T = \frac{5.66\times10^{-4}}{2\pi\times\left(24.73+1.73\right)\times10^{-15}} = 3.404\times10^{9}\text{ Hz} = \boxed{3.40\,\text{GHz}}$$

P.6 ➡ *Solution*

Firstly, if $C_{gs} \gg C_{gd}$, the unity-gain bandwidth simplifies to

$$f_T = \frac{g_m}{2\pi\left(C_{gs}+C_{gd}\right)} \approx \frac{g_m}{2\pi C_{gs}}$$

Here, the transconductance g_m is given by

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

while gate-source capacitance C_{gs} is, neglecting the overlap component,

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ox}C_{ox} \approx \frac{2}{3}WLC_{ox}$$

so that, substituting in f_T,

$$f_T = \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}}{2\pi\times\frac{2}{3}WLC_{ox}} = \frac{1}{2\pi}\sqrt{\frac{2\mu_n C_{ox}\frac{W}{L}I_D}{\frac{4}{9}W^2L^2C_{ox}^2}}$$

$$\therefore f_T = \frac{1}{2\pi}\sqrt{\frac{18\mu_n C_{ox}WI_D}{4W^2L^3C_{ox}^2}}$$

$$\therefore f_T = \frac{3}{2\pi}\sqrt{\frac{\mu_n C_{ox}WI_D}{2W^2L^3C_{ox}^2}}$$

$$\therefore \boxed{f_T \approx \frac{1.5}{\pi L}\sqrt{\frac{\mu_n I_D}{2C_{ox}WL}}}$$

P.7 ➡ *Solution*

The unity-gain bandwidth for a MOSFET is given by equation 18,

$$f_T = \frac{g_m}{2\pi\left(C_{gs}+C_{gd}\right)} = \frac{2I_D/V_{OV}}{2\pi\left(C_{gs}+C_{gd}\right)} = \frac{I_D}{\pi V_{OV}\left(C_{gs}+C_{gd}\right)}$$

$$\therefore f_T = \frac{200\times10^{-6}}{\pi\times0.3\times\left[\left(25+5\right)\times10^{-15}\right]} = 7.07\times10^{9}\text{ Hz} = \boxed{7.07\text{ GHz}}$$

P.8 ➡ *Solution*

The small-signal diffusion capacitance is obtained as the product of forward base-transit time τ_F and transconductance g_m (see equation 19),

$$C_{de} = \tau_F g_m = \tau_F\frac{I_C}{V_T} = \left(20\times10^{-12}\right)\times\frac{1.0\times10^{-3}}{25\times10^{-3}} = 8\times10^{-13}\text{ F} = \boxed{0.8\text{ pF}}$$

The base-emitter junction capacitance can be estimated as twice its value at zero EBJ voltage (see equation 20),

$$C_{je} = 2C_{je0} = 2\times20 = \boxed{40\text{ fF}}$$

The emitter-base capacitance equals the sum of C_{de} and C_{je} (equation 21),

$$C_\pi = C_{de} + C_{je} = 0.8\times10^{-12} + 40\times10^{-15} = 0.8\times10^{-12} + 0.04\times10^{-12} = \boxed{0.84\,\text{pF}}$$

Next, the depletion capacitance is given by equation 22,

$$C_\mu = \frac{C_{\mu 0}}{\left(1+\frac{V_{CB}}{V_{0c}}\right)^m} = \frac{20}{\left(1+\frac{2.0}{0.5}\right)^{0.33}} = \boxed{11.76\,\text{fF}}$$

Lastly, the unity-gain bandwidth or transition frequency f_T is expressed as (equation 23),

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)} = \frac{\left(1.0\times 10^{-3}/25\times 10^{-3}\right)}{2\pi\times\left(0.84\times 10^{-12} + 11.76\times 10^{-15}\right)} = 7.474\times 10^{9}\text{ Hz} = \boxed{7.47\text{ GHz}}$$

P.9 ➡ Solution

Problem 9.1: The unity-gain bandwidth for a BJT is given by (equation 19; using a thermal voltage of 25 mV)

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)} = \frac{I_C/V_T}{2\pi\left(C_\pi + C_\mu\right)} = \frac{I_C}{2\pi V_T\left(C_\pi + C_\mu\right)} \quad \text{(I)}$$

$$\therefore f_T = \frac{0.5\times 10^{-3}}{2\pi\times\left(25\times 10^{-3}\right)\times\left[\left(8+1\right)\times 10^{-12}\right]} = 3.54\times 10^{8}\text{ Hz} = \boxed{354\,\text{MHz}}$$

Dividing f_T by the current gain parameter yields f_β,

$$f_\beta = \frac{f_T}{\beta} = \frac{354\times 10^{6}}{100} = \boxed{3.54\text{ MHz}}$$

Problem 9.2: With an added 2-pF depletion-layer contribution to the emitter-base capacitance, C_π becomes

$$C_\pi = C_{de} + C_{je} = 2.0 + 8.0 = 10\,\text{pF}$$

so that, substituting in (I),

$$f_T = \frac{0.25\times 10^{-3}}{2\pi\times\left(25\times 10^{-3}\right)\times\left[\left(10+1\right)\times 10^{-12}\right]} = 1.45\times 10^{8}\text{ Hz} = \boxed{145\,\text{MHz}}$$

P.10 ➡ Solution

Problem 10.1: The midband gain A_M is given by equation 24,

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} g_m R_L' = -\frac{R_G}{R_G + R_{\text{sig}}} g_m \left(r_o \parallel R_D \parallel R_L\right)$$

Here, per equation 25,

$$R_L' = \frac{1}{\frac{1}{r_o} + \frac{1}{R_D} + \frac{1}{R_L}} = \frac{1}{\frac{1}{165\times 10^{3}} + \frac{1}{16\times 10^{3}} + \frac{1}{16\times 10^{3}}} = 7630\,\Omega = 7.63\,\text{k}\Omega$$

so that

$$A_M = -\frac{3.2\times 10^{6}}{3.2\times 10^{6} + 120\times 10^{3}}\times\left(1.0\times 10^{-3}\right)\times\left(7.63\times 10^{3}\right) = \boxed{-7.35\,\text{V/V}}$$

The equivalent capacitance C_{eq} is given by

$$C_{eq} = \left(1 + g_m R_L'\right)C_{gd} = \left(1 + 1.0\times 7.63\right)\times 0.4 = 3.45\,\text{pF}$$

and can be used to determine the total input capacitance C_{in} (see equation 27),

$$C_{in} = C_{gs} + C_{eq} = 1.0 + 3.45 = 4.45\,\text{pF}$$

The upper 3-dB frequency f_H is calculated to be (see equation 26)

$$f_H = \frac{1}{2\pi C_{in}\left(R_{\text{sig}} \parallel R_G\right)} = \frac{1}{2\pi\times\left(4.45\times 10^{-12}\right)\times\left[\frac{\left(120\times 10^{3}\right)\times\left(3.2\times 10^{6}\right)}{\left(120\times 10^{3}\right)+\left(3.2\times 10^{6}\right)}\right]} = \boxed{309\,\text{kHz}}$$

The transmission zero has a frequency

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{1.0\times 10^{-3}}{2\pi\times\left(0.4\times 10^{-12}\right)} = \boxed{398\,\text{MHz}}$$

The transmission zero frequency is about 1290 times greater than the upper 3-dB frequency.

Problem 10.2: Writing the expression for upper 3-dB frequency and solving for gate-drain capacitance C_{gd},

$$f_H = \frac{1}{2\pi C_{in}\left(R_{\text{sig}} \,\|\, R_G\right)} = \frac{1}{2\pi\left(C_{gs}+C_{eq}\right)\left(R_{\text{sig}} \,\|\, R_G\right)} = \frac{1}{2\pi\left[C_{gs}+\left(1+g_m R_L'\right)C_{gd}\right]\left(R_{\text{sig}} \,\|\, R_G\right)}$$

$$\therefore C_{gs}+\left(1+g_m R_L'\right)C_{gd} = \frac{1}{2\pi f_H\left(R_{\text{sig}} \,\|\, R_G\right)}$$

$$\therefore C_{gd} = \frac{\dfrac{1}{2\pi f_H\left(R_{\text{sig}} \,\|\, R_G\right)} - C_{gs}}{1+g_m R_L'}$$

The maximum C_{gd} corresponds to a f_H equal to 1 MHz; other variables remain unchanged. Accordingly,

$$\therefore C_{gd,\max} = \frac{\dfrac{1}{2\pi f_H\left(R_{\text{sig}} \,\|\, R_G\right)} - C_{gs}}{1+g_m R_L'} = \frac{\dfrac{1}{2\pi\times\left(1.0\times10^{6}\right)\times\left(116\times10^{3}\right)} - 1.0\times10^{-12}}{1+\left(1.0\times10^{-3}\right)\times7630} = \boxed{0.0431\,\text{pF}}$$

P.11 ➡ *Solution*

To establish the input capacitance, simply substitute the pertaining variables into equation 27,

$$C_{\text{in}} = C_{gd}\left(1+g_m R_L'\right)+C_{gs} = 0.1\times\left(1+39\right)+1.0 = \boxed{5.0\text{ pF}}$$

Now, note that the 3-dB frequency is given by

$$f_H = \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}'}$$

Since f_H and R_{sig}' are inversely proportional, the 3-dB frequency will be greater than 1 MHz so long as the signal-source resistance R_{sig}' is lower than a maximum value $R_{\text{sig,max}}'$ given by

$$f_H = 10^{6}\text{ Hz} = \frac{1}{2\pi C_{\text{in}} R_{\text{sig,max}}'} \rightarrow R_{\text{sig,max}}' = \frac{1}{10^{6}}\frac{1}{2\pi C_{\text{in}}}$$

$$\therefore R_{\text{sig,max}}' = \frac{1}{10^{6}}\times\frac{1}{2\pi\times\left(5.0\times10^{-12}\right)} = 31.8\,\text{k}\Omega$$

Accordingly, f_H > 1 MHz if R_{sig}' < 31.8 kΩ.

P.12 ➡ *Solution*

Computing and gleaning the hybrid-π model parameters, we have

$$g_m = \frac{I_C}{V_T} = \frac{1.0}{25} = 40\,\text{mA/V}$$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{40\times10^{-3}} = 2.5\,\text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{1.0\times10^{-3}} = 100\,\text{k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{40\times10^{-3}}{2\pi\times\left(800\times10^{6}\right)} = 7.72\,\text{pF}$$

$$C_\pi = 7.72 - C_\mu = 7.72 - 1.0 = 6.72\,\text{pF}$$

The midband voltage gain follows as (equation 28)

$$A_M = -\frac{R_B}{R_B+R_{\text{sig}}}\frac{r_\pi}{r_\pi+r_x+\left(R_B \,\|\, R_{\text{sig}}\right)}g_m \underbrace{R_L'}_{=r_o\|R_C\|R_L}$$

$$\therefore A_M = -\frac{120}{120+6}\times\frac{2.5}{2.5+0.05+\left(\dfrac{120\times6}{120+6}\right)}\times\left(40\times10^{-3}\right)\times\underbrace{\left[\left(100 \,\|\, 7.5 \,\|\, 4.8\right)\times10^{3}\right]}_{=2840\,\Omega}$$

$$\therefore A_M = \boxed{-32.7\text{ V/V}}$$

so that

$$20\log_{10}|A_M| = 20\log_{10}|-32.7| = \boxed{30.3\,\text{dB}}$$

Now, the upper 3-dB frequency is given by equation 30,

$$f_H = \frac{1}{2\pi C_{in} R'_{\text{sig}}} \quad \text{(I)}$$

Here, C_{in} is given by the sum of emitter-base capacitance C_π and Miller capacitance,

$$C_{in} = C_\pi + C_\mu\left(1 + g_m R'_L\right) = 6.72 + 1.0\times\left(1 + 40\times 2.84\right) = 121\,\text{pF}$$

and R'_{sig} is given by

$$R'_{\text{sig}} = r_\pi \,||\, \left[r_x + \left(R_B \,||\, R_{\text{sig}}\right)\right] = 2.5 \,||\, \left[0.05 + \left(120 \,||\, 6\right)\right] = 1.74\,\text{k}\Omega$$

so that, substituting in (I),

$$f_H = \frac{1}{2\pi\times\left(121\times 10^{-12}\right)\times\left(1.74\times 10^3\right)} = \boxed{756\,\text{kHz}}$$

Lastly, it can be shown that the CE amplifier has a transmission zero with frequency

$$f_Z = \frac{g_m}{2\pi C_\mu} = \frac{0.04}{2\pi\times\left(1.0\times 10^{-12}\right)} = \boxed{6.37\,\text{GHz}}$$

P.13 ➡ *Solution*

The midband gain is given by equation 28, which can be restated as

$$A_M = -\frac{R_B}{R_B + R_{\text{sig}}}\frac{r_\pi}{r_\pi + r_x + \left(R_{\text{sig}} \,||\, R_B\right)} g_m R'_L = -\left(\frac{1}{1 + \dfrac{R_{\text{sig}}}{R_B}}\right)\times\frac{r_\pi}{r_\pi + r_x + \dfrac{R_B R_{\text{sig}}}{R_B + R_{\text{sig}}}} g_m R'_L$$

Here, observing that resistance R_B is substantially greater than the signal-source resistance,

$$A_M = -\left(\frac{1}{1 + \underbrace{\dfrac{R_{\text{sig}}}{R_B}}_{\to 0}}\right)\times\frac{r_\pi}{r_\pi + r_x + \underbrace{\dfrac{R_B R_{\text{sig}}}{R_B + R_{\text{sig}}}}_{\to R_{\text{sig}}}} g_m R'_L \approx -\frac{r_\pi}{r_\pi + r_x + R_{\text{sig}}} g_m R'_L = -\frac{r_\pi}{r_\pi + r_x + R_{\text{sig}}} g_m R'_L$$

Now, if $R_{sig} \gg r_x$ and r_s, we simplify further to obtain

$$A_M = -\frac{r_\pi}{\underbrace{r_\pi + r_x + R_{\text{sig}}}_{\approx R_{\text{sig}}}} g_m R'_L \approx -\frac{r_\pi g_m R'_L}{R_{\text{sig}}} = \boxed{-\frac{\beta R'_L}{R_{\text{sig}}}}$$

Now, the upper 3-dB frequency is given by equation 30,

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}} = \frac{1}{2\pi\times\left[C_\pi + C_\mu\left(1 + g_m R'_L\right)\right]\times\left\{r_\pi \,||\, \left[r_x + \left(R_B \,||\, R_{\text{sig}}\right)\right]\right\}}$$

Here, with $g_m R'_L \gg 1$ and $R_B \gg R_{\text{sig}}$,

$$f_H = \frac{1}{2\pi\times\left[C_\pi + C_\mu\underbrace{\left(1 + g_m R'_L\right)}_{\to g_m R'_L}\right]\times\left[r_\pi \,||\, \underbrace{\left(R_B \,||\, R_{\text{sig}}\right)}_{\to R_{\text{sig}}}\right]} = \frac{1}{2\pi\times\left(C_\pi + g_m R'_L C_\mu\right)\times\left(r_\pi \,||\, R_{\text{sig}}\right)}$$

Finally, with $g_m R'_L C_\mu \gg C_\pi$ and $R_{\text{sig}} \gg r_\pi$,

$$\therefore f_H = \frac{1}{2\pi\times\left(\underbrace{C_\pi + g_m R'_L C_\mu}_{\to g_m R'_L C_\mu}\right)\times\left(\underbrace{r_\pi \,||\, R_{\text{sig}}}_{\to r_\pi}\right)} = \frac{1}{2\pi \underbrace{g_m r_\pi}_{=\beta} R'_L C_\mu} = \boxed{\frac{1}{2\pi C_\mu \beta R'_L}}$$

Gleaning the two previous results, we can establish an approximation for the gain-bandwidth product,

$$\mathrm{GBW}=|A_M|f_H\approx\frac{\cancel{\beta}\cancel{R_L'}}{R_{\mathrm{sig}}}\times\frac{1}{2\pi C_\mu\cancel{\beta}\cancel{R_L'}}=\boxed{\frac{1}{2\pi C_\mu R_{\mathrm{sig}}}}$$

With R_{sig} = 25 kΩ and C_μ = 1 pF, we get

$$\mathrm{GBW}\approx\frac{1}{2\pi\times\left(1.0\times10^{-12}\right)\times\left(25\times10^{3}\right)}=\boxed{6.37\ \mathrm{MHz}}$$

P.14 ➡ Solution

The transfer function is expressed as

$$TF=\frac{A_M}{1+\dfrac{s}{2\pi f_H}}=\frac{1000}{1+\dfrac{s}{2\pi\times\left(100\times10^{3}\right)}}=\boxed{\frac{1000}{1+\dfrac{s}{200{,}000\pi}}}$$

In turn, the gain-bandwidth product is

$$f_t=|A_M|f_H=|1000|\times\left(100\times10^{3}\right)=\boxed{10^{8}\ \mathrm{Hz}}$$

P.15 ➡ Solution

Problem 15.1: The 3-dB frequency can be estimated with the general relationship (equation 33)

$$\omega_H\approx\frac{1}{\sqrt{\left(\dfrac{1}{\omega_{P1}^2}+\dfrac{1}{\omega_{P2}^2}+\ldots\right)-2\left(\dfrac{1}{\omega_{Z1}^2}+\dfrac{1}{\omega_{Z2}^2}+\ldots\right)}}$$

The amplifier in question has two zeros at $s\to\infty$ and two poles at ω_{P1} and ω_{P2}, so we may write

$$\omega_H\approx\frac{1}{\sqrt{\dfrac{1}{\omega_{P1}^2}+\dfrac{1}{\omega_{P2}^2}-\underbrace{\dfrac{2}{\omega_{Z1}^2}}_{\to\infty}-\underbrace{\dfrac{2}{\omega_{Z2}^2}}_{\to\infty}}}=\frac{1}{\sqrt{\dfrac{1}{\omega_{P1}^2}+\dfrac{1}{\omega_{P2}^2}}}\ \text{(I)}$$

The transfer function H for the amplifier is

$$H(s)=\frac{A_M}{\left(1+\dfrac{s}{\omega_{P1}}\right)\left(1+\dfrac{s}{\omega_{P2}}\right)}$$

or, equivalently,

$$H(j\omega)=\frac{A_M}{\left(1+\dfrac{j\omega}{\omega_{P1}}\right)\left(1+\dfrac{j\omega}{\omega_{P2}}\right)}$$

which has magnitude given by

$$|H(j\omega)|=\frac{A_M}{\sqrt{\left[1+\left(\dfrac{\omega}{\omega_{P1}}\right)^2\right]\left[1+\left(\dfrac{\omega}{\omega_{P2}}\right)^2\right]}}$$

Now, if the 3-dB frequency occurs at $|H(j\omega_H)|=1/\sqrt{2}$, we may write

$$\frac{A_M^2}{\left[1+\left(\dfrac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\dfrac{\omega_H}{\omega_{P2}}\right)^2\right]}=\frac{A_M^2}{2}\to\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{\omega_{P2}}\right)^2\right]=2\ \text{(II)}$$

Now, one of the pole frequencies, ω_{P2}, must be a multiple of ω_{P1} by a proportionality constant k such that the 3-dB frequency will end up being ω_H = $0.9\omega_{P1}$. Accordingly, we substitute and solve for k,

$$\left[1+\left(\frac{0.9\omega_{P1}}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{0.9\omega_{P1}}{k\omega_{P1}}\right)^2\right]=2\rightarrow(1+0.81)\times\left(1+\frac{0.81}{k^2}\right)=2$$

$$\therefore 1+\frac{0.81}{k^2}=\frac{2}{1.81}$$

$$\therefore k=\sqrt{\frac{0.81}{\frac{2}{1.81}-1}}=2.778\approx\boxed{2.78}$$

Thus, $\omega_{P2} = 2.78\omega_{P1}$.

Problem 15.2: The exact value can determined by substituting $\omega_{P2} = k\omega_{P1} = 1.0\omega_{P1}$ in equation (II) and solving for ω_H,

$$\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{\omega_{P2}}\right)^2\right]=2\rightarrow\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{1.0\omega_{P1}}\right)^2\right]=2$$

$$\therefore\left(1+\frac{\omega_H^2}{\omega_{P1}^2}\right)\left(1+\frac{\omega_H^2}{\omega_{P1}^2}\right)=2$$

$$\therefore\left(1+\frac{\omega_H^2}{\omega_{P1}^2}\right)^2=2$$

$$\therefore 1+\frac{\omega_H^2}{\omega_{P1}^2}=\sqrt{2}$$

$$\therefore \omega_H^2=\left(\sqrt{2}-1\right)\omega_{P1}^2$$

$$\therefore \omega_H=\sqrt{\sqrt{2}-1}\omega_{P1}=\boxed{0.644\omega_{P1}}$$

To compute the *approximate* value, we substitute the pertaining quantities into equation (I), giving

$$\omega_H=\frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}}}=\frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{1.0^2\omega_{P1}^2}}}=\frac{1}{\sqrt{\frac{2}{\omega_{P1}^2}}}$$

$$\therefore \omega_H=\frac{1}{\frac{\sqrt{2}}{\omega_{P1}}}$$

$$\therefore \omega_H=\frac{\omega_{P1}}{\sqrt{2}}=\boxed{0.707\omega_{P1}}$$

The approximate solution overestimates the exact solution by 9.8%. Now, letting k = 4, we substitute in equation (I) as before to obtain

$$\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{\omega_{P2}}\right)^2\right]=2\rightarrow\left[1+\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1+\left(\frac{\omega_H}{4.0\omega_{P1}}\right)^2\right]=2$$

$$\therefore\left(1+\frac{\omega_H^2}{\omega_{P1}^2}\right)\left(1+\frac{\omega_H^2}{16\omega_{P1}^2}\right)=2$$

$$\therefore 1+\frac{\omega_H^2}{16\omega_{P1}^2}+\frac{\omega_H^2}{\omega_{P1}^2}+\frac{\omega_H^4}{16\omega_{P1}^4}=2$$

$$\therefore\frac{1}{16}\left[\left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]^2+\frac{17}{16}\left(\frac{\omega_H}{\omega_{P1}}\right)^2-1=0$$

Substituting $(\omega_H/\omega_{P1})^2 = a$ converts the biquadratic equation above into a second-degree equation,

$$\frac{1}{16}a^2+\frac{17}{16}a-1=0$$

$$\therefore a=0.894,-17.9$$

Rejecting the negative solution,

$$a=\left(\frac{\omega_H}{\omega_{P1}}\right)^2=0.894\rightarrow\omega_H=\sqrt{0.894}\,\omega_{P1}$$

$$\therefore\ \boxed{\omega_H=0.946\omega_{P1}}$$

The approximate result is

$$\omega_H=\frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}}}=\frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{4.0^2\,\omega_{P1}^2}}}=\frac{1}{\sqrt{\frac{17}{16\omega_{P1}^2}}}$$

$$\therefore\ \omega_H=\frac{4}{\sqrt{17}}\omega_H=\boxed{0.970\omega_{P1}}$$

The approximation overestimates the actual ω_H by 2.5%.

P.16 ➡ *Solution*

The gain function is given by $A(s)=A_M F_H(s)$, where A_M is the midband gain and $F_H(s)$ is a transfer function with general form,

$$F_H(s)=\frac{\left(1+\frac{s}{\omega_{Z1}}\right)\left(1+\frac{s}{\omega_{Z2}}\right)\ldots\left(1+\frac{s}{\omega_{Zn}}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)\ldots\left(1+\frac{s}{\omega_{Pn}}\right)}$$

The amplifier in question has poles at f_{P1}= 2 MHz and f_{P2} = 20 MHz and zeros at f_{Z1} = 200 MHz and $f_{Z2}\rightarrow\infty$, so we may write

$$F_H(s)=\frac{\left(1+\frac{s}{2\pi\times\left(200\times10^6\right)}\right)\left[1+\underbrace{\left(\frac{s}{2\pi\times\infty}\right)}_{\rightarrow 0}\right]}{\left(1+\frac{s}{2\pi\times\left(2.0\times10^6\right)}\right)\left(1+\frac{s}{2\pi\times\left(20\times10^6\right)}\right)}$$

$$\therefore\ F_H(s)=\frac{\left(1+\frac{s}{1.26\times10^9}\right)}{\left(1+\frac{s}{1.26\times10^7}\right)\left(1+\frac{s}{1.26\times10^8}\right)}$$

The low-frequency gain in decibels is 40 dB and can be used to determine the value of A_M,

$$20\log_{10}|A_M|=40\rightarrow A_M=10^{\left(\frac{40}{20}\right)}=100$$

so that

$$A(s)=A_M F_H(s)=\boxed{100\times\frac{\left(1+\frac{s}{1.26\times10^9}\right)}{\left(1+\frac{s}{1.26\times10^7}\right)\left(1+\frac{s}{1.26\times10^8}\right)}}$$

Now, the 3-dB frequency is given by the general relation

$$\omega_H=\frac{1}{\sqrt{\left(\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}+\ldots+\frac{1}{\omega_{Pn}^2}\right)-2\left(\frac{1}{\omega_{Z1}^2}+\frac{1}{\omega_{Z2}^2}+\ldots+\frac{1}{\omega_{Zn}^2}\right)}}$$

so that, in the case at hand,

$$\omega_H=\frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}-2\left(\frac{1}{\omega_{Z1}^2}+\frac{1}{\omega_{Z2}^2}\right)}}$$

$$\therefore \omega_H = \frac{1}{\sqrt{\frac{1}{\left[2\pi\times\left(2.0\times10^6\right)\right]^2}+\frac{1}{\left[2\pi\times\left(20\times10^6\right)\right]^2}-2\left(\frac{1}{\left[2\pi\times\left(200\times10^6\right)\right]^2}+\underbrace{\frac{1}{\infty}}_{\to 0}\right)}}$$

$$\therefore \omega_H = 1.25\times10^7 \text{ rad/s}$$

$$\therefore f_H = \frac{1.25\times10^7}{2\pi} = 1.99\times10^6 \text{ Hz} = \boxed{1.99\,\text{MHz}}$$

P.17 ➡ *Solution*

Problem 17.1: The gain function is given by

$$A(s) = A_M F_H(s) = 1000\times\left(\frac{1}{1+\frac{s}{2\pi\times\left(100\times10^3\right)}}\right) = \boxed{1000\left(\frac{1}{1+\frac{s}{6.28\times10^5}}\right)}$$

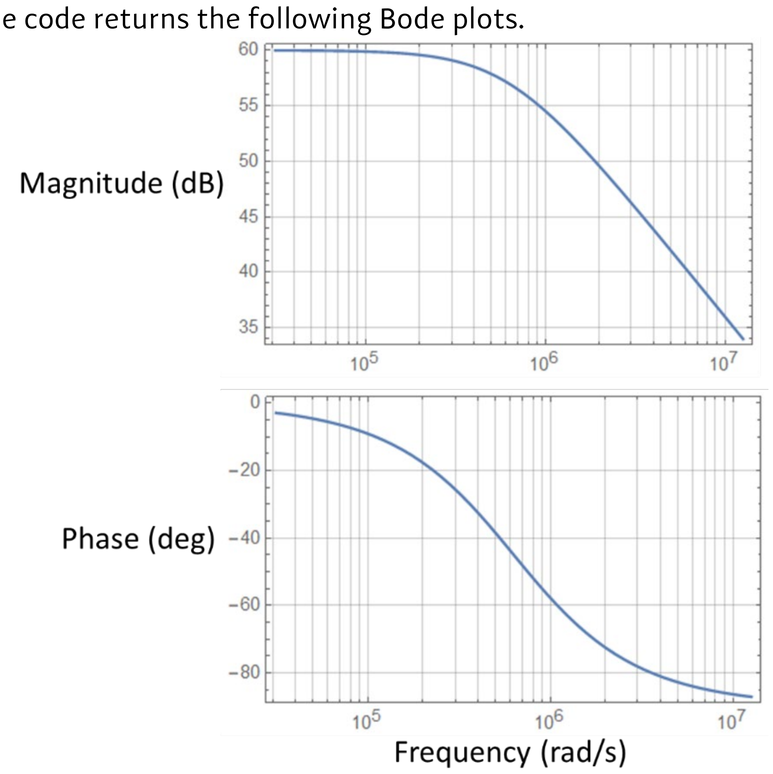
Problem 17.2: Use the technique explained in Sedra and Smith (2015) (or pretty much any control engineering textbook). In Mathematica, the following code may be used,

```
In[4]:= A = TransferFunctionModel[1000 * (1 / (1 + s / (6.28 * 10^5))), s]

Out[4]= (1000 / (1 + 1.59236 × 10^-6 s))

In[5]:= BodePlot[A]
```

The code returns the following Bode plots.



Problem 17.3: With two poles, the transfer function becomes

$$F_H(s) = \frac{1}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)} = \frac{1}{\left[1+\frac{s}{2\pi\times\left(100\times10^3\right)}\right]\left(1+\frac{s}{2\pi\times\left(1.0\times10^6\right)}\right)}$$

$$\therefore F_H(s)\frac{1}{\left(1+\frac{s}{6.28\times10^5}\right)\left(1+\frac{s}{6.28\times10^6}\right)}$$

and the gain function follows as

$$A(s) = A_M F_H(s) = \frac{1000}{\left(1+\frac{s}{6.28\times10^5}\right)\left(1+\frac{s}{6.28\times10^6}\right)}$$

To prepare the Bode plot of the system, we first expand *A(s)*,

$$A(s)=\frac{1000}{\left(\frac{6.28\times10^5+s}{6.28\times10^5}\right)\left(\frac{6.28\times10^6+s}{6.28\times10^6}\right)}$$

$$\therefore A(s)=\frac{1000\times\left(6.28\times10^5\right)\times\left(6.28\times10^6\right)}{\left(6.28\times10^5+s\right)\left(6.28\times10^6+s\right)}$$

$$\therefore A(s)=\frac{3.95\times10^{15}}{3.94\times10^{12}+6.91\times10^6 s+s^2}$$

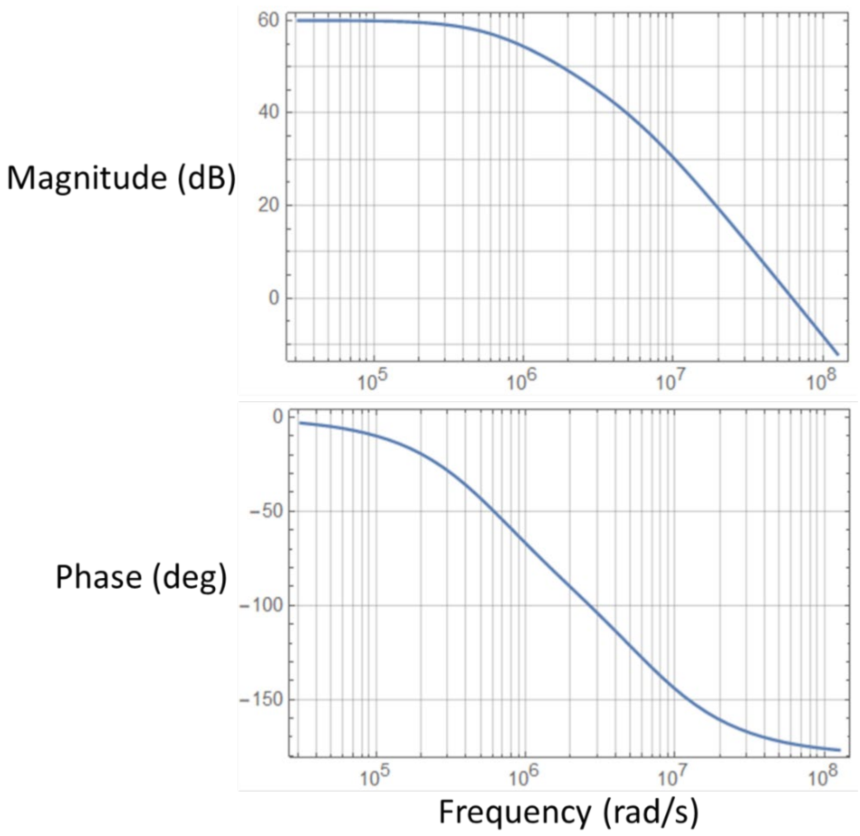
The corresponding Bode plot can be established with the following code.

```
In[7]:= A2 = TransferFunctionModel[ (3.95 * 10^15) / (3.94 * 10^12 + 6.91 * 10^6 s + s^2), s]

Out[7]= ( (3.95 × 10^15) / (3.94 × 10^12 + 6.91 × 10^6 s + s^2) ) 𝒯

In[8]:= BodePlot[A2]
```

This code returns the following graphs. Inspecting the graph and converting circular frequency to linear frequency, we see that the unity-gain frequency has changed to about 60 MHz.



P.18 ➡ *Solution*

Problem 18.1: We first determine the resistances seen by the three capacitors C_{gs}, C_{gd}, and C_L, respectively,

$$R_{gs}=R'_{\text{sig}}=12\ \text{k}\Omega$$

$$R_{gd}=R'_{\text{sig}}\left(1+g_m R'_L\right)+R'_L=12\times\left(1+1.4\times12\right)+12=226\,\text{k}\Omega$$

$$R_{C_L}=R'_L=12\,\text{k}\Omega$$

The time constants follow as (eqs. 36, 38, and 40)

$$\tau_{gs}=C_{gs}R_{gs}=\left(25\times10^{-15}\right)\times\left(12\times10^3\right)=300\,\text{ps}$$

$$\tau_{gd}=C_{gd}R_{gd}=\left(6.0\times10^{-15}\right)\times\left(226\times10^3\right)=1360\,\text{ps}$$

$$\tau_{C_L}=C_L R_{C_L}=\left(27\times10^{-15}\right)\times\left(12\times10^3\right)=324\,\text{ps}$$

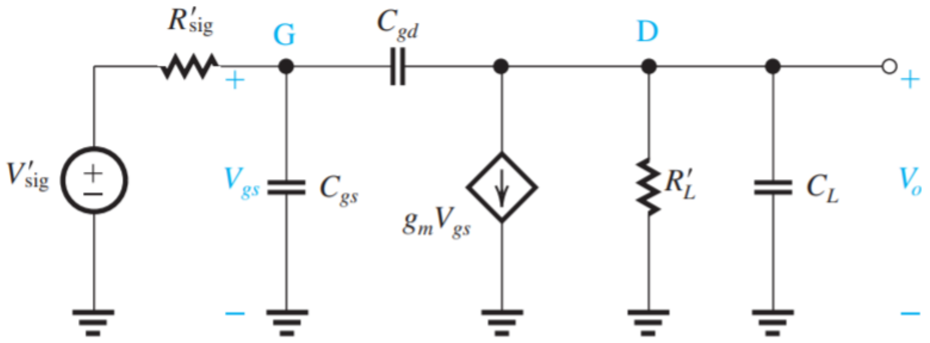
The effective high-frequency time constant τ_H is obtained by adding the three time constants, as in equation 35,

$$\tau_H=\tau_{gs}+\tau_{gd}+\tau_{C_L}=300+1360+324=1980\,\text{ps}$$

and the 3-dB frequency f_H is determined as (equation 34)

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\times\left(1980\times10^{-12}\right)} = \boxed{80.4\,\text{MHz}}$$

Now, the frequency of transmission zero can be established with reference to the generalized high-frequency equivalent circuit for the CS amp, repeated below for convenience.



Setting frequency s to its transition value s_Z, V_o will be zero and a node equation applied to the output node at $s = s_Z$ brings to

$$s_Z C_{gd}\left(V_{gs} - \cancel{V_o}\right) = g_m V_{gs} \rightarrow s_Z = \frac{g_m}{C_{gd}}$$

$$\therefore f_Z = \frac{1.4\times10^{-3}}{2\pi\times6.0\times10^{-15}} = \boxed{37.1\text{ GHz}}$$

Problem 18.2: In the Miller effect approach, the 3-dB frequency is given by equation 30,

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

Here, R'_{sig} = 12 kΩ as given, while input capacitance C_{in} can be estimated from

$$C_{\text{in}} = C_{gs} + C_{gd}\left(1+g_m R'_L\right) = 25 + 6.0\times\left(1+1.4\times12\right) = 132\text{ fF}$$

so that

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}} = \frac{1}{2\pi\times\left(132\times10^{-15}\right)\times\left(12\times10^{3}\right)} = \boxed{100\,\text{MHz}}$$

That is, the Miller effect approach predicts a 3-dB frequency about 24% greater than the value determined with open-circuit time constants. The open-circuit time constant approach is more precise because it includes the effect of the capacitance C_L between the drain node and ground.

Problem 18.3: The midband voltage gain is given by

$$A_M = -g_m R'_L = -\left(1.4\times10^{-3}\right)\times\left(12\times10^{3}\right) = -16.8\text{ V/V}$$

so that

$$\text{GBW} = \left|A_M\right| f_H = 16.8\times\left(80.4\times10^{6}\right) = \boxed{1.35\,\text{GHz}}$$

P.19 ➡ *Solution*

The midband gain is given by

$$A_M = -g_m R'_L = -\left(4\times10^{-3}\right)\times\left(20\times10^{3}\right) = \boxed{-80\text{ V/V}}$$

In Miller's approach, the input capacitance is given by equation 27,

$$C_{\text{in}} = C_{gs} + C_{gd}\left(1+g_m R'_L\right) = 2.0 + 0.1\times\left(1+4\times20\right) = \boxed{10.1\,\text{pF}}$$

and the 3-dB frequency follows as (equation 26)

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}} = \frac{1}{2\pi\times\left(10.1\times10^{-12}\right)\times\left(20\times10^{3}\right)} = 7.88\times10^{5} = \boxed{788\,\text{kHz}}$$

Switching to the method of open-circuit time constants, we first compute the three time constants (equations 36, 38, and 40)

$$\tau_{gs} = C_{gs}R_{gs} = C_{gs}R'_{\text{sig}} = \left(2.0\times10^{-12}\right)\times\left(20\times10^{3}\right) = 40\,\text{ns}$$

$$\tau_{gd} = C_{gd}R_{gd} = C_{gd}\left[R'_{\text{sig}}\left(1+g_m R'_L\right)+R'_L\right] = \left(0.1\times10^{-12}\right)\times\left\{\left(20\times10^{3}\right)\times\left[1+\left(4.0\times10^{-3}\right)\times\left(20\times10^{3}\right)\right]+\left(20\times10^{3}\right)\right\} = 164\,\text{ns}$$

$$\tau_{C_L} = C_L R_{C_L} = C_L R'_L = \left(2.0\times10^{-12}\right)\times\left(20\times10^{3}\right) = 40\,\text{ns}$$

so that (equation 35)

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L} = 40 + 164 + 40 = 244\text{ ns}$$

Finally, substituting in equation 34,

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\times\left(244\times10^{-9}\right)} = 6.52\times10^5\text{ Hz} = \boxed{652\text{ kHz}}$$

The estimate obtained from the time-constant method is more accurate than the result obtained with the Miller method because the former accounts for the output-node-to-ground capacitance C_L.

P.20 ➡ *Solution*

Time constant τ_H can be determined as (equation 42)

$$\tau_H = C_{gs}R'_{\text{sig}} + C_{gd}\left[R'_{\text{sig}}\left(1+g_mR'_L\right)+R'_L\right]+C_LR'_L$$

$$\therefore \tau_H = \left(5.0\times10^{-12}\right)\times\left(10\times10^3\right)+\left(1.0\times10^{-12}\right)\times\left[\left(10\times10^3\right)\times\left(1+5.0\times10\right)+\left(10\times10^3\right)\right]$$

$$+\left(5.0\times10^{-12}\right)\times\left(10\times10^3\right) = 6.20\times10^{-7}\text{ s} = \boxed{620\text{ ns}}$$

An estimate of the 3-dB frequency easily follows,

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\times\left(620\times10^{-9}\right)} = 2.57\times10^5\text{ Hz} = \boxed{257\text{ kHz}}$$

The input capacitance is given by equation 27,

$$C_{\text{in}} = C_{gs} + C_{gd}\left(1+g_mR'_L\right) = \left(5.0\times10^{-12}\right)+\left(1.0\times10^{-12}\right)\times\left(1+5\times10\right) = 5.6\times10^{-11}\text{ F} = 56\text{ pF}$$

and can be multiplied with the signal-source resistance R'_{sig} = 10 kΩ to yield a time value t,

$$t = C_{\text{in}}R'_{\text{sig}} = \left(56\times10^{-12}\right)\times\left(10\times10^3\right) = 5.6\times10^{-7}\text{ s} = 560\text{ ns}$$

The ratio of t to the complete time constant τ_H is

$$\frac{t}{\tau_H} = \frac{560}{620}\times100\% = 90.3\%$$

Thus, about nine-tenths of the full time constant τ_H stems from the interaction of the input capacitance C_{in} with the signal-source resistance R'_{sig}.

P.21 ➡ *Solution*

The transconductance is $g_m = I_C/V_T = 1.0\times10^{-3}/25\times10^{-3} = 0.04$ A/V = 40 mA/V. With a bias current I = 1 mA and the appropriate Early voltages, load resistance R'_L is

$$R'_L = r_{Onpn}\,||\,r_{Opnp} = \frac{V_{An}}{I}\,||\,\frac{\left|V_{Ap}\right|}{I} = \frac{130}{1.0\times10^{-3}}\,||\,\frac{50}{1.0\times10^{-3}} = 130{,}000\,||\,50{,}000$$

$$\therefore R'_L = \frac{130\times50}{130+50} = 36.1\text{k}\Omega$$

The hybrid-π resistance is

$$r_\pi = \frac{\beta(npn)}{g_m} = \frac{200}{0.04} = 5\text{k}\Omega$$

The midband voltage gain follows as

$$A_M = -\frac{r_\pi}{R_{\text{sig}}+r_x+r_\pi}g_mR'_L = -\frac{5.0}{36+0.2+5}\times0.04\times\left(36.1\times10^3\right) = -175.24 \approx \boxed{-175\text{ V/V}}$$

Now, in the Miller approach the input capacitance C_{in} is given by

$$C_{\text{in}} = C_\pi + C_\mu\left(1+g_mR'_L\right) = 16+0.3\times\left(1+40\times36.1\right) = 450\text{ pF}$$

Thévenin resistance R'_{sig} is determined next (equation 45),

$$R'_{\text{sig}} = r_\pi\,||\left(r_x+R_{\text{sig}}\right) = \frac{5.0\times\left(0.2+36\right)}{5.0+\left(0.2+36\right)} = 4.39\text{ k}\Omega$$

The 3-dB frequency is then

$$f_H = \frac{1}{2\pi C_{\text{in}}R'_{\text{sig}}} = \frac{1}{2\pi\times\left(450\times10^{-12}\right)\times\left(4.39\times10^3\right)} = 8.06\times10^4\text{ Hz} = \boxed{80.6\text{ kHz}}$$

Now, the 3-dB frequency as determined with the open-circuit time constant approach is

$$f_H = \frac{1}{2\pi\tau_H}$$

where τ_H is given by the sum of the three contributions (equation 50)

$$\tau_H = C_\pi R'_{\text{sig}} + C_\mu \left[R'_{\text{sig}} \left(1 + g_m R'_L\right) + R'_L \right] + C_L R'_L$$

$$\therefore \tau_H = \left(16\times10^{-12}\right)\times\left(4.39\times10^{3}\right) + \left(0.3\times10^{-12}\right)\times\left[\left(4.39\times10^{3}\right)\times\left(1+40\times36.1\right)+\left(36.1\times10^{3}\right)\right]$$

$$+\left(5.0\times10^{-12}\right)\times\left(36.1\times10^{3}\right) = 2.16\times10^{-6}\text{ s} = 2.16\,\mu\text{s}$$

so that

$$f_H = \frac{1}{2\pi\times\left(2.16\times10^{-6}\right)} = 73{,}700\text{ Hz} = \boxed{73.7\text{ kHz}}$$

That is, the value of f_H estimated with the open-circuit time constant method is about 9.4% lower than the value determined with the Miller effect approach.

The frequency of transmission zero is calculated as

$$f_Z = \frac{g_m}{2\pi C_\mu} = \frac{0.04}{2\pi\times\left(0.3\times10^{-12}\right)} = 2.12\times10^{10}\text{ Hz} = \boxed{21.2\text{ GHz}}$$

Lastly, the gain-bandwidth product, using the 3-dB frequency determined with the OCTC method, is found as

$$f_t = |A_M| f_H = 175\times\left(73.7\times10^{3}\right) = 1.29\times10^{7}\text{ Hz} = \boxed{12.9\text{ MHz}}$$

REFERENCE

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