Montogue

GATE Aerospace Engineering (AS):

30 Practice Questions

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Here's a set of 30 solved problems for applicants to the GATE Aerospace Engineering (AS) exam. Problems were taken from past question papers and a carefully researched assortment of textbooks. All problems are solved step by step. Enjoy! ■

PROBLEMS

Problem 1. A fighter aircraft of mass equal to 31,500 kg and wing area of 50 m^2 is cruising at 16,000 m altitude with a lift coefficient equal to 0.12. It is known that the relative air density σ at 15,000 m is approximately equal to 0.159. The equivalent airspeed of the aircraft is, most nearly:
(A) 211 m/s
(B) 290 m/s
(C) 445 m/s
(D) 727 m/s

Problem 2. The pressure on a point on the wing of an airplane is 75,800 N/m^2. The airplane is flying at 80 m/s and 2000-m altitude, at which the atmospheric pressure and density may be taken as 78.2 kPa and 0.95 kg/m^3, respectively. What is the pressure coefficient at this point on the wing?
(A) −0.79
(B) −0.39
(C) 0.12
(D) 0.39

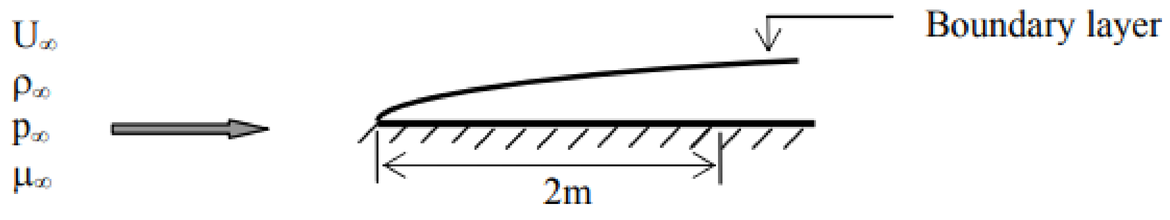
Problem 3. Consider a wing with aspect ratio of 9 and Oswald efficiency factor of 0.85 immersed in a flow at Reynolds number 5×10^6. The infinite-wing lift slope is 0.102 per degree of angle of attack, the zero-lift angle of attack is −1.5°, and the profile drag coefficient is 0.0075. If the wing is at an 8° angle of attack, compute the total drag coefficient.
(A) 0.009
(B) 0.013
(C) 0.021
(D) 0.033

Problem 4. A wing with elliptical loading has span 15 m, planform area 48 m^2, and travels in level flight at 225 m/s at an altitude where air density equals 0.7 kg/m^3. The lift coefficient is 0.243. The downwash velocity and the wing loading are, respectively:
(A) 2.44 m/s and 2590 N/m^2
(B) 2.44 m/s and 4310 N/m^2
(C) 3.71 m/s and 2590 N/m^2
(D) 3.71 m/s and 4310 N/m^2

Problem 5. A large turbofan engine has a bypass ratio of 10 and produces a static thrust of 500 kN with a jet exit velocity of 800 m/s and a fan exit velocity of 120 m/s. Assuming ideal conditions, what is the total mass flow $\dot{m}$ passing through the engine?
(A) 2500 kg/s
(B) 2600 kg/s
(C) 2750 kg/s
(D) 2950 kg/s

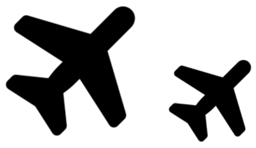
Problem 6. An aircraft with a turboprop engine produces a thrust of 500 N and flies at 100 m/s. If the propeller efficiency is 0.5, the shaft power produced by the engine is:
(A) 50 kW
(B) 100 kW
(C) 125 kW
(D) 500 kW

Problem 7. Consider a steady, two-dimensional, zero-pressure-gradient laminar flow of air over a flat plate as illustrated below. The free stream conditions are U_∞ = 120 m/s, ρ_∞ = 1.2 kg/m^3, p_∞ = 1 atm and μ_∞ = 1.8×10^{-5} Pa·s. The ratio of displacement thickness to momentum thickness of the boundary layer at a distance of 2 m from the leading edge is:



(A) 0.39
(B) 2.59
(C) 2.91
(D) 7.53

Problem 8. To estimate aerodynamic loads on an aircraft flying at 100 km/h at standard sea-level conditions, a one-fifth scale model is tested in a variable-density wind tunnel ensuring similarity of inertial and viscous forces. The pressure used in the wind tunnel is 10 times the atmospheric pressure. Assuming that the ideal gas law holds and the same temperature conditions apply to model and prototype, the velocity needed in the wind tunnel test section is __________.



(A) 20 km/h
(B) 50 km/h
(C) 100 km/h
(D) 200 km/h

Problem 9. An aircraft in climbing flight is rising steadily with a 18° climb angle and a 6° angle of attack. If the aircraft has a mass of 15,000 kg and is opposed by a drag of 12,000 N, the aircraft lift is:
(A) 120 kN
(B) 128 kN
(C) 134 kN
(D) 147 kN

Problem 10. At the start of a long distance cruise a turboprop aircraft has a total mass of 67,000 kg, of which 13,000 kg is fuel. The drag polar is given by C_D = 0.021 + 0.052C_L^2, the propeller efficiency is 0.84 and the specific fuel consumption is 1.0×10^{-7} kg/J. The speed is kept constant during the entire trajectory. The maximum range predicted with the Breguet formula is:
(A) 2790 km
(B) 3100 km
(C) 3450 km
(D) 4060 km

Problem 11. A passenger-carrying aircraft has a cylindrical cabin of 3 m diameter and is subjected in flight to a differential pressure of 0.6 N/mm^2. If the maximum allowable direct stress in the material of the cabin is 425 N/mm^2, calculate the minimum allowable thickness of the cabin wall using a factor of safety of 1.75.

(A) 2.0 mm
(B) 3.7 mm
(C) 5.4 mm
(D) 6.6 mm

Problem 12. A finite wing has a torsional stiffness of 12×10^6 $N\cdot m^2$, a span of 8.0 m, and a mean chord of 2.8 m. If the two-dimensional lift-curve slope is 3.1 and the aerodynamic center of the wing is 1.2 m forward of its flexural axis, calculate the wing torsional divergence speed at sea level. Assume a straight wing with flexural axis nearly perpendicular to the aircraft's plane of symmetry.

(A) 40 m/s
(B) 105 m/s
(C) 161 m/s
(D) 205 m/s

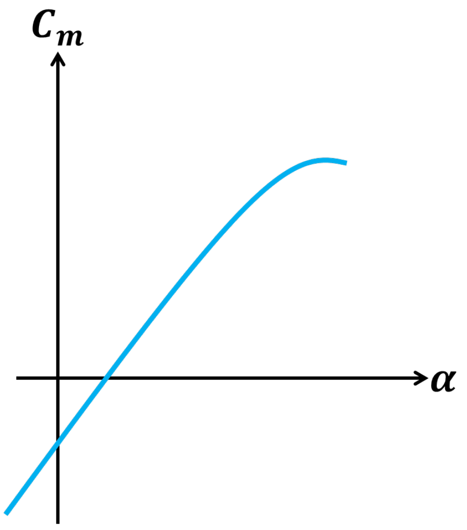
Problem 13. In a centrifugal compressor, air leaving the impeller with radial velocity 100 m/s makes an angle of 32° with the axial direction. The impeller tip speed is 450 m/s and the mass flow rate is 3.5 kg/s. The mechanical efficiency of the compressor is 0.8. What is the power required to drive the compressor?

(A) 611 kW
(B) 699 kW
(C) 764 kW
(D) 886 kW

Problem 14. The X-30 hypersonic aerospace plane is flying at 3000 m/s in an altitude of 30 km, wherein the density and viscosity of air may be taken as 0.018 kg/m^3 and 1.5×10^{-5} Pa·s, respectively. The undersurface of the vehicle compresses the flow ahead of the supersonic combustion ramjet engine. Assuming this undersurface forebody is 32 m in length, the heat flux to the forebody surface is:

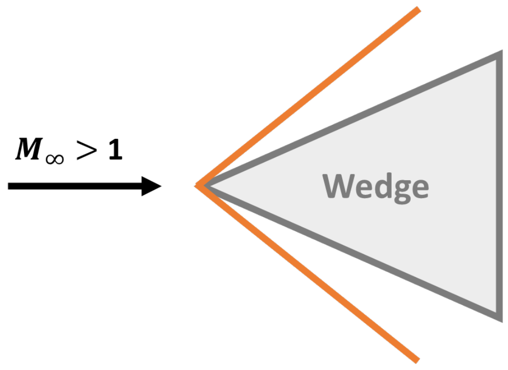
(A) 170 kW/m^2
(B) 220 kW/m^2
(C) 270 kW/m^2
(D) 320 kW/m^2

Problem 15. The following curve describes the relationship between moment coefficient C_M and angle of attack α for a certain aircraft. Which of the following statements is true for this aircraft?



(A) The aircraft can trim at a positive α, and it is stable.
(B) The aircraft can trim at a positive α, but it is unstable.
(C) The aircraft can trim at a negative α, and it is stable.
(D) The aircraft can trim at a negative α, but it is unstable.

Problem 16. Consider a 2-D wedge in supersonic flow with an attached oblique shock as shown below. The wedge half-angle is known to be greater than the maximum deflection angle of the oblique shock. In this case, if the Mach number M_∞ is increased, the oblique shock wave will:



(A) move closer to the body.
(B) move away from the body.
(C) detach from the body.
(D) become a normal shock.

Problem 17. Consider a supersonic stream at a Mach number $M = 2$ undergoing a gradual expansion. The stream is turned by an angle of 3 degrees due to the expansion. The following Prandtl-Meyer function data is given.

M	ν (Prandtl-Meyer function)
1.8	20.73
1.9	23.59
2.0	26.38
2.1	29.10
2.2	31.73
2.3	34.28
2.4	36.75

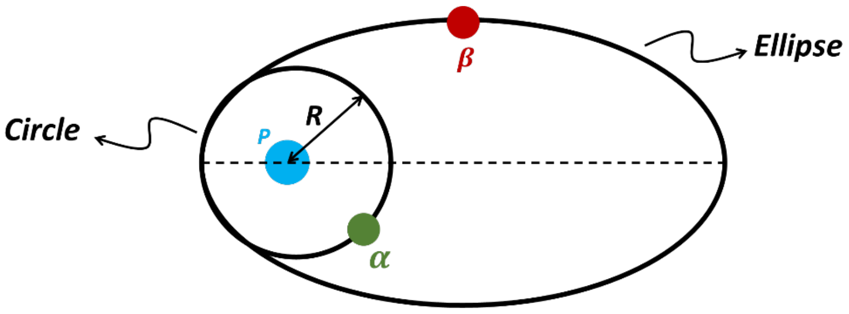
The Mach number downstream of the expansion is:
(A) 1.88
(B) 2.00
(C) 2.11
(D) 2.33

Problem 18. Regarding aspects of rocket propellant science, which of the following is **false**?
(A) Liquid fluorine is highly energetic and noncorrosive, which makes it a promising candidate for future rocket applications.
(B) Liquid hydrogen-liquid oxygen mixtures are only liquid at very low temperatures and hence qualify as cryogenic propellants.
(C) NTO/MMH, a combination of monomethyl hydrazine and nitrogen tetroxide, is an example of hypergolic liquid propellant.
(D) Beryllium powders constitute very energetic solid propellants, but have fallen out of use for being highly toxic.

Problem 19. A rocket is propelled by the expansion of room-temperature molecular hydrogen from a pressure of 2 MPa to a pressure of 0.1 MPa. Expansion is assumed to take place at a constant temperature of 300 K. Noting that the specific heat ratio and the molecular mass of H_2 are 1.4 and 2 kg/kmol, respectively, the exit jet velocity for this rocket is ________.
(A) 1730 m/s
(B) 1910 m/s
(C) 2240 m/s
(D) 2580 m/s

Problem 20. A two-stage rocket has the following masses: 1st-stage propellant mass 150,000 kg, 1st-stage dry mass 12,000 kg, 2nd-stage propellant mass 32,000 kg, 2nd-stage dry mass 3000 kg, and payload mass 3000 kg. The specific impulses of the first and second stages are 250 s and 340 s, respectively. Calculate the total variation in velocity afforded by the two stages of the rocket.
(A) 7130 m/s
(B) 8540 m/s
(C) 9560 m/s
(D) 10,800 m/s

Problem 21. Two satellites α and β have orbits around the same planet P, as illustrated below. Satellite α describes a circular orbit of radius R, whereas satellite β describes an elliptical orbit. Knowing that satellite β has a period 8 times greater than satellite α, the apogee of satellite β is:



(A) $4R$
(B) $6R$
(C) $7R$
(D) $12R$

Problem 22. A thin airfoil has a camber line defined by the relation $y_c = kc\xi(\xi - 1)(\xi - 2)$, where c is chord length, ξ varies from 0 to 1, and k is an unknown constant. If the maximum camber is 2% of the chord, the value of k is ________ (◆ *dimensionless*).

Problem 23. The lift coefficient for incompressible flow over a NACA 2412 airfoil at 5° angle of attack is 0.8. Using the Prandtl-Glauert rule, the lift coefficient for this airfoil in a freestream at Mach number M_∞ = 0.85 is calculated to be _________ (◆ *dimensionless*).

Problem 24. An unpowered glider is described by the following data. The glider is initially in level flight at 30 m/s and begins its descent at an altitude of 2000 m, for which the air density is 1.0 kg/m^3. Its rate of descent is initially equal to ________ (◆ *m/s, absolute value*).

Total weight, W	3920 N
Wing area, S	15 m^2
Zero-lift drag coefficient, $C_{D,0}$	0.015
Lift-dependent drag coef. factor, K	0.03

Problem 25. A normal shock produced by supersonic flow of air (γ = 1.4) in a circular tube has pressure ratio equal to 2. The corresponding entropy change across the shock is equal to __________ (◆*J/K*).

Problem 26. A ramjet provides net thrust equal to 10 kN to drive a missile at V_0 = 1000 m/s at an altitude of 33.5 km, where air density and temperature may be taken as 0.01 kg/m^3 and 232 K, respectively. If the missile inlet captures a streamtube of air A_0 = 1 m^2 in area, the fuel-to-air ratio is 0.04, and the fuel heating value is 50 MJ/kg, the overall efficiency equals _______ (◆%).

Problem 27. A rocket using a 4 to 1 mixture of LOX and LH_2 as a propellant has a specific impulse of 400 sec. The LH_2 mass flow rate is 10 kg/s. The thrust developed by the rocket is _______ (◆ *kN*).

Problem 28. A satellite forms a circular orbit at an altitude of 150 km above the surface of a spherical Earth. Assuming the gravitational parameter μ = 3.986×10^{14} m^3/s^2 and radius of the Earth R_E = 6400 km, the velocity required for injection of the spacecraft, parallel to the local horizon, is _________ (◆ *km/s*).

Common data question

Two tests with the same rocket solid-propellant grain under two different chamber pressures and burn rates produced the following data.

Test No.	Chamber pressure (MPa)	Burn rate (mm/s)
1	17	23.4
2	8	13.1

Problem 29. Find the combustion chamber pressure for a burning rate of 16 mm/s.
(A) 8.36 MPa
(B) 9.33 MPa
(C) 10.3 MPa
(D) 10.9 MPa

Problem 30. Calculate the propellant consumption rate in per square meter of burning surface if the density of propellant ρ_p = 1800 kg/m^3 and the grain diameter is 0.15 m. Assume that the burning rate is kept fixed at 16 mm/s.
(A) 5.10 (kg/s)/m^2
(B) 10.2 (kg/s)/m^2
(C) 19.4 (kg/s)/m^2
(D) 28.8 (kg/s)/m^2

ANSWER KEY

Problem	Answer	Problem	Answer
1	B	16	C
2	A	17	C
3	D	18	A
4	D	19	C
5	C	20	C
6	B	21	C
7	B	22	0.0520
8	B	23	1.25
9	C	24	1.30
10	A	25	198.7
11	B	26	50
12	C	27	196
13	C	28	7.8
14	B	29	B
15	B	30	D

SOLUTIONS

1 ➡ B

For steady level flight, lift equals weight; thus, $L = W$ = 31,500 × 9.81 = 309,000 N. From the definition of lift coefficient, we can establish the true airspeed V_T:

$$C_L = \frac{L}{\frac{1}{2}\rho V_T^2 S} \rightarrow V_T = \sqrt{\frac{L}{\frac{1}{2}\rho S C_L}}$$

$$\therefore V_T = \sqrt{\frac{309{,}000}{\frac{1}{2}\times(0.159\times 1.225)\times 50\times 0.12}} = 727\ \mathrm{m/s}$$

where we have used ρ_{SSL} = 1.225 kg/m^3 as the air density at sea level. It remains to convert V_T to an *equivalent* airspeed:

$$V_T = \frac{V_E}{\sqrt{\sigma}} \rightarrow V_E = \sqrt{\sigma} V_T$$

$$\therefore V_E = \sqrt{0.159}\times 727 = \boxed{290\ \mathrm{m/s}}$$

2 ➡ A

We first determine the dynamic pressure q_∞:

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\times 0.95\times 80^2 = 3040\ \mathrm{Pa}$$

Then, from the definition of pressure coefficient,

$$C_P = \frac{p - p_\infty}{q_\infty} = \frac{75{,}800 - 78{,}200}{3040} = \boxed{-0.789}$$

3 ➡ D

The lift slope for the finite wing is given by

$$a=\frac{a_0}{1+57.3a_0/(\pi eA_R)}=\frac{0.102}{1+57.3\times0.102/(\pi\times0.85\times9)}=0.0820\text{ per degree}$$

At α = 8°, the lift coefficient is

$$C_L=a(\alpha-\alpha_{L=0})=0.082\times\left[8^\circ-(-1.5^\circ)\right]=0.779$$

The total drag coefficient is then

$$C_D=c_d+\frac{C_L^2}{\pi eA_R}=0.0075+\frac{0.779^2}{\pi\times0.85\times9}=\boxed{0.0328}$$

4 ➡ D

The aspect ratio of the wing is

$$A_R=\frac{2b}{c}=\frac{(2b)^2}{2b\times c}=\frac{(2b)^2}{S}=\frac{15^2}{48}=4.69$$

The downwash is given by

$$w=\frac{k_0}{4b}\ \text{(I)}$$

where k_0 is the circulation at midspan, which for an elliptical distribution reads

$$k_0=\frac{C_LVS}{\pi b}=\frac{0.243\times225\times(2b\times c)}{\pi\times b}$$

so that, substituting in (I),

$$w=\frac{k_0}{4b}=\frac{1}{4b}\times\frac{0.243\times225\times(2b\times c)}{\pi\times b}=\frac{0.243\times225}{\pi\times\underbrace{(2b/c)}_{=A_R=4.69}}=\boxed{3.71\text{ m/s}}$$

To find the wing loading, we first note that for level flight the weight must equal the lift, that is:

$$W=L=\frac{1}{2}\rho V^2SC_L=\frac{1}{2}\times0.7\times225^2\times48\times0.243=207{,}000\text{ N}$$

Finally,

$$\frac{W}{S}=\frac{207{,}000}{48}=\boxed{4310\text{ N/m}^2}$$

5 ➡ C

The total mass flow we're looking for equals the sum of the jet and fan mass flows; noting that the bypass ratio $\beta=\dot{m}_{fan}/\dot{m}_{jet}$ = 10, we may write

$$\dot{m}=\dot{m}_{jet}+\dot{m}_{fan}=\dot{m}_{jet}\left(1+\underbrace{\frac{\dot{m}_{fan}}{\dot{m}_{jet}}}_{=10}\right)=11\dot{m}_{jet}\ \text{(I)}$$

Noting that the static thrust equals 500 kN, we can solve for $\dot{m}_{jet}$ and obtain

$$F=F_{jet}+F_{fan}=\dot{m}_{jet}V_{jet}+\dot{m}_{fan}V_{fan}$$

$$\therefore F=\dot{m}_{jet}(800+10\times120)=500\text{ kN}$$

$$\therefore \dot{m}_{jet}=\frac{500{,}000}{800+10\times120}=250\text{ kg/s}$$

Substituting in (I) gives the total mass flow:

$$\dot{m}=11\times250=\boxed{2750\text{ kg/s}}$$

6 ➡ B

The shaft power is $\Pi = FV = 500 \times 100 = 50{,}000$ W. For 50% propeller efficiency, the shaft power developed by the engine becomes 50,000/0.5 = 100,000 W = 100 kW.

7 ➡ B

For a laminar flow over a flat plate such as the one considered herein, the displacement thickness δ^* is given by

$$\delta^* = 1.7208\sqrt{\frac{\nu x}{U_\infty}}$$

whereas the momentum thickness reads

$$\theta = 0.664\sqrt{\frac{\nu x}{U_\infty}}$$

Thus, the ratio we aim for is $\delta^*/\theta = 1.7208/0.664 = 2.59$.

8 ➡ B

If similarity between inertial and viscous forces holds, then the Reynolds number must be the same for model and real aircraft:

$$\left(\frac{\rho VL}{\mu}\right)_{\text{Model}} = \left(\frac{\rho VL}{\mu}\right)_{\text{Full-scale}}$$

From the ideal gas law, $\rho = p/RT$, so that

$$\left(\frac{pVL}{\mu RT}\right)_{\text{Model}} = \left(\frac{pVL}{\mu RT}\right)_{\text{Full-scale}}$$

$$\therefore (pVL)_{\text{Model}} = (pVL)_{\text{Full-scale}}$$

$$\therefore p_M V_M L_M = p_{FS} V_{FS} L_{FS}$$

$$\therefore 10 p_{FS} \times V_M \times 0.2 L_{FS} = p_{FS} \times V_{FS} \times L_{FS}$$

$$\therefore 2V_M = V_{FS}$$

$$\therefore V_M = \frac{V_{FS}}{2} = \frac{100}{2} = \boxed{50\text{ km/h}}$$

For the given similarity conditions to hold, the model in the wind tunnel should be subjected to a 50-km/h airflow.

9 ➡ C

The basic equilibrium equation for lift in steady climbing flight is

$$L + T\sin\alpha = W\cos\gamma \quad \text{(I)}$$

where L is lift, T is thrust, α is angle of attack, W is weight, and γ is climbing angle. The thrust can be found from the other force equilibrium equation, namely

$$T\cos\alpha = D + W\sin\gamma \to T = \frac{D + W\sin\gamma}{\cos\alpha}$$

$$\therefore T = \frac{12{,}000 + (15{,}000 \times 9.81) \times \sin 18^\circ}{\cos 6^\circ} = 57{,}800\text{ N}$$

Substituting the pertaining variables in (I):

$$L = W\cos\gamma - T\sin\alpha = (15{,}000 \times 9.81) \times \cos 18^\circ - 57{,}800 \times \sin 6^\circ$$

$$\therefore L = 134{,}000\text{ N} = \boxed{134\text{ kN}}$$

10 ➡ A

The Breguet formula for propeller-driven aircraft reads

$$R_{\max} = \frac{\eta}{SFC \times g}\left(\frac{C_L}{C_D}\right)_{\max} \ln\left(\frac{m_0}{m_i}\right)$$

For a drag of the (polar) form $C_D = a + bC_L^2$, the maximum-distance lift coefficient reads

$$C_{L,\text{max dist.}} = \sqrt{\frac{a}{b}} = \sqrt{\frac{0.021}{0.052}} = 0.635$$

and the corresponding drag coefficient is

$$C_{D,\text{max dist.}} = 0.021 + 0.052 \times 0.635^2 = 0.0420$$

so that C_L/C_D = 0.635/0.042 = 15.1. Substituting in the Breguet formula brings to

$$R_{\max} = \frac{0.84}{\left(1.0 \times 10^{-7}\right) \times 9.81} \times 15.1 \times \ln\left(\frac{67{,}000}{67{,}000 - 13{,}000}\right) = 2.79 \times 10^6 \text{ m}$$

$$\therefore \boxed{R_{\max} = 2790 \text{ km}}$$

11 ➡ B

From Laplace's law for cylindrical shells,

$$\sigma_C = \frac{pd}{2t} = 425 \text{ N/mm}^2 \rightarrow t = \frac{pd(FS)}{2\sigma_C}$$

$$\therefore t = \frac{0.6 \times 3000}{2 \times 425} \times 1.75 = \boxed{3.71 \text{mm}}$$

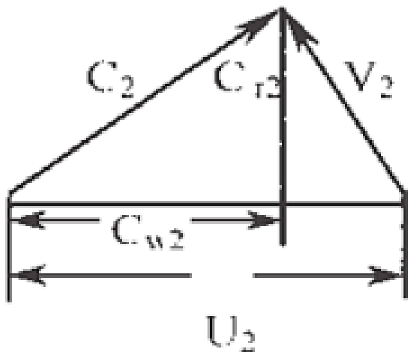
12 ➡ C

This is a straightforward application of the equation

$$V_d = \sqrt{\frac{\pi^2 GJ}{2\rho e c^2 s^2 \left(\partial c_1 / \partial \alpha\right)}} = \sqrt{\frac{\pi^2 \times \left(12 \times 10^6\right)}{2 \times 1.225 \times 1.2 \times 2.8^2 \times 8.0^2 \times 3.1}}$$

$$\therefore \boxed{V_d = 161 \text{ m/s}}$$

13 ➡ C



From the velocity triangle, we may write

$$\tan(\beta_2) = \frac{U_2 - C_{w,2}}{C_{r,2}} \rightarrow \tan(32^\circ) = \frac{450 - C_{w,2}}{100}$$

$$\therefore 100 \times \tan(32^\circ) = 450 - C_{w,2}$$

$$\therefore 62.5 = 450 - C_{w,2}$$

$$\therefore C_{w,2} = 388 \text{ m/s}$$

so that σ = 388/450 = 0.862. The theoretical power required to drive the compressor is then

$$\Pi = \sigma \dot{m} U_2^2 = 0.862 \times 3.5 \times 450^2 = 611{,}000 \text{ W}$$

$$\therefore \Pi = 611 \text{ kW}$$

Dividing this by the efficiency of the compressor gives the actual power required Π_a:

$$\Pi_a = \frac{\Pi}{\eta} = \frac{611}{0.8} = \boxed{764 \text{ kW}}$$

14 ➡ B

We first compute the Reynolds number based on the forebody length, namely

$$\text{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{0.018 \times 3000 \times 32}{1.5 \times 10^{-5}} = 1.15 \times 10^8$$

Thus, flow is turbulent and the skin friction coefficient can be estimated as

$$C_f = \frac{0.074}{\text{Re}_L^{0.2}} = \frac{0.074}{\left(1.15 \times 10^8\right)^{0.2}} = 0.00181$$

Using the analogy between momentum and convective heat transfer, the Stanton number *St* is

$$St \approx \frac{C_f}{2} = \frac{0.00181}{2} = 9.05 \times 10^{-4}$$

Finally, the heat flux at the wall is determined as

$$\dot{Q} = St \times \frac{1}{2} \rho_\infty V_\infty^3 = \left(9.05 \times 10^{-4}\right) \times 0.5 \times 0.018 \times 3000^3 = 220{,}000 \text{ W/m}^2$$

$$\therefore \boxed{\dot{Q} = 220 \text{ kW/m}^2}$$

15 ➡ B

The aircraft can trim at a positive α, but it is unstable.

16 ➡ C

When the wedge half-angle is greater than the maximum turning angle of the wave, an additional increase in upstream Mach number will cause the shock to detach from the nose of the wedge, forming a detached oblique shock or bow wave.

17 ➡ C

The Prandtl-Meyer function for the expanded flow is given by θ = 3° plus ν(2.0) = 26.38°, that is,

$$\nu(M_2) = \theta + \nu(M_1) = 3^\circ + 26.38^\circ = 29.38^\circ$$

With reference to the table that accompanies the problem, a Mach number with Prandtl-Meyer function equal to 29.38 should be between 2.1 (for which ν = 29.10) and 2.2 (for which ν = 31.73). Thus, the only viable choice is M_2 = 2.11.

18 ➡ A

Liquid fluorine is a high-energy fuel that can afford specific impulse values greater than most liquid propellants known to man. However, it is also highly toxic, reactive, and corrosive, which has led the aerospace community to abandon it altogether.

19 ➡ C

This is a straightforward application of the jet velocity formula

$$V_j = \sqrt{\frac{2\gamma R_0 T_e}{(\gamma - 1) M} \left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$V_j = \sqrt{\frac{2 \times 1.4 \times 8314 \times 300}{(1.4 - 1) \times 2} \times \left[1 - \left(\frac{0.1}{2}\right)^{\frac{1.4-1}{1.4}}\right]} = \boxed{2240.65 \text{ m/s}}$$

20 ➡ C

The pertaining masses are calculated below.

$$m_{0,1} = 150,000 + 12,000 + 32,000 + 3000 + 3000 = 200,000\,\text{kg}$$
$$m_{f,1} = 12,000 + 32,000 + 3000 + 3000 = 50,000\,\text{kg}$$
$$m_{0,2} = 32,000 + 3000 + 3000 = 38,000\,\text{kg}$$
$$m_{f,2} = 3000 + 3000 = 6000\,\text{kg}$$

The variation in velocity afforded by the first stage is

$$\Delta V_1 = gI_{sp,1}\ln\left(m_{0,1}/m_{f,1}\right) = 9.81\times 250\times\ln\left(200,000/50,000\right)$$
$$\therefore \Delta V_1 = 3400\,\text{m/s}$$

while for the second stage,

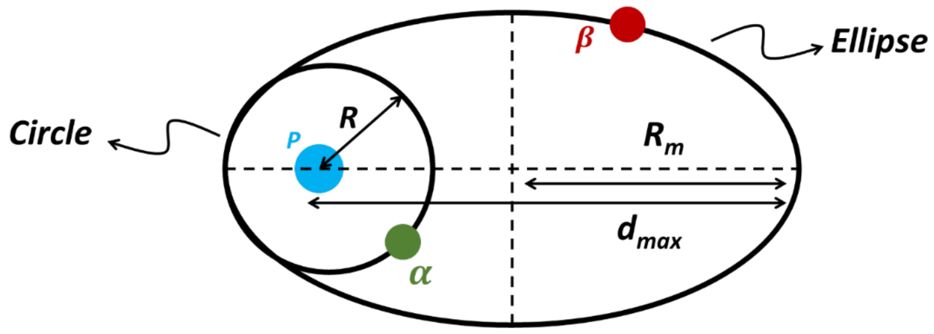
$$\Delta V_2 = gI_{sp,2}\ln\left(m_{0,2}/m_{f,2}\right) = 9.81\times 340\times\ln\left(38,000/6000\right)$$
$$\therefore \Delta V_2 = 6160\,\text{m/s}$$

The total ΔV is

$$\Delta V = \Delta V_1 + \Delta V_2 = \boxed{9560\text{ m/s}}$$

21 ➡ C

Referring to the following figure, we have $d_{\max} = 2R_m - R$.



From Kepler's third law,

$$\frac{T_\alpha^2}{R_\alpha^3} = \frac{T_\beta^2}{R_\beta^3} \rightarrow \frac{T^2}{R^3} = \frac{(8T)^2}{R_m^2}$$
$$\therefore \frac{\cancel{T}^2}{R^3} = \frac{64\cancel{T^2}}{R_m^3}$$
$$\therefore R_m = 4R$$

The corresponding apogee is then

$$d_{\max} = 2R_m - R = 2\times 4R - R = \boxed{7R}$$

22 ➡ 0.0520

The equation that describes y can be expanded to yield a cubic polynomial in ξ:

$$y = \frac{y_c}{kc} = \xi(\xi-1)(\xi-2) = \xi^3 - 3\xi^2 + 2\xi$$

Differentiating,

$$\frac{dy}{d\xi} = 3\xi^2 - 6\xi + 2$$

Setting this to zero and solving for ξ gives ξ = 1.58, which is preposterous, and ξ = 0.423, which is a viable solution. Inserting this newly found root in the equation for y gives the maximum

$$y_c = kc\xi(\xi-1)(\xi-2) = kc\times 0.423\times(0.423-1)\times(0.423-2)$$
$$\therefore y_{c,\max} = 0.385kc$$

It follows that, if the maximum camber is 2% of the chord,

$$\frac{y_{c,\max}}{c} = 0.385k = 0.02 \rightarrow \boxed{k = 0.0520}$$

23 ➡ 1.25

This is a straightforward application of the P-G rule:

$$c_\ell = \frac{c_{\ell,0}}{\sqrt{1-M_\infty^2}} = \frac{0.75}{\sqrt{1-0.8^2}} = \boxed{1.25}$$

24 ➡ 1.30 m/s

For an altitude of 2000 m, we have σ = 1.0/1.225 = 0.816. Substituting the pertaining data into the gliding rate of descent formula,

$$-\dot{h} = \frac{\rho_{SSL}\sigma C_{D,0}V^3}{2(W/S)} + \frac{2K(W/S)}{\rho_{SSL}\sigma V}$$

$$\therefore -\dot{h} = \frac{1.225\times 0.816\times 0.015\times 30^3}{2\times(3920/15)} + \frac{2\times 0.03\times(3920/15)}{1.225\times 0.816\times 30} = \boxed{1.30 \text{ m/s}}$$

25 ➡ 198.7 J/K

This is a straightforward application of the formula

$$\Delta s = R\ln(p_2/p_1) = 286.7\times\ln(2) = \boxed{198.7 \text{ J/K}}$$

26 ➡ 50%

The efficiency of the ramjet is given by the usual ratio

$$\eta_o = \frac{F_n V_0}{\dot{m}_f \Delta Q} = \frac{10{,}000\times 1000}{\dot{m}_f \times (50\times 10^6)} = \frac{0.2}{\dot{m}_f} \quad \text{(I)}$$

To find the mass flow of fuel, we write, using the given fuel-to-air ratio and thermophysical properties for flight at 33.5 km,

$$\dot{m}_f = \frac{\dot{m}_f}{\dot{m}_o}\dot{m}_o = 0.04\times\rho_0 A_0 V_0 = 0.04\times 0.01\times 1.0\times 1000 = 0.4\,\text{kg/s}$$

Substituting in (I) yields

$$\eta_o = \frac{0.2}{0.4} = 0.5 = \boxed{50\%}$$

27 ➡ 196 kN

We first compute the total mass flow of propellant:

$$\dot{m} = \dot{m}_o + \dot{m}_f = \dot{m}_f\left(1+\frac{\dot{m}_o}{\dot{m}_f}\right) = 10\times(1+4) = 50\,\text{kg/s}$$

Then, noting that $F_n = \dot{m}gI_{sp}$, we obtain

$$F_n = \dot{m}gI_{sp} = 50\times 9.81\times 400 = 196{,}000\,\text{N}$$

$$\therefore \boxed{F_n = 196\,\text{kN}}$$

28 ➡ 7.8 km/s

The velocity we aim for is

$$v = \sqrt{\frac{\mu}{R_E+z}} = \sqrt{\frac{3.986\times 10^{14}}{6400\times 10^3 + 150\times 10^3}} = 7800\,\text{m/s}$$

$$\therefore \boxed{v = 7.80 \text{ km/s}}$$

29 ➡ B

Since the burning rate may be described by the power law

$$\dot{t} = aP_c^n$$

we may divide $\dot{t}_1 = aP_{c,1}^n$ (where subscript 1 denotes test 1) by $t_2 = aP_{c,2}^n$ (test 2) to obtain

$$\frac{\dot{t}_1}{\dot{t}_2} = \left(\frac{P_{c,1}}{P_{c,2}}\right)^n$$

$$\therefore \ln\left(\frac{\dot{t}_1}{\dot{t}_2}\right) = n \times \ln\left(\frac{P_{c,1}}{P_{c,2}}\right)$$

$$\therefore n = \frac{\ln\left(P_{c,1}/P_{c,2}\right)}{\ln\left(t_1/t_2\right)} = \frac{\ln\left(17/8\right)}{\ln\left(23.4/13.1\right)} = 1.30$$

Also,

$$a = \frac{\dot{t}}{P_c^n} = \frac{13.1}{8^{1.30}} = 0.878$$

For a burning rate of 16 mm/s, the combustion chamber pressure is

$$\dot{t} = aP_c^n \rightarrow P_c = \left(\frac{\dot{t}}{a}\right)^{1/n}$$

$$\therefore P_c = \left(\frac{16}{0.878}\right)^{1/1.30} = \boxed{9.33\,\text{MPa}}$$

30 ➡ D

The mass flow rate of propellant is given by

$$\dot{m}_g = \rho_p A_b \dot{r}$$

Solving for $\dot{m}_g/A_b$ and substituting ρ_p = 1700 kg/m^3, $\dot{r}$ = 16 mm/s brings to

$$\dot{m}_g = \rho_p A_b \dot{r} \rightarrow \frac{\dot{m}_g}{A_b} = \rho_p \dot{r}$$

$$\therefore \frac{\dot{m}_g}{A_b} = 1800 \times 0.016 = \boxed{28.8 \frac{\text{kg/s}}{\text{m}^2}}$$

Note that the propellant consumption rate per area of burning surface is independent of the grain diameter.

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