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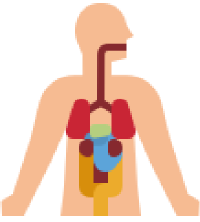

GATE Biomedical Engineering (BM):

◆30 Practice Questions

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Here's a set of 30 fully solved problems for applicants to the GATE Biomedical Engineering (BM) exam. Problems were taken from a carefully researched assortment of textbooks. All problems are solved step by step. Enjoy! ■

Problem Range	Subject
1 – 9	Physiology and Bioinstrumentation
10 – 17	Medical Imaging Systems
18 – 24	Biomechanics
25 – 30	*Numerical Answer Problems*

PROBLEMS

Problem 1. Which of the following statements related to biomedical measurements is true?

(A) In the EEG, theta waves have greater frequency than alpha, beta, or gamma waves.

(B) In the ECG, a normal P-wave is associated with ventricular repolarization.

(C) The ENG is used to examine involuntary eye movements.

(D) Impedance cardiography is used to measure electrical parameters of cardiac function but offers no insight into hemodynamic properties such as stroke volume.

Problem 2. An arterial blood gas test for a patient in respiratory care gave the following measurements:

pH	7.36
$PaCO_2$	40 mmHg
PaO_2	82 mmHg
SaO_2	96%
Hb	13 g/dL

What is the patient's total arterial O_2 content (i.e., accounting for both O_2 bound to Hb and dissolved in plasma)? Assume that 1 g of Hb carries 1.34 mL of oxygen; assume further that there is 0.003 mL of O_2 dissolved in plasma for every 1 mmHg of O_2 tension.

(A) 15.44

(B) 16.72

(C) 16.97

(D) 17.14

Problem 3. Using Krogh's model, calculate the oxygen concentration at a distance of 20 μm from a capillary of 4 μm radius if the diffusion coefficient for oxygen is 2.4×10^{-5} cm^2/s, the plasma oxygen concentration is 4.4×10^{-8} mol/cm^2, the oxygen reaction rate is 6×10^{-8} mol/cm$^3\cdot$sec, and the Krogh radius is 35 μm.

(A) 1.8×10^{-8} mol/cm^3

(B) 2.3×10^{-8} mol/cm^3

(C) 2.8×10^{-8} mol/cm^3

(D) 3.3×10^{-8} mol/cm^3

Problem 4. Hemodiafiltration (HDF) combines both diffusion and convection solute transport. It is used in some regions of the world, notably Europe. Which of the following clinical benefits of HDF is **false**?

(A) Phosphate removal in HDF is much higher than in high-efficiency and high-flux dialysis.

(B) Removal of inflammatory cytokines is better or higher with HDF than high-efficiency and high-flux dialysis.

(C) Preservation of residual renal function is much better with HDF than high-efficiency and high-flux dialysis.

(D) Improvement in albumin and other markers of nutrition is better with HDF than high-efficiency and high-flux dialysis.

Problem 5. In which phase of the cell cycle is a cell most sensitive to ionizing radiation?

(A) M

(B) G1

(C) S

(D) G2

Problem 6. A body is placed between the plates of a 20 MHz short-wave diathermy unit that behaves as a capacitor at a voltage of 2.4 kV rms. The capacitance between each plate and the body is 8 pF, and the part of the body between the plates can be represented as a cylinder of diameter 18 cm and length 30 cm; the resistivity of the tissue is 5 Ω ·m. What is the power deposited in the body?

(A) 45 W

(B) 86 W

(C) 122 W

(D) 140 W

Problem 7. In a patch-clamp experiment, the conductance of a single Ca^{2+} channel was measured to be 30 pS. The cell membrane thickness is 8 nm, and the difference between membrane potential and Nernst potential for calcium may be taken as 50 mV. Assuming that the resistivity of the fluid in the channel is 0.42 Ω ·m, determine the channel radius, a. Also, for a conductance per unit area of 1240 S/m^2, determine the No. of pores per unit area, N.

(A) $a = 0.09$ nm, $N = 20$ pores μm^{-2}

(B) $a = 0.09$ nm, $N = 41$ pores μm^{-2}

(C) $a = 0.18$ nm, $N = 20$ pores μm^{-2}

(D) $a = 0.18$ nm, $N = 41$ pores μm^{-2}

Problems 8 and 9.

8. Consider the flow of sodium ions through the voltage-gated sodium channels in a cell membrane during depolarization. The cell membrane has a surface area of 1 μm^2 and the equilibrium membrane potential for sodium ions is 62 mV. During depolarization the membrane potential shifts from a threshold value of −65 mV to a peak value 35 mV, so that an average potential of −15 mV may be taken for calculations. Determine the flow (absolute value) of sodium ions. Use 7×10^{-12} S/channel as the membrane conductance for Na^+ ions, 0.01 pF μm^{-2} as the membrane capacitance, assume that there are 75 channels per square micrometer of membrane surface area, and note that one coulomb (C) corresponds to 6.2×10^{18} univalent ions.

(A) 1.12×10^{8} Na s^{-1} μm^{-2}

(B) 2.51×10^{8} Na s^{-1} μm^{-2}

(C) 1.12×10^{9} Na s^{-1} μm^{-2}

(D) 2.51×10^{9} Na s^{-1} μm^{-2}

9. If the membrane has capacitance equal to 0.03 pF, determine the time required to change the membrane potential by 100 mV.

(A) 0.07 ms

(B) 0.19 ms

(C) 0.30 ms

(D) 0.60 ms

Problem 10. Most diagnostic X-rays use photon energies in the range of 20 to 100 keV. For carbon, which energy loss mechanisms are most important in this range?
(A) Photoelectric effect and Compton scattering.
(B) Photoelectric effect and pair production.
(C) Compton scattering and pair production.
(D) Compton scattering only.

Problem 11. A 48 keV X-ray photon strikes the phosphor of a CR plate, producing light photons with a wavelength of 400 nm. The energy conversion coefficient for this process is 20%. How many light photons are produced?
(A) 2790
(B) 3110
(C) 4050
(D) 5200

Problem 12. What is the Doppler frequency f_D when the incident ultrasound frequency is 3 MHz, the probe angle is 37°, the blood velocity is 0.41 m/s, and the speed of sound in the tissue under examination is 1610 m/s?
(A) 3.65 kHz
(B) 7.3 kHz
(C) 10.2 kHz
(D) 20.4 kHz

Problem 13. A certain MRI machine ($\gamma = 2.68\times10^8$) has a static magnetic field of 2.0 T. Spins are excited while applying a field gradient of 3 mT m^{-1}. If the slice is to be 5 mm thick, what is the Larmor frequency, ω_0, and the spread in frequencies, $\Delta\omega$, that is required?
(A) $\omega_0 = 1.13\times10^8$ Hz, $\Delta\omega = 1005$ Hz
(B) $\omega_0 = 1.13\times10^8$ Hz, $\Delta\omega = 2010$ Hz
(C) $\omega_0 = 5.36\times10^8$ Hz, $\Delta\omega = 1005$ Hz
(D) $\omega_0 = 5.36\times10^8$ Hz, $\Delta\omega = 2010$ Hz

Problem 14. In a cardiac SPECT study, the acquired planar projections are first reconstructed into which body plane?
(A) Transaxial
(B) Long axis
(C) Coronal
(D) Planar

Problem 15. Calculate the required storage capacity in MB required to store an uncompressed 12 second, 8-bit grayscale ultrasound video file of 512 × 512 pixel dimensions, given that the frame rate of the video is 30 frames per second.
(A) 75 MB
(B) 94 MB
(C) 110 MB
(D) 130 MB

Problem 16. An optical cavity under consideration for biomedical applications consists of two mirrors with 98% reflectivity. The amplifier length is 0.125 m and the mirror separation is 0.4 m. Assuming there are no losses in the cavity other than mirror transmission losses, the corresponding minimum gain required to reach lasing threshold is:
(A) 5.2%
(B) 10.4%
(C) 16.2%
(D) 21.0%

Problem 17. Molybdenum-99 (half-life = 66 hours) and technetium-99m (half-life = 6 hours) are in transient equilibrium in a Mo generator. If 850 MBq of Mo-99 is present in the generator, what would be the activity of Tc-99m after three days?
(A) 132 MBq
(B) 329 MBq
(C) 439 MBq
(D) 601 MBq

Problem 18. A man is running at 4 m/s while exposed to air at 15°C. Neglecting the contribution from clothing, the heat transfer coefficient for skin-air interface can be estimated as $h_c = 5.9v^{0.8}$, where v is velocity in m/s and h_c is in $\text{W/m}^2\cdot\text{K}$. The skin temperature is 35°C and the body surface area of the man may be taken as 1.8 m^2. What is the convective heat loss rate for the skin-air system?

(A) 115 W
(B) 209 W
(C) 418 W
(D) 644 W

Problem 19. The rupture strength of bone may be taken as 10^9 dyn/cm^2. From what minimum height can a 1-kg falling object cause fracture of a person's skull if the area of contact between the falling object and the skull is 0.6 cm^2 and the duration of impact is 1.8 milliseconds?

(A) 3 m
(B) 6 m
(C) 9 m
(D) 12 m

Problem 20. A knee replacement is described by the *S-N* curve $\sigma = A \times N^{-b}$, where A = 5000 kPa and b = 0.24. The replacement has been subjected to 15,000 loading cycles at 400 kPa. How many additional cycles, now at 600 kPa, can the knee replacement withstand before failing?

(A) 2050
(B) 4100
(C) 6150
(D) 8200

Problem 21. In the 19th century, Thomas Young posited that in the human circulation, the radius of a given daughter vessel is about 0.794 times the radius of the parent vessel. Assuming that Young's model is indeed valid for the human circulation, and knowing that the smallest capillary and the aorta have radii of 5×10^{-4} cm and 1.5 cm, respectively, what is the approximate number of generations in the human arterial tree?

(A) 31
(B) 35
(C) 37
(D) 39

Problem 22. A short person has heart rate of 96 bpm and stroke volume of 75 mL. The body surface area for this person may be calculated from the Haycock formula,

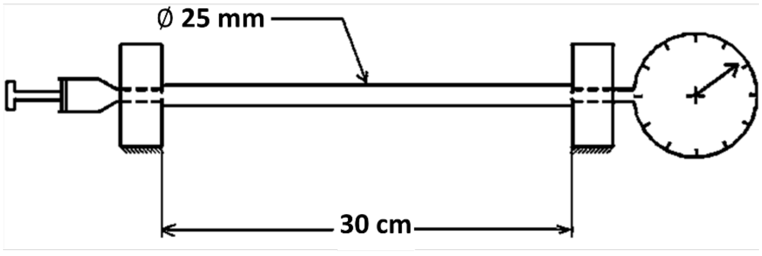
$$BSA = 0.024 \times H^{0.4} \times M^{0.54}$$

where BSA is body surface area in m^2, H is the person's height in cm, and M is the person's mass in kg. Knowing that the person is 1.65 m tall and weighs 70 kg, what is their cardiac index?

(A) 2.4 L/min/m^2
(B) 2.9 L/min/m^2
(C) 3.4 L/min/m^2
(D) 3.9 L/min/m^2

Problem 23. In the following illustration, an elastic tube with unknown modulus of elasticity *E* is anchored between two stationary ports and injected with a known volume of water. The wall thickness of the tube is 1 mm. For the following data, the modulus of elasticity is:

Initial pressure, P_o	12 kPa
Pressure increase, $P_f - P_o$	8 kPa
Initial tube diameter, D	25 mm
Initial length, L_o	30 cm
Volume injected, V	15 mL



(A) 490 kPa
(B) 980 kPa
(C) 1.96 MPa
(D) 3.92 MPa

Problem 24. Figures **1**, **2**, and **3** show the evolution of axial velocity profiles for three different pulsatile flows in a rigid tube. The thermomechanical conditions in all three profiles are similar, with the exception of flow frequency. Which of the following alternatives correctly ranks the three flows in order of **increasing** Womersley number?

Figure 1

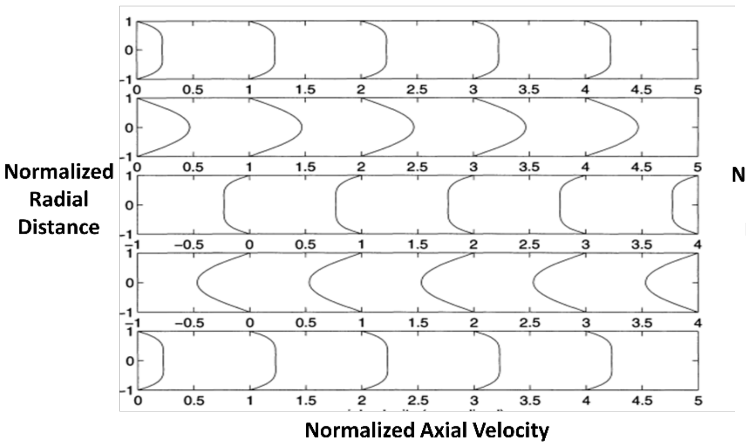


Figure 2

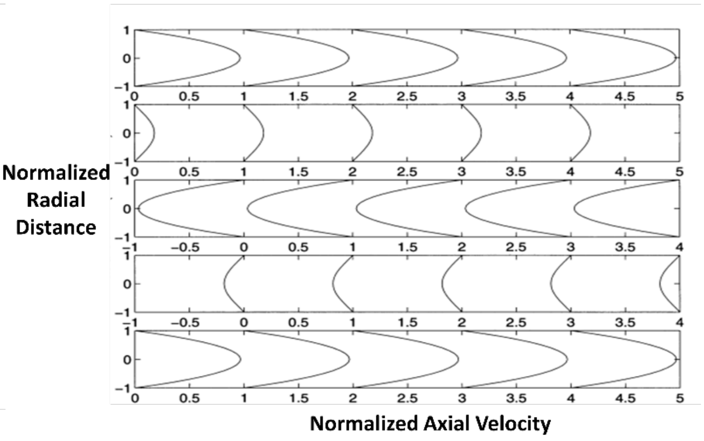
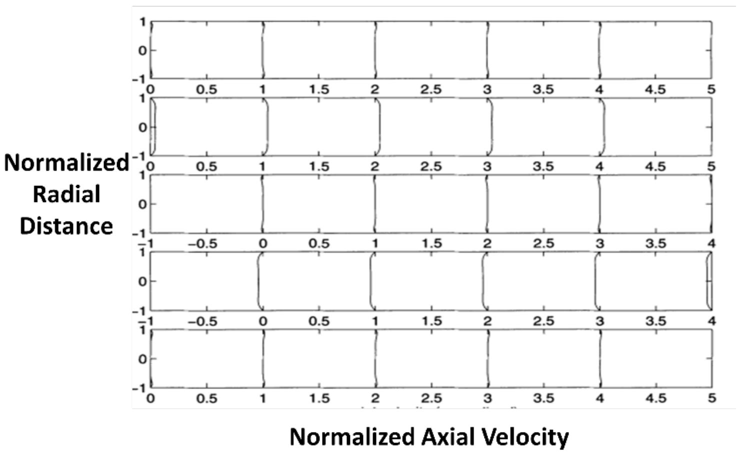


Figure 3



(A) *Wo*(3) < *Wo*(1) < *Wo*(2)
(B) *Wo*(2) < *Wo*(1) < *Wo*(3)
(C) *Wo*(1) < *Wo*(2) < *Wo*(3)
(D) *Wo*(2) < *Wo*(3) < *Wo*(1)

Problem 25. Consider an idealized bacterium swimming in water. The bacterium may be modelled as a sphere of radius 2 μm and density 1.04 g cm^{-3}. The bacterium swims at 1.3×10^{-5} m/s while immersed in water of viscosity $\eta = 10^{-3}$ Pa·s. The drag force experienced by this bacterium is ________ (◆ pN, *rounded to one decimal place*).

Problem 26. A patient has a cardiac output of 5 L/min. The systolic ejection period is 360 ms with a heart rate of 84 beats per minute. The mean aortic gradient as measured by echocardiography is 80 mmHg. Using the Gorlin equation, the aortic valve area is ______ (◆ cm^2, *rounded to two decimal places*).

Problem 27. The Beer-Lambert law was once used for measurement of light absorbance in pulse oximetry. In a certain photochemical experiment, a sample was known to have absorbance of 0.78. The molar absorptivity of the sample is 7500 $M^{-1}cm^{-1}$ and the width of the cuvette used in the experiment is 2 cm. The concentration of the sample is _______ (◆ μmol/L, *no decimal places*).

Problem 28. The intensity of a 4 MHz ultrasound beam entering tissue is 12 mW cm^{-2}. If the attenuation factor is 1 dB cm^{-1} MHz^{-1}, the intensity of the beam at a depth of 3 cm can be calculated to be _______ (◆ mW cm^{-2}, *rounded to one decimal place)*.

Problem 29. Radionuclide sample A is constituted of 300 MBq of iodine-123 (half-life = 13.2 h). Radionuclide sample B is made up of 1500 MBq of technetium-99m (half-life = 6 h). The time required for the two samples to reach the same activity is _______ (◆ hours, *rounded to one decimal place*).

Problem 30. Assume that the vertebral column of a tall man can be idealized as a slender column with pinned ends. The height of the spine is 50 cm, the moment of inertia about the buckling axis is 5×10^4 mm^4, and the elastic modulus for bone under compression is 10 GPa. Thus, the critical compressive load the spine can withstand before buckling ensues is _________ (◆ kN, *rounded to two decimal places*).

ANSWER KEY

Problem	Answer	Problem	Answer
1	C	16	C
2	C	17	C
3	C	18	D
4	D	19	B
5	A	20	B
6	B	21	B
7	D	22	D
8	B	23	C
9	A	24	B
10	A	25	0.5
11	B	26	0.42
12	B	27	52
13	D	28	1.3
14	A	29	25.4
15	B	30	19.74

SOLUTIONS

1 ➡ C

In the EEG, theta waves are actually associated with lower frequencies than alpha, beta, or gamma waves. In the ECG, a normal P-wave is related to ventricular depolarization. Impedance cardiography may be employed in indirect measurements of stroke volume. The electronystagmogram, or ENG, is used to examine involuntary eye movements.

2 ➡ C

One gram of hemoglobin is capable of carrying 1.34 mL of O_2. To find the volume of O_2 carried by Hb, we write

$$O_2 \text{ bound to Hb} = 1.34\times \text{Hb}\times \text{SaO}_2 = 1.34\times 13\times 0.96 = 16.72 \text{ mL/dL}$$

In turn, the volume of O_2 dissolved in plasma is

$$\begin{aligned}\text{O}_2\text{ dissolved in plasma} &= 0.003\times\text{PaO}_2 = 0.003\times 82\\ &= 0.246\text{ mL/dL}\end{aligned}$$

The total arterial O_2 content is then 16.72 + 0.246 = 16.97 mL/dL.

3 ➡ C

In the Krogh model, the concentration of a certain molecule a radial distance *r* away from the center of a capillary is given by

$$C(r) = C_p + \frac{r_K^2 R}{4D}\left[\frac{r^2 - r_C^2}{r_K^2} + 2\ln\left(\frac{r_C}{r}\right)\right]$$

where $C_p = 4.4\times10^{-8}$ mol/cm² is the concentration of oxygen in plasma, $r_K = 35\times10^{-6}$ m is the Krogh radius, $R = 6\times10^{-8}$ mol/cm³·sec is the oxygen reaction rate, $r_C = 4.4\times10^{-6}$ m is the radius of the capillary, and $D = 2.4\times10^{-5}$ cm²/s = 2.4×10^{-9} m²/s is the diffusion coefficient for oxygen, giving

$$C(20\ \mu\text{m}) = 4.4\times10^{-8} + \frac{\left(35\times10^{-6}\right)^2\times\left(6\times10^{-8}\right)}{4\times\left(2.4\times10^{-9}\right)}\left[\frac{\left(20\times10^{-6}\right)^2 - \left(4\times10^{-6}\right)^2}{\left(35\times10^{-6}\right)^2} + 2\ln\left(\frac{4}{20}\right)\right]$$

$$\therefore \boxed{C(20) = 2.80\times10^{-8}\ \text{mol/cm}^3}$$

4 ➡ D

Removal of phosphate mass with HDF is 15 – 20% higher than other HD modalities. It has been shown that cytokine removal is much higher with HDF, and preservation of residual renal function is prolonged with HDF. However, most studies have not found any significant benefit of HDF in improving nutritional markers, as measured by either albumin or prealbumin concentrations.

5 ➡ A

Cells are most vulnerable to ionizing radiation in the M phase of the cell cycle.

6 ➡ B

Firstly, the resistance of the body segment is

$$R = \frac{\rho\ell}{A} = \frac{(5\ \Omega\cdot\text{m})\times(0.30\ \text{m})}{\left(\pi\times0.09^2\ \text{m}^2\right)} = 59.0\ \Omega$$

A capacitive path of 2 × 8 pF is equivalent to 4 pF, which at 20 MHz presents an impedance of

$$Z = \frac{1}{2\pi fC} = \frac{1}{2\pi\times\left(20\times10^6\right)\times\left(4.0\times10^{-12}\right)} = 1990\ \Omega$$

The current that flows will therefore be dominated by the capacitance, so that

$$I = C\frac{dV}{dt} = C\times2\pi fE = \left(4.0\times10^{-12}\right)\times2\pi\times\left(20\times10^6\right)\times2400 = 1.21\ \text{A}$$

The power Π deposited in the tissue is

$$\Pi = (59.0\ \Omega)\times(1.21\ \text{A})^2 = \boxed{86.4\ \text{W}}$$

7 ➡ D

The conductance *G* is given by

$$G = \frac{\pi a^2}{\rho b}$$

where *a* is channel radius, *ρ* is resistivity, and *b* is membrane thickness. Solving for *a*, we obtain

$$G = \frac{\pi a^2}{\rho b} \rightarrow a = \sqrt{\frac{G\rho b}{\pi}}$$

$$\therefore a=\sqrt{\frac{\left(30\times10^{-12}\right)\times0.42\times\left(8\times10^{-9}\right)}{\pi}}=1.79\times10^{-10}\text{ m}$$

$$\therefore \boxed{a=0.179\text{ nm}}$$

To find the No. of pores per unit area, we write

$$g=NG \;\rightarrow\; N=\frac{g}{G}$$

$$\therefore N=\frac{1240}{30\times10^{-12}}=4.13\times10^{13}\text{ pores/m}^2$$

$$\therefore \boxed{N=41.3\text{ pores/μm}^2}$$

8 ➡ B

The equation to use is

$$i_{\text{Na}}=g_{\text{Na}}\left(V_M-V_{\text{Na}}\right)$$

so that

$$i_{\text{Na}}=7\times10^{-12}\frac{\text{S}}{\text{channel}}\times1\frac{\text{A V}^{-1}}{\text{S}}\times1\frac{\text{C s}^{-1}\text{V}^{-1}}{\text{A}}\times6.2\times10^{18}\frac{\text{ions}}{\text{C}}\times\frac{1\text{ V}}{1000\text{ mV}}$$
$$\times\frac{1\text{ Na}}{\text{ions}}\times75\,\frac{\text{channels}}{\text{μm}^2}\times\left(-15-62\right)\text{ mV}=\boxed{-2.51\times10^{8}\text{ Na s}^{-1}\text{ μm}^{-2}}$$

9 ➡ A

In the previous part, we established that the current of sodium ions is −2.51×10^{8} Na s^{-1} μm^{-2}. Noting that one coulomb corresponds to 6.2×10^{18} univalent cations, we obtain a current of −40.5 pA μm^{-2}. We can relate this change in charge and membrane potential to the membrane capacitance using

$$C_{\text{memb}}=\frac{i_{\text{Na}}t}{\Delta V_M}$$

so that, solving for time,

$$C_{\text{memb}}=\frac{\left|i_{\text{Na}}\right|t}{\Delta V_M}\;\rightarrow\; t=\frac{C_{\text{memb}}\times\Delta V_M}{\left|i_{\text{Na}}\right|}$$

$$\therefore t=\frac{\left(0.03\times10^{-12}\right)\times0.1}{40.5\times10^{-12}}=\boxed{0.0741\text{ ms}}$$

The membrane potential will change the specified amount within less than 0.08 milliseconds.

10 ➡ A

In the 20 – 100 keV range, photoelectric effect dominates at the lower end and Compton scattering dominates at the upper end. Pair production is altogether improbable.

11 ➡ B

We first convert the energy content of the X-ray photon from keV to joules:

$$\varepsilon_X=\left(48\times10^{3}\right)\times\left(1.6\times10^{-19}\right)=7.68\times10^{-15}\text{ J}$$

Since only 20% of the X-ray photons is converted to light,

$$\overline{\varepsilon}_X=0.20\varepsilon_X=1.54\times10^{-15}\text{ J}$$

Now, the energy associated with a 415-nm light photon is

$$\varepsilon_{\text{light}}=\frac{hc}{\lambda}=\frac{\left(6.6\times10^{-34}\right)\times\left(3\times10^{8}\right)}{400\times10^{-9}}=4.95\times10^{-19}\text{ J}$$

Thus, the number of light photons produced, *N*, is

$$N=\frac{1.54\times10^{-15}\text{ J}}{4.95\times10^{-19}\text{ J/photon}}=\boxed{3110\text{ photons}}$$

12 ➡ B

This is a straightforward application of the formula

$$f_D = \frac{2f_i v\cos\theta}{c} = \frac{2\times 3\times 0.41\times\cos 37^{\circ}}{1610} = 0.00732\text{ MHz}$$

$$\therefore \boxed{f_D = 7.32\text{ kHz}}$$

Note that this is within the audible range for humans (20 < *f* < 20,000 Hz), which explains why Doppler ultrasound is often accompanied by an easily recognizable pulsating sound.

13 ➡ D

The Larmor frequency is

$$\omega_0 = \gamma B = \left(2.68\times 10^8\right)\times 2.0 = \boxed{5.36\times 10^8\text{ Hz}}$$

The required spread in frequencies is, in turn,

$$\Delta\omega = \gamma\Delta B = \frac{\gamma G_z\Delta z}{2} = \frac{\left(2.68\times 10^8\right)\times\left(3\times 10^{-3}\right)\times 0.005}{2} = \boxed{2010\text{ Hz}}$$

14 ➡ A

In a SPECT study, 32 to 64 planar projections are acquired 180 degrees around the patient perpendicular to the long axis of the body, and the initial reconstruction is into the transaxial plane. From these triaxial images, the long axis of the heart is defined, and subsequently, the conventional vertical and horizontal long-axis and the short-axis images are reconstructed for analysis.

15 ➡ B

The storage capacity for 1 frame is (512 × 512) pixels × 8 bit/pixel = 2,097,152 bits. The required storage capacity then becomes

$$\text{Storage capacity} = 2{,}097{,}152\ \frac{\text{bit}}{\text{frame}}\times 30\text{ frame/sec}\times 12\text{ sec}$$

$$\therefore \text{Storage capacity} = 7.55\times 10^8\text{ bit} = 9.44\times 10^7\text{ byte}$$

$$\therefore \boxed{\text{Storage capacity} \approx 94.4\text{ MB}}$$

16 ➡ C

With *L* = 0.125 m and *R* = 0.98, the threshold gain becomes.

$$g_{\text{th}} = \frac{1}{2L}\ln\left(\frac{1}{R^2}\right) = \frac{1}{2\times 0.125}\times\ln\left(\frac{1}{0.98^2}\right) = 0.162$$

$$\therefore \boxed{g_{\text{th}} = 16.2\%}$$

17 ➡ C

For two radionuclides in transient equilibrium, the activity A_d of the daughter nuclide at a given instant in time is related to the activity A_p of the parent nuclide by

$$\left(A_D\right)_t = \frac{\left(t_{1/2}\right)_P}{\left(t_{1/2}\right)_P - \left(t_{1/2}\right)_D}\left(A_P\right)_t \quad \text{(I)}$$

where

$$\left(A_P\right)_t = \left(A_P\right)_0\exp\left(-\lambda_P t\right) = 850\times\exp\left[-\frac{0.693}{66}\times\left(3\times 24\right)\right] = 399\text{ MBq}$$

so that, substituting in (I),

$$\left(A_D\right)_t = \frac{66}{66-6}\times 399 = \boxed{439\text{ MBq}}$$

18 ➡ D

The convective heat transfer coefficient is $h_c = 5.9\times 4^{0.8} = 17.9\text{ W/m}^2\cdot\text{s}$. Substituting into Newton's law of cooling, we obtain

$$\dot{q} = h_c A\Delta T = 17.9\times 1.8\times\left(35-15\right) = \boxed{644\text{ W}}$$

19 ➡ B

The rupture strength of bone is 100×10^7 dyn/cm^2. Multiplying this by the contact area (= 0.6 cm^2) gives 0.6×10^9 dyn, or 6000 N. From the impulse theorem, the impulsive force *F* is given by

$$F = \frac{mV}{\Delta t}$$

But $V = \sqrt{2gh}$, so that

$$F = \frac{m\sqrt{2gh}}{\Delta t} \quad \to \quad h = \frac{1}{2g}\frac{F^2 \Delta t^2}{m^2}$$

$$\therefore h = \frac{1}{2 \times 9.81} \times \frac{6000^2 \times \left(1.8 \times 10^{-3}\right)^2}{1.0^2} = 5.94 \text{ m} \approx \boxed{6.0 \text{ m}}$$

20 ➡ B

Per the Palmgren-Miner rule, we have

$$\Sigma\left(\frac{n}{N}\right) = 1$$

That is,

$$\frac{n_{400}}{N_{400}} + \frac{n_{600}}{N_{600}} = 1 \quad \text{(I)}$$

We compute the maximum number of cycles at a stress level of 400 kPa:

$$\sigma = A \times N^{-b} \quad \to \quad N_{400} = \left(400/A\right)^{-1/b}$$

$$\therefore N_{400} = \left(400/5000\right)^{-1/0.24} = 37{,}193 \text{ cycles}$$

Similarly, the maximum number of cycles at a stress level of 600 kPa is

$$N_{600} = \left(600/5000\right)^{-1/0.24} = 6867 \text{ cycles}$$

Substituting in (I), we obtain

$$\frac{15{,}000}{37{,}193} + \frac{n_{600}}{6867} = 1$$

$$\therefore 0.403 + \frac{n_{600}}{6867} = 1$$

$$\therefore n_{600} = 6867 \times \left(1 - 0.403\right) = \boxed{4100 \text{ cycles}}$$

21 ➡ B

According to Young's hypothesis, the radius of a vessel in the *n*-th generation is given by

$$R_n = \left(0.794\right)^n R_0$$

where R_0 is the radius of the largest vessel, which, in the case of mammals, is the aorta. Substituting R_n = 5×10^{-4} cm, R_0 = 1.5 cm, and solving for the number of generations *n*, we get

$$R_n = \left(0.794\right)^n R_0 \quad \to \quad 5 \times 10^{-4} = \left(0.794\right)^n \times 1.5$$

$$\therefore \ln\left(5 \times 10^{-4}\right) = n \times \ln\left(0.794\right) + \ln\left(1.5\right)$$

$$\therefore -7.60 = -0.231n + 0.405$$

$$\therefore n = 34.7 \approx \boxed{35}$$

22 ➡ D

We first determine the body surface area:

$$BSA = 0.024 \times 165^{0.4} \times 70^{0.54} = 1.835 \text{ m}^2$$

Now, the cardiac output is given by the product of heart rate and stroke volume:

$$CO \equiv HR \times SV = 96 \times 0.075 = 7.2 \text{ L/min}$$

Dividing cardiac output by body surface area gives the cardiac index:

$$CI = \frac{CO}{BSA} = \frac{7.2}{1.835} = \boxed{3.92\ \text{L}/\left(\text{min}\cdot\text{m}^2\right)}$$

A normal cardiac index ranges from about 2.5 L/(min·m^2) to 4.0 L/(min·m^2); this person is just below the upper bound for normal conditions.

23 ➡ C

The change in cross-sectional area resulting from an injection of 15 mL into the tube is the change in volume divided by the fixed tube length:

$$\Delta A = \frac{\Delta V}{L_o} = \frac{15}{30} = 0.5\ \text{cm}^2$$

The compliance of the tube can be calculated as the change in area divided by the change in pressure,

$$C = \frac{\Delta A}{\Delta P} = \frac{0.5\times10^{-4}}{8000} = 6.25\times10^{-9}\ \text{m}^2/\text{Pa}$$

The incremental modulus of elasticity follows as

$$E = \frac{2\pi r_o^3}{Ch_o} = \frac{2\pi\times0.0125^3}{\left(6.25\times10^{-9}\right)\times0.001} = 1.96\times10^6\ \text{Pa} = \boxed{1.96\ \text{MPa}}$$

24 ➡ B

The Womersley number is essentially a dimensionless frequency; the greater its value, the quicker the oscillations will be, and, as a result, the less time the flow will have to adjust itself to these oscillations. Notice that flow 2 manages to develop itself to a reasonable degree before the pulsations change direction, which indicates that this is a low-Womersley number flow. Flow 1, in turn, cannot quite reach a fully-developed state before the pulsations change direction, which indicates that this is an intermediate-Womersley number flow. The most extreme situation applies to flow 3, which is subjected to such breakneck oscillations that even the most incipient motion cannot be established; thus, figure 3 corresponds to a high-Womersley number flow.

25 ➡ 0.4 – 0.6

Stokes' law gives the drag force imparted on objects moving at creeping speed:

$$F = 6\pi\eta Rv = 6\pi\times10^{-3}\times\left(2\times10^{-6}\right)\times\left(1.3\times10^{-5}\right) = 4.90\times10^{-13}\ \text{N}$$

$$\therefore \boxed{F = 0.5\ \text{pN}}$$

where we have used 1 pN = 10^{-12} N.

26 ➡ 0.32 – 0.52

The Gorlin equation can be used to compute the aortic valve area:

$$A_V = \frac{CO}{T_E\times HR\times44.3\sqrt{\Delta P}}$$

Here, *CO* is cardiac output in mL/min, T_E is the time of ejection per beat in seconds, *HR* is the heart rate in beats/min, and Δ*P* is the mean pressure gradient in mmHg. Note that the factor 44.3 in the denominator applies to aortic valves; in the case of mitral valves, we'd replace it with 37.7. Substituting the given data, we have

$$A_V = \frac{CO}{T_E\times HR\times44.3\sqrt{\Delta P}} = \frac{5}{0.360\times84\times44.3\times\sqrt{80}} = \boxed{0.42\ \text{cm}^2}$$

27 ➡ 52

According to the Beer-Lambert law, absorbance *A* is given by the product of molar absorptivity *ε*, cuvette width *b*, and concentration *c*. Solving for *c* brings to

$$A = \varepsilon bc \ \rightarrow\ 0.70 = 7500\times2.0\times c$$

$$\therefore c = 5.2\times10^{-5}\ \text{mol/L}$$

$$\therefore \boxed{c = 52\ \mu\text{mol/L}}$$

28 ➡ 1.2 – 1.4

The attenuation coefficient is 1 dB cm^{-1} Hz^{-1}, and so has a value of 4 dB cm^{-1} for an ultrasound wave of 4 MHz. At a depth of 3 cm, the attenuation is 12 dB, so the intensity of the beam after penetrating to this depth can be calculated to be $10^{-1.2} \times 20 = 1.26$ mW cm^{-2}, as shown below.

$$\text{Attenuation} = 10\log_{10}\left(\frac{I_1}{I_0}\right) [\text{dB}]$$

$$\therefore 10\log_{10}\left(\frac{I_1}{I_0}\right) = -12\ \text{dB}$$

$$\therefore \log_{10}\left(\frac{I_1}{I_0}\right) = -1.2\ \text{dB}$$

$$\therefore \frac{I_1}{I_0} = 10^{-1.2}$$

$$\therefore I_1 = 10^{-1.2} \times I_0$$

$$\therefore I_1 = 10^{-1.2} \times 20 = 1.26 \approx \boxed{1.3\ \text{mW cm}^{-2}}$$

29 ➡ 25.0 – 26.0

The decay constant for iodine-123 is $\lambda_A = \ln 2/13.2 = 0.0525\ \text{h}^{-1}$. The decay constant for technetium-99m is $\ln 2/6 = 0.116\ \text{h}^{-1}$. Equating the radioactive decay functions for the two samples and solving for time, we have

$$A_A e^{-\lambda_A t} = A_B e^{-\lambda_B t}$$

$$\therefore 300e^{-0.0525t} = 1500e^{-0.116t}$$

$$\therefore \frac{e^{-0.0525t}}{e^{-0.116t}} = \frac{1500}{300}$$

$$\therefore e^{-0.0525t+0.116t} = 5$$

$$\therefore e^{0.0635t} = 5$$

$$\therefore 0.0635t = \ln 5$$

$$\therefore t = \frac{\ln 5}{0.0635} = \boxed{25.4\ \text{h}}$$

The two samples should reach the same activity within just over one day and one hour.

30 ➡ 19.50 – 20.00

For a column with pinned ends, the Euler (critical) load is given by

$$P_{\text{cr}} = \frac{\pi^2 EI}{L^2}$$

where $E = 10$ GPa = 10,000 N/mm^2 is the modulus of elasticity, $I = 5\times10^4$ mm^4 is the moment of inertia about the buckling axis, and $L = 50$ cm = 500 mm is the height of the column, giving

$$P_{\text{cr}} = \frac{\pi^2 \times 10{,}000 \times \left(5\times10^4\right)}{500^2} = 19{,}740\ \text{N} = \boxed{19.74\ \text{kN}}$$

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