Quiz VB102
Harmonic Excitation
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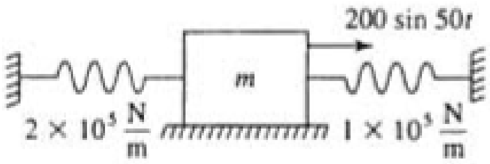
PROBLEMS

Problem 1 (Kelly, 1996)

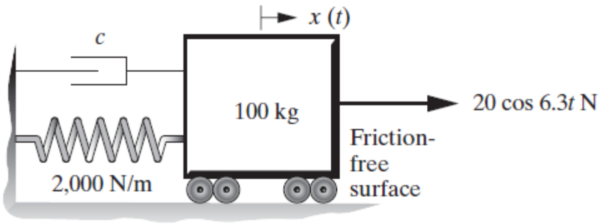
The system shown below is excited by a force that follows the harmonic relationship $F(t) = 200\ \sin 50t$ N. The spring to the left has a stiffness of k = 2×10⁵ N/m, while the one to the right has k = 1×10^5 N/m. For what value of m will resonance occur in this system?



A) m = 80 kg
B) m = 100 kg
C) m = 120 kg
D) m = 140 kg

Problem 2 (Inman, 2014, w/ permission)

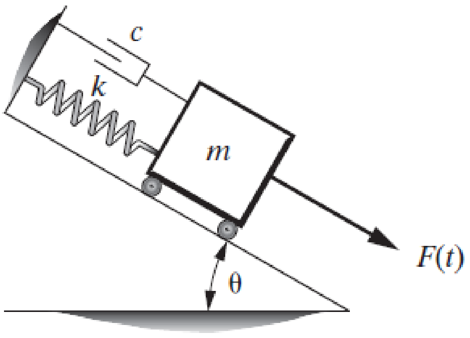
Compute the value of the damping coefficient c such that the steady-state amplitude of the system shown is 0.01 m.



A) c = 53.9 kg/s
B) c = 65.6 kg/s
C) c = 74.8 kg/s
D) c = 81.5 kg/s

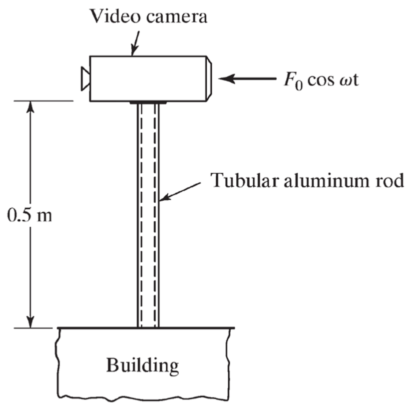
Problem 3 (Inman, 2014, w/ permission)

Write the equation of motion for the system given below for the case that $F(t) = F \cos \omega t$ and the surface is friction free. Does the angle θ affect the magnitude of oscillation?



Problem 4 (Rao, 2011, w/ permission)

A video camera of mass 2.0 kg is mounted on the top of a bank building for surveillance. The video camera is fixed at one end of a tubular aluminum rod whose other end is fixed to the building as shown. The wind-induced force acting on the video camera, *f(t)*, is found to be harmonic with *f(t)* = 25 cos 75.4*t* N. Determine the diameter of the aluminum ($E = 71\times10^9$ N/m²) tube if the maximum applicable amplitude of the video camera is to be limited to 0.005 m.



A) $D = 1.3$ cm
B) $D = 2.1$ cm
C) $D = 3.2$ cm
D) $D = 4.4$ cm

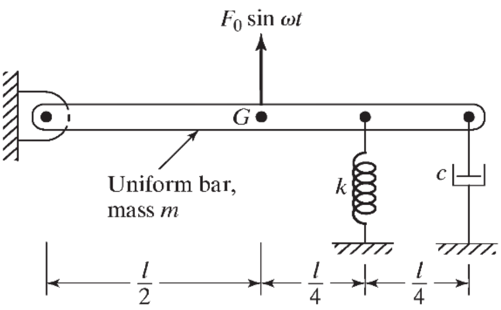
Problem 5 (Rao, 2011, w/ permission)

A thin disk of mass 0.8 kg and radius 60 mm is attached to the end of a 1.2-m steel ($G = 80\times10^9$ N/m², $\rho = 7500$ kg/m³) shaft of diameter 20 mm. The disk is subject to a harmonic torque of amplitude 12.5 N·m at a frequency of 700 rad/s. What is the steady-state amplitude of angular oscillations of the disk?

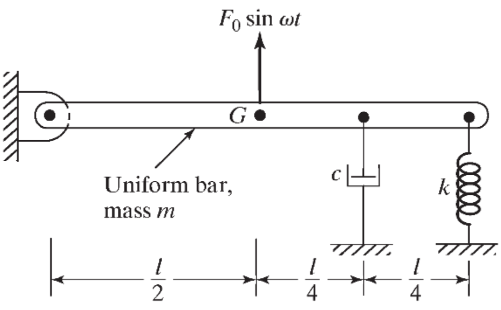
A) $\theta = 1.31°$
B) $\theta = 2.26°$
C) $\theta = 3.18°$
D) $\theta = 4.40°$

Problem 6 (Rao, 2011, w/ permission)

A uniform slender bar of mass m may be supported in one of the two ways illustrated below. Determine the arrangement that results in a reduced steady-state response of the bar under a harmonic force, $F_0 \sin \omega t$, applied to the middle of the bar as shown.



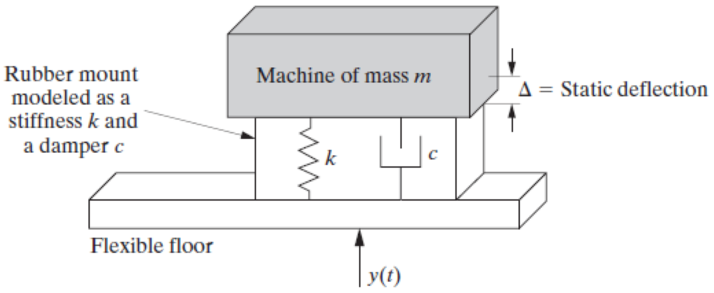
Arrangement A



Arrangement B

Problem 7 (Inman, 2014, w/ permission)

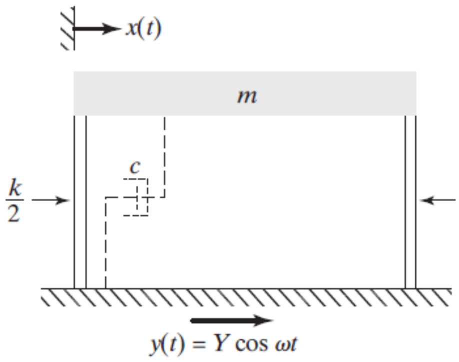
A machine weighing 2000 N rests on a support as illustrated below. The support deflects about 5 cm as a result of the weight of the machine. The floor under the support is somewhat flexible and moves, because of the motion of the nearby machine, harmonically near resonance ($r = 1$) with an amplitude of 0.2 cm. Model the floor as base motion, assume a damping ratio of $\zeta = 0.01$, and calculate the amplitude of the transmitted displacement.



A) $X = 0.03$ m
B) $X = 0.10$ m
C) $X = 0.17$ m
D) $X = 0.24$ m

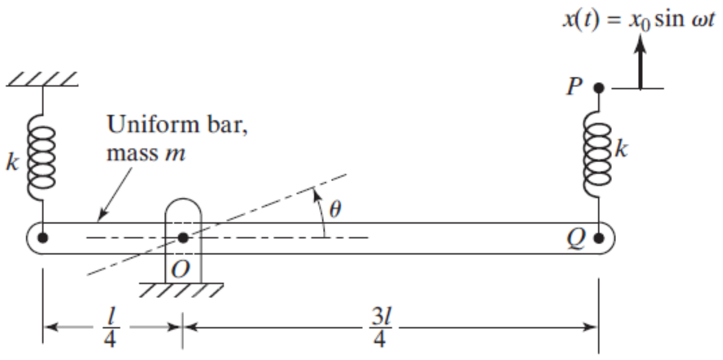
Problem 8 (Rao, 2011, w/ permission)

A single-story building frame is modeled by a rigid floor of mass m and columns of stiffness $k/2$, as shown in the following figure. It is proposed that the damper shown in the figure is attached to absorb vibrations due to a horizontal ground motion, $y(t) = Y \cos \omega t$. Derive an expression for the damping constant of the damper that absorbs maximum power.



Problem 9 (Rao, 2011, w/ permission)

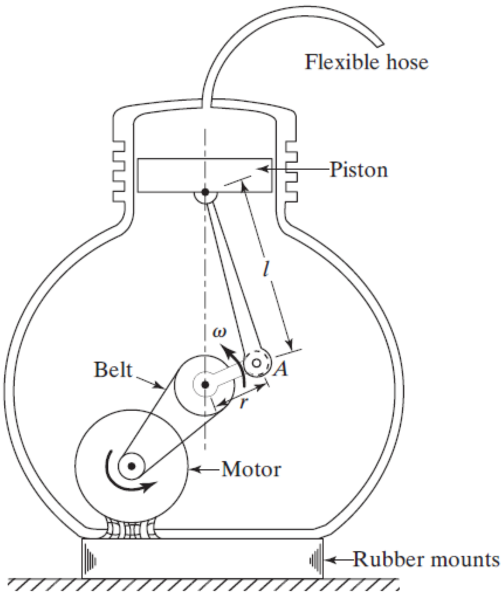
A uniform bar of mass m is pivoted at point O and supported at the ends by two springs as shown. End P of spring PQ is subjected to a sinusoidal displacement $x(t) = x_o \sin \omega t$. Find the steady-state angular displacement of the bar when $l = 1$ m, $k = 1000$ N/m, $m = 10$ kg, $x_o = 1$ cm, and $\omega = 10$ rad/s.



A) $\theta(t) = 0.0056 \sin 10t$
B) $\theta(t) = 0.0107 \sin 10t$
C) $\theta(t) = 0.0157 \sin 10t$
D) $\theta(t) = 0.0206 \sin 10t$

Problem 10 (Rao, 2011, w/ permission)

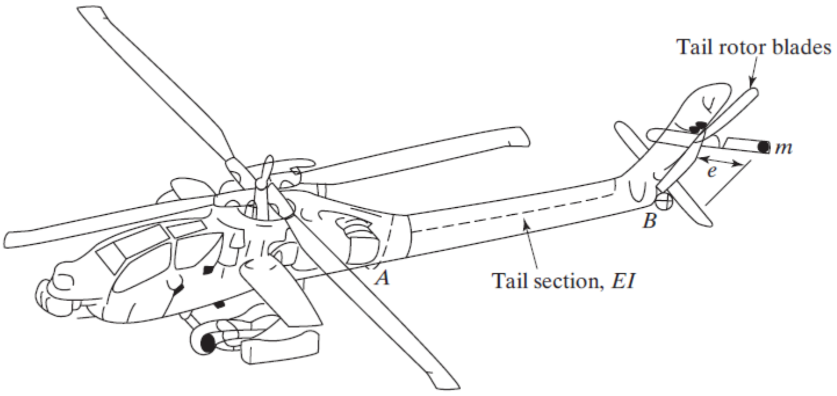
A single-cylinder air compressor of mass 100 kg is mounted on rubber mounts, as illustrated below. The stiffness and damping constants of the rubber mounts are 10^6 N/m and 2000 N·s/m, respectively. If the unbalance of the compressor is equivalent to a mass of 0.1 kg located at the end of the crank (point A), determine the amplitude of the compressor's motion, assuming that it functions at a crank speed of 3000 rpm. Assume r = 10 cm and l = 40 cm.



A) X = 0.11 mm
B) X = 0.19 mm
C) X = 0.27 mm
D) X = 0.35 mm

Problem 11 (Rao, 2011, w/ permission)

One of the tail rotor blades of a helicopter has an unbalanced mass of m = 0.5 kg at a distance of e = 0.15 m from the axis of rotation, as shown in the figure below. The tail section has a length of 4 m, a mass of 240 kg, a flexural stiffness EI of 2.5 MN·m², and a damping ratio of 0.15. The mass of the tail rotor blades, including their drive system, is 20 kg. Determine the forced response of the tail section when the blades rotate at 1500 rpm.



A) $x_p(t) = 2\sin(157.08t + 4.6°)$
B) $x_p(t) = 2\sin(157.08t + 9.2°)$
C) $x_p(t) = 4\sin(157.08t + 4.6°)$
D) $x_p(t) = 4\sin(157.08t + 9.2°)$

Problem 12 (Inman, 2014, w/ permission)

A spring-mass system (m = 10 kg, k = 4×10³ N/m) vibrates harmonically on a surface with coefficient of friction μ = 0.15. When excited harmonically at 5 Hz, the steady-state displacement of the mass is 5 cm. Calculate the amplitude of the harmonic force applied.

A) F_0 = 141 N
B) F_0 = 192 N
C) F_0 = 243 N
D) F_0 = 294 N

Problem 13 (Inman, 2014, w/ permission)

A system of unknown damping mechanism is driven harmonically at 10 Hz with an adjustable magnitude. The magnitude is changed, and the energy lost per cycle and amplitudes are measured for different amplitudes. The measured quantities are given in the following table. Is the damping viscous or Coulomb?

ΔE (J)	0.25	0.45	0.80	1.18	1.56	2.18	2.55	3.54	4.35
X (m)	0.01	0.02	0.03	0.04	0.06	0.08	0.10	0.14	0.18

Problem 14 (Inman, 2014, w/ permission)

Derive a formula for equivalent viscous damping if the damping force has the form $F_d = c \times \dot{x}^n$.

SOLUTIONS

P.1 › Solution

The springs attached to the block act in parallel, leading to an equivalent stiffness $k_{\text{eq}} = 2 \times 10^5 + 1 \times 10^5 = 3 \times 10^5$ N/m. Resonance occurs when the excitation frequency of 50 rad/s is equal to the natural frequency; that is,

$$\omega_n = \sqrt{\frac{k_{\text{eq}}}{m}} = 50\,\text{rad/s}$$

$$\therefore \omega_n^2 = \frac{k_{\text{eq}}}{m}$$

$$\therefore m = \frac{k_{\text{eq}}}{\omega_n^2} = \frac{300,000}{50^2} = \boxed{120\,\text{kg}}$$

▸ The correct answer is **C**.

P.2 › Solution

The natural frequency of the system is $\omega_n = (k/m)^{0.5} = (2000/100)^{0.5} = 4.47$ rad/s. From the equation of the driving force, $F(t) = 20\cos 6.3t$, we obtain F_o = 20 N and ω = 6.3 rad/s. The amplitude of the steady-state response, X, follows as

$$X = \frac{F_0/m}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}} = \frac{F_0/m}{\sqrt{\left(1-r^2\right)^2 + \left(\frac{c}{m}\times\omega\right)^2}}$$

$$\therefore \sqrt{\left(\omega_n^2-\omega^2\right)^2 + \left(\frac{c}{m}\times\omega\right)^2} = \left(\frac{F_0}{mX}\right)$$

$$\therefore \left(\omega_n^2-\omega^2\right)^2 + \left(\frac{c}{m}\times\omega\right)^2 = \left(\frac{F_0}{mX}\right)^2$$

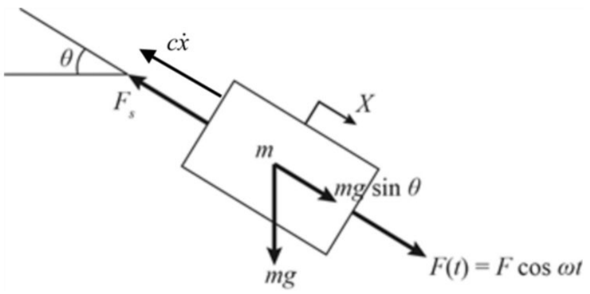
$$\therefore \left(\frac{c}{m}\times\omega\right) = \sqrt{\left(\frac{F_0}{mX}\right)^2 - \left(\omega_n^2-\omega^2\right)^2}$$

$$\therefore c = \sqrt{\frac{1}{\omega^2}\left(\frac{F_0}{X}\right)^2 - \frac{m^2}{\omega^2}\left(\omega_n^2-\omega^2\right)^2} = \sqrt{\frac{1}{6.3^2}\left(\frac{20}{0.01}\right)^2 - \frac{100^2}{6.3^2}\left(4.47^2-6.3^2\right)^2} = \boxed{53.9\text{ kg/s}}$$

▸ The correct answer is **A**.

P.3 › Solution

The free-body diagram for the system is shown below.



From Hooke's law, the static deflection in the spring is

$$k\delta = mg\sin\theta$$

$$\therefore \delta = \frac{mg\sin\theta}{k}$$

We then consider equilibrium of forces along the plane,

$$F(t) + mg\sin\theta - F_s - c\dot{x} = m\ddot{x}$$

where $F(t) = F\cos\omega t$, and F_s is the elastic force, given by $F_s = k(x+\delta)$. Substituting, it follows that

$$F + mg\sin\theta - F_s - c\dot{x} = m\ddot{x}$$

$$\therefore m\ddot{x} + c\dot{x} + k\left(x + \frac{mg\sin\theta}{k}\right) = F\cos\omega t + mg\sin\theta$$

$$\therefore m\ddot{x} + c\dot{x} + kx = F\cos\omega t$$

Clearly, this equation of motion is independent of the angle of the inclined surface θ. This indicates that the magnitude of the response is not affected by the angle of the incline.

P.4 › Solution

The force function is $f(t) = 25\cos 75.4t$ N. Comparing this with the general form $f(t) = F_0 \sin\omega t$, we have the amplitude F_o = 25 N and the frequency ω = 75.4 rad/s. Recall that, in this case, the displacement amplitude is related to other variables by the expression

$$X = \frac{F_0}{\left(k - m\omega^2\right)}$$

Substituting the maximum force F_o = 25 N, the mass m = 2.0 kg, and the vibration frequency ω = 75.4 rad/s, along with the maximum allowable displacement amplitude X = 0.005 m, we can solve for the transverse stiffness k,

$$X = \frac{F_0}{\left(k - m\omega^2\right)} \to k = \frac{F_0}{X} + m\omega^2$$

$$\therefore k = \frac{25}{0.005} + 2\times 75.4^2 = 16{,}370 \text{ N/m}$$

However, the transverse stiffness of the tubular rod is given by

$$k = \frac{3EI}{\ell^3}$$

where ℓ = 0.5 m (the height of the rod), E = 71×10^9 N/m^2 for aluminum, and k = 16,370 N/m as obtained just now. Thus,

$$k = \frac{3EI}{\ell^3} \to I = \frac{k\ell^3}{3E}$$

$$\therefore I = \frac{16{,}370\times 0.5^3}{3\times\left(71\times 10^9\right)} = 9.607\times 10^{-9} \text{ m}^4$$

The moment of inertia (with respect to a horizontal or vertical axis) for a circular cross-section is $I = \pi D^4/64$, where D is the diameter of the section. Accordingly,

$$I = \frac{\pi D^4}{64} \to D = \sqrt[4]{\frac{64I}{\pi}}$$

$$D = \sqrt[4]{\frac{64\times 9.607\times 10^{-9}}{\pi}} = 0.021 \text{ m} = \boxed{2.1 \text{ cm}}$$

▸ The correct answer is **B**.

P.5 › Solution

The torsional stiffness of the shaft is

$$K = \frac{JG}{L} = \frac{\left(\frac{\pi}{2}\times 0.01^4\right)\times\left(80\times 10^9\right)}{1.2} = 1047.2 \text{ N-m/rad}$$

The mass moment of inertia of the shaft, in turn, is

$$I_m = \frac{1}{2}\rho\pi L R^4 = \frac{1}{2}\times 7500\times\pi\times 1.2\times 0.01^4 = 1.414\times 10^{-4} \text{ kg-m}^2$$

The inertia effects of the shaft are included in a 1-DOF model by

$$I_{\text{eq}} = I_d + \frac{1}{3}I_m = \frac{1}{2}\times 0.8\times 0.06^2 + \frac{1}{3}\times 1.414\times 10^{-4} = 1.49\times 10^{-3} \text{ kg-m}^2$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{I_{\text{eq}}}} = \sqrt{\frac{1047.2}{1.49\times 10^{-3}}} = 838.34 \text{ rad/s}$$

The frequency ratio r is such that $r = (\omega/\omega_n) = (700/838.34) = 0.835$, and the magnification factor follows as

$$M = \frac{1}{1-r^2} = \frac{1}{1-0.835^2} = 3.303$$

Let Θ be the steady-state amplitude of torsional oscillation. We then have

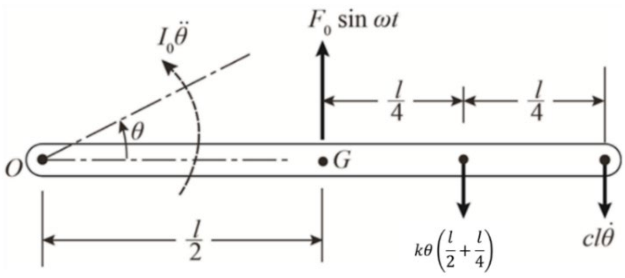
$$M = \frac{I_{\text{eq}}\omega_n^2\Theta}{T_0} \rightarrow \Theta = \frac{MT_0}{I_{\text{eq}}\omega_n^2}$$

$$\therefore \Theta = \frac{3.303\times 12.5}{\left(1.49\times 10^{-3}\right)\times 838.34^2} = 0.0394 \text{ rad} = \boxed{2.26^{\circ}}$$

▶ The correct answer is **B**.

P.6 › Solution

The figure below represents the forces acting on the system under arrangement A.



With reference to this figure, we take moments about hinge point O,

$$F_0 \sin\omega t\left(\frac{\ell}{2}\right) - \left[k\theta\left(\frac{\ell}{2}+\frac{\ell}{4}\right)\left(\frac{\ell}{2}+\frac{\ell}{4}\right)\right] - c\ell\dot{\theta}\times\ell = I_0\ddot{\theta}$$

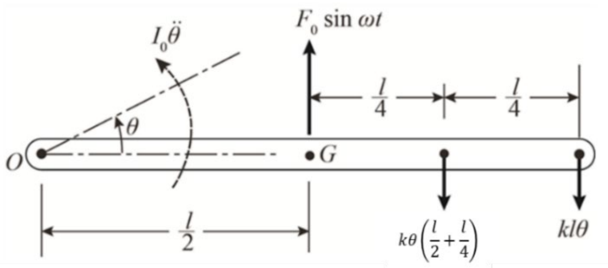
$$\therefore \frac{F_0\ell}{2}\sin\omega t - \frac{9}{16}k\ell^2\theta - c\ell^2\dot{\theta} = I_0\ddot{\theta}$$

$$\therefore \frac{F_0\ell}{2}\sin\omega\text{t} = I_0\ddot{\theta} + c\ell^2\dot{\theta} + \frac{9}{16}k\ell^2\theta$$

In this expression, F_o is the magnitude of the applied force, ω is the frequency of the applied force, ℓ is the length of the bar, k is the stiffness of the spring, I_o is the moment of inertia about hinge point O, c is the damping constant, and θ is the angular displacement of the bar. Using the equation of motion above, we can write the magnitude Θ_A of the angular displacement of system arrangement A as follows,

$$\Theta_A = \frac{\dfrac{F_0\ell}{2}}{\left[\left(\dfrac{9}{16}k\ell^2 - I_0\omega^2\right)^2 + \left(c\ell^2\omega\right)^2\right]^{1/2}}$$

We proceed to assess the movement of arrangement B, which is illustrated below.



As before, we take moments about hinge point *O*,

$$F_0 \sin \omega t\left(\frac{\ell}{2}\right) - \left[c\dot{\theta}\left(\frac{\ell}{2}+\frac{\ell}{4}\right)\right]\left(\frac{\ell}{2}+\frac{\ell}{4}\right) - k\ell\theta \times \ell = I_0\ddot{\theta}$$

$$\therefore \frac{F_0\ell}{2}\sin\omega t - \frac{9}{16}c\ell^2\dot{\theta} - k\ell^2\theta = I_0\ddot{\theta}$$

$$\therefore \frac{F_0\ell}{2}\sin\omega t = I_0\ddot{\theta} + \frac{9}{16}c\ell^2\dot{\theta} + k\ell^2\theta$$

The equation of motion above can be used to express the magnitude Θ_B of angular displacement for system arrangement B,

$$\Theta_B = \frac{\dfrac{F_0\ell}{2}}{\left[\left(k\ell^2 - I_0\omega^2\right)^2 + \left(\dfrac{9}{16}c\ell^2\omega\right)^2\right]^{1/2}}$$

In many situations, the value of the damping constant *c* is small in comparison to the spring constant *k*. Thus, in order to compare the steady-state response of the two arrangements, we neglect the *c* term. In this manner, the equation for Θ_A becomes

$$\Theta_A = \frac{\dfrac{F_0\ell}{2}}{\left[\left(\dfrac{9}{16}k\ell^2 - I_0\omega^2\right)^2 + \left(\cancel{c\ell^2\omega}\right)^2\right]^{1/2}} = \frac{\dfrac{F_0\ell}{2}}{\left[\left(\dfrac{9}{16}k\ell^2 - I_0\omega^2\right)^2\right]^{1/2}} = \frac{F_0\ell/2}{\dfrac{9}{16}k\ell^2 - I_0\omega^2}$$

The expression for Θ_B, in turn, is simplified as

$$\Theta_B = \frac{\dfrac{F_0\ell}{2}}{\left[\left(k\ell^2 - I_0\omega^2\right)^2 + \left(\cancel{\dfrac{9}{16}c\ell^2\omega}\right)^2\right]^{1/2}} = \frac{\dfrac{F_0\ell}{2}}{\left[\left(k\ell^2 - I_0\omega^2\right)^2\right]^{1/2}} = \frac{F_0\ell/2}{k\ell^2 - I_0\omega^2}$$

The numerator of both equations is the same, but the denominator of Θ_A is bound to be less than that of Θ_B and, consequently, the steady-state response of A will be greater than the steady-state response of B.

P.7 › Solution

The stiffness of the machine can be obtained with Hooke's law,

$$W = k \times \Delta$$

The weight *W* of the machine is 2000 N and the deflection Δ is 5 cm. Therefore,

$$W = k \times \Delta$$

$$\therefore 2000 = k \times 0.05$$

$$\therefore k = \frac{2000}{0.05} = 40{,}000 \text{ N/m}$$

The amplitude of the transmitted displacement, X, is given by

$$X = Y\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$$

Substituting Y = 0.2 cm, ζ = 0.01, and r = 1 gives

$$X = Y\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2} = (0.2\times10^{-2})\left[\frac{1+(2\times0.01\times1)^2}{(1-1^2)^2+(2\times0.01\times1)^2}\right]^{1/2} = \boxed{0.10\text{ m}}$$

The correct answer is **B**.

P.8 Solution

The horizontal ground motion is described by the harmonic function $y(t) = Y\cos\omega t$. The first derivative of this expression is $\dot{y}(t) = -\omega Y\sin\omega t$, while the second derivative is $\ddot{y}(t) = -\omega^2 Y\cos\omega t$. The equivalent stiffness k_{eq} of the system is such that

$$k_{\text{eq}} = \frac{k}{2}+\frac{k}{2} = k$$

The equation of motion of the rigid floor having mass m with a moving base is

$$m\ddot{x}+c(\dot{x}-\dot{y})+k_{\text{eq}}(x-y)=0$$

$$\therefore m\ddot{x}+c(\dot{x}-\dot{y})+k(x-y)=0$$

We assume $z = x - y$ and manipulate the equation above, giving

$$m(\ddot{z}+\ddot{y})+c\dot{z}+kz=0$$

$$\therefore m\ddot{z}+c\dot{z}+kz=-m\ddot{y}$$

Note, however, that $\ddot{y} = -\omega^2 Y\cos\omega t$. Substituting in the expression above brings to

$$m\ddot{z}+c\dot{z}+kz=-m(-\omega^2 Y\cos\omega t)=m\omega^2 Y\cos\omega t$$

On the basis of the equation above, we are able to write the response of the system as

$$z(t)=Z\cos(\omega t-\phi)$$

where

$$Z=\frac{m\omega^2 Y}{\left[(k-m\omega^2)^2+(c\omega)^2\right]^{1/2}}$$

and

$$\phi=\tan^{-1}\left(\frac{c\omega}{k-m\omega^2}\right)$$

We utilize the following expression for the damping force F_d,

$$F_d = c\frac{dz}{dt} = c\frac{d}{dt}\left[Z\cos(\omega t-\phi)\right] = -c\omega Z\sin(\omega t-\phi)$$

Then, we compute the energy E absorbed by the damper per cycle,

$$E=\int_{t=0}^{t=2\pi/\omega}F_d\,dz=\int_{t=0}^{t=2\pi/\omega}F_d\left(\frac{dz}{dt}\right)dt$$

$$\therefore E=\int_{t=0}^{t=2\pi/\omega}\left[-c\omega Z\sin(\omega t-\phi)\right]\left[-\omega Z\sin(\omega t-\phi)\right]dt$$

$$\therefore E = c\omega^2 Z^2 \underbrace{\int_{t=0}^{t=2\pi/\omega} \sin^2(\omega t - \phi)\, dt}_{=\frac{\pi}{\omega}}$$

$$\therefore E = c\omega^2 Z^2 \times \frac{\pi}{\omega} = \pi c\omega Z^2$$

Substituting the expression for *Z* obtained earlier yields

$$E = \pi c\omega \left[\frac{m\omega^2 Y}{\left[(k - m\omega^2)^2 + (c\omega)^2 \right]^{1/2}} \right]^2 = \frac{\pi c\omega \times m^2 \omega^4 Y^2}{(k - m\omega^2)^2 + (c\omega)^2}$$

$$\therefore E = \frac{\pi c m^2 \omega^5 Y^2}{(k - m\omega^2)^2 + (c\omega)^2}$$

To obtain the value of *c* at which maximum power is absorbed by the damper, we differentiate *E* with respect to *c* and equate it to zero,

$$\frac{dE}{dc} = -\frac{2\pi c^2 m^2 Y^2 \omega^7}{\left[(k - m\omega^2)^2 + (c\omega)^2 \right]^2} + \frac{\pi m^2 Y^2 \omega^5}{(k - m\omega^2)^2 + (c\omega)^2}$$

$$\therefore \frac{dE}{dc} = \frac{\pi m^2 Y^2 \omega^5 \left[(k - m\omega^2)^2 - (c\omega)^2 \right]}{\left[(k - m\omega^2)^2 + (c\omega)^2 \right]^2} = 0$$

Setting the expression in square brackets in the numerator to zero, the optimum value of *c* is determined to be

$$(k - m\omega^2)^2 - (c\omega)^2 = 0$$

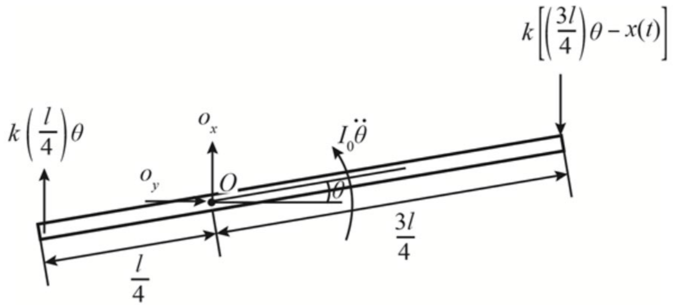
$$\therefore (k - m\omega^2)^2 = (c\omega)^2$$

$$\therefore k - m\omega^2 = c\omega$$

$$\therefore \boxed{c = \frac{k - m\omega^2}{\omega}}$$

P.9 › Solution

When the bar rotates by an angle θ counterclockwise, point *Q* will move upwards by a distance $(3l/4)\theta$ and the net compression in the spring PQ will be $[(3l/4)\theta - x(t)]$. The free-body diagram of the bar is shown below.



With reference to the figure above, we write the equilibrium of moments for the bar about point *O*,

$$-\left[k\left(\frac{l}{4}\right)\theta \right]\left(\frac{l}{4}\right) - k\left[\left(\frac{3l}{4}\right)\theta - x(t) \right]\left(\frac{3l}{4}\right) = I_0 \ddot{\theta}$$

$$\therefore -\frac{1}{16} k l^2 \theta - \frac{9}{16} k l^2 \theta + \frac{3}{4} k l x(t) = I_0 \ddot{\theta}$$

$$\therefore -\frac{5}{8} k l^2 \theta + \frac{3}{4} k l x_0 \sin \omega t = I_0 \ddot{\theta}$$

$$\therefore \left(\frac{3}{4}klx_0\right)\sin\omega t = I_0\ddot{\theta} + \left(\frac{5}{8}kl^2\right)\theta \quad \text{(I)}$$

The moment of inertia of the bar about point *O* is given by

$$I_0 = \frac{1}{12}ml^2 + m\left(\frac{l}{4}\right)^2 = \frac{1}{12}ml^2 + \frac{1}{16}ml^2 = \frac{7}{48}ml^2$$

$$\therefore I_0 = \frac{7}{48}\times 10\times 1^2 = 1.46\ \text{kg}\cdot\text{m}^2$$

As usual, the steady-state response of the system has the general form

$$\theta(t) = \Theta\sin\omega t$$

where Θ is the amplitude of the bar's angular displacement. Taking coefficients from equation (I), this quantity is found to have the form

$$\Theta = \frac{\frac{3}{4}klx_0}{\frac{5}{8}kl^2 - I_0\omega^2} = \frac{\frac{3}{4}\times 1000\times 1\times 0.01}{\frac{5}{8}\times 1000\times 1^2 - 1.46\times 10^2} = 0.0157\ \text{rad}$$

Finally, the steady-state angular displacement of the bar is

$$\boxed{\theta(t) = 0.0157\sin 10t}$$

▸ The correct answer is **C**.

P.10 › Solution

The equation of motion for an unbalanced rotating mass is

$$M\ddot{x} + c\dot{x} + kx = me\omega^2\sin\omega t$$

where, in the case at hand, *M* is the mass of the compressor, *m* is the unbalanced mass, *e* is the eccentricity, and ω is the rotational speed of the compressor. The steady-state response of the compressor is described by

$$x_p(t) = X\sin(\omega t - \phi)$$

In which the amplitude *X* is

$$X = \frac{mr\omega^2}{\sqrt{\left(k - M\omega^2\right)^2 + (c\omega)^2}}$$

and the phase angle is given by

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - M\omega^2}\right)$$

The frequency ω, expressed in radians, is

$$\omega = 3000\times\frac{2\pi}{60} = 314.16\ \text{rad/s}$$

We substitute m = 0.1 kg, r = 0.1 m, ω = 314.16 rad/s, $k = 10^6$ N/m, M = 100 kg, and c = 2000 N·s/m in the equation for *X*, giving

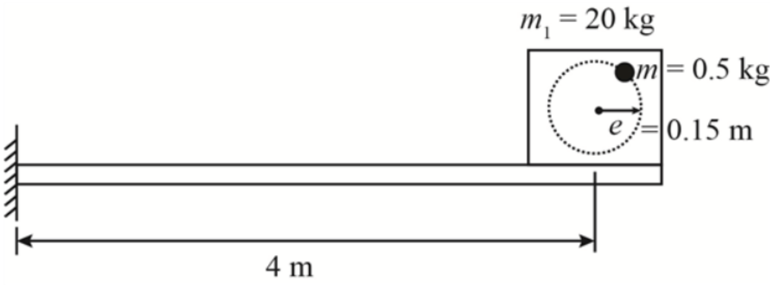
$$X = \frac{mr\omega^2}{\sqrt{\left(k - M\omega^2\right)^2 + (c\omega)^2}} = \frac{0.1\times 0.1\times 314.16^2}{\sqrt{\left(10^6 - 100\times 314.16^2\right)^2 + (2000\times 314.16)^2}} = 0.000111\ \text{m}$$

$$\therefore \boxed{X = 0.11\ \text{mm}}$$

▸ The correct answer is **A**.

P.11 Solution

Assume the tail of the helicopter to be acting like a cantilever beam of mass m_b, with an end mass m_1 of 20 kg placed at its free end and an unbalanced rotating mass of 0.5 kg at its free end having an eccentricity of 0.15 m, as shown.



Since the tail is modeled as a cantilever beam, its stiffness is given by

$$k = \frac{3EI}{\ell^3} = \frac{3\times\left(2.5\times10^6\right)}{4^3} = 117{,}188 \text{ N/m}$$

As usual, the natural frequency is $\omega_n = \left(k/m_{\text{eq}}\right)^{0.5}$, where m_{eq} is the equivalent mass of the system. With m_b as the mass of the beam and m_l as the end mass, we have

$$m_{\text{eq}} = m_1 + \frac{33}{140}m_b = 20 + 0.236\times240 = 76.64 \text{ kg}$$

We can then compute the natural frequency,

$$\omega_n = \sqrt{\frac{k}{m_{\text{eq}}}} = \sqrt{\frac{117{,}188}{76.64}} = 39.10 \text{ rad/s}$$

Knowing that the rotational speed of the blades is 1500 rpm, the operating frequency can be calculated as $\omega = 2\pi N/60 = 2\pi \times 1500/60 = 157.08$ rad/s. The frequency ratio follows as

$$r = \frac{\omega}{\omega_n} = \frac{157.08}{39.10} = 4.017$$

The forced response of the tail section of the helicopter is described by the relation

$$x_p(t) = X\sin\left(\omega t - \phi\right)$$

where the amplitude X is calculated as

$$X = \left(\frac{me}{m_1}\right)\left(\frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}\right)$$

Substituting m = 0.5 kg, e = 0.15 m, r = 4.017, ζ = 0.15, and m_1 = 20 kg gives

$$X = \left(\frac{0.5\times0.15}{20}\right)\left(\frac{4.017^2}{\sqrt{\left(1-4.017^2\right)^2 + \left(2\times0.15\times4.017\right)^2}}\right) = 0.004 \text{ m} = 4 \text{ mm}$$

Next, the phase angle component of the response is

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = \tan^{-1}\left(\frac{2\times0.15\times4.017}{1-4.017^2}\right) = \tan^{-1}\left(-0.0796\right) = -4.6^\circ$$

Lastly, the forced response of the tail section of the helicopter is

$$\boxed{x_p(t) = 4\sin\left(157.08t + 4.6^\circ\right) \text{ mm}}$$

The correct answer is **C**.

P.12 › Solution

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4000}{10}} = 20 \text{ rad/s}$$

The frequency of the motion of the mass is $\omega = 2\pi f = 2\pi \times 5 = 31.42$ Hz. The frequency ratio r is $r = \omega/\omega_n = 31.42/20 = 1.571$. The amplitude of the vibration of a harmonically excited system under Coulomb damping is given by

$$X = \frac{F_0/k}{\left[\left(1-r^2\right)^2 + \left(4\mu mg/\pi kX\right)^2\right]^{1/2}}$$

Solving for the harmonic force amplitude and substituting, we obtain

$$F_0 = kX\left[\left(1-r^2\right)^2 + \left(4\mu mg/\pi kX\right)^2\right]^{1/2}$$

$$\therefore F_0 = 4000\times 0.05\times\left[\left(1-1.571^2\right)^2 + \left(4\times 0.15\times 10\times 9.81/(\pi\times 4000\times 0.05)\right)^2\right]^{1/2} = \boxed{294 \text{ N}}$$

▸ The correct answer is **D**.

P.13 › Solution

Recall that, for viscous damping, the expression for energy lost per cycle is

$$\Delta E = \pi k \beta X^2$$

in which k is the stiffness, β is the hysteresis damping constant, and X is the amplitude of vibration. In contrast, the energy lost per cycle in the case of Coulomb damping is

$$\Delta E = 4\mu mgX$$

where μ is the coefficient of friction, m is the mass, g is the acceleration of gravity, and X is the amplitude of vibration. Using a CAS such as Mathematica, we prepare plots of ΔE versus X^2 and ΔE versus X. The graph in which a linear trend is observed will decide the form of damping. We begin by entering a list for the amplitudes.

$$X = \{0.01,0.02,0.03,0.04,0.06,0.08,0.1,0.14,0.18\}$$

Next, we type a list for the energy lost per cycle,

$$\Delta\text{E} = \{0.25,0.45,0.8,1.18,1.56,2.18,2.55,3.54,4.35\}$$

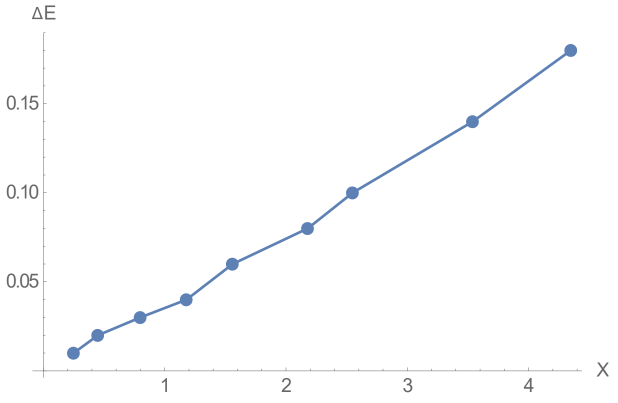
In addition to these, we need a list for the squared amplitudes,

$$\text{X2} = X^2$$

$$\{0.0001,0.0004,0.0009,0.0016,0.0036,0.0064,0.01,0.0196,0.0324\}$$

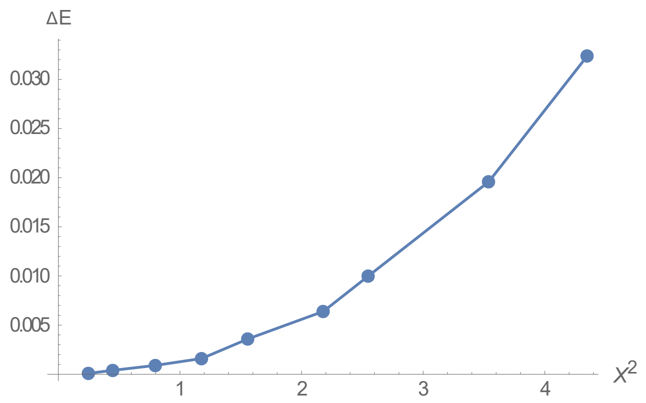
We then proceed to plot ΔE versus X.

```
list1 =Transpose@{ΔE,X};
ListPlot[list1,Joined->True,PlotMarkers->{Automatic,Medium},AxesLabel->{"X","ΔE"}]
```



Similarly, we plot ΔE versus X^2.

```
list2 =Transpose@{ΔE,X2};
ListPlot[list2,Joined->True,PlotMarkers->{Automatic,Medium},AxesLabel->{"X²","ΔE"}]
```



Clearly, it is seen that the plot of ΔE versus X yields a straight line and the plot of ΔE versus X^2 yields a curve. Hence, the damping should follow an energy loss of the form $\Delta E = 4\mu mgX$ and is Coulomb in nature.

P.14 › Solution

The steady-state response of the system is expected to have the form

$$x = X\sin\omega t$$

Accordingly, we have

$$\dot{x} = \omega X\cos\omega t$$

The energy lost per cycle is given by

$$\Delta E = \oint F_d\,dx$$

which, in the situation at hand, is written as

$$\Delta E = c\int_0^{\frac{2\pi}{\omega}} \dot{x}^n dx$$

Substituting $dx = \dot{x}dt$, we have

$$\Delta E = \int_0^{2\pi/\omega} c\dot{x}^n\left(\dot{x}dt\right) = \int_0^{2\pi/\omega} c\dot{x}^{n+1}dt$$

Substituting the equation for $\dot{x}$ gives

$$\Delta E = c\int_0^{2\pi/\omega}\left[\omega^{n+1}X^{n+1}\cos^{n+1}\omega t\right]dt$$

$$\therefore \Delta E = c\omega^n X^{n+1}\int_0^{2\pi}\cos^{n+1}u\,du$$

The energy loss for viscous damping as in the present case is

$$\Delta E = \pi c_{\text{eq}}\omega X^2$$

Equating the two foregoing expressions for ΔE and manipulating, we obtain

$$\pi c_{\text{eq}}\omega X^2 = c\omega^n X^{n+1}\int_0^{2\pi}\cos^{n+1}u\,du$$

$$\therefore c_{\text{eq}} = \frac{c\omega^n X^{n+1}}{\pi\omega X^2}\int_0^{2\pi}\cos^{n+1}u\,du$$

$$\therefore \boxed{c_{\text{eq}} = \frac{c\omega^{n-1}X^{n-1}}{\pi}\int_0^{2\pi}\cos^{n+1}u\,du}$$

ANSWER SUMMARY

Problem 1	C
Problem 2	A
Problem 3	*Open-ended pb.*
Problem 4	B
Problem 5	B
Problem 6	*Open-ended pb.*
Problem 7	B
Problem 8	*Open-ended pb.*
Problem 9	C
Problem 10	A
Problem 11	C
Problem 12	D
Problem 13	*Open-ended pb.*
Problem 14	*Open-ended pb.*

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