Quiz HT108

Heat Transfer by Radiation

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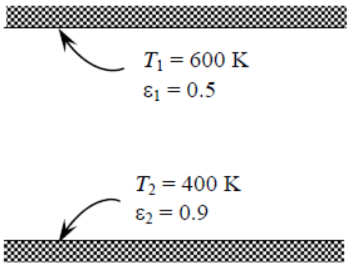
Problems

Problem 1 (Çengel & Ghajar, 2015, w/ permission)

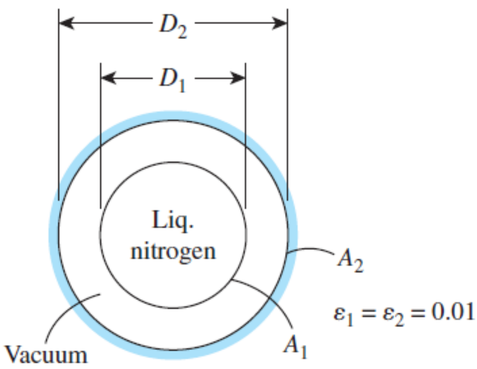
Two very large parallel plates are maintained at uniform temperatures of T_1 = 600 K and T_2 = 400 K, and have emissivities of ε_1 = 0.5 and ε_2 = 0.9, respectively. Determine the net rate of radiation heat transfer between the two surfaces per unit area of the plates (i.e., the heat flux).



A) $\dot{Q}_{12}/A_s$ = 1682 W/m^2
B) $\dot{Q}_{12}/A_s$ = 2793 W/m^2
C) $\dot{Q}_{12}/A_s$ = 3864 W/m^2
D) $\dot{Q}_{12}/A_s$ = 4975 W/m^2

Problem 2 (Çengel & Ghajar, 2015, w/ permission)

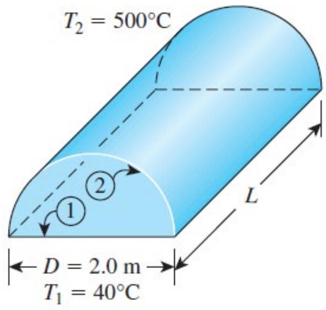
Two-phase gas-liquid oxygen is stored in a 1-m-diameter spherical tank whose surface is maintained at 80 K. The spherical tank is enclosed by a 1.6-m diameter concentric spherical surface at 273 K. Both spherical surfaces have an emissivity of 0.01, and the gap between the inner sphere and the outer sphere is vacuumed. Assuming that the spherical tank surface has the same temperature as the oxygen, determine the heat transfer rate at the spherical tank surface.



A) $|\dot{Q}_{12}|$ = 2.8 W
B) $|\dot{Q}_{12}|$ = 5.3 W
C) $|\dot{Q}_{12}|$ = 7.8 W
D) $|\dot{Q}_{12}|$ = 10.3 W

Problem 3 (Çengel & Ghajar, 2015, w/ permission)

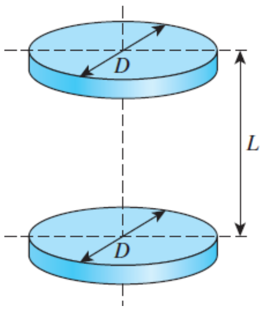
A dryer is shaped like a long semi-cylindrical duct of diameter 2 m. The base of the dryer is occupied with water-soaked materials that are to be dried. The dome of the dryer has a constant temperature of 500°C, while the materials at the base are at 40°C. Both surfaces can be approximated as blackbodies. Determine the length of the dryer so that the materials are dried at a rate of 0.1 kg/s. Use h_{fg} = 2257 kJ/kg for water.



A) L = 5.7 m
B) L = 7.0 m
C) L = 8.4 m
D) L = 9.9 m

Problem 4 (Çengel & Ghajar, 2015, w/ permission)

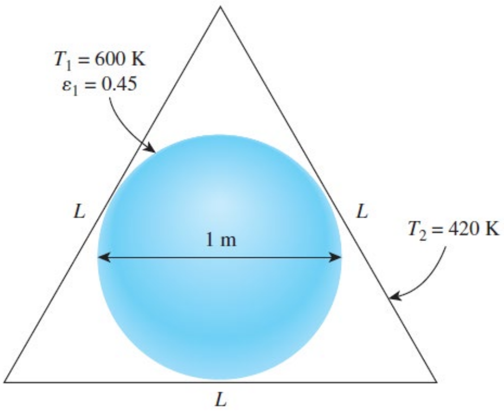
Two coaxial parallel disks of equal diameter D = 1 m are originally placed a distance of 1 m apart. If both disks behave as black surfaces, determine a new distance between the disks such that there is a 75% reduction in radiation heat transfer rate from the original distance of 1 m.



A) L^* = 1.4 m
B) L^* = 2.3 m
C) L^* = 3.4 m
D) L^* = 4.3 m

Problem 5A (Çengel & Ghajar, 2015, w/ permission)

A solid sphere of 1 m diameter at 600 K is kept in an evacuated triangular enclosure (a tetrahedron) with side length of L. Note that for the sphere to touch the tetrahedron's surfaces, the tetrahedron's side length should be $L = \sqrt{6}D$. The emissivity of the sphere is 0.45 and the temperature of the enclosure is 420 K. Determine the view factor F_{21} from the tetrahedron face to the right, surface 2, to the sphere, surface 1.



A) $F_{21} = 0$
B) $F_{21} = 0.30$
C) $F_{21} = 0.60$
D) $F_{21} = 1.0$

■ Problem 5B

If heat is generally uniformly within the sphere at a rate of 3100 W, determine the emissivity of the enclosure.

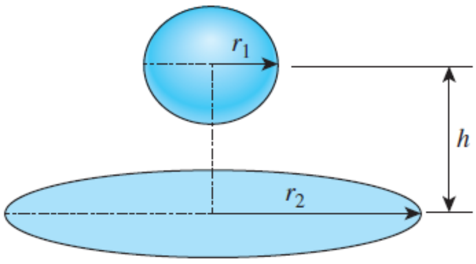
A) $\varepsilon_2 = 0.081$
B) $\varepsilon_2 = 0.122$
C) $\varepsilon_2 = 0.163$
D) $\varepsilon_2 = 0.204$

■ Problem 6A (Çengel & Ghajar, 2015, w/ permission)

This question deals with steady state radiation heat transfer between a sphere (r_1 = 30 cm) and a circular disk (r_2 = 120 cm), which are separated by a center-to-center distance h = 60 cm. When the normal to the center of the disk passes through the center of the sphere, the radiation view factor is given by

$$F_{12} = 0.5\left\{1 - \left[1 + \left(\frac{r_2}{h}\right)^2\right]^{-1/2}\right\}$$

Surface temperatures of the sphere and the disk are 600°C and 200°C, respectively, and their emissivities are 0.9 and 0.5, respectively. Calculate view factors F_{12} and F_{21}.



A) $F_{12} = 0.14$ and $F_{21} = 0.07$
B) $F_{12} = 0.14$ and $F_{21} = 0.20$
C) $F_{12} = 0.28$ and $F_{21} = 0.07$
D) $F_{12} = 0.28$ and $F_{21} = 0.20$

■ Problem 6B

Calculate the net rate of radiation heat exchange between the sphere and the disk.

A) $\dot{Q}_{12} = 2350$ W
B) $\dot{Q}_{12} = 4450$ W
C) $\dot{Q}_{12} = 6550$ W
D) $\dot{Q}_{12} = 8650$ W

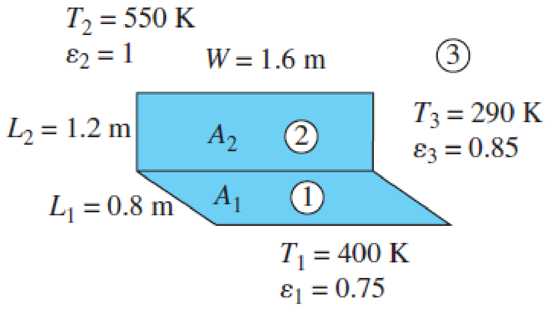
■ Problem 6C

For the given radii and temperatures of the sphere and the disk, the following three modifications could be used to increase the net rate of radiation heat exchange: (1) paint each of the two surfaces to alter their emissivities; (2) adjust the distance between them; and (3) provide an (refractory) enclosure. Calculate the net rate of radiation heat exchange between the two bodies if the *best values* are selected for each of the above modifications.

A) $\dot{Q}^* = 8600$ W
B) $\dot{Q}^* = 10{,}810$ W
C) $\dot{Q}^* = 12{,}920$ W
D) $\dot{Q}^* = 14{,}600$ W

Problem 7 (Çengel & Ghajar, 2015, w/ permission)

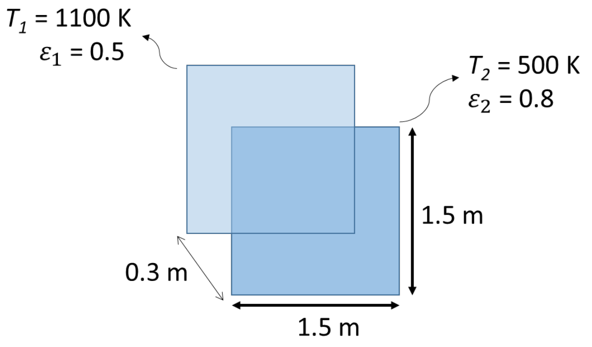
Consider two rectangular surfaces perpendicular to each other with a common edge that is 1.6 m long. The horizontal surface is 0.8 m wide and the vertical surface is 1.2 m high. The horizontal surface has an emissivity of 0.75 and is maintained at 400 K. The vertical surface is black and is maintained at 550 K. The back sides of the surfaces are insulated. The surrounding surfaces are at 290 K, and can be considered to have an emissivity of 0.85. Determine the rate of radiation heat transfer between the horizontal surface and the surroundings.



A) $|\dot{Q}_{13}|$ = 425 W
B) $|\dot{Q}_{13}|$ = 575 W
C) $|\dot{Q}_{13}|$ = 725 W
D) $|\dot{Q}_{13}|$ = 875 W

Problem 8A (Kreith et al., 2011, w/ permission)

Two 1.5 m-square and parallel flat plates are 0.3 m apart. Plate A_1 is maintained at a temperature of 1100 K and A_2 is maintained at 500 K. The emissivities of the plates are 0.5 and 0.8, respectively. Considering the surroundings to be black at 0 K and including multiple reflections, determine the net radiant exchange between the plates.



A) $\dot{Q}_{12}$ = 23,210 W
B) $\dot{Q}_{12}$ = 34,420 W
C) $\dot{Q}_{12}$ = 45,630 W
D) $\dot{Q}_{12}$ = 56,840 W

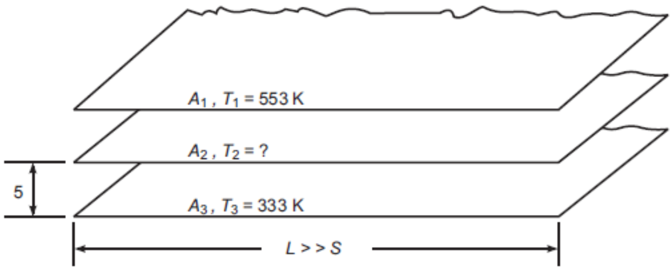
Problem 8B

Reconsidering the system in the previous problem, determine the heat input required by surface A_1 to maintain its temperature. The outer-facing surfaces of the plates are adiabatic.

A) $\dot{Q}_1$ = 20,010 W
B) $\dot{Q}_1$ = 42,020 W
C) $\dot{Q}_1$ = 64,040 W
D) $\dot{Q}_1$ = 86,050 W

Problem 9 (Kreith et al., 2011, w/ permission)

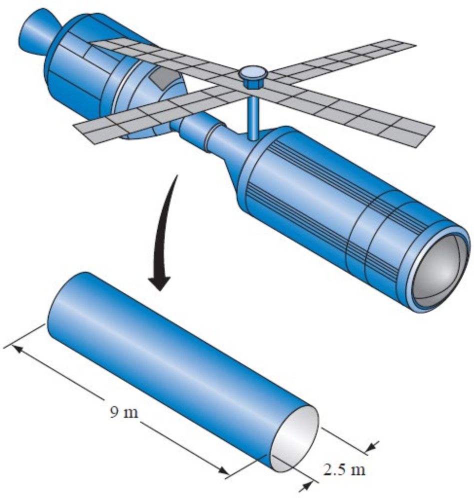
Three thin sheets of polished aluminum are placed parallel to each other so that the distance between them is very small compared to the size of the sheets. If one of the outer sheets is at 280°C, and the other outer sheet is at 60°C, calculate the net rate of heat flow by radiation stemming from surface 2. (Note that, due to symmetry, the heat flux from surface 2 to surface 1, $|Q_{21}/A|$, is bound to be the same as the heat flux from surface 2 to surface 3, $|Q_{23}/A|$).



A) $|\dot{Q}_{21}/A|$ = 34.5 W/m^2
B) $|\dot{Q}_{21}/A|$ = 58.7 W/m^2
C) $|\dot{Q}_{21}/A|$ = 76.3 W/m^2
D) $|\dot{Q}_{21}/A|$ = 92.1 W/m^2

Problem 10 (Kreith et al., 2011, w/ permission)

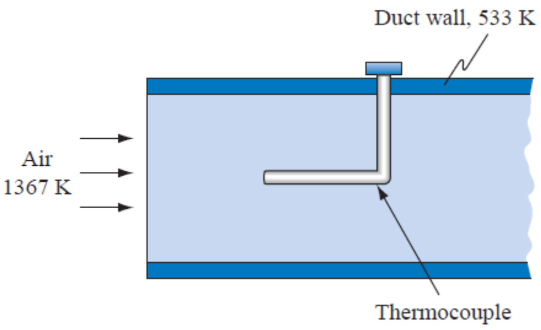
A manned spacecraft has a shape of a cylinder 2.5 m in diameter and 9 m long (see the sketch below). The air inside the capsule is maintained at 20°C, and the convection heat transfer coefficient on the interior surface is 17 W/m^{2}K. Between the outer skin and the inner surface is a 15-cm layer of glass-wool insulation having a thermal conductivity of 0.017 W/mK. If the emissivity of the skin is 0.05 and there is no aerodynamic heating or irradiation from astronomical bodies, calculate the total heat transfer rate into space at 0 K.



A) $\dot{Q}$ = 203.5 W
B) $\dot{Q}$ = 402.6 W
C) $\dot{Q}$ = 601.7 W
D) $\dot{Q}$ = 800.8 W

Problem 11A (Kreith et al., 2011, w/ permission)

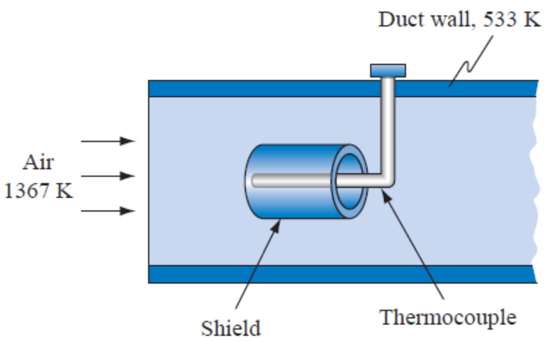
Calculate the equilibrium temperature of a thermocouple in a large air duct if the temperature is 1367 K, the duct-wall temperature is 533 K, the emissivity of the thermocouple is 0.5, and the convection heat-transfer coefficient, h_c, is 1114 W/m^{2}K. Only one of the following statements is true. Which one is it?



A) The thermocouple will overestimate the temperature of the airflow by more than 30%.
B) The thermocouple will overestimate the temperature of airflow by more than 15% but less than 30%.
C) The thermocouple will underestimate the temperature of airflow by more than 15% but less than 30%.
D) The thermocouple will underestimate the temperature of airflow by more than 30%.

■ Problem 11B

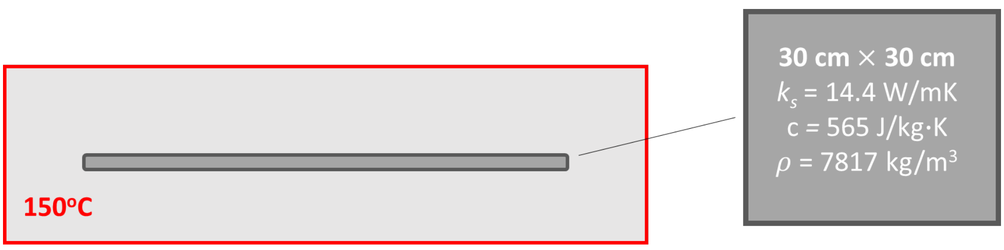
Reconsider the previous problem, this time noting that the thermocouple is endowed with a radiation shield of emissivity ε_s = 0.1. What is the new thermocouple reading?



A) T_{tc} = 1107 K
B) T_{tc} = 1208 K
C) T_{tc} = 1319 K
D) T_{tc} = 1436 K

■ Problem 12A (Kreith et al., 2011, w/ permission)

A 6-mm-thick sheet of polished 304 stainless steel (emissivity ε = 0.15) is suspended in a comparatively large vacuum-drying oven with black walls. The dimensions of the sheet are 30 cm × 30 cm. The steel has thermal conductivity k_s = 14.4 W/mK, specific heat c = 565 J/kg·K, and density ρ = 7817 kg/m^3. If the walls of the oven are uniformly at 150°C and the metal is to be heated from 0 to 120°C, estimate how long the sheet should be left in the oven if heat transfer by convection can be neglected.



A) t = 65 min
B) t = 96 min
C) t = 127 min
D) t = 158 min

Problem 12B

Estimate how long the sheet should be left in the oven if the heat transfer coefficient is 3 W/m²K.

A) *t* = 53 min

B) *t* = 84 min

C) *t* = 108 min

D) *t* = 120 min

Additional Information

Table 1 View factor expressions for some common geometries of finite size

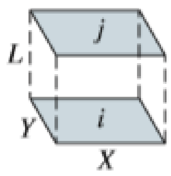
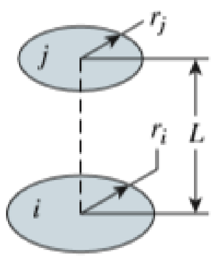
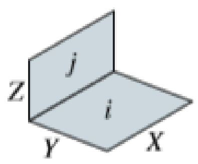
<table>
<tr><th>Geometry</th><th>Relation</th></tr>
<tr><td>Aligned parallel rectangles
</td><td>$\bar{X} = X/L$ and $\bar{Y} = Y/L$
$$F_{i \to j} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$</td></tr>
<tr><td>Coaxial parallel disks
</td><td>$R_i = r_i/L$ and $R_j = r_j/L$
$$S = 1 + \frac{1+R_j^2}{R_i^2}$$
$$F_{i \to j} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2\right]^{1/2}\right\}$$
For $r_i = r_j = r$ and $R = r/L$: $\quad F_{i \to j} = F_{j \to i} = 1 + \dfrac{1 - \sqrt{4R^2+1}}{2R^2}$</td></tr>
<tr><td>Perpendicular rectangles with a common edge
</td><td>$H = Z/X$ and $W = Y/X$
$$F_{i \to j} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$$</td></tr>
</table>

Table 2 Radiation heat transfer relations for some familiar two-surface arrangements

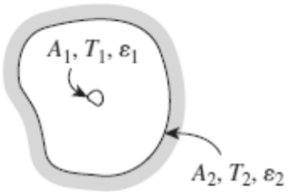
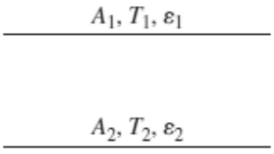
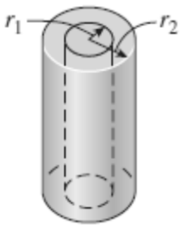
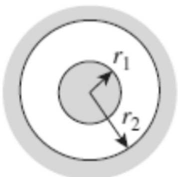
Small object in a large cavity 	$\frac{A_1}{A_2} \approx 0$ $F_{12} = 1$	$\dot{Q}_{12} = A_1 \sigma \varepsilon_1 (T_1^4 - T_2^4)$	**(13–37)**
Infinitely large parallel plates 	$A_1 = A_2 = A$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$	**(13–38)**
Infinitely long concentric cylinders 	$\frac{A_1}{A_2} = \frac{r_1}{r_2}$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2}\right)}$	**(13–39)**
Concentric spheres 	$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2}\right)^2}$	**(13–40)**

Figure 1 View factor between two perpendicular rectangles with a common edge.

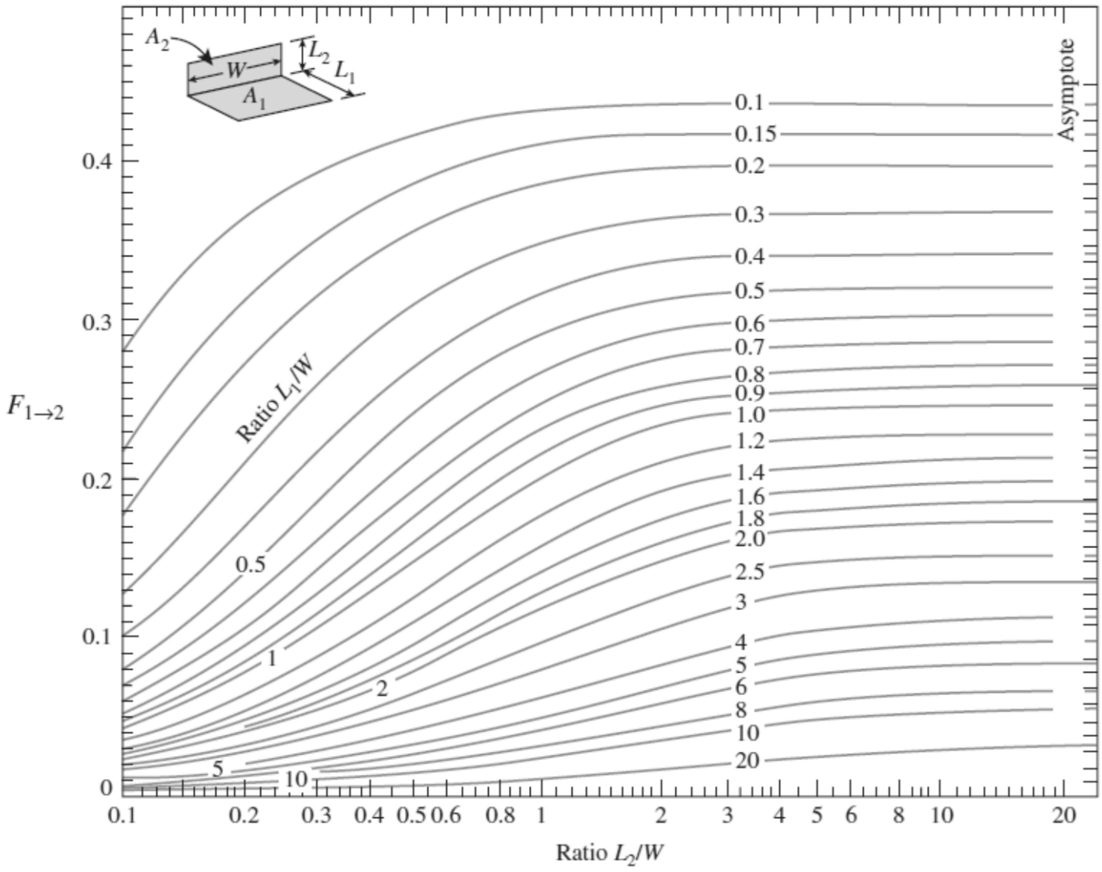
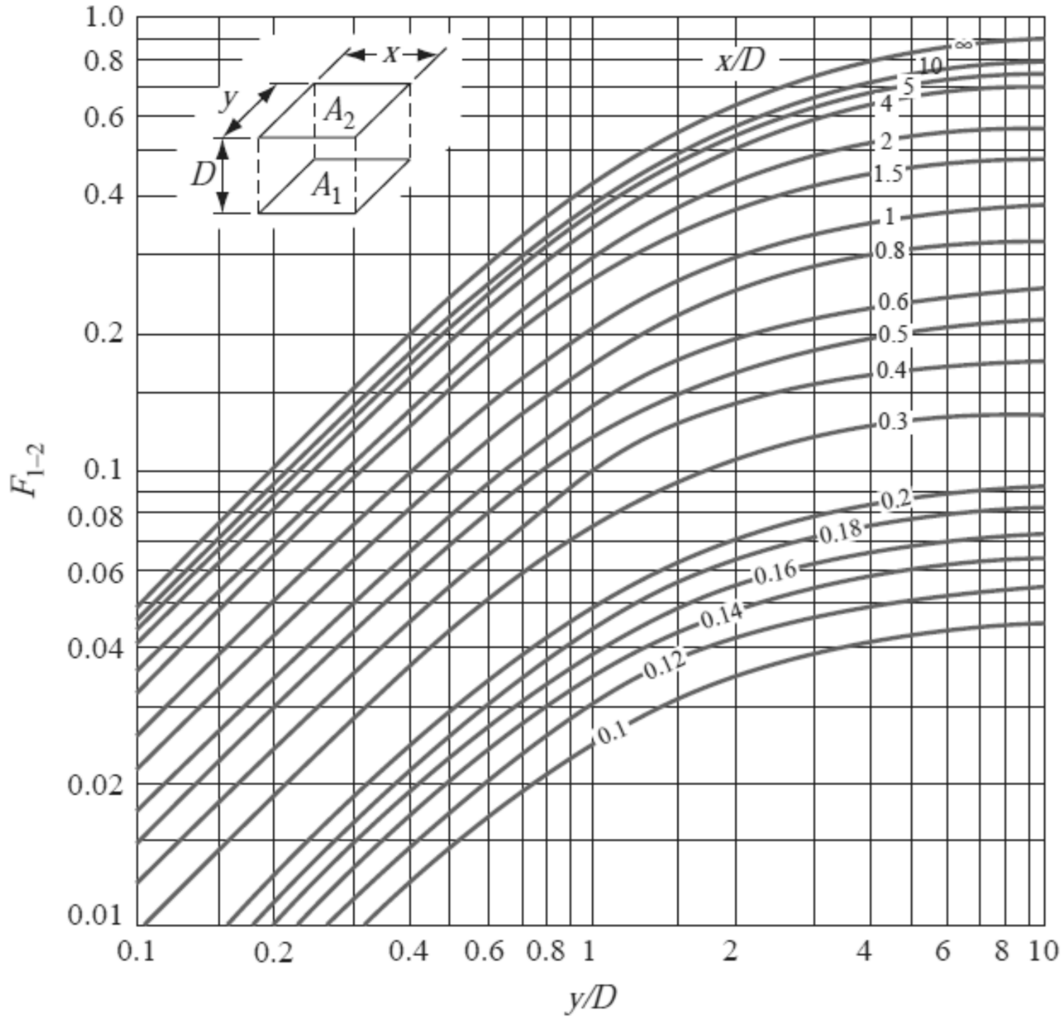


Figure 2 View factor for directly opposed rectangles.



Solutions

P.1 Solution

The net rate of radiation heat transfer between the two surfaces per unit area of the plates is determined as (see *infinitely large parallel plates* in Table 2),

$$\left|\dot{Q}_{12}\right| = \left|A_s \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}\right| \rightarrow \left|\frac{\dot{Q}_{12}}{A_s}\right| = \left|\frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}\right|$$

$$\therefore \left|\frac{\dot{Q}_{12}}{A_s}\right| = \left|\frac{\left(5.67\times10^{-8}\right)\left(600^4 - 400^4\right)}{\dfrac{1}{0.5} + \dfrac{1}{0.9} - 1}\right| = \boxed{2793\ \mathrm{W/m^2}}$$

♦ The correct answer is **B**.

P.2 Solution

The net rate of radiation heat transfer from the enclosure to the liquid nitrogen tank is determined as (see *concentric spheres* in Table 2),

$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^2}{r_2^2}\right)}$$

$$\therefore \dot{Q}_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^2}{r_2^2}\right)} = \frac{\left(4\pi\times0.5^2\right)\times\left(5.67\times10^{-8}\right)\times\left(241^4 - 273^4\right)}{\dfrac{1}{0.01} + \dfrac{1-0.01}{0.01}\times\left(\dfrac{0.5^2}{0.8^2}\right)} = -2.8\ \mathrm{W}$$

$$\therefore \boxed{\left|\dot{\mathrm{Q}}_{12}\right| = 2.8\ \mathrm{W}}$$

The negative sign implies that the heat flux goes from surface 2 to surface 1, not the other way around.

♦ The correct answer is **A**.

P.3 Solution

The latent heat of vaporization of water is h_{fg} = 2257 kJ/kg. Since all the radiation leaving the surface 1 cannot strike itself (the surface is flat), view factor F_{11} is equal to zero, F_{11} = 0. Applying the summation rule for view factors to surface 1, the following relation is obtained,

$$F_{11} + F_{12} = 1$$

Since we have established that F_{11} = 0, it follows that

$$F_{11} + F_{12} = 1 \rightarrow 0 + F_{12} = 1$$

$$\therefore F_{12} = 1$$

Then, we can use the reciprocity relation to obtain the corresponding view factor F_{21},

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{DL}{\pi DL/2}\times1 = \frac{2}{\pi}$$

The heat transfer rate by radiation from surface 2 to surface 1, $\dot{Q}_{21}$, must equal the rate of latent heat of evaporation, $\dot{Q}_{\mathrm{evap}}$. Mathematically,

$$\dot{Q}_{21} = \dot{Q}_{evap}$$

$$\therefore A_2 F_{21} \sigma \left(T_2^4 - T_1^4 \right) = \dot{m} h_{fg}$$

Substituting $A_2 = \pi DL/2$ and $F_{21} = 2/\pi$ gives

$$A_2 F_{21} \sigma \left(T_2^4 - T_1^4 \right) = \dot{m} h_{fg}$$

$$\therefore \left(\frac{\cancel{\pi} DL}{\cancel{2}} \right) \times \left(\frac{\cancel{2}}{\cancel{\pi}} \right) \times \sigma \left(T_2^4 - T_1^4 \right) = \dot{m} h_{fg}$$

$$\therefore DL\sigma \left(T_2^4 - T_1^4 \right) = \dot{m} h_{fg}$$

Solving for the length L, we obtain

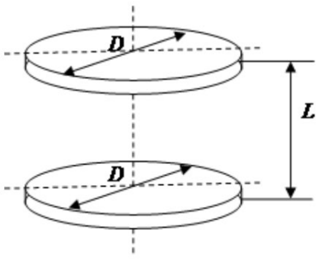
$$DL\sigma \left(T_2^4 - T_1^4 \right) = \dot{m} h_{fg} \rightarrow L = \frac{\dot{m} h_{fg}}{D\sigma \left(T_2^4 - T_1^4 \right)}$$

$$\therefore L = \frac{0.1 \times \left(2257 \times 10^3 \right)}{2 \times \left(5.67 \times 10^{-8} \right) \times \left(773^4 - 313^4 \right)} = \boxed{5.7 \text{ m}}$$

♦ The correct answer is **A**.

P.4 Solution

Consider the following illustration.



The view factor from surface 1 (either disk) to surface 2 (the opposite disk) is obtained with the equation (see *coaxial parallel disks* in Table 1)

$$F_{12} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2}$$

where R is a factor such that $R = D/2L$, in which D is the diameter of the disk and L is the distance that separates the disks. Here, we have $R = 1/(2\times1) = 0.5$ and, consequently,

$$F_{12} = 1 + \frac{1 - \sqrt{4 \times 0.5^2 + 1}}{2 \times 0.5^2} = 0.17$$

The net rate of radiation heat transfer between the two surfaces, $\dot{Q}_{12}$ (or, equivalently, $\dot{Q}_{21}$, because $|\dot{Q}_{12}| = |\dot{Q}_{21}|$ owing to symmetry), is given by

$$\dot{Q}_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4 \right)$$

The expression above shows the variables to which $\dot{Q}_{12}$ is proportional – namely, the surface area A_1, the view factor F_{12}, the Stefan-Boltzmann constant σ, and the temperatures of each surface. In the present case, the area of the disks and the temperature should remain the same, hence the 75% reduction in heat transfer rate can be only due to variation in the view factor F_{12}. The new factor F_{12}' is $F_{12}' = 0.25 \times 0.17 = 0.043$ A new value of R will ensue, and can be obtained with the expression for F_{12},

$$F_{12} = 1 + \frac{1-\sqrt{4R^2+1}}{2R^2} = 0.043$$

$$\therefore \frac{1-\sqrt{4R^2+1}}{2R^2} = -0.957$$

$$\therefore 1-\sqrt{4R^2+1} = -1.91R^2$$

$$\therefore 1+1.91R^2 = \sqrt{4R^2+1} \;\; (\hat{}2)$$

$$\therefore 1+3.82R^2+3.65R^4 = 4R^2+1$$

$$\therefore 3.65R^4 - 0.18R^2 = 0$$

This equation can be easily solved by isolating the squared radius R^2, so that

$$3.65R^4 - 0.18R^2 = 0 \rightarrow \cancel{R^2}\left(3.65R^2 - 0.18\right) = 0$$

$$\therefore R = \sqrt{\frac{0.18}{3.65}} = 0.22$$

Recalling that $R = D/2L$, the new distance L that separates one surface from the other is

$$R = 0.22 = \frac{D}{2L^*} \rightarrow L^* = \frac{1}{2\times 0.22} = \boxed{2.3\text{ m}}$$

Which is to say that, in order to reduce the radiation heat transfer by 75%, the distance between the two disks must be increased by 130%, i.e., more than doubled.

♦ The correct answer is **B**.

P.5 Solution

Part A: The surface area of the tetrahedron can be obtained with knowledge of its edge L only,

$$A_2 = 4\times\left(\frac{\sqrt{3}}{4}L^2\right) = \sqrt{3}L^2$$

where $L = \sqrt{6}D = \sqrt{6}\times 1 = 2.45$ m, and A_2 is determined to be

$$A_2 = \sqrt{3}\times 2.45^2 = 10.4\text{ m}^2$$

We shall consider the sphere as surface 1 and the triangular surface as surface 2. Any radiation that leaves surface 1 will strike surface 2, so that the view factor from surface 1 to surface 2 equals unity, $F_{12} = 1$. The corresponding view factor F_{21} is not necessarily the same as F_{12}, but can be obtained with the reciprocity rule. Given $A_1 = \pi \times 1^2 = 3.14$ m^2, it follows that

$$A_1F_{12} = A_2F_{21} \rightarrow F_{21} = \frac{A_1}{A_2}\times F_{12}$$

$$\therefore F_{21} = \frac{3.14}{10.40}\times 1 = \boxed{0.30}$$

Thus, view factor from surface 2 to surface 1 is 0.30.

♦ The correct answer is **B**.

Part B: the emissivity of the enclosure can be determined with the formula for heat transfer rate in such a system,

$$\dot{Q}=\frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\therefore 3100=\frac{\left(5.67\times10^{-8}\right)\times\left(600^4-420^4\right)}{\frac{1-0.45}{3.14\times0.45}+\frac{1}{3.14\times1}+\frac{1-\varepsilon_2}{10.40\times\varepsilon_2}}$$

Despite being seemingly complicated, the expression above is in fact a first-degree equation in the emissivity ε_2 of the enclosure. Solving it yields ε_2 = 0.081. This is a rather low value, compatible with materials such as polished metals (say, polished chromium, for which $\varepsilon \approx$ 0.06, or polished nickel, for which $\varepsilon \approx$ 0.07).

♦ The correct answer is **A**.

P.6 Solution

Part A: Noting that the radius of the disk r_2 = 120 cm and the distance separating one center from the other h = 60 cm, the view factor from surface 1 to surface 2 can be easily obtained by means of the formula we were given,

$$F_{12}=0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-1/2}\right\}$$

$$\therefore F_{12}=0.5\left\{1-\left[1+\left(\frac{120}{60}\right)^2\right]^{-1/2}\right\}=\boxed{0.28}$$

The corresponding factor F_{21} can be determined with the reciprocity rule, which requires the surface area of the sphere, $A_1=\pi D_1^2=\pi\times0.6^2=1.13$ m^2, and the area of the disk, $A_2=\pi r_2^2=\pi\times1.2^2=$ 4.52 m^2. Thus,

$$A_1F_{12}=A_2F_{21}\rightarrow F_{21}=\frac{A_1}{A_2}\times F_{12}$$

$$\therefore F_{21}=\frac{1.13}{4.52}\times0.28=\boxed{0.07}$$

♦ The correct answer is **C**.

Part B: The net rate of radiation heat transfer between the surfaces can be determined with the formula

$$\dot{Q}_{12}=\frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$\therefore \dot{Q}_{12}=\frac{\left(5.67\times10^{-8}\right)\times\left(873^4-473^4\right)}{\frac{1-0.9}{1.13\times0.9}+\frac{1}{1.13\times0.28}+\frac{1-0.5}{4.52\times0.5}}=\boxed{8650\text{ W}}$$

♦ The correct answer is **D**.

Part C: If we were to paint the surfaces to alter their emissivities, we'd have a paint capable of making the emissivities of the surfaces equal to unity, $\varepsilon_1=\varepsilon_2=1$. Furthermore, the optimal distance for radiative heat transfer between the surfaces would occur if the lower pole of the sphere were tangent to the disk, mathematically implying that $h = r_1$ = 0.30 m. The new view factor F_{12} would then be

$$F_{12}=0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-1/2}\right\}=0.5\left\{1-\left[1+\left(\frac{1.2}{0.30}\right)^2\right]^{-1/2}\right\}=0.38$$

The net rate of radiation becomes

$$\dot{Q}^* = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right) = 1.13 \times 0.38 \times \left(5.67 \times 10^{-8}\right) \times \left(873^4 - 473^4\right) = \boxed{12{,}920 \text{ W}}$$

♦ The correct answer is **C**.

P.7 Solution

The horizontal rectangle is surface 1, the vertical rectangle is surface 2, and the surroundings are associated with number 3. The view factor can be determined with the chart in Figure 1, which requires ratios L_1/W = 0.8/1.6 = 0.5 and L_2/W = 1.2/1.6 = 0.75 (each length is specified in the drawing at the top-left corner of the figure). Using these parameters, we establish the view factor F_{12} = 0.27. We also require the area of rectangle 1, which is $A_1 = L_1 W = 0.8 \times 1.6 = 1.28 \text{ m}^2$, the area of rectangle 2, which is $A_2 = L_2 W = 1.2 \times 1.6 = 1.92 \text{ m}^2$, and the area projections that complete the surface area A_3 of the enclosure,

$$A_3 = \frac{2 \times 1.2 \times 0.8}{2} + \left(\sqrt{0.8^2 + 1.2^2}\right) \times 1.6 = 3.27 \text{ m}^2$$

Applying the summation rule to surface 1, we can determine the view factor F_{13} from surface 1 to surface 3 (i.e., the surroundings),

$$F_{11} + F_{12} + F_{13} = 1$$
$$\therefore 0 + 0.27 + F_{13} = 1$$
$$\therefore F_{13} = 1 - 0.27 = 0.73$$

where F_{11} = 0 because surface 1 is flat. Using F_{12} and the areas for surfaces 1 and 2, we can establish the corresponding view factor F_{21}; that is,

$$A_1 F_{12} = A_2 F_{21} \rightarrow F_{21} = \frac{A_2}{A_1} \times F_{12}$$

$$\therefore F_{21} = \frac{1.28}{1.92} \times 0.27 = 0.18$$

Then, we can apply the summation rule a second time, this time to surface 2,

$$F_{21} + F_{22} + F_{23} = 1$$
$$\therefore 0.18 + 0 + F_{23} = 1$$
$$\therefore F_{23} = 1 - 0.18 = 0.82$$

where we have used F_{22} = 0 yet again because this surface is flat. The reciprocity rule can be used to calculate F_{31},

$$A_1 F_{13} = A_3 F_{31} \rightarrow F_{31} = \frac{A_1}{A_3} \times F_{13}$$

$$\therefore F_{31} = \frac{1.28}{3.27} \times 0.73 = 0.29$$

Finally, using the reciprocity rule a third time, we can determine view factor F_{32},

$$A_2 F_{23} = A_3 F_{32} \rightarrow F_{32} = \frac{A_2}{A_3} \times F_{23}$$

$$\therefore F_{32} = \frac{1.92}{3.27} \times 0.82 = 0.48$$

Since we have the temperatures and view factors, we are able to use the "direct method" of solution for a radiation problem, which simply consists of taking all the available expressions and solving the ensuing system of algebraic equations simultaneously. Applying the equation of heat transfer to surface 1, for example, we have

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}\left(J_1 - J_2\right) + F_{13}\left(J_1 - J_3\right)\right]$$

where J_i are the radiosities associated with surfaces *i*, ε_1 = 0.75 is the emissivity of surface 1, $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}$ is the Stefan-Boltzmann constant, and F_{ab} is the view factor from surface *a* to surface *b*. Substituting σ = 5.67×10^{-8} W/m^{2}K^4, T_1 = 400 K, ε_1 = 0.75, F_{12} = 0.27 and F_{13} = 0.73, we get

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}\left(J_1 - J_2\right) + F_{13}\left(J_1 - J_3\right)\right]$$

$$\therefore \left(5.67\times10^{-8}\right)\times 400^4 = J_1 + \frac{1-0.75}{0.75}\times\left[0.27\left(J_1 - J_2\right) + 0.73\left(J_1 - J_3\right)\right] \text{ (I)}$$

Similarly, we apply the radiation heat equation to surface 2, which is much more straightforward,

$$\sigma T_2^4 = J_2$$

$$\therefore J_2 = \left(5.67\times10^{-8}\right)\times 550^4 \text{ (II)}$$

Finally, we apply the same equation to surface 3,

$$\sigma T_3^4 = J_3 + \frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}\left(J_3 - J_1\right) + F_{32}\left(J_3 - J_2\right)\right]$$

$$\therefore \left(5.67\times10^{-8}\right)\times 290^4 = J_3 + \frac{1-0.85}{0.85}\left[0.29\times\left(J_3 - J_1\right) + 0.48\times\left(J_3 - J_2\right)\right] \text{ (III)}$$

Notice that we have three equations and three unknowns. Solving them simultaneously (for example, by using the command *Solve* in Mathematica), the results are

$$J_1 = 1587\ \mathrm{W/m^2}\ ;\ J_2 = 5188\ \mathrm{W/m^2}\ ;\ J_3 = 811\ \mathrm{W/m^2}$$

To obtain the net rate of radiation heat transfer between the horizontal surface and the surroundings, we resort to the formula

$$\left|Q_{13}\right| = \left|A_1 F_{13}\left(J_1 - J_3\right)\right| = \left|\left(0.8\times1.6\right)\times0.73\times\left(1587-811\right)\right| = \boxed{725\ \mathrm{W}}$$

The heat transfer rate between the horizontal surface and the surroundings is somewhat above 700 watts.

♦ The correct answer is **C**.

P.8 Solution

Part A: The gap between the plates can be considered to be a blackbody square prismatic surface. We have $\rho_3 = 0$ (the reflectivity of medium 3 is zero) and, consequently,

$$\rho_3 = 0 \rightarrow \varepsilon_3 = 1$$

since

$$J_3 = E_{b,3} = \sigma T_3^4 = 0$$

Further, 1 and 2 are flat surfaces, so F_{11} = 0 and F_{22} = 0. Given the symmetry of the problem, the reciprocity rule enables us to write $F_{12} = F_{21}$. The product $A_i G_i$ for a given body, G_i being the incident radiation, follows the sum

$$A_i G_i = \sum J_i A_i F_{ij}$$

Hence, we have the equations

$$A_1G_1 = \cancel{J_1A_1F_{11}} + J_2A_2F_{21} + \cancel{J_3A_3F_{31}} \rightarrow A_1G_1 = J_2A_2F_{21} \quad \text{(I)}$$

$$A_2G_2 = J_1A_1F_{12} + \cancel{J_2A_2F_{22}} + \cancel{J_3A_3F_{32}} \rightarrow A_2G_2 = J_1A_1F_{12} \quad \text{(II)}$$

$$A_3G_3 = J_1A_1F_{13} + J_2A_2F_{23} + \cancel{J_3A_3F_{33}} \rightarrow A_3G_3 = J_1A_1F_{13} + J_2A_2F_{23} \quad \text{(III)}$$

Similarly, for opaque bodies that transmit no radiation, the radiosity from a typical surface *i* can be expressed as

$$J_i = \rho_i G_i + \varepsilon_i E_{b,i}$$

which, when applied to the three participating media, yields

$$J_1 = \rho_1 G_1 + \varepsilon_1 E_{b,1} \quad \text{(IV)}$$

$$J_2 = \rho_2 G_2 + \varepsilon_2 E_{b,2} \quad \text{(V)}$$

$$J_3 = 0 \quad \text{(VI)}$$

Substituting equations (IV) and (V) into (I) and (II), the equations for the product of area and irradiation become

$$A_1G_1 = \left(\rho_2 G_2 + \varepsilon_2 E_{b,2}\right) A_2 F_{21}$$

$$A_2G_2 = \left(\rho_1 G_1 + \varepsilon_1 E_{b,1}\right) A_1 F_{12}$$

Then, we substitute G_1 from the first equation into the second and take advantage of the fact that $F_{12} = F_{21}$, giving

$$A_2G_2 = \left[\rho_1 \frac{A_2}{A_1} F_{12}\left(\rho_2 G_2 + \varepsilon_2 E_{b,2}\right) + \varepsilon_1 E_{b,1}\right] A_1 F_{12}$$

and, since $A_1 = A_2$ and $E_{b,i} = \sigma T_i^4$, irradiation G_2 follows as

$$G_2 = \frac{\sigma F_{12}\left(\varepsilon_2 T_2^4 F_{12}\rho_1 + \varepsilon_1 T_1^4\right)}{1-\left(F_{12}\right)^2 \rho_2\rho_1}$$

Note that we need factor F_{12} to proceed. This can be obtained with the graph in Figure 2; given *x/D* = *y/D* = 5 we read F_{12} = 0.71. Further, we have reflectivities ρ_1 = 1 – ε_1 = 1 – 0.5 = 0.5 and ρ_2 = 1 – ε_2 = 0.5. Substituting the available data in the expression for G_2, the result is

$$G_2 = \frac{\sigma F_{12}\left(\varepsilon_2 T_2^4 F_{12}\rho_1 + \varepsilon_1 T_1^4\right)}{1-\left(F_{12}\right)^2 \rho_2\rho_1} = \frac{\left(5.67\times10^{-8}\right)\times0.71\times\left(0.8\times500^4\times0.71\times0.5+0.5\times1100^4\right)}{1-0.71^2\times0.2\times0.5} = 31,787 \text{ W/m}^2$$

We now have enough information to determine the radiosity J_2 by means of equation (V); that is,

$$J_2 = \rho_2 G_2 + \varepsilon_2 E_{b,2} = \rho_2 G_2 + \varepsilon_2 \sigma T_2^4$$

$$\therefore J_2 = 0.2\times31,787 + 0.8\times\left(5.67\times10^{-8}\right)\times500^4 = 9192 \text{ W/m}^2$$

From equation (I), G_1 is determined to be

$$\cancel{A_1}G_1 = J_2\cancel{A_2}F_{21} \rightarrow G_1 = J_2F_{21} = 9192\times0.71 = 6526 \text{ W/m}^2$$

Radiosity J_1 is calculated with equation (IV),

$$J_1 = \rho G_1 + \varepsilon_1\sigma T_1^4 = 0.5\times6526 + 0.5\times\left(5.67\times10^{-8}\right)\times1100^4 = 44,770 \text{ W/m}^2$$

Then, the net heat exchange from surface 1 to surface 2, Q_{12}, is calculated as

$$\dot{Q}_{12} = \left(J_1 - J_2\right) A_1 F_{12}$$

where the radiosities are J_1 = 44,770 W/m^2 and J_2 = 9192 W/m^2, the area A_1 = 1.5×1.5 = 2.25 m^2, and F_{12} = 0.71. Accordingly,

$$\dot{Q}_{12} = \left(J_1 - J_2\right) A_1 F_{12} = \left(44,770 - 9192\right) \times 2.25 \times 0.71 = \boxed{56,840\ \text{W}}$$

♦ The correct answer is **D**.

Part B: The required input to surface A_1 is equal to the rate of radiative loss from surface A_1, which is given by the product $A_1(J_1 - G_1)$; that is,

$$Q_1 = A_1\left(J_1 - G_1\right) = \left(1.5 \times 1.5\right) \times \left(44,770 - 6526\right) = \boxed{86,050\ \text{W}}$$

♦ The correct answer is **D**.

P.9 Solution

We have three thin sheets of polished aluminum, each separated from the other by a fixed distance. The plates may be taken as infinitely large plates, so that the view factors F_{12} = F_{21} = F_{23} = F_{32} = 1.0. In a simple system such as the present one, the net rate of heat transfer from surface 2 must total zero, so we can state that

$$\dot{Q}_2 = \dot{Q}_{21} + \dot{Q}_{23} = A_2 F_{21}\left(E_{b,2} - E_{b,1}\right) + A_2 F_{23}\left(E_{b,2} - E_{b,3}\right) = 0$$

Due to symmetry, however, we know that the view factor from 2 to 1, F_{21}, must equal the view factor from 2 to 3, F_{23}. Applying this result to the equation above, we obtain

$$A_2 F_{21}\left(E_{b,2} - E_{b,1}\right) + A_2 F_{23}\left(E_{b,2} - E_{b,3}\right) = 0$$

$$\therefore E_{b,2} - E_{b,1} + E_{b,2} - E_{b,3} = 0$$

$$\therefore -E_{b,1} + 2E_{b,2} - E_{b,3} = 0 \quad \times(-1)$$

$$\therefore E_{b,1} - 2E_{b,2} + E_{b,3} = 0$$

$$\therefore T_1^4 - 2T_2^4 + T_3^4 = 0$$

where the terms with the same color have the same value and were therefore canceled. The equation above can be easily solved for the temperature T_2 of the middle sheet,

$$T_1^4 - 2T_2^4 + T_3^4 = 0$$

$$\therefore 2T_2^4 = T_1^4 + T_3^4$$

$$\therefore T_2 = \left(\frac{T_1^4 + T_3^4}{2}\right)^{1/4} = \left(\frac{333^4 + 553^4}{2}\right)^{1/4} = 480\ \text{K}$$

For infinitely large parallel plates, the view factor F_{12} is a function of the emissivities of the surfaces involved. Noting that $\varepsilon_1 = \varepsilon_2 = \varepsilon$, we have (refer to *infinitely large parallel plates* in Table 2),

$$F_{12} = \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{1}{\dfrac{2}{\varepsilon} - 1} = \frac{1}{\dfrac{2}{(0.05)} - 1} = 0.0256$$

Finally, the rate of heat transfer per unit area is

$$\frac{\dot{Q}_{12}}{A} = F_{12}\sigma\left(T_1^4 - T_2^4\right) = 0.0256 \times \left(5.67 \times 10^{-8}\right) \times \left(553^4 - 480^4\right) = \boxed{58.7\ \text{W/m}^2}$$

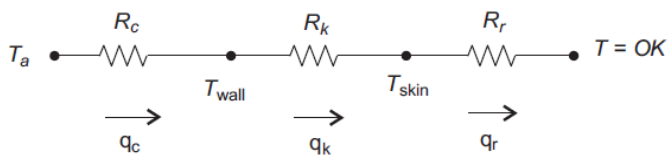
♦ The correct answer is **B**.

P.10 Solution

Since $D \gg t$, the effect of the cylinder's curvature can be neglected, and the total surface area, A, will be

$$A = \pi DL + 2\times\frac{\pi\times D^2}{4} = \pi\times 2.5\times 9 + 2\times\frac{\pi\times 2.5^2}{4} = 80.5\ \text{m}^2$$

The thermal circuit for the problem is shown below.



Here, the resistance components are the convective thermal resistance R_c, the conductive thermal resistance of the insulation R_k, and the radiative thermal resistance R_r. In addition, q_c is the convective heat transfer to the interior wall (= $h_c A(T_a - T_{\text{wall}})$), q_k is the conductive heat transfer to the insulation (= $(k/t)A(T_{\text{wall}} - T_{\text{skin}})$), and q_r is the radiative heat transfer from the skin (= $\sigma\varepsilon T_{\text{skin}}^4$). For steady state, all three rates of heat transfer must be equal, so we can write

$$h_c\left(T_a - T_{\text{wall}}\right) = \frac{k}{t}\left(T_{\text{wall}} - T_{\text{skin}}\right) = \sigma\varepsilon T_{\text{skin}}^4$$

Taking the first two equations (left to right) and solving for T_{wall}, we obtain

$$T_{\text{wall}} = \frac{T_a + \dfrac{k}{th_c}T_{\text{skin}}}{1+\dfrac{k}{th_c}} = \frac{T_a + BT_{\text{skin}}}{1+B}$$

where $B = k/th_c = 0.017/(0.15\times 17) = 0.00667$. (Since this is merely a simplifying factor meant to facilitate calculations, we ignore its dimensions.) Then, equating the first and last equations and substituting T_{wall} with the foregoing relation, we obtain

$$h_c\left(T_a - T_{\text{wall}}\right) = \sigma\varepsilon T_{\text{skin}}^4$$

$$\therefore h_c\left(T_a - \frac{T_a + BT_{\text{skin}}}{1+B}\right) = \sigma\varepsilon T_{\text{skin}}^4$$

$$\therefore \sigma\varepsilon T_{\text{skin}}^4 - h_c\left(T_a - \frac{T_a + BT_{\text{skin}}}{1+B}\right) = 0$$

Note that we are bound to obtain a fourth-degree polynomial for the skin temperature T_{skin}. Substituting σ = 5.67×10^{-8} W/m^{2}K, ε = 0.05, h_c = 17 W/m^{2}K, T_a = 293 K, and the value of B obtained just now, we have

$$\left(5.67\times10^{-8}\right)\times0.05\times T_{\text{skin}}^4 - 17\times\left(293 - \frac{293 + 0.00667\times T_{\text{skin}}}{1+0.00667}\right) = 0$$

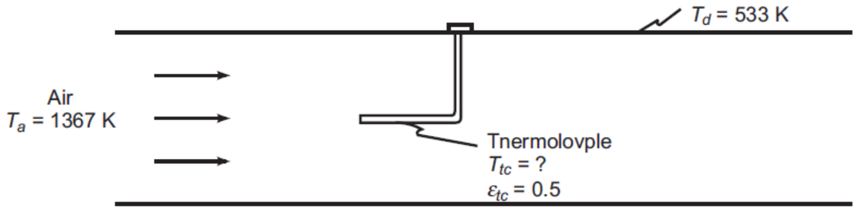
The only feasible solution for this polynomial is T_{skin} = 226.6 K. Finally, the heat transfer into space can be determined with the expression

$$\dot{Q} = \sigma\varepsilon A T_{\text{skin}}^4 = \left(5.67\times10^{-8}\right)\times0.05\times80.5\times226.6^4 = \boxed{601.7\ \text{W}}$$

♦ The correct answer is **C**.

P.11 Solution

Part A: The system in question is illustrated below.



In steady state, the heat gain by radiation must equal the heat loss by convection, so we can write

$$Q_{\text{rad}} = Q_{\text{conv}} \rightarrow h_c \cancel{A}\left(T_a - T_{tc}\right) = \sigma\varepsilon_{tc}\cancel{A}\left(T_{tc}^4 - T_d^4\right)$$

$$\therefore h_c\left(T_a - T_{tc}\right) = \sigma\varepsilon_{tc}\left(T_{tc}^4 - T_d^4\right)$$

Substituting each variable gives

$$h_c\left(T_a - T_{tc}\right) = \sigma\varepsilon_{tc}\left(T_{tc}^4 - T_d^4\right)$$

$$\therefore 114\times\left(1367 - T_{tc}\right) = \left(5.67\times10^{-8}\right)\times0.5\times\left(T_{tc}^4 - 533^4\right)$$

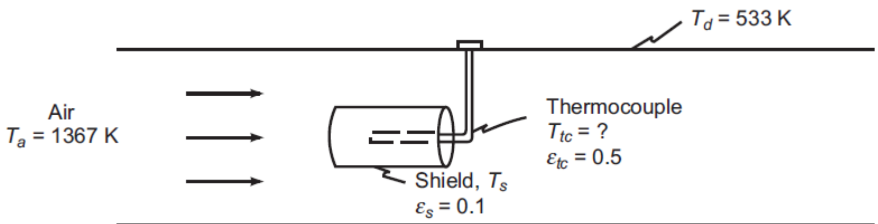
$$\therefore 155{,}838 - 114T_{tc} = -2288.03 + 2.84\times10^{-8}T_{tc}^4$$

$$\therefore 2.84\times10^{-8}T_{tc}^4 + 114T_{tc} - 158{,}126 = 0$$

This is a fourth-order polynomial in T_{tc}. Its solutions are two imaginary numbers, one negative number, and T_{tc} = 1066 K, which is the answer we seek. Relatively to the air temperature of 1367 K, we have 1066/1367 ≈ 78%, which means that the temperature we have obtained underestimates the actual air temperature by 22%. Assuming the purpose of the thermocouple is to measure the temperature of air flowing in the duct, we have an error of 301 K – an undeniably expressive deviation from the real value. This so-called *thermocouple radiation error* can be reduced by increasing the convective heat transfer coefficient via higher air velocity, by reducing the thermocouple emissivity, or by the addition of a radiation shield, as shown in the following part.

♦ The correct answer is **C**.

Part B: The updated system is illustrated below.



Let A_s = inside area of the shield ≈ outside area of the shield. A heat balance on the radiation shield yields

$$Q_s = h_c \times 2A_s\left(T_a - T_s\right) = \sigma\,\varepsilon_s A_s\left(T_s^4 - T_d^4\right)$$

in which $h_{c,s}$ = 114 W/m^{2}K is the heat transfer coefficient, T_a = 1367 K, T_s = 1367 K, T_d = 533 K and σ = 5.67×10^{-8} W/m^{2}K. Q_s is the radiative heat transfer from the shield to its surroundings, which, in the case of long concentric cylinders, is given by (this is a small modification of *infinitely long concentric cylinders* in Table 2)

$$Q_s = \frac{A_{tc}\left(E_{b,tc} - E_{b,s}\right)}{\dfrac{1}{\varepsilon_{tc}} + \dfrac{A_{tc}}{A_s}\left(\dfrac{1-\varepsilon_s}{\varepsilon_s}\right)}$$

However, the area term for the surface of the shield, A_s, is substantially greater than A_{tc}, i.e., $A_{tc}/A_s \ll 1$. This enables us to simplify the equation above to

$$Q_s = \frac{A_{tc}\left(E_{b,tc}-E_{b,s}\right)}{\dfrac{1}{\varepsilon_{tc}} + \underbrace{\cancel{\dfrac{A_{tc}}{A_s}}}_{\approx 0}\left(\dfrac{1-\varepsilon_s}{\varepsilon_s}\right)} = A_{tc}\varepsilon_{tc}\left(E_{b,tc}-E_{b,s}\right) \to Q_s = A_{tc}\varepsilon_{tc}\sigma\left(T_{tc}^4 - T_s^4\right)$$

Substituting this expression for Q_s in the energy balance and dividing by A_s, it follows that

$$\underbrace{\frac{A_{tc}}{A_s}}_{\approx 0}\varepsilon_{tc}\sigma\left(T_{tc}^4 - T_s^4\right) + 2h_c\left(T_a - T_s\right) = \sigma\varepsilon_s\left(T_s^4 - T_d^4\right)$$

$$\therefore 2h_c\left(T_a - T_s\right) \approx \sigma\varepsilon_s\left(T_s^4 - T_d^4\right)$$

It is shown that since the thermocouple is small compared to the shield, the area ratio A_{tc}/A_s approaches zero and the leftmost term in the equation can be ignored altogether. Substituting the pertaining variables, we get

$$2h_s\left(T_a - T_s\right) = \sigma\varepsilon_s\left(T_s^4 - T_d^4\right)$$

$$\therefore 2\times 114\times\left(1367 - T_s\right) = \left(5.67\times 10^{-8}\right)\times 0.1\times\left(T_s^4 - 533^4\right)$$

$$\therefore 5.67\times 10^{-9}T_s^4 + 228T_s - 312{,}134 = 0$$

As before, we arrive at a fourth-degree polynomial in the surface temperature T_s. Its solutions are two imaginary numbers, one negative number, and T_s = 1298 K, which can only be the answer we want. Finally, in order to determine the thermocouple temperature T_{tc}, we perform a heat balance in that region of the system, giving

$$\dot{Q}_{\text{conv}} = \dot{Q}_{\text{rad}}$$

$$\therefore h_c \cancel{A_{tc}}\left(T_s - T_{tc}\right) = \sigma\varepsilon_{tc}\cancel{A_{tc}}\left(T_{tc}^4 - T_s^4\right)$$

$$\therefore h_c\left(T_s - T_{tc}\right) = \sigma\varepsilon_{tc}\left(T_{tc}^4 - T_s^4\right)$$

$$\therefore 114\times\left(1367 - T_{tc}\right) = \left(5.67\times 10^{-8}\right)\times 0.5\times\left(T_{tc}^4 - 1298^4\right)$$

$$\therefore 155{,}838 - 114T_{tc} = -80{,}473 + 2.84\times 10^{-8}T_{tc}^4$$

$$\therefore 2.84\times 10^{-8}T_{tc}^4 + 114T_{tc} - 236{,}311 = 0$$

This time, the feasible solution is T_{tc} = 1319 K. The thermocouple error is now 1319.0/1367.0 = 0.965; that is, the thermocouple measurement is now less than 4% away from the actual air temperature. There is an appreciable improvement relatively to the previous configuration.

♦ The correct answer is **C**.

P.12 Solution

Part A: Neglecting convection, the rate of heat transfer is given by

$$Q_r = \sigma\varepsilon A\left(T_w^4 - T_s^4\right)$$

$$\therefore h_r = \frac{Q_r}{A\left(T_w - T_s\right)} = \sigma\,\varepsilon\left[\frac{T_w^4 - T_s^4}{T_w - T_s}\right]$$

Hence, for final conditions, with T_s = 120°C = 393 K, along with T_w = 150°C = 423 K, the heat transfer coefficient is determined as

$$h_{r,f} = \left(5.67\times 10^{-8}\right)\times 0.15\times\left(\frac{423^4 - 393^4}{423 - 393}\right) = 2.31\ \text{W/m}^2\text{K}$$

For initial conditions, the convection coefficient is

$$h_{r,i} = \left(5.67\times10^{-8}\right)\times0.15\times\left(\frac{423^4-283^4}{423-283}\right) = 1.56\ \mathrm{W/m^2K}$$

The Biot number based on half of the sheet thickness is

$$\mathrm{Bi} = \frac{\overline{h_{r,\max}}s}{2k_s}$$

where $h_{r,\max}$ = 2.31 W/m^{2}K, s = 6 mm is the thickness of the sheet, and k_s = 14.4 W/mK is the thermal conductivity of steel. Thus,

$$\mathrm{Bi} = \frac{\overline{h_{r,\max}}t}{2k_s} = \frac{2.31\times\left(6\times10^{-3}\right)}{2\times14.4} = 0.000481 \ll 0.1$$

Consequently, the internal thermal resistance of the steel sheet may be neglected. The temperature change of the sheet over a small time step follows as

$$\Delta T = \frac{q\Delta t}{mc} = \frac{\sigma\varepsilon A\left(T_w^4-T_s^4\right)\Delta t}{\rho\forall c} = \frac{2\sigma\varepsilon\left(T_w^4-T_s^4\right)\Delta t}{\rho sc}$$

where ∀ denotes volume. Substituting the pertaining variables, we obtain

$$\Delta T = \frac{\sigma\varepsilon\left(T_w^4-T_s^4\right)\Delta t}{\rho sc} = \frac{2\times\left(5.67\times10^{-8}\right)\times0.15\times\left(424^4-T_s^4\right)\Delta t}{7817\times\left(6\times10^{-3}\right)\times565} = 6.42\times10^{-13}\left(424^4-T_s^4\right)\Delta t$$

$$\therefore \Delta T = \left(0.0207-6.42\times10^{-13}T_s^4\right)\Delta t$$

As the plate heats up, the rate of heat transfer will diminish. The following procedure should be employed until $T_s = T_{s,f}$:

1. Let Δt = 20 min = 1200 s;
2. Calculate ΔT using $T_{s,i}$ = 283 K (the initial temperature of the metal);
3. Update T_s by using the relationship $T_s = T_{s,i} + \Delta T$;
4. Use the new T_s to calculate a new ΔT and repeat the procedure.

These calculations are carried out in the following table.

***t* (min)**	***DeltaT* (K)**	***Ts* (K)**
0.0	–	283.0
20.0	19.9	302.9
40.0	18.4	321.3
60.0	16.6	337.9
80.0	14.8	352.7
100.0	12.9	365.6
120.0	11.1	376.7
140.0	9.3	386.0
158.0	7.0	393.0

Computations halt at t = 158.0 min or about 2 hours and 40 minutes, at which point the metal surface temperature will have reached its final value of 120°C (= 393.0 K).

♦ The correct answer is **D**.

Part B: The rate of heat transfer, in this case, is the sum of heat transfer rates due to convection and radiation,

$$\dot{Q} = \dot{Q}_c + \dot{Q}_r = h_c A\left(T_w - T_s\right) + \sigma\varepsilon A\left(T_w^4 - T_s^4\right)$$

$$\therefore \Delta T = \frac{(Q_c+Q_r)\Delta t}{mc} = \frac{2h_c\left(T_w-T_s\right)+2\sigma\varepsilon\left(T_w^4-T_s^4\right)}{\rho sc}\Delta t$$

Substituting the pertaining variables (which, in addition to the preceding variables, includes the heat transfer coefficient h_c = 3 W/mK), it follows that

$$\Delta T = \frac{2\times3\times\left(423-T_s\right)+2\times\left(5.67\times10^{-8}\right)\times0.15\times\left(423^4-T_s^4\right)}{7817\times\left(6\times10^{-3}\right)\times565}\Delta t$$

$$\therefore \Delta T = \left(-6.42\times10^{-13}T_s^4-0.000226T_s+0.12\right)\Delta t$$

Following the procedure of the previous part, we let Δt = 10 min = 600 s initially (the temperature should be reached in a shorter period of time, because the convection component adds further heat to the system). Then, the table below is prepared.

t **(min)**	***DeltaT*** **(K)**	***Ts*** **(K)**
0.0	–	283.0
10.0	31.2	314.2
20.0	25.6	339.8
30.0	20.8	360.6
40.0	16.6	377.2
50.0	13.1	390.2
52.0	2.0	392.3
53.0	1.0	393.2

Computations halt at 53 minutes, at which point the surface will have surpassed the desired value of 120 K (= 393°C). The decrease in the time needed to attain the desired temperature is sensible relatively to the previous problem.

♦ The correct answer is **A**.

Answer Summary

Problem 1		B
Problem 2		A
Problem 3		A
Problem 4		B
Problem 5	5A	B
	5B	A
Problem 6	6A	C
	6B	D
	6C	C
Problem 7		C
Problem 8	8A	D
	8B	D
Problem 9		B
Problem 10		C
Problem 11	11A	C
	11B	C
Problem 12	12A	D
	12B	A

References

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