Quiz HD102

Hydraulic Structures & Design of Open Channels

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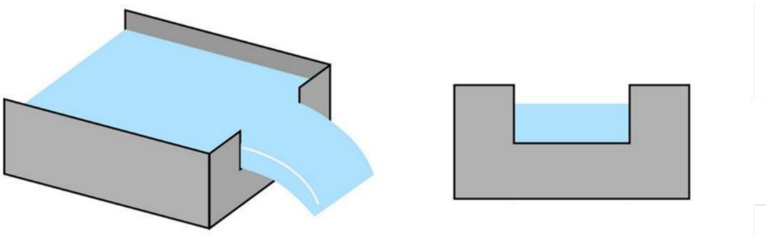
Problems

Problem 1

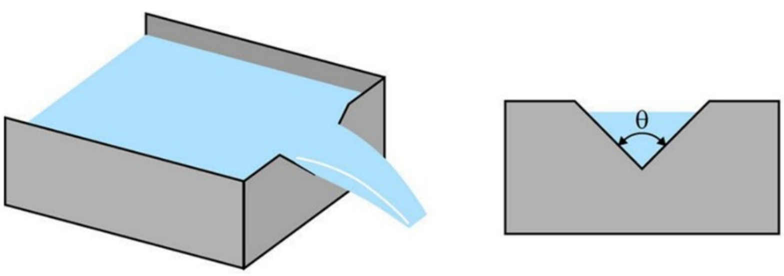
A trapezoidal irrigation canal has a bottom width of b = 1.6 m, side slopes of m = 2 (2H:1V), and a longitudinal bottom slope of S_0 = 0.0008. A rectangular sharp-crested weir is installed in this channel. The structure has a crest height of p = 0.80 m and a crest length of L_w = 1.04 m. The water surface elevation at the approach section is h_0 = 0.96 m above the weir crest. Determine the discharge in the canal.



A) Q = 0.26 m^3/s
B) Q = 0.78 m^3/s
C) Q = 1.73 m^3/s
D) Q = 2.24 m^3/s

Problem 2

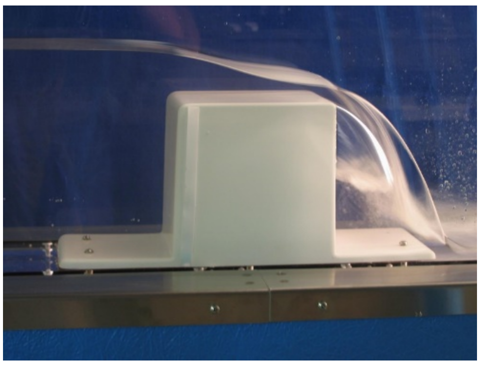
A 45° V-notch weir is installed in a 1.2-m wide rectangular laboratory flume. The crest height is 0.6 m and the water surface elevation at the approach section is 0.23 m above the crest. Determine the discharge in the weir.



A) Q = 9.42 L/s
B) Q = 14.6 L/s
C) Q = 19.1 L/s
D) Q = 24.7 L/s

Problem 3

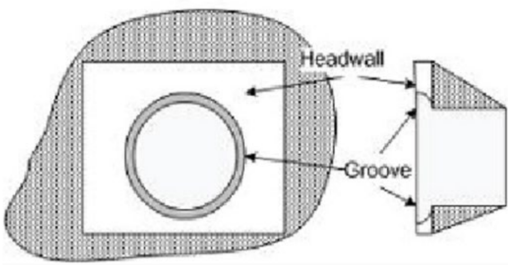
A broad-crested weir has a crest length L_b = 0.85 m, crest width L_w = 1.2 m, and crest height P = 0.30 m. The water surface is at a height of 0.26 m above the crest. Find the discharge.



A) Q = 202 L/s
B) Q = 273 L/s
C) Q = 347 L/s
D) Q = 396 L/s

Problem 4

A circular concrete culvert has diameter D = 1 m and slope S = 0.0025. The culvert has a groove end with headwall, as illustrated below. The inlet is not mitered to an embankment slope. Determine the headwater depth, HW, when the culvert conveys Q = 0.8 m^3/s under inlet control conditions.



A) HW = 0.34 m
B) HW = 0.73 m
C) HW = 1.12 m
D) HW = 1.47 m

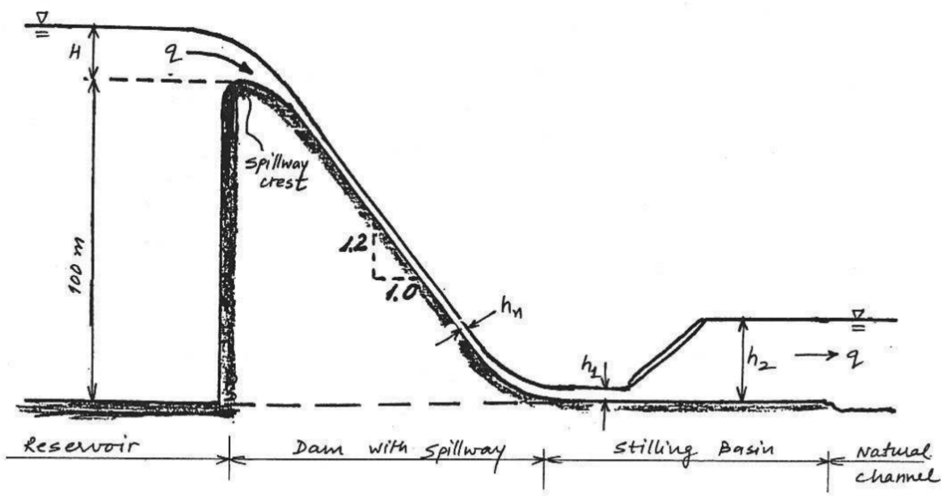
Problem 5

A rectangular box culvert has the following properties: flow depth D = 1.0 m, width b = 1.5 m, length L = 60 m, and slope S = 0.0035. The culvert is made of reinforced concrete. The inlet is square-edged on three edges with a headwall parallel to the embankment, and the outlet is submerged with tailwater depth TW = 1.3 m. Determine the headwater depth, HW, when the culvert is flowing full at Q = 3.2 m^3/s.

A) HW = 1.67 m
B) HW = 2.13 m
C) HW = 2.65 m
D) HW = 3.16 m

Problem 6

Design a spillway section for a design discharge of 1200 m^3/s, assuming that the upstream water surface level is at elevation 320 m, and the upstream channel floor is at an elevation of 280 m. The spillway has a vertical face and is 65-m long.



Problem 7

An unlined channel to be excavated in gravelly soil overlain by Bermuda grass will convey a discharge of 125 m^3/s over a slope of 0.001. Proportion the channel dimensions using the maximum permissible velocity method. Assume the channel to be straight.

Problem 8

Using the tractive force approach, design a slightly sinuous channel in fine gravel that conveys a discharge of 65 m^3/s. The bottom slope is 0.0002, and the particle size is 7.5 mm. Assume that the soil particles are slightly rounded.

Problem 9

Design a trapezoidal roadside channel lined with very coarse gravel mulch. The following data are given: discharge Q = 0.42 m^3/s, bottom width b = 0.4 m, side slope m = 3, longitudinal slope S_0 = 0.008 m/m, and d_{50} = 50 mm.

Problem 10

A trapezoidal grass-lined channel section has a bottom width of 6 m and side slopes of 3:1. The channel slope is 0.009 and the proposed channel lining is good stand 45-cm lespedeza sericea. What is the maximum allowable discharge for this lining?

A) Q = 26.7 m^3/s

B) Q = 37.8 m^3/s

C) Q = 48.3 m^3/s

D) Q = 58.2 m^3/s

Additional Information

► Theory for weirs and culverts (probs. 1 to 5) can be obtained in Chapter 6 of Akan's *Open Channel Hydraulics* (2006), pp. 200 to 209 and 212 to 224.
► Theory for spillways (prob. 6) can be obtained in Part 7 of the Lecture Notes prepared by Prof. Atil Bulu of Istanbul Technical University (https://web.itu.edu.tr/~bulu/water_resources_files/lecture_notes_07.pdf, retrieved 09/23/19).
► Theory for channel design by the permissible velocity (probs. 7 and 8) and tractive force methods can be found in Chapter 9 of Chaudhry's *Open-Channel Flow* (2008), pp. 286 to 295.
► Theory for design of roadside channels with small discharge (prob. 9) can be obtained in the FHWA's publication *Design of Roadside Channels with Flexible Linings* (2005) (https://www.fhwa.dot.gov/engineering/hydraulics/pubs/05114/05114.pdf, retrieved 09/23/19).
► Theory for design of channels with vegetative linings (prob. 10) can be found in Chapter 4 of Sturm's *Open Channel Hydraulics* (2009), pp. 132 to 136.

Figure 1 Weir length correction.

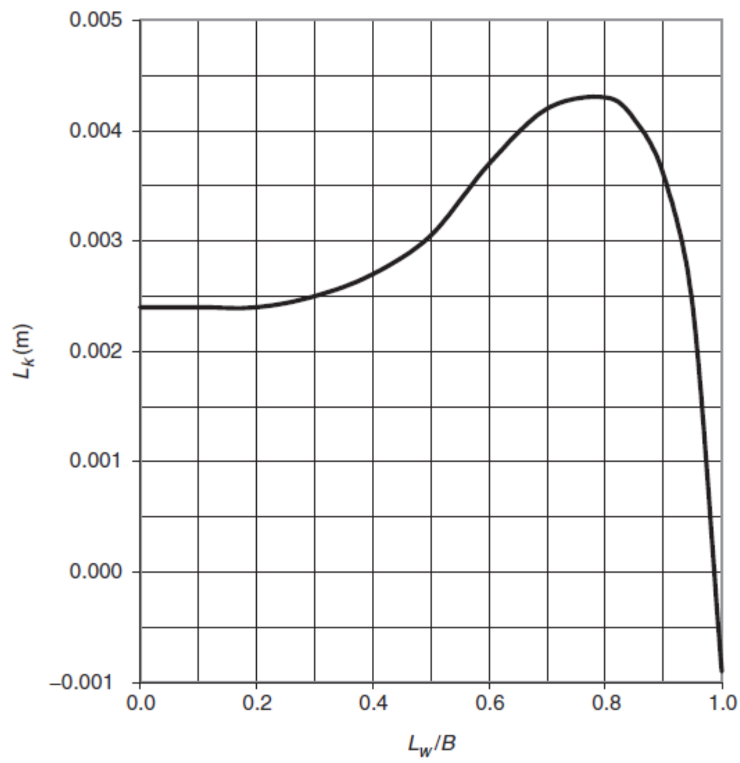


Figure 2 Weir discharge coefficient for sharp-crested rectangular weirs.

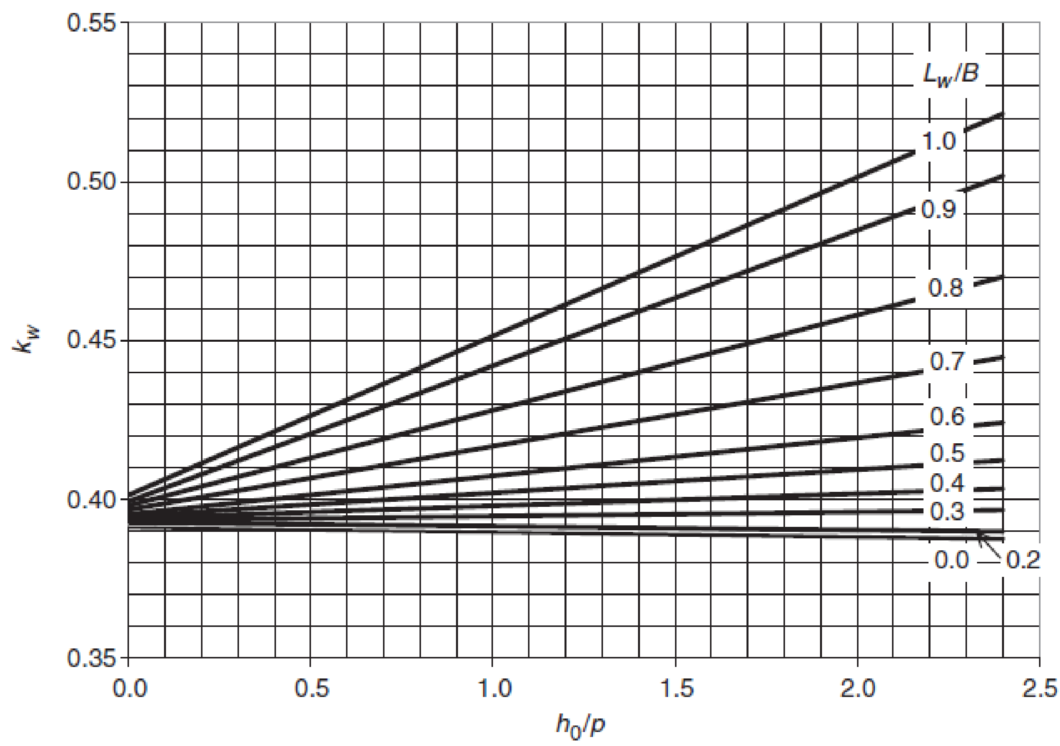


Figure 3 Head correction for sharp-crested V-notch weirs.

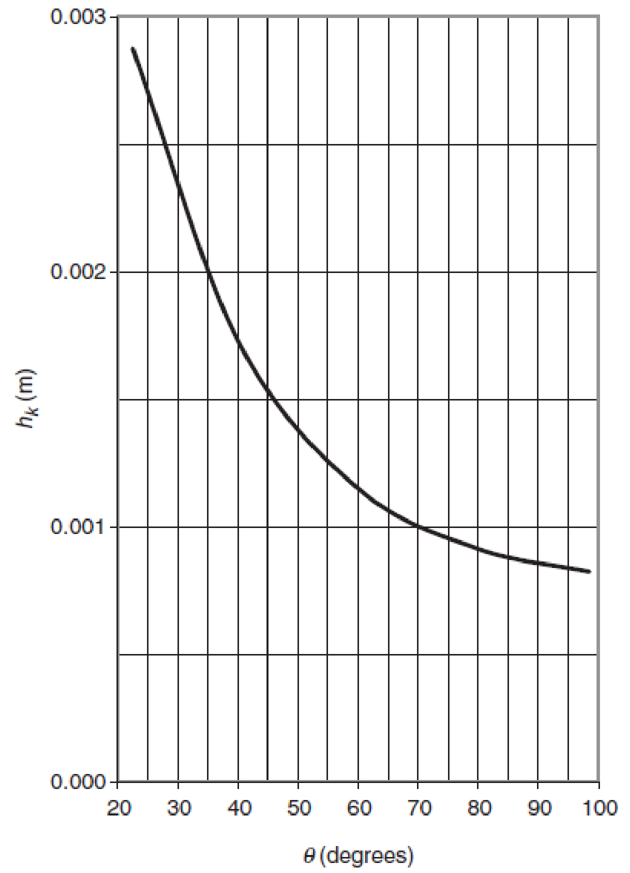


Figure 4 Discharge coefficient for fully-contracted sharp-crested V-notches.

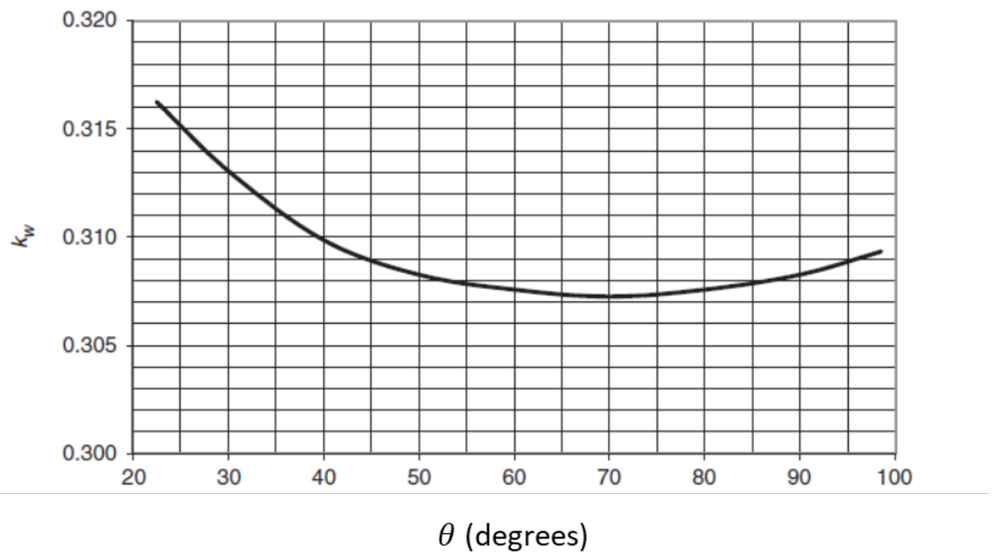


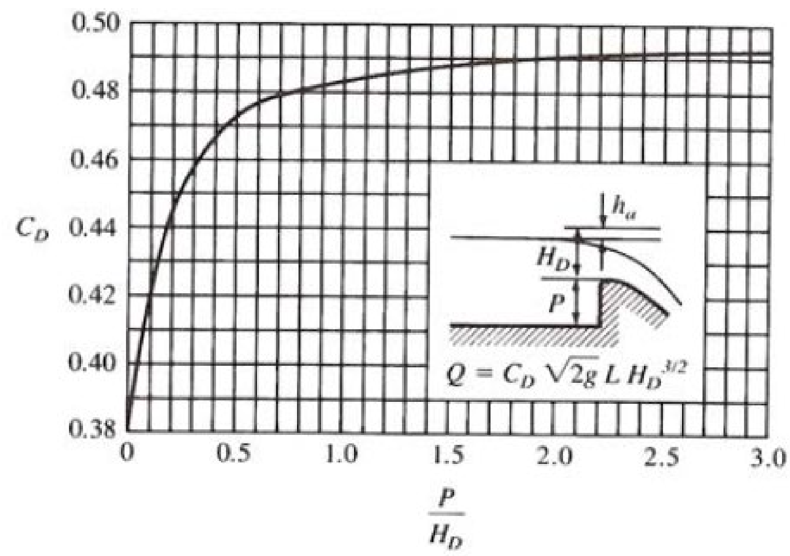
Table 1 Culvert inlet control flow coefficients

Shape and material	Inlet edge description	K_I	M_I	K_{II}	M_{II}	c	Y
Circular concrete	Square edge with headwall	0.3155	2.0			1.28	0.67
Circular concrete	Groove end with headwall	0.2512	2.0			0.94	0.74
Circular corrugated metal	Headwall	0.2512	2.0			1.021	0.69
Circular	Beveled ring, 45° bevels	0.1381	2.50			0.966	0.74
Rectangular box	30 – 75° wingwall flares	0.1475	1.00			1.117	0.81
Rectangular box	90° and 15° wingwall flares	0.2243	0.75			1.288	0.80
Rectangular box	0° wingwall flare	0.2243	0.75			1.362	0.82
Horizontal ellipse concrete	Square edge with headwall	0.3220	2.0			1.282	0.67
Horizontal ellipse concrete	Groove end with headwall	0.1381	2.5			0.940	0.74
Vertical ellipse concrete	Square edge with headwall	0.3220	2.0			1.282	0.67
Vertical ellipse concrete	Groove end with headwall	0.1381	2.5			0.940	0.74
Rectangular box	45° wingwall flare d = 0.043D			1.623	0.667	1	0.8
Rectangular box	18 – 33.7° wingwall flare d = 0.083D			1.547	0.667	0.8	0.83
Rectangular box	90° headwall with ¾" chamfers			1.639	0.667	1.21	0.79
Rectangular box	90° headwall with 45° bevels			1.576	0.667	0.8	0.83
Rectangular box	90° headwall with 33.7° bevels			1.547	0.667	0.81	0.865
Rectangular box with top bevels	45° wingwall flares – offset			1.582	0.667	0.97	0.835
Rectangular box with top bevels	33.7° wingwall flares – offset			1.576	0.667	0.81	0.88

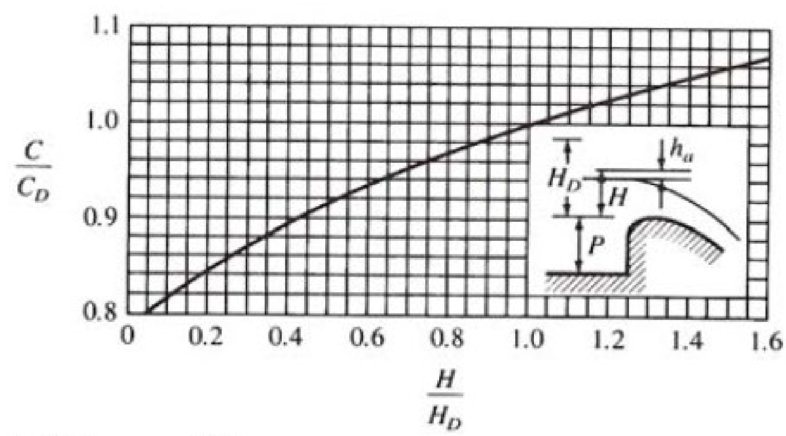
Table 2 Entrance loss coefficients

Type of structure and design of entrance	Coefficient k_e
→ *Pipe, concrete:*	
Projecting from fill socket end (groove-end)	0.2
Projecting from fill, sq. cut end	0.5
Headwall or headwall and wingwalls: socket end of pipe (groove-end)	0.2
Headwall or headwall and wingwalls: - square edge	0.5
Mitered to conform to fill slope	0.7
End-section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
→ *Pipe or pipe-arch, corrugated metal:*	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls square-edge	0.5
Mitered to conform to fill slope, paved or unpaved slope	0.7
→ *Box, reinforced concrete:*	
Headwall parallel to embankment (no wingwalls) - square-edged on three edges	0.5
Headwall parallel to embankment (no wingwalls) - rounded on three edges to radius of 1/12 barrel dimension, or beveled edges on three sides	0.2
Wingwalls at 30-75° to barrel: - square-edged at crown	0.4
Wingwalls at 30-75° to barrel: - crown edge rounded to radius of 1/12 barrel dimension or beveled top edge	0.2
Wingwalls at 10-25° to barrel: - square-edged at crown	0.5
Wingwalls parallel (extension of sides): - square-edged at crown	0.7
Side- or slope-tapered inlet	0.2

Figure 5 Coefficient of discharge for ogee crests with vertical faces.



(*a*) Discharge coefficient C_D versus P/H_D for design head



(*b*) C/C_D versus H/H_D

Table 3 Relationship between the design head and the head on the crest corresponding to maximum reservoir level

$H_e, H_d \geq$ **10 m**	H_e, H_d **< 10 m**
Without piers: $H_D = 0.43H_e^{1.22}$	*Without piers*: $H_D = 0.70H_e$
With piers: $H_D = 0.39H_e^{1.22}$	*With piers*: $H_D = 0.74H_e$

Figure 6 Coordinate coefficients.

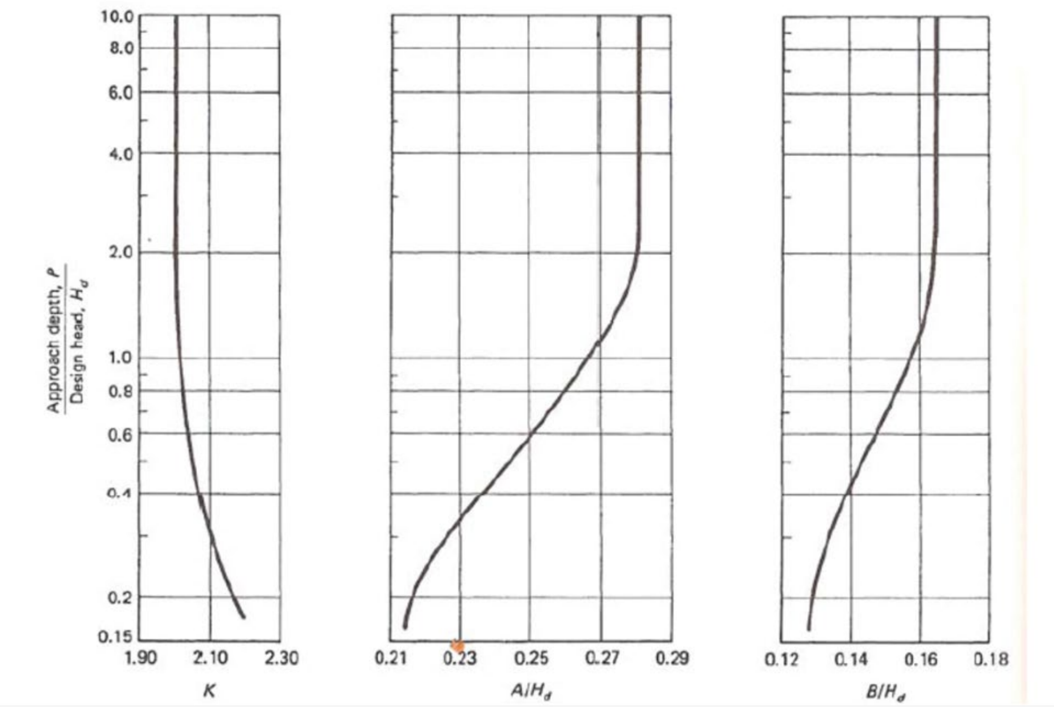


Table 4 Suggested side slopes

Material	**Recommended Side Slope**
Rock	Nearly vertical to 0.25 : 1
Earth with concrete lining	0.5 : 1
Stiff clay	0.5 : 1 to 1 : 1
Firm soil	1 : 1
Loose sandy soil	2 : 1
Light sand, sandy loam	3 : 1

Table 5 Suggested freeboard

Discharge (m^3/s)	< 0.75	0.75 to 1.5	1.5 to 85	> 85
Freeboard (m)	0.45	0.60	0.75	0.90

Table 6 Selected values of Manning's *n* for open channels

Surface Material	**Manning's Roughness Coefficient *n***
Cement	0.011
Wood	0.012
Concrete	0.013
Soil cement	0.022
Earth channel, clean	0.022
Earth channel, firm gravel	0.023
Earth channel, gravelly	0.025
Stone masonry	0.032
Gravel mulch	0.033

Table 7 Recommended permissible velocities

Material		*V* (m/s)
Fine gravel		1.5
Coarse gravel		1.8
Earth	Sandy silt	0.6
	Silt clay	1.1
	Clay	1.8
Grass-lined earth (slopes < 5%)	Bermuda grass + sandy silt	1.8
	Bermuda grass + silt clay	2.4
	Kentucky Blue grass + sandy silt	1.5
	Kentucky Blue grass + silt clay	2.1
Poor rock (usually sedimentary)	Soft sandstone	2.4
	Soft shale	1.1
Good rock (usually igneous or hard metamorphic)		6.1

Table 8 Changes in permissible velocity

Channel type	Change permissible velocity by…
Slightly sinuous	– 5%
Moderately sinuous	– 13%
Very sinuous	– 22%

Figure 7 Angles of repose for non-cohesive material.

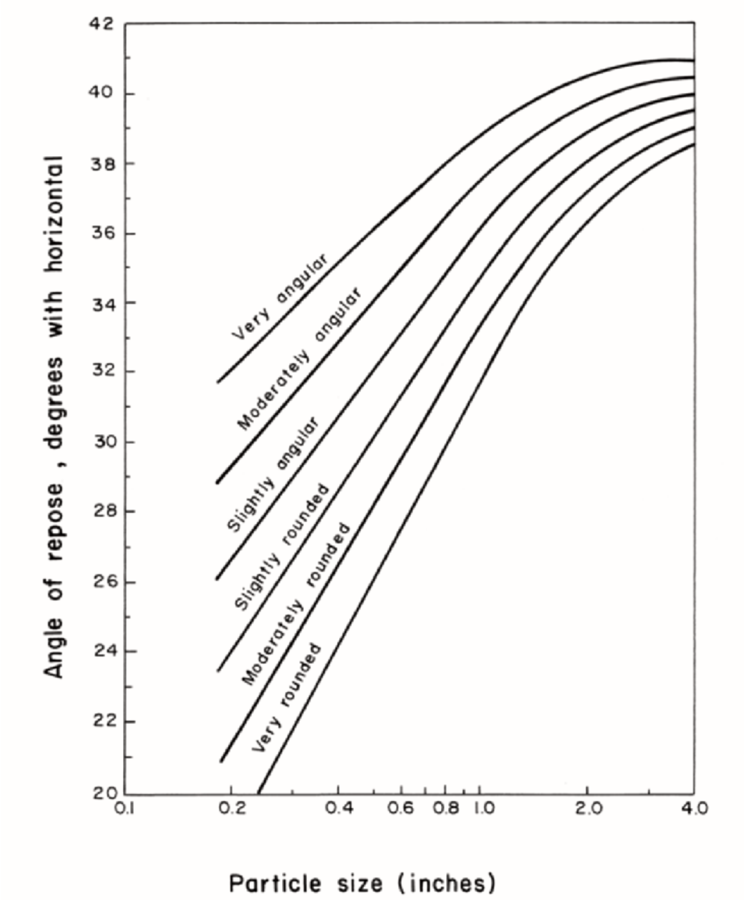


Figure 8 Permissible shear stress for noncohesive materials.

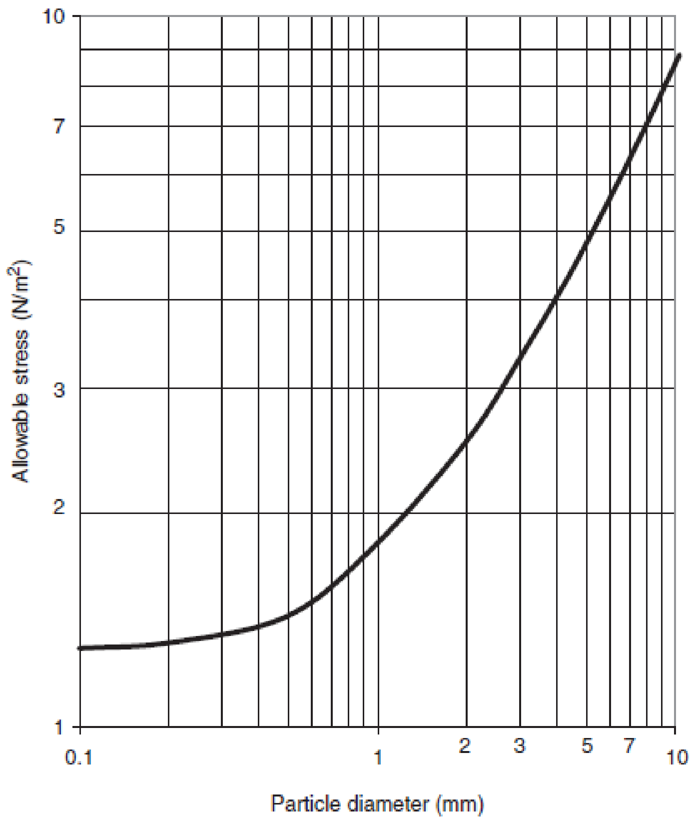


Figure 9 Permissible shear stress for cohesive materials (versus void ratio).

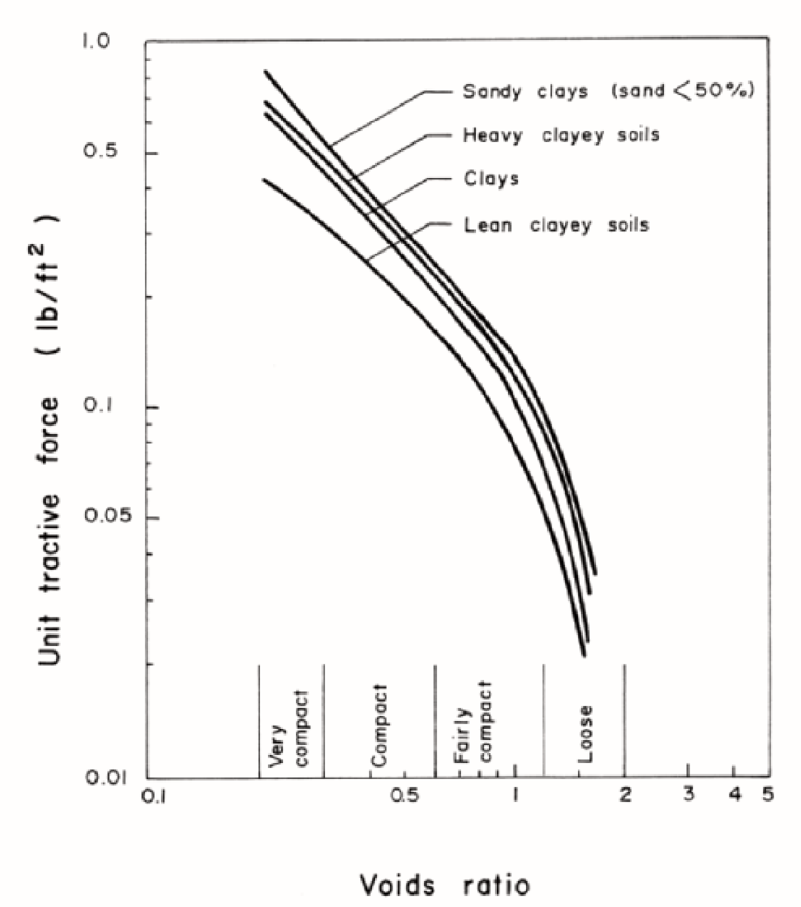


Table 9 Changes in permissible shear according to channel type

Channel type	Change permissible shear by...
Slightly sinuous	– 10%
Moderately sinuous	– 25%
Very sinuous	– 40%

Table 10 Typical permissible shear stresses
For bare soil and stone linings in roadside channels

Lining Category		Permissible Shear Stress	
		N/m^2	lb/ft^2
Gravel Mulch	Coarse gravel D_{50} = 25 mm	19	0.4
	Very coarse gravel D_{50} = 50 mm	38	0.8
Rock Riprap	D_{50} = 0.15 m	113	2.4
	D_{50} = 0.30 m	227	4.8

Table 11 Permissible shear stresses and constant a_0 for vegetative linings[1]

Retardance Class	Permissible τ_p, psf	Permissible τ_p, Pa	a_0 in Resistance Equation
A	3.70	177	24.7
B	2.10	100	30.7
C	1.00	48	36.4
D	0.60	29	40.0
E	0.35	17	42.7

[1] a_0 is a coefficient used in the computation of Manning's n with the formula

$$n = \frac{R^{1/6}}{a_0 + 16.4\log\left(R^{1.4}S^{0.4}\right)}$$

Table 12 Classification of vegetal cover as to degree of retardance

Vegetal Retardance Class	Cover and Condition
A	→Weeping lovegrass - *Excellent stand, tall (avg. 30 in. [76 cm])* →Yellow bluestem →Ischaemum – *Excellent stand, tall (avg. 36 in. [91 cm])*
B	→Kudzu – *Very dense growth, uncut* →Bermuda grass – *Good stand, tall (avg. 12 in. [30 cm]*) →Native grass mixture (little bluestem, bluestem, blue gamma, and other long and short Midwest grasses) - *Good stand, unmowed* → Weeping lovegrass – *Good stand, tall (avg. 24 in. [61 cm])* → Lespedeza sericea – *Good stand, not woody, tall (avg. 19 in. [48 cm])* → Alfalfa – *Good stand, uncut (avg. 11 in. [28 cm])* → Weeping lovegrass – *Good stand, unmowed (avg. 13 in. [33 cm])* → Kudzu – *Dense growth, uncut* → Blue gamma – *Good stand, uncut (avg. 13 in. [33 cm])*
C	→ Crabgrass – *Fair stand, uncut (10 to 48 in.) (25 to 120 cm)* →Bermuda grass – *Good stand, mowed (avg. 6 in. [15 cm])* →Common lespedeza – *Good stand, uncut (avg. 11 in. [28 cm])* →Grass-legume mixture – summer (orchard grass, redtop, Italian ryegrass, and common lespedeza) – *Good stand, uncut (6 to 8 in. [15 to 20 cm])* →Centipede grass – *Very dense cover (avg. 6 in. [15 cm])* →Kentucky bluegrass – *Good stand, headed (6 to 12 in.) [15 to 30 cm]*
D	→ Bermuda grass – *Good stand, cut to 2.5 in. height (6 cm)* →Common lespedeza – *Excellent stand, uncut (avg. 4.5 in. [11 cm])* →Buffalo grass – *Good stand, uncut (3 to 6 in.) (8 to 15 cm)* →Grass-legume mixture – fall, spring (orchard grass, redtop, Italian ryegrass, and common lespedeza) – *Good stand, uncut (4 to 5 in.) (10 to 13 cm)* →Lespedeza sericea – *After cutting to 2 in. height (5 cm), very good stand before cutting*
E	→ Bermuda grass – *Good stand, cut to 1.5 in. (4 cm)* →Bermuda grass – *Burned stubble*

Solutions

P.1 ■ Solution

The flow depth at the approach section is y = 0.80 + 0.96 = 1.76 m. The top width of the flow at the weir section is $B = b + 2my$ = 1.6 + 2 × 2 × 1.76 = 8.64 m. Thus, the average channel width is $\bar{b} = (b+B)/2$ = (1.6 + 8.64)/2 = 5.12 m. With $L_w/\bar{b}$ = 1.04/5.12 = 0.2, h_0/p = 0.96/0.80 = 1.2, we obtain L_k = 0.0024 and k_w = 0.39 from Figures 1 and 2, respectively. The effective crest length is

$$L_{ew} = L_w + L_k = 1.04 + 0.0024 = 1.0424\,\text{m}$$

and, knowing that the head correction is h_k = 0.001 m, the effective head follows as

$$h_{e0} = h_0 + h_k = 0.96 + 0.001 = 0.961\,\text{m}$$

Finally, substituting the available variables into the equation for weir discharge, we obtain

$$Q = k_w\sqrt{2g}L_{ew}h_{e0}^{3/2} = 0.39\times\sqrt{2\times 9.81}\times 1.0424\times 0.961 = \boxed{1.73\text{ m}^3/\text{s}}$$

Note that, in this case, the corrections for the crest length and the head are negligible and could have been ignored altogether.

★ The correct answer is **C**.

P.2 ■ Solution

From the problem statement, we have B = 1.2 m, p =0.6 m, θ = 45°, and h_0 = 0.23 m. For the problem at hand, h_0/p = 0.23/0.6 = 0.38 < 0.40, and h_0/B = 0.23/1.2 = 0.19 < 0.20. Therefore, the V-notch is fully contracted. For θ = 45°, we have h_k = 0.0015 m and k_w = 0.309 from Figures 3 and 4, respectively. We are then ready to substitute the available data into the equation for discharge,

$$Q = k_w\sqrt{2g}\tan\left(\frac{\theta}{2}\right)h_{e0}^{5/2} = 0.309\times\sqrt{2\times 9.81}\times\tan\left(\frac{45^\circ}{2}\right)(0.23+0.0015)^{5/2}$$

$$\therefore Q = 0.0146\,\text{m}^3/\text{s} = \boxed{14.6\text{ L/s}}$$

★ The correct answer is **B**.

P.3 ■ Solution

Initially, we neglect the velocity head of the approach flow. This implies that

$$E_0 = h_0$$

Then, since $C_V = (E_0/h_0)^{3/2}$, we initially have C_V = 1. This coefficient accounts for the approach velocity head. We proceed to obtain coefficient k_d, which is given by

$$k_d = 0.358 + 0.038\frac{E_0}{L_b} = 0.358 + 0.038\times\frac{0.26}{0.85} = 0.370$$

k_d is related to the discharge coefficient k_w by

$$k_w = k_d C_v = 0.370\times 1 = 0.370$$

We may then determine the discharge Q,

$$Q = k_w\sqrt{2g}L_w h_0^{3/2} = 0.370\times\sqrt{2\times 9.81}\times 1.2\times 0.26^{3/2} = 0.261\text{ m}^3/\text{s} = 261\text{ L/s}$$

Next, we shall refine this solution by taking the approach velocity head into account. The total depth at the approach section is 0.30 + 0.26 = 0.56 m, and the velocity is

$$V=\frac{0.261}{0.56\times 1.2}=0.388 \text{ m/s}$$

The corresponding velocity head is

$$\frac{V_0^2}{2g}=\frac{0.388^2}{2\times 9.81}=0.0077 \text{ m}$$

Thus, E_0 is actually

$$E_0=0.26+0.0077=0.2677 \text{ m}$$

Let us update coefficient C_v, which is no longer equal to unity,

$$C_v=\left(\frac{E_0}{h_0}\right)^{\frac{3}{2}}=\left(\frac{0.2677}{0.26}\right)^{\frac{3}{2}}=1.045$$

k_d is then updated as well,

$$k_d=0.358+0.038\times\frac{0.2677}{0.85}=0.370$$

Note that k_d is unchanged relative to its previous value. Finally, the discharge coefficient k_w is given by

$$k_w=C_v k_d=1.045\times 0.370=0.387$$

Having found k_w, we can return to the expression for the discharge Q, which is determined to be

$$Q=0.387\times\sqrt{2\times 9.81}\times 1.2\times 0.26^{3/2}=0.273 \text{ m}^3/\text{s}=\boxed{273 \text{ L/s}}$$

This is the updated discharge, which now accounts for the approach velocity head as well.

★ The correct answer is **B**.

P.4 ■ Solution

Before anything else, we need to verify whether the culvert is submerged or unsubmerged.

$$\frac{Q}{AD^{1/2}g^{1/2}}=\frac{0.8}{\left(\frac{\pi\times 1^2}{4}\right)\times 1^{1/2}\times 9.81^{1/2}}=0.325\le 0.62$$

Thus, it is safe to conclude that the culvert is unsubmerged. Next, we establish the value of the critical depth. We first require angle θ,

$$A\sqrt{D_H}=\frac{Q}{\sqrt{g}}\rightarrow\frac{(\theta-\sin\theta)D^2}{8}\sqrt{\frac{\frac{(\theta-\sin\theta)D^2}{8}}{D\sin\left(\frac{\theta}{2}\right)}}=\frac{0.8}{\sqrt{9.81}}$$

Substituting D = 1 m, we have

$$\left(\frac{\theta-\sin\theta}{8}\right)^{\frac{3}{2}}\sqrt{\sin\left(\frac{\theta}{2}\right)}=\frac{0.8}{\sqrt{9.81}}$$

This can be solved in Mathematica with the *FindRoot* function, namely,

$$\text{FindRoot}\left[\left(\frac{\theta - \text{Sin}[\theta]}{8}\right)^{\frac{3}{2}}\sqrt{\text{Sin}\left[\frac{\theta}{2}\right]} - \frac{0.8}{\sqrt{9.81}}, \{\theta, 1\}\right]$$

which returns 3.18 for θ. The critical depth can be obtained with the relation

$$\theta = 2\cos^{-1}\left(1 - \frac{2y}{D}\right)$$

To obtain y, we apply the command *Solve*,

$$\text{Solve}\left[3.18 == 2\text{ArcCos}\left[1 - \frac{2y}{1}\right], y\right]$$

The critical depth is found as y = 0.51 m. Next, we find the velocity V_c from the relation

$$V_c = \frac{Q}{\left[\frac{(\theta - \sin\theta)D^2}{8}\right]} = \frac{0.8}{\left[\frac{(3.18 - \sin 3.18)\times 1^2}{8}\right]} = 1.99 \text{ m/s}$$

We may then apply the form I equation for inlet control flow,

$$\frac{HW}{D} = \frac{E_c}{D} + K_I\left(\frac{Q}{AD^{0.5}g^{0.5}}\right)^{M_I} + k_s S$$

where *HW* is the headwater depth above the upstream invert of the culvert, E_c is the specific energy at critical conditions, k_s is a constant equal to 0.7 for mitered inlets and −0.5 for non-mitered inlets, *S* is the culvert barrel slope, and K_I and M_I are empirical constants. Using Table 1, we see that the pertaining coefficients are K_I = 0.2512 and M_I = 2.0. Substituting these and other appropriate variables in the equation above gives

$$\frac{HW}{D} = \frac{y_c}{D} + \frac{V_c^2}{2gD} + 0.2512\left(\frac{Q}{AD^{0.5}g^{0.5}}\right)^2 + k_s S$$

$$\therefore \frac{HW}{D} = \frac{0.51}{1} + \frac{1.99^2}{2\times 9.81} + 0.2512\left(\frac{0.8}{\frac{\pi\times 1^2}{4}\times 1^{0.5}\times 9.81^{0.5}}\right)^2 - 0.5\times 0.025 = 0.726$$

Thus, the headwater depth equals

$$HW = 0.726\times 1 \approx \boxed{0.73 \text{ m}}$$

★ The correct answer is **B**.

P.5 ■ Solution

From Table 2, we obtain k_e = 0.5. For a box culvert such as the one considered in this problem, we have $A = bD = 1.5\times 1.0 = 1.5$ m², $P = 2(b + D) = 2(1.5 + 1.0) = 5$ m, and $R = A/P = 1.5/5 = 0.3$ m. Since the culvert is made of reinforced concrete, we have n = 0.013 from Table 6. Finally, we resort to the equation for a culvert under outlet control flow with full-flow conditions,

$$HW = TW - S\times L + \left(1 + k_e + \frac{2gn^2L}{k_n^2R^{4/3}}\right)\frac{Q^2}{2gA^2}$$

Substituting the pertaining variables, we find that

$$HW = 1.3 - 0.0035\times 60 + \left(1 + 0.5 + \frac{2\times 9.81\times 0.013^2\times 60}{1^2\times 0.3^{4/3}}\right)\times\frac{3.2^2}{2\times 9.81\times 1.5^2} = \boxed{1.67 \text{ m}}$$

★ The correct answer is **A**.

P.6 ■ Solution

Assuming a high overflow spillway section, for $P/H_D \geq 3$, the discharge coefficient C_D = 0.49 (Figure 5). The head on the crest controls the discharge through the equation

$$Q = C_D\sqrt{2g}LH_e^{\frac{3}{2}} \rightarrow H_e^{3/2} = \frac{Q}{C_D\sqrt{2g}L}$$

$$\therefore H_e = \left(\frac{1200}{0.49\times\sqrt{2\times 9.81}\times 65}\right)^{\frac{2}{3}} = 4.17 \text{ m}$$

The depth of water upstream is 320 – 280 = 40 m. The approach velocity can be found from the definition of discharge,

$$Q = VA \rightarrow V = \frac{Q}{A}$$

$$\therefore V = \frac{1200}{40\times 65} = 0.46 \text{ m/s}$$

which is a quite small approach velocity, thus suggesting that our assumption of a high overflow spillway was correct. The velocity head is

$$\text{V. Head} = \frac{V^2}{2g} = \frac{0.46^2}{2\times 9.81} = 0.011 \text{ m}$$

The maximum water head, therefore, is

$$\text{Maximum water head} = 4.17 - 0.011 = 4.16 \text{ m}$$

The height of the crest, in turn, is designed to be

$$P = 40 - 4.16 = 35.84\,\text{m}$$

Now, because H_e is smaller than 10 m and there are no piers, we shall take the following equation among the relations presented in Table 3,

$$H_D = 0.70H_e \rightarrow H_D = 0.70\times 4.17 = 2.92 \text{ m}$$

Our design head is 2.92 m. Next, we compute the ratio P/H_D to ensure ourselves that its value is greater than 1.33 and the high overflow assumption is correct.

$$\frac{P}{H_D} = \frac{35.84}{2.92} = 12.27 > 1.33$$

Thence, let us calculate the shape of the downstream quadrant, which follows from the equation

$$X^{1.85} = 2H_D^{0.85}Y = 2\times 2.92^{0.85}Y = 4.97Y$$

Then, to find the point of tangency, we resort to the formula

$$x_{DT} = 0.485H_D\left(K_{sp}\alpha\right)^{1.176} \rightarrow x_{DT} = 0.485\times 2.92\times\left(2\times 2\right)^{1.176} = 7.23 \text{ m}$$

Finally, we find the ellipse that fits the upstream quadrant, which is given by

$$\frac{x^2}{A^2} + \frac{\left(B-y\right)^2}{B^2} = 1$$

To find this shape, we use the graphs presented in Figure 6. For P/H_D = 12, the ratio A/H_D equals 0.28. Therefore,

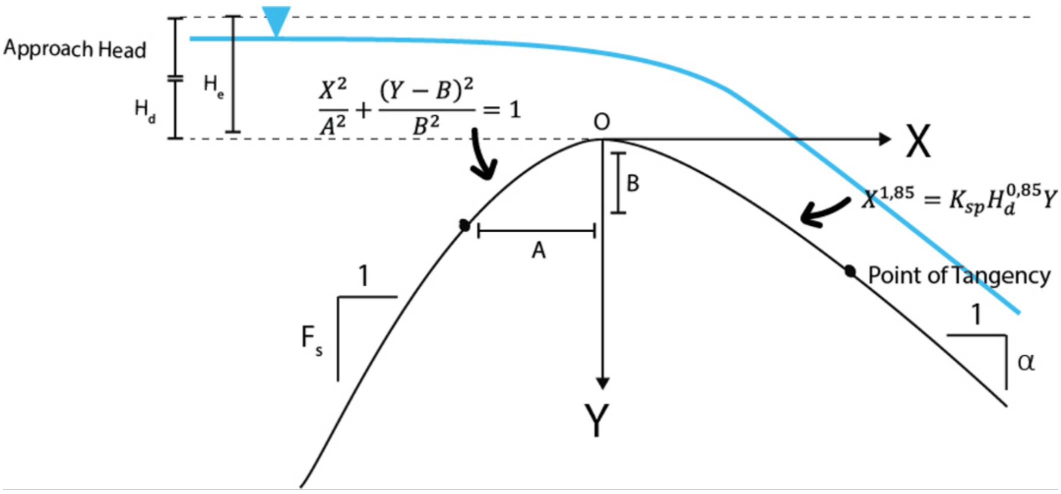
$$\frac{A}{H_D} = 0.28 \rightarrow A = 0.28\times 2.92 = 0.818 \text{ m}$$

Similarly, coefficient B can be obtained from the other graph, noting that B/H_D = 0.165 for P/H_D = 12. Hence,

$$\frac{B}{H_D} = 0.165 \rightarrow B = 0.165 \times 2.92 = 0.482 \text{ m}$$

Finally, the ellipse that describes the upstream quadrant is

$$\frac{x^2}{0.818^2} + \frac{(0.482 - y)^2}{0.482^2} = 1$$



P.7 ■ Solution

First, we select the Manning n, which has a value of 0.025 for a coarse gravel. Let us set the side slope as m = 2. Using Table 7, we take 1.8 m/s as the permissible velocity. Since the channel is going to convey a relatively large discharge, however, it is reasonable to assume that the flow depth will be greater than 1 m, and hence we may increase V_{max} by 0.15 m/s; it follows that V_{max} = 1.8 + 0.15 = 1.95 m/s. Because the channel is straight, there is no need to reduce the permissible velocity (as would be prescribed by Table 8). We then determine the hydraulic radius with the Manning formula,

$$R = \left(\frac{nV_{\max}}{k_n S_0^{1/2}}\right)^{\frac{3}{2}} = \left(\frac{0.025 \times 1.95}{1.0 \times 0.001^{1/2}}\right)^{\frac{3}{2}} = 1.91 \text{m}$$

Next, we find the required flow area from conservation of mass,

$$A = \frac{Q}{V} = \frac{125}{1.95} = 64.10 \text{ m}^2$$

We then obtain the wetted perimeter,

$$P = \frac{A}{R} = \frac{64.10}{1.91} = 33.56 \text{ m}$$

Thence, we want to find a bottom width and a flow depth that satisfy the equations for area and wetted perimeter simultaneously. For a trapezoidal channel, the former is

$$A = (b + my)y$$

$$\therefore (b + 2y)y = 64.10$$

while the latter is

$$P = b + 2y\sqrt{1+m^2}$$

$$\therefore b + 2y\sqrt{1+2^2} = P$$

$$\therefore b + 4.47y = 33.56$$

Using Mathematica, the two equations can be solved simultaneously with the command *Solve*,

$$\text{Solve}[(b+2*y)y == 64.1\&\&b+4.47y == 33.56,\{b,y\}]$$

This returns {b = 23.3, y = 2.30} and {b = −16.9, y = 11.3}. The latter solution is impossible, which leaves us with a width b = 23.3 m and a depth y = 2.30 m. Note that we have confirmed our assumption that the flow depth was going to be greater than 1 m, which we considered to justify the increase in the permissible velocity. Finally, because the discharge is greater than 85 m^3/s, we can prescribe a freeboard of 0.9 m (Table 5). It follows that the total depth of the channel will be 2.30 + 0.90 = 3.20 m.

P.8 ■ Solution

The average particle size in inches is 0.75/2.54 = 0.3 in. We shall adopt a roughness coefficient of 0.023 and a slope of 3 H : 1 V, which corresponds to a side slope angle of

$$\tan^{-1}\frac{1}{3}=18.4^{\circ}$$

Using the graph of angle of repose versus particle size (Figure 7), we find that ϕ for this material is approximately 26.5°. We then find the tractive force ratio, K,

$$K=\sqrt{1-\frac{\sin^2 18.4^{\circ}}{\sin^2 26.5^{\circ}}}=0.71$$

To find the maximum allowable stress, we make use of the graph in Figure 8, which gives the permissible stress as a function of particle diameter. For a particle diameter of 7.5 mm, the allowable stress is 6.5 N/m^2. Since the channel is slightly sinuous, the permissible shear must be reduced by 10% (Table 9). The result is

$$\tau_p=0.90\times 6.5=5.85\ \text{N/m}^2$$

The permissible tractive force on the channel sides is

$$\tau_{p,s}=0.71\times 5.85=4.15\ \text{N/m}^2$$

The actual unit tractive force on the side is

$$\tau_s=0.76\gamma y S_0=0.76\times 9810\times 0.0002y=1.49y$$

Equating this to the permissible stress on the sides, we find

$$\tau_s=\tau_{p,s}\rightarrow 1.49y=4.15$$

$$\therefore y=\frac{4.15}{1.49}=2.79\ \text{m}$$

The channel bottom width can be found by solving the Manning equation for b,

$$Q=\frac{k_n\sqrt{S_0}}{n}\times\frac{\left[(b+my)y\right]^{\frac{5}{3}}}{\left(b+2y\sqrt{1+m^2}\right)^{\frac{2}{3}}}$$

$$\therefore 65=\frac{\sqrt{0.0002}}{0.023}\times\frac{[(b+3\times 2.79)\times 2.79]^{\frac{5}{3}}}{(b+2\times 2.79\times\sqrt{1+3^2})^{\frac{2}{3}}}$$

In Mathematica, we make use of the *NSolve* command with the *Reals* option activated,

$$\text{NSolve}\left[65==\frac{\sqrt{0.0002}}{0.023}\frac{((b+3*2.79)*2.79)^{\frac{5}{3}}}{(b+2*2.79\sqrt{1+3^2})^{\frac{2}{3}}},b,\text{Reals}\right]$$

This yields the solution b = 15.44 m. Finally, we must verify whether the channel bed will be scoured or not. To do so, we compare $\tau = \gamma y S_0$ with the maximum permissible value τ_p; that is,

$$\tau = 9810 \times 2.79 \times 0.0002 = 5.47 < \tau_p = 5.85 \text{ N/m}^2$$

The result is below the permissible limit of 5.85 N/m^2, which implies that the channel bottom will not be eroded. Our design is stable for the channel sides and for the channel bed. Finally, we could predict a freeboard for the channel; using Table 5, we conclude that 0.75 m is an adequate value. The channel depth will be 2.79 + 0.75 = 3.54 m.

P.9 ■ Solution

We use the design approach presented in the FHWA's *Design of Roadside Channels with Flexible Linings*. We begin by assuming a depth of flow y = 0.5 m. The area and wetted perimeter are, respectively,

$$A = (b + my)y = (0.4 + 3 \times 0.5) \times 0.5 = 0.95 \text{ m}^2$$

$$P = b + 2y\sqrt{m^2 + 1} = 0.4 + 2 \times 0.5 \times \sqrt{3^2 + 1} = 3.56 \text{ m}$$

The hydraulic radius is R = 0.95/3.56 = 0.267 m. From Table 6, we note that Manning's n for a channel lined with gravel mulch is 0.033. The discharge is calculated with the Manning equation,

$$Q = \frac{k_n}{n} AR^{2/3} S_0^{1/2} = \frac{1.0}{0.033} \times 0.95 \times 0.267^{2/3} \times 0.008^{1/2} = 1.07 \text{ m}^3/\text{s}$$

This value is more than 5 percent greater than the design flow, and thus another iteration is needed. The new supposed flow depth can be established with the simple relation

$$y_2 = y_1 \left(\frac{Q_1}{Q_0} \right)^{0.4} = 0.5 \left(\frac{1.07}{0.42} \right)^{0.4} = 0.34 \text{ m}$$

We proceed to compute the new flow area and wetted perimeter,

$$A = (b + my)y = (0.4 + 3 \times 0.34) \times 0.34 = 0.48 \text{ m}^2$$

$$P = b + 2y\sqrt{m^2 + 1} = 0.4 + 2 \times 0.34 \times \sqrt{3^2 + 1} = 2.55 \text{ m}$$

The hydraulic radius, in turn, is R = 0.48/2.55 = 0.188 m. The new discharge follows as

$$Q = \frac{k_n}{n} AR^{2/3} S_0^{1/2} = \frac{1.0}{0.033} \times 0.48 \times 0.188^{2/3} \times 0.008^{1/2} = 0.43 \text{ m}^3/\text{s}$$

This result is within 5 percent of the initially supposed discharge. We may then determine the shear stress at maximum depth with the usual relation

$$\tau_d = \gamma y S_0 = 9810 \times 0.34 \times 0.008 = 26.7 \text{ N/m}^2$$

From Table 10, the permissible shear stress for very coarse gravel mulch is τ_d = 38 N/m^2. We take a safety factor SF = 1.0. Comparing the calculated shear with the permissible shear, we have

$$\text{SF} \times \tau_d \le \tau_p \rightarrow 1.0 \times 26.7 \le 38$$

The equality checks, and we conclude that the lining is stable. The design process is complete.

P.10 ■ Solution

From Table 12, the Vegetal Retardance Class is B, and, from Table 11, the permissible shear is 100 Pa. The maximum allowable depth comes from setting $\tau_{\max} = \gamma y_0 S = \tau_p$ and solving for y_0,

$$y_0 = \frac{\tau_p}{\gamma S} = \frac{100}{9810 \times 0.009} = 1.13\text{ m}$$

The area and wetted perimeter of the cross-section for this depth are

$$A = (bm + y)y = (6 \times 3 + 1.13) \times 1.13 = 21.62\text{ m}^2$$

$$P = b + 2y\sqrt{1+m^2} = 6 + 2 \times 1.13 \times \sqrt{1+3^2} = 13.15\text{ m}$$

Then, the hydraulic radius is R = 21.62/13.15 = 1.64 m. Noting that a_0 = 30.7 for a cover belonging to the retardance class in question, the Manning n becomes

$$n = \frac{R^{1/6}}{a_0 + 16.4\log\left(R^{1.4}S^{0.4}\right)} = \frac{1.64^{1/6}}{30.7 + 16.4\log\left(1.64^{1.4}0.009^{0.4}\right)} = 0.049$$

Finally, the allowable discharge can be calculated with the Manning equation,

$$Q = \frac{k_n}{n} A R^{2/3} S_0^{1/2} = \frac{1.0}{0.049} \times 21.62 \times 1.64^{2/3} \times 0.009^{0.5} = \boxed{58.2\text{ m}^3/\text{s}}$$

★ The correct answer is **D**.

Answer Summary

Problem 1	C
Problem 2	B
Problem 3	B
Problem 4	B
Problem 5	A
Problem 6	Open-ended pb.
Problem 7	Open-ended pb.
Problem 8	Open-ended pb.
Problem 9	Open-ended pb.
Problem 10	D

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