QUIZ CE202
Ideal Reactors – Part 2
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PROBLEMS

Problem 1

The reaction

$$A \rightarrow B$$

is to be carried out isothermally in a continuous flow reactor. The entering volumetric flow rate v_0 is 10 dm³/h. Calculate both the CSTR and PFR reactor volumes necessary to consume 99% of A (i.e., $C_A = 0.01C_{A,0}$) when the entering molar flow rate is 5 mol/h, assuming the reaction rate $-r_A$ is

$$-r_A = k \;\; ; \;\; k = 0.05 \; \frac{\text{mol}}{\text{h} \cdot \text{dm}^3}$$

True or false?

1.( ) The volume of a CSTR that accommodates this reaction is greater than 100 dm³.

2.( ) The volume of a PFR that accommodates this reaction is greater than 90 dm³.

Suppose now that the reaction rate is described by

$$-r_A = kC_A \;\; ; \;\; k = 0.0001 \text{ s}^{-1}$$

3.( ) The volume of a CSTR that accommodates this reaction is greater than 2600 dm³.

4.( ) The volume of a PFR that accommodates this reaction is greater than 150 dm³.

Suppose now that the reaction rate is described by

$$-r_A = kC_A^2 \;\; ; \;\; k = 300 \; \frac{\text{dm}^3}{\text{mol} \cdot \text{h}}$$

5.( ) The volume of a CSTR that accommodates this reaction is greater than 700 dm³.

6.( ) The volume of a PFR that accommodates this reaction is greater than 5 dm³.

Problem 2

In an isothermal batch reactor, 70% of a liquid reactant is converted in 13 minutes. The reaction, $A \rightarrow R$, is first order on reactant A. What space-time is required to effect this conversion in a plug flow reactor and in a mixed flow reactor? Consider the following statements.

Statement 1: The space-time needed to effect the reaction in a plug flow reactor is greater than 10 minutes.

Statement 2: The space-time needed to effect the reaction in a mixed flow reactor is greater than 30 minutes.

A) Both statements are true,

B) Statement 1 is true and statement 2 is false.

C) Statement 1 is false and statement 2 is true.

D) Both statements are false.

▶ Problem 3

An aqueous feed of A and B (400 liter/min, 100 mmol A/liter, 200 mmol B/liter) is to be converted to product in a **mixed flow reactor**. The kinetics of the reaction is represented by

$$A + B \rightarrow R \;\; ; \;\; -r_A = 200 C_A C_B \; \frac{\text{mol}}{\text{liter} \cdot \text{min}}$$

Find the volume of reactor required to for 99.9% conversion of A to product.

A) V = 3880 L

B) V = 6540 L.

C) V = 11,100 L

D) V = 20,000 L

▶ Problem 4

An aqueous feed of A and B (400 liter/min, 100 mmol A/liter, 200 mmol B/liter) is to be converted to product in a **plug flow reactor**. The kinetics of the reaction are represented by

$$A + B \rightarrow R \;\; ; \;\; -r_A = 200 C_A C_B \; \frac{\text{mol}}{\text{liter} \cdot \text{min}}$$

Find the volume of reactor needed for 99.9% conversion of A to product.

A) V = 124 L

B) V = 248 L.

C) V = 372 L

D) V = 496 L

▶ Problem 5

A gaseous feed of pure A (2 mol/liter, 100 mol/min) decomposes to give a variety of products in a plug flow reactor. The kinetics of the conversion is represented by

$$A \rightarrow 2.5(\text{Products}) \;\; ; \;\; -r_A = \left(10 \text{ min}^{-1}\right) C_A$$

Find the expected conversion in a 22-liter reactor.

A) X_A = 0.44

B) X_A = 0.56

C) X_A = 0.71

D) X_A = 0.90

▶ Problem 6

We plan to replace our present mixed flow reactor with one having double the volume. For the same aqueous feed (10 mol A/liter) and the same feed rate, find the new conversion. The reaction kinetics are represented by

$$A \rightarrow R \;\; ; \;\; -r_A = k C_A^{1.5}$$

and the present conversion is 70%.

A) X_A' = 0.444

B) X_A' = 0.510

C) X_A' = 0.795

D) X_A' = 0.951

▶ Problem 7

A stream of pure gaseous reactant A ($C_{A,0}$ = 660 mmol/liter) enters a plug flow reactor at a flow rate of $F_{A,0}$ = 540 mmol/min and polymerizes there as follows.

$$3\text{A} \rightarrow \text{R} \;\; ; \;\; -r_A = 54 \, \frac{\text{mmol}}{\text{liter} \cdot \text{min}}$$

How large a reactor is needed to lower the concentration of A in the exit stream to $C_{A,f}$ = 330 mmol/liter?

A) V = 1.90 L

B) V = 3.81 L

C) V = 7.50 L

D) V = 11.4 L

▶ Problem 8

An aqueous feed containing A (1 mol/liter) enters a 2-liter plug flow reactor and reacts away (2A → R, $-r_A = 0.05C_A^2$ mol/liter·s). Find the outlet concentration of A for a feed rate of 0.5 liter/min.

A) $C_{A,f}$ = 0.0339 mol/L

B) $C_{A,f}$ = 0.0769 mol/L

C) $C_{A,f}$ = 0.112 mol/L

D) $C_{A,f}$ = 0.341 mol/L

▶ Problem 9

At 650°C, phosphine vapor decomposes as follows:

$$4\text{PH}_3 \rightarrow P_{4(\text{g})} + 6\text{H}_2 \;\; ; \;\; -r_{\text{phos}} = \left(10 \text{ hr}^{-1}\right) C_{\text{phos}}$$

What size of plug flow reactor operating at 649°C and 11.4 atm is needed for 75% conversion of 10 mol/hr of phosphine in a 2/3 phosphine-1/3 inert feed?

A) V = 16.9 L

B) V = 31.1 L

C) V = 40.4 L

D) V = 49.2 L

▶ Problem 10

1 liter/s of a 20% ozone-80% air mixture at 1.5 atm and 93°C passes through a plug flow reactor. Under these conditions ozone decomposes by the homogeneous reaction

$$2O_3 \rightarrow 3O_2 \;\; ; \;\; -r_{\text{ozone}} = kC_{\text{ozone}}^2 \;\; ; \;\; k = 0.05 \, \frac{\text{liter}}{\text{mol} \cdot \text{s}}$$

What size reactor is needed for 50% decomposition of ozone?

A) V = 1080 L

B) V = 1520 L

C) V = 2130 L

D) V = 2690 L

▶ Problem 11

Pure gaseous A at about 3 atm and 30°C (120 mmol/liter) is fed into a 1-liter mixed flow reactor at various flow rates. There it decomposes, and the exit concentration of A is measured for each flow rate. From the following data, find a rate equation to represent the kinetics of the decomposition of A. Assume that reactant A alone affects the rate.

$$A \rightarrow 3R$$

v_0 **(liter/min)**	0.06	0.48	1.5	8.1
C_A **(mmol/liter)**	30	60	80	105

A) $-r_A$ = 0.002C_A mmol/liter·min
B) $-r_A$ = 0.004C_A mmol/liter·min
C) $-r_A$ = 0.002C_A^2 mmol/liter·min
D) $-r_A$ = 0.004C_A^2 mmol/liter·min

▶ Problem 12

A mixed flow reactor is being used to determine the kinetics of a reaction whose stoichiometry is A → R. For this purpose, various flow rates of an aqueous solution of 100 mmol A/liter are fed to a 1-liter reactor, and for each run the outlet concentration of A is measured. Find a rate equation to represent the following data. Also assume that reactant A alone affects the rate.

v **(liter/min)**	1	6	24
C_A **(mmol/liter)**	4	20	50

A) $-r_A$ = 12C_A mmol/liter·min
B) $-r_A$ = 24C_A mmol/liter·min
C) $-r_A$ = 12C_A^2 mmol/liter·min
D) $-r_A$ = 24C_A^2 mmol/liter·min

→ Problem 13.1

We are planning to operate a batch reactor to convert A into R. This is a liquid reaction, the stoichiometry is A → R, and the rate of reaction is given in the following table. How long must we react each batch for the concentration to drop from $C_{A,0}$ = 1.3 mol/liter to $C_{A,f}$ = 0.3 mol/liter?

C_A **(mol/liter)**	$-r_A$ **(mol/liter·min)**
0.1	0.1
0.2	0.3
0.3	0.5
0.4	0.6
0.5	0.5
0.6	0.25
0.7	0.10
0.8	0.06
1.0	0.05
1.3	0.045
2.0	0.042

A) t = 8.41 min
B) t = 12.7 min
C) t = 19.4 min
D) t = 23.2 min

→ Problem 13.2

For the reaction in the previous problem, what size of plug flow reactor would be needed for 80% conversion of a feed stream of 1000 mol A/hr at $C_{A,0}$ = 1.5 mol/liter?

A) V = 70.9 L
B) V = 110 L
C) V = 191 L
D) V = 275 L

▶ Problem 14

The data in the following table have been obtained on the decomposition of gaseous reactant A in a constant volume batch reactor at 100°C. The stoichiometry of the reaction is 2A → R + S. What size mixed flow reactor operating at 100°C and 1 atm can treat 100 mol A/hr in a feed consisting of 20% inerts to obtain 95% conversion of A?

t (sec)	p_A (atm)	t (sec)	p_A (atm)
0	1.0	140	0.25
20	0.80	200	0.14
40	0.68	260	0.08
60	0.56	330	0.04
80	0.45	420	0.02
100	0.37		

A) V = 585 L

B) V = 1050 L

C) V = 1540 L

D) V = 2160 L

ADDITIONAL INFORMATION

Table 1 Performance equations for n-th order kinetics and $\varepsilon_A = 0$

	Plug Flow or Batch	Mixed Flow
$n = 0$, $-r_A = k$	$\frac{k\tau}{C_{A0}} = \frac{C_{A0} - C_A}{C_{A0}} = X_A$ (20)	$\frac{k\tau}{C_{A0}} = \frac{C_{A0} - C_A}{C_{A0}} = X_A$
$n = 1$, $-r_A = kC_A$	$k\tau = \ln\frac{C_{A0}}{C_A} = \ln\frac{1}{1 - X_A}$ (3.12)	$k\tau = \frac{C_{A0} - C_A}{C_A} = \frac{X_A}{1 - X_A}$ (14a)
$n = 2$, $-r_A = kC_A^2$	$k\tau C_{A0} = \frac{C_{A0} - C_A}{C_A} = \frac{X_A}{1 - X_A}$ (3.16)	$k\tau = \frac{(C_{A0} - C_A)}{C_A^2} = \frac{X_A}{C_{A0}(1 - X_A)^2}$ (15)
any n, $-r_A = kC_A^n$	$(n-1)C_{A0}^{n-1}k\tau = \left(\frac{C_A}{C_{A0}}\right)^{1-n} - 1 = (1 - X_A)^{1-n} - 1$ (3.29)	$k\tau = \frac{C_{A0} - C_A}{C_A^n} = \frac{X_A}{C_{A0}^{n-1}(1 - X_A)^n}$
$n = 1$, $A \underset{2}{\overset{1}{\rightleftarrows}} R$, $C_{R0} = 0$	$k_1\tau = \left(1 - \frac{C_{Ae}}{C_{A0}}\right)\ln\left(\frac{C_{A0} - C_{Ae}}{C_A - C_{Ae}}\right) = X_{Ae}\ln\left(\frac{X_{Ae}}{X_{Ae} - X_A}\right)$	$k_1\tau = \frac{(C_{A0} - C_A)(C_{A0} - C_{Ae})}{C_{A0}(C_A - C_{Ae})} = \frac{X_A X_{Ae}}{X_{Ae} - X_A}$
General rate	$\tau = \int_{C_A}^{C_{A0}} \frac{dC_A}{-r_A} = C_{A0}\int_0^{X_{Ae}} \frac{dX_A}{-r_A}$ (19)	$\tau = \frac{C_{A0} - C_A}{-r_{Af}} = \frac{C_{A0}X_A}{-r_{Af}}$ (13)

Table 2 Performance equations for n-th order kinetics and $\varepsilon_A \neq 0$

	Plug Flow		Mixed Flow	
$n = 0$, $-r_A = k$	$\frac{k\tau}{C_{A0}} = X_A$	**(20)**	$\frac{k\tau}{C_{A0}} = X_A$	
$n = 1$, $-r_A = kC_A$	$k\tau = (1+\varepsilon_A)\ln\frac{1}{1-X_A} - \varepsilon_A X_A$	**(21)**	$k\tau = \frac{X_A(1+\varepsilon_A X_A)}{1-X_A}$	**(14*b*)**
$n = 2$, $-r_A = kC_A^2$	$k\tau C_{A0} = 2\varepsilon_A(1+\varepsilon_A)\ln(1-X_A) + \varepsilon_A^2 X_A + (\varepsilon_A+1)^2 \cdot \frac{X_A}{1-X_A}$	**(23)**	$k\tau C_{A0} = \frac{X_A(1+\varepsilon_A X_A)^2}{(1-X_A)^2}$	**(15)**
any n, $-r_A = kC_A^n$			$k\tau C_{A0}^{n-1} = \frac{X_A(1+\varepsilon_A X_A)^n}{(1-X_A)^n}$	
$n = 1$, $A \underset{2}{\overset{1}{\rightleftarrows}} rR$, $C_{R0} = 0$	$\frac{k\tau}{X_{Ae}} = (1+\varepsilon_A X_{Ae})\ln\frac{X_{Ae}}{X_{Ae}-X_A} - \varepsilon_A X_A$	**(22)**	$\frac{k\tau}{X_{Ae}} = \frac{X_A(1+\varepsilon_A X_A)}{X_{Ae}-X_A}$	
General expression	$\tau = C_{A0}\int_0^{X_A}\frac{dX_A}{-r_A}$	**(17)**	$\tau = \frac{C_{A0}X_A}{-r_A}$	**(11)**

SOLUTIONS

P.1 ➡ *Solution*

1. False. The design equation for the CSTR is

$$V = \frac{F_{A,0} - F_A}{-r_A}$$

$$\therefore V = \frac{C_{A,0}v_0 - C_A v_0}{-r_A}$$

so that

$$V = \frac{C_{A,0}v_0 - C_A v_0}{-r_A} = \frac{0.5\times10-(0.01\times0.5)\times10}{0.05} = \boxed{99\text{ dm}^3}$$

2. True. Consider now the PFR under the same conditions. The reaction is modeled as

$$-v_0 \frac{dC_A}{dV} = k \rightarrow -\frac{v_0}{k} dC_A = dV$$

$$\therefore \int_0^V dV = -\frac{v_0}{k} \int_{C_{A,0}}^{C_A} dC_A$$

$$\therefore V = -\frac{v_0}{k}\left(C_A - C_{A,0}\right)$$

Substituting the numerical quantities brings to

$$V = -\frac{10}{0.05} \times \left(0.01 \times 0.5 - 0.5\right) = \boxed{99 \text{ dm}^3}$$

3. True. With the rate of reaction $-r_A = kC_A$, the CSTR volume becomes

$$V = \frac{C_{A,0}v_0 - C_A v_0}{kC_A} = \frac{1 \text{ h}}{3600 \text{ s}} \times \left(\frac{0.5 \times 10 - 0.01 \times 0.5 \times 10}{0.0001 \times 0.01 \times 0.5}\right) = \boxed{2750 \text{ dm}^3}$$

4. False. We proceed to consider the PFR under the same conditions. The reaction is modeled as

$$-v_0 \frac{dC_A}{dV} = kC_A \rightarrow \frac{v_0}{k} \frac{dC_A}{C_A} = \int_0^V dV$$

$$\therefore V = -\frac{v_0}{k} \ln\left(\frac{C_{A,0}}{C_A}\right)$$

Substituting the numerical quantities, we obtain

$$V = -\frac{1 \text{ h}}{3600 \text{ s}} \times \frac{10}{0.0001} \times \ln\left(\frac{0.01 \times 0.5}{0.5}\right) = \boxed{128 \text{ dm}^3}$$

5. False. With the rate of reaction $-r_A = kC_A^2$, the CSTR volume becomes

$$V = \frac{C_{A,0}v_0 - C_A v_0}{kC_A^2} = \frac{0.5 \times 10 - 0.01 \times 0.5 \times 10}{300 \times \left(0.01 \times 0.5\right)^2} = \boxed{660 \text{ dm}^3}$$

6. True. We proceed to consider the PFR under the same conditions. The reaction is modeled as

$$-v_0 \frac{dC_A}{dV} = kC_A^2 \rightarrow -\frac{v_0}{k} \int_{C_{A,0}}^{C_A} \frac{dC_A}{C_A^2} = \int_0^V dV$$

$$\therefore V = \frac{v_0}{k}\left(\frac{1}{C_A} - \frac{1}{C_{A,0}}\right)$$

Substituting the numerical quantities, we get

$$V = \frac{10}{300} \times \left(\frac{1}{0.01 \times 0.5} - \frac{1}{0.5}\right) = \boxed{6.6 \text{ dm}^3}$$

P.2 ➡ *Solution*

In the batch reactor, 70% conversion occurs in 13 min. Thus, the holding time or mean residence time in the batch reactor is 13 min. For a constant fluid density system, the space time is equivalent to the holding time of the reactor. In a plug flow reactor, the space time required would be the same as the holding time in the batch reactor; accordingly, we surmise that the space time for the plug flow reactor to achieve 70% conversion is equal to 13 minutes.

Consider now the same reaction being carried out in a mixed flow reactor. For a first order reaction, the rate expression is $r_A = kC_A$. Substituting in the performance equation brings to

$$t = -\int_0^{X_{A,f}} \frac{dX_A}{-r_A} = \int_0^{X_{A,f}} \frac{dX_A}{kC_{A,0}\left(1 - X_A\right)} = -\frac{\ln\left(1 - X_A\right)}{k}$$

$$\therefore k = -\frac{\ln(1-X_A)}{t}$$

$$\therefore k = -\frac{\ln(1-0.7)}{13} = 0.0926 \text{ min}^{-1}$$

Equipped with the rate constant, we can determine the space time τ,

$$\frac{\tau}{C_{A,0}} = \frac{X_A}{-r_A} \rightarrow \tau = \frac{C_{A,0}X_A}{kC_A}$$

$$\therefore \tau = \frac{\cancel{C_{A,0}}X_A}{k\cancel{C_{A,0}}(1-X_A)}$$

$$\therefore \tau = \frac{X_A}{k(1-X_A)}$$

$$\therefore \tau = \frac{0.7}{0.0926\times(1-0.7)} = 25.2 \text{ min}$$

♦ The correct answer is **B**.

P.3 ➡ *Solution*

The rate equation for the reaction can be rewritten as

$$-r_A = 200(C_{A,0} - C_{A,0}X_A)(C_{B,0} - C_{B,0}X_A)$$

Taking $M = C_{B,0}/C_{A,0}$ = 200/100 = 2.0 as the initial molar ratio of reactants, we have

$$-r_A = 200C_{A,0}^2(1-X_A)(M-X_A)\ \frac{\text{mol}}{\text{liter}\cdot\text{min}}$$

The space time τ is given by

$$\tau = \frac{C_{A,0}X_A}{-r_A}$$

Since $\tau = V/v_0$, we have

$$\frac{V}{v_0} = \frac{C_{A,0}X_A}{-r_A} \rightarrow V = \frac{v_0 C_{A,0}X_A}{-r_A}$$

Finally, we substitute our data to obtain

$$V = \frac{v_0 C_{A,0}X_A}{-r_A} \rightarrow V = \frac{v_0 C_{A,0}X_A}{200C_{A,0}^2(1-X_A)(M-X_A)}$$

$$\therefore V = \frac{v_0 X_A}{200C_{A,0}(1-X_A)(M-X_A)}$$

$$\therefore V = \frac{400\times0.999}{200\times(100/1000)\times(1-0.999)\times(2-0.999)} = \boxed{20{,}000 \text{ L}}$$

♦ The correct answer is **D**.

P.4 ➡ *Solution*

For a second-order reaction such as the present one, we have

$$k\tau C_{A,0}(M-1) = \ln\left[\frac{M-X_A}{M(1-X_A)}\right]$$

where the initial molar ratio of reactants $M = C_{B,0}/C_{A,0}$ = 200/100 = 2.0. Inserting this and other data in the equation gives

$$200\times\tau\times0.1\times(2-1) = \ln\left[\frac{2-0.999}{2\times(1-0.999)}\right]$$

$$\therefore 20\tau = 6.22$$

$$\therefore \tau = 0.311 \text{ min}$$

The reactor volume required is determined to be

$$V = \tau v = 0.311 \times 400 = \boxed{124 \text{ L}}$$

♦ The correct answer is **A**.

P.5 ➡ *Solution*

The expansion factor is ε_A = (2.5 − 1.0)/1.0 = 1.5. The performance equation for a first-order reaction with $\varepsilon_A \neq 0$ is given by equation (21) of Table 2,

$$k\tau = (1+\varepsilon_A)\ln\left(\frac{1}{1-X_A}\right) - \varepsilon_A X_A$$

Substituting $\tau = VC_{A,0}/F_{A,0}$ gives

$$\frac{kVC_{A,0}}{F_{A,0}} = (1+\varepsilon_A)\ln\left(\frac{1}{1-X_A}\right) - \varepsilon_A X_A$$

Inserting the numerical data, we have

$$\frac{10\times 22\times 2}{100} = (1+1.5)\ln\left(\frac{1}{1-X_A}\right) - 1.5X_A$$

$$\therefore 4.4 = 2.5\ln\left(\frac{1}{1-X_A}\right) - 1.5X_A$$

The result above is a nonlinear equation in X_A. One way to solve it is to apply the *FindRoot* command in Mathematica, using an initial guess of X_A = 0.5,

$$\text{FindRoot}[4.4 - 2.5\text{Log}[1/(1-x_A)] + 1.5x_A, \{x_A, 0.5\}]$$

The code above returns X_A = 0.90. Thus, the conversion in the plug flow reactor is about 90%.

♦ The correct answer is **D**.

P.6 ➡ *Solution*

The performance equation for the reactor is

$$\frac{V}{F_{A,0}} = \frac{X_A}{-r_A}$$

which, substituting the relation we have for r_A, becomes

$$\frac{V}{F_{A,0}} = \frac{X_A}{-r_A} \rightarrow \frac{V}{F_{A,0}} = \frac{X_A}{kC_{A,0}^{1.5}(1-X_A)^{1.5}}$$

$$\therefore \frac{Vk}{F_{A,0}} = \frac{X_A}{C_{A,0}^{1.5}(1-X_A)^{1.5}}$$

Inserting $C_{A,0}$ = 10 mol/L and X_A = 0.7 in the right-hand side gives

$$\frac{Vk}{F_{A,0}} = \frac{0.7}{10^{1.5}\times(1-0.7)^{1.5}} = 0.135 \text{ (I)}$$

Now, suppose we had a mixed flow reactor with twice the volume of the original. We restate the performance equation as

$$\frac{2V}{F_{A,0}} = \frac{X'_A}{kC_{A,0}^{1.5}(1-X'_A)^{1.5}} \rightarrow 2\left(\frac{Vk}{F_{A,0}}\right) = \frac{X'_A}{10^{1.5}(1-X'_A)^{1.5}}$$

$$\therefore 2\left(\frac{Vk}{F_{A,0}}\right) = \frac{X'_A}{31.6(1-X'_A)^{1.5}}$$

From equation (I), we know that the term in parentheses in the left-hand side equals 0.135. Substituting and squaring both sides, we obtain

$$2\times\left(\frac{Vk}{F_{A,0}}\right)=\frac{X'_A}{31.6\left(1-X'_A\right)^{1.5}}\rightarrow 2\times 0.135=\frac{X'_A}{C_{A,0}^{1.5}\left(1-X'_A\right)^{1.5}}$$

$$\therefore 0.27=\frac{X'_A}{31.6\left(1-X'_A\right)^{1.5}}$$

$$\therefore\left(0.27\times 31.6\right)^2=\left[\frac{X'_A}{\left(1-X'_A\right)^{1.5}}\right]^2$$

$$\therefore 72.8=\frac{X'^{2}_A}{\left(1-X'_A\right)^3}$$

The result above is a third-degree equation in X'_A, which can be expanded to give

$$-72.8X'^3_A+217.4X'^2_A-218.4X'_A+72.8=0$$

One way to solve this equation is to employ Mathematica's *Solve* command, using the following syntax,

$$\text{Solve}[-72.8x^3+217.4x^2-218.4x+72.8==0,x]$$

This returns $x = X'_A = 0.795$. The conversion of the larger reactor is about 79.5%.

♦ The correct answer is **C**.

P.7 ➡ *Solution*

The expansion factor is $\varepsilon_A = (1-3)/3 = -0.667$. From the expression for r_A, it is easy to see that the reaction in question is a zero order reaction. In this case, the pertinent performance equation is

$$\frac{k\tau}{C_{A,0}}=\frac{kV}{F_{A,0}}=X_{A,f}\rightarrow V=\frac{F_{A,0}X_{A,f}}{k}\ \text{(I)}$$

The final concentration of reactant is given by

$$\frac{C_{A,f}}{C_{A,0}}=\frac{1-X_{A,f}}{1+\varepsilon_A X_{A,f}}$$

Substituting $C_{A,0}$ = 660 mmol/L, $C_{A,f}$ = 330 mmol/L, and $\varepsilon_A = -0.667$, we have

$$\frac{330}{660}=\frac{1-X_{A,f}}{1-0.667X_{A,f}}\rightarrow X_{A,f}=0.750$$

Substituting this and other data into equation (I) gives

$$V=\frac{540\times 0.750}{54}=\boxed{7.5\text{ L}}$$

♦ The correct answer is **C**.

P.8 ➡ *Solution*

From the expression we were given for r_A, it is easy to conclude that the reaction is a second-order reaction. The appropriate performance equation is then

$$k\tau C_{A,0}=\frac{C_{A,0}-C_{A,f}}{C_{A,f}}$$

Substituting $\tau = V/v_0$ and solving for $C_{A,f}$, we get

$$k\tau C_{A,0}=\frac{C_{A,0}-C_{A,f}}{C_{A,f}}\rightarrow k\frac{V}{v_0}C_{A,0}=\frac{C_{A,0}-C_{A,f}}{C_{A,f}}$$

$$\therefore k\frac{V}{v_0}C_{A,0}=\frac{C_{A,0}}{C_{A,f}}-1$$

$$\therefore\frac{C_{A,0}}{C_{A,f}}=k\frac{V}{v_0}C_{A,0}+1$$

$$\therefore C_{A,f} = \frac{C_{A,0}}{k\frac{V}{v_0}C_{A,0}+1}$$

Entering the numerical values, we ultimately obtain

$$C_{A,f} = \frac{1.0}{1+\frac{(0.05\times 60)\times 1.0\times 2.0}{0.5}} = \boxed{0.0769 \text{ mol/L}}$$

Factor 60 was added to change the dimensions of k from liter/mol·sec to liter/mol·min.

♦ The correct answer is **B.**

P.9 ➡ *Solution*

For the reaction in question, equation (21) of Table 2 holds,

$$k\frac{C_{A,0}V}{F_{A,0}} = (1+\varepsilon_A)\ln\left(\frac{1}{1-X_A}\right) - \varepsilon_A X_A \text{ (I)}$$

We must first evaluate the initial concentration of phosphine,

$$C_{A,0} = \frac{P_{A,0}}{RT} = \frac{2/3\times 11.4}{0.0821\times(649+273)} = 0.101 \text{ mol/L}$$

To determine the expansion factor, consider the following volume balance.

	4A → 7R
Reactants	40 → 70
Inerts	20 → 20

Accordingly,

$$\varepsilon_A = \frac{90-60}{60} = 0.5$$

We can now rearrange equation (I) and substitute our data, giving

$$V = \frac{F_{A,0}}{kC_{A,0}}\left[(1+\varepsilon_A)\ln\left(\frac{1}{1-X_A}\right) - \varepsilon_A X_A\right]$$

$$\therefore V = \frac{10}{10\times 0.101}\times\left[(1+0.5)\times\ln\left(\frac{1}{1-0.75}\right) - 0.5\times 0.75\right] = \boxed{16.9 \text{ L}}$$

♦ The correct answer is **A.**

P.10 ➡ *Solution*

For a second order reaction with expansion, the appropriate performance equation is equation (23) in Table 2,

$$V = \frac{v}{C_{A,0}k}\left[2\varepsilon_A(1+\varepsilon_A)\ln(1-X_A) + \varepsilon_A^2 X_A + (\varepsilon_A+1)^2\frac{X_A}{1-X_A}\right]$$

The initial concentration of ozone (A) is given by the ideal gas law,

$$C_{A,0} = \frac{P_{A,0}}{RT} = \frac{0.2\times 1.5}{0.0821\times(273+93)} = 0.00998 \text{ mol/L}$$

The expansion factor is ε_A = (11 − 10)/10 = 0.1. Substituting these and other data in the equation for V, we get

$$V = \frac{1.0}{0.00998\times 0.05}\times\left[2\times 0.1\times(1+0.1)\ln(1-0.5) + 0.1^2\times 0.5 + (0.1+1)^2\times\frac{0.5}{1-0.5}\right]$$

$$\therefore \boxed{V = 2130 \text{ L}}$$

♦ The correct answer is **C.**

P.11 ➡ *Solution*

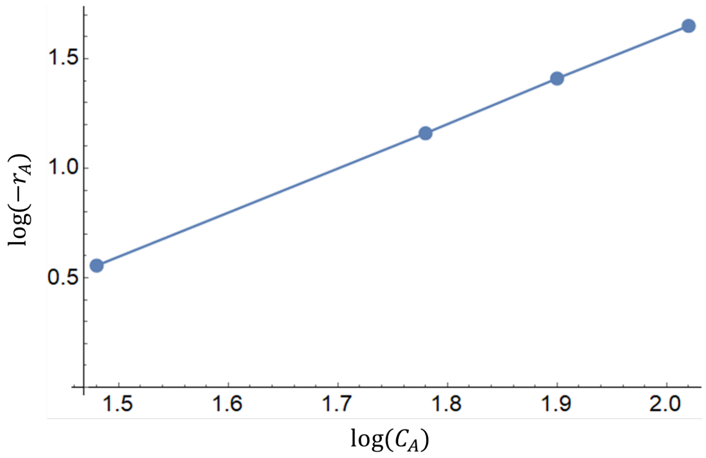
The rate $-r_A$ is given by

$$-r_A = \frac{C_{A,0}X_A v_0}{V} = \frac{C_{A,0}\left(C_{A,0}-C_A\right)v_0}{\left(C_{A,0}+\varepsilon_A C_A\right)V} = \frac{120\left(120-C_A\right)v_0}{\left(120+2C_A\right)\times 1.0} \quad \text{(I)}$$

where we have substituted $C_{A,0}$ = 120 mmol/liter, V = 1 liter, and ε_A = (3 − 1)/1 = 2.0. The data are processed in the following table.

v_0 (L/min)	C_A (mmol/L)	$-r_A$ (Eq. I)	$\log(-r_A)$	$\log C_A$
0.06	30	3.6	0.556	1.48
0.48	60	14.4	1.16	1.78
1.5	80	25.7	1.41	1.90
8.1	105	44.2	1.65	2.02

The next step is to plot $\log(-r_A)$ (the blue column) versus $\log C_A$ (the red column), as follows.



The line is described by the relation

$$\log\left(-r_A\right) = \log k + n\log C_A$$

where n is the slope of the line (and the reaction order), which can be easily seen to be n = 2.0. Thus,

$$\log\left(-r_A\right) = \log k + 2.0\log C_A$$

Substituting any of the data points available, we can determine rate constant k,

$$0.556 = \log k + 2.0\times 1.48$$

$$\therefore \log k = -2.40$$

$$\therefore k = 10^{-2.4} = 0.004$$

The appropriate expression for the reaction rate is

$$-r_A = kC_A^n = \boxed{0.004C_A^2 \text{ mmol/lit}\cdot\text{min}}$$

♦ The correct answer is **D**.

P.12 ➡ *Solution*

To begin, we write an expression for conversion X_A, as follows.

$$C_A = C_{A,0}\left(1-X_A\right) \rightarrow X_A = 1-\frac{C_A}{C_{A,0}}$$

$$\therefore X_A = 1-\frac{C_A}{100} \quad \text{(I)}$$

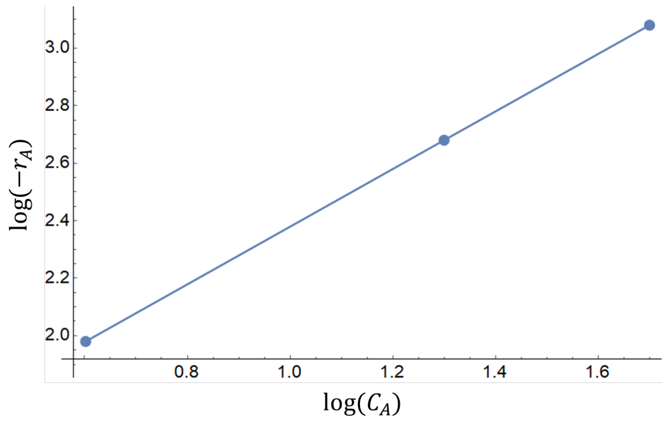
The reaction rate, in turn, is given by

$$-r_A = \frac{v_0 C_{A,0} X_A}{V} = \frac{v_0\times 100\times X_A}{1.0} = 100v_0X_A \quad \text{(II)}$$

In order to establish the reaction order, we must plot $\log -r_A$ versus $\log C_A$. The data are processed in the following table.

v_0 (L/min)	C_A (mmol/L)	X_A (Eq. I)	$-r_A$ (Eq. II)	$\log(-r_A)$	$\log C_A$
1	4	0.96	96	1.98	0.602
6	20	0.8	480	2.68	1.30
24	50	0.5	1200	3.08	1.70

The graph we are looking for is one $\log(-r_A)$ (the blue column) versus $\log C_A$ (the red column), as follows.



The slope of the line is approximately 1. Thus, the reaction in question is a first-order reaction. It remains to compute the rate constant k, which is given by

$$-r_A = kC_A \rightarrow k = \frac{-r_A}{C_A}$$

Substituting one of the available data points, we obtain

$$k = \frac{96}{4} = 24\ \text{min}^{-1}$$

Thus, the rate equation is established as

$$\boxed{-r_A = 24C_A\ \frac{\text{mmol}}{\text{liter}\cdot\text{min}}}$$

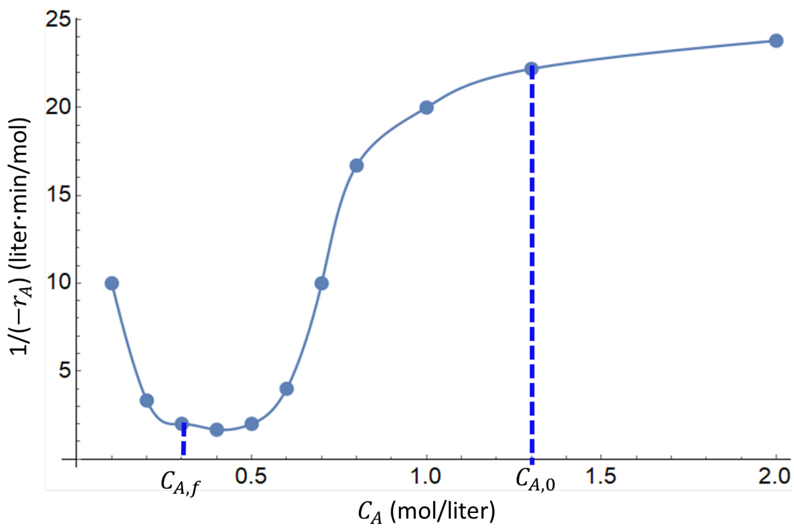
♦ The correct answer is **B.**

P.13 ➡ *Solution*

Part 1: To begin, we tabulate values of $1/(-r_A)$.

C_A (mol/liter)	$-r_A$ (mol/liter·min)	$1/(-r_A)$
0.1	0.1	10
0.2	0.3	3.33
0.3	0.5	2
0.4	0.6	1.67
0.5	0.5	2
0.6	0.25	4
0.7	0.10	10
0.8	0.06	16.7
1.0	0.05	20
1.3	0.045	22.2
2.0	0.042	23.8

Then, a rate-concentration plot of $1/(-r_A)$ (the blue column) versus C_A (the red column) is prepared.



It can be easily seen that the time required to produce the concentration drop in question is given by

$$t = \int_{C_{A,f}}^{C_{A,0}} \frac{dC_A}{-r_A}$$

The result in the right-hand side is the area below the graph between abscissae C_{Af} = 0.3 mol/L and $C_{A,0}$ = 1.3 mol/L, which, using numerical integration, is found to equal 12.7 min. Accordingly,

$$t = \int_{C_{A,f}}^{C_{A,0}} \frac{dC_A}{-r_A} \approx \boxed{12.7 \text{ min}}$$

Note

In Mathematica, a quick way to go is to combine the *Integrate* and *Interpolation* commands. We first type the data,

dataSet = {{0.1,10}, {0.2,3.33}, {0.3,2}, {0.4,1.67}, {0.5,2}, {0.6,4}, {0.7,10}, {0.8,16.7}, {1.,20}, {1.3,22.2}, {2.,23.8}}

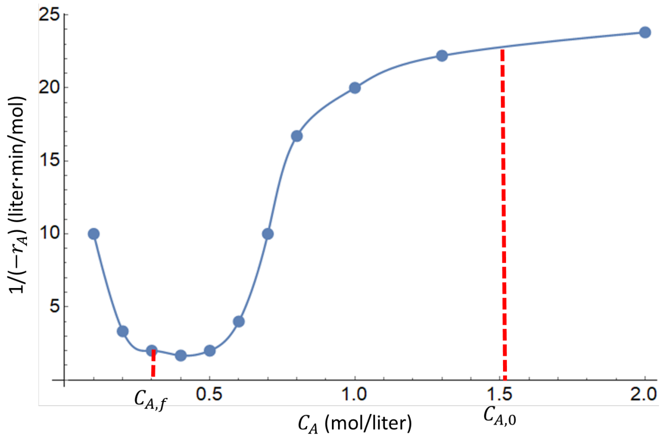
Then, we type

Integrate[Interpolation[dataSet][x], {x, 0.3,1.3}]

This returns 12.6978.

♦ The correct answer is **B.**

Part 2: The rate-concentration curve is replotted below.



The final concentration is, in this case,

$$C_A = C_{A,0}(1 - X_A) = 1.5 \times (1 - 0.8) = 0.3 \text{ mol/L}$$

For a plug flow reactor of constant density, we write

$$\tau = C_{A,0} \int_0^{X_{A,f}} \frac{dX_A}{-r_A} = -\int_{C_{A,0}}^{C_{A,f}} \frac{dC_A}{-r_A}$$

However, the space time $\tau = C_{A,0}V/F_{A,0}$, so that

$$\tau = \frac{C_{A,0}V}{F_{A,0}} = -\int_{C_{A,0}}^{C_{A,f}} \frac{dC_A}{-r_A} \rightarrow V = \frac{F_{A,0}}{C_{A,0}} \left[-\int_{1.5}^{0.3} \frac{dC_A}{-r_A} \right]$$

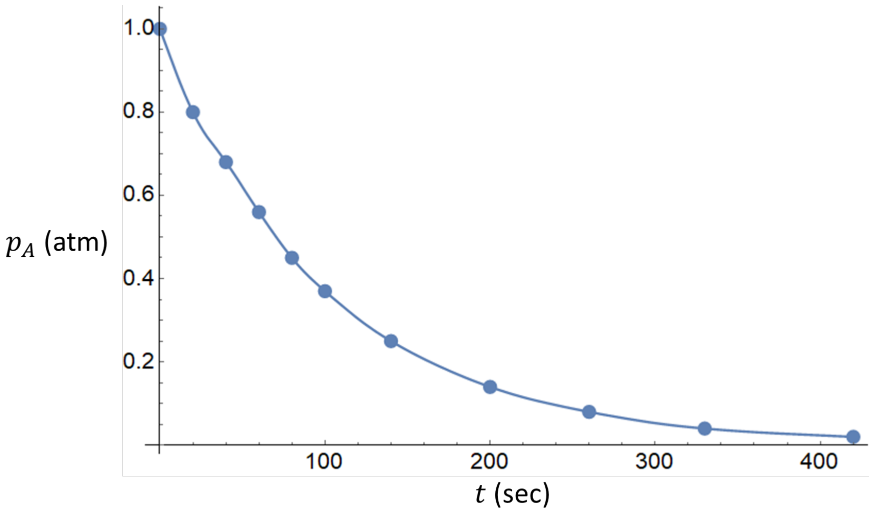
The integral in brackets equals the area below the rate-concentration curve between abscissae $C_{A,0}$ = 1.5 mol/L and $C_{A,f}$ = 0.3 mol/L. Using numerical integration, the result is found to be 17.2 min. Accordingly, the reactor volume becomes

$$V = \frac{(1000/60)}{1.5} \times 17.2 = \boxed{191 \text{ L}}$$

♦ The correct answer is **C.**

P.14 ➡ *Solution*

Since there are 20% inerts in the feed, $p_{A,0}$ = 0.8×1.0 = 0.8 atm. For 95% conversion, the pressure at the outlet must be $p_{A,f}$ = 0.05×0.8 = 0.04 atm. For mixed flow, we must first find the rate at the exit conditions. Then we can proceed to determine the reactor size. Accordingly, we draw a p_A versus t curve and find the slope (and hence the rate) at p_A = 0.04 atm.



The slope at p_A = 0.04 atm can be approximated as

$$\text{rate} = \frac{0.02 - 0.08}{420 - 260} = -\frac{0.06}{160} \text{ atm/s}$$

The space time, written in pressure units, follows as

$$\tau = \frac{p_{A,0} - p_A}{-r_A} = \frac{0.8 - 0.04}{(0.06/160)} = 2030 \text{ s}$$

The molar feed, taking the flow of inerts into account, is n_{feed} = 100×100/80 = 125 mol/hr. The volumetric flow rate can be obtained from the ideal gas law,

$$v = \frac{n_{\text{feed}} RT}{p} = \frac{125 \times 0.0821 \times 373}{1.0} = 3830 \text{ L/hr}$$

Finally, the reactor volume is calculated to be

$$V = \tau v = 2030 \times \left(3830 \times \frac{1}{3600}\right) = \boxed{2160 \text{ L}}$$

♦ The correct answer is **D**.

ANSWER SUMMARY

Problem 1		T/F
Problem 2		B
Problem 3		D
Problem 4		A
Problem 5		D
Problem 6		C
Problem 7		C
Problem 8		B
Problem 9		A
Problem 10		C
Problem 11		D
Problem 12		B
Problem 13	13.1	B
	13.2	C
Problem 14		D

REFERENCES

- FOGLER, H. (2018). *Essentials of Chemical Reaction Engineering*. 2nd edition. Upper Saddle River: Pearson.
- LEVENSPIEL, O. (1999). *Chemical Reaction Engineering*. 3rd edition. Hoboken: John Wiley and Sons.

Got any questions related to this quiz? We can help! Send a message to contact@montogue.com and we'll answer your question as soon as possible.