Quiz SM211

IMPACT LOADS

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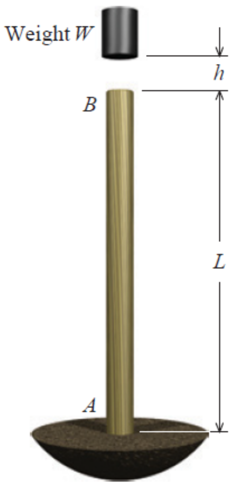
PROBLEMS

Problem 1 (Philpot, 2013, w/ permission)

A weight $W = 4000$ lbs falls from a height of $h = 18$ in. onto the top of a 10-in. diameter wood pole, as shown in the figure. The pole has a length of $L = 24$ ft and a modulus of elasticity of $E = 1.5 \times 10^6$ psi. For this problem, disregard any potential buckling effects. Calculate the maximum shortening of the pole and the maximum compression stress in the pole.



A) δ_{max} = 0.305 in. and σ_{max} = 3150 psi

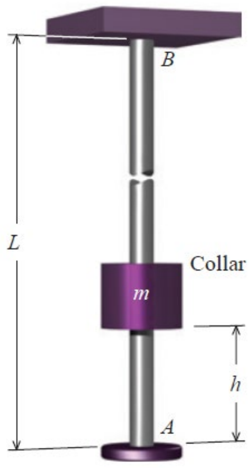
B) δ_{max} = 0.305 in. and σ_{max} = 4550 psi

C) δ_{max} = 0.603 in. and σ_{max} = 3150 psi

D) δ_{max} = 0.603 in. and σ_{max} = 4550 psi

Problem 2 (Philpot, 2009, w/ permission)

A 19-mm diameter steel (E = 200 GPa) rod is required to absorb the energy of a 25-kg collar that falls h = 75 mm, as shown in the next figure. Determine the required rod length L so that the maximum stress in the rod does not exceed 210 MPa.



A) L = 0.264 m

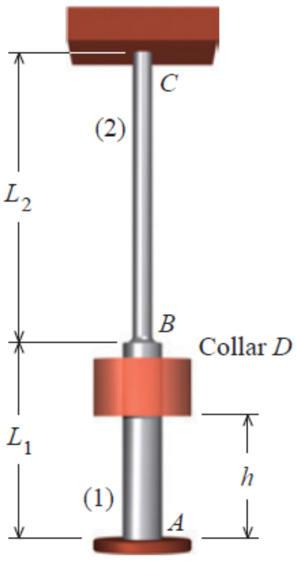
B) L = 0.592 m

C) L = 0.924 m

D) L = 1.26 m

Problem 3 (Philpot, 2013, w/ permission)

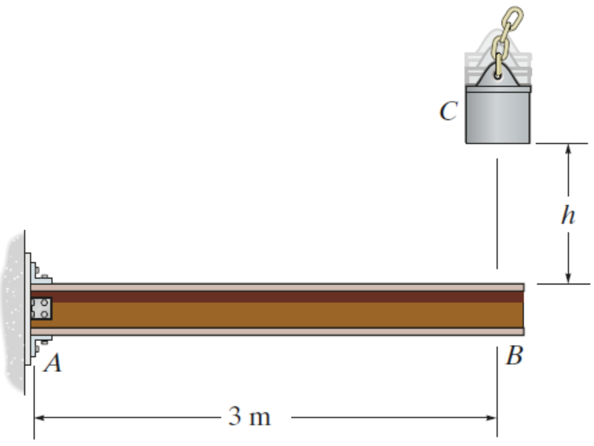
In the following figure, collar D, released from rest, slides without friction downward a distance of $h = 2.5$ in. where it strikes a head fixed to the end of compound rod ABC, which is made of aluminum ($E = 10{,}000$ ksi). Rod segment (1) has a length of $L_1 = 7$ in. and a diameter of $d_1 = 0.75$ in. and rod segment (2) has a length of $L_2 = 13$ in. and a diameter of $d_2 = 0.50$ in. Collar D weighs 20 lbs. Determine the equivalent static load for this impact case and the maximum normal stress attained in the whole assembly.



A) $P_{max} = 3520$ lbs and $\sigma_{max} = 7.96$ ksi
B) $P_{max} = 3520$ lbs and $\sigma_{max} = 18.0$ ksi
C) $P_{max} = 6120$ lbs and $\sigma_{max} = 7.96$ ksi
D) $P_{max} = 6120$ lbs and $\sigma_{max} = 18.0$ ksi

Problem 4A (Hibbeler, 2014, w/ permission)

The 50 kg block is dropped from $h = 0.9$ m onto the cantilever beam. If the beam is made from steel ($E = 200$ GPa, $\sigma_Y = 345$ MPa) determine the maximum bending stress developed in the beam. The beam has a wide-flange section with depth $d = 0.2$ m and moment of inertia about the axis of interest of $I = 46 \times 10^{-6}$ m^4.



A) $\sigma_{max} = 124$ MPa
B) $\sigma_{max} = 198$ MPa
C) $\sigma_{max} = 259$ MPa
D) $\sigma_{max} = 302$ MPa

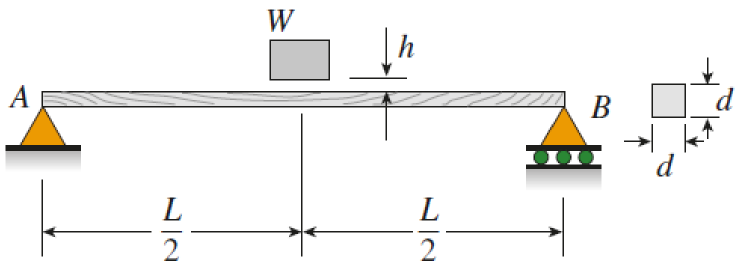
Problem 4B

What is the maximum height from which the 50-kg block can be dropped without causing yielding in the cantilever beam?

A) $h_{max} = 1.23$ m
B) $h_{max} = 1.94$ m
C) $h_{max} = 2.78$ m
D) $h_{max} = 3.65$ m

Problem 5 (Gere & Goodno, 2009, w/ permission)

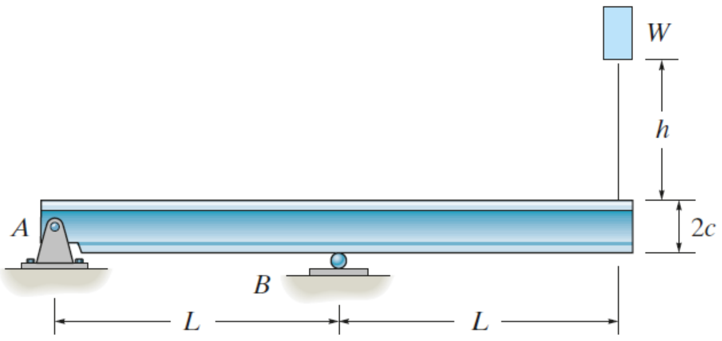
A weight $W = 20$ kN falls through a height $h = 1.0$ mm onto the midpoint of a single beam of length $L = 3$ m (see figure). The beam is made of wood with square cross-section (dimension d on each side) and $E = 12$ GPa. If the allowable bending stress is $\sigma_{\text{allow}} = 10$ MPa, what is the minimum required dimension d?



A) $d = 160$ mm
B) $d = 220$ mm
C) $d = 280$ mm
D) $d = 340$ mm

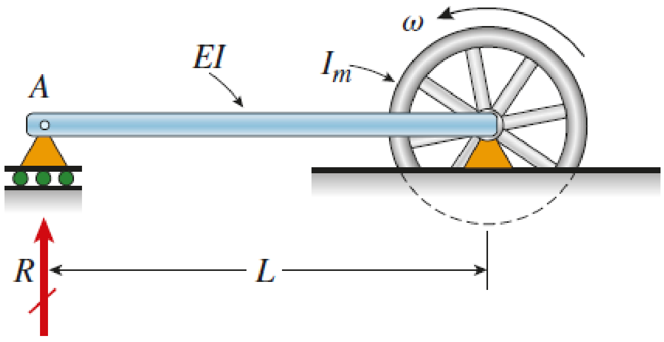
Problem 6 (Hibbeler, 2014, w/ permission)

The wide-flange beam has a length of $2L$, a depth $2c$, and a constant flexural rigidity EI. Determine the maximum height h at which a weight W can be dropped on its end without exceeding a maximum elastic stress $\sigma_{\max}$ in the beam.



Problem 7 (Gere & Goodno, 2009, w/ permission)

A heavy flywheel rotates at an angular speed ω (radians per second) around an axle (see figure). The axle is rigidly attached to the end of a simply supported beam of flexural rigidity EI and length L. The flywheel has mass moment of inertia I_m about its axis of rotation. If the flywheel suddenly freezes to the axle, what will be the reaction R at support A of the beam?



A) $R = \sqrt{\dfrac{EII_m}{3L^3}}\,\omega$

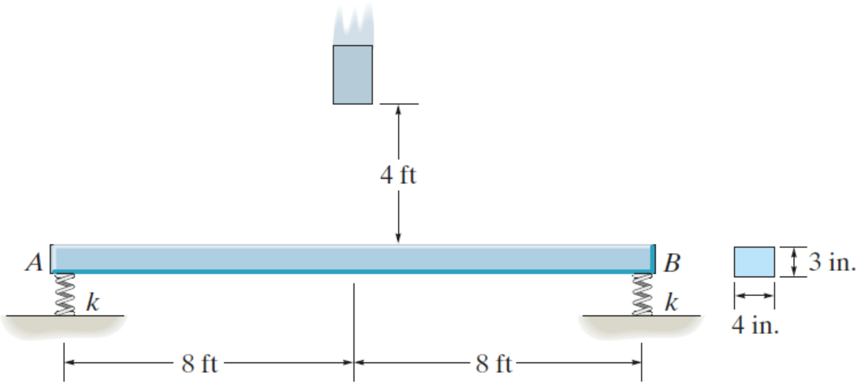
B) $R = \sqrt{\dfrac{EII_m}{2L^3}}\,\omega$

C) $R = \sqrt{\dfrac{2EII_m}{L^3}}\,\omega$

D) $R = \sqrt{\dfrac{3EII_m}{L^3}}\,\omega$

Problem 8 (Hibbeler, 2014, w/ permission)

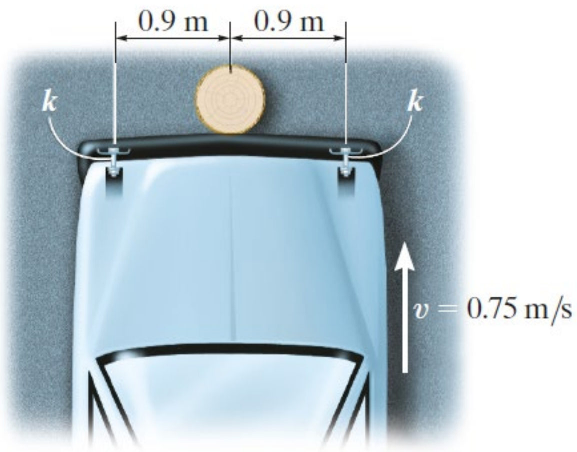
The weight of 175 lb is dropped from a height of 4 ft from the top of the steel beam ($E = 29$ ksi, $\sigma_Y = 50$ ksi). Determine the maximum deflection and maximum stress in the beam if the supporting springs at A and B each have a stiffness of $k = 500$ lb/in. The beam is 3 in. thick and 4 in. wide.



A) $\delta_{max} = 2.71$ in. and $\sigma_{max} = 13.2$ ksi
B) $\delta_{max} = 2.71$ in. and $\sigma_{max} = 27.7$ ksi
C) $\delta_{max} = 5.40$ in. and $\sigma_{max} = 13.2$ ksi
D) $\delta_{max} = 5.40$ in. and $\sigma_{max} = 27.7$ ksi

Problem 9 (Hibbeler, 2014, w/ permission)

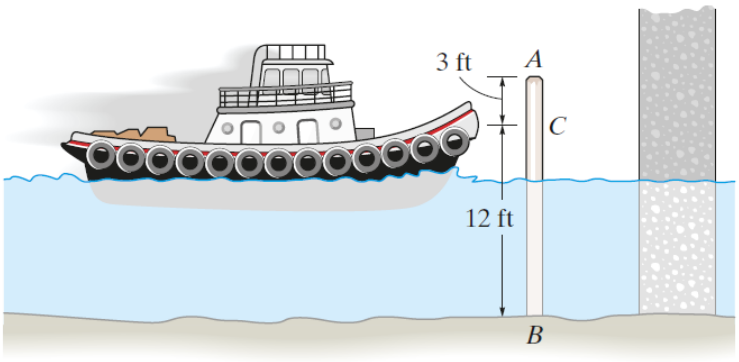
The car bumper is made of polycarbonate-polybutylene terephthalate. If $E = 2.0$ GPa, determine the maximum deflection and maximum stress in the bumper if it strikes the rigid post when the car is coasting at $v = 0.75$ m/s. The car has a mass of 1.80 Mg, and the bumper can be considered simply supported on two spring supports connected to the rigid frame of the car. For the bumper take $I = 300 \times 10^6$ mm⁴, $c = 75$ mm, $\sigma_Y = 30$ MPa and $k_{sp} = 1.5$ MN/m.



A) $\delta_{max} = 23.3$ mm. and $\sigma_{max} = 4.90$ MPa
B) $\delta_{max} = 23.3$ mm. and $\sigma_{max} = 9.83$ MPa
C) $\delta_{max} = 40.5$ mm. and $\sigma_{max} = 4.90$ MPa
D) $\delta_{max} = 23.3$ mm. and $\sigma_{max} = 9.83$ MPa

Problem 10 (Hibbeler, 2014, w/ permission)

The tugboat has a weight of 120,000 lb and is traveling forward at 2 ft/s when it strikes the 12-in. diameter fender post *AB* used to protect a bridge pier. If the post is made from treated white spruce ($E = 1.40 \times 10^6$ psi) and is assumed fixed at the river bed, determine the maximum horizontal distance the top of the post will move due to the impact. Assume the tugboat is rigid and neglect the effect of the water.

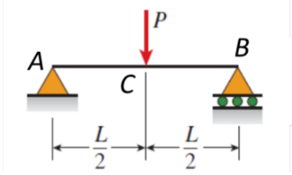


A) $(\delta_A)_{max}$ = 5.23 in.
B) $(\delta_A)_{max}$ = 10.2 in.
C) $(\delta_A)_{max}$ = 15.4 in.
D) $(\delta_A)_{max}$ = 20.6 in.

ADDITIONAL INFORMATION

Figure 1 Deflections and slopes of beams

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$



$$\delta_C = \delta_{max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$

SOLUTIONS

P.1 ➡ *Solution*

The cross-sectional area of the pole is $A = \pi \times 10^2/4 = 78.5$ in.² and the static deformation is

$$\delta_{st} = \frac{F_{st}L}{AE} = \frac{4000\times(24\times12)}{78.5\times(1.5\times10^6)} = 9.78\times10^{-3}\text{ in.}$$

The static normal stress is

$$\sigma_{st} = \frac{F_{st}}{A} = \frac{4000}{78.5} = 51\text{ psi}$$

The impact factor follows as

$$n = 1+\sqrt{1+\frac{2h}{\delta_{st}}} = 1+\sqrt{1+\frac{2\times18}{9.78\times10^{-3}}} = 61.7$$

The maximum shortening of the pole is given by the product of impact factor and static deformation; that is,

$$\delta_{max} = n\delta_{st} = 61.7\times(9.78\times10^{-3}) = \boxed{0.603\text{ in.}}$$

Similarly, the maximum compression stress is given by the product of impact factor and static normal stress,

$$\sigma_{max} = n\sigma_{st} = 61.7\times51 = \boxed{3150\text{ psi}}$$

The correct answer is **C**.

P.2 ➡ *Solution*

The cross-sectional area of the rod is $A = \pi \times 0.019^2/4 = 2.84 \times 10^{-4}$ m² and the static norm1al stress is

$$\sigma_{st} = \frac{F_{st}}{A} = \frac{(25\times 9.81)}{2.84\times 10^{-4}} = 0.864 \text{ MPa}$$

The static deformation δ_{st} can be calculated with the axial load formula

$$\delta_{st} = \frac{F_{st}L}{AE} = \frac{(25\times 9.81)\times L}{(2.84\times 10^{-4})\times(200\times 10^9)} = 4.32\times 10^{-6} L$$

Given the maximum normal stress $\sigma_{max} = 210$ MPa, the impact factor is determined as

$$n = \frac{\sigma_{max}}{\sigma_{st}} = \frac{210}{0.864} = 243$$

However, the impact factor can also be obtained from the formula

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

Substituting the pertaining variables and solving for the length L, we have

$$n = 1 + \sqrt{1 + \frac{2\times 0.075}{4.32\times 10^{-6} L}} = 243$$

$$\therefore \boxed{L = 0.592 \text{ m}}$$

The correct answer is **B**.

P.3 ➡ *Solution*

The cross-sectional area of segment (1) is $A_1 = \pi \times 0.75^2/4 = 0.442$ in.² and that of segment 2 is $A_2 = \pi \times 0.5^2/4 = 0.196$ in.² The total static deformation is due to contributions from segments (1) and (2). In mathematical terms,

$$\delta_{st} = \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 E_2} = \frac{F}{E}\times\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right)$$

$$\therefore \delta_{st} = \frac{20}{(10\times 10^6)}\times\left(\frac{7}{0.442} + \frac{13}{0.196}\right) = 1.64\times 10^{-4} \text{ in.}$$

The impact factor n follows from the usual formula

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{2\times 2.5}{(1.64\times 10^{-4})}} = 176$$

The equivalent static load is then

$$P_{max} = nP_{st} = 176\times 20 = \boxed{3520 \text{ lb}}$$

The maximum normal stress in segment (1) is given by

$$\sigma_{max,1} = \frac{P_{max}}{A_1} = \frac{3520}{0.442} = 7.96 \text{ ksi}$$

while the maximum normal stress in segment (2) is

$$\sigma_{max,2} = \frac{P_{max}}{A_2} = \frac{3520}{0.196} = 18.0 \text{ ksi}$$

Thus, the maximum stress in the assembly is $\sigma_{max} = 18$ ksi.

The correct answer is **B**.

P.4 ➡ *Solution*

Part A: The weight of the block is $W = 50 \times 9.81 = 491$ N. From the Additional Information section, the static displacement of end B is determined to be

$$\delta_{st} = \frac{PL^3}{3EI} = \frac{491 \times 3^3}{3 \times \left(200 \times 10^9\right) \times \left(45.5 \times 10^{-6}\right)} = 4.86 \times 10^{-4} \text{ m}$$

The impact factor follows as

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{2 \times 0.9}{4.86 \times 10^{-4}}} = 61.9$$

The maximum force on the beam is $P_{max} = nP = 61.9 \times 491 = 30.4$ kN. The maximum moment occurs at fixed support A, where $M_{max} = P_{max}L = 30.4 \times 10^3 \times 3 = 91.2$ kN·m. The corresponding stress is obtained with the flexure formula,

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{\left(91.2 \times 10^3\right) \times (0.2/2)}{46 \times 10^{-6}} = \boxed{198 \text{ MPa}}$$

Since $\sigma_{max} < \sigma_Y = 345$ MPa, the result is valid.

The correct answer is **B**.

Part B: The maximum force on the beam is $P_{max} = nW = n \times 50 \times 9.81 = 491n$. The maximum moment occurs at the fixed support A, where $M_{max} = P_{max}L = 491n \times 3 = 1470n$. Applying the flexure formula, we have

$$\sigma_Y = \frac{M_{max}c}{I} \rightarrow 345 \times 10^6 = \frac{1470n \times (0.2/2)}{46 \times 10^{-6}}$$

$$\therefore n = 108$$

The static deflection remains $\delta_{max} = 4.86 \times 10^{-4}$ m. Substituting in the equation for impact factor gives

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \rightarrow 108 = 1 + \sqrt{1 + \frac{2h}{4.86 \times 10^{-4}}}$$

$$\therefore \boxed{h_{max} = 2.78 \text{ m}}$$

The correct answer is **C**.

P.5 ➡ *Solution*

From the definition of impact factor, we have

$$n = \frac{\sigma_{max}}{\sigma_{st}} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

The static normal stress is obtained with the flexure formula,

$$\sigma_{st} = \frac{M}{S} = \frac{WL}{4} \bigg/ \frac{d^3}{3} = \frac{3WL}{2d^3}$$

The static deflection, in turn, is determined as (as given in the Additional Information section)

$$\delta_{st} = \frac{WL^3}{48EI} = \frac{WL^3}{48E} \times \frac{1}{\left(\frac{d^4}{12}\right)} = \frac{WL^3}{4Ed^4}$$

Inserting these results into the equation for n, we find that

$$\frac{\sigma_{max}}{\frac{3WL}{2d^3}} = 1 + \sqrt{1 + \frac{2h}{\frac{WL^3}{4Ed^4}}}$$

$$\therefore \frac{2\sigma_{\max} d^3}{3WL} = 1 + \sqrt{1 + \frac{8Ehd^4}{WL^3}}$$

Lastly, we insert the numerical data and solve for *d*,

$$\frac{2\times\left(10\times10^6\right)\times d^3}{3\times\left(20\times10^3\right)\times 3} = 1 + \sqrt{1 + \frac{8\times\left(12\times10^9\right)\times 0.001\times d^4}{\left(20\times10^3\right)\times 3^3}}$$

$$\therefore 111d^3 = 1 + \sqrt{1 + 178d^4}$$

$$\therefore \boxed{d = 280\ \text{mm}}$$

The correct answer is **C**.

P.6 ➡ *Solution*

The static deflection at the end of the beam due to weight *W* can be determined by integration,

$$\frac{1}{2}P\delta_{st} = 2\times\left(\frac{1}{EI}\right)\int_0^L (-Px)^2\,dx$$

$$\therefore \delta_{st} = \frac{2PL^3}{3EI}$$

The maximum static normal stress is given by

$$\sigma_{st} = \frac{WLc}{I}$$

Using the impact factor, we can easily associate this quantity to the maximum elastic stress $\sigma_{\max}$,

$$n = \frac{\sigma_{\max}}{\sigma_{st}} \rightarrow \sigma_{\max} = n\sigma_{st}$$

Recall that *n* is defined as

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

so that

$$\sigma_{\max} = n\sigma_{st} \rightarrow \sigma_{\max} = \left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right)\frac{WLc}{I}$$

Manipulating this equation brings to

$$\frac{\sigma_{\max} I}{WLc} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

$$\therefore \frac{\sigma_{\max} I}{WLc} - 1 = \sqrt{1 + \frac{2h}{\delta_{st}}}$$

$$\therefore \left(\frac{\sigma_{\max} I}{WLc} - 1\right)^2 = 1 + \frac{2h}{\delta_{st}}$$

$$\therefore h = \frac{\delta_{st}}{2}\left[\left(\frac{\sigma_{\max} I}{WLc} - 1\right)^2 - 1\right]$$

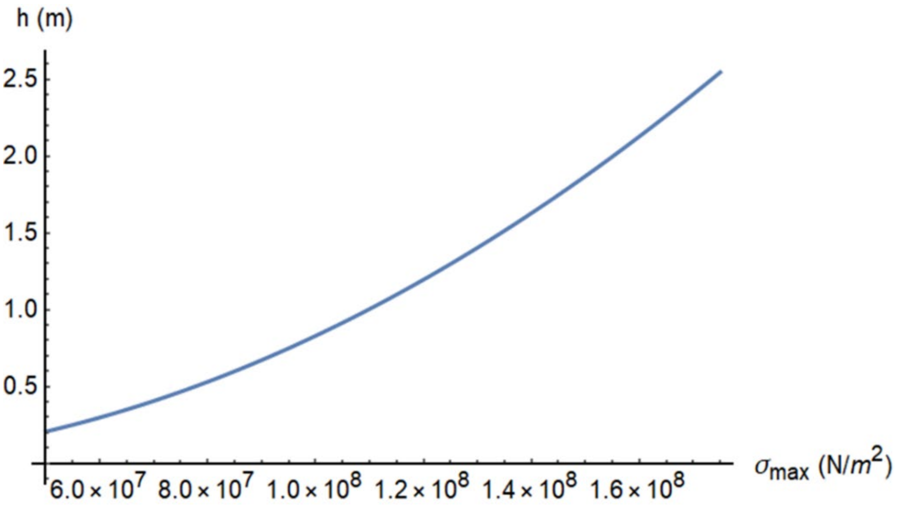
$$\therefore h = \frac{\delta_{st}}{2}\left[\left(\frac{\sigma_{\max} I}{WLc}\right)^2 - \frac{2\sigma_{\max} I}{WLc}\right]$$

$$\therefore h = \frac{WL^3}{3EI}\left[\left(\frac{\sigma_{\max} I}{WLc}\right)^2 - \frac{2\sigma_{\max} I}{WLc}\right]$$

$$\therefore h = \frac{\cancel{W} L^{\cancel{3}}}{3E\cancel{I}} \times \frac{\sigma_{\max} \cancel{I}}{\cancel{W}\cancel{L}c} \times \left(\frac{\sigma_{\max} I}{WLc} - 2 \right)$$

$$\therefore \boxed{h = \frac{\sigma_{\max} L^2}{3Ec} \left(\frac{\sigma_{\max} I}{WLc} - 2 \right)}$$

From this relation, we see that the maximum allowable height increases with increasing beam length (doubling *L* will cause *h* to increase four-fold) and moment of inertia, but decreases with increasing Young's modulus and block weight. As an illustration, the following graph displays the relationship between maximum normal stress $\sigma_{\max}$ and drop height *h* for a falling weight of 100 N and a steel beam with square cross-section (*c* = 0.2 m), covering a range of hypothetical $\sigma_{\max}$ going from 50 MPa to 175 MPa.



P.7 ➡ *Solution*

We will disregard the mass of the beam and all energy losses due to the sudden stopping of the rotating flywheel. Further, we assume that *all* of the kinetic energy of the flywheel is transformed into strain energy of the beam. The kinetic energy of the rotating flywheel is

$$KE = \frac{1}{2} I_m \omega^2$$

The strain energy of the beam, in turn, is obtained with the usual relation

$$U = \int \frac{M^2 dx}{2EI}$$

With moment *M* = *Rx* (*x* is measured from support *A*), we get

$$U = \frac{1}{2EI} \int_0^L (Rx)^2 \, dx = \frac{R^2 L^3}{6EI}$$

Equating kinetic energy to strain energy and solving for reaction *R*, it follows that

$$KE = U$$

$$\therefore \frac{1}{2} I_m \omega^2 = \frac{R^2 L^3}{6EI}$$

$$\therefore R^2 = \frac{3 I_m \omega^2 EI}{L^3}$$

$$\therefore \boxed{R = \sqrt{\frac{3EII_m}{L^3}} \omega}$$

The correct answer is **D**.

P.8 ➡ *Solution*

From the Additional Information section, we see that the static deflection of the midpoint of the beam under these loading conditions is

$$\delta_{st} = \frac{PL^3}{48EI}$$

Since $F = k_{\text{beam}}\delta_{\text{st}}$, the equivalent stiffness of the beam is stated as

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48\times(29\times10^3)\times\left(\frac{1}{12}\times4\times3^3\right)}{(16\times12)^3} = 1.77 \text{ kip/in.}$$

Refer to the free-body diagram illustrated below. From the equilibrium of forces in the vertical direction, we can write

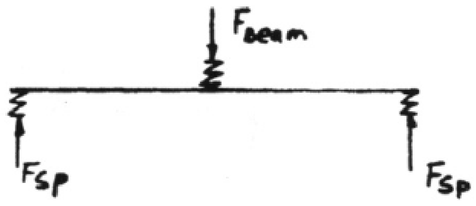
$$2F_{\text{sp}} = F_{\text{beam}}$$

$$\therefore 2k_{\text{sp}}\delta_{\text{sp}} = k_{\text{beam}}\delta_{\text{beam}}$$

$$\therefore \delta_{\text{sp}} = \frac{k_{\text{beam}}}{2k_{\text{sp}}}\times\delta_{\text{beam}}$$

$$\therefore \delta_{\text{sp}} = \frac{(1.77\times10^3)}{2\times500}\times\delta_{\text{beam}}$$

$$\therefore \delta_{\text{sp}} = 1.77\delta_{\text{beam}} \text{ (I)}$$



Consider now the conservation of energy in the beam. The work done by the falling weight must be equal to the sum of kinetic energies of the system. In mathematical terms,

$$U_e = U_i$$

$$\therefore W\left(h+\delta_{\text{sp}}+\delta_{\text{beam}}\right) = \frac{1}{2}k_{\text{beam}}\delta_{\text{beam}}^2 + 2\left(\frac{1}{2}\right)k_{\text{sp}}\delta_{\text{sp}}^2$$

Substituting the pertaining variables and using equation (I), we have

$$U_e = U_i$$

$$\therefore W\left(h+\delta_{\text{sp}}+\delta_{\text{beam}}\right) = \frac{1}{2}k_{\text{beam}}\delta_{\text{beam}}^2 + 2\left(\frac{1}{2}\right)k_{\text{sp}}\delta_{\text{sp}}^2$$

$$\therefore 175\times\left[(4\times12)+1.77\delta_{\text{beam}}+\delta_{\text{beam}}\right] = \frac{1}{2}\times\left(1.77\times10^3\right)\delta_{\text{beam}}^2 + 500\times\left(1.77\delta_{\text{beam}}\right)^2$$

$$\therefore 8400+310\delta_{\text{beam}}+175\delta_{\text{beam}} = 885\delta_{\text{beam}}^2 + 1570\delta_{\text{beam}}^2$$

$$\therefore 2455\delta_{\text{beam}}^2 - 485\delta_{\text{beam}} - 8400 = 0$$

$$\therefore \delta_{\text{beam}} = 1.95 \text{ in.}$$

In view of equation (I), we find that

$$\delta_{\text{sp}} = 1.77\delta_{\text{beam}} = 1.77\times1.95 = 3.45 \text{ in.}$$

The maximum deflection is then

$$\delta_{\max} = \delta_{\text{beam}} + \delta_{\text{sp}} = 1.95+3.45 = \boxed{5.40 \text{ in.}}$$

The force exerted on the beam is

$$F_{\text{beam}} = k_{\text{beam}}\delta_{\text{beam}} = 1.77\times1.95 = 3.45 \text{ kip}$$

From an analysis of internal forces, the maximum bending moment can be shown to be

$$M_{\max} = \frac{F_{\text{beam}}L}{4} = \frac{3.45\times(16\times12)}{4} = 166 \text{ kip-in.}$$

The corresponding maximum stress follows from the flexure formula,

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{166\times(3/2)}{\left(\frac{1}{12}\times4\times3^3\right)} = \boxed{27.7 \text{ ksi}}$$

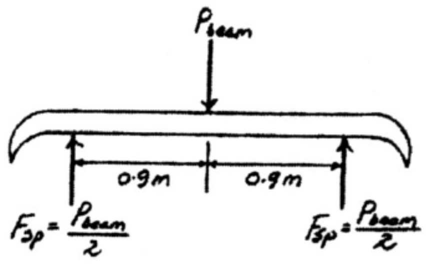
Since $\sigma_{max} < 50$ ksi, the result is valid.

The correct answer is **D**.

P.9 ➡ *Solution*

Refer to the figure below. For equilibrium, the force on the spring F_{sp} = $P_{beam}/2$. Then,

$$k_{\text{sp}}\delta_{\text{sp}} = \frac{k\delta_{\text{beam}}}{2} \rightarrow \delta_{\text{sp}} = \frac{k}{2k_{\text{sp}}}\delta_{\text{beam}} \quad \text{(I)}$$



The equivalent spring constant for the beam can be determined using one of the cases of beam deflection in the Additional Information section,

$$k = \frac{48EI}{L^3} = \frac{48\times(2\times10^9)\times(300\times10^{-6})}{1.8^3} = 4.94 \text{ MN/m}$$

From conservation of energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}k\delta_{\text{beam}}^2 + \frac{k^2}{4k_{\text{sp}}}\delta_{\text{beam}}^2$$

$$\therefore \frac{1}{2}\times1800\times0.75^2 = \frac{1}{2}\times(4.94\times10^6)\times\delta_{\text{beam}}^2 + \frac{(4.94\times10^6)^2}{4\times(1.5\times10^6)}\delta_{\text{beam}}^2$$

$$\therefore 506 = 2.47\times10^6\,\delta_{\text{beam}}^2 + 4.07\times10^6\,\delta_{\text{beam}}^2$$

$$\therefore \delta_{\text{beam}} = 8.80\times10^{-3} \text{ m}$$

Backsubstituting into equation (I), we obtain

$$\delta_{\text{sp}} = \frac{k}{2k_{\text{sp}}}\delta_{\text{beam}} = \frac{4.94\times10^6}{2\times(1.5\times10^6)}\times(8.80\times10^{-3}) = 0.0145 \text{ m}$$

The maximum displacement follows as

$$\delta_{\max} = \delta_{\text{sp}} + \delta_{\text{beam}} = 0.0145 + 8.80\times10^{-3} = \boxed{23.3 \text{ mm}}$$

The maximum force on the beam is $P_{beam} = k\delta_{beam} = (4.94 \times 10^6) \times (8.8 \times 10^{-3}) = 43.5$ kN. The maximum moment occurs at midspan and equals $M_{max} = P_{max}L/4 = (43.5 \times 10^3) \times 1.8/4 = 19.6$ kN·m. Appealing to the flexure formula, the maximum normal stress is calculated as

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{(19.6\times10^3)\times0.075}{300\times10^{-6}} = \boxed{4.90 \text{ MPa}}$$

Since $\sigma_{max} < \sigma_Y = 30$ MPa, the above analysis is valid.

The correct answer is **A**.

P.10 ➡ *Solution*

Appealing to one of the deflection cases in the Additional Information section and solving for the maximum load P_{max}, we have

$$(\delta_C)_{max} = \frac{P_{max}L_{BC}^3}{3EI} \rightarrow P_{max} = \frac{3EI(\delta_C)_{max}}{L_{BC}^3} \quad \text{(I)}$$

From conservation of energy, we find that

$$\frac{1}{2}mv^2 = \frac{1}{2}P_{max}(\delta_C)_{max}$$

$$\therefore \frac{1}{2}mv^2 = \frac{1}{2}\times\left(\frac{3EI(\delta_C)_{max}}{L_{BC}^3}\right)\times(\delta_C)_{max}$$

$$\therefore mv^2 = \frac{3EI(\delta_C)_{max}^2}{L_{BC}^3}$$

$$\therefore (\delta_C)_{max} = \sqrt{\frac{mv^2L_{BC}^3}{3EI}}$$

Substituting the numerical data gives

$$(\delta_C)_{max} = \sqrt{\frac{(120{,}000/32.2)\times 2^2\times 12^3}{3\times(1.40\times 10^6\times 144)\times\left(\frac{\pi}{4}\times 0.5^4\right)}} = 0.931 \text{ ft} = 11.2 \text{ in.}$$

Backsubstituting into equation (I) yields, for the maximum load P_{max},

$$P_{max} = \frac{3\times(1.40\times 10^6)\times\left(\frac{\pi}{4}\times 6^4\right)\times 11.2}{144^3} = 16.0 \text{ kips}$$

The slope at point *C*, also taken from the Additional Information section, is given by

$$\theta_C = \frac{P_{max}L_{BC}^2}{2EI} = \frac{(16.0\times 10^3)\times(12\times 12)^2}{2\times(1.40\times 10^6)\times\left(\frac{\pi}{4}\times 6^4\right)} = 0.116 \text{ rad}$$

It remains to determine the maximum displacement of the top of the post, which we compute with the geometric relation

$$(\delta_A)_{max} = (\delta_C)_{max} + \theta_C L_{CA} = 11.2 + 0.116\times(3\times 12) = \boxed{15.4 \text{ in.}}$$

The correct answer is **C**.

ANSWER SUMMARY

Problem 1		**C**
Problem 2		**B**
Problem 3		**B**
Problem 4	**4A**	**B**
	4B	**C**
Problem 5		**C**
Problem 6		Open-ended pb.
Problem 7		**D**
Problem 8		**D**
Problem 9		**A**
Problem 10		**C**

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