Quiz HT105

Internal Convection

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Problems

Problem 1 (Çengel & Ghajar, 2015, w/ permission)

A 10-m-long and 10-mm-inner diameter pipe made of commercial steel is used to heat a liquid in an industrial process. The liquid enters the pipe with T_i = 25°C, V = 2.0 m/s. A uniform heat flux is maintained by an electric resistance heater wrapped around the outer surface of the pipe, so that the fluid exits at 75°C. Assume fully developed flow and taking the average flow properties to be ρ = 1000 kg/m^3, c_p = 4000 J/kg·K, μ = 2 × 10^{-3} kg/m·s, k = 0.48 W/m·K, and Pr = 10. The Darcy friction factor is f = 0.045. True or false?

1.( ) The heat flux produced by the heater is greater than 75 kW/m^2.
2.( ) The heat transfer coefficient is greater than 4500 W/m^{2}K.
3.( ) The surface temperature at the exit is greater than 105°C.
4.( ) The pumping power required to convey the flow is greater than 12 W.

Problem 2 (Çengel & Ghajar, 2015, w/ permission)

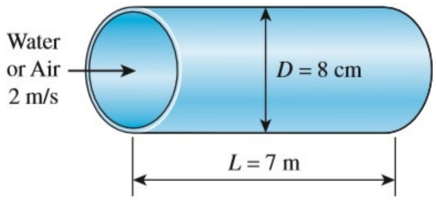
Cooling water available at 10°C is used to condense steam at 30°C in the condenser of a power plant at a rate of 0.15 kg/s by circulating the cooling water through a bank of 5-m-long, 1.2-cm-internal diameter thin copper tubes. Water enters the tubes at a mean velocity of 4 m/s and leaves at a temperature of 24°C. The tubes are nearly isothermal at 30°C. Determine the average heat transfer coefficient between the water and the tubes, and the number of tubes needed to achieve the indicated heat transfer rate in the condenser. Use as properties ρ = 998.7 kg/m^3, c_p = 4183 J/kg°C, and a heat of vaporization h_{fg} = 2430 kJ/kg.

A) h = 6500 W/m^{2}K and N = 14 tubes
B) h = 6500 W/m^{2}K and N = 22 tubes
C) h = 12,000 W/m^{2}K and N = 14 tubes
D) h = 12,000 W/m^{2}K and N = 22 tubes

Problem 3 (Çengel & Ghajar, 2015, w/ permission)

Determine the convection heat transfer coefficient for the flow of (1) air and (2) water at a velocity of 2 m/s in an 8-cm-diameter and 7-m-long tube when the tube is subjected to uniform heat flux from all surfaces. What is the ratio h_1/h_2? Use the fluid properties listed below.

	ν (m^2/s)	k (W/mK)	Pr
Air	1.56 × 10^{-5}	0.0255	0.730
Water	8.94 × 10^{-7}	0.607	6.14



A) h_1/h_2 = 0.0002
B) h_1/h_2 = 0.002
C) h_1/h_2 = 0.02
D) h_1/h_2 = 0.2

Problem 4 (Kreith et al., 2011, w/ permission)

Compute the heat transfer coefficient for 10°C water flowing at 4 m/s in a long, 2.5-cm-ID pipe (surface temperature 40°C) by three different equations: the Dittus-Boelter correlation, the Sieder-Tate correlation, and the Petukhov-Popov correlation. Which correlation gives the highest results? Use as properties ν = 1.3 × 10^{-6} m^2/s, k = 0.577 W/mK, Pr = 9.5, absolute viscosity μ_b = 1296 × 10^{-6} Ns/m^2 and viscosity at surface temperature μ_s = 658 × 10^{-6} Ns/m^2.

A) The Dittus-Boelter correlation gives the highest results.
B) The Sieder-Tate correlation gives the highest results.
C) The Petukhov-Popov correlation gives the highest results.
D) The correlations are not valid because the flow is not turbulent.

Problem 5 (Kreith et al., 2011, w/ permission)

Calculate the heat transfer coefficient for water at a bulk temperature of 32°C flowing at a velocity of 1.5 m/s through a 2.54-cm-ID duct with a wall temperature of 43°C. Use three different equations: the Dittus-Boelter correlation, the Sieder-Tate correlation, and the Petukhov-Popov correlation. Which correlation gives the lowest results? Use as properties ν = 0.773 × 10^{-6} m^2/s, k = 0.619 W/mK, Pr = 5.16, absolute viscosity μ_b = 763 × 10^{-6} Ns/m^2, and viscosity at surface temperature μ_s = 626 × 10^{-6} Ns/m^2.

A) The Dittus-Boelter correlation gives the lowest results.
B) The Sieder-Tate correlation gives the lowest results.
C) The Petukhov-Popov correlation gives the lowest results.
D) The correlations are not valid because the flow is not turbulent.

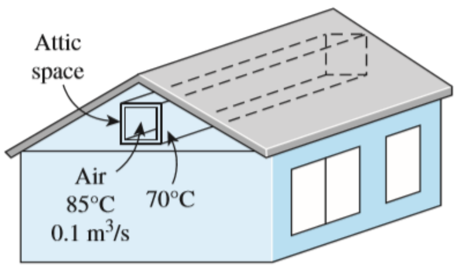
Problem 6 (Çengel & Ghajar, 2015, w/ permission)

The components of an electronic system dissipating 180 W are located in a 1-m-long horizontal duct whose cross-section is 16 cm × 16 cm. The components in the duct are cooled by forced air, which enters at 27°C at a rate of 0.65 m^3/min. Assuming 85 percent of the heat generated inside is transferred to air flowing through the duct and the remaining 15 percent is lost through the outer surfaces of the duct, determine the exit temperature of air and the highest component surface temperature in the duct. Use as properties ρ = 1.15 kg/m^3, c_p = 1007 J/kg°C, ν = 1.66 × 10^{-5} m^2/s, k = 0.0263 W/mK, and Pr = 0.727.

A) T_e = 39.3°C and T_{max} = 132°C
B) T_e = 39.3°C and T_{max} = 180°C
C) T_e = 51.4°C and T_{max} = 132°C
D) T_e = 51.4°C and T_{max} = 180°C

Problem 7 (Çengel & Ghajar, 2015, w/ permission)

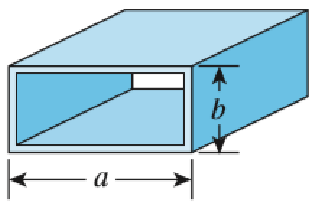
Hot air at atmospheric pressure and 85°C enters a 10-m-long uninsulated square duct of section 0.15 × 0.15 m that passes through the attic of a house at a rate of 0.1 m^3/s. The duct is observed to be nearly isothermal at 70°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the air space in the attic. Use as properties ρ = 1.01 kg/m^3, ν = 2.05 × 10^{-5} m^2/s, k = 0.0292 W/mK, c_p = 1007 J/kg·K, and Pr = 0.720.



A) T_e = 75.7°C and $\dot{Q}$ = 452 W
B) T_e = 75.7°C and $\dot{Q}$ = 951 W
C) T_e = 88.2°C and $\dot{Q}$ = 452 W
D) T_e = 88.2°C and $\dot{Q}$ = 951 W

Problem 8 (Çengel & Ghajar, 2015, w/ permission)

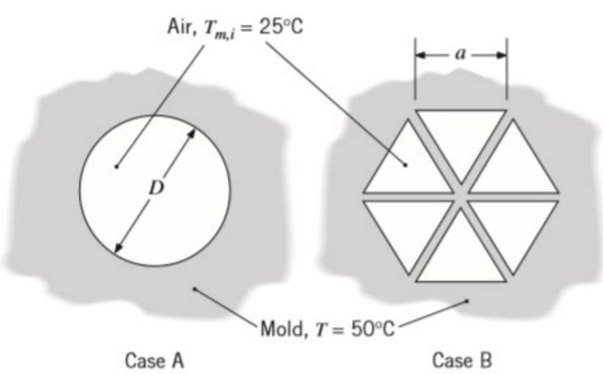
Consider a 10-m-long smooth rectangular tube, with a = 50 mm and b = 25 mm, that is maintained at a constant surface temperature. Liquid water enters the tube at 20°C with a mass flow rate of 0.01 kg/s. Determine the tube surface temperature necessary to heat the water to the desired outlet temperature of 80°C. Use as properties ρ = 988 kg/m^3, ν = 5.54 × 10^{-7} m^2/s, Pr = 3.55, k = 0.644 W/mK, and c_p = 4180 J/kgK.



A) T_s = 86.3°C
B) T_s = 94.5°C
C) T_s = 102°C
D) T_s = 110°C

Problem 9A (Bergman et al., 2011, w/ permission)

Air at 25°C flows at 30 × 10^{-6} kg/s within 100-mm-long channels used to cool a high thermal conductivity metal mold. Assume the flow is hydrodynamically and thermally fully developed. Determine the heat transferred to the air for a circular channel (D = 10 mm) when the mold temperature is 50°C. Use as properties μ = 189 × 10^{-7} Ns/m^2.ρ = 1.13 kg/m^3, k = 0.027 W/mK, and c_p = 1007 J/kgK.



A) $\dot{q}$ = 0.112 W
B) $\dot{q}$ = 0.237 W
C) $\dot{q}$ = 0.325 W
D) $\dot{q}$ = 0.486 W

■ Problem 9B

Using new manufacturing methods, channels of complex cross-section can be readily fabricated within metal objects, such as molds. Consider air flowing under the same conditions as in case A, except now the channel is segmented into six smaller triangular sections (see above). The flow area of case A is equal to the flow area of case B. Determine the heat transferred to the air for the segmented channel.

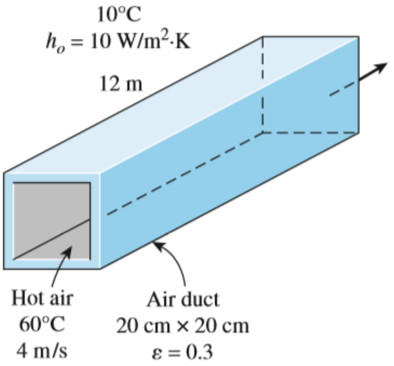
A) $\dot{q}$ = 0.601 W
B) $\dot{q}$ = 0.752 W
C) $\dot{q}$ = 0.901 W
D) $\dot{q}$ = 1.04 W

■ Problem 9C

Compare the pressure drops for cases A and B.

Problem 10 (Çengel & Ghajar, 2015, w/ permission)

Hot air at 60°C leaving the furnace of a house enters a 12-m-long section of a sheet metal duct of rectangular cross-section 20 cm × 20 cm at an average velocity of 4 m/s. The thermal resistance of the duct is negligible, and the outer surface of the duct, whose emissivity is 0.3, is exposed to the cold air at 10°C in the basement, with a convection heat transfer coefficient of 10 W/m²K. Taking the walls of the basement to be at 10°C also, determine the temperature at which the hot air will leave the basement. Use as properties ρ = 1.09 kg/m³, $\nu = 1.80 \times 10^{-5}$ m²/s, k = 0.0274 W/mK, c_p = 1007 J/kg·K, and Pr = 0.723.



A) T_e = 16.4°C
B) T_e = 25.0°C
C) T_e = 33.1°C
D) T_e = 45.0°C

Problem 11 (Kreith et al., 2011, w/ permission)

Mercury at an inlet bulk temperature of 90°C flow through a 1.2-cm-ID tube at a flow rate of 4535 kg/h. The tube is part of a nuclear reactor in which heat can be generated uniformly at any desired rate by adjusting the neutron flux level. Determine the length of tube required to raise the bulk temperature of the mercury to 230°C without generating any mercury vapor, and determine the corresponding heat flux. Use the following correlation for liquid metals,

$$\mathrm{Nu} = 4.82 + 0.0185\left(\mathrm{Re}\,\mathrm{Pr}\right)^{0.827}$$

which is valid provided that $Re \times Pr > 100$. Use as properties $\mu = 11.16 \times 10^{-4}$ Ns/m². ρ = 13,240 kg/m³, k = 11.7 W/mK, Pr = 0.0130, and c_p = 141 J/kgK. The boiling point of mercury is 355°C.

A) L = 0.203 m and $\dot{q}''$ = 0.727 MW/m^2
B) L = 0.203 m and $\dot{q}''$ = 1.58 MW/m^2
C) L = 0.417 m and $\dot{q}''$ = 0.727 MW/m^2
D) L = 0.417 m and $\dot{q}''$ = 1.58 MW/m^2

Problem 12 (Kreith et al., 2011, w/ permission)

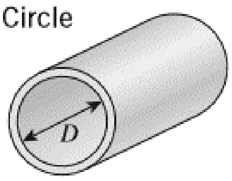
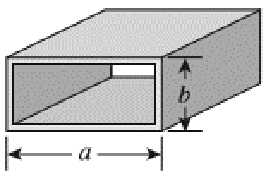
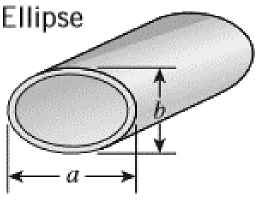
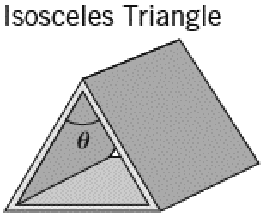
Mercury flows inside a copper tube 9 m long with a 5.1 cm inside diameter at an average velocity of 7 m/s. The temperature at the inside surface of the tube is 38°C uniformly throughout the tube, and the arithmetic mean bulk temperature of the mercury is 66°C. Assuming the velocity and temperature profiles are fully developed, calculate the rate of heat transfer by convection for the 9 m length by considering the mercury as (1) an ordinary liquid and (2) a liquid metal. In the former case, use the Dittus-Boelter correlation. In the latter case, it has been found that *Nu* for flow of liquid metals with a constant surface temperature boundary can be estimated as

$$\mathrm{Nu} = 5.0 + 0.025(\mathrm{Re}\,\mathrm{Pr})^{0.8}$$

Use as properties ν = 0.105 × 10^{-6} m^2/s, k = 9.76 W/mK, and Pr = 0.0193.

Additional Information

Table 1 Nusselt number and friction factor for fully developed laminar flow in tubes of various cross-sections

		Nusselt Number		
Tube Geometry	a/b or $\theta°$	T_s = Const.	$\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Solutions

P.1 Solution

1. True. The cross-sectional area of the pipe is $A_s = \pi \times 0.01^2/4 = 7.85 \times 10^{-5}$ m^2, and the mass flow rate follows as

$$\dot{m} = \rho A_c V = 1000 \times \left(7.85 \times 10^{-5}\right) \times 2.0 = 0.157 \text{ kg/s}$$

The rate of heat transfer from the pipe is determined next,

$$\dot{Q} = \dot{m} c_p \Delta T = 0.157 \times 4000 \times \left(75 - 25\right) = 31,400 \text{ kW}$$

Given the pipe surface area $A_s = \pi \times 0.01 \times 10 = 0.314$ m^2, the heat flux is calculated to be

$$\dot{q}'' = \frac{\dot{Q}}{A_s} = \frac{31,400}{0.314} = 100 \text{ kW/m}^2$$

2. False. The Reynolds number is given by

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{1000 \times 2.0 \times 0.01}{2 \times 10^{-3}} = 10,000$$

This value places us in the turbulent range of Reynolds numbers. Accordingly, the Nusselt number can be determined with the Dittus-Boelter correlation,

$$\text{Nu} = 0.023 \, \text{Re}^{0.8} \, \text{Pr}^{0.4} = 0.023 \times 10,000^{0.8} \times 10^{0.4} = 91.5$$

The heat transfer coefficient is then

$$\text{Nu} = \frac{h \times D}{k} \rightarrow h = \frac{\text{Nu} \times k}{D}$$

$$\therefore h = \frac{91.5 \times 0.48}{0.01} = 4390 \text{ W/m}^2\text{K}$$

3. False. The surface temperature at the exit is obtained by rearranging Newton's law of cooling,

$$\dot{q}'' = h\left(T_s - T_\infty\right) \rightarrow T_s = T_\infty + \frac{\dot{q}''}{h}$$

$$\therefore T_s = 75 + \frac{100,000}{4390} = 97.8^\circ\text{C}$$

4. True. The pressure drop through the pipe is calculated as

$$\Delta p = f \frac{L}{D} \frac{\rho V^2}{2} = 0.045 \times \frac{10}{0.01} \times \frac{1000 \times 2.0^2}{2} = 90 \text{ kPa}$$

The volume flow rate, in turn, is $\dot{v} = V \times A_c = 2.0 \times 7.85 \times 10^{-5} = 1.57 \times 10^{-4}$ m^3/s. The pumping power required to convey this flow is then

$$P = \Delta p \times \dot{v} = 90,000 \times \left(1.57 \times 10^{-4}\right) = 14.1 \text{ W}$$

P.2 Solution

The cross-sectional area of the pipe is $A_c = \pi \times 0.012^2/4 = 1.13 \times 10^{-4}$ m^2, and the mass flow rate follows as

$$\dot{m} = \rho A_c V = 998.7 \times \left(1.13 \times 10^{-4}\right) \times 4 = 0.451 \text{ kg/s}$$

The rate of heat transfer for one tube is

$$\dot{Q} = \dot{m} c_p \Delta T = 0.451 \times 4183 \times \left(24 - 10\right) = 26,400 \text{ W}$$

The logarithmic mean temperature difference is

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{24-10}{\ln\left(\dfrac{30-24}{30-10}\right)} = 11.6^{\circ}\mathrm{C}$$

The surface area of the pipe is $A_s = \pi \times 0.012 \times 5 = 0.189$ m^2. The average heat transfer coefficient is determined by isolating h in the equation

$$\dot{Q} = hA_s\Delta T_{\ln} \rightarrow h = \frac{\dot{Q}}{A_s \Delta T_{\ln}}$$

$$\therefore h = \frac{26,400}{0.189 \times 11.6} = \boxed{12,000\ \mathrm{W/m^2K}}$$

The total rate of heat transfer is given by the product

$$\dot{Q}_{\mathrm{total}} = \dot{m}_{\mathrm{cond}} h_{fg} = 0.15 \times 2430 = 365\ \mathrm{kW}$$

The required number of tubes is obtained by dividing $\dot{Q}_{\mathrm{total}}$ by the heat transfer rate per tube. Accordingly,

$$N = \frac{\dot{Q}_{\mathrm{total}}}{\dot{Q}} = \frac{365,000}{26,400} = 13.8 \approx \boxed{14\ \mathrm{tubes}}$$

♦ The correct answer is **C**.

P.3 Solution

Let us establish the Reynolds number for this air flow,

$$\mathrm{Re}_1 = \frac{VD}{\nu_1} = \frac{2.0 \times 0.08}{1.56 \times 10^{-5}} = 10,300$$

Accordingly, the flow can be assumed to be fully turbulent. Then, the equation to apply for the Nusselt number is

$$\mathrm{Nu}_1 = 0.023\,\mathrm{Re}_1^{0.8}\,\mathrm{Pr}^{0.4} = 0.023 \times 10,300^{0.8} \times 0.730^{0.4} = 32.9$$

Appealing to9 the definition of *Nu*, the heat transfer coefficient becomes

$$\mathrm{Nu}_1 = \frac{h_1 \times D}{k} \rightarrow h_1 = \frac{\mathrm{Nu}_1 \times k}{D}$$

$$\therefore h_1 = \frac{32.9 \times 0.0255}{0.08} = 10.5\ \mathrm{W/m^2K}$$

Suppose now that the flowing fluid is water. The Reynolds number is now

$$\mathrm{Re}_2 = \frac{VD}{\nu_2} = \frac{2.0 \times 0.08}{8.94 \times 10^{-7}} = 179,000$$

As in the previous case, flow is fully turbulent. The corresponding Nusselt number is

$$\mathrm{Nu}_2 = 0.023 \times 179,000^{0.8} \times 6.14^{0.4} = 757$$

The heat transfer coefficient, in sequence, is

$$h_2 = \frac{757 \times 0.607}{0.08} = 5740\ \mathrm{W/m^2K}$$

Finally, the ratio of the two obtained heat transfer coefficients is calculated to be

$$\frac{h_1}{h_2} = \frac{10.5}{5740} = 1.83 \times 10^{-3} \approx \boxed{0.002}$$

That is, the heat transfer coefficient obtained with water is about 1/0.002 = 500 times greater than the coefficient obtained with air.

♦ The correct answer is **B**.

P.4 Solution

The Reynolds number for this flow is

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{4\times 0.025}{1.3\times 10^{-6}} = 76{,}900$$

Thus, flow is fully turbulent. Using the Dittus-Boelter correlation, we have

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4} = 0.023\times 76{,}900^{0.8}\times 9.5^{0.4} = 459$$

The corresponding heat transfer coefficient follows as

$$\mathrm{Nu} = \frac{h\times D}{k} \rightarrow k = \frac{\mathrm{Nu}\times k}{D}$$

$$\therefore h = \frac{\mathrm{Nu}\times k}{D} = \frac{459\times 0.577}{0.025} = 10{,}600\ \mathrm{W/m^2K}$$

For situations where significant property variations due to a large temperature difference exist, the Sieder-Tate correlation may be a better choice. In this approach, the Nusselt number is given by

$$\mathrm{Nu} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027\times 76{,}900^{0.8}\times 9.5^{1/3}\times\left(\frac{1296\times 10^{-6}}{658\times 10^{-6}}\right)^{0.14} = 510$$

The corresponding heat transfer coefficient follows as

$$h = \frac{510\times 0.577}{0.025} = 11{,}800\ \mathrm{W/m^2K}$$

The Petukhov-Popov correlation is a third option for assessing heat transfer in turbulent internal flow,

$$\mathrm{Nu} = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{K_1 + K_2 (f/8)^{1/2}\left(\mathrm{Pr}^{2/3}-1\right)}$$

where friction factor *f* can be estimated as

$$f = \left(1.82\log_{10}\mathrm{Re} - 1.64\right)^{-2}$$

and coefficients K_1 and K_2 are such that

$$K_1 = 1 + 3.4f$$

$$K_2 = 11.7 + \frac{1.8}{\mathrm{Pr}^{1/3}}$$

In the present case,

$$f = \left(1.82\times\log_{10}76{,}900 - 1.64\right)^{-2} = 0.0190$$

Further,

$$K_1 = 1 + 3.4\times 0.019 = 1.06$$

$$K_2 = 11.7 + \frac{1.8}{9.5^{1/3}} = 12.6$$

The value of *Nu* is computed next,

$$\mathrm{Nu} = \frac{(0.0190/8)\times 76{,}900\times 9.5}{1.06 + 12.6\times(0.0190/8)^{1/2}\times\left(9.5^{2/3}-1\right)} = 542$$

Finally, the heat transfer coefficient is calculated to be

$$h = \frac{542 \times 0.577}{0.025} = 12,500 \text{ W/m}^2\text{K}$$

The computed Nusselt numbers and heat transfer coefficients are summarized below. Clearly, the Petukhov-Popov correlation provides the highest results.

Correlation	*Nu*	*h* (W/m²K)
Dittus-Boelter	459	10,600
Sieder-Tate	510	11,800
Petukhov-Popov	542	12,500
Average	504	11,600

♦ The correct answer is **C**.

P.5 Solution

The Reynolds number for this flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{1.5 \times 0.0254}{0.773 \times 10^{-6}} = 49,300$$

Thus, flow is fully turbulent. From the Dittus-Boelter correlation, we have

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023 \times 49,300^{0.8} \times 5.16^{0.4} = 252$$

The corresponding heat transfer coefficient follows as

$$\text{Nu} = \frac{h \times D}{k} \rightarrow k = \frac{\text{Nu} \times k}{D}$$

$$\therefore h = \frac{\text{Nu} \times k}{D} = \frac{252 \times 0.619}{0.0254} = 6140 \text{ W/m}^2\text{K}$$

In the Sieder-Tate approach, the Nusselt number is given by

$$\text{Nu} = 0.027\,\text{Re}^{0.8}\,\text{Pr}^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 0.027 \times 49,300^{0.8} \times 5.16^{1/3} \times \left(\frac{763 \times 10^{-6}}{626 \times 10^{-6}}\right)^{0.14} = 272$$

The corresponding heat transfer coefficient follows as

$$h = \frac{\text{Nu} \times k}{D} = \frac{272 \times 0.619}{0.0254} = 6630 \text{ W/m}^2\text{K}$$

The Petukhov-Popov correlation is a third option for assessing heat transfer in turbulent internal flow,

$$\text{Nu} = \frac{(f/8)\,\text{Re}\,\text{Pr}}{K_1 + K_2 (f/8)^{1/2} \left(\text{Pr}^{2/3} - 1\right)}$$

where, as established in the previous problem, friction factor f can be estimated as

$$f = \left(1.82 \log_{10} \text{Re} - 1.64\right)^{-2}$$

and coefficients K_1 and K_2 are such that

$$K_1 = 1 + 3.4f$$

$$K_2 = 11.7 + \frac{1.8}{\text{Pr}^{1/3}}$$

In the present case,

$$f = \left(1.82 \times \log_{10} 49,300 - 1.64\right)^{-2} = 0.0210$$

In addition,

$$K_1 = 1 + 3.4 \times 0.0210 = 1.07$$

$$K_2 = 11.7 + \frac{1.8}{9.5^{1/3}} = 12.7$$

The value of *Nu* is established next,

$$\mathrm{Nu} = \frac{(0.0210/8) \times 49{,}300 \times 5.16}{1.07 + 12.7 \times (0.0210/8)^{1/2} \times (5.16^{2/3} - 1)} = 283$$

Lastly, the heat transfer coefficient is calculated to be

$$h = \frac{285 \times 0.619}{0.0254} = 6900 \ \mathrm{W/m^2K}$$

The Nusselt numbers and heat transfer coefficients are summarized below. Clearly, the Dittus-Boelter correlation provides the lowest results.

Correlation	*Nu*	*h* (W/m²K)
Dittus-Boelter	252	6140
Sieder-Tate	272	6630
Petukhov-Popov	283	6900
Average	269	6560

♦ The correct answer is **A**.

P.6 Solution

The hydraulic diameter of the section is

$$D_h = \frac{4A_c}{P} = \frac{4 \times 0.16^2}{4 \times 0.16} = 0.16 \ \mathrm{m}$$

The mass flow rate of air is given by

$$\dot{m} = \rho\dot{v} = 1.15 \times (0.65/60) = 0.0125 \ \mathrm{kg/s}$$

The exit temperature of air can be obtained by manipulating the equation for heat transfer rate,

$$\dot{Q} = \dot{m}c_p (T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}c_p}$$

$$\therefore T_e = 27 + \frac{(0.85 \times 180)}{0.0124 \times 1007} = \boxed{39.3^\circ \mathrm{C}}$$

The mean flow velocity is

$$V = \frac{\dot{v}}{A} = \frac{(0.65/60)}{0.16^2} = 0.423 \ \mathrm{m/s}$$

and the Reynolds number follows as

$$\mathrm{Re} = \frac{VD_h}{\nu} = \frac{0.423 \times 0.16}{1.66 \times 10^{-5}} = 4080$$

Since 2300 < *Re* < 10,000, the flow is transitional in nature. However, contact with the electronic components is likely to induce turbulence in the flow. Thus, it seems reasonable to consider the flow to be fully turbulent. With this assumption, the Nusselt number can be determined as

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4} = 0.023 \times 4080^{0.8} \times 0.727^{0.4} = 15.7$$

Appealing to the definition of *Nu*, the heat transfer coefficient is calculated as

$$\text{Nu} = \frac{hD_h}{k} \rightarrow h = \frac{\text{Nu} \times k}{D_h}$$

$$\therefore h = \frac{15.7 \times 0.0263}{0.16} = 2.58 \text{ W/m}^2\text{K}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform surface heat flux, its value is computed as

$$\dot{Q} = hA_s\left(T_{\max} - T_e\right) \rightarrow T_{\max} = T_e + \frac{\dot{Q}}{hA_s}$$

$$\therefore T_{\max} = 39.3 + \frac{\left(0.85 \times 180\right)}{2.58 \times \left(4 \times 0.16 \times 1.0\right)} = \boxed{132^{\circ}\text{C}}$$

♦ The correct answer is **A**.

P.7 Solution

The hydraulic diameter of the section is

$$D_h = \frac{4A_c}{P} = \frac{4 \times 0.15^2}{4 \times 0.15} = 0.15 \text{ m}$$

The average flow velocity is, in turn,

$$V = \frac{\dot{v}}{A_c} = \frac{0.1}{0.15^2} = 4.44 \text{ m/s}$$

The Reynolds number is determined next,

$$\text{Re} = \frac{VD_h}{\nu} = \frac{4.44 \times 0.15}{2.05 \times 10^{-5}} = 32{,}500$$

This result indicates that flow in the duct is turbulent. Accordingly, the Nusselt number can be determined with the correlation

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023 \times 32{,}500^{0.8} \times 0.720^{0.3} = 84.8$$

The heat transfer coefficient follows from the definition of Nusselt number,

$$\text{Nu} = \frac{h \times D_h}{k} \rightarrow h = \frac{\text{Nu} \times k}{D_h}$$

$$\therefore h = \frac{84.8 \times 0.0292}{0.15} = 16.5 \text{ W/m}^2\text{K}$$

The mass flow rate of air is

$$\dot{m} = \rho\dot{v} = 1.01 \times 0.1 = 0.101 \text{ kg/s}$$

The surface area of the duct is A_s = 4 × 0.15 × 10 = 6 m^2. The exit temperature of air can be obtained with the relation

$$T_e = T_s - \left(T_s - T_i\right)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right)$$

$$\therefore T_e = 70 - \left(70 - 85\right)\exp\left(-\frac{16.5 \times 6.0}{0.101 \times 1007}\right) = \boxed{75.7^{\circ}\text{C}}$$

The log-mean temperature difference is

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{75.7 - 85}{\ln\left(\dfrac{70 - 75.7}{70 - 85}\right)} = 9.61^{\circ}\text{C}$$

Lastly, the rate of heat loss from the duct is computed as

$$\dot{Q} = hA_s \Delta T_{\ln} = 16.5 \times 6.0 \times 9.61 = \boxed{951 \text{ W}}$$

♦ The correct answer is **B**.

P.8 Solution

The surface area of the duct is $A_s = 2(a + b)L = 2 \times (0.05 + 0.025) \times 10 = 1.5$ m^2. The hydraulic diameter of the duct, in turn, is

$$D_h = \frac{4A_c}{P} = \frac{4 \times ab}{2(a+b)} = \frac{2ab}{a+b} = \frac{2 \times 0.05 \times 0.025}{0.05 + 0.025} = 0.0333 \text{ m}$$

The average flow velocity is determined as

$$\dot{m} = \rho A_c V \rightarrow V = \frac{\dot{m}}{\rho A_c}$$

$$\therefore V = \frac{0.01}{988 \times (0.05 \times 0.025)} = 0.00810 \text{ m/s}$$

The Reynolds number is determined next,

$$\text{Re} = \frac{VD_h}{\nu} = \frac{0.00810 \times 0.0333}{5.54 \times 10^{-7}} = 487$$

Since *Re* < 2300, the flow is laminar. The hydrodynamic entry length is estimated as

$$L_e = 0.05 \,\text{Re}\, D_h = 0.05 \times 487 \times 0.0333 = 0.811 \text{ m}$$

while the thermal entry length is calculated to be

$$L_t = \text{Pr} \times L_e = 3.55 \times 0.811 = 2.88 \text{ m}$$

Since both entry lengths are shorter than the tube length (= 10 m), we conclude that the flow is fully-developed. The ratio of cross-sectional dimensions a/b = 0.05/0.025 = 2.0, the surface temperature is constant, and hence, referring to Table 1, the Nusselt number is read as 3.39. We proceed to compute the heat transfer coefficient,

$$\text{Nu} = \frac{h \times D_h}{k} \rightarrow h = \frac{\text{Nu} \times k}{D_h}$$

$$\therefore h = \frac{3.39 \times 0.644}{0.0333} = 65.6 \text{ W/m}^2\text{K}$$

The surface temperature of the tube follows as

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \rightarrow 80 = T_s - (T_s - 20)\exp\left(-\frac{65.6 \times 1.5}{0.01 \times 4180}\right)$$

$$\therefore \boxed{T_s = 86.3^\circ\text{C}}$$

♦ The correct answer is **A**.

P.9 Solution

Part A: The Reynolds number is

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times (30 \times 10^{-6})}{\pi \times 0.01 \times (189 \times 10^{-7})} = 202$$

Accordingly, flow is laminar. The Nusselt number for a circular conduit with constant surface temperature is *Nu* = 3.66 (Table 1). The heat transfer coefficient follows as

$$\mathrm{Nu}=\frac{h\times D}{k}\rightarrow h=\frac{\mathrm{Nu}\times k}{D}$$

$$\therefore h=\frac{3.66\times 0.027}{0.01}=9.88\ \mathrm{W/m^2K}$$

The outlet temperature is determined next,

$$T_o=T_s+\left(T_i-T_s\right)\exp\left(-\frac{PLh}{\dot{m}c_p}\right)$$

$$\therefore T_o=50+\left(25-50\right)\exp\left[-\frac{\left(\pi\times 0.01\right)\times 0.1\times 9.88}{\left(30\times 10^{-6}\right)\times 1007}\right]=41.1^{\circ}\mathrm{C}$$

The heat transfer rate is then

$$\dot{q}=\dot{m}c_p\left(T_o-T_i\right)=\left(30\times 10^{-6}\right)\times 1007\times\left(41.1-25\right)=\boxed{0.486\ \mathrm{W}}$$

♦ The correct answer is **D**.

Part B: Since the cross-sectional area is the same in both cases, the dimension *a* of each triangle can be established as

$$\frac{\pi D^2}{4}=6\times\frac{a^2}{2}\rightarrow a=\sqrt{\frac{\pi}{12}}D$$

$$\therefore a=\sqrt{\frac{\pi}{12}}\times 10=5.12\ \mathrm{mm}$$

The corresponding hydraulic diameter is

$$D_h=\frac{4A_c}{P}=\frac{4\times\left(a^2/2\right)}{3a}=\frac{2a}{3}=3.41\ \mathrm{mm}$$

The Reynolds number, in turn, is

$$\mathrm{Re}=\frac{4\dot{m}}{\pi D_h\mu}=\frac{4\times\left(30/6\times 10^{-6}\right)}{\pi\times\left(3.41\times 10^{-3}\right)\times\left(189\times 10^{-7}\right)}=98.8$$

Thus, flow is laminar. With reference to Table 1, the Nusselt number for an equilateral triangle (i.e., such that θ = 60°) is seen to be 2.47. The heat transfer coefficient follows as

$$h=\frac{2.47\times 0.027}{0.00341}=19.6\ \mathrm{W/m^2K}$$

The outlet temperature then becomes

$$T_o=50+\left(25-50\right)\exp\left[-\frac{\left(3\times 0.00512\right)\times 0.1\times 19.6}{\left(5\times 10^{-6}\right)\times 1007}\right]=49.9^{\circ}\mathrm{C}$$

Using the total mass flow to account for all six channels, the heat transfer rate for the segmented channel becomes

$$\dot{q}=\dot{m}c_p\left(T_o-T_i\right)=\left(30\times 10^{-6}\right)\times 1007\times\left(49.9-25\right)=\boxed{0.752\ \mathrm{W}}$$

♦ The correct answer is **B**.

Part C: The average flow velocity is

$$V=\frac{\dot{m}}{\rho A_c}=\frac{30\times 10^{-6}}{1.13\times\left(\pi\times 0.01^2/4\right)}=0.338\ \mathrm{m/s}$$

With recourse to Table 1, the friction factor for case A is f = 64/202 = 0.317. Applying the Darcy-Weisbach formula gives

$$\Delta p_A = f\frac{\rho V^2}{2D_h}L = 0.317\times\frac{1.13\times 0.338^2}{2\times 0.01}\times 0.1 = 0.205\text{ Pa}$$

For case B, f = 53/98.8 = 0.536 (Table 1), the flow velocity remains as 0.338 m/s, and

$$\Delta p_B = 0.536\times\frac{1.13\times 0.338^2}{2\times 0.00341}\times 0.1 = 1.01\text{ Pa}$$

In summary, segmenting the channel into six smaller sections increases the heat transfer rate by 54.7%, but comes at the expense of a nearly five-fold increase in pressure drop. Another relevant observation is that, for the circular duct, the hydrodynamic entry length is x_{fd} = 0.05ReD_h = 0.05 × 202 × 0.01 = 0.101 m, which happens to be just above the length of the channel. Accordingly, the flow may not be fully developed as assumed. For the triangular duct, x_{fd} = 0.05 × 98.8 × 0.00341 = 0.0168 m, which indicates that the fully-developed flow assumption is more appropriate.

P.10 Solution

The hydraulic diameter of the section is

$$D_h = \frac{4A_c}{P} = \frac{4\times 0.2^2}{(4\times 0.2)} = 0.2\text{ m}$$

The Reynolds number is determined next,

$$\text{Re} = \frac{VD_h}{\nu} = \frac{4.0\times 0.2}{1.80\times 10^{-5}} = 44,400$$

This result implies that we are well into the turbulent region of internal flow. Accordingly, the Nusselt number can be estimated as

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023\times 44,400^{0.8}\times 0.723^{0.3} = 109$$

Resorting to the definition of Nu, the heat transfer coefficient is calculated to be

$$\text{Nu} = \frac{h\times D_h}{k} \rightarrow h = \frac{\text{Nu}\times k}{D_h}$$

$$\therefore h = \frac{109\times 0.0274}{0.2} = 14.9\text{ W/m}^2\text{K}$$

The mass flow rate of air is

$$\dot{m} = \rho A_c V = 1.09\times(0.2\times 0.2)\times 4.0 = 0.174\text{ kg/s}$$

In steady operation, heat transfer from hot air to the duct must be equal to heat transfer from the duct to the surroundings by convection and radiation. This, in turn, must be equal to the energy loss of the hot air in the duct. In mathematical terms, we write

$$\dot{Q}_{\text{conv,in}} = \dot{Q}_{\text{conv+rad,out}} = \Delta\dot{E}_{\text{hot air}}$$

The leftmost term can be stated as

$$\dot{Q} = hA_s\Delta T_{\text{ln}} = 14.9\times(4\times 0.2\times 12)\times\frac{T_e - 60}{\ln\left(\dfrac{T_s - T_e}{T_s - 60}\right)}$$

$$\therefore \dot{Q} = \frac{143(T_e - 60)}{\ln\left(\dfrac{T_s - T_e}{T_s - 60}\right)} \quad \text{(I)}$$

The middle term, in sequence, is stated as

$$\dot{Q} = h_o A_s \left(T_s - T_o\right) + \varepsilon A_s \sigma \left(T_s^4 - T_o^4\right)$$

$$\therefore \dot{Q} = 10 \times 9.6 \times \left(T_s - 10\right) + 0.3 \times 9.6 \times \left(5.67 \times 10^{-8}\right) \times \left[\left(T_s + 273\right)^4 - \left(10 + 273\right)^4\right]$$

$$\therefore \dot{Q} = 96\left(T_s - 10\right) + 16.3 \times 10^{-8} \times \left[\left(T_s + 273\right)^4 - 283^4\right] \text{ (II)}$$

As for the rightmost term, we have

$$\dot{Q} = \dot{m} c_p \left(T_e - T_i\right) = 0.174 \times 1007 \times \left(60 - T_e\right)$$

$$\therefore \dot{Q} = 175\left(60 - T_e\right) \text{ (III)}$$

Expressions (I), (II) and (III) constitute a system of equations with 3 unknowns, namely the heat transfer rate $\dot{Q}$, the surface temperature T_s, and the exit temperature T_e. Solving this system with a CAS such as Mathematica yields $\dot{Q}$ = 2617 W, T_s = 33.2°C, and T_e = 45.0°C. Therefore, we conclude that the hot air will lose heat at a rate of about 2620 W and exit the duct at 45.0°C.

♦ The correct answer is **D**.

P.11 Solution

The Reynolds number for this flow is determined as

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times \left(4535/3600\right)}{\pi \times 0.012 \times \left(11.16 \times 10^{-4}\right)} = 120{,}000$$

Thus, flow is turbulent. The product of Reynolds number and Prandtl number is 120,000 × 0.0130 = 1560, which is greater than 100 and hence implies that the correlation is valid. The Nusselt number is then

$$\text{Nu} = 4.82 + 0.0185\left(120{,}000 \times 0.013\right)^{0.827} = 12.9$$

The heat transfer coefficient is determined next,

$$\text{Nu} = \frac{h \times D}{k} \rightarrow h = \frac{\text{Nu} \times k}{D}$$

$$\therefore h = \frac{12.9 \times 11.7}{0.012} = 12{,}600 \text{ W/m}^2\text{K}$$

The maximum allowable heat flux is determined by the outlet conditions. The outlet wall temperature must not be higher than the mercury boiling point of 355°C. Appealing to Newton's law of cooling, we have

$$\dot{Q} = hA\left(T_{\max} - T_{b,\text{out}}\right) \rightarrow \dot{q}'' = \dot{Q}/A = h\left(T_{\max} - T_{b,\text{out}}\right)$$

$$\therefore \dot{q}'' = 12{,}600 \times \left(355 - 230\right) = \boxed{1.58 \text{ MW/m}^2}$$

The required length of tube can be determined with the heat balance

$$q = \dot{m} c_p \left(T_{b,\text{out}} - T_{b,\text{in}}\right) = \frac{q}{A} \times \pi D L$$

Solving for L and substituting, we obtain

$$\dot{m} c_p \left(T_{b,\text{out}} - T_{b,\text{in}}\right) = \frac{q}{A} \times \pi D L \rightarrow L = \frac{\dot{m} c_p \left(T_{b,\text{out}} - T_{b,\text{in}}\right)}{\left(q/A\right)\pi D}$$

$$\therefore L = \frac{\left(4535/3600\right) \times 141 \times \left(230 - 90\right)}{\left(1.58 \times 10^6\right) \times \pi \times 0.012} = \boxed{0.417 \text{ m}}$$

♦ The correct answer is **D**.

P.12 Solution

To begin, we compute the Reynolds number for the specified parameters,

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{7.0\times 0.051}{0.105\times 10^{-6}} = 3.40\times 10^{6}$$

Accordingly, the flow is fully turbulent. Assume first that the mercury discharge is modeled as a flow of an ordinary fluid. In this case, the Nusselt number can be estimated with the Dittus-Boelter correlation,

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.3} = 0.023\times\left(3.40\times 10^{6}\right)^{0.8}\times 0.0193^{0.3} = 1180$$

Note that the exponent that accompanies *Pr* for a cooling process is 0.3, not 0.4. The corresponding heat transfer coefficient is determined next,

$$\mathrm{Nu} = \frac{h\times D}{k} \rightarrow h = \frac{\mathrm{Nu}\times k}{D}$$

$$\therefore h = \frac{1180\times 9.76}{0.051} = 226{,}000\ \mathrm{W/m^2K}$$

Finally, the rate of heat transfer is found as

$$\dot{q}_1 = hA\left(T_b - T_s\right) = h\pi DL\left(T_b - T_s\right)$$

$$\therefore \dot{q}_1 = 226{,}000\times\pi\times 0.051\times 9\times\left(66-38\right) = 9.12\ \mathrm{MW}$$

Suppose next that the mercury is modeled with the correlation provided for liquid metals. In this case, the Nusselt number becomes

$$\mathrm{Nu} = 5.0 + 0.025\left(\mathrm{Re}\,\mathrm{Pr}\right)^{0.8} = 5.0 + 0.025\times\left[\left(3.40\times 10^{6}\right)\times 0.0193\right]^{0.8} = 184$$

while the heat transfer coefficient follows as

$$h = \frac{\mathrm{Nu}\times k}{D} = \frac{184\times 9.76}{0.051} = 35{,}200\ \mathrm{W/m^2K}$$

Lastly, the rate of heat transfer is found as

$$\dot{q}_2 = 35{,}200\times\pi\times 0.051\times 9\times\left(66-38\right) = 1.42\ \mathrm{MW}$$

Applying the Dittus-Boelter correlation – which is valid for $Pr > 0.5$ only – to mercury (for which $Pr \approx 0.02$) leads to a 642% overestimation in the rate of heat transfer to the pipe. This illustrates the fact that application of empirical correlations outside the limits of experimental verification can lead to serious errors.

Answer Summary

Problem 1		T/F
Problem 2		C
Problem 3		B
Problem 4		C
Problem 5		A
Problem 6		A
Problem 7		B
Problem 8		A
Problem 9	9A	D
	9B	B
	9C	Open-ended pb.
Problem 10		D
Problem 11		D
Problem 12		Open-ended pb.

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