Montogue

Quiz SM104

Internal Forces and Moments

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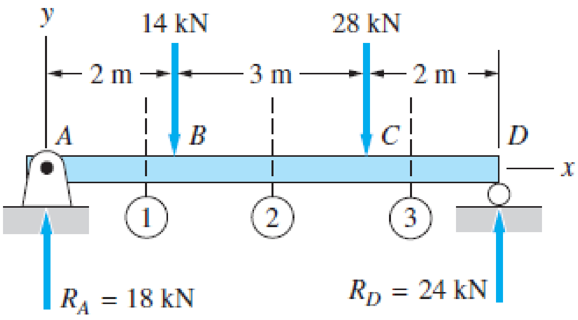
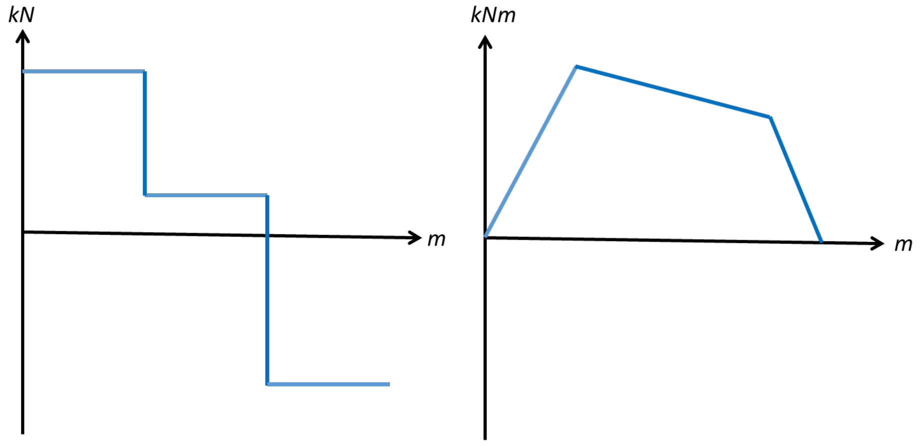
Problems

PROBLEM 1

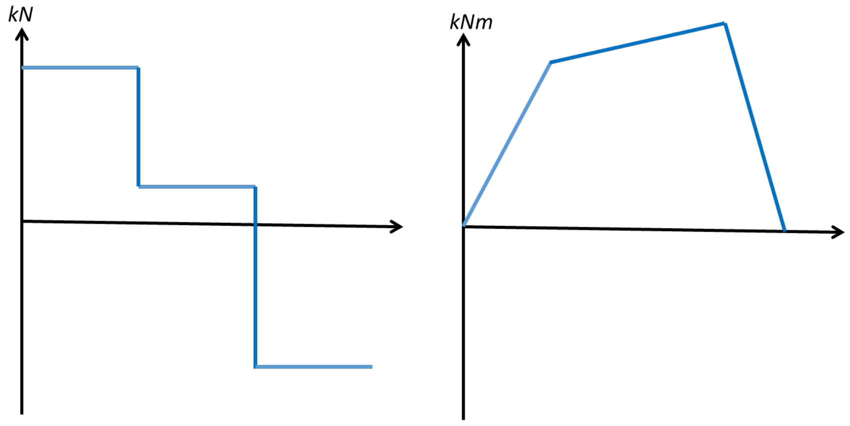
The simply supported beam shown in the figure below carries two concentrated loads. Which of the following pairs of graphs best approximates the shear and bending moment diagrams for this system?



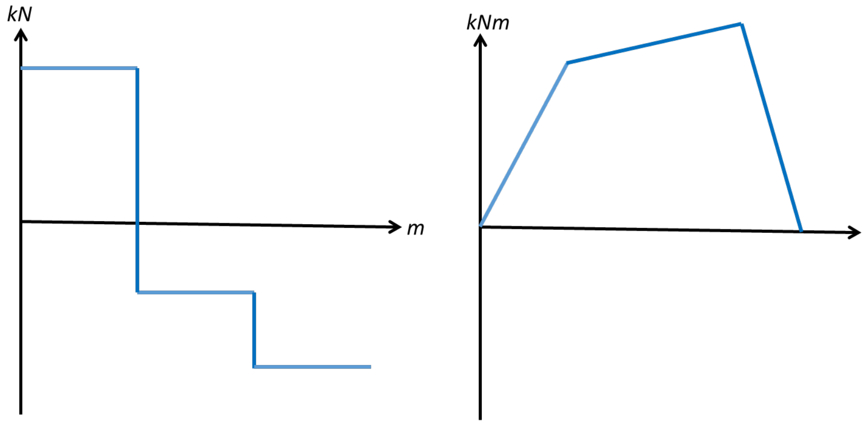
A)



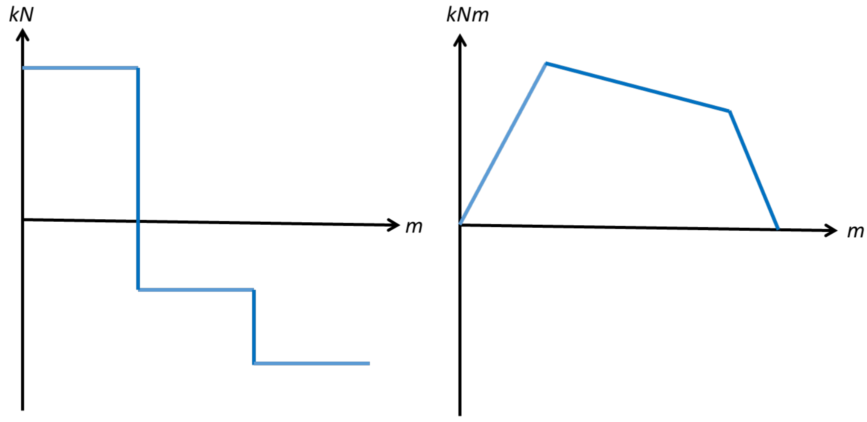
B)



C)

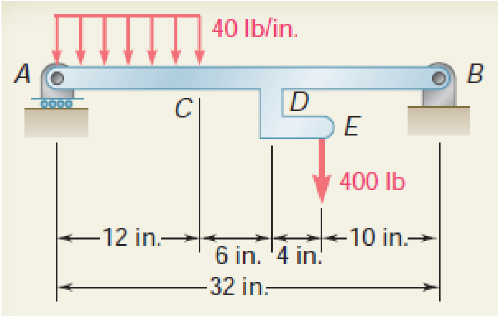


D)



PROBLEM 2 **(Beer et al., 2013, w/ permission)**

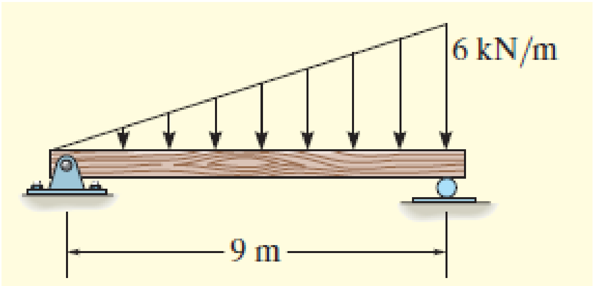
Consider beam *AB* shown below. What are the maximum values for shear force and bending moment for this system?



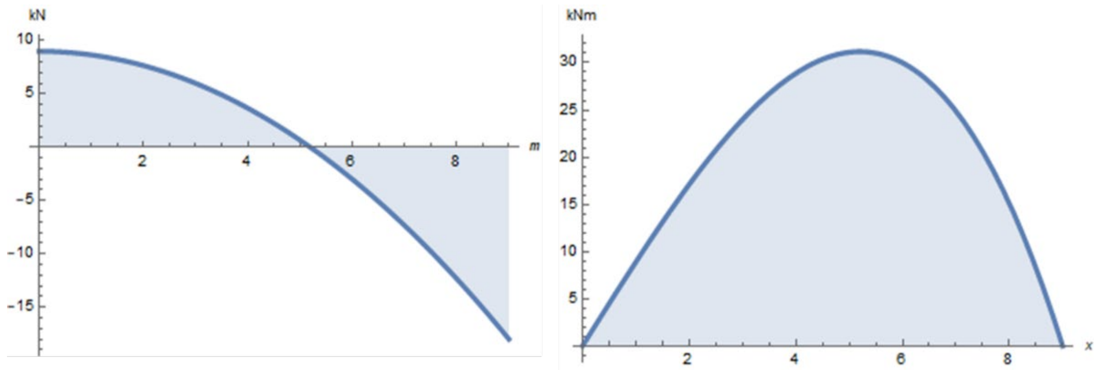
A) $|V_{max}|$ = 365 lb and $|M_{max}|$ = 3510 lb·in.
B) $|V_{max}|$ = 365 lb and $|M_{max}|$ = 5110 lb·in.
C) $|V_{max}|$ = 515 lb and $|M_{max}|$ = 3510 lb·in.
D) $|V_{max}|$ = 515 lb and $|M_{max}|$ = 5110 lb·in.

PROBLEM 3 **(Hibbeler, 2010, w/ permission)**

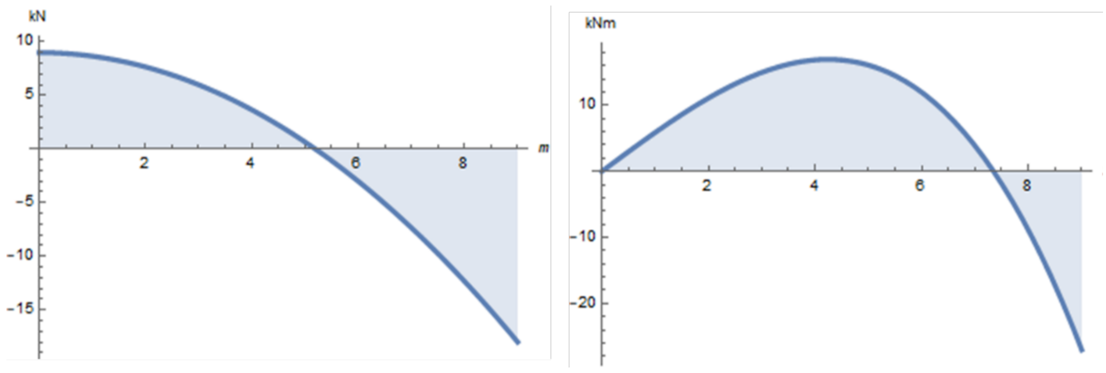
Which of the following graphs best illustrates the shear force and bending moment diagrams for the beam illustrated below?



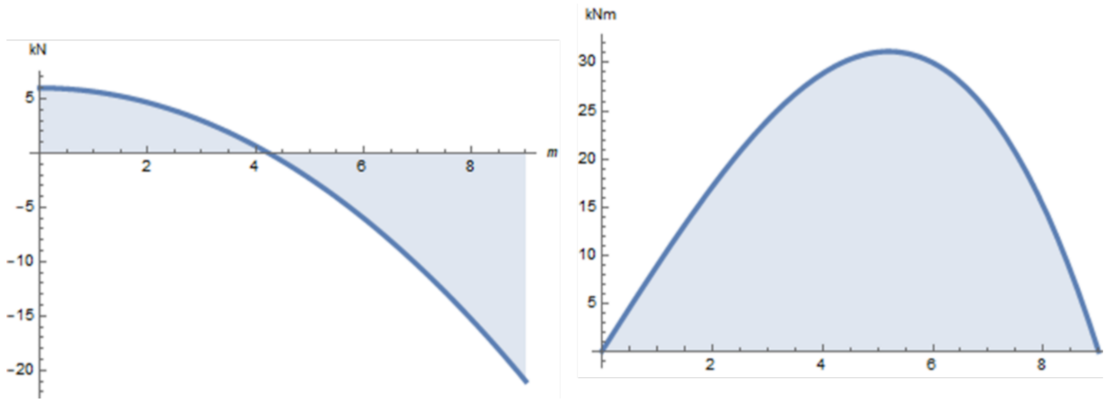
A)



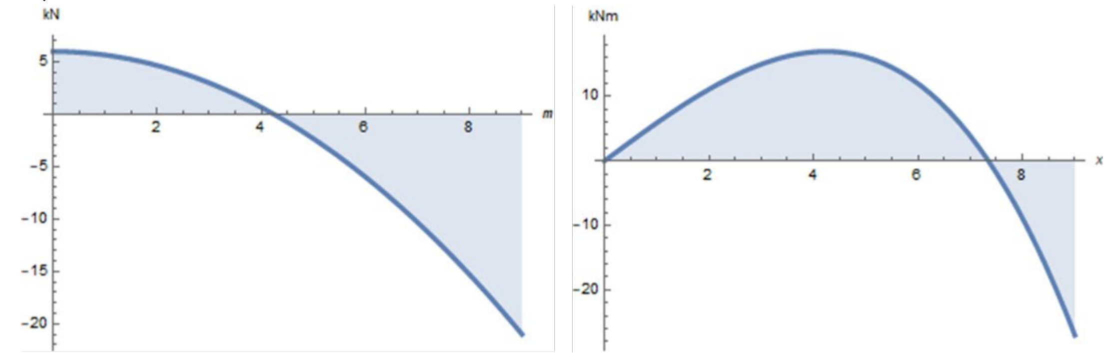
B)



C)

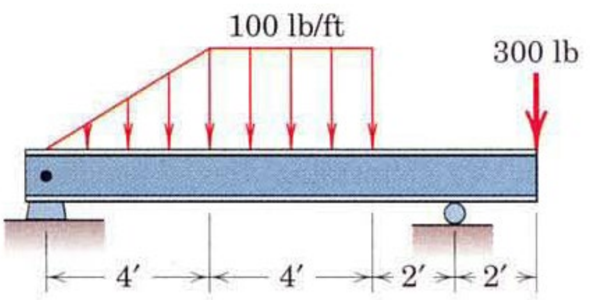


D)



PROBLEM 4 (Merriam & Kraige, 2002, w/ permission)

Consider a beam with the loading pattern shown below. Find the distance *x*, measured from the left end, for which the bending moment is maximum.



A) x = 4.47 ft

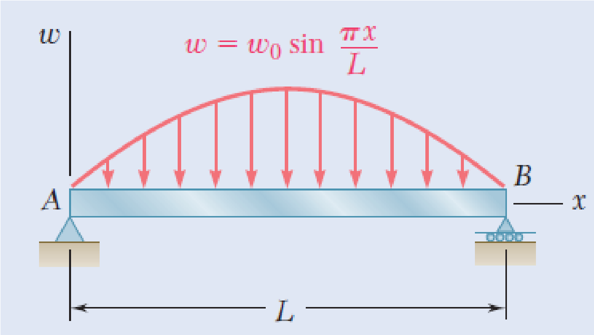
B) x = 6.12 ft

C) x = 7.54 ft

D) x = 10.0 ft

PROBLEM 5 (Beer et al., 2013, w/ permission)

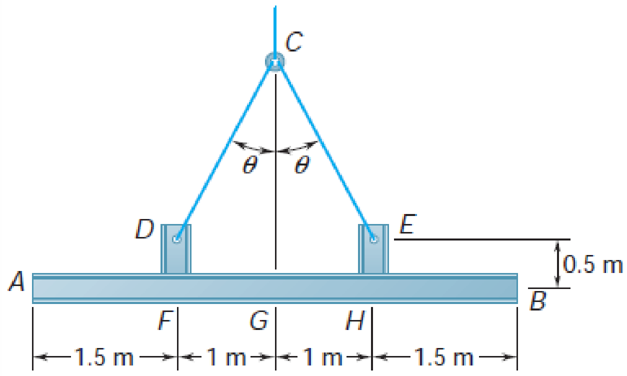
Determine the maximum absolute value of the bending moment in the beam illustrated below.



A) $M_{\max} = (w_o L^2)/(2\pi)$
B) $M_{\max} = (w_o L^2)/\pi$
C) $M_{\max} = (w_o L^2)/(2\pi^2)$
D) $M_{\max} = (w_o L^2)/\pi^2$

PROBLEM 6 (Beer et al., 2015, w/ permission)

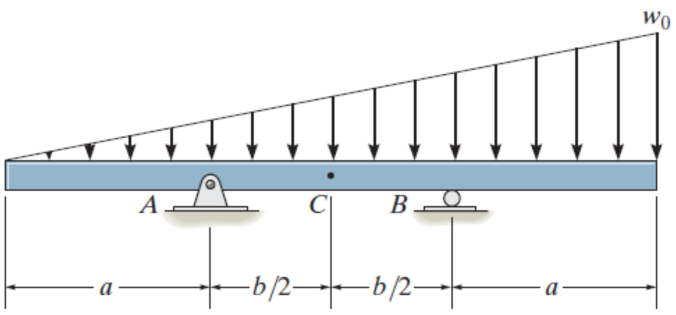
Two small channel sections DF and EH have been welded to the uniform beam AB of weight *w* = 3 kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that θ = 30° and neglecting the weight of the channel sections, determine the maximum values of the shear force and bending moment in the beam.



A) $|V_{\max}|$ = 600 N and $|M_{\max}|$ = 624 N·m
B) $|V_{\max}|$ = 600 N and $|M_{\max}|$ = 924 N·m
C) $|V_{\max}|$ = 900 N and $|M_{\max}|$ = 624 N·m
D) $|V_{\max}|$ = 900 N and $|M_{\max}|$ = 924 N·m

PROBLEM 7 (Hibbeler, 2010, w/ permission)

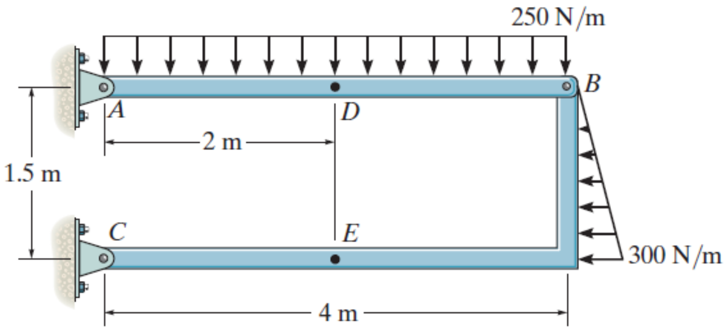
Determine the ratio of *a*/*b* for which the shear force will be zero at the midpoint *C* of the double-overhang beam.



A) a/b = 1/8
B) a/b = 1/6
C) a/b = 1/4
D) a/b = 1/2

PROBLEM 8 (Hibbeler, 2010, w/ permission)

Determine the internal normal force, shear force, and moment at point *E* of the two-member frame shown. True or false?



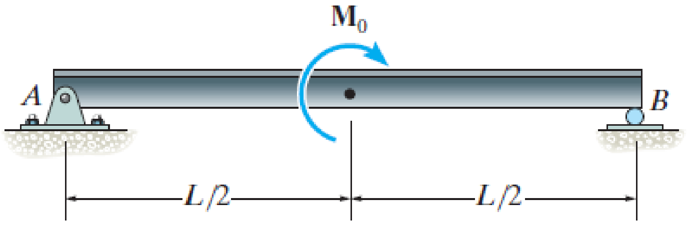
1. ( ) The absolute value of the internal normal force at *E* is less than 1 kN.

2.( ) The absolute value of the shear force at *E* is less than 1 kN.

3. ( ) The absolute value of the internal moment at *E* is less than 1.2 kN·m.

PROBLEM 9 (Hibbeler, 2010, w/ permission)

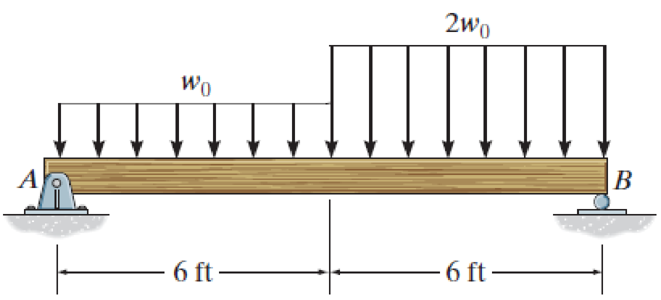
If L = 9 m, the beam will fail when the maximum shear force is V_{max} = 5 kN or the maximum bending moment is M_{max} = 22 kN·m. Determine the largest couple moment M_0 the beam will support.



A) M_0 = 22 kN·m
B) M_0 = 44 kN·m
C) M_0 = 66 kN·m
D) M_0 = 88 kN·m

PROBLEM 10 (Hibbeler, 2010, w/ permission)

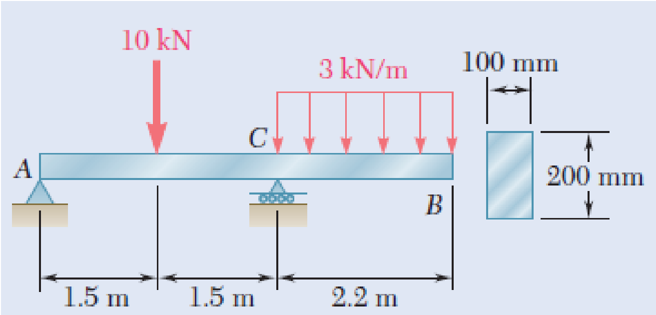
Determine the largest intensity w_0 of the distributed load that the beam can support if the beam can withstand a maximum shear force V_{max} = 1200 lb and a maximum bending moment of M_{max} = 600 lb·ft.



A) w = 13.9 lb/ft
B) w = 21.8 lb/ft
C) w = 32.5 lb/ft
D) w = 44.1 lb/ft

PROBLEM 11 (Beer et al., 2015, w/ permission)

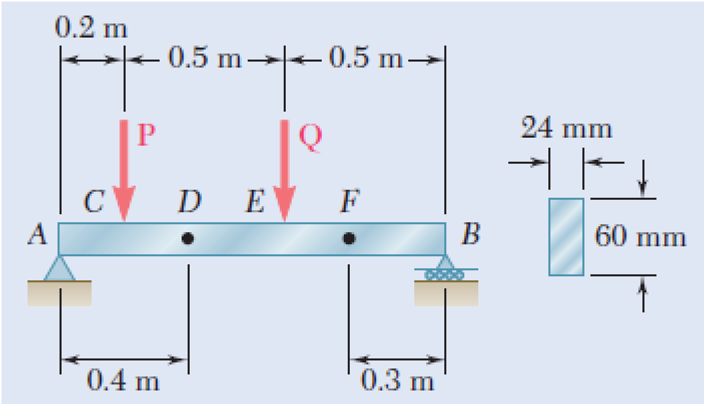
For the beam and loading shown, determine the maximum normal stress due to bending in a transverse section at *C*.



A) σ = 10.9 MPa
B) σ = 21.2 MPa
C) σ = 30.4 MPa
D) σ = 41.0 MPa

PROBLEM 12 (Beer et al., 2015, w/ permission)

The beam *AB* supports two concentrated loads *P* and *Q*. The normal stress due to bending in the bottom edge of the beam is +55 MPa at D and +37.5 MPa at F. Determine the maximum normal stress due to bending that occurs in the beam.

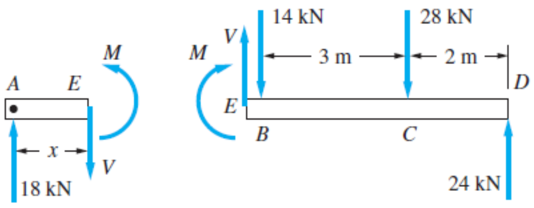


A) σ = 37.5 MPa
B) σ = 50.0 MPa
C) σ = 62.5 MPa
D) σ = 75.0 MPa

Solutions

P.1 ■ Solution

The determination of the expressions for *V* and *M* follows. First, consider segment AB, which encompasses 0 < *x* < 2 m. The figure below shows the forces and moments acting on the beam when it is divided into two segments by a point E, located between A and B, and a distance *x* away from the left support.



Applying an equilibrium of forces in the *y*-direction, we have

$$\Sigma F_y = 0 \rightarrow 18 - V = 0$$

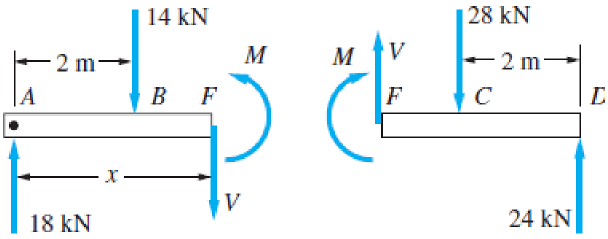
$$\therefore V = 18 \text{ kN}$$

and, from the equilibrium of moments relative to point *E*,

$$\Sigma M_E = 0 \rightarrow M - 18x = 0$$

$$\therefore M = 18x \text{ kN}\cdot\text{m } (0 < x < 2 \text{ m})$$

Next, we consider the forces and moments at segment BC, 2 < x < 5 m, as shown below.



Applying the two equations of equilibrium as before, we have

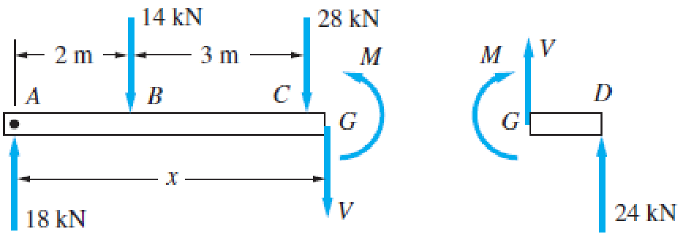
$$\Sigma F_y = 0 \rightarrow 18 - 14 - V = 0$$

$$\therefore V = 4 \text{ kN}$$

$$\Sigma M_F = 0 \rightarrow M - 18x + 14(x-2) = M - 18x + 14x - 28 = 0$$

$$\therefore M = 4x + 28 \text{ kN}\cdot\text{m } (2 < \text{x} < 5 \text{ m})$$

Finally, we consider segment CD, 5 < x < 7 m.



Applying the two equations of equilibrium gives

$$\Sigma F_y = 0 \rightarrow 18 - 14 - 28 - V = 0$$

$$\therefore V = -24 \text{ kN}$$

$$\Sigma M_G = 0 \rightarrow M + 28(x-5) + 14(x-2) - 18x$$

$$\therefore M + 42x - 140 - 28 - 18x = 0$$

$$\therefore M + 24x - 168 = 0$$

$$\therefore M = 168 - 24x \text{ kN}\cdot\text{m}$$

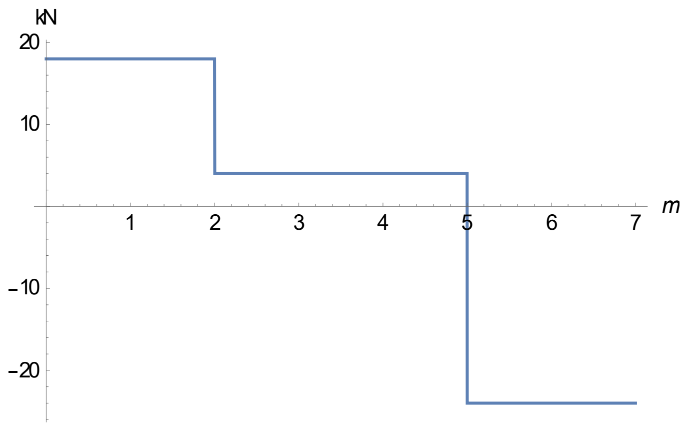
We are now ready to assemble these results in moment and shear force diagrams. The shear force varies according to the laws

$$V(x) = 18 \text{ kN} \quad (0 < x < 2 \text{ m})$$

$$V(x) = 4 \text{ kN } (2 < \text{x} < 5 \text{ m})$$

$$V(x) = -24 \text{ kN } (5 < x < 7 \text{ m})$$

The plot thus obtained with Mathematica is shown below.



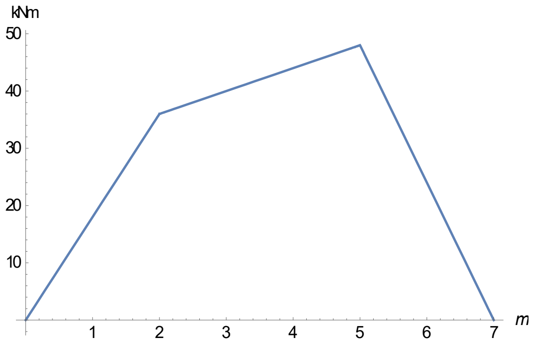
Next, we plot the bending moment diagram, which follows the piecewise function

$$M(x) = 18x \ (0 < x < 2 \text{ m})$$

$$M(x) = 4x + 28 \;\; (2 < x < 5 \text{ m})$$

$$M(x) = 168 - 24x \;\; (5 < x < 7 \text{ m})$$

The plot thus obtained with Mathematica is given below.



Bear in mind that the moment *M* at any section equals the area under the shear diagram up to that section; that is,

$$V = \frac{dM}{dx} \rightarrow M = \int V dx$$

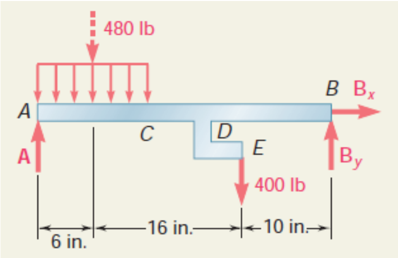
For instance, in the interval 0 < *x* < 2 m we have

$$M = \int 18dx = 18x \; (0 < x < 2 \text{ m})$$

■ The correct answer is **B**.

P.2 ■ Solution

The reactions are easily determined with a free body diagram.



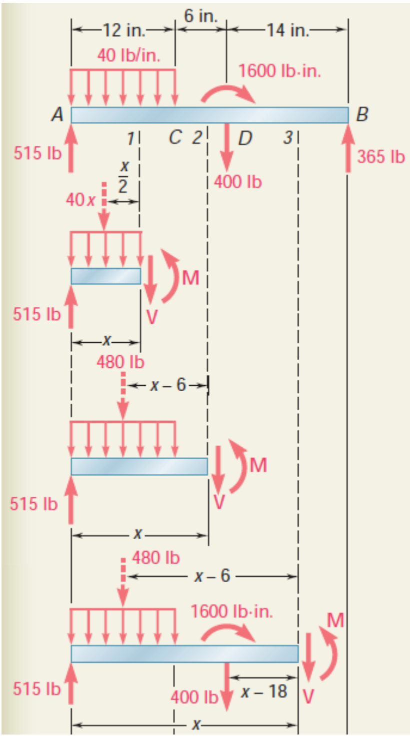
First, we sum moments relative to point A,

$$\Sigma M_A = 0 \rightarrow B_y(32) - 400(22) - 480(6) = 0 \therefore B_y = 365 \text{ lb}$$

Then, summing forces in the *y*-direction, we have

$$\Sigma F_y = 0 \rightarrow A - 400 - 480 + 365 = 0 \therefore A = 515 \text{ lb}$$

and, because B_x is the only force acting on the *x*-direction, we immediately have $B_x = 0$.



We proceed to investigate the internal forces and moments acting on the beam. From point A to C, we consider the portion of the beam to the left of section 1 in the figure shown above. The distributed load is replaced with an equivalent concentrated force. Writing equations for equilibrium of forces in the *y*-direction and for equilibrium of moments, we see that

$$\Sigma F_y = 0 \rightarrow 515 - 40x - V = 0$$

$$\therefore V(x) = 40x - 515$$

$$\Sigma M_O = 0 \rightarrow -515x + 40x\left(\frac{x}{2}\right) + M$$

$$\therefore M(x) = 515x - 20x^2 \ (0 < x < 12 \text{ in.})$$

These equations describe the variation of shear force and bending moment with distance *x* from the left support. Next, we consider the equilibrium of forces and moments to the left of section 2 in the previous figure. As before, we replace the distributed load with an equivalent force. The equations of equilibrium yield

$$\Sigma F_y = 0 \rightarrow 515 - 480 - V = 0$$

$$\therefore V(x) = 35 \text{ lb}$$

$$\Sigma M_O = 0 \rightarrow -515x + 480(x-6) + M = 0$$

$$\therefore M(x) = 35x + 2880 \ \ (12 < x < 18 \text{ in.})$$

Finally, consider the segment that goes from D to B. Applying the equations of equilibrium gives

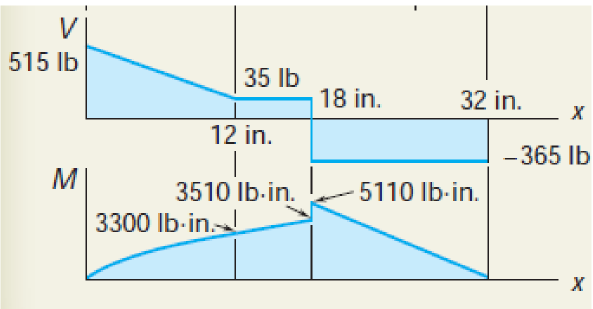
$$\Sigma F_y = 0 \rightarrow 515 - 480 - 400 - V = 0$$

$$\therefore V(x) = 365 \text{ lb}$$

$$\Sigma M_O = 0 \rightarrow -515x + 480(x-6) - 1600 + 400(x-18) + M = 0$$

$$\therefore M(x) = -365x + 11680 \ (18 < x < 32 \text{ in.})$$

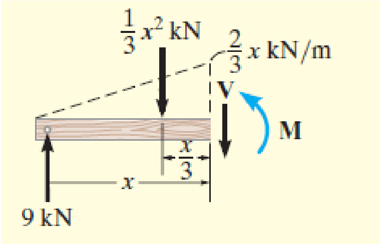
The shear and bending moment diagrams for the entire beam can now be prepared. Notice that the couple of moment 1600 lb·in applied at point D introduces a discontinuity into the bending moment diagram. Inspecting the graphs below, we verify that $|V_{max}|$ = 515 lb and $|M_{max}|$ = 5110 lb-in.



■ The correct answer is **D**.

P.3 ■ Solution

A free body diagram for a segment of the beam having length *x* is illustrated in continuation.



Using proportional triangles, we see that the distributed loading on the beam segment has an intensity of w/x = 6/9, so that w = (2/3)x. The magnitude of the resultant force that can replace this distributed load is equal to (1/2)x(2/3)x =

(1/3)x^2. This force acts through the centroid of the distributed loading area, a distance (1/3)x from the right end, as shown in the previous figure. Writing equations for equilibrium of forces in the y-direction and for equilibrium of moments, we see that

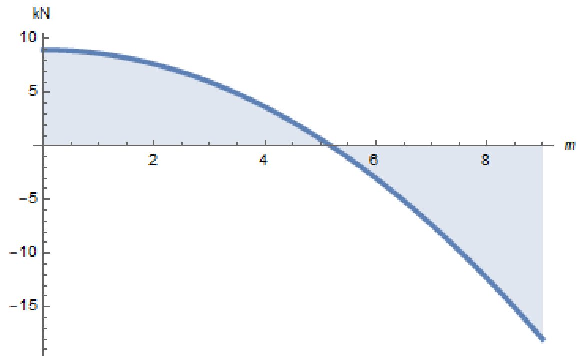
$$\Sigma F_y = 0 \rightarrow 9 - \frac{1}{3}x^2 - V = 0$$

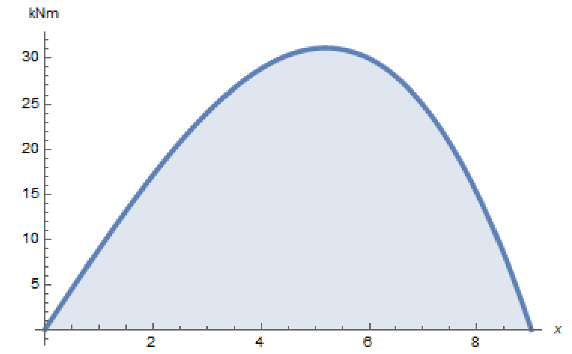
$$\therefore V(x) = 9 - \frac{1}{3}x^2$$

$$\Sigma M_O = 0 \rightarrow -9x + \frac{1}{3}x^2\left(\frac{x}{3}\right) + M = 0$$

$$\therefore M(x) = 9x - \frac{x^3}{9}\ (0 < x < 9\text{ m})$$

Plots of shear and bending moment are given below.





Note that the shear force diagrams in options *A* and *C* are quite similar. However, they differ in the point of zero shear, which can be easily obtained by setting the equation of *V(x)* to zero; that is,

$$V(x) = 9 - \frac{1}{3}x^2 = 0 \rightarrow \frac{1}{3}x^2 = 9$$

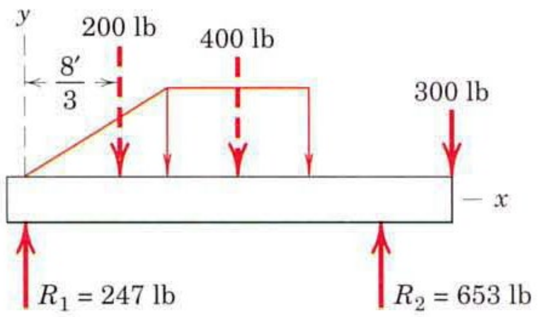
$$\therefore x = 27^{1/2} = 5.2\text{ m}$$

Hence, the graph in *A* is the correct shear force diagram.

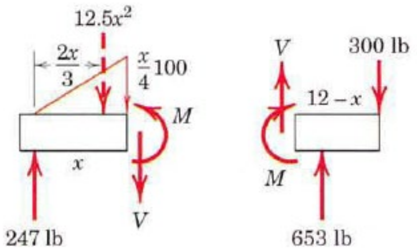
■ The correct answer is **A**.

P.4 ■ Solution

The support reactions are most easily obtained by considering the resultants of the distributed loads on the whole beam. In doing so, the reactions are determined as R_1 = 247 lb and R_2 = 653 lb.



Consider first the free body diagram for the beam segment that goes from the left end (i.e., from x = 0) to x = 4 ft, as shown.



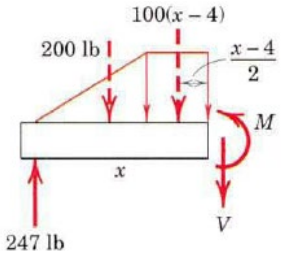
Summing forces in the vertical direction and moments relative to the cut section, we have

$$\Sigma F_y = 0 \rightarrow V = 247 - 12.5x^2$$

$$\Sigma M = 0 \rightarrow M + 12.5x^2\left(\frac{x}{3}\right) - 247x = 0$$

$$\therefore M(x) = 247x - 4.17x^3 \; (0 < x < 4 \text{ ft})$$

Next, we move on to interval 4 < x < 8 ft.



A vertical force summation and a moment summation about the right end yield, respectively,

$$\Sigma F_y = 0 \rightarrow V + 100(x-4) + 200 - 247 = 0$$

$$\therefore V + 100x - 400 + 200 - 247 = 0$$

$$\therefore V(x) = 447 - 100x$$

$$\Sigma M_0 = 0 \rightarrow M + 100(x-4)\left(\frac{x-4}{2}\right) + 200\left[x - \frac{2}{3}(4)\right] - 247x = 0$$

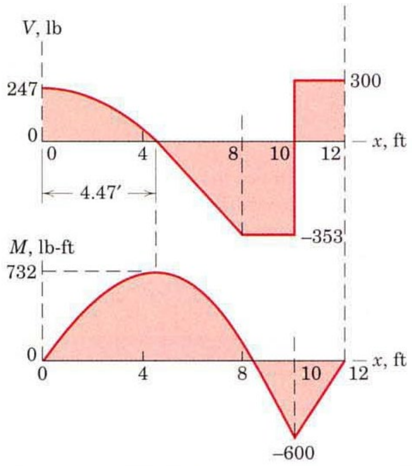
$$\therefore M(x) = -267 + 447x - 50x^2 \; (4 < x < 8 \text{ ft})$$

Consider now the beam segment such that 8 < x < 10 ft. Summing forces and moments, we have

$$V(x) = -353 \text{ lb}$$

$$M(x) = 2930 - 353x \; (8 < x < 10 \text{ ft})$$

Finally, the last portion is analyzed by inspection. The shear is constant at +300 lb, and the moment follows a straight-line relation beginning with zero at the right end of the beam. The final shear force and bending moment diagrams are shown below.



After graphing the diagrams, we note that the maximum positive bending moment occurs somewhere in the interval 0 < x < 8 ft, while the maximum negative bending moment is readily seen to be −600 lb·ft at x = 10 ft. To obtain the maximum positive bending moment, recall that the internal moment is maximum when the shear force equals zero. In segment 0 < x < 8 ft, we have $V(x)$ = 447 – 100x. Setting this to zero, we obtain x = 4.47 m. The bending moment at this position is

$$M(4.47) = -267 + 447(4.47) - 50(4.47)^2 = 732.0 \text{ lb} \cdot \text{ft}$$

which is greater in absolute value than 600 lb·ft; therefore, it is taken as the maximum bending moment acting on the structure.

■ The correct answer is **A**.

P.5 ■ Solution

We begin with the assertion that the derivative of shear with respect to x equals the distributed load w, while the derivative of bending moment with respect to x yields the shear force. Mathematically,

$$\frac{dV}{dx} = -w \rightarrow \frac{dV}{dx} = -w_0 \sin\frac{\pi x}{L}$$

$$\therefore\ V(x) = -\int w_0 \sin\frac{\pi x}{L} dx = \frac{w_0 L}{\pi}\cos\frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$\therefore\ M(x) = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

We have two boundary conditions, namely, $M(0)$ = 0 and $M(L)$ = 0. Applying these gives

$$M(0) = \frac{w_0 L^2}{\pi^2}\sin 0 + C_1(0) + C_2 = 0 \rightarrow C_2 = 0$$

$$M(L) = \frac{w_0 L^2}{\pi^2}\underbrace{\sin\pi}_{=0} + C_1 L + \underbrace{C_2}_{=0} = 0 \rightarrow C_1 = 0$$

Thus the expression for shear force is simply

$$V(x) = \frac{w_0 L}{\pi}\cos\frac{\pi x}{L}$$

while for bending moment,

$$M(x) = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L} \quad (0 < x < L)$$

We set dM/dx = V = 0. Inspecting the equation for $V(x)$, we see that $V(L/2)$ = 0, in which case we have

$$V\left(\frac{L}{2\pi}\right) = \frac{w_0 L}{\pi}\cos\left[\frac{\pi}{L}\left(\frac{L}{2}\right)\right] = \frac{w_0 L}{\pi}\underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} = 0$$

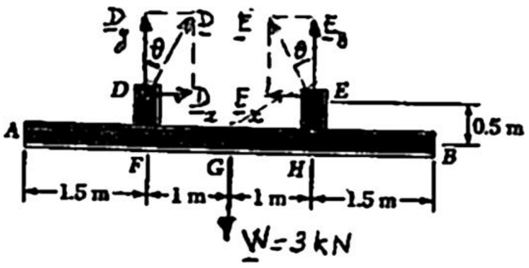
The maximum bending moment is then

$$M_{\max} = M\left(\frac{L}{2}\right) = \frac{w_0 L^2}{\pi^2}\sin\left[\frac{\pi}{L}\left(\frac{L}{2}\right)\right] = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi}{2} = \boxed{\frac{w_0 L^2}{\pi^2}}$$

■ The correct answer is **D**.

P.6 ■ Solution

Consider the free body diagram for this structure.

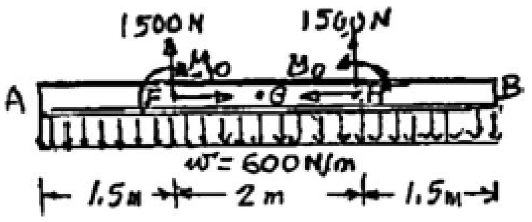


From symmetry, we can infer that E_y = D_y. Further, E_x = D_x = $D_y \tan\theta$. Summing forces in the y-direction, we have

$$\Sigma F_y = 0 \rightarrow D_y + E_y - 3 = 0$$

$$\therefore D_y = E_y = 1.5 \text{ kN}$$

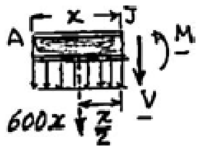
From the previous equality, we have $D_x = 1.5\tan\theta \rightarrow$ and $E_x = 1.5\tan\theta \leftarrow$. We replace the forces at D and E with equivalent force-couple systems, as illustrated below.



The equivalent moment M_0 is such that $M_0 = 1.5\tan\theta \times 0.5$ = (750 N·m)$\tan\theta$. Note that the weight of the beam per unit length is

$$w = \frac{W}{L} = \frac{3 \text{ kN}}{5 \text{ m}} = 0.6\frac{\text{kN}}{\text{m}} = 600\frac{\text{N}}{\text{m}}$$

We are now ready to prepare the shear and bending moment diagrams. First, consider the segment that goes from A to F.



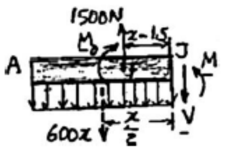
The sum of forces in the y-direction is such that

$$\Sigma F_y = 0 \rightarrow -V - 600x = 0 \therefore V(x) = -600x$$

The sum of moments relative to the right end of this segment is

$$\Sigma M_0 = 0 \rightarrow M + 600x\left(\frac{x}{2}\right) = 0 \therefore M(x) = -300x^2 \ (0 < x < 1.5 \text{ m})$$

Note that, for x = 0, we have $V(0)$ = 0 and $M(0)$ = 0, while, for x = 1.5 m, it is seen that $V(1.5)$ = −900 N and $M(1.5)$ = −675 N·m. Next, consider the segment that goes from F to H. The sum of vertical forces and the sum of moments relative to the right end yield, respectively,



$$\Sigma F_y = 0 \rightarrow 1500 - 600x - V = 0$$

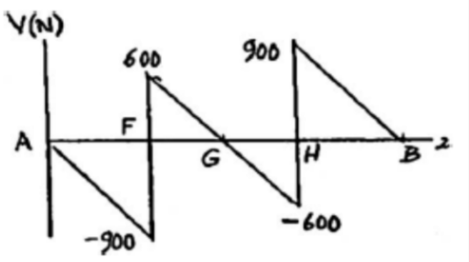
$$\therefore V(x) = 1500 - 600x$$

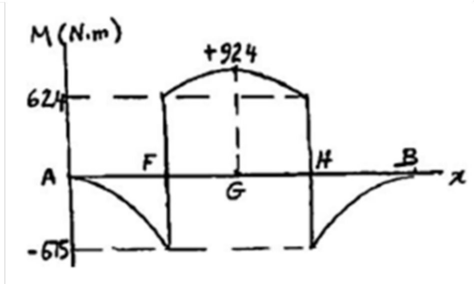
$$\Sigma M_0 = 0 \rightarrow -1500(x - 1.5) + 600x\left(\frac{x}{2}\right) + M - M_0 = 0$$

$$\therefore -1500x + 2250 + 300x^2 - M_0 + M = 0$$

$$\therefore M(x) = M_0 - 300x^2 + 1500x - 2250 \ \ (1.5 < x < 2.5 \text{ m})$$

At x = 1.5 m, we have $V(1.5)$ = 600 N and $M(1.5) = M_0$ – 675 N·m, while, at x = 2.5 m, we note that $V(2.5)$ = 0 and $M(2.5) = M_0$ – 375 N·m. From G to B, the diagrams can be easily obtained through symmetry. The ensuing shear and bending moment diagrams are as shown.





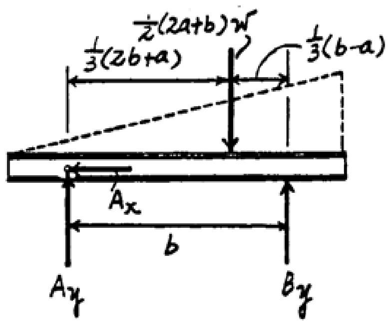
It is easy to see that the largest shear force is $|V_{\text{max}}|$ = 900 N at x = 1.5 m and 3.5 m. Now, making θ = 60°, moment M_0 becomes $M_0 = 750\times\tan 60°$ = 1299 N·m. Hence, we have, just to the right of point F, $M(1.5) = M_0$ – 675 = 1299 – 675 =

624 N·m, while, just to the right of point *G*, we have $M(2.5) = M_0 - 375 = 1299 - 375 = 924$ N·m. Accordingly, $|M_{max}| = 924$ N·m.

■ The correct answer is **D**.

P.7 ■ Solution

Consider the following free body diagram for the beam segment that extends itself from the left end to the right of point *B*.

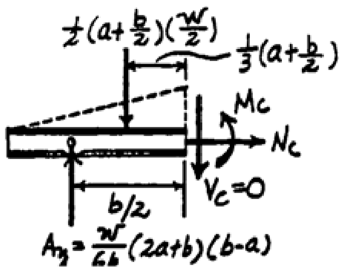


Taking moments about point *B* in the beam, we have

$$\Sigma M_B = 0 \rightarrow \frac{1}{2}(2a+b)\times w\left[\frac{1}{3}(b-a)\right] - A_y b = 0$$

$$\therefore A_y = \frac{w}{2b}(2a+b)(b-a)$$

Consider now the following free body diagram.



The problem requires that $V_C = 0$. Summing forces in the vertical direction, then, we have

$$\Sigma F_y = 0 \rightarrow \frac{w}{6b}(2a+b)(b-a) - \frac{1}{2}\left(a+\frac{b}{2}\right)\frac{w}{2} = 0$$

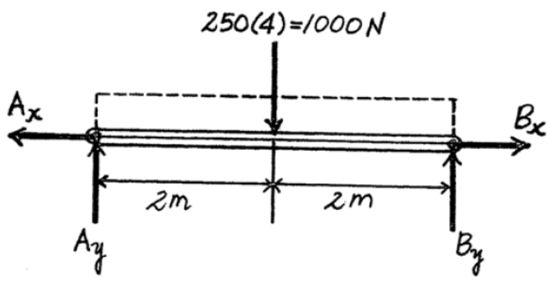
$$\therefore \frac{w}{6b}(2a+b)(b-a) = \frac{w}{8}(2a+b)$$

$$\therefore \boxed{\frac{a}{b} = \frac{1}{4}}$$

■ The correct answer is **C**.

P.8 ■ Solution

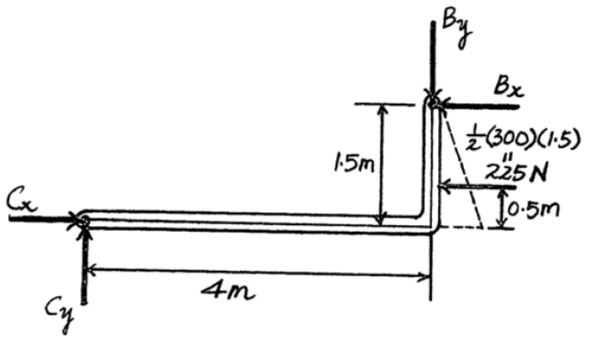
Consider first the free body diagram for member AB.



The vertical component of the reaction at *B* can be obtained by taking moments about point *A*; that is,

$$\Sigma M_B = 0 \rightarrow B_y \times 4 - 1000 \times 2 = 0 \therefore B_y = 500 \text{ N}$$

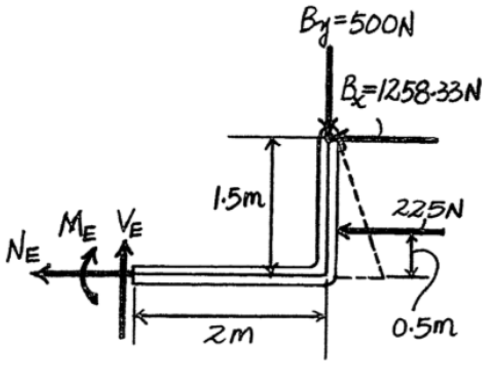
Similarly, the horizontal component of *B* can be obtained by taking moments about point *C*, as shown.



Mathematically,

$$\Sigma M_C = 0 \rightarrow -500 \times 4 + 225 \times 0.5 + B_x \times 1.5 = 0 \therefore B_x = 1258.3 \text{ kN}$$

Consider now the free body diagram for segment BE, as illustrated in continuation.



Summing forces in the horizontal direction, we have

$$\Sigma F_x = 0 \rightarrow -N_E - 1258.3 - 225 = 0$$

$$\therefore N_E = -1483.3 \text{ N}$$

Similarly, summing forces in the *y*-direction, we see that

$$\Sigma F_y = 0 \rightarrow V_E - 500 = 0$$

$$\therefore V_E = 500 \text{ N}$$

Finally, the bending moment at *E* can be determined by applying the second condition of equilibrium to this point, giving

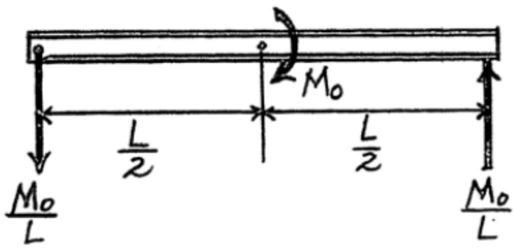
$$\Sigma M_E = 0 \rightarrow -M_E + 225 \times 0.5 + 1258.3 \times 1.5 - 500 \times 2 = 0$$

$$\therefore M_E = 1000 \text{ N} \cdot \text{m}$$

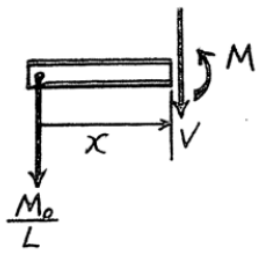
■ Statements **2** and **3** are true, whereas statement **1** is false.

P.9 ■ Solution

The free body diagram of the structure is illustrated below.



First, consider the segment in the interval $0 < x < L/2$.

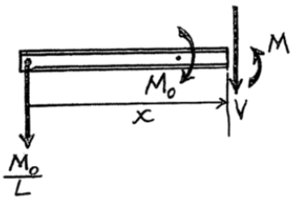


Applying the two conditions of equilibrium, we have

$$\Sigma F_y = 0 \rightarrow -\frac{M_0}{L} - V = 0 \therefore V(x) = -\frac{M_0}{L}$$

$$\Sigma M_0 = 0 \rightarrow M + \frac{M_0}{L}x = 0 \therefore M(x) = -\frac{M_0}{L}x \;\; (0 < x < L/2)$$

Next, consider the segment located in the interval $L/2 < x < L$. Applying the two conditions of equilibrium, we see that



$$\Sigma F_y = 0 \to -\frac{M_0}{L} - V = 0 \therefore V(x) = -\frac{M_0}{L}$$

$$\Sigma M_0 = 0 \to M + \frac{M_0}{L}x - M_0 = 0 \therefore M(x)$$

$$= M_0 - \frac{M_0}{L}x$$

$$= M_0\left(1 - \frac{x}{L}\right) \quad \left(\frac{L}{2} < x < L\right)$$

When M_0 = 500 N·m and L = 9 m, and noting that V_{max} = 5 kN and M_{max} = 22 kN·m, we make use of the expressions

$$V_{\max} = \frac{M_0}{L} \to 5 = \frac{M_0}{9}$$

$$\therefore M_0 = 45 \text{ kN} \cdot \text{m}$$

and

$$M_{\max} = \frac{M_0}{2} \to 22 = \frac{M_0}{2}$$

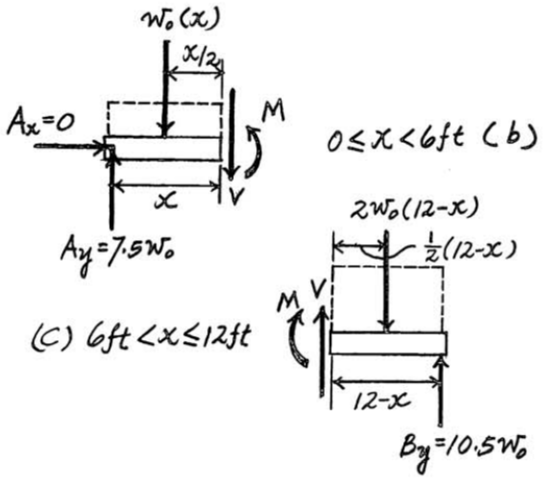
$$\therefore M_0 = 44 \text{ kN} \cdot \text{m}$$

The lower result controls; thus, the largest couple moment that the beam is capable of withstanding is M_0 = 44 kN·m.

■ The correct answer is **B**.

P.10 ■ Solution

Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 < x < 6$ ft and $6 < x < 12$ ft of the beam. The free-body diagrams of the beam's segment sectioned through the arbitrary point within these two regions are shown in figures (b) and (c) below. The reactions at A and B were found as $A_y = 7.5w_0$ and $B_y = 10.5w_0$.



Consider first the segment for which $0 < x < 6$ ft. Summing forces in the vertical direction and moments, we obtain

$$\Sigma F_y = 0 \to 7.5w_0 - w_0 x - V = 0$$

$$\therefore V(x) = w_0(7.5 - x) \quad \text{(I)}$$

$$\Sigma M_0 = 0 \to w_0 x\left(\frac{x}{2}\right) - 7.5w_0 x + M = 0$$

$$\therefore M(x) = 7.5w_0 x - 0.5w_0 x^2 = \frac{w_0}{2}(15x - x^2) \quad (0 < x < 6 \text{ ft}) \quad \text{(II)}$$

Next, consider the segment that encompasses $6 < x < 12$ ft. Summing forces and moments as before, we see that

$$\Sigma F_y = 0 \rightarrow 10.5w_0 - 2w_0(12-x) + V = 0 \therefore V(x)$$
$$= 2w_0(12-x) - 10.5w_0 = 24w_0 - 2w_0x - 10.5w_0$$
$$= w_0(13.5 - 2x) \quad \text{(III)}$$

$$\Sigma M_0 = 0 \rightarrow 10.5w_0(12-x) - 2w_0(12-x)\left[\frac{1}{2}(12-x)\right] - M = 0$$

$$\therefore M(x) = w_0(-x^2 + 13.5x - 18) \quad (6 < x < 12 \text{ ft}) \quad \text{(IV)}$$

The shear diagram is plotted using Equations (I) and (III). The value of the shear force at x = 6 ft can be evaluated using either equation,

$$V(6) = w_0(7.5-6) = 1.5w_0$$

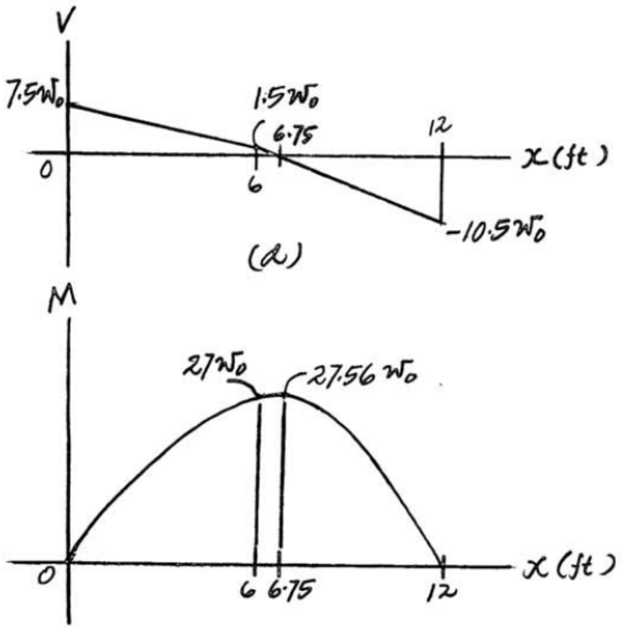
The location at which the shear is equal to zero is obtained by setting V = 0 in Equation (III),

$$w_0(13.5-2x) = 0$$

$$\therefore 13.5w_0 - 2w_0x = 0$$

$$\therefore x = \frac{13.5}{2} = 6.75 \text{ ft}$$

The moment diagram in the figure below is plotted using Equations (II) and (IV).



The value of the moment at x = 6 ft is evaluated as

$$M(6) = \frac{w_0}{2}(15 \times 6 - 6^2) = 27w_0$$

The value of the moment at x = 6.75 ft is evaluated using Equation (IV),

$$M(6.75) = w_0(-6.75^2 + 13.5 \times 6.75 - 18) = 27.56w_0$$

By observing the shear and moment diagrams, we verify that V_{max} = 10.5w_0 and M_{max} = 27.56w_0. We can then propose two equalities,

$$V_{\text{max}} = 10.5w_0 = 1200$$

$$\therefore w_0 = 114.3 \text{ lb} \cdot \text{ft}^{-1}$$

and

$$M_{\text{max}} = 27.56w_0 = 600$$

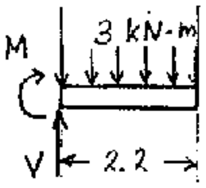
$$\therefore w_0 = \frac{600}{27.56} = 21.8 \text{ lb} \cdot \text{ft}^{-1}$$

The second result governs, and the largest intensity of the distributed load is taken as w_0 = 21.8 lb/ft.

The correct answer is **B**.

P.11 ■ Solution

Consider segment CB, as shown.



The sum of moments relative to point C (the left end of the cut off segment) must equal zero; that is,

$$\Sigma M_C = 0 \rightarrow -M + 2.2 \times 3 \times 10^3 \times 1.1 = 0$$

$$\therefore M = 7.26 \text{ kN}$$

Next, we compute the section modulus for the rectangular section, which is given by

$$S = \frac{bh^2}{6} = \frac{100 \times 200^2}{6} = 6.67 \times 10^5 \text{ mm}^3 = 6.67 \times 10^{-4} \text{ m}^3$$

Finally, the normal stress can be obtained as the ratio of moment to section modulus (i.e., the flexure formula),

$$\sigma = \frac{M}{S} = \frac{7260}{6.67 \times 10^{-4}} = 1.088 \times 10^7 \text{ Pa} = \boxed{10.9 \text{ MPa}}$$

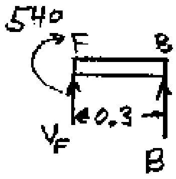
■ The correct answer is **A**.

P.12 ■ Solution

As in Problem 11, the cross-section of the beam is rectangular, and its section modulus equals

$$S = \frac{bh^2}{6} = \frac{24 \times 60^2}{6} = 14{,}400 \text{ mm}^3 = 1.44 \times 10^{-5} \text{ m}^3$$

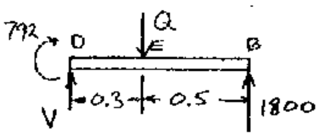
Using the flexure formula, we can write $\sigma = M/S$, so that $M = S\sigma$. At point D, we have $M_D = 1.44 \times 10^{-5}(55 \times 10^6) = 792 \text{ N} \cdot \text{m}$, while at point F we have $M_F = 1.44 \times 10^{-5}(37.5 \times 10^6) = 540 \text{ N} \cdot \text{m}$. Now, consider segment FB of the beam.



The sum of moments relative to point F must equal zero. Hence,

$$\Sigma M_F = 0 \rightarrow -540 + 0.3B = 0 \therefore B = \frac{540}{0.3} = 1800 \text{ N}$$

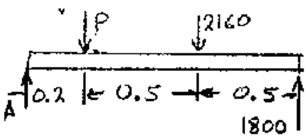
Next, using free body DEFB, we have the sum of moments



$$\Sigma M_D = 0 \rightarrow -792 - 0.3Q + 0.8(1800) = 0$$

$$\therefore Q = \frac{(-792 + 0.8 \times 1800)}{0.3} = 2160 \text{ N}$$

Then, consider the beam as a whole.



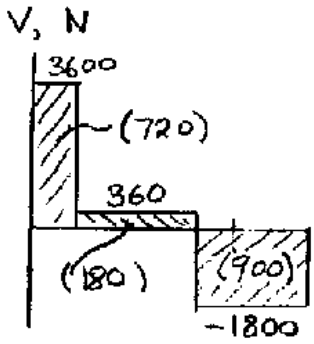
$$\Sigma M_A = 0 \rightarrow -0.2P - 0.7 \times 2160 + 1.2 \times 1800 = 0$$

$$P = \frac{-0.7 \times 2160 + 1.2 \times 1800}{0.2} = 3240 \text{ N}$$

Summing forces in the *y*-direction, we obtain the value of reaction A,

$$\Sigma F_y = 0 \rightarrow A - 3240 - 2160 + 1800 = 0 \therefore A = 3600 \text{ N}$$

We are now ready to consider the shear diagram and its areas, from which the moments can be determined.



From A to C⁻ (just to the left of C), $V(x) = 3600$ N and $A_{AC} = 0.2(3600) = 720\ \mathrm{N\cdot m} = M$; from C⁺ (just to the right of C) to E⁻, we have $V(x) = 3600 - 3240 = 360$ N and $A_{CE} = 0.5(360) = 180\ \mathrm{N\cdot m} = M$; finally, from E⁺ to B, it follows that $V(x) = 360 - 2160 = -1800$ N and $A_{\mathrm{EB}} = 0.5(-1800) = -900\ \mathrm{N\cdot m}$. Now, the bending moments are found to be $M_A = 0, M_C = 0 + 720 = 720\ \mathrm{N\cdot m}$, $M_E = 720 + 180 = 900\ \mathrm{N\cdot m}$, and $M_B = 900 - 900 = 0$. Clearly, the maximum bending moment is $|M_{\max}| = 900\ \mathrm{N\cdot m}$ at point E. The maximum normal stress follows from the flexure formula,

$$\sigma_{\max} = \frac{|M_{\max}|}{S} = \frac{900}{1.44\times 10^{-5}} = \boxed{62.5\ \mathrm{MPa}}$$

■ The correct answer is **C**.

Answer Summary

Problem 1	B
Problem 2	D
Problem 3	A
Problem 4	A
Problem 5	D
Problem 6	D
Problem 7	C
Problem 8	T/F
Problem 9	B
Problem 10	B
Problem 11	A
Problem 12	C

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