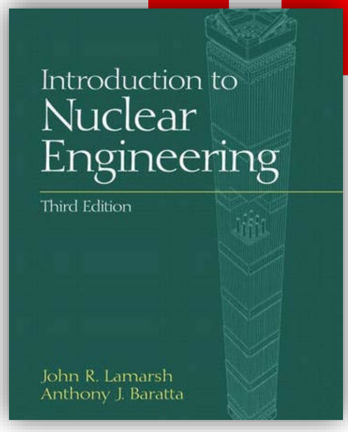

Montogue

Quiz NUC02

Reviewed Solutions to *Introduction to Nuclear Engineering*, 3rd Ed., by Lamarsh and Baratta

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Problem Distribution

Chapter	Problems Covered
5	5.3, 5.4, 5.7(a,b), 5.8, 5.9, 5.11a, 5.14, 5.15, 5.18, 5.19, 5.22, 5.30
6	6.1, 6.3, 6.4, 6.6, 6.9, 6.11, 6.17, 6.20, 6.25, 6.28, 6.29, 6.32(a, b)
7	7.4, 7.8, 7.12, 7.13, 7.19, 7.22, 7.25, 7.27, 7.28, 7.30, 7.38, 7.39, 7.40, 7.41
8	8.1, 8.2, 8.4, 8.7, 8.8, 8.9, 8.10, 8.12, 8.17, 8.19

Problems

■ Chapter 5 – Neutron Diffusion and Moderation

Problem 5.3

Using equations (5.10) and (5.11), estimate the diffusion coefficients of **(a)** beryllium, **(b)** graphite, for monoenergetic 0.0253 eV neutrons.

Problem 5.4

The neutron flux in a bare spherical reactor of radius 50 cm is given by

$$\phi = 5\times 10^{13}\frac{\sin(0.0628r)}{r}\ \text{neutrons/cm}^2\text{-sec}$$

where r is measured from the center of the reactor. The diffusion coefficient for the system is 0.80 cm.

(a) What is the maximum value of the flux in the reactor? **(b)** Calculate the neutron current density as a function of position in the reactor. **(c)** How many neutrons escape the reactor per second?

Problem 5.7 **[(a) and (b) only]**

An isotropic point source emits S neutrons/sec in an infinite moderator. **(a)** Compute the net number of neutrons passing per second through a spherical surface of radius r centered on the source. **(b)** Compute the number of neutrons absorbed per second within the sphere.

Problem 5.8

Two infinite planar sources each emitting S neutrons/cm^2 are placed parallel to one another in an infinite moderator at the distance a apart. Calculate the flux and current as a function of distance from a plane midway between the two.

Problem 5.9

Suppose the two planar sources in the preceding problem are placed at right angles to one another. Derive expressions for the flux and current as a function of distance from the line of intersection of the sources in a plane bisecting the angle between the sources.

Problem 5.11 **[(a) only]**

An infinite bare slab of moderator of thickness 2*a* contains uniformly distributed sources emitting *S* neutrons/cm^3-sec. **(a)** Show that the flux in the slab is given by

$$\phi = \frac{S}{\Sigma_a}\left[1 - \frac{\cosh(x/L)}{\cosh\left(\frac{x+d}{L}\right)}\right]$$

where *x* is measured from the center of the slab.

> **Hint:** *The solution to an inhomogeneous differential equation is the sum of solutions to the homogeneous equation plus a particular solution. Try a constant for the particular solution.*

Problem 5.14

A sphere of moderator of radius *R* contains uniformly distributed sources emitting *S* neutrons/cm^3-sec. **(a)** Show that the flux in the sphere is given by

$$\phi = \frac{S}{\Sigma_a}\left[1 - \frac{R+d}{r}\frac{\sinh(r/L)}{\sinh\left(\frac{R+d}{L}\right)}\right]$$

(b) Derive an expression for the current density at any point in the sphere.
(c) How many neutrons leak from the sphere per second?
(d) What is the average probability that a source neutron will escape from the sphere?

Problem 5.15

The three-group fluxes for a bare spherical fast reactor of radius *R* = 50 cm are given by the following expressions:

$$\phi_1(r) = \frac{3\times 10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$
$$\phi_2(r) = \frac{2\times 10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$
$$\phi_3(r) = \frac{1\times 10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$

The group-diffusion coefficients are D_1 = 2.2 cm, D_2 = 1.7 cm, and D_3 = 1.05 cm. Calculate the total leakage of neutrons from the reactor in all three groups. [**Note:** *Ignore the extrapolation distance.*]

Problem 5.18

The thermal flux in a bare cubical reactor is given approximately by the function

$$\phi_T(x,y,z) = A\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)$$

where *A* is a constant, *a* is the length of a sided of the cube, $\tilde{a}$ is *a* plus 2*d*, *d* is the extrapolation distance, and *x*, *y*, and *z* are measured from the center of the reactor. Derive expressions for the **(a)** thermal neutron current as a function of position in the reactor; **(b)** number of thermal neutrons leaking per second from each side of the reactor; and **(c)** total number of thermal neutrons leaking per second from the reactor.

Problem 5.19

A planar source at the center of an infinite slab of graphite 2 meters thick emits 10^8 thermal neutrons per cm^2/sec. Given that the system is at room temperature, calculate the: **(a)** total number of thermal neutrons in the slab per cm^2 at any time; **(b)** number of thermal neutrons absorbed per cm^2/sec of the slab; **(c)** neutron current as a function of position in the slab; **(d)** total number of neutrons leaking per cm^2/sec from the two surfaces of the slab; and **(e)** probability that a source neutron does not leak from the slab.

Problem 5.22

The thermal flux in a bare spherical reactor 1 m in diameter is given approximately by

$$\phi_T(r) = 2.29\times10^{14}\,\frac{\sin(0.0628r)}{r}\ \text{neutrons/cm}^2\text{-sec}$$

If the reactor is moderated and cooled by unit density water that takes up to one-third of the reactor volume, how many grams of ^2H (deuterium) are produced per year in the reactor? Assume that the water is only slightly warmed by the heat of the reactor.

Problem 5.30

An infinite slab of ordinary water 16 cm thick contains a planar source at its center emitting 10^8 thermal neutrons per cm^2/sec. Compute and plot the thermal flux within the slab.

■ Chapter 6 – Nuclear Reactor Theory

Problem 6.1

Calculate the fuel utilization and infinite multiplication factor for a fast reactor consisting of a mixture of liquid sodium and plutonium, in which the plutonium is present to 3.0 w/o. The density of the mixture is approximately 1.0 g/cm^3.

Problem 6.3

A bare-cylinder reactor of height 100 cm and diameter 100 cm is operating at a steady-state power of 20 MW. If the origin is taken at the center of the reactor, what is the power density at the point $r = 7$ cm, $Z = -22.7$ cm?

Problem 6.4

In a spherical reactor of radius 45 cm, the fission rate density is measured as 2.5×10^{11} fissions/cm^3-sec at a point 35 cm from the center of the reactor. **(a)** At what steady-state power is the reactor operating? **(b)** What is the fission rate density at the center of the reactor?

Problem 6.6

The core of a certain reflected reactor consists of a cylinder 10-ft high and 10-ft in diameter. The measured maximum-to-average flux is 1.5. When the reactor is operated at a power level of 825 MW, what is the power density in the reactor in kW/liter?

Problem 6.9

(a) Estimate the critical radius of a hypothetical bare spherical reactor having the same composition as the reactor in Problem 6.1. **(b)** If the reactor operates at a thermal power level of 500 MW, what is the maximum value of the flux? **(c)** What is the probability that a fission neutron will escape from the reactor?

Problem 6.11

A large research reactor consists of a cubical array of natural uranium rods in a graphite moderator. The reactor is 25 ft on a side and operates at a power of 20 MW. The average value of $\bar{\Sigma}_f$ is 2.5×10^{-3} cm^{-1}. **(a)** Calculate the buckling. **(b)** What is the maximum value of the thermal flux? **(c)** What is the average value of the thermal flux?

Problem 6.17

Consider a critical bare slab reactor 200 cm thick consisting of a homogeneous mixture of ^{235}U and graphite. The maximum thermal flux is 5×10^{12} neutrons/cm^2-sec. Using modified one-group theory, calculate: **(a)** the buckling of the reactor; **(b)** the critical atomic concentration of uranium; **(c)** the thermal diffusion area; **(d)** the value of k_∞; **(e)** the thermal flux and current throughout the slab; **(f)** the thermal power produced per cm^2 of this slab.

Problem 6.20

A bare-spherical reactor 50 cm in radius is composed of a homogeneous mixture of ^{235}U and beryllium. The reactor operates at a power level of 50 thermal kilowatts. Using modified one-group theory, compute: **(a)** the critical mass of ^{235}U; **(b)** the thermal flux throughout the reactor; **(c)** the leakage of neutrons from the reactor; **(d)** the rate of consumption of ^{235}U.

Problem 6.25

A bare-thermal reactor in the shape of a cube consists of a homogeneous mixture of ^{235}U and graphite. The ratio of atom densities is $N_F/N_M = 1.0\times10^{-5}$ and the fuel temperature is 250°C. Using modified one-group theory, calculate: **(a)** the critical dimensions; **(b)** the critical mass; **(c)** the maximum thermal flux when the reactor operates at a power of 1 kW.

Problem 6.28

The core of a spherical reactor consists of a homogeneous mixture of ^{235}U and graphite with a fuel-moderator atom ratio $N_T/N_M = 6.8\times10^{-6}$. The core is surrounded by an infinite graphite reflector. The reactor operates at a thermal power of 100 kW. Calculate the: **(a)** value of k_∞; **(b)** critical core radius; **(c)** critical mass; **(d)** reflector savings; **(e)** thermal flux throughout the reactor; **(f)** maximum-to-average flux ratio.

Problem 6.29

Estimate the new critical radius and critical mass of a reactor with the same composition as described in Problem 6.20 when the core is surrounded by an infinite beryllium reflector.

Problem 6.32 **[(a) and (b) only]**

The core of an infinite planar thermal reactor consists of a solution of ^{238}Pu and H_2O with a plutonium concentration of 8.5 g/liter. The core is reflected on both faces by infinitely thick H_2 reflectors. Calculate the **(a)** reflector savings; and **(b)** critical thickness of the core;

■ Chapter 7 – The Time-Dependent Reactor

Problem 7.4

Express the following reactivities of a ^{235}U-fueled thermal reactor in percent: **(a)** 0.001, **(b)** $2, **(c)** −50 cents.

Problem 7.8

A ^{235}U-fueled reactor originally operating at a constant power of 1 milliwatt is placed on a positive 10-minute period. At what time will the reactor power level reach 1 megawatt?

Problem 7.12

The reactor in Problem 7.8 is scrammed by the instantaneous insertion of 5 dollars of negative reactivity after having reached a constant power level of 1 megawatt. Approximately how long does it take the power level to drop to 1 milliwatt?

Problem 7.13

When a certain research reactor operating at a constant power of 2.7 megawatts is scrammed, it is observed that the power drops to a level of 1 watt in 15 minutes. How much reactivity was inserted when the reactor was scrammed?

Problem 7.19

An experimental reactor facility is a bare square cylinder 100 cm high, composed of small beryllium blocks with thin foils of ^{235}U placed in between, so that the system can be considered to be a homogeneous mixture of Be and ^{235}U. The reactor is to be controlled with a single black control rod 2.5 cm in radius and located along the axis of the system. **(a)** If the reactor is critical with the rod fully withdrawn, how much negative reactivity is introduced into the system when the rod is fully inserted? **(b)** Assuming that the rod moves into the reactor instantaneously, on what period does the reactor go?

Problem 7.22

A certain pressurized-water reactor is to be controlled by 61 cluster control assemblies, each assembly containing 20 black rods 1.15 cm in diameter. The reactor core is a cylinder 320 cm in diameter. The average thermal diffusion length in the core is 1.38 cm, $D = 0.21$ cm, and Σ_f in the core material is approximately 2.6 cm^{-1}. Calculate the total worth of the rods.

Problem 7.25

Suppose the fast reactor described in Example 6.3 is controlled with 50 rods, each rod containing approximately 500 g of natural boron. Estimate the total worth of the rods.

Problem 7.27

A control rod 100 cm long has an integral worth of 50 cents when totally inserted. **(a)** How much reactivity is introduced into the reactor when the rod is pulled one-quarter of the way out? **(b)** At what rate is reactivity introduced at this point per cm motion of the rod?

Problem 7.28

Suppose that, at some time during its operating history, the reactor described in Example 7.7 is critical with the rod withdrawn one-half of its full length. If the rod is now suddenly withdrawn another 10 cm, **(a)** how much reactivity is introduced? **(b)** On what period does the reactor power rise?

Problem 7.30

An infinite ^{235}U-fueled, water-moderated reactor contains 20% more ^{235}U than required to become critical. What concentration of **(a)** boron in ppm or **(b)** boric acid in g/liter is required to hold down the excess reactivity of the system?

Problem 7.38

What is the effective half-life of ^{135}Xe in a thermal flux of 10^{14} neutrons/cm^2-sec at a temperature of 800°C?

Problem 7.39

Compute and plot the equilibrium xenon reactivity as a function of thermal flux from $\phi_T = 5\times10^{12}$ to $\phi_T = 5\times10^{14}$.

Problem 7.40

Using Fig. 7.14, plot the maximum post-shutdown xenon reactivity as a function of thermal flux from $\phi_T = 10^{13}$ to $\phi_T = 5\times10^{14}$.

Problem 7.41

A ^{235}U-fueled reactor operating at a thermal flux of 5×10^{13} neutrons/cm^2-sec is scrammed at a time when the reactor has 5% in reverse reactivity. Compute the time to the onset of the deadtime and its duration.

■ Chapter 8 – Heat Removal from Nuclear Reactors

Problem 8.1

The nuclear ship Savannah was powered by a PWR that operated at a pressure of 1750 psia. The coolant water entered the reactor vessel at a temperature of 497°F, exited at 519°F, and passed through the vessel at a rate of 9.4×10^6 lb/hr. What was the thermal power output of this reactor?

Problem 8.2

An experimental LMFBR operates at 750 MW. Sodium enters the core at 400°C and leaves at 560°C. At what rate must the sodium be pumped through the core?

Problem 8.4

A BWR operates at a thermal power of 1593 MW. Water enters the bottom of the core at 526°F and passes through the core at a rate of 48×10^6 lb/hr. The reactor pressure is 1025 psia. Using the result of the previous problem, compute the rate in lb/hr at which steam is produced in this reactor.

Problem 8.7

A small PWR plant operates at a power of 485 MWt. The core, which is approximately 75.4 in. in diameter and 91.9 in. high, consists of a square lattice of 23,142 fuel tubes of thickness 0.021 in. and inner diameter of 0.298 in. on a 0.422-in. pitch. The tubes are filled with 3.40 w/o-enriched UO_2. The core is cooled by water, which enters at the bottom at 496°F and passes through the core at a rate of 34×10^6 lb/hr at 2015 psia. Compute **(a)** the average temperature of the water leaving the core; **(b)** the average power density in kW/liter: **(c)** the maximum heat production rate, assuming the reactor core is bare.

Problem 8.8

The core of a BWR consists of 764 fuel assemblies, each containing a square array of 49 fuel rods on a 0.748-in. pitch. The fuel rods are 175 in. long, but contain fuel over only 144 in. of their length. The outside diameter of the fuel rods is 0.563 in., the cladding (fuel tube) is 0.032 in. thick, and the UO_2 fuel pellets are 0.487 in. in diameter. This leaves a gap of (0.563 – 2 × 0.032 – 0.487)/2 = 0.006 in. between the pellets and the cladding. The UO_2 has an average density of approximately 10.3 g/cm^3. The radius of the core is 93.6 in., and the reactor is designed to operate at 3293 MW. The peak-to-average power density is 2.62. Calculate for this reactor: **(a)** the total weight of UO_2 and uranium in the core; **(b)** the specific power in kW/kg U; **(c)** the average power density in kW/liter; **(d)** the average linear rod power q'_{avg} in kW/ft; **(e)** the maximum heat production rate.

Problem 8.9

The core of an LMFBR consists of a square lattice of 13,104 fuel rods 0.158 in. in diameter, 30.5 in. long, on a 0.210-in. pitch. The fuel rods are 26 w/o-enriched uranium clad in 0.005-in. stainless steel. Liquid sodium enters the core at approximately 300°C and passes through the core at an average speed of 31.2 ft/sec. The core produces 270 MW of thermal power, with a maximum-to-average power density of 1.79. Calculate: **(a)** the maximum heat production rate; **(b)** the maximum neutron flux.

Problem 8.10

The variation of the neutron flux and/or heat production rate along the *z*-direction in the core of a reflected reactor is often approximated by the function

$$\phi = \text{constant} \times \cos\left(\frac{\pi z}{\tilde{H}}\right)$$

where $\tilde{H}$ is the distance between the extrapolated boundaries and is somewhat larger than H, the actual height of the core. Show that the maximum-to-average heat production ratio in the *z*-direction, Ω_z, is given by

$$\Omega_z = \frac{\pi H/2\tilde{H}}{\sin\left(\pi H/2\tilde{H}\right)}$$

Problem 8.12

The core of a fast reactor is a cylinder 38.8 cm in radius and 77.5 cm high. Two-dimensional (*r*,*z*) multigroup calculations show that the power density distribution in the core can be represented approximately by the expression

$$P(r,z) = P_0\left[1-\left(\frac{r}{51}\right)^2\right]\cos\left(\frac{\pi z}{109}\right)$$

where P_0 is a constant and *r* and *z* are the distances in centimeters from the axis and the midplane of the core, respectively. **(a)** Evaluate P_0 in terms of the total core power *P*. **(b)** What is the maximum-to-average power ratio in the core? **(c)** Calculate the maximum-to-average power ratios in the radial and axial directions.

Problem 8.17

The reactor core in Problem 8.12 produces 400 MW, of which 8 MW is due to γ-ray heating of the coolant. The total heat transfer area of the fuel is 1,580 ft^2. Compute **(a)** the average power density in the core in kW/liter and kW/ft^3; **(b)** the average heat flux; **(c)** the maximum heat flux.

Problem 8.19

The temperature at the center of the fuel rod in Problem 8.9, where *q''* is the largest, is 1220°F. Calculate the temperatures at the fuel-cladding interface and at the outer surface of the cladding.

Solutions

■ P5.3

Part (a): Referring to Table II.3, Σ_s = 0.7589 cm^{-1} for beryllium. Given the atomic number *A* = 4 for this element, we substitute into (5.12) to obtain

$$\bar{\mu} = \frac{2}{3A} = \frac{2}{3\times 4} = 0.167$$

Referring to equation (5.11),

$$\lambda_{\text{tr}} = \frac{1}{\Sigma_s(1-\bar{\mu})} = \frac{1}{0.7589\times(1-0.167)} = 1.58 \text{ cm}$$

Substituting into (5.10),

$$D = \frac{\lambda_{\text{tr}}}{3} = \frac{1.58}{3} = \boxed{0.527 \text{ cm}}$$

Part (b): Referring to Table II.3, Σ_s = 0.3811 cm^{-1} for carbon. The process to calculate the diffusion coefficient is identical to part (a):

$$\bar{\mu} = \frac{2}{3A} = \frac{2}{3\times 6} = 0.111$$

$$\lambda_{\text{tr}} = \frac{1}{\Sigma_s(1-\bar{\mu})} = \frac{1}{0.3811\times(1-0.111)} = 2.95 \text{ cm}$$

$$D = \frac{\lambda_{\text{tr}}}{3} = \frac{2.95}{3} = \boxed{0.983 \text{ cm}}$$

■ P5.4

Part (a): The maximum flux occurs in the center of the spherical reactor, that is, in $r \to 0$. Noting that

$$\lim_{x\to 0}\frac{\sin(kx)}{x} = k$$

we proceed to write

$$\phi_{\max}(r) = \lim_{r\to 0}\phi(r) = \lim_{r\to 0}\left[5\times 10^{13}\frac{\sin(0.0628r)}{r}\right] = \left(5\times 10^{13}\right)\times 0.0628 = \boxed{3.14\times 10^{12} \text{ n/cm}^2\cdot\text{s}}$$

Part (b): The neutron current density is given by the usual relation $J(r) = -Dd\phi(r)/dr$, which in the present case yields

$$J(r) = -0.80\times\frac{d}{dr}\left[5\times 10^{13}\frac{\sin(0.0628r)}{r}\right] = \boxed{-4\times 10^{13}\left[\frac{0.0628\cos(0.0628r)}{r} - \frac{\sin(0.0628r)}{r^2}\right]}$$

Part (c): The leakage rate is obtained by integrating the neutron current density over the surface of the spherical reactor:

$$\int_A J_r dA = 4\pi R^2 J(R) = 4\pi R^2\left\{-4\times 10^{13}\left[\frac{0.0628\cos(0.0628R)}{R} - \frac{\sin(0.0628R)}{R^2}\right]\right\}$$

$$\therefore \int_A J_r dA = 4\pi\times 50^2\times\left\{-4\times 10^{13}\left[\frac{0.0628\cos(0.0628\times 50)}{50} - \frac{\sin(0.0628\times 50)}{50^2}\right]\right\}$$

$$\therefore \int_A J_r dA = \boxed{1.58\times 10^{15} \text{ n/sec}}$$

■ P5.7

Part (a): Per equation (5.33), the flux associated with a point source emitting S neutrons per second isotropically in an infinite medium is

$$\phi(r) = \frac{Se^{-r/L}}{4\pi Dr}$$

The current density is identical to the result given on page 243,

$$J(r) = \frac{S}{4\pi}\left(\frac{1}{rL} + \frac{1}{r^2}\right)e^{-r/L}$$

The net number of neutrons passing per second through a spherical surface of radius r centered at the point source is

$$4\pi r^2 J(r) = 4\pi r^2\times\frac{S}{4\pi}\left(\frac{1}{rL} + \frac{1}{r^2}\right)e^{-r/L} = \boxed{S\left(1 + \frac{r}{L}\right)e^{-r/L}}$$

Part (b): In this part, we first set up the integral

$$\int_0^r \Sigma_a\phi(r)dV = \int_0^r \Sigma_a\frac{Se^{-r/L}}{4\pi Dr}\times 4\pi r^2 dr = \frac{\Sigma_a S}{D}\int_0^r re^{-r/L}dr$$

The rightmost integral can be evaluated using integration by parts; we can speed things up using Mathematica:

```
In[121]:= Expand[Integrate[r * Exp[-r / L], {r, 0, r}, Assumptions → r > 0]]
```

$$\text{Out[121]= } L^2 - e^{-\frac{r}{L}} L^2 - e^{-\frac{r}{L}} L\, r$$

As shown,

$$\int_0^r \Sigma_a \phi(r)\,dV = \frac{\Sigma_a S}{D}\left(-Lre^{-r/L} - L^2 e^{-r/L} + L^2\right)$$

Simplifying,

$$\int_0^r \Sigma_a \phi(r)\,dV = \boxed{\frac{\Sigma_a SL^2}{D}\left[1-\left(1+\frac{r}{L}\right)e^{-r/L}\right]}$$

■ P5.8

The flux afforded by an infinite planar source is given by equation (5.29),

$$\phi = \frac{SL}{2D}e^{-|x|/L}$$

Setting $x = a/2$ and doubling the flux to account for both planar sources,

$$\phi(x=0) = 2\times\frac{SL}{2D}e^{-\frac{(a/2)}{L}} = \boxed{\frac{SL}{D}e^{-\left(\frac{a}{2L}\right)}}$$

Now, note that the current vectors in a region midway between the two planes have the same magnitude and opposite directions. As a result, the current density at $x = 0$ is zero.

$$\boxed{J(x=0)=0}$$

■ P5.9

The flux at a distance r from the line of intersection of the sources is

$$\phi(r) = 2\times\frac{SL}{2D}e^{-r/L} = \boxed{\frac{SL}{D}e^{-r/L}}$$

The current density associated with either of the two planar sources is

$$J_1 = J_2 = -D\frac{d}{dr}\left(\frac{SL}{2D}e^{-r/L}\right) = -D\times\frac{SL}{2D}\times\left(-\frac{1}{L}e^{-r/L}\right)$$

$$\therefore J_1 = J_2 = \frac{1}{2}Se^{-r/L}$$

The total current density can be established with the law of cosines:

$$J = \sqrt{J_1^2 + J_2^2 - 2J_1(-J_1)\cos\theta} = \sqrt{2J_1^2 + 2J_1^2\cos\theta} = \sqrt{2}J_1\sqrt{1+\cos\theta}$$

$$\therefore J = \sqrt{2}\times\frac{1}{2}Se^{-r/L}\times\sqrt{1+\cos\theta}$$

$$\therefore \boxed{J = \frac{\sqrt{2}}{2}Se^{-r/L}\sqrt{1+\cos\theta}}$$

where θ is the angle between the sources.

■ P5.11

The ODE that describes neutron diffusion in an infinite bare slab is

$$\frac{d^2\phi}{dx^2} - \frac{1}{L^2}\phi = -\frac{S}{D} \quad \text{(I)}$$

The solution to the homogeneous form of this ODE is

$$\phi_h(x) = A\sinh\left(\frac{x}{L}\right) + B\cosh\left(\frac{x}{L}\right)$$

In turn, we follow the hint in the problem statement and take a constant C as the particular solution:

$$\phi_p(x) = C$$

Substituting ϕ_p into (I) and solving for C, we get

$$0-\frac{1}{L^2}\times C=-\frac{S}{D}$$

$$\therefore C=\frac{SL^2}{D}$$

so that

$$\phi(x)=\phi_h(x)+\phi_p(x)=A\sinh\left(\frac{x}{L}\right)+B\cosh\left(\frac{x}{L}\right)+\frac{SL^2}{D}$$

Next, we use the symmetry boundary condition $\phi(a+d)=\phi(-(a+d))$, giving

$$A\sinh\left(\frac{a+d}{L}\right)+B\cosh\left(\frac{a+d}{L}\right)+\frac{SL^2}{D}=A\sinh\left[-\frac{(a+d)}{L}\right]+B\cosh\left[-\frac{(a+d)}{2}\right]+\frac{SL^2}{D}$$

$$\therefore A\sinh\left(\frac{a+d}{L}\right)+B\cosh\left(\frac{a+d}{L}\right)=A\sinh\left[-\frac{(a+d)}{L}\right]+B\cosh\left[-\frac{(a+d)}{2}\right]$$

But $sinh(-x)=-sinh(x)$ (i.e., the hyperbolic sine is an odd function) and $cosh(-x)=cosh(x)$ (i.e., the hyperbolic cosine is an even function), hence

$$A\sinh\left(\frac{a+d}{L}\right)+B\cosh\left(\frac{a+d}{L}\right)=-A\sinh\left[\frac{(a+d)}{L}\right]+B\cosh\left[\frac{(a+d)}{2}\right]$$

$$\therefore 2A\sinh\left(\frac{a+d}{L}\right)=0$$

$$\therefore A=0$$

Then, the solution $\phi(x)$ further reduces to

$$\phi(x)=B\cosh\left(\frac{x}{L}\right)+\frac{SL^2}{D}$$

Next, we employ the boundary condition $\phi(a+d)=0$,

$$\phi(a+d)=B\cosh\left(\frac{a+d}{L}\right)+\frac{SL^2}{D}=0$$

$$\therefore B=-\frac{SL^2}{D\cosh\left[(a+d)/L\right]}$$

Lastly,

$$\phi(x)=\frac{SL^2}{D}\left[1-\frac{\cosh(x/L)}{\cosh\left(\frac{a+d}{L}\right)}\right]=\boxed{\frac{S}{\Sigma_a}\left[1-\frac{\cosh(x/L)}{\cosh\left(\frac{a+d}{L}\right)}\right]}$$

■ P5.14

Part (a): The ODE we need to solve is

$$\frac{1}{r^2}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right)-\frac{\phi}{L^2}=-\frac{S}{D}$$

Letting $w=r\phi$, the equation simplifies to

$$\frac{d^2w}{dr^2}-\frac{1}{L^2}w=-\frac{Sr}{D}$$

The general solution to this equation is

$$w_h=A\sinh\left(\frac{r}{L}\right)+B\cosh\left(\frac{r}{L}\right)$$

or, equivalently,

$$\phi_h(r) = \frac{A}{r}\sinh\left(\frac{r}{L}\right) + \frac{B}{r}\cosh\left(\frac{r}{L}\right)$$

where *A* and *B* are constants. To find the particular solution, we try *w* = *Cr*, where *C* is a constant, so that

$$\frac{d^2w}{dr^2} - \frac{1}{L^2}w = -\frac{Sr}{D} \;\rightarrow\; 0 - \frac{Cr}{L^2} = -\frac{Sr}{D}$$

$$\therefore w_p = \frac{SrL^2}{D}$$

$$\therefore \phi_p = \frac{SL^2}{D}$$

The final solution has the form

$$\phi(r) = \phi_h + \phi_p = \frac{A}{r}\sinh\left(\frac{r}{L}\right) + \frac{B}{r}\cosh\left(\frac{r}{L}\right) + \frac{SL^2}{D} \quad \text{(I)}$$

As an initial boundary condition, $\lim_{r\to 0}\phi$ must be finite. In the right-hand side of (I), this limit is defined for the first and third terms but not for the one in the middle, because:

```
In[133]:= Limit[B/r * Cosh[r / L], r → 0]

Out[133]= Indeterminate
```

Thus, for the boundary condition to hold, we must have *B* = 0. Then, (I) simplifies to

$$\phi(r) = \phi_h + \phi_p = \frac{A}{r}\sinh\left(\frac{r}{L}\right) + \frac{SL^2}{D} \quad \text{(II)}$$

The second boundary condition is $\phi(R+d) = 0$. Substituting in (II) and solving for *A*,

$$\phi(r = R+d) = \frac{A}{R+d}\sinh\left(\frac{R+d}{L}\right) + \frac{SL^2}{D} = 0$$

$$\therefore \phi(r = R+d) = \frac{A}{R+d}\sinh\left(\frac{R+d}{L}\right) = -\frac{SL^2}{D}$$

$$\therefore \frac{A}{R+d}\sinh\left(\frac{R+d}{L}\right) = -\frac{SL^2}{D}$$

$$\therefore A = -\frac{SL^2}{D}\frac{R+d}{\sinh\left(\frac{R+d}{L}\right)}$$

Finally,

$$\phi(r) = \phi_h + \phi_p = \frac{SL^2}{D}\frac{R+d}{\sinh\left(\frac{R+d}{L}\right)}\frac{1}{r}\sinh\left(\frac{r}{L}\right) + \frac{SL^2}{D}$$

$$\therefore \boxed{\phi(r) = \frac{S}{\Sigma_a}\left\{1 - \frac{R+d}{r}\frac{\sinh(r/L)}{\sinh\left[(R+d)/L\right]}\right\}}$$

In the final passage, we've used the definition of diffusion area $L^2 = D/\Sigma_a$.
Part (b): The current density is given by equation (5.8),

$$J(r) = -D\nabla\phi = -D \times \frac{d}{dr}\left\langle \frac{S}{\Sigma_a}\left\{1 - \frac{R+d}{r}\frac{\sinh(r/L)}{\sinh\left[(R+d)/L\right]}\right\}\right\rangle$$

The rightmost derivative can be obtained with Mathematica:

```
In[136]:= Simplify[D[S/Σa * (1 - (R + d)/r * Sinh[r / L]/Sinh[(R + d) / L]), r]]

Out[136]= ((d + R) S Csch[(d+R)/L] (-r Cosh[r/L] + L Sinh[r/L]))/(L r^2 Σa)
```

Thus,

$$J(r) = -D\nabla\phi = \boxed{D \times \frac{S(d+R)}{\Sigma_a L r^2}\frac{1}{\sinh\left[(d+R)/L\right]}\left[r\cosh\left(\frac{r}{L}\right) - L\sinh\left(\frac{r}{L}\right)\right]}$$

Part (c): The leakage rate is given by

$$4\pi R^2 \times J = \boxed{\frac{4\pi R^2 DS(d+R)}{\Sigma_a L r^2}\frac{1}{\sinh\left[(d+R)/L\right]}\left[r\cosh\left(\frac{r}{L}\right) - L\sinh\left(\frac{r}{L}\right)\right]}$$

Part (d): The number of neutrons that escape per cm^2/sec is given by the leakage rate determined in part (c):

$$\frac{4\pi R^2 DS(d+R)}{\Sigma_a L r^2}\frac{1}{\sinh\left[(d+R)/L\right]}\left[r\cosh\left(\frac{r}{L}\right) - L\sinh\left(\frac{r}{L}\right)\right]$$

The number of neutrons emitted per cm^2/sec is simply *S*. We proceed to compute the probability that a neutron escapes:

$$\text{Probability that a neutron escapes} = \frac{\text{No. that escape per cm}^2\text{/sec}}{\text{No. emitted per cm}^2\text{/sec by the source}}$$

$$\therefore \left[\begin{array}{c}\text{Probability that}\\ \text{a neutron escapes}\end{array}\right] = \frac{\dfrac{4\pi R^2 DS(d+R)}{\Sigma_a L r^2}\dfrac{1}{\sinh\left[(d+R)/L\right]}\left[r\cosh\left(\dfrac{r}{L}\right) - L\sinh\left(\dfrac{r}{L}\right)\right]}{S}$$

$$\therefore \left[\begin{array}{c}\text{Probability that}\\ \text{a neutron escapes}\end{array}\right] = \boxed{\frac{4\pi R^2 D(d+R)}{\Sigma_a L r^2}\frac{1}{\sinh\left[(d+R)/L\right]}\left[r\cosh\left(\frac{r}{L}\right) - L\sinh\left(\frac{r}{L}\right)\right]}$$

■ P5.15

The total leakage of neutrons is given by

$$\sum\left(-D\nabla^2\phi\right) = -D_1\nabla^2\phi_1 - D_2\nabla^2\phi_2 - D_3\nabla^2\phi_3 \quad \text{(I)}$$

We begin by computing $\nabla^2\phi_1$

$$\nabla^2\phi_1 = \left(3\times10^{15}\right)\frac{d^2}{dr^2}\left[\frac{3\times10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)\right]$$

$$\therefore \nabla^2\phi_1 = \left(3\times10^{15}\right)\left[-\frac{2\pi}{r^2 R}\cos\left(\frac{\pi r}{R}\right) + \frac{2}{r^3}\sin\left(\frac{\pi r}{R}\right) - \frac{\pi^2}{rR^2}\sin\left(\frac{\pi r}{R}\right)\right]$$

Substituting *r* = *R* brings to

$$\nabla^2\phi_1\Big|_{r=R} = \left(3\times10^{15}\right)\left[-\frac{2\pi}{R^2\times R}\cos\left(\frac{\pi\times R}{R}\right) + \cancel{\frac{2}{R^3}\sin\left(\frac{\pi\times R}{R}\right)} - \cancel{\frac{\pi^2}{R\times R^2}\sin\left(\frac{\pi\times R}{R}\right)}\right]$$

$$\therefore \nabla^2\phi_1\Big|_{r=R} = \left(3\times10^{15}\right)\times\frac{2\pi}{R^3}$$

$$\therefore \nabla^2\phi_1\Big|_{r=R} = \frac{6\pi\times10^{15}}{R^3}$$

Similarly for $\nabla\phi_2$ and $\nabla\phi_3$,

$$\nabla^2\phi_2\Big|_{r=R} = \left(2\times10^{15}\right)\times\frac{2\pi}{R^3} = \frac{4\pi\times10^{15}}{R^3}$$

$$\nabla^2\phi_3\Big|_{r=R} = \left(1\times10^{15}\right)\times\frac{2\pi}{R^3} = \frac{2\pi\times10^{15}}{R^3}$$

Substituting in (I), we obtain

$$\sum\left(-D\nabla^2\phi\right) = -2.2\times\left(\frac{6\pi\times10^{15}}{50^3}\right) - 1.7\times\left(\frac{4\pi\times10^{15}}{50^3}\right) - 1.05\times\left(\frac{2\pi\times10^{15}}{50^3}\right)$$

$$\therefore \sum\left(-D\nabla^2\phi\right) = \boxed{-5.55\times10^{11}\text{ neutrons}}$$

■ P5.18

Part (a): The neutron current density is given by $J(x,y,z) = -\nabla\phi$, which can be written vectorially as

$$\mathbf{J}(x,y,z) = -D\frac{\partial}{\partial x}\phi(x,y,z)\mathbf{i} - D\frac{\partial}{\partial y}\phi(x,y,z)\mathbf{j} - D\frac{\partial}{\partial z}\phi(x,y,z)\mathbf{k}$$

$$\mathbf{J}(x,y,z) = \left\{ \begin{array}{c} -\frac{DA\pi}{\tilde{a}}\left[\sin\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right]\mathbf{i} - \frac{DA\pi}{\tilde{a}}\left[\cos\left(\frac{\pi x}{\tilde{a}}\right)\sin\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right]\mathbf{j} \\ -\frac{DA\pi}{\tilde{a}}\left[\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\sin\left(\frac{\pi z}{\tilde{a}}\right)\right]\mathbf{k} \end{array} \right\}$$

Part (b): Due to symmetry, the No. of neutrons leaking per second is the same for any of the six sides of the cubical reactor. To find this leakage rate Q, we integrate the neutron current density over the area of any of the faces; for example:

$$Q = \int_{\text{Surface}} J(x,y,z)\,dA = \int_{-\tilde{a}/2}^{\tilde{a}/2}\int_{-\tilde{a}/2}^{\tilde{a}/2}\frac{DA\pi}{\tilde{a}}\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)dydz$$

$$\therefore Q = \frac{DA\pi}{\tilde{a}}\int_{-\tilde{a}/2}^{\tilde{a}/2}\int_{-\tilde{a}/2}^{\tilde{a}/2}\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)dydz$$

$$\therefore Q = \frac{DA\pi}{\tilde{a}}\int_{-a/2}^{a/2}\cos\left(\frac{\pi z}{\tilde{a}}\right)dz\times\frac{\tilde{a}}{\pi}\sin\left(\frac{\pi y}{\tilde{a}}\right)\Bigg|_{y=-\tilde{a}/2}^{y=\tilde{a}/2}$$

$$\therefore Q = \frac{DA\pi}{\tilde{a}}\int_{-a/2}^{a/2}\cos\left(\frac{\pi z}{\tilde{a}}\right)dz\times\frac{2\tilde{a}}{\pi}$$

$$\therefore Q = \frac{DA\pi}{\tilde{a}}\times\frac{\tilde{a}}{\pi}\sin\left(\frac{\pi z}{\tilde{a}}\right)\Bigg|_{z=-\tilde{a}/2}^{z=\tilde{a}/2}\times\frac{2\tilde{a}}{\pi}$$

$$\therefore Q = \frac{DA\pi}{\tilde{a}}\times\frac{2\tilde{a}}{\pi}\times\frac{2\tilde{a}}{\pi}$$

$$\therefore \boxed{Q = \frac{4\tilde{a}AD}{\pi}\text{ neutrons/sec}}$$

Part (c): To find the total No. of thermal neutrons leaking per second from the reactor, we multiply the leaking rate obtained in part (*b*) by six, which is the number of faces of a cubical reactor:

$$Q_{\text{Total}} = 6\times\frac{4\tilde{a}AD}{\pi} = \boxed{24\frac{\tilde{a}AD}{\pi}\text{ neutrons/sec}}$$

■ P5.19

Part (a): For the situation at hand, a = 200/2 = 100 cm, $S = 10^8$ n/cm^2-sec, L = 59 cm (Table 5.2), $\overline{D}$ = 0.84 cm (Table 5.2), and $d = 2.13\overline{D} = 2.13 \times 0.84 = 1.79$ cm (equation 5.22). Substituting into equation (5.36) yields

$$\phi = \frac{SL}{2D}\frac{\sinh\left[\left(a+d-|x|\right)/L\right]}{\cosh\left[\left(a+d\right)/L\right]}$$

$$\therefore \phi = \frac{10^8 \times 59}{2 \times 0.84} \times \frac{\sinh\left[\left(100+1.79-|x|\right)/59\right]}{\cosh\left[\left(100+1.79\right)/59\right]}$$

$$\therefore \phi(x) = 1.21\times10^9 \sinh(1.73-0.0170x)$$

Integrating this relationship over the slab thickness, we have:

```
In[532]:= Integrate[1.21 * 10^9 * Sinh[1.73 - 0.017 * x], {x, -100, 100}]

Out[532]= 1.02879 × 10^12
```

That is,

$$\phi_T = 1.03\times10^{12}\ \text{n/cm}^2\cdot\text{sec}$$

Dividing by $v = 2200\times10^2$ cm/s,

$$\rho_N = \frac{\phi_T}{v} = \frac{1.03\times10^{12}}{2200\times10^2} = \boxed{4.68\times10^6\ \text{n/cm}^2}$$

Part (b): Taking $\bar{\Sigma}_a = 2.4\times10^{-4}$ cm^{-1} from Table 5.2, we have

$$\bar{\Sigma}_a\phi_T = \left(2.4\times10^{-4}\right)\times\left(1.03\times10^{12}\right) = \boxed{2.47\times10^8\ \text{n/cm}^2\cdot\text{s}}$$

Part (c): To express current density as a function of position, first recall that

$$J(x) = -D\frac{d\phi(x)}{dx}$$

Replacing $\phi(x)$ with the expression obtained in part (a) and carrying out the differentiation,

$$J(x) = -D\frac{d\phi(x)}{dx} = -0.84\times\frac{d}{dx}\left[1.21\times10^9\sinh(1.73-0.0170x)\right]$$

$$\therefore J(x) = -0.84\times(-0.0170)\times1.21\times10^9\cosh(1.73-0.0170x)$$

$$\therefore \boxed{J(x) = 1.73\times10^7\cosh(1.73-0.0170x)}$$

Part (d): The rate at which neutrons leak from the two surfaces of the slab is 2 × $J(x = 100)$, that is,

$$2J(x=100) = 2\times\left[1.73\times10^7\cosh(1.73-0.0170\times100)\right] = \boxed{3.46\times10^7\ \text{n/cm}^2\text{-sec}}$$

Part (e): Taking the rate of emission $S = 10^8$ neutrons/cm^2-sec and the leakage rate calculated in part (d), the probability that a source neutron will not leak from the slab becomes

$$\text{Pr} = 1-\frac{2J(a)}{S} = 1-\frac{3.46\times10^7}{10^8} = \boxed{0.654}$$

■ P5.22

The number density of ordinary water may be taken as

$$N'_{H_2O} = \frac{1.0\times\left(6.02\times10^{23}\right)}{18.0153} = 3.342\times10^{22}\ \text{cm}^{-3}$$

If the water takes up to one-third of the reactor volume, the number density may be corrected as

$$N_{H_2O} = \frac{1}{3}N'_{H_2O} = \frac{1}{3}\times\left(3.342\times10^{22}\right) = 1.114\times10^{22}\ \text{cm}^{-3}$$

The corresponding density of protium atoms is

$$N_H = 2N_{H_2O} = 2\times\left(1.114\times10^{22}\right) = 2.228\times10^{22}\ \text{cm}^{-3}$$

Referring to Table II.2, we read $\sigma_a(^1\text{H})$ = 333 mb = 0.333 b. The macro absorption cross-section follows as

$$\Sigma_a = N_H \sigma_a\left({}^1\text{H}\right) = \left(2.228\times10^{22}\right)\times\left(0.333\times10^{-24}\right) = 0.00742\ \text{cm}^{-1}$$

We proceed to compute the hydrogen absorption rate:

$$\text{Absorption rate} = \int_0^{50} \Sigma_a \phi_T\left(r\right)\times 4\pi r^2 dr$$

$$\therefore \text{Absorption rate} = \int_0^{50} 0.00742\times\left[\left(2.29\times10^{14}\right)\frac{\sin\left(0.0628r\right)}{r}\right]\times 4\pi r^2 dr$$

$$\therefore \text{Absorption rate} = \left(2.135\times10^{13}\right)\int_0^{50} \sin\left(0.0628r\right)\times r\times dr$$

The rightmost expression can be evaluated in Mathematica:

```
In[522]:= 2.135 * 10^13 * Integrate[Sin[0.0628 * r] * r, {r, 0, 50}]

Out[522]= 1.7007 × 10^16
```

That is, *Absorption rate* ≈ 1.701×10^{16} sec^{-1}. Noting that 1 year = 86,400 × 365 sec, we have

$$\text{Absorption rate} = \left(1.701\times10^{16}\right)\times 86,400\times365 = 5.364\times10^{23}\ \text{yr}^{-1}$$

Lastly, noting that the molar mass of deuterium ≈2 g/mol, the ^{2}H production rate becomes

$$^2\text{H production rate} = \left(5.364\times10^{23}\right)\times\frac{2.0}{6.02\times10^{23}} = \boxed{1.782\ \text{g/yr}}$$

The reactor outputs approximately 1.78 grams of ^{2}H each year.

■ P5.30

The neutron flux for a bare slab is given by equation (5.36), with the proper adaptations for thermal neutrons:

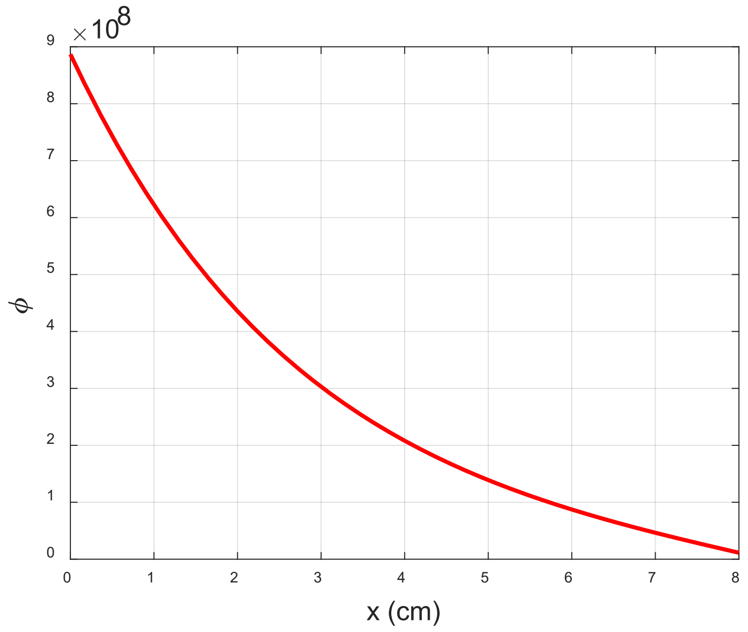
$$\phi_T = \frac{SL_T}{2\bar{D}}\frac{\sinh\left[\left(a+d-|x|\right)/L_T\right]}{\cosh\left[\left(a+d\right)/L_T\right]}$$

Here, $S = 10^8$ cm^2/sec, a = 8 cm, $L_T = \sqrt{8.1}$ = 2.85 cm (Table 5.2), and $d = 2.13\bar{D}$ = 2.13 × 0.16 = 0.341 cm (equation 5.22), giving

$$\phi_T = \frac{10^8\times2.85}{2\times0.16}\times\frac{\sinh\left[\left(8+0.341-|x|\right)/2.85\right]}{\cosh\left[\left(8+0.341\right)/2.85\right]} = \boxed{9.52\times10^7\sinh\left[0.351\left(8.341-|x|\right)\right]}$$

The boxed equation can be plotted with the following MATLAB code:

```
phi_T = @(x) 9.52e7 * sinh(0.351*(8.341 - abs(x)));
fplot(phi_T, [0 8], 'LineWidth', 2, 'Color', 'red')
ylim([0 9e8])
grid on
```



■ P6.1

The fuel utilization factor can be determined with equation (6.9), which we restate as

$$f = \frac{\Sigma_{aF}}{\Sigma_a} = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aS}} = \frac{1}{1 + \dfrac{\Sigma_{aS}}{\Sigma_{aF}}} \quad \text{(I)}$$

The ratio of macro cross-sections in the denominator is given by

$$\frac{\Sigma_{aF}}{\Sigma_a} = \frac{N_S}{N_F}\frac{\sigma_{aS}}{\sigma_{aF}} = \frac{\rho_S}{\rho_F}\frac{M_F}{M_S}\frac{\sigma_{aS}}{\sigma_{aF}}$$

In view of the specified mixture composition, we have

$$\frac{\rho_F}{\rho_S + \rho_F} = 0.03 \ \rightarrow \ (1 - 0.03)\rho_F = 0.03\rho_S$$

$$\therefore \frac{\rho_S}{\rho_F} = \frac{0.97}{0.03} = 32.3$$

Further, M_F = 239 (approximate molar mass of plutonium-239), M_S = 23 (MM of sodium), σ_{aF} = 2.11 b (micro absorption cross-section of ^{239}Pu; Table 6.1), and σ_{aS} = 0.0008 b (MACS of Na; also from Table 6.1). It follows that

$$\frac{\Sigma_{aF}}{\Sigma_a} = 32.3 \times \frac{239}{23} \times \frac{0.0008}{2.11} = 0.127$$

Substituting in (I) brings to

$$f = \frac{1}{1 + 0.127} = \boxed{0.887}$$

Taking η = 2.61 from the last column of Table 6.1, the infinite multiplication factor becomes

$$k_\infty = \eta f = 2.61 \times 0.887 = \boxed{2.32}$$

■ P6.3

Referring to Table 6.2, the flux equation for a finite cylinder reads

$$\phi(r,z) = \frac{3.63P}{VE_R\Sigma_F} J_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

Multiplying through by $E_R\Sigma_F$ gives the power density distribution:

$$P(r,z) = E_R\Sigma_F\phi(r,z) = \frac{3.63P}{V} J_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

The power rating of the reactor is P = 20 MW = 2×10^7 W. The volume of the reactor is $V = \pi R^2 H = \pi \times 50^2 \times 100$ = 785,400 cm^3. Substituting above, along with r = 7 cm and z = −22.7 cm, we obtain

$$P(r = 7, z = -22.7) = \frac{3.63 \times (2 \times 10^7)}{785{,}400} \times J_0\left(\frac{2.405 \times 7}{50}\right) \times \cos\left[\frac{\pi \times (-22.7)}{100}\right]$$

$$\therefore P(r = 7,\ z = -22.7) = \boxed{67.9\ \text{W/cm}^3}$$

■ P6.4

Part (a): We have neither the macro cross-section Σ_F nor constant A, but we can use the given information to solve for the product $\Sigma_F A$:

$$R = \Sigma_F\phi(r = 35) = \Sigma_F A \left.\frac{\sin(\pi r/R)}{r}\right|_{r=35} = 2.5 \times 10^{11}$$

$$\therefore \Sigma_F A \underbrace{\frac{\sin(\pi \times 35/45)}{35}}_{=0.0184} = 2.5 \times 10^{11}$$

$$\therefore \Sigma_F A = \frac{2.5\times10^{11}}{0.0184} = 1.36\times10^{13}$$

Noting that E_R = 200 MeV = 3.2×10^{-11} J, we proceed to write

$$P = E_R\Sigma_F\int_V \phi(r)\,dV = E_R\Sigma_F A\int_0^R \frac{1}{r}\sin\left(\frac{\pi r}{R}\right)\times 4\pi r^2 dr$$

$$\therefore P = 4\pi E_R\Sigma_F A\int_0^R \sin\left(\frac{\pi r}{R}\right) r\,dr$$

The rightmost integral can be determined with integration by parts; we can use Mathematica to speed things up:

```
In[154]:= Integrate[Sin[Pi * r / R] * r, {r, 0, R}, Assumptions → R > 0]

           R^2
Out[154]= ───
           π
```

Therefore,

$$P = 4\pi\times\left(3.2\times10^{-11}\right)\times\left(1.36\times10^{13}\right)\times\frac{45^2}{\pi} = 3.53\times10^6\text{ W} = \boxed{3.53\text{ MW}}$$

Part (b): To find the fission rate density at the center of the reactor, we compute the limit

$$\lim_{r\to0}\Sigma_F\phi(r) = \Sigma_F A\lim_{r\to0}\frac{\sin(\pi r/R)}{r} = \Sigma_F A\times\frac{\pi}{R}$$

$$\therefore \lim_{r\to0}\Sigma_F\phi(r) = \left(1.36\times10^{13}\right)\times\frac{\pi}{45} = \boxed{9.50\times10^{11}\text{ fissions/cm}^3\text{-sec}}$$

■ P6.6

As a first step, we solve the power equation for the volume-integrated flux:

$$P = E_R\Sigma_F\int_V \phi\,dV \;\rightarrow\; \int_V \phi\,dV = \frac{P}{E_R\Sigma_F}$$

The average flux follows from the mean value theorem for integrals:

$$\phi_{\text{avg}} = \frac{1}{V}\int_V \phi\,dV$$

We were given the maximum-to-average flux ratio Ω = 1.5. To find the maximum power density, we write

$$P_{\max} = E_R\Sigma_F\phi_{\max} = E_R\Sigma_F\times1.5\phi_{\text{avg}} = \frac{E_R\Sigma_F}{V}\times1.5\int_V \phi\,dV$$

$$\therefore P_{\max} = \frac{\cancel{E_R\Sigma_F}}{V}\times1.5\frac{P}{\cancel{E_R\Sigma_F}}$$

$$\therefore P_{\max} = \frac{1.5P}{V} = \frac{1.5\times825}{\pi\times5^2\times10} = 1.58\,\frac{\text{MW}}{\text{ft}^3}\times\frac{0.0353}{1}\,\frac{\text{ft}^3}{\text{L}}\times\frac{1000}{1}\,\frac{\text{kW}}{\text{MW}} = \boxed{55.8\text{ kW}/\ell}$$

■ P6.9

Part (a): For completeness, we will repeat the calculations for thermal utilization and infinite multiplication factor even though the results are unchanged relatively to Problem 6.1. We first compute the number densities for plutonium and sodium; the data used are the same as in Problem 6.1:

$$N_F = \frac{\left(6.02\times10^{23}\right)\times0.97}{23} = 2.54\times10^{22}\text{ n/cm}^2$$

$$N_M = \frac{\left(6.02\times10^{23}\right)\times0.03}{239} = 7.56\times10^{19}\text{ n/cm}^2$$

Using the same cross-sections as in Problem 6.1, we proceed to determine the thermal utilization:

$$f = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM}} = \frac{N_F \sigma_{aF}}{N_F \sigma_{aF} + N_M \sigma_{aM}}$$

$$\therefore f = \frac{\left(7.56\times10^{19}\right)\times\left(2.11\times10^{-24}\right)}{\left(7.56\times10^{19}\right)\times\left(2.11\times10^{-24}\right)+\left(2.54\times10^{22}\right)\times\left(0.0008\times10^{-24}\right)} = 0.887$$

Taking η = 2.61 from Table 6.1, k_∞ becomes

$$k_\infty = \eta f = 2.61\times0.887 = 2.32$$

With reference to Table 6.1, we read $\sigma_{\text{tr,Na}}$ = 3.3 b and $\sigma_{\text{tr,F}}$ = 6.8 b, so that

$$\Sigma_{\text{tr}} = \left(7.56\times10^{19}\right)\times\left(6.8\times10^{-24}\right)+\left(2.54\times10^{22}\right)\times\left(3.3\times10^{-24}\right) = 0.0843\ \text{cm}^{-1}$$

and

$$D = \frac{\lambda_{\text{tr}}}{3} = \frac{1}{3\Sigma_{\text{tr}}} = \frac{1}{3\times0.0843} = 3.95\ \text{cm}$$

The diffusion area is

$$L^2 = \frac{D}{\Sigma_a} = \frac{3.95}{\left(7.56\times10^{19}\right)\times\left(2.11\times10^{-24}\right)+\left(2.54\times10^{22}\right)\times\left(0.0008\times10^{-24}\right)} = 21{,}960\ \text{cm}^2$$

The bare critical radius is

$$\tilde{R}_c = \pi\sqrt{\frac{L^2}{k_\infty - 1}} = \pi\times\sqrt{\frac{21{,}960}{2.32-1}} = 405.2\ \text{cm}$$

and $d = 2.13D = 2.13 \times 3.95 = 8.41$ cm, so that

$$R_c = \tilde{R}_c - d = 405.2 - 8.41 = \boxed{396.8\ \text{cm}}$$

Part (b): In general, the neutron flux is described by

$$\phi(r) = \frac{P}{4E_R\Sigma_f R^2}\frac{\sin\left(\pi r/\tilde{R}\right)}{r}$$

To find the *maximum* flux, we write

$$\phi_{\max} = \frac{P}{4E_R\Sigma_f R^2}\times\frac{\pi}{\tilde{R}}\underbrace{\lim_{r\to0}\frac{\sin\left(\pi r/\tilde{R}\right)}{\pi r/\tilde{R}}}_{=1} = \frac{\pi P}{4E_R\Sigma_f R^2\tilde{R}}$$

$$\therefore \phi_{\max} = \frac{\pi\times\left(500\times10^6\right)}{4\times\left(3.2\times10^{-11}\right)\times\left[\left(7.56\times10^{19}\right)\times\left(1.4\times10^{-24}\right)\right]\times396.8^2\times405.2}$$

$$\therefore \boxed{\phi_{\max} = 1.82\times10^{15}\ \text{n/cm}^2\cdot\text{sec}}$$

Note that σ_f = 1.4 b = 1.4×10^{-24} cm^2 was read from Table 6.1.

Part (c): We first resort to equation (6.15) to compute the product of buckling and diffusion area:

$$B^2L^2 = k_\infty - 1 = 2.32 - 1 = 1.32$$

so that

$$P = \frac{1}{1+B^2L^2} = \frac{1}{1+1.32} = 0.431$$

The probability that a fission neutron will escape from the reactor is then

$$\text{Pr} = 1 - 0.431 = \boxed{0.569}$$

■ P6.11

Part (a): Noting that $a = b = c = 25$ ft, the buckling is straightforwardly obtained as

$$B = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = 3\left(\frac{\pi}{25}\right)^2 = \boxed{0.0474 \text{ ft}^2}$$

Part (b): The flux afforded by a rectangular parallelepiped, of which the cubical reactor is a special case, is

$$\phi(x,y,z) = A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$$

The maximum flux occurs in the centroid of the reactor, that is, at $(x,y,z) = (0, 0, 0)$, giving

$$\phi_{\max} = \phi(0,0,0) = A$$

Referring to Table 6.2, we have

$$\phi_{\max} = A = \frac{3.87P}{VE_R\Sigma_f} = \frac{3.87\times\left(20\times10^6\right)}{(25\times30.48)^3\times\left(3.2\times10^{-11}\right)\times\left(2.5\times10^{-3}\right)} = \boxed{2.19\times10^{12} \text{ n/cm}^2\text{-s}}$$

Part (c): Noting that $a = 25 \times 30.48 = 762$ cm, the average flux ϕ_{avg} follows from the mean value theorem:

$$\phi_{\text{avg}} = \frac{1}{V}\int\phi dV = \frac{A}{V}\int_{-381}^{381}\cos\left(\frac{\pi x}{762}\right)dx\int_{-381}^{381}\cos\left(\frac{\pi y}{762}\right)dy\int_{-381}^{381}\cos\left(\frac{\pi z}{762}\right)dz$$

$$\therefore \phi_{\text{avg}} = \frac{1}{V}\int\phi dV = \frac{2.19\times10^{12}}{(25\times30.48)^3}\times485.1^3 = \boxed{5.65\times10^{11} \text{ n/cm}^2\text{-s}}$$

■ P6.17

Part (a): The reactor buckling is

$$B^2 = \left(\frac{\pi}{a}\right)^2 = \left(\frac{\pi}{200}\right)^2 = \boxed{2.47\times10^{-4} \text{ cm}^{-2}}$$

Part (b): Assuming operation takes place at room temperature, we can refer to Table 6.3 and read $\eta_T = 2.065$. From Table 5.2, $L_T^2 = 3500$ cm^2. From Table 5.3, $\tau_T = 368$ cm^2. Inserting these data into equation (6.83), we obtain

$$Z = \frac{1+B^2\left(L_{TM}^2+\tau_{TM}\right)}{\eta_T-1-B^2\tau_{TM}} = \frac{1+\left(2.47\times10^{-4}\right)\times(3500+368)}{2.065-1-\left(2.47\times10^{-4}\right)\times368} = 2.01$$

From Table 3.4, $\sigma_{aF} = 680.8$ b; from Table II.2, $\sigma_{aM} = 3.4$ mb $= 0.0034$ b; from Table 3.2, $g_{aF}(20°\text{C}) = 0.9780$. The density of graphite is ≈1.6 g/cm^3. We proceed to compute ρ_F:

$$\frac{\rho_F}{\rho_M} = Z\frac{M_F\sigma_{aF}}{M_M\sigma_{aM}} = Z\frac{M_F\sigma_{aM}(E_0)}{M_M g_{aF}(20^\circ\text{C})\sigma_{aF}}$$

$$\therefore \rho_F = 1.60\times2.01\times\frac{235\times0.0034}{12\times0.978\times680.8} = \boxed{3.22\times10^{-4} \text{ g/cm}^3}$$

Part (c): We first compute the thermal utilization f:

$$f = \frac{Z}{Z+1} = \frac{2.01}{2.01+1} = 0.668$$

Then, we substitute into equation (6.82) to obtain

$$L_T^2 = (1-f)L_{TM}^2 = (1-0.668)\times3500 = \boxed{1160 \text{ cm}^2}$$

Part (d): The infinite multiplication factor is

$$k_\infty = \eta_T f = 2.065\times0.668 = \boxed{1.38}$$

Part (e): The thermal flux is given by the appropriate formula from Table 6.2:

$$\phi_{\text{th}}(x) = \phi_{\max}\cos\left(\frac{\pi x}{a}\right) = \boxed{5\times 10^{12}\cos\left(\frac{\pi x}{200}\right)}$$

The current density follows as

$$J(x) = -D\frac{d}{dx}\phi_{\text{th}}(x) = \boxed{5\times 10^{12}\times\frac{\pi D}{200}\sin\left(\frac{\pi x}{200}\right)}$$

Part (f): Referring to Table 3.2, we have $g_f(20^0\text{C}) = 0.9759$. With reference to Table 3.4, we read $\sigma_f = 582.2$ b. The macro fission cross-section Σ_f is

$$\Sigma_f = \Sigma_{fF} + \Sigma_{fM} = \rho_F\frac{N_A}{M_F}\times g_{fF}\left(20^{\text{o}}\text{C}\right)\sigma_f\left(E_0\right)$$

$$\therefore \Sigma_f = \left(3.22\times 10^{-4}\right)\times\frac{6.02\times 10^{23}}{235}\times 0.9759\times\left(582.2\times 10^{-24}\right) = 4.69\times 10^{-4}\ \text{cm}^{-1}$$

Per Table 6.2, the power and the maximum flux in an infinite slab reactor are related by

$$A = \phi_{\max} = \frac{1.57P}{aE_R\Sigma_f}$$

Solving for power,

$$\phi_{\max} = \frac{1.57P}{aE_R\Sigma_f} \rightarrow P = \frac{\phi_{\max}aE_R\Sigma_f}{1.57}$$

$$\therefore P = \frac{\left(5\times 10^{12}\right)\times 200\times\left(3.2\times 10^{-11}\right)\times\left(4.69\times 10^{-4}\right)}{1.57} = \boxed{9.56\ \text{W/cm}^2}$$

■ P6.20

Part (a): The buckling of a spherical reactor is given by

$$B^2 = \left(\frac{\pi}{R}\right)^2 = \left(\frac{\pi}{50}\right)^2 = 0.00395\ \text{cm}^{-2}$$

Assuming operation takes place at room temperature, we can refer to Table 6.3 and read $\eta_T = 2.065$. From Table 5.2, $L_T^2 = 480$ cm^2. From Table 5.3, $\tau_T = 102$ cm^2. Substituting into equation (6.83), we get

$$Z = \frac{1 + B^2\left(L_{TM}^2 + \tau_{TM}\right)}{\eta_T - 1 - B^2\tau_{TM}} = \frac{1 + 0.00395\times\left(480 + 102\right)}{2.065 - 1 - 0.00395\times 102} = 4.98$$

The density of beryllium may be taken as 1.85 g/cm^3. The mass of moderator (Be) follows as

$$m_M = \frac{4}{3}\pi R^3\rho_M = \frac{4}{3}\pi\times 50^3\times 1.85 = 9.69\times 10^5\ \text{g}$$

Further, $\sigma_{aF} = 680.8$ b (Table 3.4) and $\sigma_{aM} = 0.0092$ b (Table II.3). Referring to Table 3.2, we have $g_a(20^0\text{C}) = 0.9780$. The critical mass of fuel follows as

$$m_F = Z\frac{\sigma_{aM}M_F}{g_{aF}(T)\sigma_{aF}(E_0)M_M}m_M = \frac{4.98\times 0.0092\times 235}{0.9780\times 680.8\times 9}\times\left(9.69\times 10^5\right) = \boxed{1740\ \text{g}}$$

Part (b): Referring to Table 3.2, we have $g_{fF}(20^0\text{C}) = 0.9759$. With reference to Table 3.4, we read $\sigma_f = 582.2$ b. We proceed to calculate macro cross-section Σ_f:

$$\Sigma_f = \frac{m_F N_A}{M_F V}\frac{\sqrt{\pi}}{2}\sigma_{fF}\left(E_0\right)g_{fF}\left(T\right)$$

$$\therefore \Sigma_f = \frac{1740\times\left(6.02\times 10^{23}\right)}{235\times\left(\frac{4}{3}\pi\times 50^3\right)}\times\frac{\sqrt{\pi}}{2}\times\left(582.2\times 10^{-24}\right)\times 0.9759 = 0.00429\ \text{cm}^{-1}$$

Using the appropriate results from Table 6.2, we obtain the flux distribution

$$\phi(r) = A\frac{1}{r}\sin\left(\frac{\pi r}{R}\right) = \frac{P}{4R^2 E_R \Sigma_f}\frac{1}{r}\sin\left(\frac{\pi r}{R}\right)$$

$$\therefore \phi(r) = \frac{50{,}000}{4\times 50^2 \times \left(3.2\times 10^{-11}\right)\times 0.00429}\frac{1}{r}\sin\left(\frac{\pi r}{50}\right)$$

$$\therefore \boxed{\phi(r) = 3.64\times 10^{13}\frac{1}{r}\sin\left(\frac{\pi r}{50}\right)}$$

Part (c): Before computing the leakage rate, we need the current density at the surface of the spherical reactor (note that we have taken diffusivity D = 0.50 cm from Table 5.2):

$$J(r=R) = -D\frac{d}{dr}\phi(r)\Big|_{r=R} = -0.5\times\left(3.64\times 10^{13}\right)\times\frac{d}{dr}\left[\frac{1}{r}\sin\left(\frac{\pi r}{50}\right)\right]\Bigg|_{r=R}$$

$$\therefore J(r=R) = -0.5\times\left(3.64\times 10^{13}\right)\times\frac{1}{50R^2}\left[\pi R\cos\left(\frac{\pi R}{50}\right) - \sin\left(\frac{\pi R}{50}\right)\right]$$

$$\therefore J(r=R) = -0.5\times\left(3.64\times 10^{13}\right)\times\frac{1}{50\times 50^2}\left[\pi\times 50\times\cos\left(\frac{\pi\times 50}{50}\right) - \sin\left(\frac{\pi\times 50}{50}\right)\right]$$

$$\therefore J(r=R) = 2.29\times 10^{10}\ \text{n/cm}^2\text{-sec}$$

Therefore,

$$\text{Total leakage} = 4\pi R^2 J(R) = 4\pi\times 50^2\times\left(2.29\times 10^{10}\right) = \boxed{7.19\times 10^{14}\ \text{n/sec}}$$

Part (d): The consumption rate is

$$\text{Consumption rate} = 1.05(1+\alpha)P = 1.05\times(1+0.169)\times\left(50\times 10^{-3}\right)$$

$$\therefore \text{Consumption rate} = \boxed{0.0614\ \text{g/day}}$$

■ P6.25

Part (a): Interpolating data from Table 3.2, we have $g_{fF}(250^0\text{C})$ = 0.9416. With reference to Table 3.4, we read σ_{aF} = 680.8 b. Resorting to Table II.2, we have σ_{aM} = 3.4 mb = 0.0034 b. To establish the fuel utilization f, we first write

$$Z = \frac{N_F\sigma_{aF}}{N_M\sigma_{aM}} = \left(1.0\times 10^{-5}\right)\times\frac{0.9416\times 680.8}{0.0034} = 1.885 \approx 1.89$$

so that

$$f = \frac{Z}{1+Z} = \frac{1.89}{1+1.89} = 0.654$$

Taking η_T = 2.065 from Table 6.3, the infinite multiplication factor k_∞ becomes

$$k_\infty = \eta_T f = 2.065\times 0.654 = 1.35$$

Taking L_T^2 = 3500 cm^2 from Table 5.2 and τ_T = 368 cm^2 from Table 5.3, we can determine the buckling B^2:

$$B^2 = \frac{k_\infty - 1}{M_T^2} = \frac{k_\infty - 1}{(1-f)L_T^2 + \tau_T} = \frac{1.35-1}{(1-0.654)\times 3500 + 368} = 2.22\times 10^{-4}\ \text{cm}^{-2}$$

But $B^2 = 3(\pi/a)^2$ for a cubic reactor. Solving for dimension a,

$$B^2 = 3\left(\frac{\pi}{a}\right)^2 \rightarrow B^2 = \frac{3\pi^2}{a^2}$$

$$\therefore a = \pi\sqrt{\frac{3}{B^2}} = \pi\times\sqrt{\frac{3}{2.22\times 10^{-4}}} = \boxed{365\ \text{cm}}$$

Part (b): To find the critical mass, we first estimate the number of grams ρ_F of fuel in the mixture (note that we are using $\rho_M \approx$ 1.6 g/cm^3 as the density of graphite):

$$\rho_F = \rho_M \frac{N_F}{N_M}\frac{M_F}{M_M} = 1.6\times\left(1.0\times10^{-5}\right)\times\frac{235}{12} = 3.13\times10^{-4}\ \text{g/cm}^3$$

The critical mass is then

$$m_M = \rho_F V = \left(3.13\times10^{-4}\right)\times365^3 = 15,220\ \text{g} \approx \boxed{15.2\ \text{kg}}$$

Part (c): We must now determine the macro fission cross-section Σ_f. Taking σ_f = 582.2 b from Table 3.4, we write

$$\Sigma_f = \rho_F \frac{N_A}{M_F}\times g_f\left(250^\text{o}\text{C}\right)\sigma_f\left(E_0\right)$$

$$\therefore \Sigma_f = \left(3.13\times10^{-4}\right)\times\frac{6.02\times10^{23}}{235}\times0.9416\times\left(582.2\times10^{-24}\right) = 4.4\times10^{-4}\ \text{cm}^{-1}$$

With reference to Table 6.2, the maximum flux is related to power as

$$\phi_{\max} = A = \frac{3.87P}{VE_R\Sigma_f} = \frac{3.87\times1000}{365^3\times\left(3.2\times10^{-11}\right)\times\left(4.4\times10^{-4}\right)} = \boxed{5.65\times10^9\ \text{n/cm}^2}$$

■ P6.28

Part (a): With reference to Table 3.4, we read σ_{aF} = 680.8 b. Resorting to Table II.2, we have σ_{aM} = 3.4 mb = 0.0034 b. From Table 3.2, we read $g_{aF}(E_0)$ = 0.9780. To compute the fuel utilization f, we first write

$$Z = \frac{N_F}{N_M}\frac{\sigma_{aF}}{\sigma_{aM}} = \left(6.8\times10^{-6}\right)\times\frac{0.9780\times680.8}{0.0034} = 1.33$$

so that

$$f = \frac{Z}{Z+1} = \frac{1.33}{1.33+1} = 0.571$$

Taking η_T = 2.065 from Table 6.3,

$$k_\infty = \eta_T f = 2.065\times0.571 = \boxed{1.18}$$

Part (b): Taking L_T^2 = 3500 cm^2 from Table 5.2. we have

$$L_{T,c}^2 = \left(1-f\right)L_{T,M}^2 = \left(1-0.571\right)\times3500 = 1502\ \text{cm}^2$$

The reactor buckling is then

$$B^2 = \frac{k_\infty - 1}{L_{T,c}^2} = \frac{1.18-1}{1502} = 1.19\times10^{-4}\ \text{cm}^{-2}$$

Next, we take $L_T = \sqrt{3500}$ = 59 cm. Since the moderator and reflector are the same, we can avoid using transcendental equations and instead appeal to equation (6.100):

$$B\cot\left(BR\right) = -\frac{1}{L_r}$$

$$\therefore \cot\left(BR\right) = -\frac{1}{BL_r} = -\frac{1}{\sqrt{1.19\times10^{-4}}\times59} = -1.55$$

We can solve for $x = BR$ using the following MATLAB code:

```
>> f = @(x) cot(x)+1.55;
x0 = 2
fzero(f,x0)

ans =

    2.5686
```

That is, $BR \approx 2.57$. Solving for R,

$$R = \frac{2.57}{B} = \frac{2.57}{\sqrt{1.19\times10^{-4}}} = \boxed{236\ \text{cm}}$$

Part (c): The amount ρ_F of grams of fuel per unit volume of mixture is

$$\rho_F = \rho_M \frac{N_F}{N_M}\frac{M_F}{M_M} = 1.6\times\left(6.8\times10^{-6}\right)\times\frac{235}{12} = 2.13\times10^{-4}\ \text{g/cm}^3$$

Multiplying this by the reactor volume gives the critical mass m_F:

$$m_F = \rho_F V = \left(2.13\times10^{-4}\right)\times\frac{4}{3}\pi\times236^3 = 11{,}730\ \text{g} = \boxed{11.7\ \text{kg}}$$

Part (d): Noting that

$$B = \frac{\pi}{R_0} \rightarrow R_0 = \frac{\pi}{B}$$

$$\therefore R_0 = \frac{\pi}{\sqrt{1.19\times10^{-4}}} = 288\ \text{cm}$$

the reflector savings follow as

$$\delta = R_0 - R = 288 - 236 = \boxed{52\ \text{cm}}$$

Part (e): The first step here is to determine the macro fission cross-section Σ_f; taking g_F(20°C) = 0.9759 from Table 3.2 and σ_f = 582.2 b from Table 3.4, we have

$$\Sigma_f = \frac{\rho_F N_A}{M_F} g_F\left(20^\circ\text{C}\right)\sigma_f = \frac{\left(2.13\times10^{-4}\right)\times\left(6.02\times10^{23}\right)}{235}\times\left[0.9759\times\left(582.2\times10^{-24}\right)\right]$$

$$\therefore \Sigma_f = 3.10\times10^{-4}\ \text{cm}^{-1}$$

Inserting this and other data into equation (6.103) brings to

$$A = \frac{PB^2}{4\pi E_R \Sigma_f\left[\sin(BR) - BR\cos(BR)\right]}$$

$$\therefore A = \frac{100{,}000\times\left(1.19\times10^{-4}\right)}{4\pi\times\left(3.2\times10^{-11}\right)\times\left(3.1\times10^{-4}\right)\times\left[\sin\left(\sqrt{1.19\times10^{-4}}\times236\right) - \sqrt{1.19\times10^{-4}}\times236\cos\left(\sqrt{1.19\times10^{-4}}\times236\right)\right]}$$

$$\therefore A = 3.52\times10^{13}\ \text{cm}^{-1}$$

Then, the neutron flux is described by

$$\phi_c(r) = A\frac{\sin(Br)}{r} = \boxed{3.52\times10^{13}\frac{\sin(Br)}{r}}$$

Part (f): The maximum flux is given by the usual limit

$$\phi_{\max} = \lim_{r\to0}\phi(r) = \left(3.52\times10^{13}\right)\lim_{r\to0}\left[\frac{\sin(Br)}{r}\right] = \left(3.52\times10^{13}\right)\times B$$

$$\therefore \phi_{\max} = \left(3.52\times10^{13}\right)\times\sqrt{1.19\times10^{-4}} = 3.84\times10^{11}\ \text{n/cm}^2\text{-sec}$$

The average flux, in turn, is

$$\phi_{\text{avg}} = \frac{1}{V}\int_V \phi_c\,dV = \frac{P}{VE_R\Sigma_f} = \frac{100{,}000}{\left(\frac{4}{3}\pi\times236^3\right)\times\left(3.2\times10^{-11}\right)\times\left(3.1\times10^{-4}\right)} = 1.83\times10^{11}\ \text{n/cm}^2\text{-sec}$$

Finally, we compute the ratio

$$\Omega = \frac{\phi_{\max}}{\phi_{\text{avg}}} = \frac{3.84}{1.83} = \boxed{2.10}$$

■ P6.29

Firstly, Z remains unchanged at 4.98, so the fuel utilization becomes

$$f = \frac{Z}{Z+1} = \frac{4.98}{4.98+1} = 0.833$$

The infinite multiplication factor is

$$k_\infty = \eta_F f = 2.065 \times 0.833 = 1.72$$

Also,

$$L_{TC}^2 = (1-f)L_{TM}^2 = (1-0.833)\times 480 = 80.2 \text{ cm}^2$$

The updated buckling is

$$B^2 = \frac{k_\infty - 1}{L_{TC}^2} = \frac{1.72-1}{80.2} = 0.00898 \text{ cm}^{-2}$$

$$\therefore B = \sqrt{0.00898} = 0.0948 \text{ cm}^{-1}$$

Taking L_{Tr} = 21 cm for beryllium (Table 5.2), we appeal to equation (6.100):

$$B\cot BR = -\frac{1}{L_{Tr}} \rightarrow \cot BR = -\frac{1}{BL_{Tr}}$$

$$\therefore \cot BR = -\frac{1}{0.0948\times 21} = -0.502$$

We can solve for $x = BR$ using the following MATLAB code:

```
>> f = @(x) cot(x) + 0.502;
x0 = 2;
fzero(f,x0)

ans =
    2.0360
```

That is, BR = 2.036. Solving for radius,

$$BR = 2.036 \rightarrow R = \frac{2.036}{B}$$

$$\therefore R = \frac{2.036}{0.0948} = \boxed{21.5 \text{ cm}}$$

Using m_F = 1740 g from Problem 6.20, we can determine the amount ρ_F of fuel per unit volume of mixture:

$$\rho_F = \frac{1740}{\frac{4\pi}{3}\times 50^3} = 3.32\times 10^{-3} \text{ g/cm}^3$$

The critical mass is

$$m_F = \left(\frac{4\pi}{3}\times 21.5^3\right)\times\left(3.32\times 10^{-3}\right) = \boxed{138 \text{ g}}$$

■ P6.32

Part (a): As usual, the first step is to compute the fuel utilization f; to do so, we need parameter Z. Taking g_{aF}(20°C) = 1.0723 from the penultimate column of Table 3.2, σ_{aF} = 1011.3 b from Table 3.4, and σ_{aM} = 0.664 b from Table II.2, we may write

$$Z = \frac{\rho_F M_M g_{aF}(20^\circ\text{C})\sigma_{aF}}{\rho_M M_F \sigma_{aM}} = \frac{\left(8.5\times 10^{-3}\right)\times 18\times 1.0723\times 1011.3}{1.0\times 239\times 0.664} = 1.045$$

so that

$$f = \frac{Z}{Z+1} = \frac{1.045}{1+1.045} = 0.511$$

Then, taking L_T^2 = 8.1 cm^2 for water (Table 5.2), we have

$$L_T^2 = (1-f)L_{TM}^2 = (1-0.511)\times 8.1 = 3.96 \text{ cm}^2$$

and, with $\tau_T \approx$ 27 cm^2 (Table 5.3), we compute the thermal migration area

$$M_T^2 = L_T^2 + \tau_T = 3.96 + 27 = 30.96 \text{ cm}^2$$

The reflector savings follow from equation (6.107):

$$\delta = 7.2 + 0.10\left(M_T^2 - 40.0\right) = 7.2 + 0.1\left(30.96 - 40\right) = \boxed{6.30\text{ cm}}$$

Part (b): Solving equation (6.105) for reflected core thickness,

$$\delta = \frac{\tilde{a}_0}{2} - \frac{a}{2} \rightarrow a = \tilde{a}_0 - 2\delta \quad \text{(I)}$$

We can relate the unreflected core thickness $\tilde{a}_0$ to reactor buckling:

$$B^2 = \left(\frac{\pi}{\tilde{a}_0}\right)^2 \rightarrow \left(\frac{\pi}{\tilde{a}_0}\right)^2 = \frac{k_\infty - 1}{M_T^2}$$

$$\therefore \frac{\pi^2}{\tilde{a}_0^2} = \frac{k_\infty - 1}{M_T^2}$$

$$\therefore \frac{\pi^2}{\tilde{a}_0^2} = \frac{\eta_T f - 1}{M_T^2}$$

$$\therefore \tilde{a}_0 = \sqrt{\frac{\pi^2 M_T^2}{\eta_T f - 1}} = \pi\sqrt{\frac{30.96}{2.035 \times 0.511 - 1}} = 87.5\text{ cm}$$

where η_T = 2.035 was taken from Table 6.3. Substituting in (I),

$$a = 87.5 - 2 \times 6.30 = \boxed{74.9\text{ cm}}$$

■ P7.4

Part (a): Obviously, 0.001 = 0.1%.

Part (b): With reference to Table 7.2, the delayed neutron fraction for ^{235}U in thermal neutron fission is β = 0.0065. The reactivity r in dollars is related to the decimal reactivity ρ by $r = \rho/\beta$, so that

$$r = \frac{\rho}{\beta} \rightarrow \rho = \beta r$$

$$\therefore \rho = 0.0065 \times \$2 = 0.013 = \boxed{1.3\%}$$

Part (c): The same reasoning for part (b) applies here as well:

$$r = \frac{\rho}{\beta} \rightarrow \rho = \beta r$$

$$\therefore \rho = 0.0065 \times \left(-\$0.5\right) = -0.00325 = \boxed{-0.325\%}$$

■ P7.8

The power Π rises exponentially with time and has a period T = 10 minutes; mathematically,

$$\Pi(t) = \Pi_0 \exp(t/T) \rightarrow 10^6 = 10^{-3}\exp(t/10)$$

$$\therefore 10^9 = \exp(t/10)$$

$$\therefore \ln 10^9 = \frac{t}{10}$$

$$\therefore t = 10 \times \ln 10^9 = \boxed{207\text{ min}}$$

The power level will reach 1 MW within three hours and 27 minutes.

■ P7.12

For a ^{235}U-fueled reactor, 5 dollars of negative reactivity amount to ρ = –5 × 0.0065 = –0.0325. This added reactivity will cause the power to drop to a level P_J such that

$$P_J = \frac{\beta(1-\rho)}{\beta - \rho}P_0 = \frac{0.0065 \times \left(1 + 0.0325\right)}{0.0065 + 0.0325} \times 1.0 = 0.172\text{ MW}$$

Upon being lowered to 0.172 MW, the power decreases further according to the exponential law

$$P(t) = P_J \exp\left(-\frac{t}{80}\right) = 0.172\times10^6 \exp\left(-\frac{t}{80}\right)$$

The time T_0 required for the power to drop to 1 mW = 10^{-3} W is determined as

$$P(T_0) = 10^{-3} = 0.172\times10^6 \exp\left(-\frac{T_0}{80}\right)$$

$$\therefore \frac{10^{-3}}{0.172\times10^6} = \exp\left(-\frac{T_0}{80}\right)$$

$$\therefore \ln\left(\frac{10^{-3}}{0.172\times10^6}\right) = -\frac{T_0}{80}$$

$$\therefore T_0 = -80\times\ln\left(\frac{10^{-3}}{0.172\times10^6}\right) = 1517 \text{ sec} = \boxed{25.3 \text{ min}}$$

The power will drop to 1 mW within less than half an hour.

■ P7.13

After the reactor was scrammed, its power was lowered from P_0 = 2.7 MW to P_1, which is unknown. Fifteen minutes later, the reactor drops from P_1 to P_2 = 1 W. We can use this latter information to determine P_1:

$$P_2 = P_1 \exp(-t/T) \;\rightarrow\; P_1 = P_2 \exp(t/T)$$

$$\therefore P_1 = 1.0\times\exp\left[(15\times60)/80\right] = 76{,}880 \text{ W}$$

Then, we can estimate the reactivity ρ inserted when the reactor was scrammed:

$$P_1 = P_0 \frac{\beta(1-\rho)}{\beta-\rho} \;\rightarrow\; \rho = \frac{(P_0-P_1)\beta}{\beta P_0 - P_1}$$

$$\therefore \rho = \frac{\left(2.7\times10^6 - 76{,}880\right)\times0.0065}{0.0065\times\left(2.7\times10^6\right)-76{,}880} = -0.287 = \boxed{-\$44.15}$$

■ P7.19

Part (a): The reactor buckling B_0 is

$$B_0^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \left(\frac{2.405}{50}\right)^2 + \left(\frac{\pi}{100}\right)^2 = 3.30\times10^{-3} \text{ cm}^{-2}$$

To compute the fuel utilization, we take L_{TM}^2 = 480 cm^2 (Table 5.2), γ_{TH} = 102 cm^2 (Table 5.3), and η_T = 2.065 (Table 6.3), giving

$$f = \frac{1+B_0^2\left(L_{TM}^2+\gamma_{TH}\right)}{\eta_T + B_0^2 L_{TM}^2} = \frac{1+\left(3.3\times10^{-3}\right)\times(480+102)}{2.065+\left(3.3\times10^{-3}\right)\times480} = 0.800$$

The thermal migration area is

$$M_T^2 = L_T^2 + \gamma_T = (1-f)L_{TM}^2 + \gamma_T = (1-0.8)\times480+102 = 198 \text{ cm}^2$$

Taking $\bar{D}$ = 0.50 cm (Table 5.2) and the total macro cross-section $\Sigma_f = \Sigma_a + \Sigma_s$ = 0.001137 + 0.7589 = 0.760 cm^{-1} (Table II.3), we substitute into (7.56) to obtain

$$d = 2.131\bar{D}\frac{a\Sigma_f + 0.9354}{a\Sigma_f + 0.5098} = 2.131\times0.5\times\frac{2.5\times0.760+0.9354}{2.5\times0.760+0.5098} = 1.25 \text{ cm}$$

Substituting all pertaining data into (7.57) gives

$$\rho_\omega = \frac{7.43M_T^2}{\left(1+B_0^2M_T^2\right)R^2}\left[0.116+\ln\left(\frac{R}{2.405a}\right)+\frac{d}{a}\right]^{-1}$$

$$\rho_\omega = \frac{7.43\times198}{\left[1+\left(3.30\times10^{-3}\right)\times198\right]\times50^2}\left[0.116+\ln\left(\frac{50}{2.405\times2.5}\right)+\frac{1.25}{2.5}\right]^{-1} = 0.130 = \boxed{13.0\%}$$

Part (b): The infinite multiplication factor is

$$k_\infty = \frac{1}{1-\rho} = \frac{1}{1-0.13} = 1.15$$

Taking I_p = 3.9×10^{-3} sec from Table 7.1 and β = 0.0065 from Table 7.2, the reactor period becomes

$$T = \frac{I_p}{(1-\beta)k_\infty - 1} = \frac{3.9\times10^{-3}}{(1-0.0065)\times1.15-1} = \boxed{0.0274\text{ s}}$$

or 27.4 msec.

■ P7.22

The radius R_c of a control cell is calculated as

$$\pi R_c^2 = \frac{\pi\times(320/2)^2}{61\times20}$$

$$\therefore R_c = 4.58\text{ cm}$$

Also, a = 1.15/2 = 0.575 cm, $y = a/L_T$ = 0.575/1.38 = 0.417, and $z = R_C/L_T$ = 4.58/1.38 = 3.32. We proceed to compute extrapolation distance d, which is given by (7.56):

$$d = 2.131\bar{D}\frac{a\Sigma_f+0.9354}{a\Sigma_f+0.5098} = 2.131\times0.21\times\frac{0.575\times2.6+0.9354}{0.575\times2.6+0.5098} = 0.543\text{ cm}$$

Substituting into equation (6.116),

$$E(y,z) = \frac{z^2-y^2}{2y}\left[\frac{I_0(y)K_1(z)+K_0(y)I_1(z)}{I_1(z)K_1(y)-K_1(z)I_1(y)}\right]$$

$$\therefore E(y,z) = \frac{3.32^2-0.417^2}{2\times0.417}\times\left[\frac{I_0(0.417)K_1(3.32)+K_0(0.417)I_1(3.32)}{I_1(3.32)K_1(0.417)-K_1(3.32)I_1(0.417)}\right] = 6.79$$

Inserting results into (7.60) gives the rod utilization:

$$\frac{1}{f_R} = \frac{(z^2-y^2)d}{2a}+E(y,z) = \frac{\left(3.32^2-0.417^2\right)\times0.543}{2\times0.575}+6.79 = 11.9$$

$$\therefore f_R = \frac{1}{11.9} = 0.0840$$

The total worth ρ_∞ follows as

$$\rho_\infty = \frac{0.0840}{1-0.0840} = 0.0917 = \boxed{9.17\%}$$

■ P7.25

The worth of the rods is given by

$$\rho_\omega = \frac{\Sigma_{aB}}{\Sigma_{aF}+\Sigma_{aC}}$$

Here, Σ_{aF} = 8.33×10^{-3} cm^{-1} and Σ_{ac} = 1.9×10^{-5} cm^{-1} have been established in Problem 6.3. To compute Σ_{aB}, note first that the total mass of boron in 50 rods is 50 × 500 = 25,000 g, and the corresponding No. of atoms is

$$n_B = \frac{25{,}000}{10.811}\times\left(6.02\times10^{23}\right) = 1.39\times10^{27}$$

Noting that the critical radius in Example 6.3 was determined to be 49.5 cm, the number density of B atoms becomes

$$N_A = \frac{1.39\times10^{27}}{\frac{4\pi}{3}\times49.5^3} = 2.74\times10^{21}\ \text{cm}^{-3}$$

Given the absorption cross-section σ_{aB} = 0.276 b, we have

$$\Sigma_{aB} = \left(2.74\times10^{21}\right)\times\left(0.276\times10^{-24}\right) = 7.56\times10^{-4}\ \text{cm}^{-1}$$

Finally,

$$\rho_\omega = \frac{\Sigma_{aB}}{\Sigma_{aF}+\Sigma_{aC}} = \frac{7.56\times10^{-4}}{8.33\times10^{-3}+1.9\times10^{-5}} \approx 0.0905 = \boxed{9.05\%}$$

■ P7.27

Part (a): The worth of a partially inserted rod can be estimated with equation (7.63):

$$\rho_\omega(x) = \rho_\omega(H)\left[\frac{x}{H} - \frac{1}{2\pi}\sin\left(\frac{2\pi x}{H}\right)\right]$$

Noting that $\rho_\omega(H)$ = 50 cents, the worth associated with $x = 3H/4$ is

$$\rho_\omega\left(\frac{3H}{4}\right) = 50\times\left[\frac{3\cancel{H}/4}{\cancel{H}} - \frac{1}{2\pi}\sin\left(\frac{2\pi\times3\cancel{H}/4}{\cancel{H}}\right)\right] = \boxed{45.5\text{ cents}}$$

Part (b): To find the rate at which reactivity is introduced at the point in question per cm motion of the rod, we differentiate (7.63) with respect to x:

$$\frac{d}{dx}\rho_\omega\left(3H/4\right) = \frac{\rho_\omega(H)}{H}\left[1-\cos\left(\frac{2\pi\times 3H/4}{H}\right)\right] = \boxed{0.5\text{ cents/cm}}$$

■ P7.28

Part (a): Let H = 70 cm. In Example 7.7, we calculated $\rho_\omega(H/2)$ = 0.065. With reference to Figure 7.11, we have $\rho_\omega(H) = 2\times\rho_\omega(H/2) = 0.130$. We are looking for the reactivity associated with $H/2 + 10 = 45$ cm. Appealing to equation (7.63), we write

$$\rho_\omega(x) = \rho_\omega(H)\left[\frac{x}{H} - \frac{1}{2\pi}\sin\left(\frac{2\pi x}{H}\right)\right]$$

$$\therefore \rho_\omega(45) = \rho_\omega(H)\left[\frac{45}{70} - \frac{1}{2\pi}\sin\left(\frac{2\pi\times45}{70}\right)\right] = 0.0997 = \boxed{9.97\%}$$

Part (b): The period in question is given by (7.50),

$$T = \frac{1}{\rho}\sum_i \beta_i \bar{t}_i = \frac{1}{0.0997}\times0.0848 = \boxed{0.851\text{ sec}}$$

■ P7.30

Part (a): Taking η_T = 2.065 from Table 6.3, we compute the steady-state fuel utilization f:

$$k_\infty = \eta_T f \quad\rightarrow\quad f = \frac{k_\infty}{\eta_T}$$

$$\therefore f = \frac{1.0}{2.065} = 0.484$$

Referring to Table II.3, we can read Σ_a = 0.0220 cm^{-1} for water. It follows that

$$\Sigma_{aM} = \frac{\sqrt{\pi}}{2}\times0.0220 = 0.0195\ \text{cm}^{-1}$$

The corresponding macro absorption cross-section for fuel is

$$f = \frac{\Sigma_{aF}}{\Sigma_{aF}+\Sigma_{aM}} \quad\rightarrow\quad \Sigma_{aF} = \frac{f}{1-f}\Sigma_{aM}$$

$$\therefore \Sigma_{aF} = \frac{0.484}{1-0.484} \times 0.0195 = 0.0183 \text{ cm}^{-1}$$

If we have 20% more ^{235}U than is required to become critical, the fuel utilization becomes

$$f = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM}} = \frac{1.2 \times 0.0183}{1.2 \times 0.0183 + 0.0195} = 0.530$$

The corresponding infinite multiplication factor is

$$k_\infty = \eta_T f = 2.065 \times 0.530 = 1.094$$

and the worth ρ_ω is

$$\rho_\omega = \frac{k_\infty - 1}{k_\infty} = \frac{1.094-1}{1.094} = 0.0859$$

The concentration of boron required to sustain this system is determined as

$$C = \frac{\rho_\omega}{1.92(1-f)\times 10^{-3}} = \frac{0.0859}{1.92 \times (1-0.530)\times 10^{-3}} = \boxed{95.2 \text{ ppm}}$$

Part (b): The molar mass of boric acid is 3 × 1 + 1 × 10.8 + 3 × 16 = 61.8 g/mol. It follows that the concentration of H_3BO_3 required to sustain the system in focus is

$$c(H_3BO_3) = \frac{M(H_3BO_3)}{M(B)} \times 95.2 = \frac{61.8}{10.8} \times 95.2 = 545 \text{ ppm} = \boxed{0.545 \text{ g/L}}$$

■ P7.38

Referring to Table 7.8, we read $\lambda_X = 2.09\times10^{-5}$ sec^{-1}. With reference to Table II.3, we read σ_a = 24.5 b. From the fourth column of Table 3.2, we read g_a(800°C) = 0.9887. Substituting into the effective half-life formula given on page 379, we obtain

$$\left(T_{1/2}\right)_{\text{eff}} = \frac{0.693}{\lambda_X + \bar{\sigma}_{aX}\phi_T} = \frac{0.693}{\left(2.09\times10^{-5}\right) + \frac{\sqrt{\pi}}{2} \times 0.9887 \times 24.5 \times \left(\frac{293}{1073}\right)^{\frac{1}{2}} \times 10^{14}}$$

$$\therefore \left(T_{1/2}\right)_{\text{eff}} = 6.18\times10^{-16} \text{ s} = \boxed{0.618 \text{ fs}}$$

As discussed in the text, the half-life of ^{135}Xe is quite short.

■ P7.39

The micro cross-section of ^{135}Xe is 2.65×10^6 b (page 377) and $\lambda_X = 2.09\times10^{-5}$ (Table 7.6). It follows that

$$\phi_X = \frac{\lambda_X}{\phi_{aX}} = \frac{2.09\times10^{-5}}{\left(2.65\times10^6\right)\times10^{-24}} = 7.89\times10^{12}$$

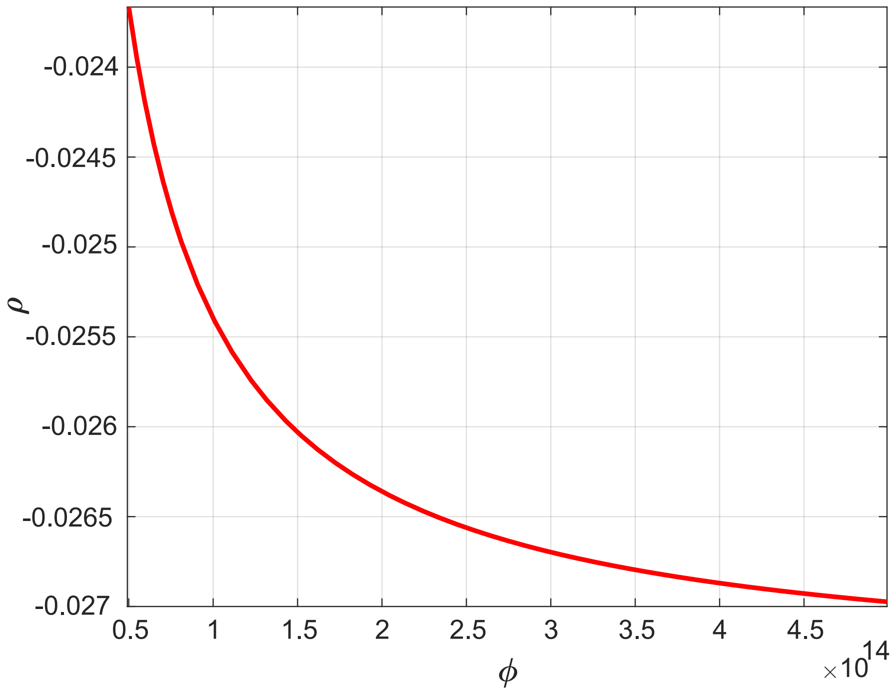
From Table 7.5, we have γ_I = 0.0639 and γ_X = 0.00237. For ^{235}U, ν = 2.42 and $p = \varepsilon$ = 1.0. With reference to equation (7.98), we write

$$\rho = -\frac{\gamma_I + \gamma_X}{\nu p \varepsilon}\frac{\phi_T}{\phi_X + \phi_T} = -\frac{0.0639 + 0.00237}{2.42 \times 1.0 \times 1.0} \times \frac{\phi_T}{7.89\times10^{12} + \phi_T}$$

$$\therefore \rho = -0.0274\frac{\phi_T}{7.89\times10^{12} + \phi_T}$$

This equation can be plotted for $\phi_T \in (5\times10^{12}, 5\times10^{14})$ with help of the following MATLAB code; for better visualization, I've chosen to truncate the lower bound at 5×10^{13} instead of 5×10^{12}:

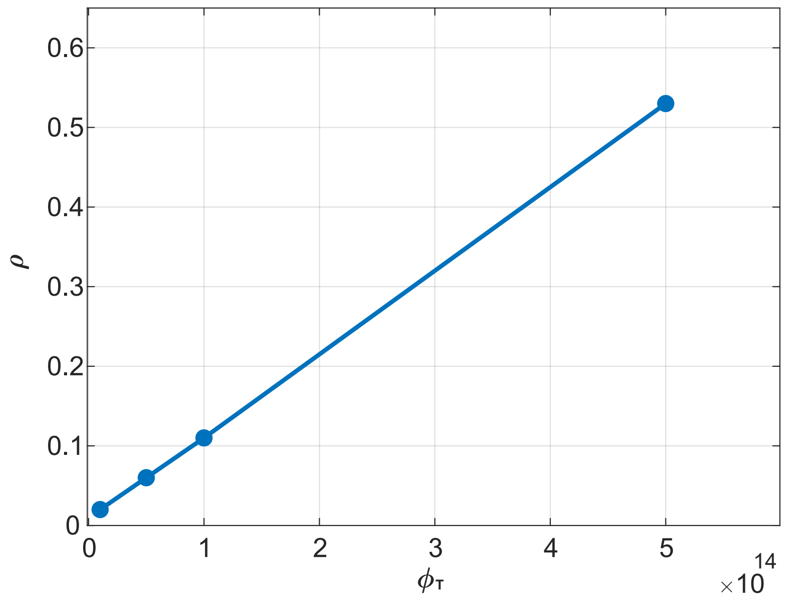
```
rho = @(phi) -0.0274*(phi/(7.89e12 + phi));
fplot(rho, [5e13, 5e14], 'LineWidth', 2, 'Color', 'red')
grid on
```



■ P7.40

This is basically an eyeballing exercise. Figure 7.14 has four curves describing negative reactivity as a function of time after shutdown. For ϕ_T = 10^{13} n/cm^2-sec, the negative reactivity peaks at ≈0.02. For ϕ_T = 5×10^{13}, the NR peaks at ≈0.06. For ϕ_T = 10^{14}, the NR peaks at ≈0.11. Finally, for ϕ_T = 5×10^{14} the NR peaks at ≈0.53. The four data points are plotted below; we see that the maximum xenon-135 buildup increases steadily with thermal flux.

```
phiT = [1e13, 5e13, 1e14, 5e14];
rho = [0.02, 0.06, 0.11, 0.53];
plot(phiT, rho, 'LineWidth', 2, 'Marker', '.', 'MarkerSize', 20)
xlim([0, 6e14]);
ylim([0, 0.65]);
grid on
```



■ P7.41

The reactivity equivalent of xenon is given by equation (7.103), which we repeat here for convenience:

$$\rho = -\frac{1}{\nu p\varepsilon}\left[\frac{(\gamma_I + \gamma_X)\phi_T}{\phi_X + \phi_T} e^{-\lambda_X t} + \frac{\gamma_I \phi_T}{\phi_I - \phi_X}\left(e^{-\lambda_X t} - e^{-\lambda_I t}\right)\right]$$

Here, we have ν = 2.42, $p\varepsilon$ = 1, $\gamma_I + \gamma_X$ = 0.0663 (Table 7.5), γ_I = 0.0639 (Table 7.5), ϕ_T = 5×10^{13} n/cm^2-sec (given), ϕ_X = 0.77×10^{13} n/cm^2-sec (equation (7.97)), ϕ_I = 1.055×10^{13} n/cm^2-sec (equation (7.104)), λ_X = 0.0753 hr^{-1} (Table 7.6), and λ_I = 0.1035 hr^{-1} (Table 7.6), so that

$$\rho = -\frac{1}{2.42\times 1.0}\times\left[\begin{array}{l}\dfrac{0.0663\times\left(5\times 10^{13}\right)}{0.77\times 10^{13}+5\times 10^{13}}e^{-0.0753\times t}\\+\dfrac{0.0639\times\left(5\times 10^{13}\right)}{\left(1.055\times 10^{13}\right)-\left(0.77\times 10^{13}\right)}\times\left(e^{-0.0753t}-e^{-0.1035t}\right)\end{array}\right]$$

$$\therefore \rho = 0.463e^{-0.1035t} - 0.487e^{-0.0755t}$$

Setting ρ = –0.05, we can solve for t with the following MATLAB code:

```
>> rho = @(t) 0.463*exp(-0.1035*t) - 0.487*exp(-0.0755*t) + 0.05;
t0 = 1;
fzero(rho, t0)

ans =
    3.1977
```

That is, $t \approx$ 3.20 h; this is the time to the onset of deadtime. The second nearest root of `rho` is $t \approx$ 19.59 h, which I found by inspecting the graph of `rho` and running *fzero* with an initial guess `t0` = 20. The duration of deadtime is then

$$\Delta t = 19.59 - 3.20 = \boxed{16.4\text{ h}}$$

■ P8.1

Referring to Table IV.1, we read saturated liquid enthalpies of 475.9 Btu/lb and 487.7 lb/hr for temperatures of 490°F and 500°F, respectively. Interpolating between these two values yields ≈484.2 Btu/lb, as shown in the following MATLAB code:

```
x = [490, 500];
v = [475.9, 487.7];
interp1(x, v, 497)

ans =
  484.1600
```

Proceeding similarly with a temperature of 519°F, we interpolate linearly between h(510) = 499.6 Btu/hr and h(520) = 511.7 Btu/hr to obtain h(519) ≈ 510.5 Btu/hr. Then, the power output of the reactor is determined as

$$\dot{q} = \dot{w}\left(h_{\text{out}} - h_{\text{in}}\right) = \left(9.4\times 10^{6}\right)\times\left(510.5-484.2\right) = 2.47\times 10^{8}\ \text{Btu/h}$$

$$\therefore \boxed{\dot{q} = 72.4\ \text{MW}}$$

where we have used 1 Btu/hr = 2.93×10^{-7} MW.

■ P8.2

With the appropriate unit conversions, we see that the sodium enters the core at 752°F and leaves at 1040°F. With reference to Table IV.5, we see that the enthalpy of sodium at 752°F is 381.4 Btu/lb. In turn, we can interpolate between $h(T = 932)$ = 436.0 Btu/lb and $h(T = 1112)$ = 490.1 Btu/lb to obtain $h(T = 1040)$ = 468.46 ≈ 468.5 Btu/lb. The reactor operates at 750 MW or 2.56×10^{9} Btu/hr. It remains to compute the rate $\dot{w}$ at which the sodium must be pumped:

$$\dot{q} = \dot{w}\left(h_{\text{out}} - h_{\text{in}}\right) \rightarrow \dot{w} = \frac{\dot{q}}{h_{\text{out}} - h_{\text{in}}}$$

$$\therefore \dot{w} = \frac{2.56\times 10^{9}}{468.5-381.4} = \boxed{2.94\times 10^{7}\ \text{lb/hr}}$$

■ P8.4

With reference to Table IV.1, interpolating between h_f(520°F) = 511.7 and h_f(530°F) = 523.9 Btu/lb gives h_{in} = 519.0 Btu/lb. Referring to Table IV.2, we interpolate between h_f(1000 psia) = 542.4 and h_f(1100 psia) = 557.4 Btu/lb to obtain h_f = 546.2 Btu/lb. From the same table, we interpolate between h_{fg}(1000 psia) = 650.0 and h_{fg}(1100 psia) = 631.0 Btu/lb to find h_{fg} = 645.3 Btu/lb. Noting that q = 1593 MW = 5.436×10^{9} Btu/hr, we can substitute the pertaining data into the formula derived in Problem 3 to yield

$$w_g = \frac{\dot{q} - w\left(h_f - h_{\text{in}}\right)}{h_{fg}} = \frac{5.436\times 10^{9} - \left(48\times 10^{6}\right)\times\left(546.2-519.0\right)}{645.3} = \boxed{6.40\times 10^{6}\ \text{lb/hr}}$$

P8.7

Part (a): Interpolating data from Table IV.1, we find h_{in}(496°F) = 483.0 Btu/lb. The reactor operates at 485 MWt or, equivalently, 1.64×10^9 Btu/hr, and the mass flow rate of water is 34×10^6 lb/hr. We can use this information to compute heat value h_{out}:

$$h_{\text{out}} = \frac{\dot{q}}{\dot{w}} + h_{\text{in}} = \frac{1.64\times10^9}{34\times10^6} + 483.0 = 531.2 \text{ Btu/lb}$$

With reference to Table IV.1, we glean that $T(h = 523.9)$ = 530°F and $T(h = 536.4)$ = 540°F. Interpolating between these two values gives $T(h = 531.2)$ = 535.8 ≈ 536°F. This is the average temperature of water leaving the core.

Part (b): The inner diameter of a fuel tube is a = 0.298 in.; the number of fuel tubes is n = 23,142; the height of the core is H = 91.9 in.: the power rating of the reactor is P = 485 MW = 485,000 kW; a conversion factor 1/(30.48 × 2) = 0.0164 must be included for dimensional homogeneity. We proceed to compute the average power density q'''_{avg}:

$$q'''_{\text{avg}} = \frac{\dot{q}}{0.0164\dfrac{\pi a^2}{4}Hn} = \frac{485{,}000}{0.0164\times\dfrac{\pi\times0.298^2}{4}\times91.9\times23{,}142} = \boxed{199.4 \text{ kW/liter}}$$

Part (c): Assuming that E_d = 180 MeV and E_R = 200 MeV,

$$q(0) = \frac{2.32PE_d}{nE_R} = \frac{2.32\times485\times180}{23{,}142\times200} = 0.0443 \text{ MW} = 151{,}200 \text{ Btu/hr}$$

The maximum heat production rate follows as

$$q''_{\text{max}} = \frac{q(0)}{2Ha^2} = \frac{151{,}200}{2\times(75.4/12)\times(0.298/2\times1/12)^2} = \boxed{1.56\times10^8 \text{ Btu/hr}\cdot\text{ft}^3}$$

P8.8

Part (a): The total number of fuel rods is 764 × 49 = 37,436. Noting that 1 in.^{3} = 5.79×10^{-4} ft^3, the total volume of fuel in the BWR is found as

$$V_F = 37{,}436\times\left[144\times\pi\times\left(\frac{0.487}{2}\right)^2\right]\times5.79\times10^{-4}\frac{\text{ft}^3}{\text{in}^3} = 581 \text{ ft}^3$$

or 1.65×10^7 cm^3. Multiplying this volume by the density of UO_2 gives the mass of uranium dioxide in the core:

$$m(\text{UO}_2) = \rho V_F = 10.3\times(1.65\times10^7) = \boxed{1.70\times10^8 \text{ g}} = 374{,}800 \text{ lb}$$

Assuming 3% enrichment, the average atomic mass of uranium may be taken as

$$M(\text{UO}_2) = \left(\frac{0.03}{235} + \frac{0.97}{238}\right)^{-1} = 237.91$$

so that the proportion of U in a given amount of UO_2 may be estimated as

$$\frac{M(\text{U})}{M(\text{UO}_2)} = \frac{237.91}{237.91 + 2\times16} = 0.881$$

giving

$$m(\text{U}) = 0.881\times(1.70\times10^8) = \boxed{1.5\times10^8 \text{ g}} = 331{,}000 \text{ lb}$$

Part (b): The specific power is given by

$$\Pi_s = \frac{3293\times10^3 \text{ kW}}{150{,}000 \text{ kg U}} = \boxed{22.0 \text{ kW/kg U}}$$

Part (c): The average power density is

$$q'''_{\text{avg}} = \frac{P}{V_F} = \frac{3293\times10^3 \text{ kW}}{1.65\times10^7 \text{ cm}^3}\times1000\frac{\text{cm}^3}{\text{liter}} = \boxed{200.0 \text{ kW/liter}}$$

Part (d): Noting that 175 in. = 14.6 ft and that there are 37,436 fuel rods, the average linear rod power becomes

$$q'_{r,\text{avg}} = \frac{3293\times10^3}{37{,}436\times14.6} = \boxed{6.02 \text{ kW/ft}}$$

Part (e): Appealing to equation (8.15) and noting that 1 kW ≈ 3412 Btu/hr,

$$q_r(0) = \frac{2.32PE_I}{nE_R} = \frac{2.32\times\left(3293\times10^3\right)\times180}{37{,}436\times200}\times3412 = 626{,}700 \text{ Btu/hr}$$

Finally,

$$q_r'''(0) = \frac{q_r(0)}{2Ha^2} = \frac{626{,}700}{2\times14.6\times(0.487/12)^2} = \boxed{1.30\times10^7 \text{ Btu/hr-ft}^3}$$

■ P8.9

Part (a): The reactor operates at 270,000 kW, or 9.21×10^8 Btu/hr. Also, recall that 1 in.^{3} = 0.000579 ft^3. The maximum heat production per unit volume is given by equation (8.12):

$$q'''_{\max} = \frac{1.16PE_d}{Ha^2nE_R} = \frac{1.16\times\left(9.21\times10^8\right)\times180}{30.5\times\left[(0.158/2)^2\times0.000579\right]\times13{,}104\times200} = \boxed{6.66\times10^8 \text{ Btu/hr-ft}^3}$$

Part (b): Taking $\rho_F(^{235}\text{U})$ = 19.1 g/cm^3 as the density of uranium-235 and noting that the fuel rods are enriched at 26 w/o, the number density of ^{235}U atoms becomes

$$N\left(^{235}\text{U}\right) = \frac{0.26\times19.1\times\left(6.02\times10^{23}\right)}{235} = 1.27\times10^{22} \text{ cm}^{-3}$$

The fission micro cross-section of ^{235}U is 582.2 b, and the corresponding macro CS is

$$\Sigma_{\text{fr}} = \left(1.27\times10^{22}\right)\times\left(582.2\times10^{-24}\right) = 7.39 \text{ cm}^{-1}$$

Taking g_f = 0.9309 by interpolating data from Table 3.2, we can correct the CS above for a temperature of 300°C (= 573 K):

$$\bar{\Sigma}_{\text{fr}} = \frac{\sqrt{\pi}}{2}\times0.9309\times7.39\times\left(\frac{293}{273+300}\right)^{1/2} = 4.36 \text{ cm}^{-1}$$

Finally, the maximum neutron flux is

$$\phi_{\max} = \frac{q'''_{\max}}{\bar{\Sigma}_{\text{fr}}\times E_d} = \frac{6.66\times10^8 \dfrac{\text{Btu}}{\text{hr-ft}^3}\times6.46\times10^7 \dfrac{\text{MeV/sec-cm}^3}{\text{Btu/hr-ft}^3}}{4.36\times180} = \boxed{5.48\times10^{13} \text{ n/cm}^3\text{-sec}}$$

■ P8.10

Let C denote the constant that multiplies $\cos(\pi z/\tilde{H})$ in the flux function we were given. It is immediately apparent that

$$\phi_{\max} = \phi(z=0) = C$$

The average flux, in turn, is determined as

$$\phi_{\text{avg}} = \frac{\int_{-H/2}^{H/2}\phi(z)\,dz}{\int_{-H/2}^{H/2}dz} = \frac{C\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{\tilde{H}}\right)dz}{H} \quad \text{(I)}$$

Writing down the integral in the numerator separately,

$$\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{\tilde{H}}\right)dz = \frac{\tilde{H}}{\pi}\sin\left(\frac{\pi z}{\tilde{H}}\right)\Bigg|_{-H/2}^{H/2} = \frac{2\tilde{H}}{\pi}\sin\left(\frac{\pi H}{2\tilde{H}}\right)$$

Substituting in (I),

$$\phi_{\text{avg}} = \frac{C\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{\tilde{H}}\right)dz}{H} = \frac{2C\tilde{H}}{\pi H}\sin\left(\frac{\pi H}{2\tilde{H}}\right)$$

Lastly, we have the ratio Ω_z:

$$\Omega_z = \frac{\phi_{max}}{\phi_{avg}} = \frac{\cancel{C}}{\frac{2\cancel{C}\tilde{H}}{\pi H}\sin\left(\frac{\pi H}{2\tilde{H}}\right)} = \boxed{\frac{\pi H/2\tilde{H}}{\sin\left(\pi H/2\tilde{H}\right)}}$$

■ P8.12

Part (a): We first integrate the power distribution over the reactor volume:

$$P = \int_V P(r,z)\,dV = 2\pi P_0 \int_{-H/2}^{H/2}\int_0^R \left[1-\left(\frac{r}{51}\right)^2\right]\cos\left(\frac{\pi z}{109}\right) r\,dr\,dz$$

Writing down the first integral separately,

$$2\pi P_0 \int_0^R \left[1-\left(\frac{r}{51}\right)^2\right] r\,dr = 2\pi P_0 \int_0^R \left(r-\frac{r^3}{51^2}\right)dr = \pi P_0\left(r^2-\frac{r^4}{2\times 51^2}\right)\Bigg|_0^R$$

$$\therefore 2\pi P_0 \int_0^R \left[1-\left(\frac{r}{51}\right)^2\right] r\,dr = \pi P_0\left(R^2-\frac{R^4}{2\times 51^2}\right)$$

$$\therefore 2\pi P_0 \int_0^R \left[1-\left(\frac{r}{51}\right)^2\right] r\,dr = \pi P_0\left(38.8^2-\frac{38.8^4}{2\times 51^2}\right) = 3361 P_0$$

Carrying out the second integration,

$$2\pi P_0 \int_{-H/2}^{H/2}\int_0^R \left[1-\left(\frac{r}{51}\right)^2\right]\cos\left(\frac{\pi z}{109}\right) r\,dr\,dz = 3361 P_0 \int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{109}\right)dz$$

$$\therefore 2\pi P_0 \int_{-H/2}^{H/2}\int_0^R \left[1-\left(\frac{r}{51}\right)^2\right]\cos\left(\frac{\pi z}{109}\right) r\,dr\,dz = 3361 P_0 \times \frac{109}{\pi}\sin\left(\frac{\pi z}{109}\right)\Bigg|_{-H/2}^{H/2}$$

$$\therefore 2\pi P_0 \int_{-H/2}^{H/2}\int_0^R \left[1-\left(\frac{r}{51}\right)^2\right]\cos\left(\frac{\pi z}{109}\right) r\,dr\,dz = 3361 P_0 \times \frac{218}{\pi}\sin\left(\frac{\pi H}{218}\right)$$

$$\therefore 2\pi P_0 \int_{-H/2}^{H/2}\int_0^R \left[1-\left(\frac{r}{51}\right)^2\right]\cos\left(\frac{\pi z}{109}\right) r\,dr\,dz = 3361 P_0 \times \frac{218}{\pi}\sin\left(\frac{\pi \times 77.5}{218}\right)$$

$$\therefore 2\pi P_0 \int_{-H/2}^{H/2}\int_0^R \left[1-\left(\frac{r}{51}\right)^2\right]\cos\left(\frac{\pi z}{109}\right) r\,dr\,dz = 209{,}600 P_0$$

Expressing P_0 in terms of total power P, we obtain

$$P = 209{,}600 P_0 \;\rightarrow\; P_0 = \frac{P}{209{,}600}$$

$$\therefore \boxed{P_0 = 4.77\times 10^{-6} P}$$

Part (b): The maximum power P_{max} occurs at the centroid of the cylindrical reactor, that is, at (r = 0, z = 0):

$$P_{max} = P(r=0, z=0) = P_0$$

The average power P_{avg} follows from the mean value theorem for integrals:

$$P_{avg} = \frac{1}{\pi R^2 H}\int_V P(r,z)\,dV$$

In part (a), we've established that the rightmost integral above equals 209,600P_0; therefore,

$$P_{max} = \frac{209{,}600 P_0}{\pi R^2 H} = \frac{209{,}600 \times P_0}{\pi \times 38.8^2 \times 77.5} = 0.572 P_0$$

The ratio that we're looking for is

$$\Omega = \frac{P_{max}}{P_{avg}} = \frac{P_0}{0.572 P_0} = \boxed{1.748}$$

Part (c): The radial component of power is given by

$$P_r(r) = P_{r,0}\left[1-\left(\frac{r}{51}\right)^2\right]$$

and clearly reaches a maximum such that $P_{r,max} = P_{r,0}$. The *average* radial component of power is given by the ratio

$$P_{r,\text{avg}}(r) = \frac{\int P(r)\,dA}{\int dA} = \frac{3361P_{r,0}}{\pi R^2} = \frac{3361P_{r,0}}{\pi \times 38.8^2} = 0.711P_{r,0}$$

so that

$$\Omega_r = \frac{P_{r,\max}}{P_{r,\text{avg}}} = \frac{\cancel{P_{r,0}}}{0.711\cancel{P_{r,0}}} = \boxed{1.406}$$

To find the corresponding ratio Ω_z for the axial direction, we can avoid more tedious integration by evoking the relationship $\Omega = \Omega_r\Omega_z$:

$$\Omega = \Omega_r\Omega_z \;\;\rightarrow\;\; \Omega_z = \frac{\Omega}{\Omega_r}$$

$$\therefore \Omega_z = \frac{1.748}{1.406} = \boxed{1.243}$$

■ P8.17

Part (a): The reactor produces 400 – 8 = 392 MW of power. The radius and height are 38.8 cm and 77.5 cm, respectively. We proceed to compute the average power density P_{avg}:

$$P_{\text{avg}} = \frac{1}{V}\int_V P(r,z)\,dV = \frac{1}{\pi R^2 H}\int_V P(r,z)\,dV$$

$$\therefore P_{\text{avg}} = \frac{1}{\pi \times 38.8^2 \times 77.5} \times 392 = 0.00107\text{ MW/cm}^3$$

Noting that 1 MW = 1000 kW and 1 ℓ = 1000 cm^3,

$$P_{\text{avg}} = 0.00107\frac{\text{MW}}{\text{cm}^3}\times 1000\frac{\text{kW}}{\text{MW}}\times 1000\frac{\text{cm}^3}{\text{liter}} = \boxed{1070\text{ kW}/\ell}$$

Noting that 1 liter = 0.0353 ft^3, it follows that

$$P_{\text{avg}} = 1070\frac{\text{kW}}{\ell}\times\frac{1}{0.0353}\frac{\ell}{\text{ft}^3} = \boxed{30{,}300\text{ kW/ft}^3}$$

Part (b): The average heat flux can be established by multiplying P_{avg}, which we determined in part (*a*), by the heat transfer length λ:

$$q''_{\text{avg}} = P_{\text{avg}}\lambda \;\;\text{(I)}$$

To find λ, we compare the cylinder volume with the product 1580 × λ, giving

$$\left(\pi \times 38.8^2 \times 77.5\right)\times\left(3.53\times10^{-5}\right) = 1580\times\lambda$$

$$\therefore \lambda = 0.00819\text{ ft}$$

where we have used 1 cm^3 ≈ 3.53×10^{-5} ft^3 in the first passage. Substituting in (I),

$$q''_{\text{avg}} = 30{,}300\times0.00819 = \boxed{248\text{ kW/ft}^2}$$

Part (c): To find the maximum heat flux, we first use our result from Part (*a*) of Problem 8.12 to estimate the maximum power P_{max}:

$$P_{\max} = 4.77\times10^{-6}P\pi R^2H = \left(4.77\times10^{-6}\right)\times392\times\pi\times38.8^2\times77.5 = 685\text{ MW}$$

Then, we divide $P_{\max}$ by the heat transfer area while noting that 1 ft^2 = 929 cm^2:

$$q''_{\max} = \frac{685}{1580\times929} = 4.67\times10^{-4}\text{ MW/cm}^2 = \boxed{0.467\text{ kW/cm}^2}$$

■ P8.19

The fuel rods have radius a = 0.158/2 = 0.079 in. and the cladding thickness is b = 0.005 in. The fuel rod length is H = 30.5 in = 2.54 ft. Referring to Table IV.6, we see that the thermal conductivity of uranium fuel at 1220°F can be obtained by interpolating between $k(T = 1200°\text{F}) = 21.20$ and $k(T = 1400°\text{F}) = 22.00$ to give

≈21.28 Btu/hr-ft-ºF. The thermal resistance of the fuel then becomes (equation 8.48)

$$R_f = \frac{1}{4\pi H k_f} = \frac{1}{4\pi \times 2.54 \times 21.28} = 1.47 \times 10^{-3} \text{ ºF-hr/Btu}$$

In turn, the thermal resistance of the cladding is given by equation (8.51); the thermal conductivity of steel, $k_c \approx$ 13.5 Btu/hr-ft-ºF, may be taken from Table IV.6:

$$R_c = \frac{\ln(1+b/a)}{2\pi H k_c} = \frac{\ln(1+0.005/0.079)}{2\pi \times 2.54 \times 13.5} = 2.85 \times 10^{-4} \text{ ºF-hr/Btu}$$

Using $q'''_{\max}$ = 6.66×10^8 Btu/hr-ft^3 as determined in Problem 8.9, we compute the power

$$q = \pi a^2 H \dot{q}''' = \pi \times \left(0.079 \times \frac{1}{12}\right)^2 \times 2.54 \times \left(6.66 \times 10^8\right) = 230{,}300 \text{ Btu/hr}$$

The temperature T_s at the fuel-cladding interface is

$$q = \frac{T_m - T_s}{R_f} \rightarrow T_s = T_m - qR_f$$

$$\therefore T_s = 1220 - 230{,}300 \times \left(1.47 \times 10^{-3}\right) = \boxed{882^\circ \text{F}}$$

In turn, the temperature T_c at the outer surface of the cladding is

$$q = \frac{T_m - T_s}{R_f + R_c} \rightarrow T_s = T_m - q\left(R_f + R_c\right)$$

$$\therefore T_s = 1220 - 230{,}300 \times \left[\left(1.47 \times 10^{-3}\right) + \left(2.85 \times 10^{-4}\right)\right] = \boxed{816^\circ \text{F}}$$