Montogue

Microeconomics | Quiz ECN1

Consumer Theory

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◆ Problem List

Problem	Title
1	Utility with No Budget Constraint
2	Budget Line
3	Constrained Choice and Utility Maximization I
4	Constrained Choice and Utility Maximization II
5	Another Constrained Choice Problem
6	Income Elasticities for Different Utility Functions
7	Income Elasticity of Demand and the Engel Curve I
8	Income Elasticity of Demand and the Engel Curve II
9	Price Consumption Curve
10	Elasticities
11	Cross-Elasticity of Demand
12	Loss of Individual Consumer Surplus I
13	Loss of Individual Consumer Surplus II
14	Consumer Risk Behavior I
15	Consumer Risk Behavior II

◆ Problems

Problem 1 (Modified from Nicholson and Snyder, 2016)

Utility with No Budget Constraint. Carlos consumes steaks (*s*) and beer cans (*b*), which afford him a utility *U* given by

$$U(s,b) = 24s - 2s^2 + 16b - b^2$$

(a) Assuming that the costs of *s* and *b* have no bearing on Carlos's choice, how many steaks and beer cans should he consume in order to maximize his utility? What is the corresponding maximum utility?
(b) Assume now that Carlos is on a diet, so that the sum of steaks and beer cans he consumes must be no greater than 8. How many units of each product does he consume under these circumstances?

Problem 2 (Modified from Nicholson and Snyder, 2016)

Budget Line. (a) Phoebe regularly buys beer cans (*b*) and wine bottles (*w*). If she spends her entire income on these two products, she can afford 30 beer cans and 6 wine bottles. Alternatively, she can trade her entire income for 12 beers and 10 wine bottles. A beer can costs \$2. Find the price of a wine bottle and Phoebe's income *Y*.
(b) Assume that Phoebe's utility function is $U(b,w) = b + w$. How many beer cans and wine bottles will she consume?

Problem 3 (Modified from Nicholson and Snyder, 2016)

Constrained Choice and Utility Maximization I. Anthony is an office worker and an avid consumer of burgers and milk-shakes. Each day, he spends his lunch break at the same restaurant, which sells burgers at \$2 each, shakes at \$1 each, and no other products. Burgers (*b*) and shakes (*s*) provide Anthony with a utility *U* such that

$$U(b,s) = (bs)^{1/2}$$

If Antony has $12 available for lunch, how should he spend his money so as to maximize his utility *U*?

Problem 4 (Modified from Nicholson and Snyder, 2016)

Constrained Choice and Utility Maximization II. Bernard is an avid reader looking for fiction books. He likes only Victorian-era novels (*v*) and Latin-American magic realism novels (*m*), and his utility function is

$$U(v, m) = v^{0.75} m^{0.25}$$

The typical price of *v* is $6, and a typical *m* costs $12. If Bernard has $240 to spend on books, what is his optimal bundle?

Problem 5

Another Constrained Choice Problem. Bernard, the intense reader described in Problem 4, likes to drink coffee (good *x*) with exactly one spoonful of sugar (good *y*). Bernie does not vary the 1:1 proportion of coffee and sugar. His preferences for the two goods are described by the utility function $U(x,y) = \min(x,y)$. Notice that the utility function is not differentiable at $x = y$. Bernie has an income of $5, the price of a cup of coffee is $0.40, and the price of a spoonful of sugar is $0.10.
(a) Find Bernie's optimal consumption bundle.
(b) Suppose that, due to new import taxes, the prices of coffee and sugar rise to $0.45 and $0.15, respectively. Find Bernie's updated consumption bundle. How much will Bernie be paying in taxes?
(c) Derive Bernie's demand functions for cups of coffee and spoonfuls of sugar. Are the goods normal or inferior, ordinary or Giffen, substitutes or complements?

Problem 6 (Modified from Perloff, 2017)

Income Elasticities for Different Utility Functions. Derive the income elasticity of demand for individuals with the following utility functions:
(a) Cobb-Douglas.
(b) Perfect substitutes.
(c) Perfect complements.

Problem 7 (Modified from Perloff, 2017)

Income Elasticity of Demand and the Engel Curve I. A consumer faces prices for hot dogs and hamburgers at $1 each. Consumption of the two commodities at three different income levels are tabulated below.

Income	Hot dogs	Hamburgers
$20	8	12
$25	11	14
$30	15	15

(a) Use the information to sketch the income consumption curve.
(b) Sketch Engel curves for hot dogs and hamburgers.
(c) What is the income elasticity of hot dogs for this consumer as income increases from $25 to $30?

Problem 8 (Modified from Salvatore, 2006)

Income Elasticity of Demand and the Engel Curve II. A family's annual consumption of poultry meat varies with income as shown in the following table.

Income ($/year)	3800	6100	8000	10,200	12,000	14,000	16,000	18,000
Quantity (kg/year)	45	90	135	150	160	180	160	90

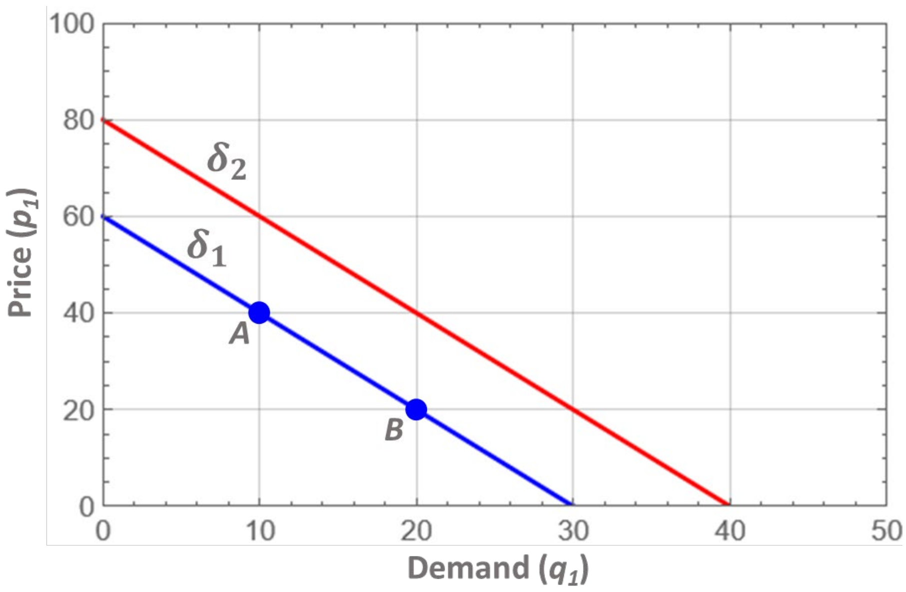
(a) Find the income elasticity of demand of this family for poultry meat at various successive levels of income. Over what range of income is poultry meat a luxury, a necessity, or an inferior good?
(b) Plot the Engel curve for poultry meat in this family.

Problem 9 (Modified from Perloff, 2017)

Price Consumption Curve. Andrew, a teenager, usually spends his allowance on detective novels and board games. To analyze his budget-constrained product choices, draw a graph with detective novels on the vertical axis and board games on the horizontal axis. Andrew is given \$200 a month by his parents. In Andrew's economy, the price of detective novels is fixed at \$5 per book, whereas the price of board games is somewhat volatile. When the price of a typical board game is \$5, Andrew purchases 16 board games and 24 detective novels. When the price of a typical board game rises to \$10, Andrew purchases 10 board games and 20 detective novels. When the price of a typical board game rises to \$20, Andrew purchases 7 board games and 12 detective novels.
(a) Draw the utility-maximizing points and sketch the price-consumption curve.
(b) Draw the individual demand curve for board games.
(c) Use the information provided to calculate Andrew's elasticity of demand for board games between \$5 and \$10, and between \$10 and \$20.

Problem 10

Elasticities. In the graph shown in continuation, δ_1 and δ_2 are demand curves for good I.
(a) Calculate the price elasticity of the good at point *A* and point *B*. Do you get the same answer at both points? Why or why not?
(b) If income increases by 20%, demand curve δ_1 shifts to δ_2. Calculate an approximate value for income elasticity at point *A*.
(c) Suppose that the price of good I is \$10, and that an increase of 10% in the price of good I caused the demand of another good, good II, to increase by 25%. Estimate the cross-price elasticity for good II. Is good II a substitute good or a complementary good relatively to good I?



Problem 11 (Modified from Salvatore, 2006)

Cross-Elasticity of Demand. The following table describes a consumer's demand for three goods – pencils (*x*), pens (*y*), and erasers (*z*) – at different prices. 'Before' and 'after' denote the consumer's change in demand after the price of one of the goods was changed. Estimate the cross-elasticity of demand between pencils and pens and between pencils and erasers; also state whether the goods are substitutes or complements.

Product	Before		After	
	Price (\$/unit)	Quantity (units/month)	Price (\$/unit)	Quantity (units/month)
Pens (*y*)	3.0	50	2.0	60
Pencils (*x*)	1.0	20	1.0	15
Erasers (*z*)	1.50	18	2.0	16
Pencils (*x*)	1.0	20	1.0	19

Problem 12 (Modified from Serrano and Feldman, 2018)

Loss of Individual Consumer Surplus I. A consumer has utility function $U(x_1, x_2) = x_1 x_2$. The consumer has budget M = \$18; the prices of goods 1 and 2 are initially p_1 = \$1 and p_2 = \$1, but p_1 eventually rises to \$2.25. What is the loss in consumer surplus incurred by this price increase?

Problem 13

Loss of Individual Consumer Surplus II. August has utility function $U(x,y) = 10x - (1/3)x^3 + y$, where x and y are the quantities of two goods that she buys regularly. In the following exercises, assume that the price of good y is $p_y = \$1$ throughout and that August has a budget M of at least $12.

(a) What is August's demand function for good x? How many units of good 1 does she demand when the prices are $p_x = p_y = \$1$?

(b) What is August's inverse demand function for x, $p_x(x)$? What is her consumer surplus?

(c) Suppose that the price of x rises to $p_x = \$6$, while p_y is unchanged at $1. How many units of x does she demand now? What is her new consumer surplus?

Problem 14 (Modified from Pindyck and Rubinfeld, 2015)

Consumer Risk Behavior I. Suppose that Jimmy's utility function is given by $U(I) = \sqrt{10I}$, where I represents annual income in thousands of dollars.

(a) Is Jimmy risk averse, risk neutral, or risk loving?

(b) Suppose that Jimmy is currently earning an income of $40,000 and can earn that income next year with certainty. Jimmy is offered a new job that can pay $44,000 with a probability of 0.6 or $33,000 with a probability of 0.4. In view of his behavior with respect to risk, will Jimmy take the new job?

(c) In (b), would Jimmy be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? [**Hint:** *What is the risk premium?*]

Problem 15 (Modified from Pindyck and Rubinfeld, 2015)

Consumer Risk Behavior II. Suppose that two investments have the same three payoffs, but the probabilities associated with each payoff differ, as indicated in the following table.

Payoff	Probability (Investment A)	Probability (Investment B)
$300	0.10	0.30
$250	0.80	0.40
$200	0.10	0.30

(a) Find the expected return and standard deviation of each investment.

(b) Alfred has utility function $U(I) = 5I$, where I denotes the payoff. Which investment will he choose?

(c) Beto has utility function $U(I) = 5\sqrt{I}$, where I denotes the payoff. Which investment will he choose?

(d) Gammo has utility function $U(I) = 5I^2$. Which investment will he choose?

◆ Solutions

■ Problem 1

Part (a): The first-order conditions are

$$\frac{\partial U}{\partial s} = 24 - 4s = 0$$

$$\frac{\partial U}{\partial b} = 16 - 2b = 0$$

Solving the first equation gives $s = 6$; solving the second equation gives $b = 8$. Thus, Carlos should consume 6 steaks and 8 beer cans. The corresponding utility afforded by this number of products is

$$U(s=6, b=8) = 24\times 6 - 2\times 6^2 + 16\times 8 - 8^2 = \boxed{136}$$

Part (b): We are now in a constrained optimization scenario. The Lagrangian is

$$L(s,b,\lambda) = 24s - 2s^2 + 16b - b^2 + \lambda(8 - b - s)$$

The first-order conditions are

$$\frac{\partial U}{\partial s} = 24 - 4s - \lambda = 0$$

$$\therefore 24 - 4s = \lambda \quad \text{(I)}$$

$$\frac{\partial U}{\partial b} = 16 - 2b - \lambda = 0$$

$$\therefore 16 - 2b = \lambda \ \ \text{(II)}$$

$$\frac{\partial U}{\partial \lambda} = 8 - b - s = 0$$

$$\therefore b + s = 8 \ \ \text{(III)}$$

Equating (I) and (II), we have

$$24 - 4s = 16 - 2b$$
$$\therefore 12 - 2s = 8 - b$$
$$\therefore b = 2s - 4 \ \ \text{(IV)}$$

Substituting in (III),

$$b + s = 8 \ \rightarrow \ (2s - 4) + s = 8$$
$$\therefore 3s = 12$$
$$\therefore s = \boxed{4}$$

Substituting in (IV),

$$b = 2s - 4 = 2 \times 4 - 4 = \boxed{4}$$

Thus, Carlos should consume 4 steaks and 4 beer cans. The corresponding utility afforded by this number of products is

$$U(s = 4, b = 4) = 24 \times 4 - 2 \times 4^2 + 16 \times 4 - 4^2 = \boxed{112}$$

■ Problem 2

Part (a): Phoebe's budget constraint is $p_b b + p_w w = Y$, where Y is her income. Substituting the two given data points brings to

$$p_b b + p_w w = Y \ \rightarrow \ 2 \times 30 + p_w \times 6 = Y$$
$$\therefore 60 + 6p_w = Y \ \ \text{(I)}$$

$$p_b b + p_w w = Y \ \rightarrow \ 2 \times 12 + p_w \times 10 = Y$$
$$\therefore 24 + 10p_w = Y \ \ \text{(II)}$$

Equating (I) and (II) and solving for p_w, we obtain

$$60 + 6p_w = 24 + 10p_w$$

$$\therefore 36 = 4p_w$$

$$\therefore p_w = \frac{36}{4} = \boxed{\$9.0}$$

A wine bottle costs \$9. Substituting in either (I) or (II) gives the missing income Y,

$$60 + 6p_w = Y$$
$$\therefore Y = 60 + 6 \times 9.0 = \boxed{\$114}$$

Phoebe has an income of \$114.

Part (b): The given utility function indicates that, for Phoebe, beer cans and wine bottles are perfect substitutes in a 1:1 ratio. Accordingly, Phoebe will spend her entire income on the cheaper good, that is, she will buy \$114/\$2 = 57 beer cans and zero wine bottles.

■ Problem 3

Solution 1: Lagrange multipliers. To find the maximum utility for a fixed budget, we first set up the Lagrangian L,

$$L(b, s, \lambda) = b^{1/2}s^{1/2} + \lambda(12 - 2.0b - 1.0s)$$

The first-order conditions are

$$\frac{\partial L}{\partial b} = \frac{1}{2}\left(\frac{s}{b}\right)^{1/2} - 2.0\lambda = 0$$

$$\therefore \frac{1}{2}\left(\frac{s}{b}\right)^{1/2} = 2.0\lambda \ \ \text{(I)}$$

$$\frac{\partial L}{\partial s} = \frac{1}{2}\left(\frac{b}{s}\right)^{1/2} - 1.0\lambda = 0$$

$$\therefore \frac{1}{2}\left(\frac{b}{s}\right)^{1/2} = 1.0\lambda \ \ \text{(II)}$$

$$\frac{\partial L}{\partial \lambda} = 12 - 2.0b - 1.0s = 0$$

$$\therefore 2.0b + 1.0s = 10 \ \ \text{(III)}$$

Dividing (I) by (II),

$$\frac{\frac{1}{2}\left(\frac{s}{b}\right)^{1/2}}{\frac{1}{2}\left(\frac{b}{s}\right)^{1/2}} = \frac{2.0\cancel{\lambda}}{1.0\cancel{\lambda}} = 2.0$$

$$\therefore \frac{s}{b} = 2.0$$

$$\therefore s = 2b$$

Substituting in (III),

$$2.0b + 1.0s = 12 \ \rightarrow \ 2.0b + 1.0 \times (2b) = 12$$

$$\therefore 4b = 12$$

$$\therefore b = \frac{12}{4} = 3$$

In turn, $s = 2b = 2 \times 3 = 6$. The maximized utility is

$$U(b = 3, s = 6) = (3 \times 6)^{1/2} = \boxed{\sqrt{18}}$$

Anthony should buy 3 burgers and 6 shakes for a maximum utility of $\sqrt{18}$ utils.

Solution 2: Marginal utilities. Alternatively, we could equate the ratio of marginal utilities to the ratio of prices, giving

$$\frac{MU_b}{MU_s} = \frac{p_b}{p_s} \ \rightarrow \ \frac{(1/2)b^{-0.5}s^{0.5}}{(1/2)b^{0.5}s^{-0.5}} = \frac{2.0}{1.0}$$

$$\therefore \frac{s}{b} = \frac{2.0}{1.0}$$

$$\therefore s = 2.0b$$

Substituting in the budget constraint equation,

$$2.0b + 1.0s = 12 \ \rightarrow \ 2.0b + 1.0 \times (2b) = 12$$

$$\therefore 4b = 12$$

$$\therefore b = \frac{12}{4} = 3$$

and $s = 2.0 \times 3 = 6$. Needless to say, the solution is identical to the one obtained via Lagrange multipliers.

■ Problem 4

Solution 1: Lagrange multipliers. To find the maximum utility for a fixed budget, we first set up the Lagrangian L,

$$L(v, m, \lambda) = v^{0.75}m^{0.25} + \lambda(240 - 6v - 12m)$$

The first-order conditions are

$$\frac{\partial L}{\partial v} = 0.75\left(\frac{m}{v}\right)^{0.25} - 6.0\lambda = 0$$

$$\therefore 0.75\left(\frac{m}{v}\right)^{0.25} = 6.0\lambda \ \ \text{(I)}$$

$$\frac{\partial L}{\partial m} = 0.25\left(\frac{v}{m}\right)^{0.75} - 12.0\lambda = 0$$

$$\therefore 0.25\left(\frac{v}{m}\right)^{0.75} = 12.0\lambda \ \ \text{(II)}$$

$$\frac{\partial L}{\partial \lambda} = 240 - 6.0v - 12.0m = 0$$

$$\therefore 6.0v + 12.0m = 240 \ \ \text{(III)}$$

Dividing (I) by (II),

$$\frac{0.75\left(\frac{m}{v}\right)^{0.25}}{0.25\left(\frac{v}{m}\right)^{0.75}} = \frac{6.0\cancel{\lambda}}{12.0\cancel{\lambda}} = \frac{1}{2}$$

$$\therefore 3.0\frac{m}{v} = 0.5$$

$$\therefore m = \frac{v}{6} \ \ \text{(IV)}$$

Substituting in (III),

$$6.0v + 12.0m = 240 \ \rightarrow \ 6.0v + 12.0 \times \frac{v}{6} = 240$$

$$\therefore 6.0v + 2v = 240$$

$$\therefore v = \frac{240}{8} = \boxed{30}$$

Using (IV),

$$m = \frac{v}{6} = \frac{30}{6} = \boxed{5}$$

The maximized utility is

$$U\left(v = 30, m = 5\right) = 30^{0.75} \times 5^{0.25} \approx \boxed{19.2}$$

Bernie should purchase 30 Victorian-era novels and 5 magic realism novels for a maximum utility of 19.2 utils.

Solution 2: Lagrange multipliers. Another way to proceed is to equate the ratio of marginal utilities to the ratio of prices, giving

$$\frac{MU_v}{MU_m} = \frac{p_v}{p_m} \ \rightarrow \ \frac{(3/4)v^{-0.25}m^{0.25}}{(1/4)v^{0.75}m^{-0.75}} = \frac{6.0}{12.0}$$

$$\therefore 3.0\frac{m}{v} = \frac{1}{2}$$

$$\therefore m = \frac{v}{6}$$

Substituting in the budget constraint equation,

$$6.0v + 12.0m = 240 \ \rightarrow \ 6.0v + 12.0 \times \frac{v}{6} = 240$$

$$\therefore 6.0v + 2.0v = 240$$

$$\therefore v = \frac{240}{8.0} = 30$$

and m = 30/6 = 5. Of course, the solution is identical to the one obtained via Lagrange multipliers.

■ Problem 5

Part (a): The budget constraint is $0.40x + 0.10y = 5.0$. Since Bernie consumes coffee and sugar at equal proportions, we have $x = y$ and

$$0.40x + 0.10y = 5.0 \ \rightarrow \ 0.40x + 0.10x = 5.0$$

$$\therefore 0.50x = 5.0$$

$$\therefore \boxed{x = 10.0}$$

Thus, $x = y = 10.0$.

Part (b): Bernie's budget constraint is now $0.45x + 0.15y = 5.0$, giving

$$0.45x + 0.15y = 5.0 \quad \rightarrow \quad 0.60x = 5.0$$

$$\therefore x = \frac{5.0}{0.60} \approx 8.33 = \boxed{\frac{25}{3}}$$

As the price of coffee rose from \$0.40 to \$0.45, we have a tax of \$0.05. Similarly, as the price of spoonfuls of sugar rose from \$0.10 to \$0.15, a tax of \$0.05 is implied. Thus, Bernie is paying a tax of (25/3) × (\$0.05 + \$0.05) ≈ \$0.83.

Part (c): The demand functions are

$$x = y = \frac{M}{p_x + p_y}$$

The goods are normal (higher income results in higher consumption), ordinary (higher price results in lower consumption), and complements of one another (higher price of one results in lower consumption of the other).

Problem 6

Part (a): The income elasticity of demand (or income elasticity) is the percentage change in quantity demanded in response to a given percentage change in income, *Y*,

$$\xi = \frac{Y}{q_1}\frac{dq_1}{dY} \quad \text{(I)}$$

For a Cobb-Douglas utility function,

$$U\left(q_1, q_2\right) = q_1^{\alpha} q_2^{1-\alpha}$$

the demand for q_1 is given by

$$q_1 = \frac{\left(1-\alpha\right)Y}{p_1}$$

where p_1 is the price of good 1. Differentiating with respect to income,

$$\frac{dq_1}{dY} = \frac{\left(1-\alpha\right)}{p_1}$$

Substituting in (I),

$$\xi = \frac{Y}{q_1}\frac{dq_1}{dY} = \frac{Y}{\left[\frac{\left(1-\alpha\right)Y}{p_1}\right]} \times \frac{\left(1-\alpha\right)}{p_1} = \boxed{1.0}$$

Part (b): The utility function for perfect substitutes can be stated as

$$U\left(q_1, q_2\right) = aq_1 + bq_2 \ \ , \ a, b > 0$$

Assume the price of good 1 is less than the price of good 2. In this case, the consumer will maximize their utility at a corner solution, consuming only good 1. Then, the demand function becomes simply the consumer's income Y divided by the price of good 1,

$$q_1 = \frac{Y}{p_1}$$

Differentiating with respect to income,

$$\frac{dq_1}{dY} = \frac{1}{p_1}$$

Substituting in (I),

$$\xi = \frac{Y}{q_1}\frac{dq_1}{dY} = \frac{Y}{q_1} \times \frac{1}{p_1} = \boxed{\frac{Y}{p_1 q_1}}$$

Part (c): The utility function for perfect complements may be stated as

$$U\left(q_1, q_2\right) = \min\left(aq_1, bq_2\right) \ \ , \ a, b > 0$$

In this case, individuals will consume perfect complements in fixed proportions, *a* units of q_1 to *b* units of q_2. The demand for q_1 is

$$q_1 = \frac{Y}{p_1 + \frac{a}{b}p_2}$$

Differentiating with respect to income,

$$\frac{dq_1}{dY} = \frac{1}{p_1 + \frac{a}{b}p_2}$$

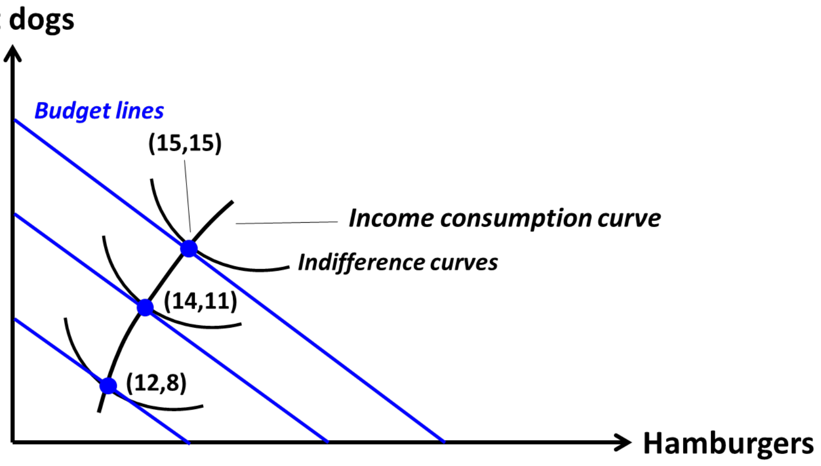
Substituting in (I),

$$\xi = \frac{Y}{q_1}\frac{dq_1}{dY} = \frac{Y}{\left(\frac{Y}{p_1 + \frac{a}{b}p_2}\right)} \times \frac{1}{p_1 + \frac{a}{b}p_2} = \boxed{1}$$

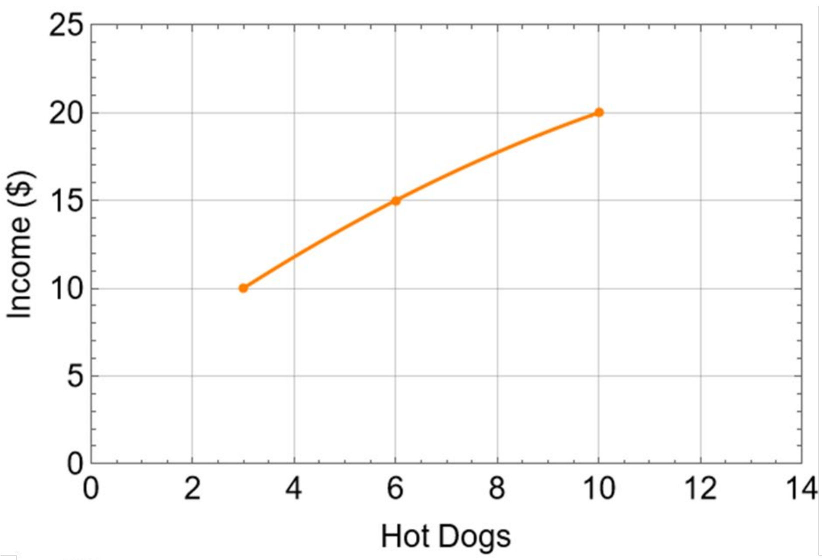
As in the case of the Cobb-Douglas utility function, the income elasticity of demand for perfect complements is equal to 1.

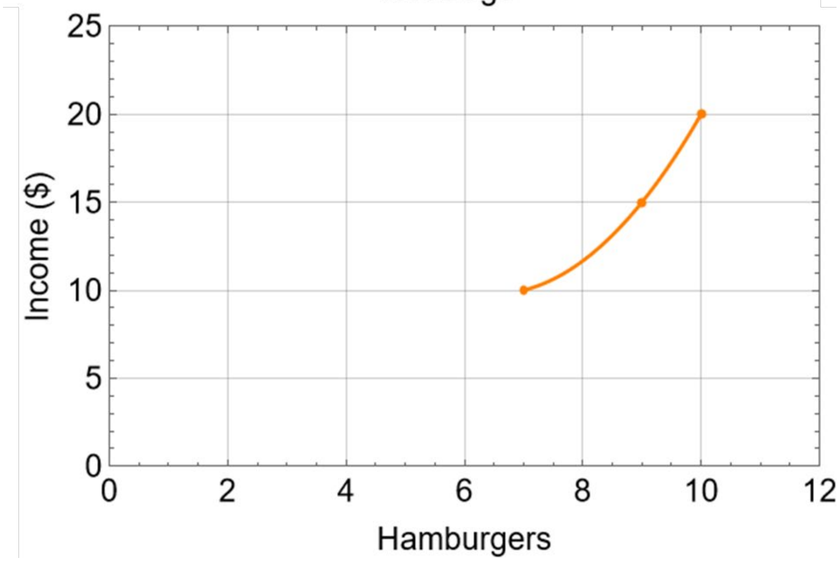
■ Problem 7

Part (a): The income consumption curve is sketched to the side.



Part (b): An Engel curve is simply a function of income versus quantity demanded. The Engel curves for hot dogs and burgers are shown below.





Part (c): The income elasticity of hot dogs as income is raised from \$25 to \$30 can be estimated as

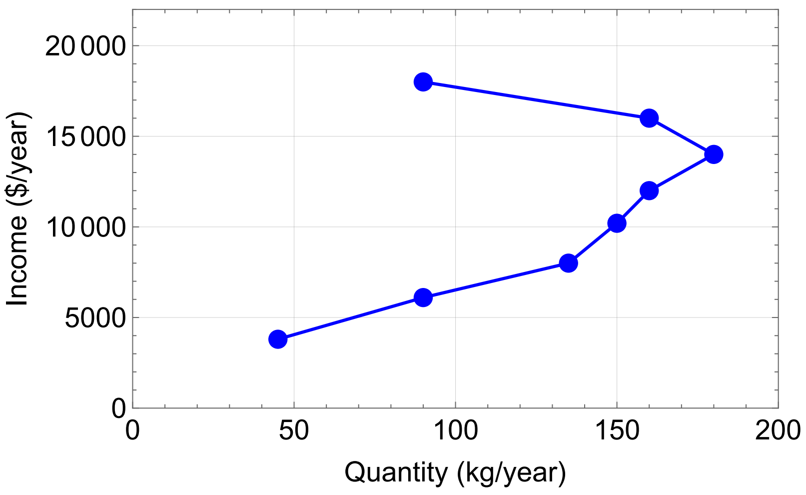
$$\xi_Y \approx \frac{Y}{q}\frac{\Delta q}{\Delta Y} = \frac{25}{11} \times \frac{15-11}{30-25} = \boxed{1.82}$$

Problem 8

Part (a): The income elasticity ξ_P calculations are summarized below. The rightmost column indicates whether the good is a necessity ($0 < \xi_P < 1$), a luxury ($\xi_P \geq 1$), or an inferior good ($\xi_P < 0$).

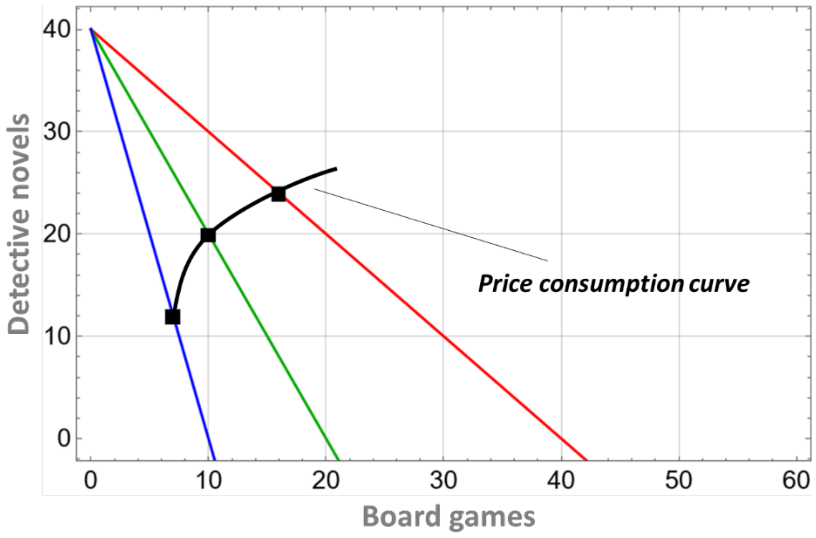
Income (\$/year)	Quantity (kg/year)	Percent change in quantity	Percent change in income	Income elasticity	Type of good
3800	45				
6100	90	100.00%	60.53%	1.65	**Luxury**
8000	135	50.00%	31.15%	1.61	**Luxury**
10200	150	11.11%	27.50%	0.40	**Necessity**
12000	160	6.67%	17.65%	0.38	**Necessity**
14000	180	12.50%	16.67%	0.75	**Necessity**
16000	160	-11.11%	14.29%	-0.78	**Inferior**
18000	90	-43.75%	12.50%	-3.50	**Inferior**

Part (b): The Engel curve is a plot of income (first column) versus quantity (second column).

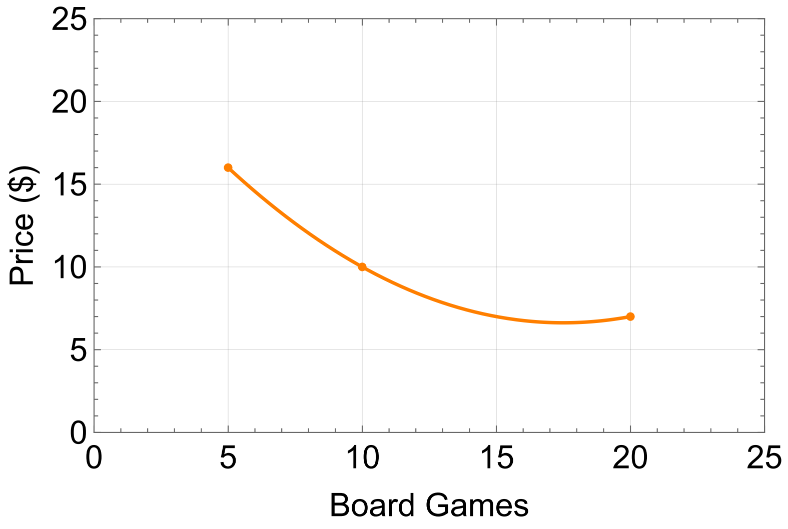


Problem 9

Part (a): The utility-maximizing points are the black squares. The red, green, and blue lines are budget lines for board games priced at \$5, \$10 and \$20, respectively. The black curve is the price consumption curve.



Part (b): The individual demand curve for board games can be outlined using the three specified data points.



Part (c): As the price of board games rises from \$5 to \$10, Andrew's elasticity of demand is estimated as

$$\xi_p \approx \frac{p}{q}\frac{\Delta q}{\Delta p} = \frac{5}{16} \times \frac{10-16}{10-5} = \boxed{-0.375}$$

In turn, as the price of board games rises from \$10 to \$20, Andrew's EOD is

$$\xi_p \approx \frac{p}{q}\frac{\Delta q}{\Delta p} = \frac{10}{10} \times \frac{7-10}{20-10} = \boxed{-0.3}$$

■ Problem 10

Part (a): The price elasticity of good I is given by

$$\xi_p = \frac{p_1}{q_1}\frac{dq_1}{dp_1}$$

The derivative dq_1/dp_1 for a straight line such as δ_1 is constant and in this case equals, using points *A* and *B* as references,

$$\frac{dq_1}{dp_1} = -\frac{20-10}{40-20} = -0.5$$

Then, to calculate the price elasticities at points *A* and *B*, we substitute in the formula for ξ_P to obtain, respectively,

$$\left(\xi_p\right)_A = \frac{40}{10} \times (-0.5) = \boxed{-2.0}$$

$$\left(\xi_p\right)_B = \frac{20}{20} \times (-0.5) = \boxed{-0.5}$$

Even though the two points lie on the same line, the price elasticities are not the same because the proportion p_1/q_1 changes depending on the line we measure it.
Part (b): Since income increases by 20%, we have $\Delta Y/Y = 0.2$. As the demand curve shifts from δ_1 to δ_2, we can draw a horizontal line through point *A* and verify that the number of units of good A demanded at a price of \$40 rises from 10 to 20. It follows that $\Delta q_1/q_1 = (20 - 10)/10 = 1.0$. Thus, we can approximate the income elasticity as

$$\xi_Y \approx \frac{Y}{q_1}\frac{\Delta q_1}{\Delta Y} = \frac{\Delta q_1/q_1}{\Delta Y/Y} = \frac{1.0}{0.2} = \boxed{5.0}$$

Part (c): Good I had a price increase $\Delta p_1/p_1 = 0.1$, and this caused the demand for good II to increase by 25%, hence $\Delta q_2/q_2 = 0.25$. The cross-price elasticity then becomes

$$\xi_{1\to 2} \approx \frac{p_1}{q_2}\frac{\Delta q_2}{\Delta p_1} = \frac{\Delta q_2/q_2}{\Delta p_1/p_1} = \frac{+0.25}{0.1} = \boxed{+2.5}$$

Since a price increase in good I increased the demand for good II, the cross-price elasticity is positive and good II is a substitute good relatively to good I.

■ Problem 11

The cross-elasticity of demand for goods *x* (pencils) and *y* (pens) is

$$\xi_{xy} \approx \frac{\Delta q_x/q_x}{\Delta p_y/p_y} = \frac{(15-20)/20}{(2.0-3.0)/3.0} = \boxed{0.75}$$

In turn, the cross-elasticity of demand for goods *x* (pencils) and *z* (erasers) is

$$\xi_{xz} \approx \frac{\Delta q_x/q_x}{\Delta p_z/p_z} = \frac{(19-20)/20}{(2.0-1.5)/1.5} = \boxed{-0.15}$$

Since ξ_{xy} is positive, we surmise that pencils and pens are substitutes for the consumer in question. Since ξ_{xz} is negative, we surmise that erasers and pencils are complements for the consumer in question.

■ Problem 12

The demand function for good 1 is $x_1 = M/(2p_1)$, which can be inverted to yield the inverse demand function $p_1 = M/(2x_1)$. When $p_1 = \$1$, the number of units of good 1 demanded by the consumer is

$$(x_1)_i = \frac{18}{2\times 1.0} = 9$$

In turn, when p_1 = \$2.25, the updated demand is

$$(x_1)_f = \frac{18}{2\times 2.25} = 4$$

The loss in consumer surplus is obtained by integrating the inverse demand function over the range defined by the foregoing values of x,

$$\text{Loss in consumer's surplus} = \int_{(x_1)_f}^{(x_1)_i} p_1(x_1)\,dx_1$$

$$\therefore \text{Loss in consumer's surplus} = \int_4^9 \frac{18}{2x_1}dx_1 = \int_4^9 \frac{9}{x_1}dx_1$$

$$\therefore \int_4^9 \frac{9}{x_1}dx_1 = 9\ln\left(\frac{9}{4}\right) \approx \boxed{7.30}$$

■ Problem 13

Part (a): The marginal rate of substitution is

$$MRS = \frac{U_x}{U_y} = \frac{p_x}{p_y} \rightarrow \frac{10-x^2}{1} = \frac{p_x}{1.0}$$

Solving for x gives the demand function for good x, namely

$$10 - x^2 = p_x \quad (*)$$
$$\therefore 10 - p_x = x^2$$
$$\therefore x = \sqrt{10 - p_x}$$

When p_x = \$1, we obtain

$$x = \sqrt{10 - 1.0} = \boxed{3}$$

Part (b): As highlighted in (*) of part (a), the inverse demand function for good x is $p_x(x) = 10 - x^2$. When p_x = \$1 and August is consuming 3 units of x, her surplus from the consumption of x is

$$\Delta = \int_{x=0}^{x=3} p_x(x)\,dx - p_x x = \int_{x=0}^{x=3}\left[10 - x^2\right]dx - 1.0\times 3$$

$$\therefore \Delta = \left(10x - \frac{x^3}{3}\right)\Bigg|_{x=0}^{x=3} - 3.0$$

$$\therefore \Delta = (30 - 9) - 3.0 = \boxed{18.0}$$

Part (c): When the price of x rises to \$6 while p_y remains at \$1, August will consume $x = \sqrt{10-6} = 2$ units of x. (This will cost 2 × \$6 = \$12, which is why we postulated that August has a budget $M \geq 12$). August's surplus from the consumption of x is now

$$\Delta = \int_{x=0}^{x=2} p_x(x)\,dx - p_x x = \int_{x=0}^{x=2}\left[10 - x^2\right]dx - 6.0\times 2$$

$$\therefore \Delta = \left(10x - \frac{x^3}{3}\right)\Bigg|_{x=0}^{x=2} - 12.0$$

$$\therefore \Delta = \left(20 - \frac{8}{3}\right) - 12.0 = \boxed{\frac{16}{3}}$$

The price increase led August to consume one less unit of good x and lose nearly 70% of her original surplus.

■ Problem 14

Part (a): Taking the first derivative of utility with respect to income,

$$U(I) = \sqrt{10I} \rightarrow \frac{dU}{dI} = \frac{\sqrt{10}}{2}I^{-1/2}$$

Taking the second derivative,

$$\frac{dU}{dI}=\frac{\sqrt{10}}{2}I^{-1/2} \quad \rightarrow \quad \frac{d^2U}{dI^2}=-\frac{\sqrt{10}}{4}I^{-3/2}$$

Since $U''(I) < 0 \; \forall \; I > 0$, function $U(I)$ exhibits diminishing marginal utility and Jimmy can be considered risk averse.

Part (b): The utility of Jimmy's current salary is $U(I = 40) = \sqrt{10 \times 40} = 20$. The expected utility of the job Jimmy was offered is

$$E[U]=0.6\times\sqrt{10\times 44}+0.4\times\sqrt{10\times 33}=19.85$$

which is less than 20. Therefore, Jimmy will not take the new job.

Part (c): This part of the problem assumes that Jimmy takes the new job for some unexplained reason. His expected salary $E[S]$ is

$$E[S]=0.6\times 44{,}000+0.4\times 33{,}000=\$39{,}600$$

The risk premium is the amount Jimmy would be willing to pay so that he receives the expected salary for certain rather than the risky salary in his new job. As calculated in part (b), the utility associated with the new salary is 19.85. Setting U = 19.85 and solving for income, we obtain

$$U(I)=\sqrt{10I} \quad \rightarrow \quad 19.85=\sqrt{10I}$$

$$\therefore I=\frac{19.85^2}{10}=39.40$$

Therefore, Jimmy would be equally satisfied with a certain salary of $39,400 or an uncertain salary with an expected value of $39,600. His risk premium is $39,600 – $39,400 = $200. Jimmy would be willing to pay $200 so as to guarantee his income would be $39,600 for certain and eliminate the risk associated with the new job.

■ Problem 15

Part (a): The expected value of the return on investment A is

$$E[V]=0.1\times 300+0.80\times 250+0.10\times 200=\$250$$

The variance on investment A is

$$\text{Var}[V]=0.1\times(300-250)^2+0.8\times(250-250)^2+0.1\times(200-250)^2=\$500$$

The expected value of the return on investment B is

$$E[V]=0.1\times 300+0.80\times 250+0.10\times 200=\$250$$

The variance on investment B is

$$\text{Var}[V]=0.3\times(300-250)^2+0.4\times(250-250)^2+0.3\times(200-250)^2=\$1500$$

Part (b): Alfred's expected utility from investment A is

$$E_A[U]=0.10\times[5\times(300)]+0.80\times[5\times(250)]+0.10\times[5\times(200)]=1250$$

Alfred's expected utility from investment B is

$$E_B[U]=0.30\times[5\times(300)]+0.40\times[5\times(250)]+0.30\times[5\times(200)]=1250$$

Since both investments yield the same expected utility, Alfred will be indifferent between the two. Note that Alfred is risk neutral, so he only cares about expected values.

Part (c): Beto's expected utility from investment A is

$$E_A[U]=0.10\times\left[5\times\sqrt{300}\right]+0.80\times\left[5\times\sqrt{250}\right]+0.10\times\left[5\times\sqrt{200}\right]=78.98$$

Beto's expected utility from investment B is

$$E_B[U]=0.30\times\left[5\times\sqrt{300}\right]+0.40\times\left[5\times\sqrt{250}\right]+0.30\times\left[5\times\sqrt{200}\right]=78.82$$

Beto will choose investment A, because it yields slightly greater expected utility. Note that Beto is risk averse, so he prefers the investment with less variability.

Part (d): Gammo's expected utility from investment A is

$$E_A[U] = 0.10 \times \left[5 \times (300)^2\right] + 0.80 \times \left[5 \times (250)^2\right] + 0.10 \times \left[5 \times (200)^2\right] = 315{,}000$$

Gammo's expected utility from investment B is

$$E_B[U] = 0.30 \times \left[5 \times (300)^2\right] + 0.40 \times \left[5 \times (250)^2\right] + 0.30 \times \left[5 \times (200)^2\right] = 320{,}000$$

Gammo will choose investment B because it yields greater expected utility. Note that Gammo is a risk lover, so he prefers the investment with greater variability.

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