Microeconomics and Game Theory

◆30 Practice Questions

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Here's a set of 30 fully solved problems on basic microeconomics and game theory. The problems were taken from a carefully researched assortment of textbooks. All problems are solved step by step. Enjoy! ■

PROBLEMS

Problem 1. A consumer has budget M = \$18; the prices of goods 1 and 2 are initially p_1 = \$1 and p_2 = \$1, but p_1 eventually rises to \$2.25. The consumer has utility function $U(x_1, x_2) = x_1x_2$. Most nearly, what is the loss in consumer surplus incurred by the price increase?

(A) 5.1
(B) 6.3
(C) 7.3
(D) 8.1

Problems 2 to 4. Bernard likes to drink coffee (good x) with exactly one spoonful of sugar (good y). Bernie does not vary the 1:1 proportion of coffee and sugar. His preferences for the two goods are described by the utility function $U(x,y) = min(x,y)$. Notice that the utility function is not differentiable at $x = y$. Bernie has an income of \$5, the price of a cup of coffee is \$0.40, and the price of a spoonful of sugar is \$0.10.

2. Find Bernie's optimal consumption bundle.
(A) $x = 8$; $y = 8$
(B) $x = 8$; $y = 10$
(C) $x = 10$; $y = 8$
(D) $x = 10$; $y = 10$

3. Suppose that, due to new import taxes, the prices of coffee and sugar rise to \$0.45 and \$0.15, respectively. Find Bernie's updated consumption bundle. Relatively to the tax-free consumption bundle of the previous problem, how much will Bernie be paying in taxes?
(A) \$0.53
(B) \$0.64
(C) \$0.83
(D) \$0.98

4. Derive Bernie's demand functions for cups of coffee and spoonfuls of sugar. Are the goods normal or inferior, ordinary or Giffen, substitutes or complements?
(A) Normal, ordinary, complements.
(B) Normal, ordinary, substitutes.
(C) Normal, Giffen, substitutes.
(D) Inferior, ordinary, complements.

Problem 5. If a 10% increase in the price of good B leads to a 20% decrease in the quantity demand of good A, then
(A) The cross-price elasticity is –0.5 and the goods are complements.
(B) The cross-price elasticity is –0.5 and the goods are substitutes.
(C) The cross-price elasticity is –2 and the goods are complements.
(D) The cross-price elasticity is –2 and the goods are substitutes.

Problem 6. In a certain year, consumers in a certain country bought 9 million TVs with combined value of \$10.8 billion and 15 million laptops for a total of \$17.2 billion. Suppose that the average consumer has a Cobb-Douglas utility function and buys these two goods only. If q_1 and q_2 denote TVs and laptops, respectively, and A is a positive constant, which of the following is a likely utility function for the average consumer in the market at hand?
(A) $U(q_1,q_2) = Aq_1^{0.386}q_2^{0.614}$
(B) $U(q_1,q_2) = Aq_1^{0.614}q_2^{0.386}$
(C) $U(q_1,q_2) = Aq_1^{0.332}q_2^{0.667}$
(D) $U(q_1,q_2) = Aq_1^{0.667}q_2^{0.332}$

Problem 7. Suppose that two investments have the same three payoffs, but the probabilities associated with each payoff differ as indicated in the following table.

Payoff	Probability (Investment A)	Probability (Investment B)
\$300	0.10	0.30
\$250	0.80	0.40
\$200	0.10	0.30

Alfred has utility function $U(I) = 5\sqrt{I}$ and Beto has utility function $U(I) = 5I^2$, where I denotes investment payoff. In view of their risk aversion behavior, which investment would Alfred and Beto choose?
(A) Both Alfred and Beto would choose investment A.
(B) Alfred would choose investment A and Beto would choose investment B.
(C) Alfred would choose investment B and Beto would choose investment A.
(D) Both Alfred and Beto would choose investment B.

Problem 8. Sarah has a quasilinear utility function $U = Y^{0.5} + 2N$ and income $Y = wH$, where N is hours of leisure, Y is other consumption, w is her wage and H is hours of work. What is the slope of her labor supply curve with respect to a change in wage?
(A) 1/2
(B) 1/4
(C) 1/8
(D) 1/16

Problem 9. The following normal-form bimatrix shows the possible payoffs for two competing firms that can decide to set their product prices as high or low. In each cell, payoffs to the left and right refer to firms 1 and 2, respectively. Which of the following statements is correct?

		Firm 2: High	Firm 2: Low
Firm 1	High	(\$60,\$45)	(\$50,\$35)
	Low	(\$40,\$10)	(\$15,\$20)

(A) Firm 1's dominant strategy is to price low.
(B) Firm 1's dominant strategy is to price high.
(C) Firm 2's dominant strategy is to price low.
(D) Firm 2's dominant strategy is to price high.

Problem 10. Suppose the production function for the automotive and parts industry is $q = L^{0.29}K^{0.18}M^{0.62}$, where L is labor, K is capital, and M is energy and materials. Similarly, the production function for the aerospace industry may be taken as $q = L^{0.24}K^{0.19}M^{0.54}$. What kind of returns to scale do the two production functions exhibit? (**IRS**: Increasing returns to scale; **DRS**: Decreasing returns to scale)

(A) Automotive: IRS; Aerospace: IRS
(B) Automotive: IRS; Aerospace: DRS
(C) Automotive: DRS; Aerospace: IRS
(D) Automotive: DRS; Aerospace: DRS

Problem 11. A perfectly competitive market is constituted of three types of firms; each type of firm operates with a different cost function, as follows:

$$\begin{cases} \text{Type 1 firms: } C_1(q_1) = 2q_1^2 + 242 \\ \text{Type 2 firms: } C_2(q_2) = 3q_2^2 + 192 \\ \text{Type 3 firms: } C_3(q_3) = 4q_3^2 + 100 \end{cases}$$

The market demand function for the good is $Q = 1200 - 3p$. In the short run, there are 24 type 1 firms, 24 type 2 firms, and 16 type 3 firms. What is the equilibrium price of the good?

(A) \$50
(B) \$60
(C) \$75
(D) \$80

Problem 12. A monopolist faces a demand curve given by $Q = 98 - p$, where Q denotes aggregate output and p denotes price. Assume that the monopolist has a cost structure given by $C(Q) = 0.3Q^2 - 6Q + 300$. What are the output Q deployed by the monopolist and the corresponding profits π?

(A) $Q = 40$; $\pi = \$1540$
(B) $Q = 45$; $\pi = \$1540$
(C) $Q = 40$; $\pi = \$1780$
(D) $Q = 45$; $\pi = \$1780$

Problem 13. Consider a typical Stackelberg duopoly in which the leader firm sets its output before the follower firm. Both produce at the same marginal cost c, have no fixed costs, and face the same inverse demand function $p = a - bQ$. What is the ratio of the leader firm's output q_L to the follower firm's output q_F?

(A) $q_L/q_F = 1$
(B) $q_L/q_F = 3/2$
(C) $q_L/q_F = 2$
(D) $q_L/q_F = 3$

Problem 14. How many mixed Nash equilibria are there in the following game?

		Player 2: L	Player 2: R
Player 1	T	(0,1)	(6,0)
	M	(2,0)	(5,2)
	B	(3,3)	(3,4)

(A) 0
(B) 1
(C) 2
(D) 3

Problems 15 and 16. Doctoral candidates Delton and Epson must choose a professor to advise them in their research. The available professors are Dr. Watson, Dr. Young, and Dr. Zhao. We assign (somewhat arbitrary) utilities according to the amount of help each professor provides to the student: Dr. Watson, a rather poor adviser, gives 40 utils; Dr. Young, a superb adviser, gives 60 utils; and Dr. Zhao, a decent adviser, gives 50 utils. If Delton and Epson happen to choose the same adviser, the professor will struggle to help both students at the same time and the utility they yield will be reduced to 70% of the aforementioned values.

15. Let δ (delta) and ε (epsilon) denote Delton and Epson, respectively. Further, let the three professors be denoted by their respective initials. Which of the following is a normal-form bimatrix representation of the game at hand?

(A)

δ \ ε	W	Y	Z
W	(28,40)	(40,60)	(40,50)
Y	(60,40)	(60,42)	(60,50)
Z	(40,50)	(50,60)	(50,35)

(B)

δ \ ε	W	Y	Z
W	(28,28)	(60,40)	(60,50)
Y	(40,60)	(42,42)	(40,50)
Z	(50,60)	(50,40)	(35,35)

(C)

δ \ ε	W	Y	Z
W	(28,28)	(60,40)	(50,40)
Y	(40,60)	(42,42)	(50,60)
Z	(40,50)	(60,50)	(35,35)

(D)

δ \ ε	W	Y	Z
W	(28,28)	(40,60)	(40,50)
Y	(60,40)	(42,42)	(60,50)
Z	(50,40)	(50,60)	(35,35)

16. Which of the following is a mixed-strategy Nash equilibrium for this game?

(A) $((\delta_W,\delta_Y,\delta_Z),(\varepsilon_W,\varepsilon_Y,\varepsilon_Z)) = ((0,\frac{25}{33},\frac{8}{33}),(0,\frac{25}{33},\frac{8}{33}))$

(B) $((\delta_W,\delta_Y,\delta_Z),(\varepsilon_W,\varepsilon_Y,\varepsilon_Z)) = ((0,\frac{7}{11},\frac{12}{33}),(0,\frac{7}{11},\frac{12}{33}))$

(C) $((\delta_W,\delta_Y,\delta_Z),(\varepsilon_W,\varepsilon_Y,\varepsilon_Z)) = ((\frac{1}{4},\frac{3}{8},\frac{3}{8}),(\frac{1}{4},\frac{3}{8},\frac{3}{8}))$

(D) $((\delta_W,\delta_Y,\delta_Z),(\varepsilon_W,\varepsilon_Y,\varepsilon_Z)) = ((\frac{1}{5},\frac{3}{5},\frac{1}{5}),(\frac{1}{5},\frac{1}{5},\frac{3}{5}))$

Problem 17. In the following three-player game, each player has two strategies: player I may choose either *T* or *B*; player II may choose either ℓ or *r*; and player III may choose either *L* or *R*. The triples inside the bimatrices indicate the payoffs to players I, II, and III, in that order. The only pure-strategy Nash equilibrium in this game is **(__,__,__)** (*Write your answer within the parentheses*).

III: *L*

I \ II	ℓ	*r*
T	(3,4,4)	(1,3,3)
B	(8,1,4)	(2,0,6)

III: *R*

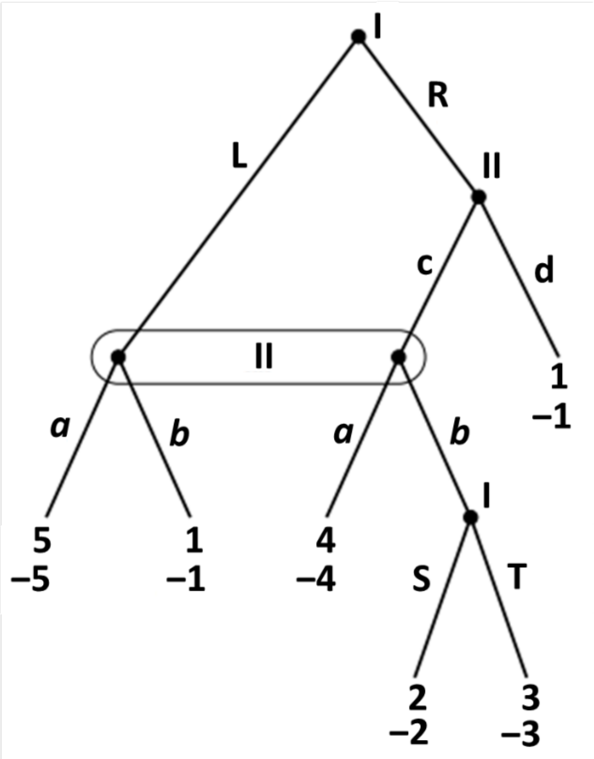
I \ II	ℓ	*r*
T	(4,0,5)	(0,1,6)
B	(5,1,3)	(1,2,5)

Problem 18. What is the value (that is, the positive equilibrium payoff) of the following zero-sum game?

I \ II	X	Y
A	(9,−9)	(−5,5)
B	(2,−2)	(7,−7)
C	(8,−8)	(−1,1)

(A) 0
(B) 2
(C) 3
(D) 7

Problem 19. In the following extensive game,



(A) Both players have perfect recall.
(B) Only player I has perfect recall.
(C) Only player II has perfect recall.
(D) Neither player has perfect recall.

Problems 20 and 21. Two political candidates are scheduled to campaign in two states, one in period $t = 1$ and the other in period $t = 2$. In each state they can either choose a positive campaign that promotes their own agenda (*P* for player 1, *p* for player 2) or a negative one that attacks their opponent (*N* for player 1, *n* for player 2). Residents of the first period state do not mind negative campaigns, which are generally effective, and payoffs in this state are given by the following bimatrix:

Player 1 \ Player 2	*p*	*n*
P	(2,2)	(0,5)
N	(5,0)	(3,3)

In the second-period state, residents dislike negative campaigns despite their effectiveness and the payoffs are given by the following bimatrix:

Player 1 \ Player 2	*p*	*n*
P	(6,6)	(1,0)
N	(0,1)	(2,2)

20. For the case of extreme discounting (that is, discount factor $\delta = 0$), how many subgame perfect equilibria are there in this multi-stage game?
(A) 5
(B) 8
(C) 16
(D) 32
21. Now, with $\delta = 1$, show yourself that there is a subgame perfect equilibrium in which the players choose (*P*,*p*) in the first stage game. What is the lowest value of δ for which this subgame perfect equilibrium survives into the second stage?
(A) 2/5
(B) 1/2
(C) 3/5
(D) 3/4

Problem 22. Consider an instance of the cooperative Gloves Game with three players; one player possesses a left glove and each of the two other players has a right glove. This game's core is:
(A) An empty set.
(B) A point.
(C) Two points.
(D) A triangle.

Problem 23. Consider a three-player coalitional game $\langle N, v\rangle$ where $N = \{1,2,3\}$ is the set of players and v is the coalitional function described by

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$$
$$v(\{1,2\}) = 4 \quad ; \quad v(\{1,3\}) = 9 \quad ; \quad v(\{2,3\}) = 16$$
$$v(\{1,2,3\}) = 21$$

Which of the following is **not** a Shapley value associated with one of the three players?
(A) 23/6
(B) 22/3
(C) 25/3
(D) 59/6

Problem 24. Consider the following bargaining problem. A "unit pie" is split into non-negative amounts x and y with $x + y \leq 1$. The utility function of player I is $u(x) = x$, and the utility function of player II is $v(y) = 1 - (1 - y)^2$. The threat point is (0,0). Find the Nash bargaining solution for this problem. What are the utilities of the players at the Nash bargaining solution?
(A) $u = 1/\sqrt{3}$; $v = 1/3$
(B) $u = 1/\sqrt{3}$; $v = 2/3$
(C) $u = 2/\sqrt{3}$; $v = 1/3$
(D) $u = 2/\sqrt{3}$; $v = 2/3$

Problem 25. Consider an evolutionary game with n pure strategies and stage game payoff π_{ij} to an i-player who meets a j-player. If $p = (p_1, p_2, \ldots, p_n)$ is the frequency of each type in the population, the expected payoff to an i-player is then $\pi_i(p) = \sum_{j=1}^{n} p_j \pi_{ij}$, and the average payoff in the game is $\bar{\pi} = \sum_{j=1}^{n} p_j \pi_i(p)$. The replicator dynamic for this game is then given by

$$\dot{p}_i = p_i\left(\pi_i(p) - \bar{\pi}(p)\right)$$

I. If p^* is a Nash equilibrium of the stage game, p^* is a fixed point of the replicator dynamic.
II. If p^* is not a Nash equilibrium of the stage game, then p^* is not an evolutionary equilibrium.
III. If p^* is an evolutionarily stable strategy (ESS) of the stage game, then p^* is an asymptotically stable equilibrium of the replicator dynamic. Conversely, if a point is an asymptotically stable equilibrium in a symmetric game, then the point is an ESS.

Which of the above statements are true?
(A) I and II.
(B) II and III.
(C) I and III.
(D) I, II and III.

Problems 26 to 30 are open-ended.

Problem 26. Production Function and Two Plants. An entrepreneur purchases two factories to produce widgets. Each factory produces identical products, and each has a production function given by

$$q_i\left(k,l\right)=\sqrt{k_i l_i} \quad ; \quad i=1,2$$

where k is capital input and l is labor input. The factories differ, however, in the amount of capital equipment each has. In particular, factory 1 has $k_1 = 49$, whereas factory 2 has $k_2 = 196$. Rental rates for k and l are given by $w = v = \$1$.

(a) If the entrepreneur wishes to minimize short-run total costs of widget construction, how should output be allocated between the two factories? **(60%)**

(b) Given that output is optimally allocated between the two factories, calculate the short-run total, average, and marginal cost curves. What is the marginal cost of the 500th widget? **(20%)**

(c) How should the entrepreneur allocate widget production between the two factories in the long run? **(10%)**

(d) How would your answer to part (c) change if both factories exhibited diminishing returns to scale? **(10%)**

Problem 27. Advertising and Product Quality. A firm produces a single product whose quality is either high or low (the firm knows the quality but cannot choose it) and sells it to consumers in each of two periods. The marginal cost of production is 4 if quality is high and 3 if quality is low. In each period there are N consumers, each of whom is interested in buying at most one unit in each period and is willing to pay 10 if quality is high and 5 if quality is low. However, consumers cannot tell the product's quality before they consume it in period 1. Suppose that the firm can advertise its product on TV in period 1. Although advertising itself does not convey direct information about the product's quality, it can serve as a signal – consumers might be able to infer the product's quality from the fact that the firm was willing to spend money on advertising. Suppose that the cost of a TV ad is A if quality is low and αA if quality is high, where $\alpha < 1$ (e.g., it is cheaper to design an ad for a high-quality product). The intertemporal discount factor is δ.

(a) What is the minimum amount of TV ads that the firm needs to sponsor in order to signal that its product quality is high? **(60%)**

(b) How does your answer depend on the discount factor δ? How does it depend on α? How does it depend on N? Explain your answer in detail. **(40%)**

Problem 28. Cournot Duopoly with Asymmetric Costs. In a two-firm Cournot duopoly, two firms face the same demand curve $p = a - bQ$. Assume that firm 1 faces marginal cost c_1 and firm 2 faces marginal cost c_2, where $c_1 < c_2$ (so firm 1 enjoys a cost advantage relative to firm 2). Also, $a > c_2$.

(a) Find the best response functions of firms 1 and 2. Compare them. **(15%)**

(b) Find the Nash equilibrium of this Cournot game of quantity competition. Which firm produces a larger output? **(40%)**

(c) Find equilibrium price and equilibrium profits for each firm. Which firm earns a larger profit? **(30%)**

(d) Assume that both firms have now become cost-symmetric, so that $c_1 = c_2 = c$. Show that the Nash equilibrium now simplifies to a symmetric Cournot equilibrium. **(15%)**

Problem 29. Oligopolies and Collusion. Industries A and B can be characterized by a series of parameters: n, the number of firms, is 8 in both industries; r, the annual interest rate, is 10% in both industries; f, the frequency with which the firms interact (number of times per year), is 1 in industry A and 12 in industry B; g, the industry growth rate, is 10% in industry A and –30% in industry B; finally, h, the likelihood that the industry will continue in existence into the next period, is 80% in industry A and 100% in industry B.

(a) In which of the two industries do you think tacit collusion in more likely to take place? Justify your answer. **(50%)**

(b) Consider an n firm homogeneous-good oligopoly with constant marginal cost c, the same for all firms. Let $\bar{\delta}$ be the minimum value of the discount factor such that it is possible to sustain monopoly prices in a collusive agreement. Show that $\bar{\delta}$ is decreasing in n. Interpret the result. **(25%)**

(c) Consider a price-setting oligopoly with n firms, all with constant marginal cost c. For a discount factor of 0.8, what is the maximum number of firms such that there exists equilibrium with monopoly pricing? **(25%)**

Problem 30. Entry and Competition. Consider an industry for a homogeneous product with a single firm (firm 1) that can produce at zero cost. The demand function in the industry is given by $Q = a - p$, where $a > 0$. Now suppose that a second firm (firm 2) considers entry into the industry. Firm 2 can also produce at zero cost. If firm 2 enters, firms 1 and 2 compete by setting prices. Consumers buy from the firm that sets the lowest price. If both firms charge the same prices, consumers buy from firm 1.

(a) Solve for the Nash equilibrium if firm 2 chooses to enter the industry. Would firm 2 wish to enter if entry required some initial investment? **(25%)**

(b) Now suppose that before it enters, firm 2 can choose a capacity, x_2, and a price p_2 (the capacity x_2 means that firm 2 can produce no more than $q_2 = x_2$ units). Given q_2 and p_2, firm 1 chooses its price and then consumers decide who to buy from. Compute the Nash equilibrium in the product market if firm 1 chooses to fight firm 2. What is firm 1's profit in this case? Would firm 2 choose to produce in that case? **(15%)**

(c) Now suppose that firm 1 decides to accommodate the entry of firm 2. Compute the residual demand that firm 1 faces after firm 2 sells q_2 units, and then write the maximization problem of firm 1 and solve it for p_1. What is firm 1's profit if it decides to accommodate firm 2's entry? Would firm 2 wish to enter in this case? **(15%)**

(d) Given your answers to (b) and (c), compute for each p_2 the largest capacity that firm 2 can choose without inducing firm 1 to fight it. (**Hint**: to answer the question you need to solve a quadratic equation. The solution is given by the small root). **(25%)**

(e) Show that the capacity you computed in (d) is decreasing with p_2. Explain the intuition for your answer. Given your answer, explain how firm 2 will choose its price. Computing p_2 is too complicated; you are just asked to explain in words how firm 2 chooses p_2. **(20%)**

ANSWER KEY

Problem	Answer	Problem	Answer
1	C	14	B
2	D	15	D
3	C	16	A
4	A	17	(B,ℓ,L)
5	C	18	C
6	A	19	B
7	B	20	C
8	D	21	D
9	B	22	B
10	B	23	C
11	D	24	B
12	C	25	A
13	C		

SOLUTIONS

1 ➡ C

The demand function for good 1 is $x_1 = M/(2p_1)$, which can be inverted to yield the inverse demand function $p_1 = M/(2x_1)$. When p_1 = \$1, the number of units of good 1 demanded by the consumer is

$$(x_1)_i = \frac{18}{2\times 1.0} = 9$$

In turn, when p_1 = \$2.25, the updated demand is

$$(x_1)_f = \frac{18}{2\times 2.25} = 4$$

The loss in consumer surplus is obtained by integrating the inverse demand function from $(x_1)_i$ to $(x_1)_f$,

$$\text{Loss in consumer surplus} = \int_{(x_1)_f}^{(x_1)_i} p_1(x_1)\,dx_1$$

$$\therefore \text{Loss in consumer surplus} = \int_4^9 \frac{18}{2x_1}dx_1 = \int_4^9 \frac{9}{x_1}dx_1$$

$$\therefore \int_4^9 \frac{9}{x_1}dx_1 = 9\ln\left(\frac{9}{4}\right) \approx \boxed{7.30}$$

2 ➡ D

Bernie's budget constraint is $0.40x + 0.10y = 5.0$. Since Bernie consumes coffee and sugar at equal proportions, we have $x = y$ and

$$0.40x + 0.10y = 5.0 \quad \rightarrow \quad 0.40x + 0.10x = 5.0$$

$$\therefore 0.50x = 5.0$$

$$\therefore \boxed{x = 10}$$

Thus, $x = y = 10$.

3 ➡ C

Bernie's budget constraint is now $0.45x + 0.15y = 5.0$, giving

$$0.45x + 0.15y = 5.0 \quad \rightarrow \quad 0.60x = 5.0$$

$$\therefore x = \frac{5.0}{0.60} = \boxed{\frac{25}{3}}$$

As the price of coffee rose from \$0.40 to \$0.45, we have a tax of \$0.05. Similarly, as the price of spoonfuls of sugar rose from \$0.10 to \$0.15, a tax of \$0.05 is implied. Thus, Bernie is paying a tax of (25/3) × (\$0.05 + \$0.05) ≈ \$0.83.

4 ➡ A

The demand functions are

$$x = y = \frac{M}{p_x + p_y}$$

The goods are normal (higher income results in higher consumption), ordinary (higher price results in lower consumption), and complements of one another (higher price of one results in lower consumption of the other).

5 ➡ C

The cross-price elasticity of demand is given by

$$XED = \frac{\%\Delta Q_D \text{ of Good A}}{\%\Delta P_D \text{ of Good B}} = \frac{-20\%}{10\%} = \boxed{-2}$$

Since the cross-price elasticity is negative, the goods are complements.

6 ➡ A

Consumers spent \$28 million in total, of which α = 10.8/28 ≈ 38.6% was for TVs and 100 – 38.6 = 61.4% was for laptops. The exponents of a Cobb-Douglas utility function determine the portion of income spent on each of the goods. Therefore, we can estimate the utility function for an average consumer in this country as

$$U(q_1,q_2) = Aq_1^{\alpha}q_2^{1-\alpha} = \boxed{Aq_1^{0.386}q_2^{0.614}}$$

7 ➡ B

Alfred's expected utility from investment A is

$$E_A[U] = 0.10\times\left[5\times\sqrt{300}\right]+0.80\times\left[5\times\sqrt{250}\right]+0.10\times\left[5\times\sqrt{200}\right] = 78.98$$

Alfred's expected utility from investment B is

$$E_B[U] = 0.30\times\left[5\times\sqrt{300}\right]+0.40\times\left[5\times\sqrt{250}\right]+0.30\times\left[5\times\sqrt{200}\right] = 78.82$$

Alfred will choose investment A, because it yields slightly greater expected utility. Note that Alfred is risk averse, so he prefers the investment with less variability.

Beto's expected utility from investment A is

$$E_A[U] = 0.10\times\left[5\times(300)^2\right]+0.80\times\left[5\times(250)^2\right]+0.10\times\left[5\times(200)^2\right] = 315{,}000$$

Beto's expected utility from investment B is

$$E_B[U] = 0.30\times\left[5\times(300)^2\right]+0.40\times\left[5\times(250)^2\right]+0.30\times\left[5\times(200)^2\right] = 320{,}000$$

Beto will choose investment B because it yields greater expected utility. Note that Beto is a risk lover, so he prefers the investment with greater variability.

8 ➡ D

Let $T = N + H$. We set up the Lagrangian

$$L = Y^{0.5} + 2N + \lambda\left[w(T-N)-Y\right]$$

Then, the first-order conditions are

$$\frac{\partial L}{\partial N} = 2 - w\lambda = 0 \quad \text{(I)}$$

$$\frac{\partial L}{\partial Y} = 0.5Y^{-0.5} - \lambda = 0 \quad \text{(II)}$$

$$\frac{\partial L}{\partial \lambda} = w(T-N) - Y = 0 \quad \text{(III)}$$

Solving (II) for λ,

$$\lambda = \frac{1}{2\sqrt{Y}}$$

Substituting in (I),

$$2 - w\lambda = 0$$

$$\therefore 2 = w\lambda$$

$$\therefore 2 = w\times\frac{1}{2\sqrt{Y}}$$

$$\therefore \sqrt{Y} = \frac{w}{4}$$

$$\therefore Y = \frac{w^2}{16}$$

Substituting in (III),

$$w(T-N) - \frac{w^2}{16} = 0$$

Dividing through by w and solving for N,

$$T - N - \frac{w}{16} = 0$$

$$\therefore N = T - \frac{w}{16}$$

But $H = T - N$,

$$H = T - N$$

$$\therefore H = T - \left(T - \frac{w}{16}\right)$$

$$\therefore H = \frac{w}{16}$$

Differentiating with respect to wage, the slope of Sarah's labor supply function is

$$\boxed{\frac{\partial H}{\partial w} = \frac{1}{16}}$$

9 ➡ B

Action *High* is a dominant strategy for Firm 1 because it should play *High* regardless of whether firm 2 plays *High* or *Low*.

10 ➡ B

These production functions are Cobb-Douglas. If the sum of exponents is greater than 1, the production function has increasing returns to scale; if the sum of exponents equals 1, the production function has constant returns to scale; if the sum of exponents is less than 1, the production function has decreasing returns to scale. In the present case, the sum of exponents for the automotive industry is 0.29 + 0.18 + 0.62 = 1.09, hence it exhibits increasing returns to scale; in turn, the sum of exponents for the aerospace industry is 0.24 + 0.19 + 0.54 = 0.97, so it exhibits decreasing returns to scale.

11 ➡ D

Setting marginal cost to price for each of the three firms and solving for the firms' individual output, we may write

$$MC_{\text{firm 1}} = 4q_1 = p \;\; \rightarrow \;\; q_1 = \frac{1}{4}p$$

$$MC_{\text{firm 2}} = 6q_2 = p \;\; \rightarrow \;\; q_2 = \frac{1}{6}p$$

$$MC_{\text{firm 3}} = 8q_3 = p \;\; \rightarrow \;\; q_3 = \frac{1}{8}p$$

The market supply then becomes

$$S = 24q_1 + 24q_2 + 16q_3 = \frac{24}{4}p + \frac{24}{6}p + \frac{16}{8}p = 12p$$

Equating this to the demand Q and solving for market price,

$$S = Q \;\; \rightarrow \;\; 12p = 1200 - 3p$$

$$\therefore 15p = 1200$$

$$\therefore p = \frac{1200}{15} = \boxed{\$80}$$

12 ➡ C

The total revenue is

$$\overline{\text{TR}} = PQ = (98 - Q) \times Q = 98Q - Q^2$$

The marginal revenue is

$$\overline{\text{MR}} = 98 - 2Q$$

The marginal cost is

$$C(Q) = 0.3Q^2 - 6Q + 300 \;\; \rightarrow \;\; \overline{\text{MC}} = 0.6Q - 6$$

Equating *MR* and *MC*,

$$\overline{MR} = \overline{MC} \;\; \rightarrow \;\; 98 - 2Q = 0.6Q - 6$$

$$\therefore 104 = 2.6Q$$

$$\therefore Q = \frac{104}{2.6} = \boxed{40}$$

The price is $P = 98 - 40 = \$58$. The monopolistic firm's profits follow as

$$\pi_m = \overline{TR} - \overline{TC} = 40 \times 58 - \left(0.3 \times 40^2 - 6 \times 40 + 300\right) = \boxed{\$1780}$$

13 ➡ C

In this Stackelberg duopoly, the profit-maximizing output of the leader is given by

$$\frac{1}{2}\left(a - c - 2bq_L\right) = 0$$

Solving for q_L,

$$q_L = \frac{a-c}{2b}$$

In turn, the follower responds by producing q_F, that is,

$$q_F = \frac{a-c}{2b} - \frac{1}{2}q_L = \frac{a-c}{2b} - \frac{1}{2} \times \left(\frac{a-c}{2b}\right)$$

$$\therefore q_F = \frac{a-c}{4b}$$

Using these results, we conclude that the leader firm produces twice as much as the follower firm,

$$\frac{q_L}{q_F} = \frac{(a-c)/2b}{(a-c)/4b} = \boxed{2}$$

14 ➡ B

First, it is easy to see that the game has no equilibrium in pure strategies. Because the best response to each pure strategy is unique and there is no pure-strategy equilibrium, both players must mix at least two strategies in any equilibrium. Hence, in equilibrium, player 2 has to mix between *L* and *R*, and therefore has to be indifferent between *L* and *R* by the best response condition. Let $(1 - q, q)$ be the mixed strategy of player II, $0 < q < 1$. Let (a,b,c) be the mixed strategy of player 1, where he plays *T*, *M*, and *B* with probabilities *a*, *b*, and *c* respectively. Suppose player 1 uses only *T* and *M* with positive probability ($c = 0$), so their mixed strategy can be stated as $(a,b,0)$. Making player II indifferent means setting $1 \times a + 0 \times b = 0 \times a + 2 \times b$, or $a = 2b = 2(1 - a)$, that is, $a = 2/3$, $b = 1/3$. Then player II gets an expected payoff of 2/3 for each of their pure strategies. In turn, player 2 chooses $(1 - q, q)$ so as to make player 1 indifferent between *T* and *M* because both have to be pure best responses to $(1 - q, q)$ and hence must have equal payoff. This means

$$6 \times q = 2 \times (1-q) + 5 \times q$$

$$\therefore 6q = 2 - 2q + 5q$$

$$\therefore 3q = 2$$

$$\therefore q = \frac{2}{3}$$

Hence, the mixed strategy is (1/3, 2/3). The expected payoff is 4 for both *T* and *M*, which is higher than the payoff 3 that player 1 would get when playing *B*. So indeed, we have the equilibrium ((2/3, 1/3, 0), (1/3, 2/3)).

Next, we try a mixed strategy $(a,0,c)$ of player 1 that only uses *T* and *B*. Equating the expected payoffs to player 2 gives the equation $a + 3c = 4c$ or $a = c$, yielding the mixed strategy (1/2,0,1/2) where player 2 gets expected payoff 2. Then player 2 has to play $(1 - q,q)$ so that player 1 is indifferent between *T* and *B*, that is, $6q = 3$ or $q = 1/2$, with expected payoff 3 to player 1. This looks like an equilibrium but in fact is *not*, because the expected payoff for *M* is 3.5, which is higher than 3. So *T* and *B* have equal payoff but are nevertheless not best responses to (1/2,1/2), which means we don't have an equilibrium.

Finally, consider the case that player 1 plays with a mixed strategy $(0,b,c)$ where only *M* and *B* have positive probability. Equal payoff to the columns of player 2 means $3c = 2b + 4c$ or $0 = 2b + c$ which is clearly not possible for probabilities *b*,

c that sum to one. This is due to the fact that against *M* and *B*, the best pure response is always to play *r*, so the indifference cannot be achieved.
There is no strategy where player 1 can play all three pure strategies with positive probability: In that case the player would have to be indifferent between *T* and *M*, which already determines the mixed strategy of player uniquely as (1/3, 2/3), but then *B* gets a different payoff. So it is not possible to make player 1 indifferent between all three pure strategies. In summary, the game has only one mixed equilibrium.

15 ➡ D

This simple observational exercise should be obvious to most students.

16 ➡ A

Firstly, note that, for player δ, strategy *W* is strictly dominated by strategy *Y*, hence we can simplify the normal-form representation you chose in the previous problem to the following bimatrix.

δ \ ε	W	Y	Z
Y	(60,40)	(42,42)	(60,50)
Z	(50,40)	(50,60)	(35,35)

Next, note that, for player ε, strategy *W* is strictly dominated by strategy *Y*.

δ \ ε	Y	Z
Y	(42,42)	(60,50)
Z	(50,60)	(35,35)

As highlighted in yellow, two pure-strategy Nash equilibria arise in this game, namely (*Y*, *Z*) and (*Z*, *Y*). there is a third equilibrium, in this case a mixed one; to find it, we can take advantage of the fact that the bimatrix is symmetric. Thus, if *p* is the probability of either of the players choosing *Y*, then the mixed strategy equilibrium must satisfy the condition

$$42p + 60(1-p) = 50p + 35(1-p)$$
$$\therefore 42p + 60 - 60p = 50p + 35 - 35p$$
$$\therefore -33p = -25$$
$$\therefore p = \frac{25}{33}$$

and 1 – *p* = 8/33. Thus, $((0,\frac{25}{33},\frac{8}{33});(0,\frac{25}{33},\frac{8}{33}))$ is a mixed Nash equilibrium for the game at hand. In practical terms, the mixed equilibrium could mean that both doctoral students randomize their choice of adviser so that Dr. Watson is not chosen at all, Dr. Young is chosen with probability 25/33 ≈ 75.8%, and Dr. Zhao is chosen with probability 8/33 ≈ 24.2%.

17 ➡ (B,ℓ,L)

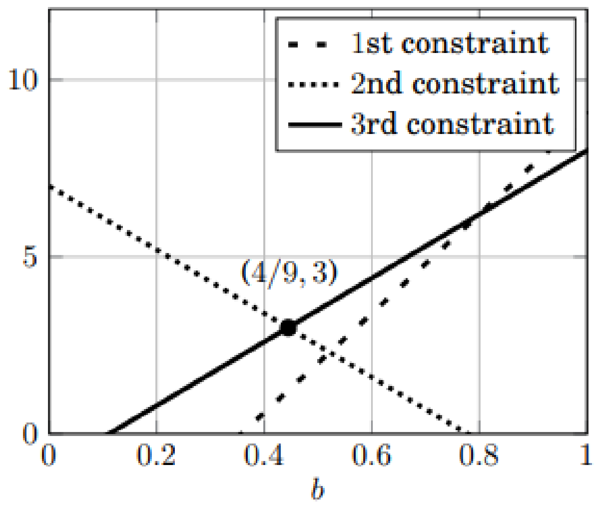
Simple iterative elimination of strictly dominated strategies applies here. First, *B* strictly dominates *T*. After eliminating *T*, player III's strategy *L* strictly dominates *R*. After eliminating *R*, player II's strategy *ℓ* strictly dominates *r*. Then, the remaining strategy is (*B*,*ℓ*,*L*), which emerges as the game's only pure-strategy Nash equilibrium.

18 ➡ C

This zero-sum game must have a Nash equilibrium in mixed strategies – and only one, since player 2 has only one way of mixing their strategies. Specifically, let *b* be the probability of player 2 playing *X*; then, the minimax conditions for player 1 are

$$\begin{cases} 9b - 5\times(1-b) \geq w \\ -2b + 7\times(1-b) \geq w \\ 8b - 1\times(1-b) \geq w \end{cases}$$

where w is a slack variable to be maximized. A graphical inspection reveals that the active constraints are the second and the third because maximizing the last two constraints also satisfies the first; see below.



Equating the second and third linear equations posed above, we have

$$7-9b=9b-1$$
$$\therefore -18b=-8$$
$$\therefore b=\frac{4}{9}$$

The payoff at the Nash equilibrium for player 2 is –3. Therefore, denoting as a the probability that player B will choose *B* within the support, we may write

$$3\times a=3\times(1-a)$$
$$\therefore 6a=3$$
$$\therefore a=\frac{1}{2}$$

In summary, the Nash equilibrium is ((0,1/2,1/2),(4/9,5/9)) and the value is 3.

19 ➡ B

In this game, player II does not have perfect recall because the two nodes on their second information set are preceded by different sequences of own moves, namely an empty sequence and action *c*. It follows that player 2 has forgotten if it made the move *c* or not.

20 ➡ C

The first stage game has a unique dominant strategy Nash equilibrium (*N*,*n*) while the second stage game has two pure strategy equilibria, (*P*,*p*) and (*N*,*n*), in which each player chooses the positive campaign with probability 1/5. In the second stage, the players must play either (*P*,*p*) or (*N*,*n*) for any history in a pure strategy subgame perfect equilibrium. With extreme discounting we cannot support play in the first stage that is not a Nash equilibrium because there is no second-stage 'punishment' that can deter first-stage deviations; it follows that in the first stage the players must play (*N*,*n*). Thus, (*N*,*n*) followed by either (*P*,*p*) or (*N*,*n*) will be the only outcomes that can be supported as subgame perfect equilibria. There are $2^4 = 16$ pure strategy equilibria because for each of the 4 outcomes of the first stage the players must specify which of the two equilibria (*P*,*p*) or (*N*,*n*) will be played in the second stage.

21 ➡ D

We can use the conditional second stage strategies in which player 1 (respectively 2) plays *P* (respectively *p*) if the choice in the first stage was (*P*,*p*) while they play *N* and *n* otherwise. In the first stage neither player wants to deviate from (*P*,*p*) because the gain of switching actions is 3 (from 2 to 5) while the loss from the punishment in the second stage is 4 (and it is not discounted so its value remains 4). The discounted punishment must be at least as high as the gain from deviation, so the inequality is $3 - 4\delta \leq 0$, and the solution is $\delta \geq 3/4$.

22 ➡ B

Let player 3 be the one with a left glove. Then, given the Gloves Game $\langle N, v\rangle$, where $N = \{1,2,3\}$ is the set of players and v is the coalitional function given by

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{1,2\}) = 0$$
$$v(\{1,3\}) = v(\{2,3\}) = v(\{1,2,3\}) = 1$$

the core turns out to be a singleton obtained by solving

$$C(v) = \left\{x \in \Re_+^3 \mid x_1 + x_3 \geq 1,\ x_2 + x_3 \geq 1,\ \sum_{i\in N} x_i = 1\right\}$$
$$\therefore C(v) = \{(0,0,1)\}$$

23 ➡ C

The following table lists the Shapley value calculations; the value is given by the usual formula

$$\Phi(v) = \frac{1}{n!}\sum_{\sigma} m^{\sigma}(v)$$

The calculations are tabulated below.

σ	$m_1^{\sigma}(v)$	$m_2^{\sigma}(v)$	$m_3^{\sigma}(v)$
(1,2,3)	0	4	17
(1,3,2)	0	12	9
(2,1,3)	4	0	17
(2,3,1)	5	0	16
(3,1,2)	9	12	0
(3,2,1)	5	16	0
	Σ = 23	Σ = 44	Σ = 59

The Shapley vector is found to be

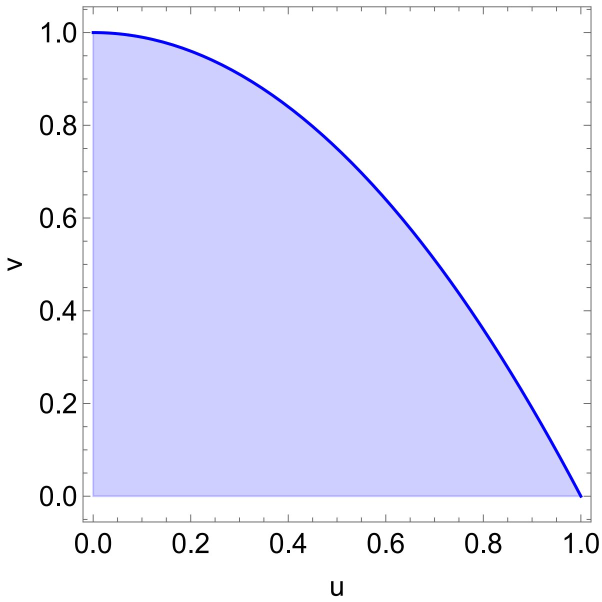
$$\Phi(v) = \frac{1}{3!}\sum_{\sigma} m^{\sigma}(v) = \frac{1}{6}(23,\ 44,\ 59)$$

None of the Shapley vector components equals 25/3 (= 50/6). Note that the Shapley values must add up to the worth of the coalition, that is,

$$\frac{23}{6} + \frac{44}{6} + \frac{59}{6} = 21 = v(N)$$

24 ➡ B

Firstly, the bargaining set is represented by the shaded area between the threat point (0,0) and the blue curve:



The Pareto frontier of the bargaining set is the blue curve itself. It is given by the set of pairs ($u(x)$, $v(1 – x)$) for $x \in [0,1]$. Because $u = u(x) = x$, we have $v(1 – x) = 1 – x^2$, so the curve is simply the graph of the function $1 – u^2$ for $u \in [0,1]$. The Nash bargaining solution is found on the Pareto frontier, by maximizing the Nash product $u(x)v(1 – x)$, namely

$$u(x)v(1-x) = x\times(1-x^2) = x - x^3$$

Differentiating with respect to x and setting the result to zero,

$$\frac{d}{dx}\left[u(x)v(1-x)\right]=1-3x^2=0$$

$$\therefore x=\frac{1}{\sqrt{3}}$$

Differentiating a second time and substituting $x = 1/\sqrt{3}$ yields a negative value, which indicates that this value of x indeed corresponds to a maximum. Therefore, the pie will be split into $x = 1/\sqrt{3}$ for player I and $1 - 1/\sqrt{3}$ for player II, with utilities given by

$$u\left(x=\frac{1}{\sqrt{3}}\right)=x=\boxed{\frac{1}{\sqrt{3}}\ \text{(Player I)}}$$

$$v(1-x)=1-x^2=1-\left(\frac{1}{\sqrt{3}}\right)^2=\boxed{\frac{2}{3}\ \text{(Player II)}}$$

25 ➡ A

I. *True.* That said, the converse is not true: there are fixed points of the replicator dynamic that are not Nash equilibria of the evolutionary game, because if an i-player does not exist in the population at one point in time, it can never appear in the future under a replicator dynamic. Therefore, for any i, the state $p_i = 1$, $p_j = 0$ for $j \neq i$ is a fixed point under the replicator dynamic.

II. *True.* To prove this assertion, assume p^* is *not* isolated. Then, there is an i and an $\varepsilon > 0$ such that $\pi_i(p^*) - \bar{\pi}(p^*) > \varepsilon$ in a ball around p^*. But then the replicator dynamic implies that p_i grows exponentially along a trajectory starting at any point in this ball, which means that p^* is not asymptotically stable.

III. *False.* To name but one example, page 284 of Gintis (2009) provides an example of an evolutionary game that has a locally stable fixed point that is not an evolutionarily stable strategy.

➡ 26

Part (a): The total output is $q = q_1 + q_2$, with

$$q_1=\sqrt{k_1 l_1}=\sqrt{49 l_1}=7\sqrt{l_1}$$

$$q_2=\sqrt{k_2 l_2}=\sqrt{196 l_2}=14\sqrt{l_2}$$

Solving for l_1 and l_2,

$$q_1=7\sqrt{l_1}\ \rightarrow\ l_1=\frac{q_1^2}{49}$$

$$q_2=14\sqrt{l_2}\ \rightarrow\ l_2=\frac{q_2^2}{196}$$

Noting that rental rates for capital and labor are $v = \$1$ and $w = \$1$, respectively, the short-run cost functions for the two factories are

$$C_1(k,l)=vk_1+wl_1=1.0\times 49+1.0\times\frac{q_1^2}{49}$$

$$\therefore C_1(k,l)=49+\frac{q_1^2}{49}\ \text{(I)}$$

$$C_2(k,l)=vk_2+wl_2=1.0\times 196+1.0\times\frac{q_2^2}{196}$$

$$\therefore C_2(k,l)=196+\frac{q_2^2}{196}\ \text{(II)}$$

The total short-run cost then becomes

$$C(q_1,q_2)=C_1(q_1)+C_2(q_2)=49+\frac{q_1^2}{49}+196+\frac{q_2^2}{196}=245+\frac{q_1^2}{49}+\frac{q_2^2}{196}$$

To minimize cost, we first set up a Lagrangian,

$$L(q_1, q_2, \lambda) = 245 + \frac{q_1^2}{49} + \frac{q_2^2}{196} + \lambda(q - q_1 - q_2)$$

The first-order conditions are computed next,

$$\frac{dL}{dq_1} = \frac{2q_1}{49} - \lambda = 0$$

$$\therefore \lambda = \frac{2q_1}{49} \text{ (I)}$$

$$\frac{dL}{dq_2} = \frac{2q_2}{196} - \lambda = 0$$

$$\therefore \lambda = \frac{q_2}{98} \text{ (II)}$$

$$\frac{dL}{d\lambda} = q - q_1 - q_2 = 0$$

$$\therefore q = q_1 + q_2 \text{ (III)}$$

Equating (I) and (II) and solving for q_2,

$$\frac{q_2}{98} = \frac{2q_1}{49} \rightarrow q_2 = 4q_1$$

Substituting in (III),

$$q = q_1 + q_2 = q_1 + 4q_1 = 5q_1$$

$$\therefore \boxed{q_1 = \frac{q}{5}}$$

Also,

$$q_2 = 4q_1 = \boxed{\frac{4q}{5}}$$

In order to minimize short-run cost, 20% of the output should come from factory 1, and 80% should come from factory 2.

Part (b): The short-run cost curve was derived in part (a) and can be stated as

$$C(q_1, q_2) = 245 + \frac{q_1^2}{49} + \frac{q_2^2}{196}$$

Using the results from part (a), we can restate this in terms of the total output *q*,

$$C(q) = 245 + \frac{(q/5)^2}{49} + \frac{(4q/5)^2}{196} = \boxed{245 + \frac{q^2}{245}}$$

This is the short-run cost curve. Differentiating it with respect to *q* gives the short-run marginal cost,

$$MC(q) = \frac{dC(q)}{dq} = \boxed{\frac{2q}{245}}$$

Dividing *C(q)* by *q* yields the short-run average cost,

$$AC(q) = \frac{C(q)}{q} = \boxed{\frac{245}{q} + \frac{q}{245}}$$

The marginal cost of the 500th widget is

$$MC(q = 500) = \frac{2 \times 500}{245} = \boxed{\$4.08}$$

Part (c): In the long run, given constant returns to scale, location doesn't really matter because the producer can change the capital *k*. The entrepreneur could split evenly or produce all output in one location, etc.

Part (d): If there are decreasing returns to scale with identical production functions, then the entrepreneur should let each firm have equal share of production. *AC* and *MC* are not constant anymore, becoming increasing functions of *q*.

➡ 27

Part (a): Suppose that consumers believe that if the firm purchases *x* TV ads, then its quality is high. If the low quality firm does not advertise, consumers will agree to pay at most 5 for its product. Then, the profit of the low quality firm will be

$$N \times (5-3) \times (1+\delta) = 2N(1+\delta)$$

If the low quality firm advertises, consumers will pay 10 in the first period and 5 in the second period. The firm's profit will then be

$$N \times (10-3) - Ax + \delta N(5-3) = 7N - Ax + 2\delta N$$

Comparing the two profit levels, it does not pay for the firm to advertise if

$$7N - Ax + 2\delta N < 2N(1+\delta)$$
$$\therefore 7N - Ax + \cancel{2\delta N} < 2N + \cancel{2\delta N}$$
$$\therefore 7N - Ax < 2N$$
$$\therefore 5N < Ax$$
$$\therefore x > \frac{5N}{A}$$

We need to check that the high quality firm is willing to sponsor 5*N*/*A* ads. If it sponsors *x* TV ads (thereby signaling its high quality), its profit becomes

$$N \times (10-4) - \alpha Ax + \delta N \times (10-4) = 6N \times (1+\delta) - \alpha Ax$$

If the firm does not advertise in period 1, consumers will believe in period 1 that its quality is low and would agree to pay only 5. Since consumers learn the firm's quality in period 2, the profit of firm 1 in this case is

$$N \times (5-4) + \delta N \times (10-4) = N + 6\delta N$$

Comparing the two profit levels, it pays for the firm to advertise if

$$6N \times (1+\delta) - \alpha Ax > N + 6\delta N$$
$$\therefore 6N + \cancel{6\delta N} - \alpha Ax > N + \cancel{6\delta N}$$
$$\therefore -\alpha Ax > -5N$$
$$\therefore x < \frac{5N}{\alpha A}$$

Since $\alpha < 1$, sponsoring 5*N*/*A* ads will enable the high quality firm to separate itself. As shown above, this is the minimal number of ads needed for separation.
Part (b): The answer does not depend on δ since the second-period profits are completely independent of advertising. Intuitively, this is because the product quality becomes common knowledge in period 2 regardless of what happened in period 1.
The answer is also independent of α because $\alpha < 1$ ensures that the high quality firm will find it profitable to sponsor 5*N*/*A* ads given that its advertising cost is smaller.
However, the answer does depend on *N* because a higher *N* increases the value of advertising given that the benefit from advertising is in the form of higher per-unit profits in period 1. The higher *N* is, the higher the total profit from advertising will be. Thus, the high quality firm needs to spend a greater amount in period 1 in order to separate itself from the low quality firm.

➡ 28

Part (a): In this setting, firm 1 chooses its output q_1 to solve the profit maximization problem

$$\max_{q_1 \geq 0} \pi_1 = (a - bq_1 - bq_2)q_1 - c_1 q_1$$

Differentiating with respect to q_1, we obtain

$$\frac{d\pi_1}{dq_1} = a - 2bq_1 - bq_2 - c_1 = 0$$

Solving for q_1 gives the best response function

$$q_1 = \frac{a-c_1}{2b} - \frac{1}{2}q_2 \quad \text{(I)}$$

In turn, firm 2 chooses its output level q_2 to solve the profit maximization problem

$$\max_{q_2 \ge 0} \pi_2 = \left(a - bq_1 - bq_2\right)q_2 - c_1 q_2$$

Differentiating with respect to q_2, we obtain

$$\frac{d\pi_1}{dq_2} = a - 2bq_2 - bq_1 - c_2 = 0$$

Solving for q_2 gives the best response function

$$q_2 = \frac{a-c_2}{2b} - \frac{1}{2}q_1 \quad \text{(II)}$$

Relative to firm 1's best response function, firm 2's BRF originates at a lower vertical intercept since $\frac{a-c_2}{2b} < \frac{a-c_1}{2b}$ given that $c_1 < c_2$ by assumption. Intuitively, the firm benefitting from a cost advantage produces more units for a given output level of its rival. Both best response functions, however, have the same slope.

Part (b): Here, we cannot make use of symmetry equilibrium because the two firms face different marginal costs. Substituting best-response function (II) into (I) and manipulating, we have

$$q_1 = \frac{a-c_1}{2b} - \frac{1}{2}q_2 = \frac{a-c_1}{2b} - \frac{1}{2} \times \left(\frac{a-c_2}{2b} - \frac{1}{2}q_1\right)$$

$$\therefore q_1 = \frac{a-c_1}{2b} - \frac{a-c_2}{4b} + \frac{1}{4}q_1$$

$$\therefore \frac{3q_1}{4} = \frac{2a - 2c_1 - a + c_2}{4b}$$

$$\therefore \boxed{q_1 = \frac{a - 2c_1 + c_2}{3b}}$$

The result above is the equilibrium output for firm 1. Then, substituting into (II) and manipulating brings to

$$q_2 = \frac{a-c_2}{2b} - \frac{1}{2}\left(\frac{a-2c_1+c_2}{3b}\right)$$

$$\therefore q_2 = \frac{3\left(a-c_2\right)}{6b} - \left(\frac{a-2c_1+c_2}{6b}\right)$$

$$\therefore q_2 = \frac{3a - 3c_2 - a + 2c_1 - c_2}{6b}$$

$$\therefore q_2 = \frac{2a + 2c_1 - 4c_2}{6b}$$

$$\therefore \boxed{q_2 = \frac{a - 2c_2 + c_1}{3b}}$$

This is the equilibrium output for firm 2. We can directly compare q_1 and q_2 to find that the firm with the lower marginal cost (namely, firm 1) will produce more output in equilibrium, that is, $q_1 > q_2$,

$$q_1 > q_2 \quad \rightarrow \quad \frac{a - 2c_1 + c_2}{\cancel{3b}} > \frac{a - 2c_2 + c_1}{\cancel{3b}}$$

$$\therefore a - 2c_1 + c_2 > a - 2c_2 + c_1$$

$$\therefore -2c_1 + c_2 > -2c_2 + c_1$$

$$\therefore 3c_2 > 3c_1$$

$$\therefore c_2 > c_1$$

This inequality matches the assumption given in the problem statement. Therefore, our starting inequality, $q_1 > q_2$, is true.

Corner solution. There is a situation in which firm 1 is the only active firm in the market, that is, $q_2 = 0$. To see when this situation holds true, we set firm 2's equilibrium output to be equal to or less than zero, giving

$$q_2 = \frac{a - 2c_2 + c_1}{3b} \leq 0$$

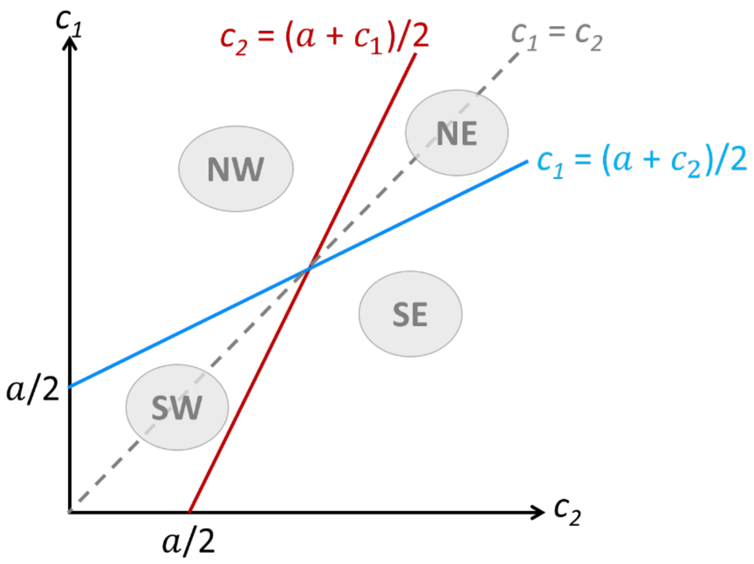
$$\therefore a - 2c_2 + c_1 \leq 0$$

$$\therefore a - 2c_2 + c_1 \leq 0$$

$$\therefore \frac{a + c_1}{2} \leq c_2$$

$$\therefore c_2 \geq \frac{a + c_1}{2}$$

This condition means that firm 2's costs are so high, relative to firm 1's, that it cannot profitably supply output to the market. As the cost differential between the two firms grows (as c_2 increases), the low-cost firm's output increases whereas the high-cost firm's output decreases. Once the cost discrepancy is large enough, the high-cost firm does not compete in the market. The following figure depicts cutoff $c_2 = (a + c_1)/2$ in the (c_2, c_1) plane, so for all cost pairs above cutoff $c_2 = (a + c_1)/2$ firm 2 remains inactive. The figure includes a symmetric cutoff for firm 1, $c_1 = (a + c_2)/2$, so for points below it firm 1 remains inactive. In summary, only firm 2 is active in the northwest (NW) region, as it is relatively more efficient than its rival; only firm 1 is active in the southeast (SE) region given its cost advantage; both firms are active in the southwest (SW) region as they are both efficient and symmetric; and no firm is active in the northeast (NE) region because both are relatively inefficient.



Part (c): Both firms face the same price. Substituting the expressions for q_1 and q_2 obtained in part (b) into the demand curve brings to

$$p = a - bQ = a - b(q_1 + q_2)$$

$$\therefore p = a - \cancel{b}\left(\frac{a - 2c_1 + c_2}{3\cancel{b}} + \frac{a - 2c_2 + c_1}{3\cancel{b}}\right)$$

$$\therefore p = a - \left(\frac{a - 2c_1 + c_2 + a - 2c_2 + c_1}{3}\right)$$

$$\therefore p = a - \left(\frac{2a - c_1 - c_2}{3}\right)$$

$$\therefore p = \left(\frac{3a - 2a + c_1 + c_2}{3}\right)$$

$$\therefore \boxed{p = \frac{a + c_1 + c_2}{3}}$$

In equilibrium, firm 1 earns profit

$$\pi_1 = pq_1 - c_1q_1 = (p - c_1)q_1$$

Substituting q_1 from part (b) and price p obtained just now, we get

$$\pi_1 = \left(\frac{a + c_1 + c_2}{3} - c_1\right) \times \frac{a - 2c_1 + c_2}{3b}$$

$$\therefore \pi_1 = \left(\frac{a + c_1 + c_2 - 3c_1}{3}\right) \times \frac{a - 2c_1 + c_2}{3b}$$

$$\therefore \pi_1 = \left(\frac{a - 2c_1 + c_2}{3}\right) \times \frac{a - 2c_1 + c_2}{3b}$$

$$\therefore \boxed{\pi_1 = \frac{(a - 2c_1 + c_2)^2}{9b}}$$

In turn, firm 2 earns profit

$$\pi_2 = pq_2 - c_2q_2 = (p - c_2)q_2$$

$$\therefore \pi_2 = \left(\frac{a + c_1 + c_2}{3} - c_2\right) \times \frac{a - 2c_2 + c_1}{3b}$$

$$\therefore \pi_2 = \left(\frac{a + c_1 + c_2 - 3c_2}{3}\right) \times \frac{a - 2c_2 + c_1}{3b}$$

$$\therefore \pi_2 = \left(\frac{a + 2c_2 + c_1}{3}\right) \times \frac{a - 2c_2 + c_1}{3b}$$

$$\therefore \boxed{\pi_2 = \frac{(a - 2c_2 + c_1)^2}{9b}}$$

We can show that $\pi_1 > \pi_2$, that is, firm 1 will earn a higher profit,

$$\pi_1 > \pi_2 \quad \rightarrow \quad \frac{(a - 2c_1 + c_2)^2}{9b} > \frac{(a - 2c_2 + c_1)^2}{9b}$$

$$\therefore (a - 2c_1 + c_2)^2 > (a - 2c_2 + c_1)^2$$

$$\therefore \cancel{a} - 2c_1 + c_2 > \cancel{a} - 2c_2 + c_1$$

$$\therefore -3c_1 > -3c_2$$

$$\therefore c_1 < c_2$$

This inequality matches the assumption given in the problem statement. We conclude that $\pi_1 > \pi_2$; intuitively, the firm benefitting from a cost advantage earns a higher profit.

Part (d): Setting $c_1 = c_2 = c$, the quantity produced by firm 1 becomes

$$q_1 = \frac{a - 2c_1 + c_2}{3b} = \frac{a - 2c + c}{3b} = \frac{a - c}{3b}$$

Similarly, the output of firm 2 becomes

$$q_2 = \frac{a - 2c_2 + c_1}{3b} = \frac{a - 2c + c}{3b} = \frac{a - c}{3b}$$

so that $q_1 = q_2 = q$, where q is simply the output in a symmetric Cournot duopoly. Next, we can substitute $c_1 = c_2 = c$ in the price formula derived in part (c) to obtain

$$p = \frac{a + c_1 + c_2}{3} = \frac{a + c + c}{3} = \frac{a + 2c}{3}$$

This is identical to the price set by the firms in a symmetric Cournot duopoly. Lastly, we set $c_1 = c_2 = c$ to the equation for profits obtained in part (c) so that yet again that results boil down to those obtained for a symmetric Cournot duopoly,

$$\pi_2 = \frac{(a - 2c_2 + c_1)^2}{9b} = \frac{(a - 2c + c)^2}{9b} = \frac{(a - c)^2}{9b}$$

➡ 29

Part (a): The effective discount factor is given by

$$\delta = \frac{h(1+g)}{1 + r/f}$$

In industry A, the likelihood of continued existence is $h_A = 0.8$; the industry growth rate is $g_A = 0.1$; the annual interest rate is $r_A = 0.1$; and the frequency with which firms interact is $f_A = 1$, giving

$$\delta_A = \frac{h_A(1+g_A)}{1 + r_A/f_A} = \frac{0.8 \times (1 + 0.1)}{1 + 0.1/1} = 0.8$$

In industry B, we have $h_B = 1.0$, $g_B = -0.3$, $r_B = 0.1$, and $f_B = 12$, giving

$$\delta_B = \frac{h_B(1+g_B)}{1 + r_B/f_B} = \frac{1.0 \times (1 - 0.3)}{1 + 0.1/12} = 0.694$$

Since $\delta_A > \delta_B$, and knowing that the number of companies is the same in both industries, we'd expect that tacit collusion is more likely in industry A than in industry B. Although interaction between firms is more frequent in industry B, the fact that it is a declining industry ultimately renders collusion very difficult – the promise of continuing collusion in the future is of little importance, leading firms to have a greater incentive to cheat on a collusion agreement.

Part (b): Let π^M be the total industry profits. Under the collusive agreement, each firm receives π^M/n. If one of the firms undercuts its rivals, then it gets approximately π^M. Finally, if firms revert to a (perpetual) price war, each firm gets zero. It follows that the condition such that it is an equilibrium for firms to price at the monopoly level is given by

$$\frac{1}{1-\delta}\frac{\pi^M}{n} \geq \pi^M$$

Solving for δ brings to

$$\frac{1}{1-\delta}\frac{\cancel{\pi^M}}{n} \geq \cancel{\pi^M}$$

$$\therefore \frac{1}{1-\delta} \times \frac{1}{n} \geq 1$$

$$\therefore \frac{1}{n} \geq 1 - \delta$$

$$\therefore \delta \geq 1 - \frac{1}{n}$$

$$\therefore \delta \geq \frac{n-1}{n}$$

It follows that collusion is stable if and only if $\delta > \bar{\delta} \equiv (n-1)/n$. Importantly, this condition is independent of the value of π^M, so the same condition would apply for any level of collusion. Taking the derivative of δ with respect to n, we obtain

In[531]:= $\text{Simplify}\left[D\left[\frac{n-1}{n}, n\right]\right]$

Out[531]= $\frac{1}{n^2}$

As shown in the Mathematica snippet,

$$\frac{d\delta}{dn}=\frac{d}{dn}\left(\frac{n-1}{n}\right)=\frac{d}{dn}\left(1-\frac{1}{n}\right)=\frac{1}{n^2}>0$$

for all positive integers n. It follows that $\bar{\delta}$ is increasing in n. The interpretation is straightforward: the more firms there are, the more difficult it is to sustain a collusive agreement. The idea is that the relative gain from cheating is greater the greater the number of firms; the profit from cheating is always the same, but the profit from collusion is lower the greater n is.

(c) The no-deviation constraint is given by

$$\frac{1}{n}\frac{\pi^M}{1-\delta}\geq\pi^M$$

where n is the number of firms, π^M are monopoly profits, and δ = 0.8, giving

$$\frac{1}{n}\frac{\cancel{\pi^M}}{1-\delta}\geq\cancel{\pi^M}$$

$$\therefore\frac{1}{1-\delta}\geq n$$

$$\therefore n\leq\frac{1}{1-\delta}$$

$$\therefore n\leq\frac{1}{1-0.8}=\boxed{5}$$

At most 5 firms must constitute the oligopoly for equilibrium monopoly prices to be viable.

➡ 30

Part (a): In a Nash equilibrium, both firms will charge prices equal to 0. This is the only pair of prices for which no firm can benefit from deviation. If prices are negative, firms lose money and are better off charging 0 (in which case they do not lose money). If prices are positive, it pays to cut the price by a cent below the price of the rival and, thereby, capture the entire market. Hence, if entry requires even a small initial investment, firm 2 will choose to stay out.

Part (b): If firm 1 fights it charges p_2 and captures the entire market because when both firms set equal prices, all consumers prefer to buy from firm 1. Firm 1's profit then is $\pi_1^F = (a - p_2)\,p_2$. Firm 2 obtains 0 profit and, hence, would prefer to stay out.

Part (c): If firm 1 accommodates firm 2, its residual demand is $Q_1 = a - q_2 - p_1$. The problem of firm 1 is to maximize $p_1 Q_1$. The price that maximizes firm 1's profit is $p_1^\star = (a - q_2)/2$, so firm 1's profit is $\pi_1^A = (a - q_2)^2/4$. In this case, firm 2 earns $p_2 q_2 > 0$ and hence would choose to enter.

Part (d): Here, we need to compare π_1^F from part (b) and π_1^A from part (c). Equating the two and solving for the largest output x_2 ($= q_2$) that firm 2 can choose without inducing firm 1 to fight it, we get

```
In[524]:= Solve[(a - p2) p2 == (a - q2)^2/4, q2]

Out[524]= {{q2 → a - 2 √(a - p2) √p2}, {q2 → a + 2 √(a - p2) √p2}}
```

As shown, the lowest root is

$$\boxed{x_2=a-2\sqrt{(a-p_2)\,p_2}}$$

Part (e): Referring to the expression at the end of part (d), we see that $x_2(p_2)$ is decreasing in p_2 if $p_2 > a/2$ and increasing otherwise. To determine whether p_2 is above or below $a/2$, note that the entrant's profit is $p_2 x_2(p_2)$. If $p_2 < a/2$, then since $x_2(p_2)$ is increasing in p_2, the entrant would want to raise p_2 as much as possible since both their capacity and their profit per unit will increase. This happens up to $a/2$. Thus, in equilibrium, it must be the case that $p_2 > a/2$ so that $x_2(p_2)$ is decreasing in p_2.

REFERENCES

- BADIA, L. and MARCHORO, T. (2022). *Game Theory: A Handbook of Problems and Exercises*. Società Editrice Esculapio.
- BAUSO, D. (2016). *Game Theory with Engineering Applications*. Society for Industrial and Applied Mathematics.
- BELLEFLAMME, P. and PEITZ, M. (2015). *Industrial Organization*. 2nd edition. Cambridge University Press.
- CABRAL, L.M.B. (2017). *Introduction to Industrial Organization*. 2nd edition. MIT Press.
- CHOI, P.-S., DUNAWAY, E. and MUÑOZ-GARCIA, F.M. (2021). *Industrial Organization: Practice Exercises with Answer Keys*. Springer.
- GINTIS, H. (2009). *Game Theory Evolving*. 2nd edition. Princeton University Press.
- MASCHLER, M., SOLAN, E. and ZAMIR, S. (2013). *Game Theory*. Cambridge University Press.
- NICHOLSON, W. and SNYDER, C. (2012). *Microeconomic Theory*. 12th edition. Cengage Learning.
- PERLOFF, J.M. (2014). *Microeconomics: Theory and Applications with Calculus*. 3rd edition. Pearson.
- SERRANO, R. (2018). *A Short Course in Intermediate Microeconomics with Calculus*. 2nd edition. Cambridge University Press.
- TADELIS, S. (2013). *Game Theory: An Introduction*. Princeton University Press.
- VON STENGEL, B. (2022). *Game Theory Basics*. Cambridge University Press.