Quiz SM102
Moment of Inertia

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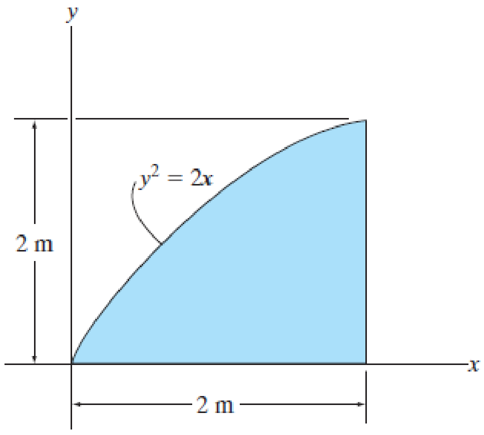
Problems

PROBLEM 1 A

The moments of inertia of the area shown about the *x*-axis and the *y*-axis are:



A) I_x = 213 (10^6) mm^4 and I_y = 457 (10^6) mm^4
B) I_x = 213 (10^6) mm^4 and I_y = 588 (10^6) mm^4
C) I_x = 331 (10^6) mm^4 and I_y = 457 (10^6) mm^4
D) I_x = 331 (10^6) mm^4 and I_y = 588 (10^6) mm^4

PROBLEM 1 B

The polar moment of inertia of the area presented in the previous part about the origin of the coordinate frame is:

A) J_0 = 549 (10^6) mm^4
B) J_0 = 670 (10^6) mm^4
C) J_0 = 801 (10^6) mm^4
D) J_0 = 919 (10^6) mm^4

PROBLEM 1 C

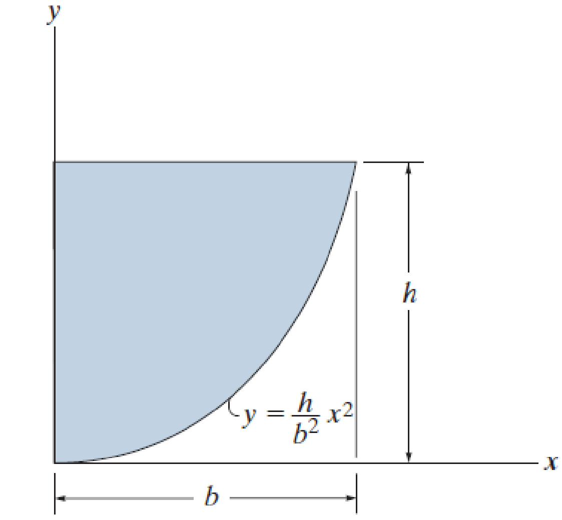
The product of inertia of the area introduced in Part A with respect to the *x*- and *y*-axes is

A) I_{xy} = 155 (10^6) mm^4
B) I_{xy} = 267 (10^6) mm^4
C) I_{xy} = 372 (10^6) mm^4
D) I_{xy} = 480 (10^6) mm^4

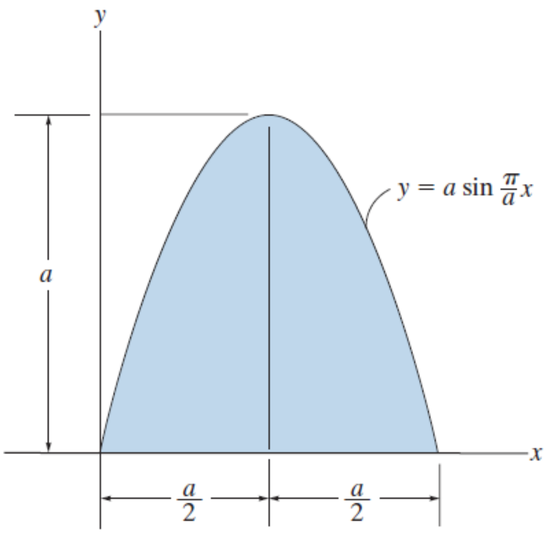
PROBLEM 2

True or false?

1. ( ) The moment of inertia of the shaded area about the x-axis is $I_x = (4/7)bh^3$.
2. ( ) The moment of inertia of the shaded area about the y-axis is $I_y = (2/15)hb^3$.



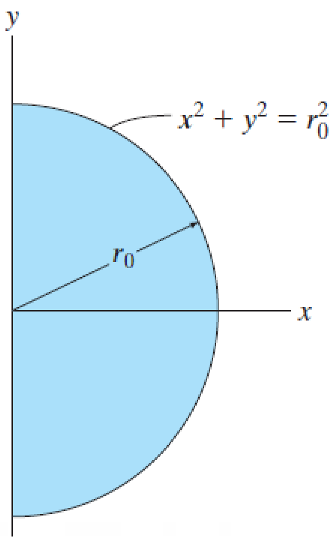
3. ( ) The moment of inertia of the shaded area about the x-axis is $I_x = (4a^4)/(9\pi)$.
4. ( ) The moment of inertia of the shaded area about the y-axis is $I_y = [a^4(\pi^2 - 2)]/\pi^3$.



PROBLEM 3

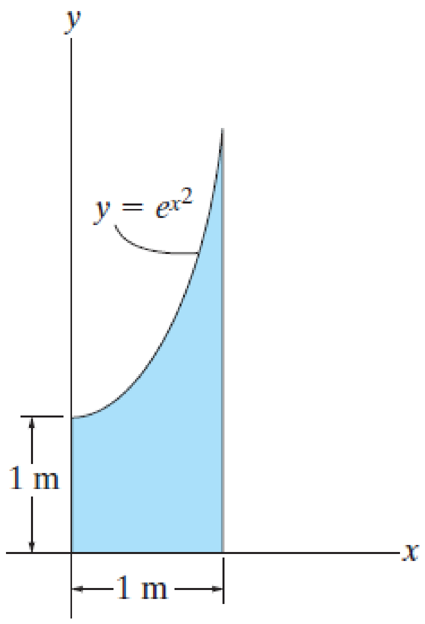
True or false?

1. ( ) The moment of inertia of the shaded area about the x-axis is $I_x = \pi r_0^4/8$.
2. ( ) The moment of inertia of the shaded area about the y-axis is $I_y = \pi r_0^4/4$.



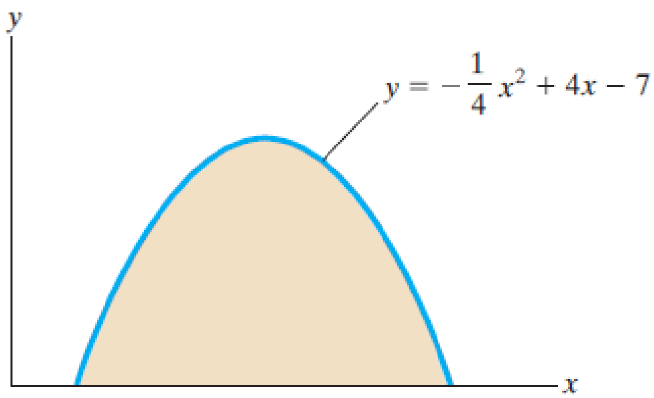
3. ( ) The moment of inertia of the shaded area about the *x*-axis is greater than 1 m^4.

4. ( ) The moment of inertia of the shaded area about the *y*-axis is greater than 1 m^4.



PROBLEM 4A

Determine the radius of gyration relative to the *y*-axis for the area shown.



A) R_y = 1.95 u.l.

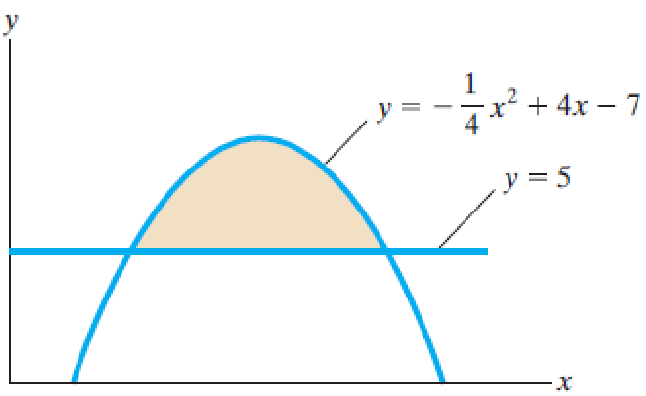
B) R_y = 4.93 u.l.

C) R_y = 8.44 u.l.

D) R_y = 12.1 u.l.

PROBLEM 4B

Determine the radius of gyration relative to the *y*-axis for the shaded area.



A) R_y = 2.77 u.l.

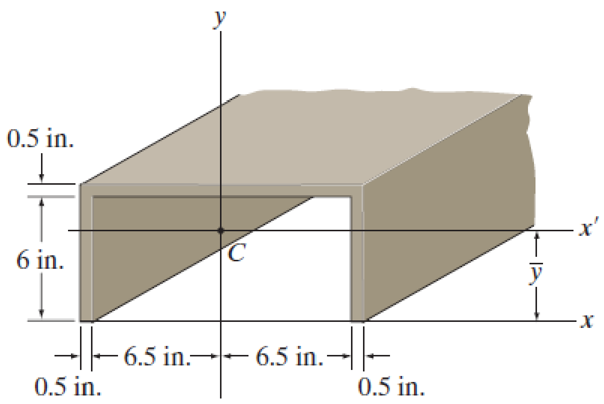
B) R_y = 4.90 u.l.

C) R_y = 8.20 u.l.

D) R_y = 11.8 u.l.

PROBLEM 5 (Hibbeler, 2010, w/ permission)

Determine the moment of inertia of the area of the channel with respect to the *y*-axis.



A) I_y = 273 in.^{4}

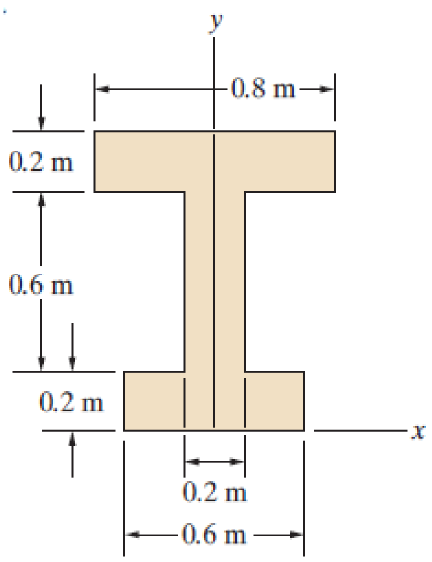
B) I_y = 388 in.^{4}.

C) I_y = 476 in.^{4}

D) I_y = 545 in.^{4}

PROBLEM 6 (Bedford & Fowler, 2008, w/ permission)

Determine the moment of inertia of the section relative to the *x*-axis.



A) I_x = 109.6 (10^9) mm^4

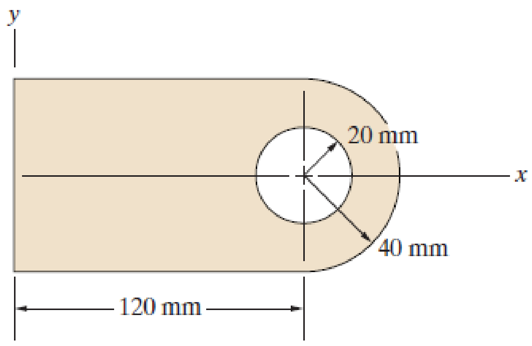
B) I_x = 163.6 (10^9) mm^4

C) I_x = 224.0 (10^9) mm^4

D) I_x = 298.5 (10^9) mm^4

PROBLEM 7 (Bedford & Fowler, 2008, w/ permission)

Determine the moment of inertia of the section relative to the *x*-axis.



A) I_x = 6.0 (10^6) mm^4

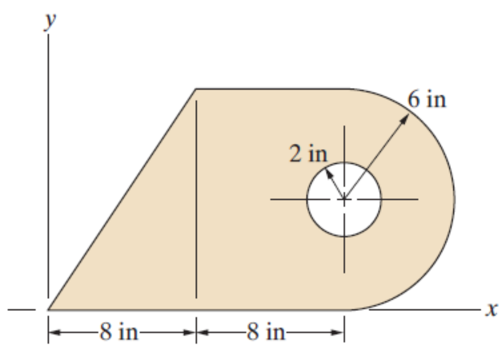
B) I_x = 9.0 (10^6) mm^4

C) I_x = 12.0 (10^6) mm^4

D) I_x = 15.0 (10^6) mm^4

PROBLEM 8 (Bedford & Fowler, 2008, w/ permission)

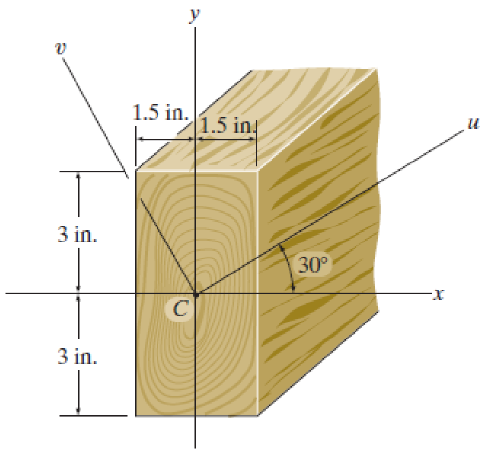
Determine the moment of inertia relative to the *y*-axis.



A) I_y = 8945 in.^{4}
B) I_y = 16,565 in.^{4}
C) I_y = 24,860 in.^{4}
D) I_y = 32,235 in.^{4}

PROBLEM 9 (Hibbeler, 2010, w/ permission)

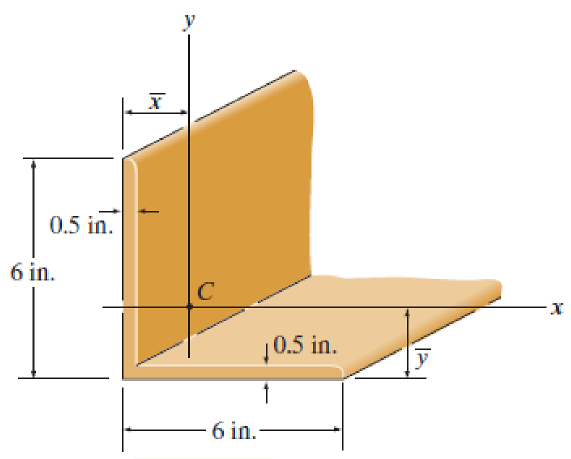
Consider the following rectangular beam section and the hypothetical axes *u* and *v*. True or false?



1. ( ) The moment of inertia I_u of the section about the *u*-axis is I_u = 52.5 in.^{4}
2. ( ) The moment of inertia I_v of the section about the *v*-axis is I_v = 23.6 in.^{4}
3. ( ) The product of inertia I_{uv} of the section relative to axes *u* and *v* is I_{uv} = 17.5 in.^{4}

PROBLEM 10 (Hibbeler, 2010, w/ permission)

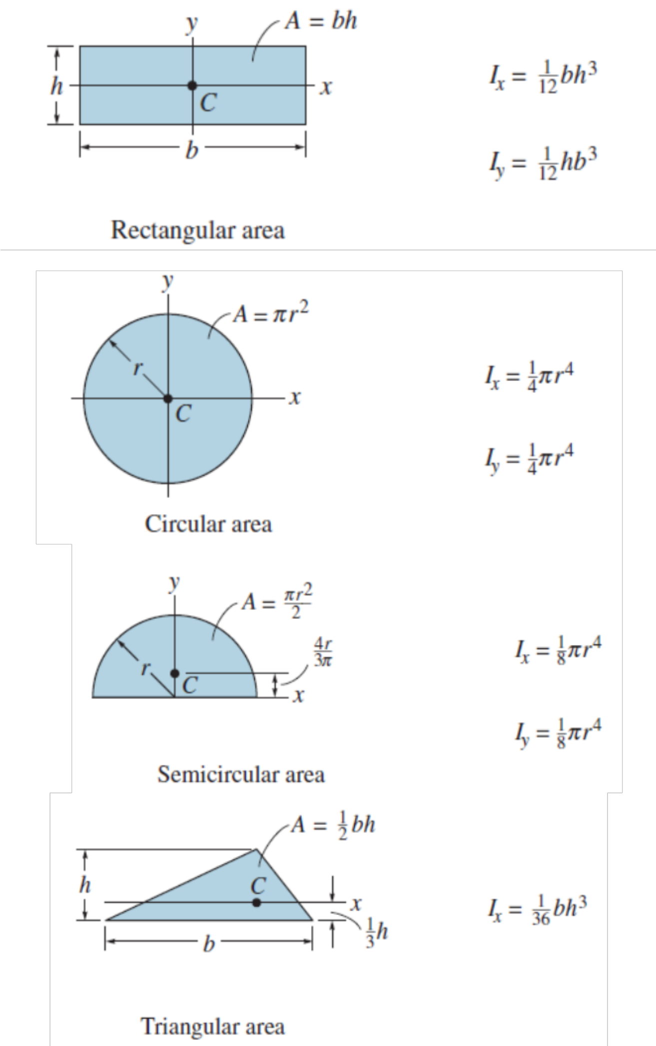
Locate the centroid $\bar{x}$ and $\bar{y}$ of the cross-sectional area and then determine the orientation of the principal axes, which have their origin at the centroid *C* of the area.



A) θ_p = 15° and θ_p = −15°
B) θ_p = 30° and θ_p = −30°
C) θ_p = 45° and θ_p = −45°
D) θ_p = 60° and θ_p = −60°

Additional Information

Figure 1 Moments of inertia for selected geometries



Solutions

P.1 ■ Solution

Part A: The moments of inertia in question are such that, for I_x,

$$I_x = \int_0^2 \int_0^{\sqrt{2}x^{\frac{1}{2}}} y^2 dydx = 2.13\ \text{m}^4 = \boxed{213\ (10^6)\ \text{mm}^4}$$

and, for I_y,

$$I_y = \int_0^2 \int_0^{\sqrt{2}x^{\frac{1}{2}}} x^2\ dydx = 4.57\ \text{m}^4 = \boxed{457\ (10^6)\ \text{mm}^4}$$

■ The correct answer is **A**.

Part B: The polar moment of inertia, J_O, is obtained with the equation

$$J_O = \int_A r^2 dA$$

where r is the radial distance from the origin of the coordinate system to the elemental area dA. Instead of evaluating this integral, however, we could use the relation $r^2 = x^2 + y^2$, so that

$$J_O = \int_A r^2 dA = \int_A (x^2 + y^2) dA = \int_A x^2 dA + \int_A y^2 dA = I_y + I_x$$

That is to say, the polar moment of inertia about the origin equals the sum of the moment of inertia with respect to the x-axis and the moment of inertia with respect to the y-axis. Using the results from Part A, we obtain

$$J_O = 213\ (10^6) + 457\ (10^6) = \boxed{670\ (10^6)\ \text{mm}^4}$$

■ The correct answer is **B**.

Part C: The product of inertia of the area in question is given by

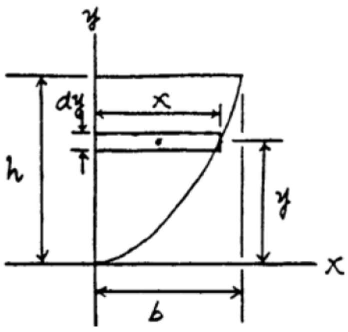
$$I_{xy} = \int_A xy dA = \int_0^2 \int_0^{\sqrt{2}x^{\frac{1}{2}}} xy\, dydx = \boxed{267\ (10^6)\text{mm}^4}$$

■ The correct answer is **B**.

P.2 ■ Solution

1. False. The area of the differential element parallel to the *x*-axis is

$$dA = (h-y)dx = \left(h - \frac{h}{b^2}x^2\right)dx$$



Applying the equation for I_x and performing the integration, we have

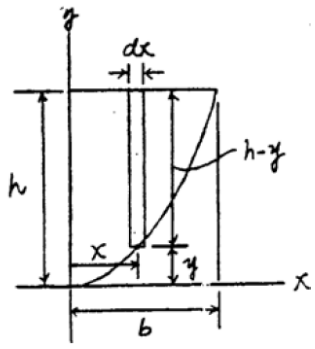
$$I_x = \int_A y^2 dA = \int_0^h y^2 \times \frac{b}{\sqrt{h}} y^{\frac{1}{2}} dy$$

$$\therefore I_x = \frac{b}{\sqrt{h}} \times \frac{2y^{7/2}}{7}\bigg|_{y=0}^{y=h} = \frac{b}{h^{\frac{1}{2}}} \times \frac{2h^{7/2}}{7}$$

$$\therefore I_x = \frac{2}{7}bh^3$$

2. True. The area of the differential element parallel to the *y*-axis is

$$dA = (h-y)dx = \left(h - \frac{h}{b^2}x^2\right)dx$$



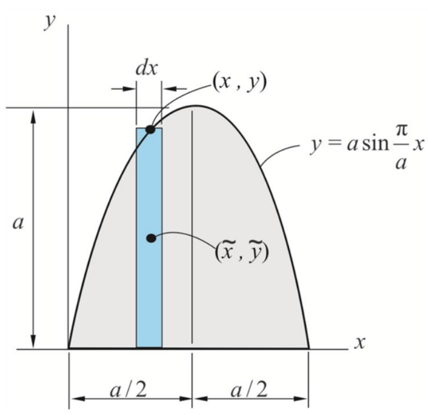
so that, substituting in the expression for I_y and performing the integration, we get

$$I_y = \int_A x^2 \left(h - \frac{h}{b^2}x^2\right)dx = \int_0^b \left(hx^2 - \frac{h}{b^2}x^4\right)dx$$

$$\therefore I_y = \left(\frac{hx^3}{3} - \frac{h}{5b^2}x^5\right)\bigg|_{y=0}^{y=b} = \frac{hb^3}{3} - \frac{hb^3}{5}$$

$$\therefore I_y = \frac{2}{15}hb^3$$

3. True. The differential area element parallel to the *y*-axis is shown in blue.



The contribution of this element to the moment of inertia I_x is given by

$$dI_x = \frac{y^3}{12}dx + (ydx)\left(\frac{y}{2}\right)^2 = \frac{y^3}{12}dx + \frac{y^3}{4}dx = \frac{1}{3}y^3 dx$$

or, substituting y,

$$dI_x = \frac{1}{3}\left(a\sin\frac{\pi}{a}x\right)^3 dx = \frac{a^3}{3}\sin^3\frac{\pi}{a}x\,dx$$

Integrating on both sides, we have

$$\int dI_x = \int_0^a \frac{a^3}{3}\sin^3\frac{\pi}{a}x\,dx = \frac{a^3}{3}\int_0^a \sin\frac{\pi}{a}x\left(1-\cos^2\frac{\pi}{a}x\right)dx$$

Let $\cos(\pi/a)x = t$. Differentiating on both sides, we have

$$-\left(\frac{\pi}{a}\sin\frac{\pi}{a}x\right)\frac{dx}{dt} = 1 \rightarrow \sin\left(\frac{\pi}{a}x\right)dx = -\frac{a}{\pi}dt$$

The bounds of the integral change from $x = 0$ to $t = 1$ and from $x = a$ to $t = -1$. Backsubstituting in the previous integral gives

$$I_x = \frac{a^3}{3}\int_1^{-1}(1-t^2)\left(-\frac{a}{\pi}dt\right) = -\frac{a^4}{3\pi}\int_1^{-1}(1-t^2)dt$$

$$\therefore I_x = -\frac{a^4}{3\pi}\left(t-\frac{t^3}{3}\right)\Bigg|_{t=1}^{t=-1} = -\frac{a^4}{3\pi}\left(-\frac{2}{3}-\frac{2}{3}\right) = \frac{4a^4}{9\pi}$$

4. False. Proceeding similarly with the moment of inertia about the y-axis, the following integral is proposed,

$$I_y = \int x^2 dA = a\int_0^a x^2\sin\frac{\pi}{a}x\,dx$$

$$\therefore I_y = a\left[\frac{2\left(\frac{\pi}{a}\right)x\sin\left(\frac{\pi}{a}\right)x + \left[2-\left(\frac{\pi}{a}\right)^2x^2\right]\cos\left(\frac{\pi}{a}\right)x}{\left(\frac{\pi}{a}\right)^3}\right]\Bigg|_{x=0}^{x=a}$$

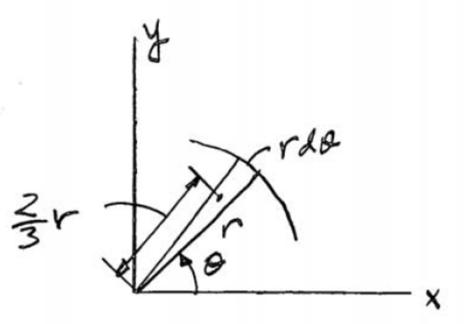
Accordingly,

$$I_y = a\left\{\left[\frac{2\left(\frac{\pi}{a}\right)a\sin\left(\frac{\pi}{a}\right)a + \left[2-\left(\frac{\pi}{a}\right)^2x^2\right]\cos\left(\frac{\pi}{a}\right)a}{\left(\frac{\pi}{a}\right)^3}\right] - \left[\frac{2\left(\frac{\pi}{a}\right)0\sin\left(\frac{\pi}{a}\right)0 + \left[2-\left(\frac{\pi}{a}\right)^2x^2\right]\cos\left(\frac{\pi}{a}\right)0}{\left(\frac{\pi}{a}\right)^3}\right]\right\}$$

$$\therefore I_y = \frac{a^4[(\pi^2-2)-2]}{\pi^3} = \frac{a^4(\pi^2-4)}{\pi^3}$$

P.3 ■ Solution

1. True. A circular cross-section such as the one considered here begs the use of polar coordinates. The area of the differential area element shown below is $dA = (rd\theta)dr$.



In polar coordinates, the y-coordinate is transformed as $y = r\sin\theta$. We can then set up the integral as follows,

$$I_x = \int_A y^2 dA = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^2\sin^2\theta\,rdrd\theta$$

$$\therefore I_x = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3\sin^2\theta\,drd\theta = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3\sin^2\theta\,drd\theta$$

$$\therefore I_x = \frac{r_0^4}{4}\int_{-\pi/2}^{\pi/2} \sin^2\theta \, dr d\theta$$

At this point, note that $\sin^2 2\theta \equiv (1/2)(1-\cos 2\theta)$. Accordingly,

$$I_x = \frac{r_0^4}{4}\int_{-\pi/2}^{\pi/2} \sin^2\theta \, dr d\theta = \frac{r_0^4}{8}\left(\theta - \frac{\sin 2\theta}{2}\right)\Bigg|_{-\pi/2}^{\pi/2} = \frac{\pi r_0^4}{8}$$

2. False. In polar coordinates, the *x*-coordinate is transformed as $x = r\cos\theta$. Substituting this quantity into the equation for the moment of inertia and integrating, it follows that

$$I_y = \int_A x^2 dA = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^2\cos^2\theta \, r dr d\theta$$

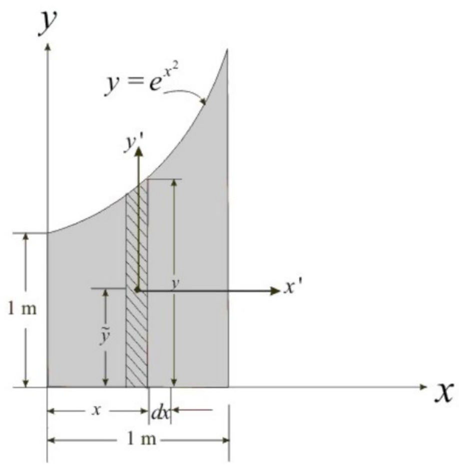
$$\therefore I_y = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3\cos^2\theta \, dr d\theta = \int_{-\pi/2}^{\pi/2}\int_0^{r_0} r^3\cos^2\theta \, dr d\theta$$

$$\therefore I_y = \frac{r_0^4}{4}\int_{-\pi/2}^{\pi/2} \cos^2\theta \, dr d\theta$$

However, $\cos^2\theta \equiv (1/2)(\cos 2\theta + 1)$. Thus,

$$I_y = \frac{r_0^4}{8}\int_{-\pi/2}^{\pi/2} (\cos 2\theta + 1) d\theta = \frac{r_0^4}{8}\left(\theta - \frac{\sin 2\theta}{2}\right)\Bigg|_{-\pi/2}^{\pi/2} = \frac{\pi r_0^4}{8}$$

3. True. Consider the following illustration of the shaded area.



The elemental contribution of the strip to the overall moment of inertia is

$$dI_x = d\bar{I}_x + dA\tilde{y}$$

which, upon substitution of the pertaining variables, becomes

$$dI_x = \frac{dx \times y^3}{12} + y \times dx \times \frac{y^2}{2} = \frac{y^3 dx}{3}$$

The overall moment of inertia follows by integrating the relation above, that is,

$$I_x = \frac{1}{3}\int_0^1 y^3 dx = \frac{1}{3}\int_0^1 \left(e^{x^2}\right)^3 dx = \frac{1}{3}\int_0^1 e^{3x^2} dx$$

There is no simple antiderivative available to evaluate the integral above. Thus, a numerical procedure is in order. Let us apply Simpson's rule,

$$\int_{x_0}^{x_n} f(x)dx \approx \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2}) + y_n]$$

The bounds of the integral are $x = 0$ and $x = 1$. Let the number of intervals be equal to 6, so that the width h of each area element is written as

$$h = \frac{\text{upper limit} - \text{lower limit}}{6} = \frac{1-0}{6} \approx 0.167$$

Then, the following table is prepared.

n	x_n	y_n
0	0	1.000
1	0.167	1.087
2	0.334	1.397
3	0.501	2.123
4	0.668	3.814
5	0.835	8.098
6	1	20.086

We are now ready to evaluate the integral via Simpson's rule, giving

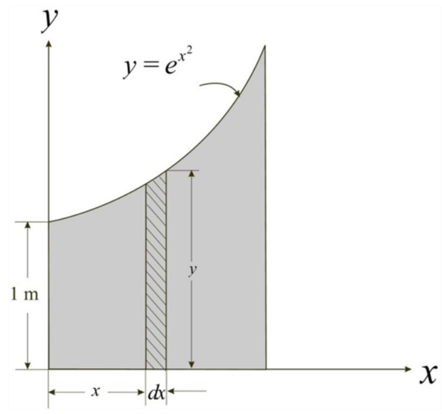
$$\int_{x_0}^{x_n} e^{3x^2}dx \approx \frac{(1/6)}{3}[1.000 + 4(1.087 + 2.123 + 8.098) + 2(1.397 + 3.814) + 20.086] = 4.263$$

Moment of inertia I_x is then

$$I_x = \frac{1}{3}\int_0^1 e^{3x^2}dx = \frac{1}{3} \times 4.263 = 1.421\ \mathrm{m}^4$$

which is greater than 1 m^4, thus implying that statement 3 is true.

4. False. As before, consider an elemental area of thickness *dx*, as shown.



The area of the elemental strip is $dA = ydx$. Then, the moment of inertia of the area about the *y*-axis is

$$I_y = \int_A x^2 dA = \int_0^1 x^2 y dx$$

Substituting $y = e^{x^2}$, we have

$$I_y = \int_0^1 x^2 e^{x^2} dx$$

This integral, much like the previous one, does not lend itself to simple analytical methods. We shall evaluate it numerically using Simpson's rule. Let the number of intervals be equal to 6, so that the width *h* of each area element becomes $h = 1/6 \approx 0.167$. The following table is prepared.

n	x_n	y_n
0	0	0.000
1	0.167	0.029
2	0.334	0.125
3	0.501	0.323
4	0.668	0.697
5	0.835	1.400
6	1	2.718

We now have enough information to evaluate the integral; that is,

$$\int_{x_0}^{x_n} x^2 e^{x^2}dx \approx \frac{(1/6)}{3}[0 + 4(0.029 + 0.323 + 1.400) + 2(0.125 + 0.697) + 2.718] = 0.632\ \mathrm{m}^4$$

which is less than 1 m^4, thus implying that statement 4 is false.

P.4 ■ Solution

Part A: The problem involves a straightforward application of the integral formula for moment of inertia, followed by use of the definition of radius of gyration with respect to the *y*-axis,

$$R_y = \sqrt{\frac{I_y}{A}}$$

where I_y is the moment of inertia about the *y*-axis and *A* is the area of the surface. Before applying this formula, however, we require the points at which the curve intercepts the *x*-axis, namely,

$$y = -\frac{1}{4}x^2 + 4x - 7 = 0 \rightarrow x = 2 \text{ and } 14$$

That is, the parabola in question intercepts the *x*-axis at $x = 2$ and $x = 14$. We can now determine the moment of inertia I_y,

$$I_y = \int_2^{14} \int_0^{-\frac{1}{4}x^2+4x-7} x^2 dydx = 5126\ [\text{u.l.}]^4$$

The area *A* of the surface is, in turn,

$$A = \int_2^{14} \left(-\frac{1}{4}x^2 + 4x - 7\right) dx = 72\ [\text{u.a.}]$$

Finally, the radius of gyration R_y is such that

$$R_y = \sqrt{\frac{5126}{72}} = \boxed{8.44\ [\text{u.l.}]}$$

■ The correct answer is **C**.

Part B: First, we need to locate the points at which the curve intersects the horizontal line,

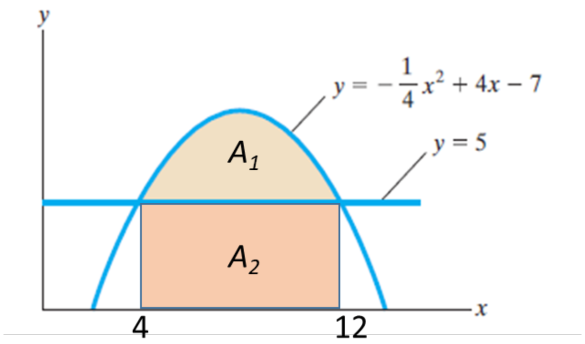
$$5 = -\frac{1}{4}x^2 + 4x - 7 \rightarrow -\frac{1}{4}x^2 + 4x - 12 = 0$$

$$x = 4 \text{ and } 12$$

That is, the curve intersects the line at $x = 4$ and $x = 12$. We can now produce the integral for the moment of inertia,

$$I_y = \int_4^{12} \int_5^{-\frac{1}{4}x^2+4x-7} x^2 dydx = 1433.6\ [\text{u.l}]^4$$

We also require area $A_1 = A - A_2$, as illustrated below.



The total area *A* is such that

$$A = \int_4^{12} \left(-\frac{1}{4}x^2 + 4x - 7\right) dx = 61.33\ [\text{u.a.}]$$

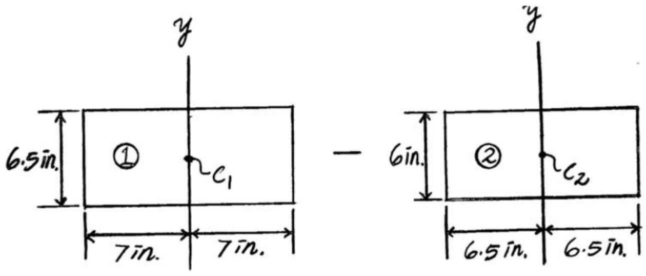
In addition, $A_2 = (12 - 4)\times 5 = 40$ u.a., so that $A_1 = A - A_2 = 61.33 - 40 = 21.33$ u.a. Finally, the radius of gyration R_y is such that

$$R_y = \sqrt{\frac{I_y}{A_1}} = \sqrt{\frac{1434}{21.33}} = \boxed{8.20\ [\text{u.l.}]}$$

■ The correct answer is **C**.

P.5 ■ Solution

The section can be represented as the sum of a solid rectangular area and a negative rectangular area, as shown.



Since the *y*-axis passes through the centroid of both rectangular segments, we have

$$I_x = I_1 - I_2$$

Here, I_1 is such that

$$I_1 = \frac{1}{2}(6.5)(14)^3 = 1486.3 \text{ in.}^4$$

and I_2 equals

$$I_2 = \frac{1}{12}(6)(13)^3 = 1098.5 \text{ in.}^4$$

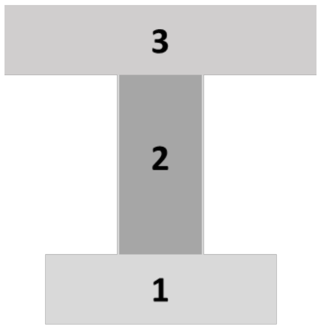
Finally, the required moment of inertia is

$$I_x = I_1 - I_2 = \boxed{388 \text{ in.}^4}$$

■ The correct answer is **B**.

P.6 ■ Solution

The area is divided into 3 rectangles, as shown below.



The total moment of inertia is the sum of the contributions of each part of the section,

$$I_x = I_1 + I_2 + I_3$$

I_1 is determined as

$$I_1 = \frac{(600)(200)^3}{12} = 400\ (10^6) \text{ mm}^4$$

I_2 can be obtained with the parallel-axis theorem,

$$I_2 = \frac{bh^3}{12} + d_y^2 A = \frac{(200)(600)^3}{12} + 500^2(200)(600) = 33{,}600\ (10^6) \text{ mm}^4$$

The same applies to I_3, which is calculated as

$$I_3 = \frac{bh^3}{12} + d_y^2 A = \frac{(800)(200)^2}{12} + 900^2(200)(800) = 129{,}600\ (10^6) \text{ mm}^4$$

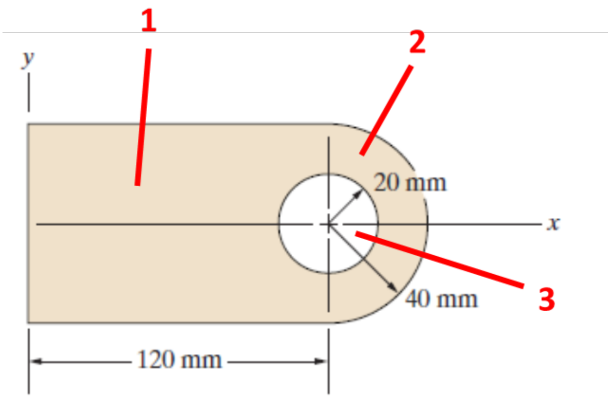
Finally, moment of inertia I_x equals

$$I_x = I_1 + I_2 + I_3 = (400 + 33{,}600 + 129{,}600)(10^6) \text{ mm}^4$$
$$= 163{,}600\ (10^6) \text{ mm}^4 = \boxed{163.6\ (10^9) \text{ mm}^4}$$

■ The correct answer is **B**.

P.7 ■ Solution

The area is subdivided into a rectangle (section 1), a semicircle (2), and a void circle (3), as shown.



Using tabulated results, the moment of inertia of part 1 about the x-axis is

$$I_1 = \frac{(120)(80)^3}{12} = 5.12\ (10^6)\ \text{mm}^4$$

Moment of inertia of part 2 about the x-axis is

$$I_2 = \frac{\pi R^4}{8} = \frac{\pi (40)^4}{8} = 1.01\ (10^6)\ \text{mm}^4$$

Finally, we have the moment of inertia of part 3, which, being a void area, must be subtracted from the total moment of inertia,

$$I_3 = -\frac{\pi R^4}{4} = -\frac{\pi (40)^4}{4} = -0.126\ (10^6)\ \text{mm}^4$$

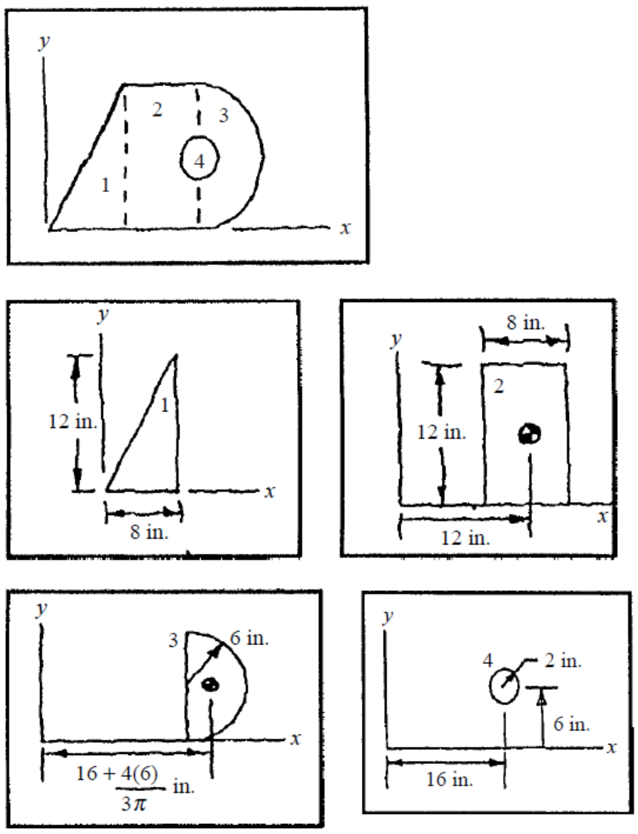
The total moment of inertia, I_x, is given by

$$I_x = I_1 + I_2 - I_3 = (5.12 + 1.01 - 0.126) \times 10^6 = \boxed{6.0\ (10^6)\ \text{mm}^4}$$

■ The correct answer is **A**.

P.8 ■ Solution

We divide the composite area into a triangle (section 1), a rectangle (2), a half-circle (3), and a circular cutout (4); see below.



The contribution to the moment of inertia due to triangle 1 is

$$I_1 = \frac{1}{4} \times 12 \times 8^3 = 1536\ \text{in.}^4$$

The contribution owing to rectangle 2, in turn, follows from the parallel-axis theorem,

$$I_2 = \frac{1}{12} \times 12 \times 8^3 + 12^2 \times 8 \times 12 = 14{,}336 \text{ in.}^4$$

The contribution relative to half-circle 3 is

$$I_3 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(6)^4 + \left(16 + \frac{4 \times 6}{3\pi}\right)^2 \times \frac{1}{2}\pi(6)^2 = 19{,}593 \text{ in.}^4$$

Finally, the contribution from circular cutout 4 is

$$I_4 = -\frac{1}{4} \times \pi(2)^4 + (16)^2 \times \pi \times 2^2 = -3230 \text{ in.}^4$$

which, being a hole, is associated with a negative sign. The final moment of inertia is

$$I_y = I_1 + I_2 + I_3 + I_4 = 1536 + 14{,}336 + 19{,}593 - 3230 = \boxed{32{,}235 \text{ in.}^4}$$

■ The correct answer is **D**.

P.9 ■ Solution

1. False. Since the rectangular beam cross-sectional area is symmetrical about the *x*- and *y*-axes, the product of inertia equals zero, $I_{xy} = 0$. As for the moments of inertia relative to the *x*- and *y*-axes, we have

$$I_x = \frac{(3)(6)^3}{12} = 54 \text{ in.}^4 \quad ; \quad I_y = \frac{(6)(3)^3}{12} = 13.5 \text{ in.}^4$$

We can then apply the following equations for the moments of inertia with respect to the *u* and *v* axes.

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

Substituting $I_x = 54$ in.^{4}, $I_y = 13.5$ in^4, $I_{xy} = 0$, and $\theta = 30°$, it follows that

$$I_u = \frac{54 + 13.5}{2} + \frac{54 - 13.5}{2}\cos 2(30) - 0\sin 2(30) = 43.9 \text{ in.}^4$$

2. True. Indeed,

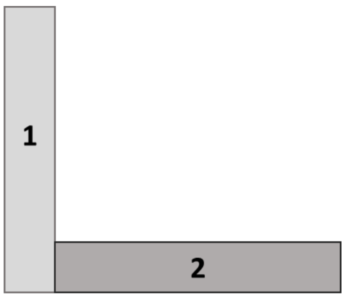
$$I_v = \frac{54 + 13.5}{2} - \frac{54 - 13.5}{2}\cos 2(30) + 0\sin 2(30) = 23.6 \text{ in.}^4$$

3. True. Indeed,

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta = \frac{54 - 13.5}{2}\sin 2(30) + 0\cos 2(30) = 17.5 \text{ in}^4$$

P.10 ■ Solution

The area is subdivided into two rectangles, as illustrated below.



The coordinates of the centroid are determined as

$$x = \frac{\sum \tilde{x}A}{\sum A} = \frac{\underbrace{0.25(0.5)(6)}_{\text{Area 1}} + \underbrace{3.25(5.5)(0.5)}_{\text{Area 2}}}{0.5(6) + 5.5(0.5)} = 1.68 \text{ in}$$

$$y = \frac{\sum \tilde{y}A}{\sum A} = \frac{0.25(0.5)(5.5) + 3(6)(0.5)}{0.5(5.5) + 6(0.5)} = 1.68 \text{ in.}$$

Hence, the location of the centroid C is $C(\bar{x},\bar{y}) = (1.68, 1.68)$. We are now ready to determine the moments of inertia relative to the axes with origin at the centroid. For I_x, we have

$$I_x = I_1 + I_2$$

where I_1 pertains to cross-sectional area 1 and I_2 pertains to cross-sectional area 2; that is,

$$I_1 = \left[\frac{(0.5)(6)^3}{12} + 0.5(6)(3 - 1.68)^2\right] = 14.23 \text{ in.}^4$$

$$I_2 = \left[\frac{(5.5)(0.5)^3}{12} + 5.5(0.5)(1.68 - 0.25)^2\right] = 5.68 \text{ in.}^4$$

so that $I_x = 14.23 + 5.68 = 19.91 \text{ in.}^4$ Similarly, for I_y, we have

$$I_y = I_1 + I_2$$

where I_1 and I_2 are given by

$$I_1 = \left[\frac{6(0.5)^3}{12} + 0.5(6)(1.68 - 0.25)^2\right] = 6.20 \text{ in.}^4$$

$$I_2 = \left[\frac{0.5(5.5)^3}{12} + 0.5(5.5)(1.57)^2\right] = 13.71 \text{ in.}^4$$

Therefore, $I_y = 6.20 + 13.71 = 19.91 \text{ in.}^4$ We also require the product of inertia I_{xy},

$$I_{xy} = 6(0.5)(-1.435)(1.32) + 5.5(0.5)(1.57)(-1.435) = -11.92 \text{ in.}^4$$

Finally, we establish the orientation of the principal axes,

$$\tan 2\theta_p = -\frac{2I_{xy}}{(I_x - I_y)} = -\frac{2(-11.92)}{\underbrace{(19.91 - 19.91)}_{=0}} \to \infty$$

Clearly, then, $2\theta_p = \arctan(\infty) \to 90^\circ \text{ and } -90^\circ$, which implies that

$$2\theta_p = 90^\circ \to \boxed{\theta_p = 45^\circ \text{ and } -45^\circ}$$

■ The correct answer is **C**.

Answer Summary

Problem 1	**1A**	A
	1B	B
	1C	B
Problem 2		T/F
Problem 3		T/F
Problem 4	**4A**	C
	4B	C
Problem 5		B
Problem 6		B
Problem 7		A
Problem 8		D
Problem 9		T/F
Problem 10		C

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