Montogue

AP Calculus AB

20+2 Practice Problems

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No calculators allowed!

→ SECTION I

1. $\lim\limits_{x\to 0}\dfrac{\sin^3(4x)}{x^3}$

(A) -4
(B) 1
(C) 4
(D) 64

2. What is $\lim\limits_{x\to \ln 3} f(x)$, if

$$f(x)=\begin{cases} e^x, \text{if } x>\ln 3 \\ 6-e^x, \text{if } x\le \ln 3\end{cases}$$

(A) Nonexistent.
(B) 3
(C) 6
(D) e^3

3. Consider the equation

$$e^{xy}-3x^2-4y^2=0$$

The value of dy/dx at (1,0) is:

(A) 4
(B) 6
(C) 8
(D) 10

4. Find dy/dx for the expression defined below.

$$y^3 = (x+5)^2 (2x-1)^3$$

(A) $\frac{y}{3}\left(\frac{2}{x+5}+\frac{3}{2x-1}\right)$

(B) $\frac{y}{3}\left(\frac{2}{x+5}+\frac{6}{2x-1}\right)$

(C) $\frac{3}{y}\left(\frac{2}{x+5}+\frac{6}{2x-1}\right)$

(D) $\frac{1}{3y^2}\left(\frac{2}{x+5}+\frac{3}{2x-1}\right)$

5. Which of the following alternatives contains all values of x for which $f(x)=\frac{x^5}{20}-\frac{x^3}{2}$ is concave down?

(A) $-\sqrt{3} < x < 0 \cup x > \sqrt{3}$

(B) $x < -\sqrt{3} \cup 0 < x < \sqrt{3}$

(C) $-\sqrt{3} < x < \sqrt{3}$

(D) $-1 < x < \sqrt{3}$

6. Find the value of c that satisfies Rolle's theorem for $f(x)$ defined below on the interval [0, 6].

$$f(x)=\frac{x^2-6x+8}{x^2-6x+5}$$

(A) 1

(B) 2

(C) 4

(D) No such value exists.

7. If $y = \sin px$, the value of $d^{78}y/dx^{78}$ is:

(A) $-p^{78}\sin px$

(B) $p^{78}\sin px$

(C) $p^{78}\cos px$

(D) $-p^{78}\cos px$

8. If $g(x)$ as defined below is differentiable for all real values, what is the value of n?

$$g(x)=\begin{cases} mx^3+2nx^2+13, & x\le 1 \\ 2mx^2+nx+3, & x>1 \end{cases}$$

(A) 2

(B) 5

(C) 10

(D) 15

9. The side of a cube is increasing at a rate of 2 centimeters per second. At the instant when the side of the cube is 8 cm long, what is the rate of change (cm^2/sec) of the surface area of the cube? Assume the initial length of the cube was zero.

(A) 96
(B) 192
(C) 288
(D) 384

10. The differentiable function g is defined for all real numbers x. Values of g and g' for various values of x are given in the following table. Regarding the behavior of g in the interval [0, 6], which of the following is true?

x	0	$0 < x < 2$	2	$1 < x < 4$	4	$2 < x < 6$	6
$g(x)$	-9	+	-7	$-$	-2	$-$	-4
$g'(x)$	5	+	0	+	0	$-$	-3

(A) $g(x)$ has a relative maximum at $x = 4$.
(B) $g(x)$ has relative maxima at $x = 2$ and 4.
(C) $g(x)$ has relative maxima at $x = 2$ and 6.
(D) $g(x)$ has relative maxima at $x = 2$, 4, and 6.

11. Find the slope of the normal line to $y = x + \cos xy$ at (0,1).

(A) -1
(B) 0
(C) 1
(D) 2

12. $\int x\sqrt{2x}dx$

(A) $\frac{\sqrt{2}}{5}x^2 + C$

(B) $\frac{\sqrt{2}}{5}x^{3/2} + C$

(C) $\frac{\sqrt{2}}{5}x^{5/2} + C$

(D) $\frac{2\sqrt{2}}{5}x^{5/2} + C$

13. $\int_0^2 \frac{2x^2dx}{\sqrt{9-x^3}}$

(A) $2\sqrt{2}/3$
(B) 4/3
(C) 8/3
(D) 16/3

14. $\int_1^{e^3} \frac{\ln^2 x}{x}dx$

(A) 4
(B) 9/2
(C) $(e^3 - 1)/2$
(D) $e^3/2$

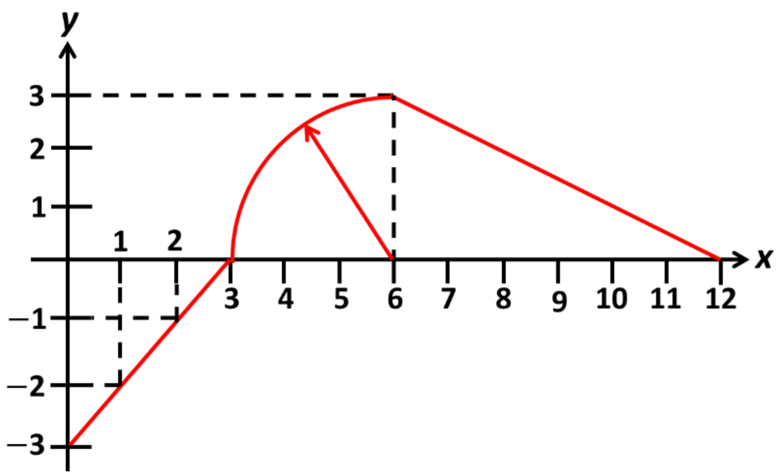
15. $\int_0^{\pi/2} \sin^5 xdx$

(A) 1/5
(B) 2/5
(C) 7/15
(D) 8/15

16. If $\int_{-K}^{K} |2x|dx = 20$ and $K > 0$, the value of K is

(A) $\sqrt{5}$
(B) $\sqrt{10}$
(C) $2\sqrt{5}$
(D) 10

17. If $F(x) = \int_1^x f(x)dx$, where $f(x)$ is plotted below, the value of $F(6)$ is:



(A) $-\frac{9}{2}+\frac{9\pi}{4}$ **(B)** $-2+\frac{9\pi}{4}$ **(C)** $-\frac{9}{2}+\frac{9\pi}{2}$ **(D)** $-2+\frac{9\pi}{2}$

18. Functions $g(x)$ and $h(x)$ are differentiable and defined for all values of x.Values of $g(x)$, $g'(x)$, $h(x)$, and $h'(x)$ for some x are tabulated below. Evaluate the integral

$$\int_1^2 h'(g(x))g'(x)dx$$

x	1	2	3	4
$g(x)$	2	3	1	−2
$g'(x)$	1	3	−3	−1
$h(x)$	−1	1	6	1
$h'(x)$	−2	4	5	0

(A) 4
(B) 5
(C) 6
(D) 7

19. Given $g(x)$ as defined below, what is the value of the integral of $g(x)$ from −2 to $\pi/2$?

$$g(x)=\begin{cases} x^2-3x, \ x<0 \\ \cos(3x), \ x\geq 0 \end{cases}$$

(A) $\frac{23}{3}$
(B) 8
(C) $\frac{25}{3}$
(D) $\frac{26}{3}$

20. For which of the following differential equations will a slope field show nothing but negative slopes in the fourth quadrant?

(A) $\frac{dy}{dx}=-\frac{x}{y}$

(B) $\frac{dy}{dx}=xy+5$

(C) $\frac{dy}{dx}=xy^2-2$

(D) $\frac{dy}{dx}=\frac{y}{x^2}-3$

SECTION II

	
Use a calculator for Problem 1!	**No calculators allowed in Problem 2!**

Problem 1. A particle moves along the x-axis following the function $x(t)$ indicated below, where x is measured in feet and t in seconds.

$$x(t) = t^3 - 13t^2 + 40t$$

A. Find the velocity and acceleration at time t.
B. Determine the velocity and acceleration at time t = 3 s. Is the particle speeding up or slowing down?
C. When is the particle at rest?
D. Determine the total distance travelled by the particle during the first 10 seconds.

Problem 2. Water is dripping at a rate of 5π cm^3/s into a conical tank that has a base diameter of 16 cm and a height of 50 cm.
A. Express the volume of the tank in terms of its radius.
B. At one point in the filling process, the radius of the water surface equals 5 cm. On this instant in time, at what rate (cm/s) will the radius of the water surface be expanding?
C. How fast is the height of the water in the tank increasing when the radius of the water surface equals 5 cm?

Solutions

→ Section I

1.D.

$$\lim_{x\to 0}\frac{\sin^3(4x)}{x^3}=\lim_{4x\to 0}4^3\frac{\sin^3(4x)}{4^3x^3}=4^3\lim_{4x\to 0}\frac{\sin^3(4x)}{4^3x^3}$$

$$\therefore 4^3\lim_{4x\to 0}\frac{\sin^3(4x)}{4^3x^3}=4^3\lim_{4x\to 0}\left[\frac{\sin(4x)}{4x}\right]^3$$

Substituting $4x = u$,

$$4^3\lim_{4x\to 0}\left[\frac{\sin(4x)}{4x}\right]^3=4^3\underbrace{\lim_{u\to 0}\left(\frac{\sin u}{u}\right)^3}_{=1}=\boxed{64}$$

2.B. As in every limit problem involving piecewise functions, the key is to evaluate both one-sided limits,

$$\lim_{x\to(\ln 3)^+}f(x)=e^{\ln 3}=3$$

$$\lim_{x\to(\ln 3)^-}f(x)=6-e^{\ln 3}=3$$

Since the one-sided limits are equal, we conclude that $\lim_{x\to\ln 3}f(x)$ = 3.

3.B. Applying implicit differentiation, we obtain

$$e^{xy}-3x^2-4y^2=0\to e^{xy}\left(x\frac{dy}{dx}+y\right)-6x-8y\frac{dy}{dx}=0$$

$$\therefore e^{xy}x\frac{dy}{dx}+e^{xy}y-6x-8y\frac{dy}{dx}=0$$

$$\therefore\left(e^{xy}x-8y\right)\frac{dy}{dx}=6x-e^{xy}y$$

$$\therefore\frac{dy}{dx}=\frac{6x-e^{xy}y}{e^{xy}x-8y}$$

Substituting (x = 1, y = 0) brings to

$$\left.\frac{dy}{dx}\right|_{(x,y)=(1,0)}=\frac{6\times 1-e^{1\times 0}\times 0}{e^{1\times 0}\times 1-8\times 0}=\boxed{6}$$

4.B. The derivative can be easily obtained through a combination of logarithmic differentiation and implicit differentiation,

$$y^3=(x+5)^2(2x-1)^3\to\ln y^3=\ln\left[(x+5)^2(2x-1)^3\right]$$

$$\therefore\ln y^3=2\ln(x+5)+3\ln(2x-1)$$

$$\therefore \frac{d(3\ln y)}{dx} = \frac{d}{dx}\left[2\ln(x+5)\right] + \frac{d}{dx}\left[3\ln(2x-1)\right]$$

$$\therefore \frac{3}{y}\frac{dy}{dx} = \frac{2}{x+5} + \frac{6}{2x-1}$$

$$\therefore \frac{dy}{dx} = \boxed{\frac{y}{3}\left(\frac{2}{x+5} + \frac{6}{2x-1}\right)}$$

5.B. A function is concave down when its second derivative is less than zero. In the case at hand, we have

$$f(x) = \frac{x^5}{20} - \frac{x^3}{2} \to f'(x) = \frac{x^4}{4} - \frac{3x^2}{2}$$

and

$$f'(x) = \frac{x^4}{4} - \frac{3x^2}{2} \to f''(x) = x^3 - 3x$$

Setting $f''(x)$ to zero and solving for x, we obtain

$$f''(x) = x^3 - 3x = 0 \to x(x^2 - 3) = 0$$

$$\therefore x(x - \sqrt{3})(x + \sqrt{3}) = 0$$

That is, the graph of $f''(x)$ should cross the x-axis at three abscissae, namely $x = 0$, $-\sqrt{3}$, and $\sqrt{3}$. In between these points, $f''(x)$ should be alternately positive and negative. Take an x value slightly greater than $\sqrt{3}$, such as $x = 2$. The value of $f''(2)$ is

$$f''(2) = 2^3 - 3\times 2 = 2$$

Since $f''(2) > 0$, it follows that $f''(x) > 0$ for all $x \in (\sqrt{3}, \infty)$. As a consequence, we can also infer that $f''(x) < 0$ for $x \in (0, \sqrt{3})$, $f''(x) > 0$ for $x \in (-\sqrt{3}, 0)$, and $f''(x) < 0$ for $x \in (-\infty, -\sqrt{3})$. The interval for which $f(x)$ will be concave down is $x < -\sqrt{3} \cup 0 < x < \sqrt{3}$.

6.D. Factor $f(x)$ and you'll obtain

$$f(x) = \frac{x^2 - 6x + 8}{x^2 - 6x + 5} = \frac{(x-2)(x-4)}{(x-1)(x-5)}$$

The appearance of the denominator indicates that there are discontinuities at $x = 1$ and $x = 5$. Since there are discontinuities in interval [0, 6], Rolle's theorem cannot be applied.

7.A. In general, the n-th derivative of $y = \sin nx$ follows the patterns outlined in the table below.

$\frac{d^n y}{dx^n} = p^n \cos px$	$n = 1, 5, 9, \ldots$

$\frac{d^n y}{dx^n} = -p^n \sin px$	n = 2, 6, 10, ...
$\frac{d^n y}{dx^n} = -p^n \cos px$	n = 3, 7, 11, ...
$\frac{d^n y}{dx^n} = p^n \sin px$	n = 4, 8, 12, ...

The derivative in question belongs to the case illustrated in the second row; that is,

$$\boxed{\frac{d^{78} y}{dx^{78}} = -p^{78} \sin px}$$

8.B. The key to solve this problem is to observe that, if $g(x)$ is to be differentiable for all values of x, it must be so at x = 1, which is the abscissa that separates the two expressions of piecewise function g. For this to be the case, both pieces of g must be equal at x = 1, and both pieces of the first derivative $g'(x)$ must be equal at x = 1. Applying the first requirement, we have

$$m \times 1^3 + 2n \times 1^2 + 13 = 2m \times 1^2 + n \times 1 + 2 \rightarrow m + 2n + 13 = 2m + n + 3$$
$$\therefore m - n = 10 \text{ (I)}$$

To apply the second requirement, we derive $g(x)$,

$$g'(x) = \begin{cases} 3mx^2 + 4nx, \ x \le 1 \\ 4mx + n, \ x > 1 \end{cases}$$

and then equate the two pieces of $g'(x)$ at x = 1,

$$3m \times 1^2 + 4n \times 1 = 4m \times 1 + n \rightarrow 3m + 4n = 4m + n$$
$$\therefore m - 3n = 0$$
$$\therefore m = 3n \text{ (II)}$$

Substituting m in (I), we find that

$$m - n = 4 \rightarrow (3n) - n = 10$$
$$\therefore \boxed{n = 5}$$

9.B. If $L(t)$ is the side of the cube at time t, we may write

$$\frac{dL}{dt} = 2$$

or, integrating,

$$\frac{dL}{dt} = 2 \rightarrow L(t) = 2t + C$$

Since $L(0)$ = 0, it is easy to see that C = 0 and $L(t)$ = $2t$. The side of the cube will equal 8 cm at t = 4 s. The surface area $S(t)$ of the cube is described by the expression

$$S(t)=6\left[L(t)\right]^2=6\times(2t)^2=24t^2$$

and varies at a rate dS/dt such that

$$S(t)=24t^2\rightarrow\frac{dS(t)}{dt}=48t$$

so that, at t = 4 s,

$$\left.\frac{dS}{dt}\right|_{t=4}=48\times 4=\boxed{192\text{ cm}^2/\text{s}}$$

10.A. As can be seen, $g'(x)$ equals zero at x = 2 and x = 4. However, only x = 4 corresponds to a local maximum, because $g'(x)$ shifts sign from positive to negative close to this value of x.

11.A. Using implicit differentiation, we find dy/dx,

$$\frac{dy}{dx}=1-\left(y+x\frac{dy}{dx}\right)\sin xy$$

Notice that the second term on the right-hand side will yield zero once we substitute x = 0, so there's no need to solve for dy/dx. We can straightforwardly substitute (x = 0, y = 1), giving

$$\frac{dy}{dx}=1-\cancel{\left(1+0\times\frac{dy}{dx}\right)\sin(0\times 1)}=1$$

The slope of the tangent line at (0,1) is 1. It follows that the slope of the normal line at (0,1) is -1/(1) = -1, that is, the negative reciprocal of the slope of the tangent line.

12.D. First, manipulate the integrand by noting that $\sqrt{x}=x^{1/2}$,

$$\int x\sqrt{2x}dx=\int x\times\sqrt{2}x^{1/2}dx=\int\sqrt{2}x^{3/2}dx$$

Next, apply the power rule,

$$\int x^n dx=\frac{x^{n+1}}{n+1}+C$$

$$\therefore\int\sqrt{2}x^{3/2}dx=\frac{\sqrt{2}x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+C=\boxed{\frac{2\sqrt{2}}{5}x^{5/2}+C}$$

13.C. Let $u=9-x^3$. Its derivative is

$$\frac{du}{dx}=-3x^2\rightarrow x^2dx=-\frac{du}{3}$$

so that, substituting in the integrand,

$$\int_0^2\frac{2x^2dx}{\sqrt{9-x^3}}=\int_0^2\frac{-2\times(du/3)}{\sqrt{u}}$$

$$\therefore\int_0^2\frac{2x^2dx}{\sqrt{9-x^3}}=-\frac{2}{3}\int_0^2 u^{-1/2}du$$

$$\therefore \int_0^2 \frac{2x^2 dx}{\sqrt{5-x^3}} = -\frac{2}{3}\left(\left.\frac{u^{1/2}}{1/2}\right|_{u(0)}^{u(2)}\right)$$

Noting that $u(0) = 9 - 0^3 = 9$ and $u(2) = 9 - 2^3 = 1$, we ultimately obtain

$$\int_0^2 \frac{2x^2 dx}{\sqrt{5-x^3}} = -\frac{4}{3}\left(\left.u^{1/2}\right|_9^1\right) = -\frac{4}{3}\left(1^{1/2} - 9^{1/2}\right)$$

$$\therefore \int_0^2 \frac{2x^2 dx}{\sqrt{5-x^3}} = -\frac{4}{3}(1-3)$$

$$\therefore \int_0^2 \frac{2x^2 dx}{\sqrt{5-x^3}} = \boxed{\frac{8}{3}}$$

14.B. Let $u = \ln x$. Thus, $du = dx/x$. Substituting in the integral brings to

$$\int_1^{e^3} \frac{\ln^2 x}{x} dx = \int_1^{e^3} (\ln x)^2 \, du = \int_1^{e^3} u du$$

$$\therefore \int_1^{e^3} \frac{\ln^2 x}{x} dx = \left.\frac{u^2}{2}\right|_{u(1)}^{u(e^3)}$$

where

$$u\left(e^3\right) = \ln e^3 = 3$$

$$u(1) = \ln 1 = 0$$

so that

$$\int_1^{e^3} \frac{\ln^2 x}{x} dx = \frac{3^2}{2} - \frac{0^2}{2} = \boxed{\frac{9}{2}}$$

15.D. We begin with some quick algebra,

$$\int_0^{\pi/2} \sin^5 x dx = \int_0^{\pi/2} \sin^4 x \sin x dx = \int_0^{\pi/2} \left(1-\cos^2 x\right)^2 \sin x dx$$

At this point, we use the substitution $u = \cos x$, so that

$$\int_0^{\pi/2} \left(1-\cos^2 x\right)^2 \sin x dx = \int_0^{\pi/2} \left(1-u^2\right)^2 (-du)$$

$$\therefore \int_0^{\pi/2} \left(1-\cos^2 x\right)^2 \sin x dx = -\int_0^{\pi/2} \left(1-2u^2+u^4\right) du$$

$$\therefore \int_0^{\pi/2} \left(1-\cos^2 x\right)^2 \sin x dx = -\left.\left(\frac{u^5}{5} - \frac{2u^3}{3} + u\right)\right|_{u(0)}^{u(\pi/2)}$$

Here, $u(\pi/2) = \cos(\pi/2) = 0$ and $u(0) = 1$, giving

$$\int_0^{\pi/2} \left(1-\cos^2 x\right)^2 \sin x dx = -\left[0 - \left(\frac{1^5}{5} - \frac{2\times 1^3}{3} + 1\right)\right]$$

$$\therefore \int_0^{\pi/2} \left(1-\cos^2 x\right)^2 \sin x dx = \frac{1}{5} - \frac{2}{3} + 1 = \frac{3-10+15}{15} = \boxed{\frac{8}{15}}$$

16.B. Since $y = |3x|$ is symmetrical with respect to the y-axis, the integral can be rewritten as

$$\int_{-K}^{K} |2x| dx = 2\int_0^K 2x dx = \cancel{2} \times \frac{2x^2}{\cancel{2}}\Bigg|_0^K = 20$$

$$\therefore 2x^2\Big|_0^K = 2K^2 = 20$$

$$\therefore K^2 = 10$$

$$\therefore \boxed{K = \sqrt{10}}$$

17.B. With $x = 6$, we have

$$F(6) = \int_1^6 f(x) dx$$

The integral of $f(x)$ from $x = 1$ to $x = 6$ can be divided into two parts. In $x \in (1, 3)$, we have a triangle of area

$$A_1 = \frac{-2 \times (3-1)}{2} = -2$$

The negative sign is used because the area is below the x-axis. In $x \in (3, 6)$, we have a quarter-circle of area

$$A_2 = \frac{\pi \times 3^2}{4} = \frac{9\pi}{4}$$

Finally,

$$F(6) = \int_1^6 f(x) dx = A_1 + A_2 = \boxed{-2 + \frac{9\pi}{4}}$$

18.B. By the chain rule, the derivative of $h(g(x))$ is $h'(g(x))g'(x)$. Accordingly, the antiderivative of $h'(g(x))g'(x)$ can only be

$$\int h'(g(x)) g'(x) dx = h(g(x)) + C$$

The definite integral at hand is

$$\int_1^2 h'(g(x)) g'(x) dx = h(g(2)) - h(g(1))$$

$$\therefore \int_2^3 h'(g(x)) g'(x) dx = h(3) - h(2)$$

$$\therefore \int_2^3 h'(g(x)) g'(x) dx = 6 - 1 = \boxed{5}$$

19.C. A piecewise function can be integrated just like any continuous function, so long as we apply the property of definite integrals

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

which is valid for any $c \in (a, b)$. In the present case,

$$\int_{-2}^{\pi} g(x)dx = \int_{-2}^{0}\left(x^2 - 3x\right)dx + \int_{0}^{\pi/2} \cos(3x)\,dx$$

$$\therefore \int_{-2}^{\pi} g(x)dx = \left(\frac{x^3}{3} - \frac{3x^2}{2}\right)\Bigg|_{-2}^{0} + \frac{\sin(3x)}{3}\Bigg|_{0}^{\pi/2}$$

$$\therefore \int_{-2}^{\pi} g(x)dx = \left\{0 - \left[\frac{(-2)^3}{3} - \frac{3\times(-2)^2}{2}\right]\right\} + \left(-\frac{1}{3} - 0\right)$$

$$\therefore \int_{-2}^{\pi} g(x)dx = -\left(-\frac{8}{3} - 6\right) + \left(-\frac{1}{3}\right)$$

$$\therefore \int_{-2}^{\pi} g(x)dx = -\left(-\frac{8}{3} - \frac{18}{3}\right) + \left(-\frac{1}{3}\right)$$

$$\therefore \int_{-2}^{\pi} g(x)dx = \frac{26}{3} - \frac{1}{3} = \boxed{\frac{25}{3}}$$

20.D. In the fourth quadrant, $x > 0$ and $y < 0$. Notice that the right-hand side of equation (D) consists of a y/x^2 term, which will be negative for any coordinate pair such that $x > 0$, $y < 0$. Subtracting 3 from this term will only make it more negative. We surmise that the fourth-quadrant region of a slope field representing equation (D) will contain only negative slopes.

Answer Summary

1	D	**11**	A
2	B	**12**	D
3	B	**13**	C
4	B	**14**	B
5	B	**15**	D
6	D	**16**	B
7	A	**17**	B
8	B	**18**	B
9	B	**19**	C
10	A	**20**	D

→ **Section II**

Problem 1

(A) The velocity is

$$v(t) = \frac{dx}{dt} = \boxed{3t^2 - 26t + 40}$$

The acceleration is

$$a(t) = \frac{d^2x}{dt^2} = \boxed{6t - 26}$$

(B) At time t = 3 s, the velocity is

$$v(3) = 3\times 3^2 - 26\times 3 + 40 = \boxed{-11 \text{ ft/s}}$$

and the acceleration is

$$a(3) = 6\times 3 - 26 = \boxed{-8 \text{ ft}^2/\text{s}}$$

Since $v(3)$ and $a(3)$ have the same sign, the particle is **speeding up** at t = 3 s.

(C) The particle is at rest when $v(t)$ = 0. Setting $v(t)$ = 0 and solving for t brings to

$$v(t) = 3t^2 - 26t + 40 = 0$$

$$\therefore t = \frac{26 \pm \sqrt{676 - 4\times 3\times 40}}{6}$$

$$\therefore t = \frac{26 \pm \sqrt{196}}{6} = \frac{26 \pm 14}{6} = \boxed{\frac{20}{3}\text{s, } 2\text{s}}$$

The particle will be at rest at t = 2 s and t = 20/3 ≈ 6.667 s.

(D) To determine the distance travelled by the particle in the first 10 seconds, we integrate $x(t)$ from t = 0 to t = 10 s,

$$\Delta x = \int_0^{10} x(t)dt = \left(\frac{t^4}{4} - \frac{13t^3}{3} + 20t^2\right)\Bigg|_0^{10}$$

$$\therefore \Delta x = \left(\frac{10^4}{4} - \frac{13\times 10^3}{3} + 20\times 10^2\right) - 0 = (2500 - 4333.33 + 2000) - 0$$

$$\boxed{\Delta x = 166.67 \text{ ft}}$$

Problem 2

(A) The volume of a cone is given by

$$V(r,h) = \frac{1}{3}\pi r^2 h$$

We can state that the general proportion between radius r and height h is maintained as the cone is filled; that is,

$$\frac{r}{h} = \frac{8}{50} = \frac{4}{25} \rightarrow h = \frac{25r}{4}$$

Substituting in $V(r,h)$ gives

$$V(r) = \frac{1}{3}\pi r^2 \times \frac{25r}{4} = \boxed{\frac{25}{12}\pi r^3}$$

(B) Differentiating $V(r)$ with respect to time yields

$$V(r) = \frac{25}{12}\pi r^3 \rightarrow \frac{dV}{dt} = \frac{25}{12}\pi r^2 \frac{dr}{dt}$$

Substituting $dV/dr = 5\pi$, $r = 5$, and solving for dr/dt, we obtain

$$\frac{dV}{dt} = \frac{25}{12}\pi r^2 \frac{dr}{dt} \rightarrow 5\pi = \frac{25}{12}\pi \times 5^2 \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{5\pi \times 12}{25\pi \times 5^2} = \frac{60\pi}{625\pi} = \boxed{\frac{12}{125}\text{ cm/s}}$$

(C) Recall from part (A) that $h = 25r/4$. Differentiating with respect to time, we obtain

$$h = \frac{25r}{4} \rightarrow \frac{dh}{dt} = \frac{25}{4}\frac{dr}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{25}{4} \times \frac{12}{125} = \boxed{\frac{3}{5}\text{cm/s}}$$

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