Montogue

AP Physics 1

24 Extra Practice Problems

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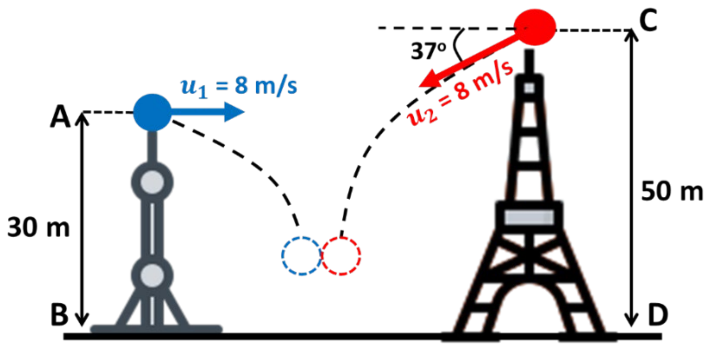
Here are 24 fully solved problems that didn't make it to my AP Physics 1 problem set, either because including these problems would mess with the subject weight distribution or because they were too similar to problems that had already been included in the main problem set. Enjoy!

Extra Problem Distribution

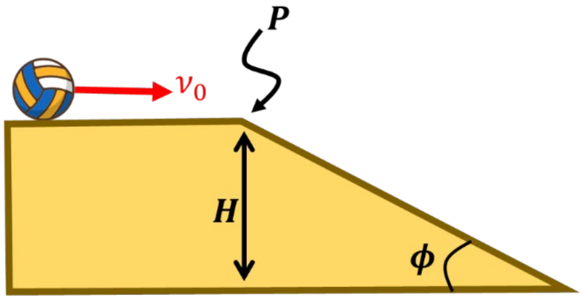
Topic	Problems
Kinematics	1 – 2
Dynamics	3 - 7
Circular Motion	8
Energy	9 - 15
Momentum	16 - 20
Torque and Rotational Motion	21 - 22
DC Circuits	23 - 24

1. AB and CD are two towers in the same vertical plane having heights 30 m and 50 m, respectively. From the top of AB, a body is thrown with velocity u_1 = 8 m/s horizontally towards CD. At the same instant, another body is thrown downwards from the top of CD towards AB at a similar speed u_2 = 8 m/s making an angle of 37° with the horizontal, as shown. If the bodies collide in air, what is the separation between the towers?



(A) 60 m
(B) 75 m
(C) 90 m
(D) 105 m

2. A volleyball rolls over a frictionless flat surface with constant speed v_0 towards an incline, as shown. The minimum value of angle ϕ so that the ball does not hit the incline after being launched from point P is:



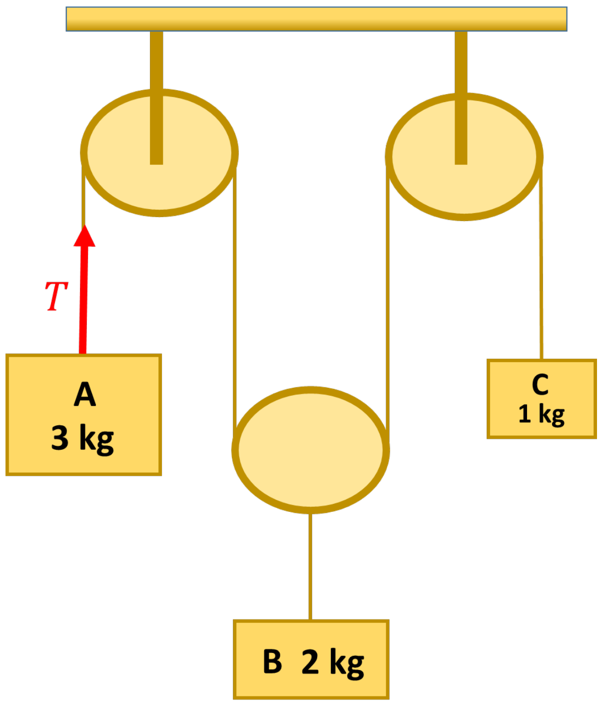
(A) $\frac{\pi}{4}$

(B) $\tan^{-1}\left(\frac{1}{v_0}\sqrt{\frac{gH}{2}}\right)$

(C) $\tan^{-1}\left(\frac{\sqrt{gH}}{v_0}\right)$

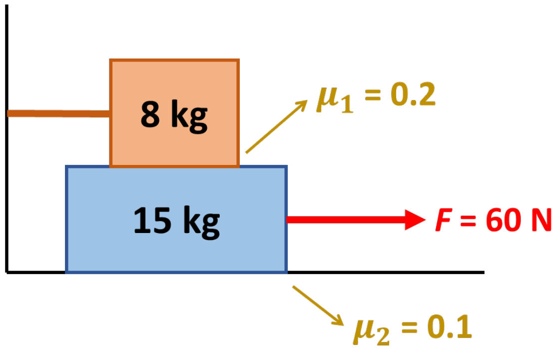
(D) $\tan^{-1}\left(\frac{\sqrt{2gH}}{v_0}\right)$

3. The following figure shows two pulleys attached to a rigid roof and a mobile pulley, all of which have negligible mass. If the masses of blocks A, B and C are 3 kg, 2 kg, and 1 kg, respectively, what is the magnitude of the tension T on block A?



(A) 5.9 N
(B) 11.8 N
(C) 17.7 N
(D) 23.6 N

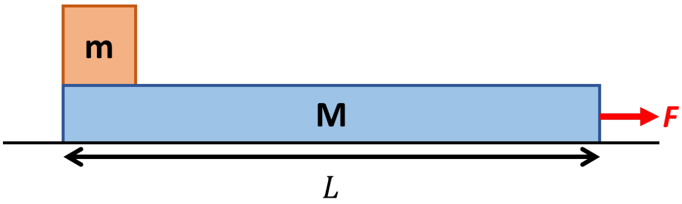
Problems 4 and 5 refer to the following figure.



4. A 8-kg block is placed on top of a 15-kg block. A horizontal force F of 60-N intensity is applied to the lower block, and the upper block is tied to the wall. The coefficient of kinetic friction between the upper block and the lower block is $\mu_1 = 0.2$, while the coefficient of kinetic friction between the lower block and the ground is $\mu_2 = 0.1$, as shown. If the upper block does not slide relatively to the lower block, the tension in the string is, most nearly:
(A) 7.84 N
(B) 15.7 N
(C) 18.3 N
(D) 21.1 N

5. The magnitude of the acceleration of the 15-kg block is, most nearly:
(A) 0.78 m/s^2
(B) 1.41 m/s^2
(C) 1.87 m/s^2
(D) 2.11 m/s^2

Problems 6 and 7 refer to the following figure.



6. A small block of mass m = 1 kg is initially at rest on one end of a board of length L = 2 m and mass M = 2 kg, as shown. The coefficients of static and kinetic friction between the block and the board are μ_s = 0.4 and μ_k = 0.3, respectively. Of the following choices, what is the greatest force F with which the board can be dragged forward on a frictionless plane without having the block slide relatively to the board?

(A) 3.3N
(B) 4.5 N
(C) 5.9 N
(D) 6.3 N

7. What is the approximate time required for the block to fall from the opposite end of the board if the board is being dragged forward with a force F = 6.94 N?

(A) 1.25 s
(B) 1.75 s
(C) 2.75 s
(D) 3.25 s

8. Two particles P and Q are set in motion along a circular trajectory in the same sense. At t = 0, they are at the ends of a diameter, as illustrated in figure 1, and have equal angular speeds of π rad s^{-1}, but travel with different tangential accelerations; Q's acceleration is twice that of P. They collide at t = 2 s. If P and Q had travelled in the opposite senses, as illustrated in figure 2, at what approximate time t would they have collided?

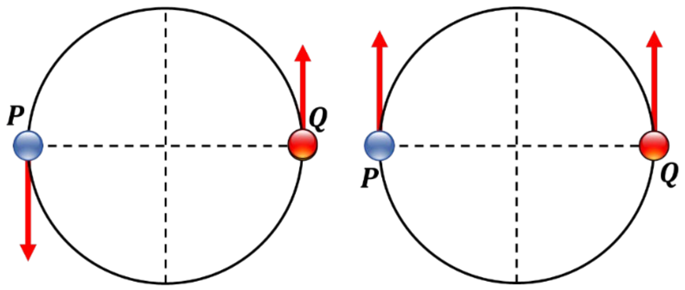
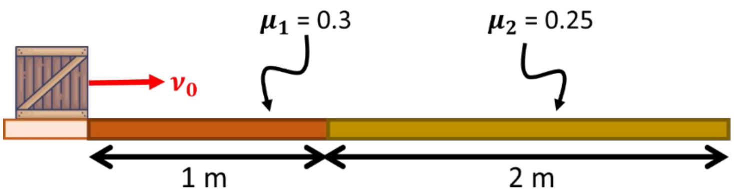


Figure 1 **Figure 2**

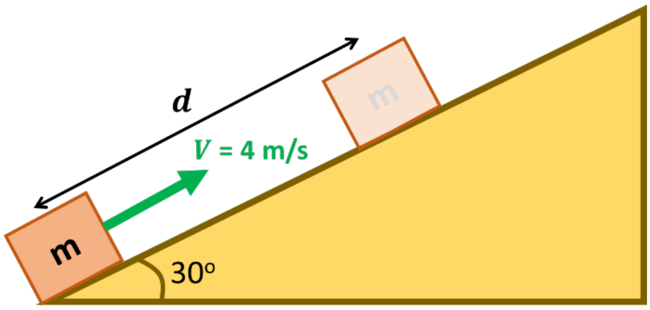
(A) 0.26 s
(B) 0.43 s
(C) 0.81 s
(D) 1.2 s

9. A wooden crate is launched on a horizontal surface with initial velocity v_0, as shown. As the crate moves forward, it first travels over a 1-m long rough segment with coefficient of kinetic friction 0.3, then it transitions onto a 2-m long rough segment with coefficient of kinetic friction 0.25. The crate stops moving at the end of the second rough section. The value of initial velocity v_0 is, most nearly:



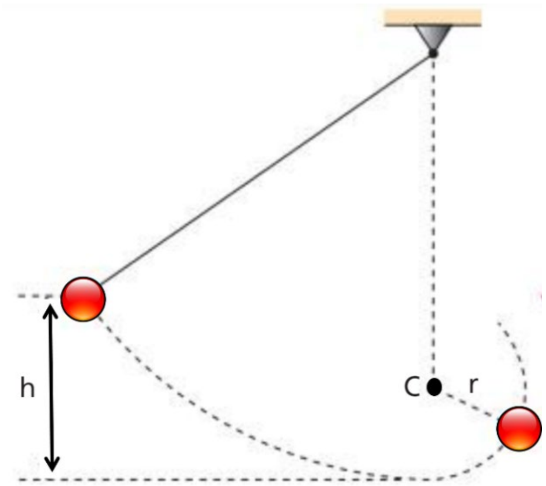
(A) 2 m/s
(B) 3 m/s
(C) 4 m/s
(D) 5 m/s

10. Beginning at the lowest point of a plane inclined at 30º relatively to the horizontal, a block of mass m = 5 kg is launched with initial velocity V = 4 m/s, as shown. The block goes up the inclined plane, stops, reverses and ultimately returns to its starting point with a velocity of 3 m/s. The distance d that the block travelled in its ascending trajectory is, most nearly:



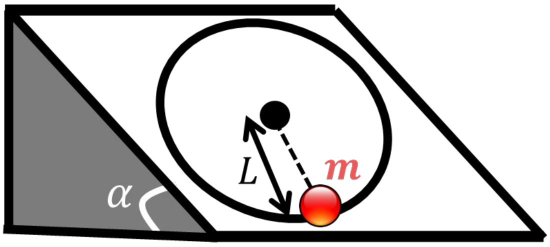
(A) 1.3 m
(B) 1.8 m
(C) 2.3 m
(D) 2.8 m

11. The following figure, which is out of scale, shows a pendulum released at a height h above the lowest point of its trajectory. In the vertical section that passes through the hanging point, a pin causes the wire to bend so that the pendulum then describes a circular trajectory of radius r and center C. The lowest value of h required for the pendulum to describe a complete circumference relatively to C is:



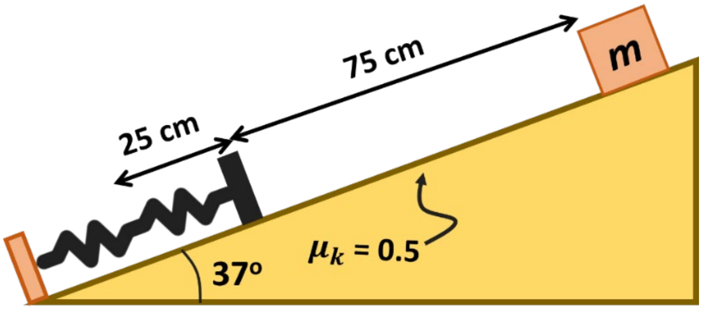
(A) $1.25r$
(B) $1,5r$
(C) $2.0r$
(D) $2.5r$

12. The figure shows a simple pendulum that outlines a circular trajectory while supported on an inclined plane that makes an angle α with the horizontal. Let T_1 and T_2 denote the tensions in the string at the highest and lowest points of the trajectory, respectively. If the mass of the sphere at the end of the string equals m, difference $T_2 - T_1$ equals:



(A) $2mg \sin \alpha$
(B) $3mg \sin \alpha$
(C) $4mg \sin \alpha$
(D) $6mg \sin \alpha$

13. A block leaves rest and begins to slide down an inclined plane towards a spring with stiffness equal to 160 N/m, compressing it by 25 cm before being brought to a halt. The block slides down 75 cm along the inclined plane before touching the spring. The inclined plane is rough with a coefficient of kinetic friction equal to 0.5. The mass of the block is most nearly:

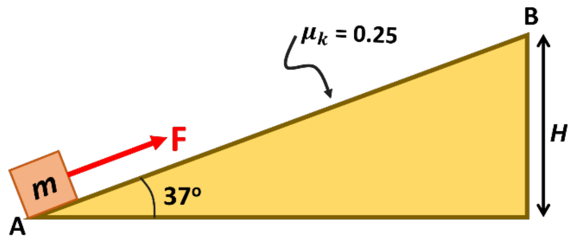


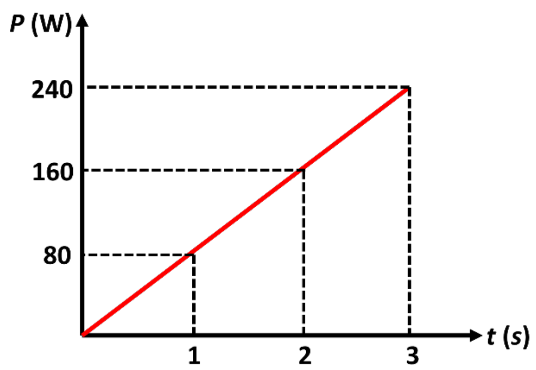
(A) 1.25 kg
(B) 2.5 kg
(C) 3.75 kg
(D) 5 kg

14. A bucket of mass m = 500 g initially contains 2.5 liters of water. The bucket is lifted through a pulley from the ground up to a height h = 10 m in 20 sec. Importantly, the bucket contains a small orifice through which water is being lost at a rate of 0.04 liter/sec. What is the work done by the pulley's operator to lift the bucket up to the height in question?

(A) 135 J
(B) 206 J
(C) 245 J
(D) 276 J

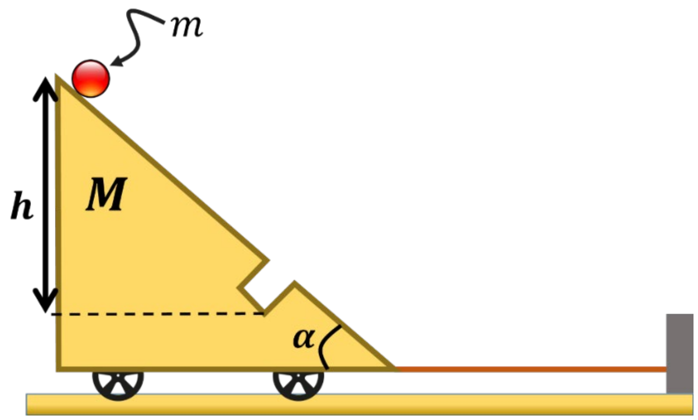
15. A 4-kg block leaves rest at the base of an inclined plane while driven by a force F, as shown. The power associated with F varies linearly with time, as shown in the accompanying graph. The kinetic friction coefficient between the block and the surface is 0.25 and the height of the inclined plane is H = 2.4 m. Knowing that the block reaches point B at t = 2 s, the final velocity of the block is most nearly:





(A) 2 m/s
(B) 3 m/s
(C) 4 m/s
(D) 5 m/s

16. A wheeled wedge of mass M is initially at rest over a smooth horizontal surface and attached to a rigid wall through a wire, as shown. The wedge makes an angle α with the horizontal. A ball of mass m is abandoned from the top of the wedge and rolls down along its inclined plane until it falls down a small hole. What is the final velocity achieved by the system?



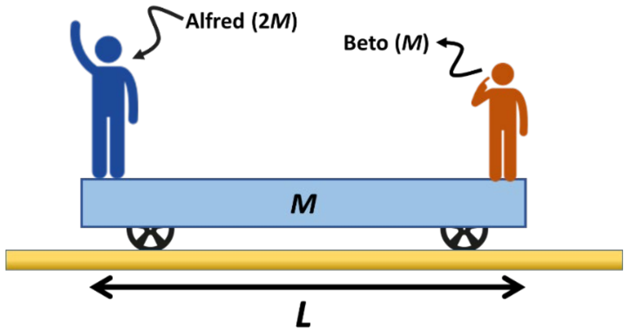
(A) $\dfrac{mg\sin\alpha}{M+m}\sqrt{\dfrac{2h}{g}}$

(B) $\dfrac{mg\cos\alpha}{M+m}\sqrt{\dfrac{2h}{g}}$

(C) $\dfrac{mg}{M+m}\sqrt{\dfrac{2h}{g\sin\alpha}}$

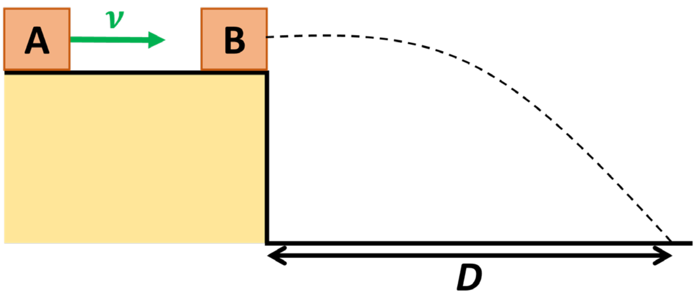
(D) $\dfrac{mg}{M+m}\sqrt{\dfrac{2h}{g\cos\alpha}}$

17. Alfred and Beto are standing at opposite ends of a wheeled plank, as shown. Alfred has mass $2M$, Beto has mass M, and the plank has mass M and length L. At one instant, Alfred and Beto begin to move in opposite directions until they have completely switched positions, with Alfred on the end of the plank Beto was originally standing on and vice versa. At the end of their movements, which of the following correctly states what happened to the wheeled plank?



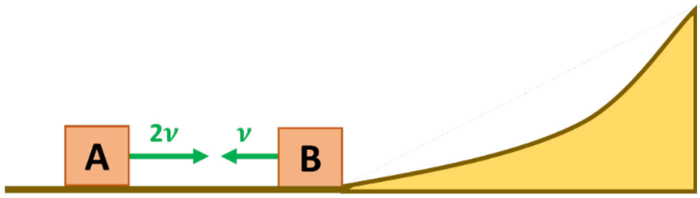
(A) The plank will have moved $L/4$ leftwards relatively to the Earth.
(B) The plank will have moved $L/3$ leftwards relatively to the Earth.
(C) The plank will have moved $L/2$ leftwards relatively to the Earth.
(D) The plank will have moved $L/3$ rightwards relatively to the Earth.

18. A block A moving forward at speed v over a smooth horizontal plane collides inelastically with another block B, identical to the first, initially at rest. The ensemble then drops off a ridge and covers a horizontal distance D before hitting the ground, as shown. If the collision had been elastic, the horizontal range achieved by block B would have been:



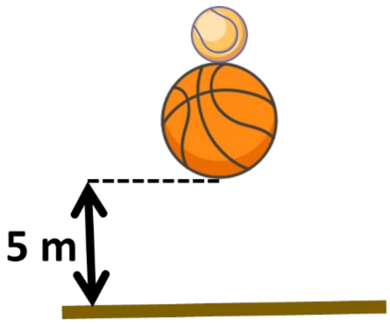
(A) D
(B) $2D$
(C) $4D$
(D) $6D$

19. The figure shows a block A moving over a frictionless horizontal surface with velocity $2v$ towards a block B, with similar mass m, moving in the opposite direction with velocity v. After colliding inelastically, the ensemble moves rightward and climbs the ramp shown, reaching a maximum height H before stopping. Had the collision been elastic, the maximum height reached by block B after the collision would have been:



(A) $2H$
(B) $4H$
(C) $8H$
(D) $16H$

20. A basketball of 600-g mass is at a height of 5 m, and right above it is a tennis ball of 60-g mass. In a given instant, the balls are released. Assuming all shocks are perfectly elastic, the maximum height reached by the tennis ball will be, most nearly:



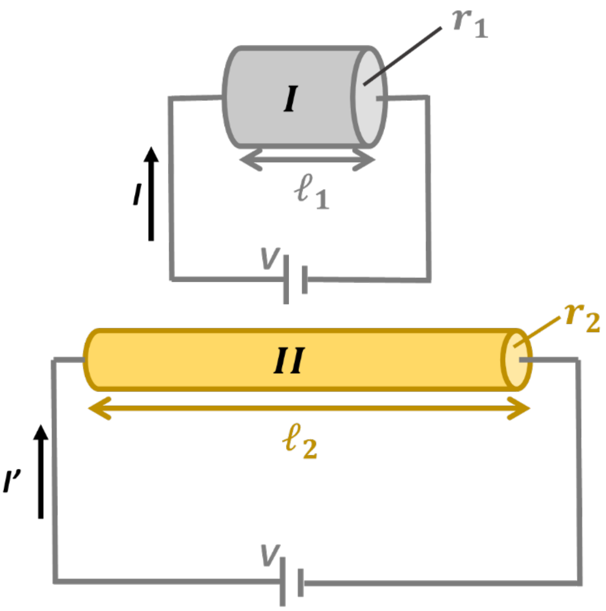
(A) 20 m
(B) 25 m
(C) 30 m
(D) 35 m

21. Five hundred joules of energy are spent to increase the rotational speed of a flywheel from 60 rpm to 360 rpm. The mass moment of inertia of the wheel is most nearly:
(A) 0.52 kg·m^2
(B) 0.62 kg·m^2
(C) 0.72 kg·m^2
(D) 0.82 kg·m^2

22. A flywheel of mass 2 kg and radius 10 cm mounted on a central axis is given a constant torque of 0.25 N·m. If the flywheel starts from rest, what is the change in kinetic energy of the flywheel after 8 seconds?

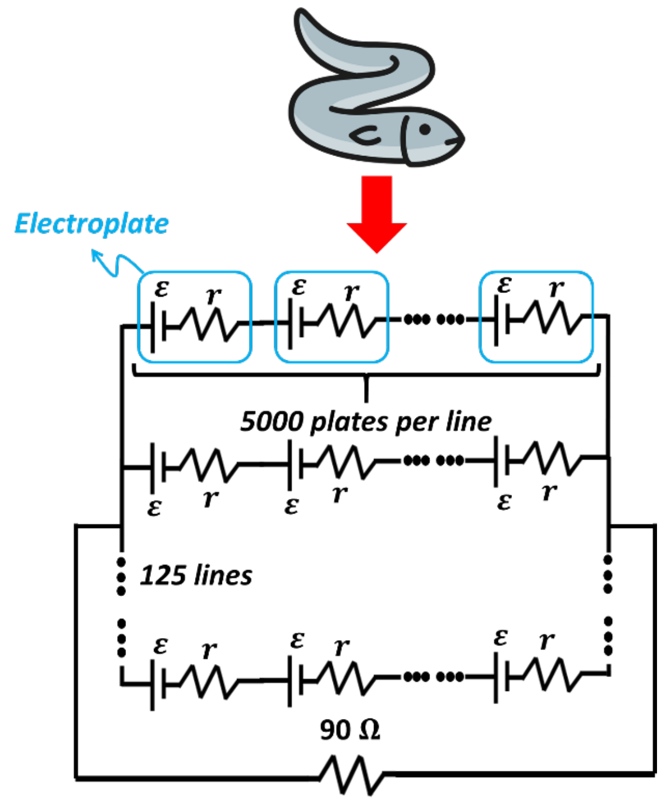
(A) 100 J
(B) 200 J
(C) 300 J
(D) 400 J

23. A resistor I of cylindrical shape has length ℓ_1 and radius r_1, as shown. The resistor is made of a material of resistivity ρ_1. When connected to a voltage source V, a current I passes through the resistor. The device is replaced by resistor II, which has three times the length and half the radius of resistor I. Further, resistor II is made of a material with resistivity $\rho_2 = \rho_1/2$. Thus, if resistor II is connected to the same voltage source V, the current flowing through it will be:



(A) $I/12$
(B) $I/6$
(C) $I/4$
(D) $I/3$

24. Some species of fish are capable of producing electric currents by means of special cells known as *electroplates*. One species endowed with these special cells is *Electrophorus electricus*, a type of eel that inhabits the Amazon rainforest. *Electrophorus* has 125 "lines" connected in parallel; each line has 5000 electroplates connected in series. In circuit-analysis terms, an electroplate consists of a tiny battery of emf ε = 0.015 V and an internal resistance r = 0.25 Ω, as illustrated below. If the water surrounding the eel can be modelled as a 90-Ω Ohmic resistor, what is the magnitude of the current generated by *Electrophorus* in the surrounding water?



(A) 625 mA
(B) 682 mA
(C) 750 mA
(D) 833 mA

Solutions

1.A. Suppose the bodies collide after t seconds. If x_1 and x_2 are the displacements in the horizontal direction of the body over AB and the body over CD, respectively, the distance d that separates the towers becomes

$$d = x_1 + x_2$$

where $x_1 = u_1 t$ and $x_2 = u_2 \cos 37^\circ t$, so that

$$d = x_1 + x_2 = \left(u_1 + u_2 \cos 37^\circ\right) t \quad \text{(I)}$$

To determine time t, we note that in this interval the two bodies cover a vertical distance $y_2 - y_1$ with no relative acceleration, so that

$$y_2 - y_1 = 50 - 30 = 20 = u_2 \sin 37^\circ t$$

$$\therefore t = \frac{20}{u_2 \sin 37^\circ} = \frac{20}{8 \times 0.6} = 4.17 \text{ s}$$

Lastly, we substitute t in (I) to obtain

$$d = \left(8 + 8 \times \cos 37^\circ\right) \times 4.17 = \boxed{60 \text{ m}}$$

2.B. After being launched from point P, the volleyball falls under the effect of gravity g; the time of flight t is

$$H = \frac{1}{2} g t^2 \rightarrow t = \sqrt{\frac{2H}{g}}$$

The horizontal range R covered by the ball is

$$R = v_0 t = v_0 \sqrt{\frac{2H}{g}}$$

However, the minimum horizontal range R' that the volleyball must cover not to hit the incline is

$$R' = \frac{H}{\tan \phi}$$

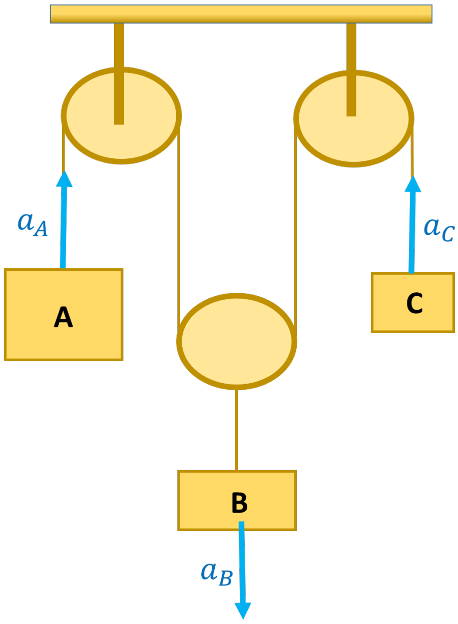
Equating R and R' brings to

$$R' = R \rightarrow \frac{H}{\tan \phi} = v_0 \sqrt{\frac{2H}{g}}$$

$$\therefore \tan \phi = \frac{H}{v_0 \sqrt{\frac{2H}{g}}} = \frac{H}{v_0} \sqrt{\frac{g}{2H}} = \frac{1}{v_0} \sqrt{\frac{gH}{2}}$$

$$\therefore \boxed{\phi = \tan^{-1} \left(\frac{1}{v_0} \sqrt{\frac{gH}{2}} \right)}$$

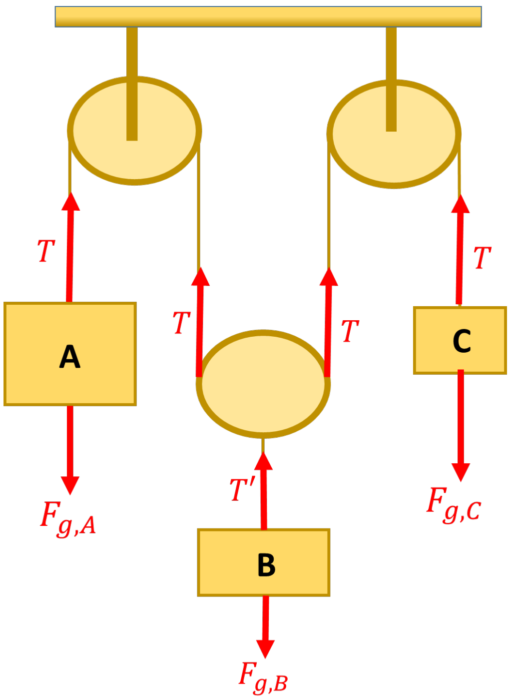
3.B. Suppose the accelerations of each block are oriented as follows. We needn't be too strict on the orientations of each acceleration, as calculations will reveal their exact alignments.



Note that accelerations a_A, a_B, and a_C are related by the simple expression

$$a_A + a_C = 2a_B \rightarrow a_B = \frac{a_A + a_C}{2} \quad \text{(I)}$$

The forces acting on each block are outlined below.



Applying Newton's second law to block A brings to

$$T - m_A g = m_A a_A \rightarrow T - 3\times 9.8 = 3\times a_A$$

$$\therefore T - 29.4 = 3a_A \quad \text{(II)}$$

Applying the N2L to block C, in turn, we get

$$T - 1\times 9.8 = 1\times a_C$$
$$\therefore T - 9.8 = a_C \quad \text{(III)}$$

From the massless mobile pulley, it is easy to see that

$$T' = 2T \quad \text{(IV)}$$

Applying the N2L to block B gives

$$m_B g - T' = m_B a_B \rightarrow 2\times 9.8 - T' = 2a_B$$
$$\therefore 19.6 - T' = 2a_B \quad \text{(V)}$$

Equation (I) can be restated as

$$2a_B = a_A + a_C \rightarrow 6a_B = 3a_A + 3a_C$$

so that, substituting a_B from (V), a_A from (II), and a_C from (III), we get

$$3\times(19.6 - T') = (T - 29.4) + 3\times(T - 9.8)$$
$$\therefore 3\times(19.6 - 2T) = (T - 29.4) + 3\times(T - 9.8)$$
$$\therefore 58.8 - 6T = T - 29.4 + 3T - 29.4$$
$$\therefore 58.8 - 6T = 4T - 58.8$$
$$\therefore 117.6 = 10T$$
$$\therefore \boxed{T = 11.8\ \text{N}}$$

Equipped with tension T, we can determine the accelerations on each block. In equation (II),

$$T - 29.4 = 3a_A \rightarrow 11.8 - 29.4 = 3a_A$$
$$\therefore -17.6 = 3a_A$$
$$\therefore \boxed{a_A = -5.87\ \text{m/s}^2}$$

The negative sign indicates that block A is actually accelerating downward, not upward, as we originally supposed. Next, substituting T into (III) yields

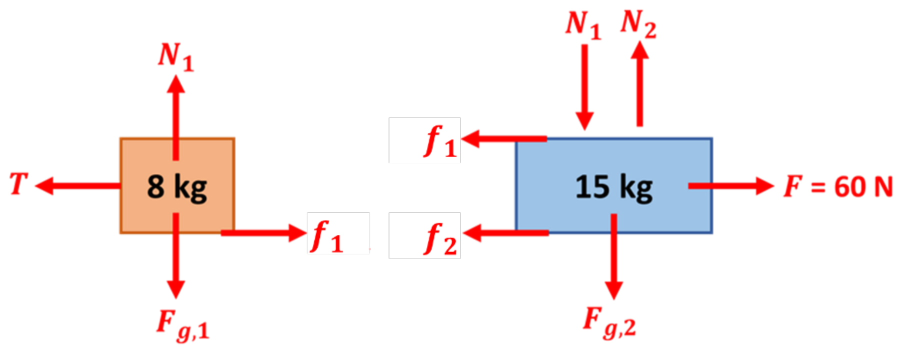
$$T - 9.8 = a_C \rightarrow a_C = 11.8 - 9.8 = \boxed{2.0\ \text{m/s}^2}$$

Block C is indeed accelerating upward as we supposed. Lastly, equation (I) yields

$$a_B = \frac{a_A + a_C}{2} = \frac{-5.87 + 2.0}{2} = \boxed{-1.94\ \text{m/s}^2}$$

Block B is actually accelerating upward, not downward.

4.B. Free-body diagrams for both blocks are drawn below.



In the upper block, normal force N_1 equals the block's weight $F_{g,1}$; that is,

$$F_{g,1} = N_1 = m_1 g = 8 \times 9.8 = 78.4 \text{ N}$$

The tension T in the rope equals friction force f_1, so that

$$T = f_1 = \mu_{k,1} N_1 = 0.2 \times 78.4 = \boxed{15.7 \text{ N}}$$

5.C. For the 15-kg block, applying Newton's second law in the horizontal gives

$$F - f_1 - f_2 = ma$$

$$\therefore 60 - f_1 - f_2 = 15a \ \ (\text{I})$$

Here, f_1 = 15.7 N and f_2 is the friction force between the lower block and the ground. Summing forces in the y-direction brings to

$$\Sigma F_y = 0 \rightarrow N_2 - N_1 - F_{g,2} = 0$$

$$\therefore N_2 - 15.7 - 15 \times 9.81 = 0$$

$$\therefore N_2 = 163 \text{ N}$$

Noting that $\mu_{k,2}$ = 0.1, friction force f_2 is determined as

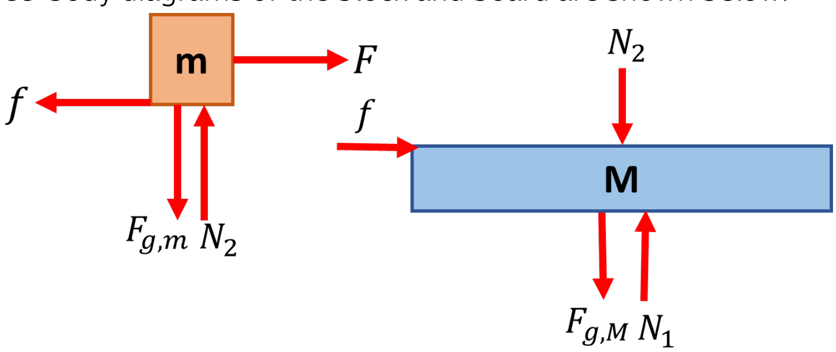
$$f_2 = \mu_{k,2} N_2 = 0.1 \times 163 = 16.3 \text{ N}$$

Substituting in (I), we can establish acceleration a,

$$60 - f_1 - f_2 = 15a \rightarrow 60 - 15.7 - 16.3 = 15a$$

$$\therefore a = \frac{28}{15} = \boxed{1.87 \text{ m/s}^2}$$

6.C. Free-body diagrams of the block and board are shown below.



From vertical equilibrium in block m, we write

$$N_2 = F_{g,m} \rightarrow N_2 = 1.0 \times 9.8 = 9.8 \text{ N}$$

From vertical equilibrium in the board, in turn, we write

$$N_1 = N_2 + F_{g,M} \rightarrow N_1 = 9.8 + 2 \times 9.8 = 29.4 \text{ N}$$

Applying Newton's second law to the block brings to

$$F - f = ma \ \ \text{(I)}$$

Likewise for the board,

$$f = Ma \ \ \text{(II)}$$

Equation (II) indicates that the acceleration that the board attains relatively to the earth stems entirely from friction force f. The friction force is maximum when

$$f_{\text{max}} = \mu_s N_2 = 0.4 \times 9.8 = 3.92 \text{ N}$$

and corresponds to an acceleration a such that

$$f = Ma \rightarrow a = \frac{f}{M} = \frac{3.92}{2} = 1.96 \text{ m/s}^2$$

Adding equations (I) and (II) gives

$$(F - f) + f = ma + Ma$$

$$F = (m + M)a = (1 + 2) \times 1.96 = \boxed{5.88 \text{ N}}$$

In order for the block not to slide relatively to the board, force F used to tow the board must be no greater than 5.88 N.

7.C. Since F = 6.94 N > 5.88 N, the block will slide relatively to the board. The friction force between block and board, noting that the kinetic friction coefficient μ_k = 0.3 should be used because the bodies are in motion, is given by

$$f = \mu_k N_2 = 0.3 \times 9.8 = 2.94 \text{ N}$$

Writing Newton's second law for the block gives

$$F - f = ma_{\text{block}} \rightarrow 6.94 - 2.94 = 1.0 \times a_{\text{block}}$$

$$\therefore a_{\text{block}} = 4 \text{ m/s}^2$$

Writing Newton's second law for the board yields

$$f = Ma_{\text{board}} \rightarrow a_{\text{board}} = \frac{f}{M} = \frac{6.94}{2} = 3.47 \text{ m/s}^2$$

The relative acceleration is then

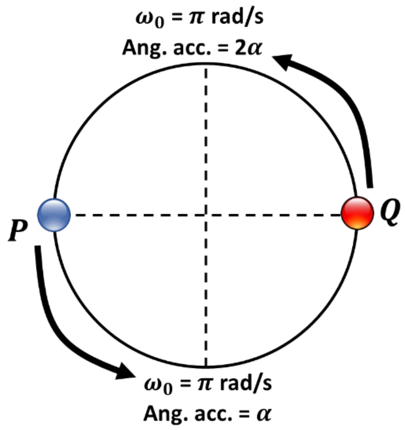
$$\overline{a} = a_{\text{block}} - a_{\text{board}} = 4 - 3.47 = 0.53 \text{ m/s}^2$$

The time required for the block to reach the other end of the board is then

$$\Delta x = \overline{a}\frac{t^2}{2} \rightarrow t = \sqrt{\frac{2\Delta x}{\overline{a}}}$$

$$\therefore t=\sqrt{\frac{2\times 2}{0.53}}=\boxed{2.75\text{ s}}$$

8.B. The initial system is illustrated below.



Before anything else, we must determine angular acceleration α. To do so, we write an equation for the relative motion of the two bodies, noting that, in relative terms, P is π rad ahead of Q until the two bodies collide. Thus,

$$\theta=\theta_0+\omega_r t+\frac{1}{2}\alpha_r t^2 \rightarrow \pi=0+(\pi-\pi)\times 2+\frac{1}{2}\times(2\alpha-\alpha)\times 2^2$$

$$\therefore \pi=2\alpha$$

$$\therefore \alpha=\frac{\pi}{2}\text{rad s}^{-2}$$

Were the two bodies to travel in opposite senses, they'd approach each other at a relative angular speed $\omega_r = \pi + \pi = 2\pi$ rad/s and converge with a relative acceleration $\alpha_r = \alpha + 2\alpha = 3\pi/2$ rad s^{-2}, so that

$$\theta=\theta_0+\omega_r t+\frac{1}{2}\alpha_r t^2 \rightarrow \pi=0+2\pi t+\frac{1}{2}\times\frac{3\pi}{2}\times t^2$$

$$\therefore \pi=2\pi t+\frac{3\pi}{4}t^2$$

$$\therefore \frac{3\pi}{4}t^2+2\pi t-\pi=0$$

$$\therefore 2.36t^2+6.28t-3.14=0$$

$$\therefore t=\frac{-6.28+\sqrt{6.28^2-4\times 2.36\times(-3.14)}}{2\times 2.36}=\boxed{0.430\text{ sec}}$$

9.C. The crate is decelerated by two retarding forces, namely the friction force f_1 that acts as the crate crosses the 1-m long rough segment and the friction force f_2 applied as it travels over the 2-m long rough section. These forces account for the variation in the crate's kinetic energy from

$KE_0 = mv_0^2/2$ at the beginning of the trajectory to $KE_1 = 0$ at the end. Accordingly, we write

$$\Sigma W = \Delta KE \rightarrow -f_1 d_1 - f_2 d_2 = \frac{mv_1^2}{2} - \frac{mv_0^2}{2}$$

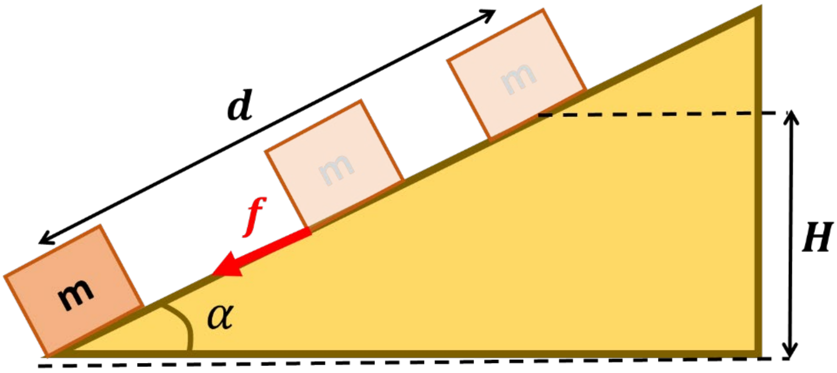
$$\therefore -f_1 d_1 - f_2 d_2 = 0 - \frac{mv_0^2}{2}$$

$$\therefore \cancel{m} g\mu_1 d_1 + \cancel{m} g\mu_2 d_2 = \frac{\cancel{m} v_0^2}{2}$$

$$\therefore v_0 = \sqrt{2g\left(\mu_1 d_1 + \mu_2 d_2\right)}$$

$$\therefore v_0 = \sqrt{2 \times 9.8 \times \left(0.3 \times 1.0 + 0.25 \times 2.0\right)} = \boxed{3.96 \text{ m/s}}$$

10.A. The conditions of the block in its ascending trajectory are detailed below.



The work of nonconservative forces accounts for the variation in mechanical energy. As shown above, the only NCF that does work in the block's ascending trajectory is the friction force f. As the block travels upward, we have

$$\Sigma W_{NCF} = \Delta E_m \rightarrow W_f = \left(E_m\right)_1 - \left(E_m\right)_0$$

$$\therefore W_f = mgH - \frac{mV_0^2}{2} \text{ (I)}$$

where W_f denotes the work done by the friction force. As the block descends, we have

$$\Sigma W_{NCF} = \Delta E_m \rightarrow W_f = \left(E_m\right)_1 - \left(E_m\right)_0$$

$$\therefore W_f = \frac{mV_1^2}{2} - mgH \text{ (II)}$$

Since the work done by the friction force is the same in the ascending and descending trajectories, we can equate (I) and (II) and, noting that $H = d\sin\alpha$, write

$$mgH - \frac{mV_0^2}{2} = \frac{mV_1^2}{2} - mgH \rightarrow gH - \frac{V_0^2}{2} = \frac{V_1^2}{2} - gH$$

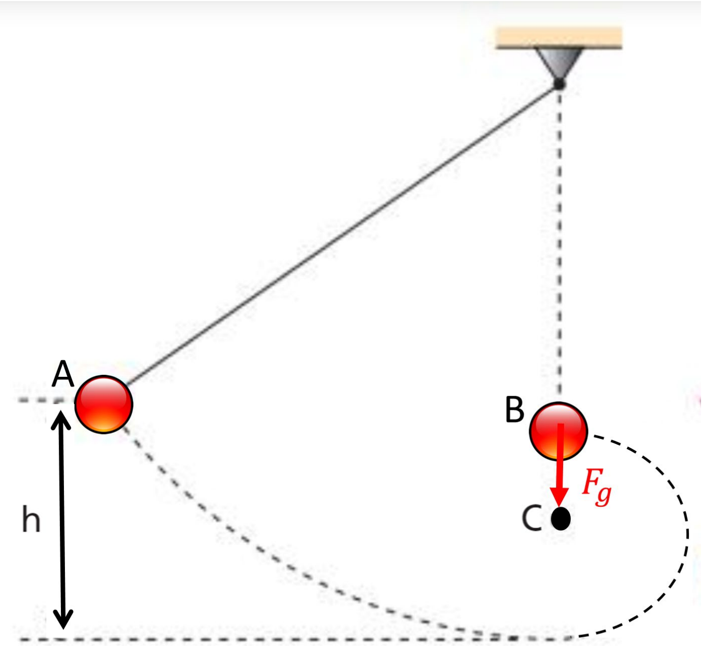
$$\therefore \frac{V_0^2+V_1^2}{2}=2gH$$

$$\therefore V_0^2+V_1^2=4gH$$

$$\therefore V_0^2+V_1^2=4gd\sin\theta$$

$$\therefore d=\frac{\left(V_0^2+V_1^2\right)}{4g\sin\theta}=\frac{\left(4^2+3^2\right)}{4\times 9.8\times\sin 30^{\circ}}=\boxed{1.28\text{ m}}$$

11.D. The pendulum reaches its topmost point at B, as shown.



At this point, the tension in the string approaches zero and the sole centripetal force acting on the mass is the weight F_g; in mathematical terms,

$$F_{cp}=F_g\rightarrow\frac{mv_B^2}{r}=mg$$

$$\therefore v_B^2=gr\ \text{(I)}$$

Since the system is conservative, the mechanical energy at A should equal the mechanical energy at B; that is,

$$\left(E_m\right)_A=\left(E_m\right)_B\rightarrow mg\left(h-2r\right)=\frac{mv_B^2}{2}$$

$$\therefore \cancel{m}g\left(h-2r\right)=\frac{\cancel{m}v_B^2}{2}$$

$$\therefore gh-2gr=\frac{v_B^2}{2}$$

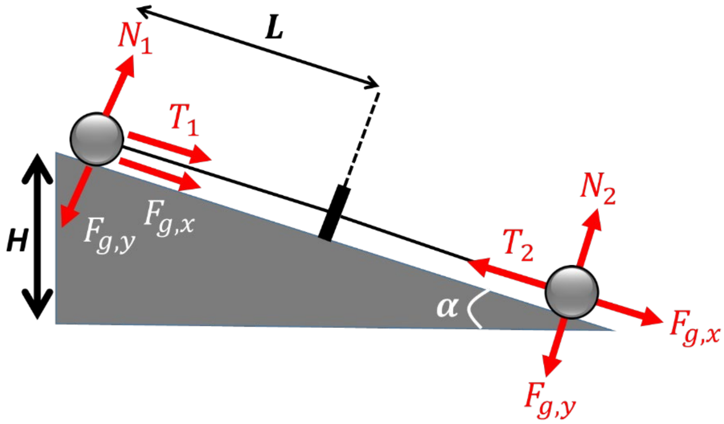
$$\therefore 2gh-4gr=v_B^2$$

Noting that $v_B^2=gr$ from equation (I), we substitute above to obtain

$$2gh - 4gr = v_B^2 \rightarrow 2gh - 4gr = gr$$

$$\therefore h = \frac{5r}{2} = \boxed{2.5r}$$

12.D. The following free-body diagram shows a side view of the pendulum.



Summing radial forces in the highest point of the trajectory, we find that

$$T_1 + F_{g,x} = \frac{mv_1^2}{r} \rightarrow T_1 + mg\sin\alpha = \frac{mv_1^2}{L}$$

Multiplying through by $L/2$ brings to

$$T_1 + mg\sin\alpha = \frac{mv_1^2}{L} \rightarrow \frac{T_1 L}{2} + \frac{mgL\sin\alpha}{2} = \frac{mv_1^2}{2} \quad \text{(I)}$$

Next, we apply the N2L to radial forces in the lowest point of the trajectory, giving

$$T_2 - F_{g,x} = \frac{mv_2^2}{r} \rightarrow T_2 - mg\sin\alpha = \frac{mv_2^2}{L}$$

Multiplying through by $L/2$, we get

$$T_2 - mg\sin\alpha = \frac{mv_2^2}{L} \rightarrow \frac{T_2 L}{2} - \frac{mgL\sin\alpha}{2} = \frac{mv_2^2}{L} \quad \text{(II)}$$

As the pendulum oscillates, the only force that does work on the system is the weight F_g; it follows that the system is conservative. Applying conservation of mechanical energy between 1, the highest point in the trajectory, and 2, the lowest point in the trajectory, we obtain

$$(E_m)_1 = (E_m)_2 \rightarrow mgH + \frac{mv_1^2}{2} = \frac{mv_2^2}{2}$$

From elementary trigonometry, $H = 2L\sin\alpha$, so that

$$mgH + \frac{mv_1^2}{2} = \frac{mv_2^2}{2} \rightarrow 2mgL\sin\alpha + \frac{mv_1^2}{2} = \frac{mv_2^2}{2}$$

Substituting the kinetic energy terms from (I) and (II) yields

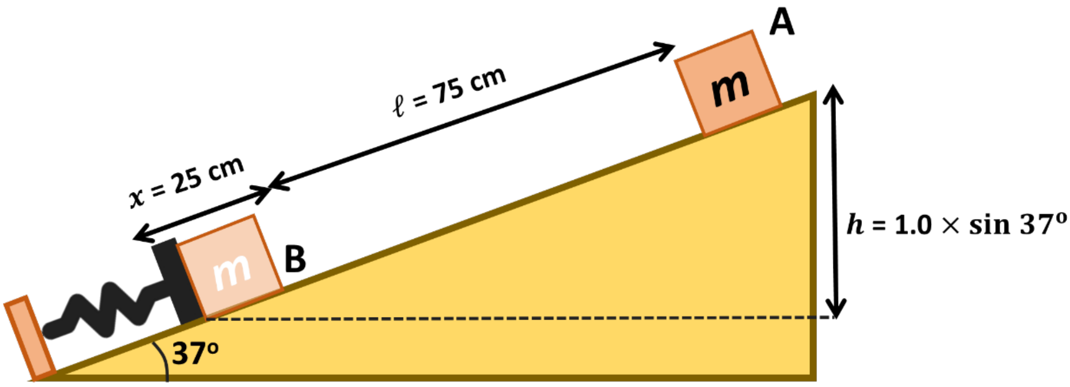
$$2mgL\sin\alpha + \left(\frac{T_1L}{2} + \frac{mgL\sin\alpha}{2}\right) = \left(\frac{T_2L}{2} - \frac{mgL\sin\alpha}{2}\right)$$

$$\therefore 2mgL\sin\alpha + \frac{T_1L}{2} + mgL\sin\alpha = \frac{T_2L}{2}$$

$$\therefore 3mgL\sin\alpha = \frac{L}{2}(T_2 - T_1)$$

$$\therefore T_2 - T_1 = \frac{6mgL\sin\alpha}{L} = \boxed{6mg\sin\alpha}$$

13.B. Let point A be the starting point of the block, and B the point at which the block stops moving after sliding down 1 m.



The variation in mechanical energy equals the work of the nonconservative forces. In the case at hand, the only NCF that does work is the friction force f; in mathematical terms,

$$W_{NCF} = \Delta E_m \rightarrow W_f = (E_m)_B - (E_m)_A$$

$$\therefore -f(x+\ell) = \frac{kx^2}{2} - mgh$$

$$\therefore -mg\mu\cos 37^\circ(x+\ell) = \frac{kx^2}{2} - mgh$$

$$\therefore mg\mu\cos 37^\circ(x+\ell) = mgh - \frac{kx^2}{2}$$

$$\therefore mg\left[h - \mu\cos 37^\circ(x+\ell)\right] = \frac{kx^2}{2}$$

$$\therefore m = \frac{kx^2}{2g\left[h - \mu\cos 37^\circ(x+\ell)\right]}$$

$$\therefore m = \frac{160\times 0.25^2}{2\times 9.8\times(0.6 - 0.5\times 0.8\times 1.0)} = \boxed{2.55\text{ kg}}$$

14.B. The leaky bucket functions as a variable mass system. The initial mass of the system is 0.5 kg from the bucket itself plus 2 L × 1 kg/L = 2 kg of water, so m_0 = 2 + 0.5 = 2.5 kg. This mass is reduced at a rate of 0.04 L/sec × 1 kg/L = 0.04 kg/s. The weight of the system is then

$$F_g = (2.5 - 0.04t) \times 9.8 = 24.5 - 0.392t \text{ (I)}$$

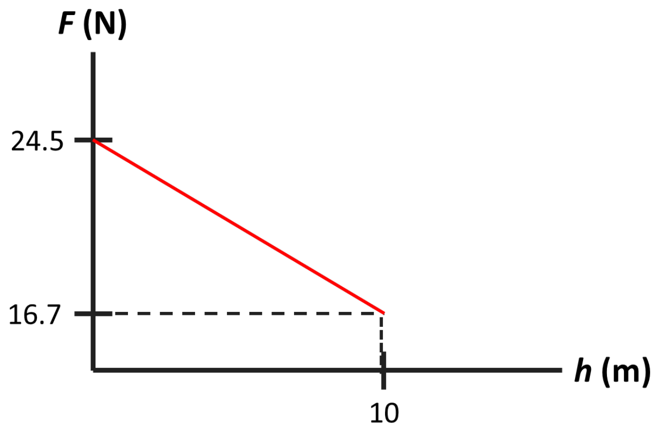
The average velocity of the bucket as it is lifted upwards is

$$V = \frac{\Delta y}{\Delta t} = \frac{10}{20} = 0.5\,\text{m/s}$$

Thus, the height of the bucket is described by h = 0.5t. Equivalently, t = 2h, which can be plugged into (I) to yield

$$F_g = 24.5 - 0.392 \times (2h) = 24.5 - 0.784h$$

The force used to convey the bucket varies from F_g = 24.5 N at h = 0 to F_g = 24.5 – 0.784 × 10 = 16.7 N at h = 10 m, as plotted below.



The work done by the operator is given by the area under the graph,

$$W = \frac{(16.7 + 24.5) \times 10}{2} = \boxed{206\text{ J}}$$

15.C. The variation in mechanical energy equals the work done by nonconservative forces. In the case at hand, two nonconservative forces act on the block: friction f and force F. In mathematical terms, the variation of mechanical energy between point A, from which the block leaves rest, and point B, the top of the inclined plane, is such that

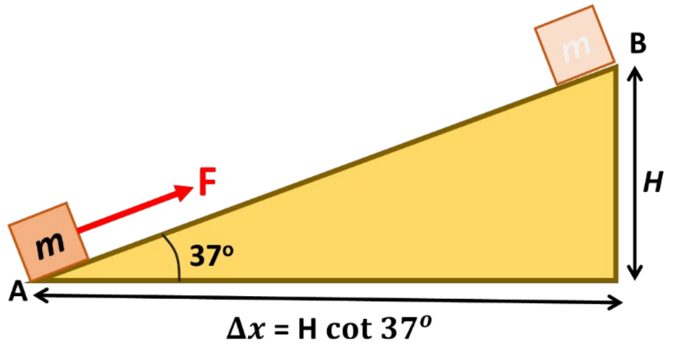
$$\Sigma W_{NCF} = \Delta E_m \to W_f + W_F = (E_m)_B - (E_m)_A$$

$$\therefore -f\ell + W_F = \left(\frac{mv_B^2}{2} + mgH\right) - 0$$

$$\therefore -g\mu_k H \cot 37^\circ + \frac{W_F}{m} = \frac{v_B^2}{2} + gH$$

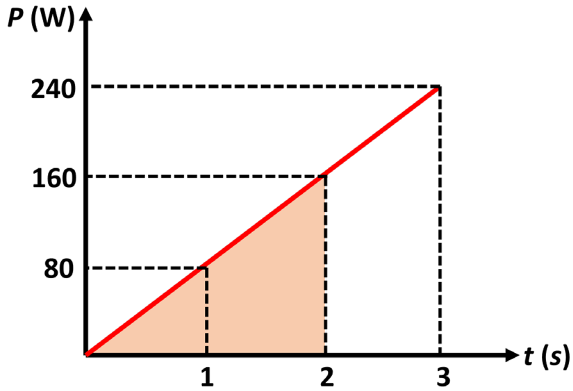
$$\therefore v_B = \sqrt{2\left(-g\mu_k H\cot 37^{\circ} + \frac{W_F}{m} - gH\right)}\ \text{(I)}$$

Note that we have used the principle that the work done by the friction force is proportional to the horizontal projection Δx of the block's trajectory.



The work W_F done by force F is given by the shaded area of the power-time graph we were given,

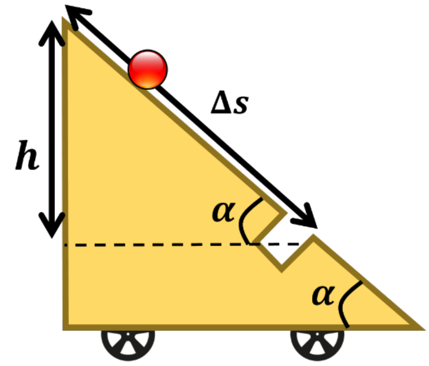
$$W_F = \frac{2\times 160}{2} = 160\ \text{J}$$



Substituting in (I) gives

$$v_B = \sqrt{2\left[-9.8\times 0.25\times 2.4\cot 37^{\circ} + \frac{160}{4} - 9.8\times 2.4\right]} = \boxed{4.17\,\text{m/s}}$$

16.B. Initially, the wedge is motionless and the ball slides down its inclined face with an acceleration a. Summing forces in the direction normal to the ball's movement, it is easy to see that $a = g\sin\alpha$.



As the ball travels downhill, it covers a distance $\Delta s = h/\sin\alpha$. From kinematics, we can determine the time required to cover this distance,

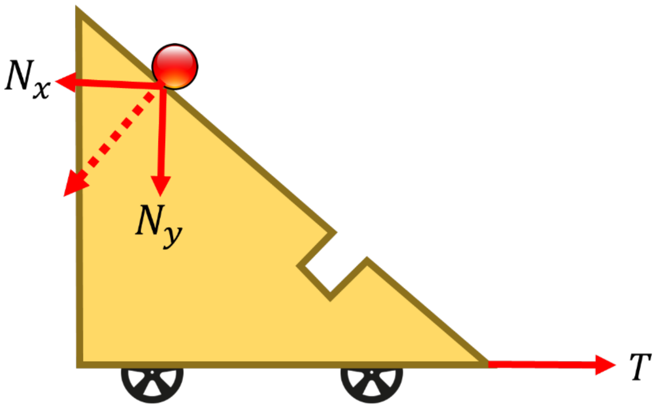
$$\Delta s = \frac{at^2}{2} \rightarrow \frac{h}{\sin\alpha} = \frac{g\,\sin\alpha\Delta t^2}{2}$$

$$\therefore \Delta t = \sqrt{\frac{2h}{g\sin^2\alpha}}$$

The tension T in the wire is balanced by the horizontal component of the normal force that the ball exerts on the wedge,

$$T = N_x \rightarrow T = N\sin\alpha$$

$$\therefore T = mg\cos\alpha\sin\alpha$$



Tension T is an external force and adds a linear momentum $T\Delta t$ to the system. In the period of observation, the system begins with a linear momentum $Q_i = 0$ and ends with a linear momentum $Q_f = (M+m)V$, where V is the velocity we aim to determine. Thus,

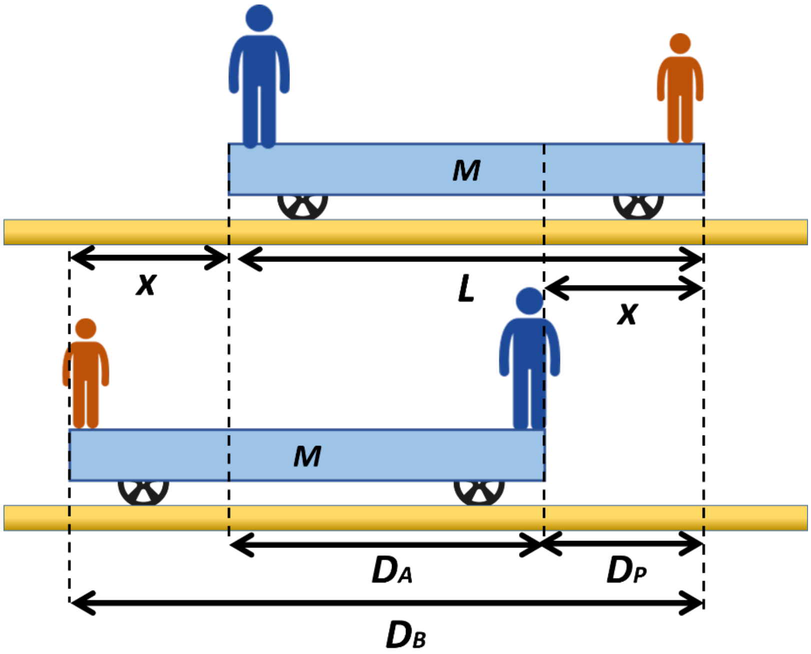
$$T\Delta t = \Sigma Q_f - \Sigma Q_i$$

$$\therefore T\Delta t = (M+m)V - 0$$

$$\therefore mg\cos\alpha\sin\alpha\sqrt{\frac{2h}{g\sin^2\alpha}} = (M+m)V - 0$$

$$\therefore \boxed{V = \frac{mg\cos\alpha}{M+m}\sqrt{\frac{2h}{g}}}$$

17.A. We have every reason to believe that the system includes no external forces, so linear momentum is conserved and the system's center of mass will have zero displacement relatively to the Earth as Alfred and Beto switch positions. Refer to the following figure.



The fact that the plank will have been displaced to the left is intuitive: Alfred, who is twice as heavy as Beto, is moving to the right, thus the force that counterbalances Alfred's movement and pushes the plank to the left should be more intense than the force that counterbalances Beto's displacement and pushes the plank to the right. From the geometry of the figure above, we see that Alfred's displacement relatively to the Earth is $D_A = L - x$; Beto's displacement is $D_B = L + x$; and the plank's displacement is $D_P = x$. From conservation of linear momentum, we can write

$$Q_{\text{final}} = Q_{\text{initial}} \to M_A D_A = M_B D_B + M_P D_P$$

$$\therefore 2M\left(L - x\right) = M\left(L + x\right) + Mx$$

$$\therefore 2ML - 2Mx = ML + Mx + Mx$$

$$\therefore ML = 4Mx$$

$$\therefore L = 4x$$

$$\therefore \boxed{x = \frac{L}{4}}$$

That is, the plank will move $L/4$ to the left relatively to the Earth.

18.B. Let m denote the mass of either block and v_x denote the horizontal speed achieved by the ensemble after the inelastic collision. Applying conservation of linear momentum in the horizontal, we have

$$Q_{\text{final}} = Q_{\text{initial}} \to mv = \left(m + m\right)v_x$$

$$\therefore mv = 2mv_x$$

$$\therefore v_x = \frac{v}{2}$$

The horizontal distance covered by the unified block is then

$$D = v_x t = \frac{vt}{2}$$

Suppose now that blocks A and B undergo an elastic collision. In this case, the blocks, being identical, "switch" velocities, with A coming to rest and B leaping off the ridge with velocity $v_x' = v$. The horizontal distance covered by block B before hitting the ground is then

$$D' = v_x' t = vt = 2\left(\frac{vt}{2}\right) = \boxed{2D}$$

19.D. We first determine the velocity V of the ensemble obtained after the inelastic collision,

$$m \times 2v - m \times v = (m+m)V$$

$$\therefore 2mv - mv = 2mV$$

$$\therefore V = \frac{v}{2}$$

Assuming all kinetic energy of the ensemble is converted into potential energy, height H is determined to be

$$\frac{(m+m)V^2}{2} = (m+m)gH \rightarrow \frac{V^2}{2} = gH$$

$$\therefore H = \frac{V^2}{2g} = \frac{1}{2g}\left(\frac{v}{2}\right)^2 = \frac{v^2}{8g}$$

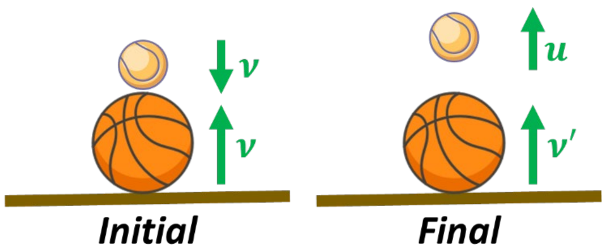
Suppose now that blocks A and B undergo an elastic collision. In this case, much like in the previous problem, the blocks will "switch" velocities, with A moving away from B at velocity v and B moving away from A at velocity $2v$. Block B climbs the ramp with this speed and reaches a height H' such that

$$\frac{m \times (2v)^2}{2} = mgH' \rightarrow 2mv^2 = mgH'$$

$$\therefore 2v^2 = gH'$$

$$\therefore H' = \frac{2v^2}{g} = 16\left(\frac{v^2}{8g}\right) = \boxed{16H}$$

20.D. As they reach the ground, the balls will have speed $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} = 9.9$ m/s. The basketball hits the floor elastically and reverses its velocity from 9.9 m/s downward to 9.9 m/s upward, as illustrated in the drawing labeled *Initial*. Shock between the basketball and the tennis ball causes the former to ascend upward with velocity v' and the latter to ascend upward with velocity u, as shown in the drawing labeled *Final*.



Since the collisions are perfectly elastic, the velocities must follow the simple relationship

$$v + v = u - v'$$
$$\therefore 2v = u - v'$$
$$\therefore v' = u - 2v \text{ (I)}$$

Noting that linear momentum in the vertical direction is conserved, we may write

$$Q_{\text{initial}} = Q_{\text{final}} \to Mv + m(-v) = Mv' + mu$$
$$\therefore Mv - mv = Mv' + mu$$

Substituting v' from (I) gives

$$Mv - mv = Mv' + mu \to Mv - mv = M(u - 2v) + mu$$
$$\therefore Mv - mv = Mu - 2Mv + mu$$
$$\therefore (3M - m)v = (M + m)u$$
$$\therefore u = \frac{(3M - m)v}{(M + m)} = \frac{(3 \times 0.6 - 0.06) \times 9.9}{0.6 + 0.06} = 26.1 \text{ m/s}$$

Assuming that all the kinetic energy acquired by the tennis ball is converted into potential energy, the maximum height h achieved by the ball is calculated to be

$$\frac{mu^2}{2} = mgh \to h = \frac{u^2}{2g}$$
$$\therefore h = \frac{26.1^2}{2 \times 9.8} = \boxed{34.8 \text{ m}}$$

21.C. Rotational kinetic energy is given by a half of the product of mass moment of inertia and squared angular velocity. In the present case, we have

$$\frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_0^2 = 500 \to \frac{1}{2}I\left(\omega_1^2 - \omega_0^2\right) = 500$$

where $\omega_1 = 360 \times 2\pi/60 = 37.7$ rad/s and $\omega_0 = 60 \times 2\pi/60 = 6.28$ rad/s, so that

$$\frac{1}{2}I\left(\omega_1^2 - \omega_0^2\right) = 500 \to \frac{1}{2}I\left(37.7^2 - 6.28^2\right) = 500$$

$$\therefore \frac{1}{2} I \times 1380 = 500$$

$$\therefore I = \frac{500 \times 2}{1380} = \boxed{0.725 \text{ kg} \cdot \text{m}^2}$$

22.B. The angular acceleration is given by the ratio of torque to mass moment of inertia,

$$\tau = \alpha I \rightarrow \alpha = \frac{\tau}{I} = \frac{\tau}{\frac{1}{2} m r^2}$$

$$\therefore \alpha = \frac{0.25}{\frac{1}{2} \times 2 \times 0.1^2} = 25 \text{ rad/s}^2$$

The angle $\Delta\theta$ that the flywheel rotates over the course of 8 sec is

$$\Delta\theta = \frac{1}{2} \alpha t^2 = 0.5 \times 25 \times 8^2 = 800 \text{ rad}$$

The change in kinetic energy is given by the work W_τ done by the torque,

$$W_\tau = \tau \Delta\theta = 0.25 \times 800 = \boxed{200 \text{ J}}$$

23.B. The resistance of the first resistor is

$$R_I = \frac{\rho_1 \ell_1}{A_1} = \frac{\rho_1 \ell_1}{\pi r_1^2}$$

and the current flowing through it is $I = V/R_I$. The resistance of resistor II is

$$R_{II} = \frac{\rho_2 \ell_2}{A_2} = \frac{\frac{\rho_1}{2} \times 3\ell_1}{\pi (r_1/2)^2} = \frac{6\rho_1 \ell_1}{\pi r_1^2} = 6R_I$$

and the current flowing through it when connected to a voltage V becomes

$$I' = \frac{V}{R_{II}} = \frac{V}{6R_I} = \frac{(V/R_I)}{6} = \boxed{\frac{I}{6}}$$

24.C. On each line, there are 5000 electroplates. Each electroplate has a 0.15-V battery connected to an internal resistance r = 0.25 Ω. Accordingly, a line can be represented by a single battery of emf equal to 0.015 × 5000 = 75 V and a resistor such that R_{eq} = 0.25 × 5000 = 1250 Ω.

There are 125 such lines in parallel. These can be interpreted as an in-parallel association of 125 batteries, each with a 75-V emf. A connection of N identical batteries in parallel can be reduced to a single equivalent battery – in this case, a single battery with ε = 75 V. Further, each of the 125 lines has a 1250-Ω resistance. A connection of N identical resistances R can be reduced to a single equivalent device of equivalent resistance equal to R/N – in this case, 1250/125 = 10 Ω. This equivalent resistance is "connected" to the water, which we were told to interpret as a 90-Ω resistor, and should lead to a current I such that

$$U = RI \to I = \frac{U}{R}$$

$$\therefore I = \frac{75}{10+90} = 0.75\text{ A} = \boxed{750\text{ mA}}$$

Answer Summary

1	**A**	**13**	**B**
2	**B**	**14**	**B**
3	**B**	**15**	**C**
4	**B**	**16**	**B**
5	**C**	**17**	**A**
6	**C**	**18**	**B**
7	**C**	**19**	**D**
8	**B**	**20**	**D**
9	**C**	**21**	**C**
10	**A**	**22**	**B**
11	**D**	**23**	**B**
12	**D**	**24**	**C**

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