Montogue

Quiz HT106

Natural Convection

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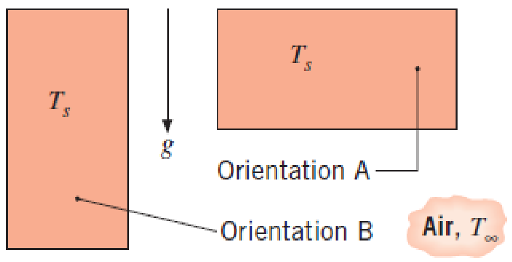
Problems

Problem 1 (Bergman et al., 2011, w/ permission)

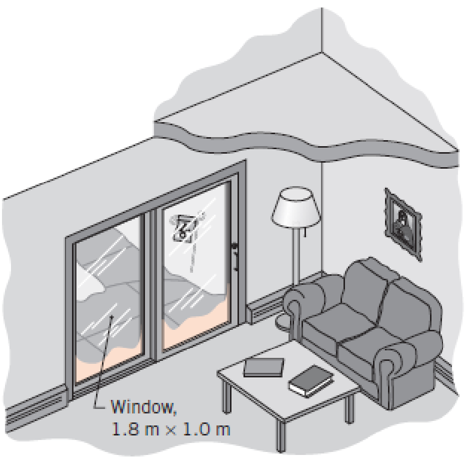
Consider a vertical plate of dimensions 0.25 m × 0.50 m that is at T_s = 100°C in a quiescent environment at T_∞ = 20°C. In the interest of minimizing heat transfer from the plate, which orientation, A or B, is preferred? What is the convection heat transfer from the front surface of the plate when it is in the preferred orientation? Use as properties $\nu = 1.92\times10^{-6}$ m²/s, k = 0.0287 W/m·K, $\alpha = 2.74\times10^{-6}$ m²/s, and Pr = 0.702.



A) The preferred orientation is A, and $\dot{q}$ = 22.9 W
B) The preferred orientation is A, and $\dot{q}$ = 45.7 W
C) The preferred orientation is B, and $\dot{q}$ = 22.9 W
D) The preferred orientation is B, and $\dot{q}$ = 45.7 W

Problem 2A (Bergman et al., 2011, w/ permission)

During a winter day, the window of a patio door with a height of 1.8 m and width 1.0 m shows a frost line near its base. The room wall and air temperatures are 15ºC. Explain why the window would show a frost layer at the base rather than at the top.



Problem 2B

Estimate the heat loss through the window due to free convection and radiation. Assume that the window has a uniform temperature of 0°C, and the emissivity of the glass surface is 0.94. Use ν = 1.41×10^{-5} m^2/s, α = 1.99×10^{-5} m^2/s k = 0.0247 W/mK, and Pr = 0.710.

A) $\dot{q}$ = 105.7 W

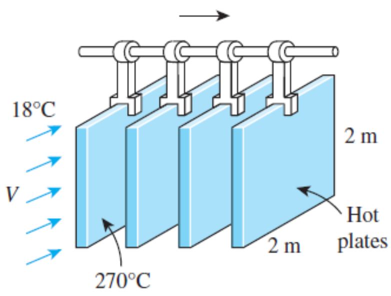
B) $\dot{q}$ = 223.2 W

C) $\dot{q}$ = 306.7 W

D) $\dot{q}$ = 414.2 W

Problem 3 (Çengel & Ghajar, 2015, w/ permission)

In a protection facility, thin square plates 2 m × 2 m in size coming out of the oven at 270°C are cooled by blowing ambient air at 18°C horizontally parallel to their surfaces. Determine the air velocity above which the natural effects on heat transfer are less than 10 percent and thus are negligible. Consider ν = 2.83×10^{-5} m^2/s at the film temperature.



A) V = 3.83 m/s

B) V = 6.61 m/s

C) V = 10.9 m/s

D) V = 14.2 m/s

Problem 4A (Bergman et al., 2011, w/ permission)

An aluminum alloy plate, heated to a uniform temperature of 227°C, is allowed to cool while vertically suspended in a room where the ambient air and surroundings are at 27°C. The plate is 0.3-m square with a thickness of 15 mm and an emissivity of 0.25. Develop an expression for the time rate of change of the plate temperature, assuming the temperature to be uniform at any time.

Problem 4B

Determine the initial rate of cooling when the plate temperature is 227°C. Use as properties of air ν = 2.64 × 10^{-5} m^2/s, α = 3.83 × 10^{-5} m^2/s, Pr = 0.690, and k = 0.0338 W/mK. For aluminum, consider ρ = 2770 kg/m^3 and c_p = 983 J/kgK.

A) dT/dt = −0.035 K/s

B) dT/dt = −0.099 K/s

C) dT/dt = −0.17 K/s

D) dT/dt = −0.34 K/s

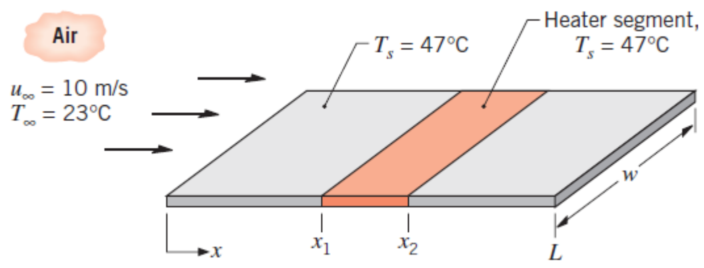
Problem 4C

The plate described in the previous problem has been used in an experiment to determine the free convection heat transfer coefficient. At an instant of time when the plate temperature was 127°C, the time rate of change of this temperature was observed to be −0.0465 K/s. Calculate the corresponding heat transfer coefficient, then compare this result with an estimate based on the adequate empirical correlation. What is the percentage difference between the two estimates? Take the properties of air to be β = 0.00286 K^{-1}, ν = 2.09 × 10^{-5} m^2/s, α = 2.99 × 10^{-5} m^2/s, Pr = 0.700, and k = 0.030 W/mK. For aluminum, use the same relations as in Part B.

A) The difference between estimates is greater than 15% but less than 25%.
B) The difference between estimates is greater than 25% but less than 35%.
C) The difference between estimates is greater than 35% but less than 45%.
D) The difference between estimates is greater than 45% but less than 55%.

■ Problem 5A (Bergman et al., 2011, w/ permission)

A highly polished aluminum plate of length 0.5 m and width 0.2 m is subjected to an airstream at a temperature of 23°C and a velocity of 10 m/s. Because of upstream conditions, the flow is turbulent over the entire length of the plate. A series of segmented, independently controlled heaters is attached to the lower side of the plate to maintain approximately isothermal conditions over the entire plate. The electrical heater covering the section between positions x_1 = 0.2 m and x_2 = 0.3 m is shown in the schematic. Estimate the electrical power that must be supplied to the designated heater segment to maintain the plate surface temperature at T_s = 47°C. Use ν = 1.67×10^{-5} m^2/s, α = 2.37×10^{-6} m^2/s, k = 0.0269 W/mK, and Pr = 0.706.



A) P_e = 18.9 W
B) P_e = 28.5 W
C) P_e = 44.4 W
D) P_e = 55.7 W

■ Problem 5B

If the blower that maintains the airstream velocity over the plate malfunctions, but the power assigned to the heater remains constant, estimate the surface temperature of the designated segment. Assume that the ambient air is extensive, quiescent, and at 23°C. Use ε = 0.03 as the emissivity of the surface.

A) T_s = 99°C
B) T_s = 174°C
C) T_s = 255°C
D) T_s = 305°C

■ Problem 6A (Bergman et al., 2011, w/ permission)

The 4 m × 4 m horizontal roof of an uninsulated melting furnace is comprised of a 0.08-m thick fireclay brick (k_1 = 1.8 W/mK) refractory covered by a 5-mm thick steel (k_3 = 48.8 W/mK) plate. The refractory plate exposed to the furnace gases is maintained at 1700 K during operation, while the outer surface of the steel is exposed to the air and walls of a large room at 25°C. The emissivity of the steel is ε = 0.3. Determine the rate of heat loss from the roof. Use as properties β = 0.0025 K^{-1}. ν = 2.64×10^{-5} m^2/s, α = 3.83×10^{-5} m^2/s, and Pr = 0.690.

A) $\dot{q}$ = 191.5 kW
B) $\dot{q}$ = 289.7 kW
C) $\dot{q}$ = 368.9 kW
D) $\dot{q}$ = 479.8 kW

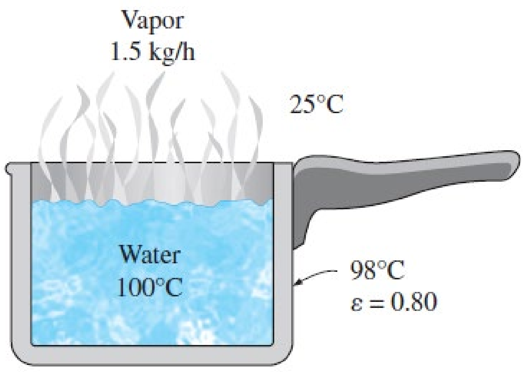
Problem 6B

If a 20-mm thick layer of alumina-silica insulation (ρ = 64 kg/m^3, k = 0.125 W/mK) is placed between the refractory and the steel, what is the new rate of heat loss from the roof? What is the temperature at the inner surface of the insulation?

A) $\dot{q}$ = 85.3 kW and $T_{ins,i}$ = 1463 K
B) $\dot{q}$ = 85.3 kW and $T_{ins,i}$ = 1678 K
C) $\dot{q}$ = 171.2 kW and $T_{ins,i}$ = 1463 K
D) $\dot{q}$ = 171.2 kW and $T_{ins,i}$ = 1678 K

Problem 7A (Çengel & Ghajar, 2015, w/ permission)

Water is boiling in a 12-cm deep pan with outer diameter of 25 cm that is placed on top of a stove. The ambient air of the surrounding surfaces is at a temperature of 25°C and the emissivity of the outer surface of the pan is 0.80. Assuming the entire pan to be at an average temperature of 98°C, determine the rate of heat loss from the cylindrical side surface of the pan to the surroundings by natural convection and radiation. Consider properties ν = 1.91×10^5 m^2/s, k = 0.0282 W/mK, and Pr = 0.720.



A) $\dot{q}_{\text{conv}}$ = 23.1 W and $\dot{q}_{\text{rad}}$ = 28.6 W
B) $\dot{q}_{\text{conv}}$ = 23.1 W and $\dot{q}_{\text{rad}}$ = 47.3 W
C) $\dot{q}_{\text{conv}}$ = 46.2 W and $\dot{q}_{\text{rad}}$ = 28.6 W
D) $\dot{q}_{\text{conv}}$ = 46.2 W and $\dot{q}_{\text{rad}}$ = 47.3 W

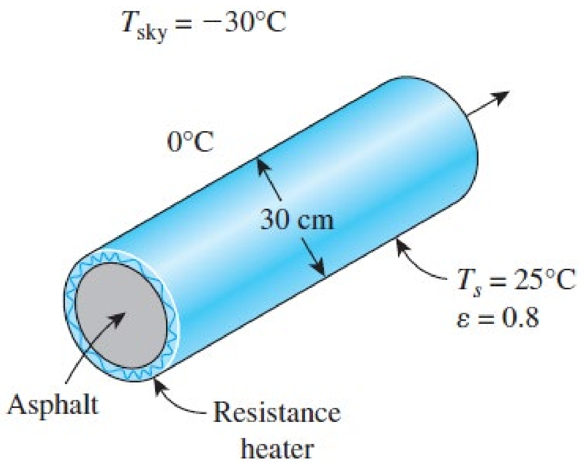
Problem 7B

If water is boiling at a rate of 1.5 kg/h at 100°C, determine the ratio of the heat lost from the sides of the pan to that by the evaporation of water. The enthalpy of vaporization of water at 100°C is 2257 kJ/kg.

A) R = 9.9%
B) R = 18.8%
C) R = 28.7%
D) R = 39.6%

Problem 8A (Çengel & Ghajar, 2015, w/ permission)

Thick fluids such as asphalt and waxes and the pipes in which they flow are often heated in order to reduce the viscosity of the fluids and thus to reduce the pumping costs. Consider the flow of such a fluid through a 100-m long pipe of outer diameter 30 cm in calm ambient air at 0°C. The pipe is heated electrically, and a thermostat keeps the outer surface temperature of the pipe constant at 25°C. The emissivity of the outer surface of the pipe is 0.8, and the effective sky temperature is −30°C. Determine the power rating of the electric resistance heater that needs to be used. Use as properties ν = 1.45×10^{-5} m^2/s, k = 0.0246 W/mK, and Pr = 0.733.



A) $\dot{q}$ = 14.1 kW
B) $\dot{q}$ = 21.5 kW
C) $\dot{q}$ = 28.9 kW
D) $\dot{q}$ = 35.8 kW

■ Problem 8B

Determine the cost of electricity associated with heating the pipe during a 10-hour period under the above conditions if the price of electricity is $0.09/kWh.

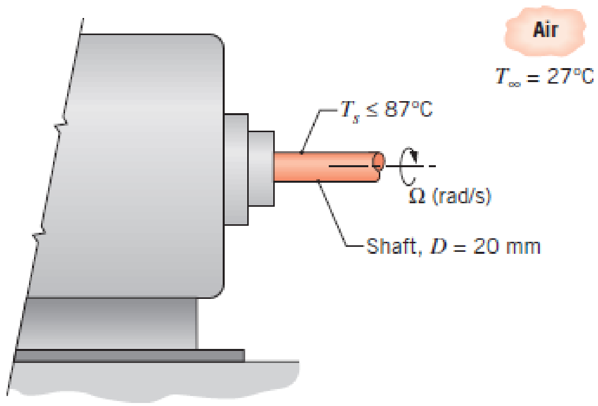
A) C = $14,00
B) C = $20,00
C) C = $26,00
D) C = $32,00

■ Problem 9A (Bergman et al., 2011, w/ permission)

The maximum surface temperature of the 20-mm diameter shaft of a motor operating in ambient air at 27°C should not exceed 87°C. Because of power dissipation within the motor housing, it is desirable to reject as much heat as possible through the shaft to the ambient air. For rotating cylinders, a suitable correlation for estimating the convection coefficient is of the form

$$\overline{Nu}_D = 0.133\,\mathrm{Re}_D^{2/3}\,\mathrm{Pr}^{1/3}$$
$$\left(\mathrm{Re}_D < 4.3\times10^5\ ;\ 0.7 < \mathrm{Pr} < 670\right)$$

where $Re_D = \Omega D^2/\nu$, with Ω being the rotational velocity (rad/s). Find the convection coefficient and the maximum heat transfer rate per unit length as a function of rotational speed in the range of 5000 to 15,000 rpm. Use β = 0.00303 K^{-1}, ν = 1.89×10^{-5} m^2/s, k = 0.0285 W/mK, and α = 2.69×10^{-6} m^2/s.



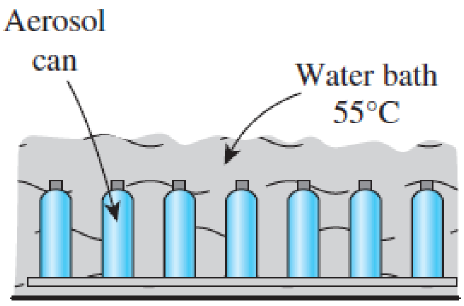
A) q'_{rot} = 133.5 W/m
B) q'_{rot} = 212.6 W/m
C) q'_{rot} = 260.8 W/m
D) q'_{rot} = 315.9 W/m

Problem 9B

Estimate the free convection coefficient and the maximum heat rate per unit length for the stationary shaft. Mixed free and forced convection coefficients may become significant for $Re < 4.7(Gr^3/Pr)^{0.137}$. Are free convection effects important for the range of rotational speeds designated in Part A? Consider Pr = 0.702 in the right-hand side of the preceding inequality.

Problem 10A (Çengel & Ghajar, 2015, w/ permission)

In a plant that manufactures canned aerosol paints, the cans are temperature-tested in water baths at 55°C before they are shipped to ensure that they withstand temperatures up to 55°C during transportation and shelving, as illustrated below. The cans, moving on a conveyor, enter the open hot water bath, which is 0.5-m deep, 1-m wide, and 3.5-m long, and move slowly in the hot water toward the other end. Some of the cans fail the test and explode in the water bath. The water container is made of sheet metal, and the entire container is at about the same temperature as the hot water. The emissivity of the outer surface of the container is 0.7. If the temperature of the surrounding air and surfaces is 20°C, determine the rate of heat loss from the four side surfaces of the container (disregard the top surface, which is open). The water is heated electrically by resistance heaters, and the cost of electricity is \$0.085/kWh. If the plant operates 24 h a day, 365 days a year, and thus 8760 h a year, determine the annual cost of the heat losses from the container for this facility. Use as properties ν = 1.68×10^{-5} m^2/s, k = 0.0264 W/mK, and Pr = 0.726.



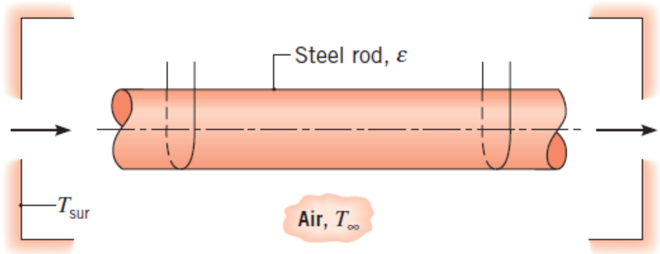
A) C = \$522/year
B) C = \$890/year
C) C = \$1116/year
D) C = \$1455/year

Problem 10B

Reconsider the previous problem. In order to reduce the heating cost of the hot water, it is proposed to insulate the side and bottom surfaces of the container with a 5-cm thick fiberglass insulation (k = 0.035 W/mK) and to wrap the insulation with aluminum foil (ε = 0.1) in order to minimize the heat loss by radiation. An estimate is obtained from a local insulation contractor, who proposes to do the insulation job for \$350, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves? Use ν = 1.54×10^{-5} m^2/s, k = 0.0254 W/mK, and Pr = 0.730.

Problem 11A (Bergman et al., 2011, w/ permission)

Long stainless steel rods of 50-mm diameter are pre-heated to a uniform temperature of 1000 K before being suspended from an overhead conveyor for transport to a hot forming operation. The conveyor is in a large room whose walls and air are at 300 K. Assuming the linear motion of the rod to have a negligible effect on convection heat transfer from its surface, determine the average convection coefficient at the start of the transport process. Use ν = 6.02×10^{-5} m^2/s, α = 8.73×10^{-5} m^2/s, k = 0.0497 W/mK, and Pr = 0.690.



A) h = 1.91 W/m²K
B) h = 4.35 W/m²K
C) h = 6.70 W/m²K
D) h = 9.84 W/m²K

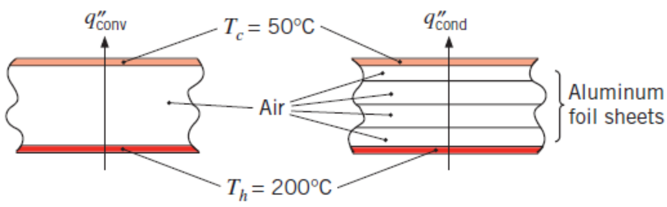
■ Problem 11B

If the surface emissivity of the rod is ε = 0.40, determine the effective radiation heat transfer coefficient at the start of the transport process. Assuming a cumulative (convection plus radiation) heat transfer coefficient corresponding to the results obtained to this point, what is the maximum allowable conveyor transit time, if the centerline temperature of the rod must exceed 900 K for the forming operation? Properties of the steel are k = 25 W/mK and $\alpha = 5.2\times10^{-6}$ m²/s.

A) T = 100 s
B) T = 160 s
C) T = 220 s
D) T = 280 s

■ Problem 12A (Bergman et al., 2011, w/ permission)

A 50-mm thick air gap separates two horizontal metal plates that form the top surface of an industrial furnace. The bottom plate is at T_h = 200°C and the top plate is at T_c = 50°C. The plant operator wishes to provide insulation between the plates to minimize heat loss. The relatively hot temperatures preclude use of foamed or felt insulation materials. Evacuated insulation materials cannot be used due to the harsh industrial environment and their expense. A young engineer suggests that equally spaced, thin horizontal sheets of aluminum foil may be inserted in the gap to eliminate natural convection and minimize heat loss through the air gap. Determine the convective heat flux across the gap when no insulation is in place. Use β = 0.00251 K^{-1}, $\nu = 2.62\times10^{-5}$ m²/s, $\alpha = 3.80\times10^{-5}$ m²/s, Pr = 0.690, and k = 0.0337 W/mK.



A) q''_{conv} = 302 W/m²
B) q''_{conv} = 525 W/m²
C) q''_{conv} = 719 W/m²
D) q''_{conv} = 934 W/m²

■ Problem 12B

Determine the minimum number of foil sheets that must be inserted in the gap to eliminate free convection.

A) N = 2
B) N = 3
C) N = 4
D) N = 5

Solutions

P.1 Solution

The film temperature is $T_f = (T_s + T_\infty)/2 = (20 + 100)/2 = 60^\circ$C, and the corresponding thermal expansion coefficient is $\beta = 1/T_f = 1/(273 + 60) = 0.00300$ K^{-1}. Of the two possible orientations, the maximum Rayleigh number is obtained with orientation B, which corresponds to a value of *Ra* such that

$$Ra_{\max} = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

with L = 0.5 m and other pertaining variables, we get

$$Ra_{\max} = \frac{9.81\times 0.00300\times\left(100-20\right)\times 0.5^3}{\left(1.92\times 10^{-5}\right)\times\left(2.74\times 10^{-5}\right)} = 5.59\times 10^8$$

which is less than the critical value $Ra_{x,c} = 10^9$. Consequently, flow conditions are laminar for both orientations. In order to reduce heat transfer from the plate, we should maximize the thickness of the boundary layer and maximize the length of the plate in the vertical direction. This is why orientation B is preferred. For laminar flow in a vertical plate, the Nusselt number can be obtained with the correlation

$$Nu = 0.68 + \frac{0.67Ra^{1/4}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{4/9}}$$

$$\therefore Nu = 0.68 + \frac{0.67\times\left(5.59\times 10^8\right)^{1/4}}{\left[1+\left(\frac{0.492}{0.702}\right)^{9/16}\right]^{4/9}} = 79.7$$

The corresponding convection coefficient is then

$$Nu = \frac{h\times L}{k} \rightarrow h = \frac{k\times Nu}{L}$$

$$\therefore h = \frac{0.0287\times 79.7}{0.5} = 4.57\ \text{W/m}^2\text{K}$$

Noting that the cross-sectional area A = 0.25×0.5 = 0.125 m^2, the convection heat transfer from the front surface of the plate in orientation B is

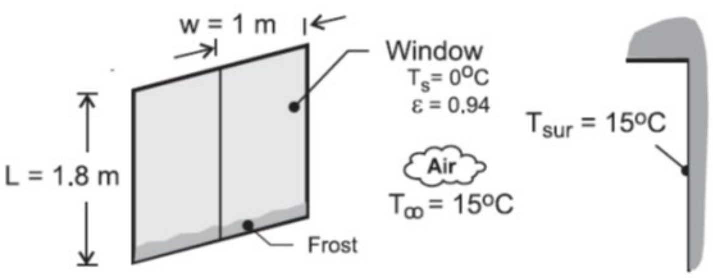
$$\dot{q} = hA\left(T_s - T_\infty\right) = 4.57\times 0.125\times\left(100-20\right) = \boxed{45.7\ \text{W}}$$

The convection heat transfer from the front surface of the plate in the preferred orientation is 45.7 watts.

♦ The correct answer is **D**.

P.2 Solution

Part A: Consider the following schematic.



For these winter conditions, a frost line could appear and it would be at the bottom of the window. The boundary layer is thinnest at the top of the window, and hence the heat flux from the warmer room is greater than compared to that bottom portion of the window, where the boundary layer is thicker. Also, the air in the room may be stratified and cooler near the floor compared to near the ceiling.

Part B: Taking into consideration both convection and radiation, the heat loss from the room to the window is

$$\dot{q} = A_s \times \left[h(T_\infty - T_s) + \varepsilon\sigma\left(T_{\text{sur}}^4 - T_s^4\right) \right]$$

The Nusselt number can be determined via the correlation

$$Nu = \left\{ 0.825 + \frac{0.387 Ra^{1/6}}{\left[1 + \left(\frac{0.492}{\text{Pr}} \right)^{9/16} \right]^{8/27}} \right\}^2$$

This in turn requires the Rayleigh number *Ra*, which is given by

$$Ra = \frac{g\beta(T_\infty - T_s)L^3}{\nu\alpha}$$

Here, β = 1/(288 + 273)/2 = 0.00357 K^{-1} and L = 1.8 m. Substituting these and other appropriate variables, we get

$$Ra = \frac{g\beta T(T_\infty - T_s)L^3}{\nu\alpha} = \frac{9.81 \times 0.00357 \times (15-0) \times 1.8^3}{(1.41 \times 10^{-5}) \times (1.99 \times 10^{-5})} = 1.09 \times 10^{10}$$

Backsubstituting this value and Pr = 0.710 into the expression for Nu, we obtain

$$Nu = \left\{ 0.825 + \frac{0.387 \times \left(1.09 \times 10^{10}\right)^{1/6}}{\left[1 + \left(\frac{0.492}{0.710} \right)^{\frac{9}{16}} \right]^{\frac{8}{27}}} \right\}^2 = 259.2$$

Then, heat transfer coefficient h is determined as

$$Nu = \frac{h \times L}{k} \rightarrow h = \frac{Nu \times k}{L}$$

$$\therefore h = \frac{259.2 \times 0.0247}{1.8} = 3.56 \text{ W/m}^2\text{K}$$

Finally, heat loss $\dot{q}$ is calculated as

$$\dot{q} = (1.8 \times 1.0) \times \left[3.56 \times 15 + 0.94 \times \left(5.67 \times 10^{-8}\right) \times \left(288^4 - 273^4\right) \right] = \boxed{223.2 \text{ W}}$$

♦ The correct answer is **B**.

P.3 Solution

The film temperature T_f = (270 + 18)/2 = 144°C and the thermal expansion coefficient β = 1/(144 + 273) = 0.00240 K^{-1}. The Grashof number can be computed with the usual equation,

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

$$\therefore Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{9.81\times 0.0024\times(270-18)\times 2^3}{(2.83\times 10^{-5})^2} = 5.93\times 10^{10}$$

Consider now the equation for the Reynolds number, *Re*,

$$\mathrm{Re} = \frac{VL}{\nu}$$

Substituting L = 2 m and $\nu = 2.83\times10^{-5}$ m^2/s gives

$$\mathrm{Re} = \frac{V\times 2}{(2.83\times 10^{-5})} = 70,671\times V$$

For the natural convection effects on heat transfer to be less than 10 percent, the ratio of the Grashof number to the square of the Reynolds number must be within a threshold of 0.1. Mathematically,

$$\frac{Gr}{\mathrm{Re}^2} \le 0.1$$

Substituting $Gr = 5.93\times10^{10}$ and $Re = 70{,}671V$, we obtain

$$\frac{(5.93\times 10^{10})}{(70,671V)^2} = 0.1$$

$$\therefore \frac{5.93\times 10^{10}}{4.99\times 10^9 V^2} = 0.1$$

$$\therefore V^2 = \frac{5.93\times 10^{10}}{4.99\times 10^9 \times 0.1}$$

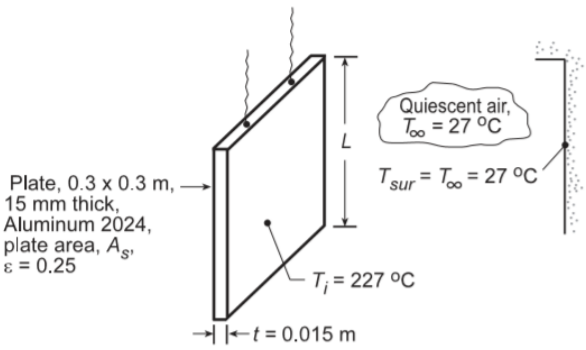
$$\therefore V = \left(\frac{5.93\times 10^{10}}{4.99\times 10^9\times 0.1}\right)^{\frac{1}{2}} = \boxed{10.9\text{ m/s}}$$

The velocity above which natural convection heat transfer from the plate is negligible is 10.9 m/s, or about 39 km/h.

♦ The correct answer is **C**.

P.4 Solution

Part A: Consider the following schematic.



From an energy balance on the plate with free convection and radiation exchange, we obtain

$$\rho A_s t c_p \frac{dT}{dt} = -h\times 2A_s(T_s - T_\infty) - \varepsilon\times 2A_s\sigma(T_s^4 - T_{\text{sur}}^4)$$

$$\therefore \frac{dT}{dt} = -\frac{2}{\rho tc}\left[h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{\text{sur}}^4)\right]$$

where T_s is the plate temperature, assumed to be uniform at any time. The equation above describes the variation in temperature with time.

Part B: The thermal expansion coefficient is β = 1/400 = 0.0025 K^{-1}. The Rayleigh number is calculated as

$$Ra = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha} = \frac{9.81\times 0.0025\times\left(227-27\right)\times 0.3^3}{\left(2.64\times 10^{-5}\right)\times\left(3.83\times 10^{-5}\right)} = 1.31\times 10^8$$

Since $Ra < 10^9$, the Nusselt number can be estimated with the correlation

$$Nu = 0.68 + \frac{0.670Ra^{1/4}}{\left[1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{4/9}}$$

$$\therefore Nu = 0.68 + \frac{0.670\times\left(1.31\times 10^8\right)^{1/4}}{\left[1+\left(0.492/0.690\right)^{9/16}\right]^{4/9}} = 55.5$$

From the definition of Nu, the heat transfer coefficient is found as

$$Nu = \frac{h\times L}{k} \rightarrow h = \frac{k\times Nu}{L}$$

$$\therefore h = \frac{0.0338\times 55.5}{0.3} = 6.25 \text{ W/m}^2\text{K}$$

Substituting this and other quantities in the relation obtained in Part A, we see that

$$\frac{dT}{dt} = -\frac{2}{2770\times 0.015\times 983}\times\left[6.25\times\left(227-27\right)+0.25\times\left(5.67\times 10^{-8}\right)\times\left(500^4-300^4\right)\right] = \boxed{-0.099 \text{ K/s}}$$

The temperature is dropping almost 0.1 K in each second.

♦ The correct answer is **B**.

Part C: The heat transfer coefficient is obtained by solving the equation obtained in Part A for h. Mathematically,

$$h = \frac{-\rho t c_p \dfrac{dT}{dt} - 2\varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)}{2\left(T_s - T_\infty\right)}$$

$$\therefore h = \frac{-2770\times 0.015\times 983\times\left(-0.0465\right)-2\times 0.25\times\left(5.67\times 10^{-8}\right)\times\left(400^4-300^4\right)}{2\times\left(127-27\right)} = 7.02 \text{ W/m}^2\text{K}$$

To select an appropriate empirical correlation, we first compute the Rayleigh number,

$$Ra = \frac{g\beta\Delta TL^3}{\nu\alpha} = \frac{9.81\times 0.00286\times 100\times 0.3^3}{\left(2.09\times 10^{-5}\right)\times\left(2.99\times 10^{-5}\right)} = 1.21\times 10^8$$

Since $Ra < 10^9$, we can use the same correlation as we used in Part A; that is,

$$Nu = 0.68 + \frac{0.670Ra^{1/4}}{\left[1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\times\left(1.21\times 10^8\right)^{1/4}}{\left[1+\left(0.492/0.700\right)^{9/16}\right]^{4/9}} = 54.5$$

so that

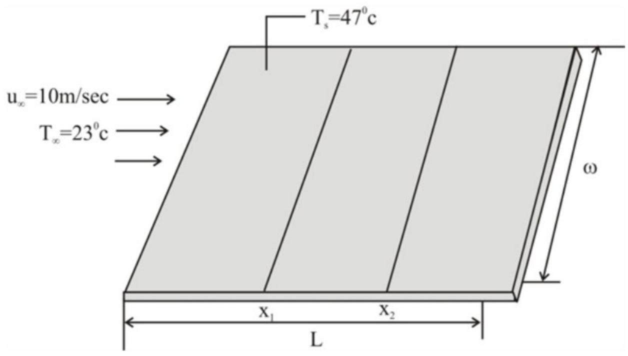
$$h = \frac{k\times Nu}{L} = \frac{0.030\times 54.5}{0.3} = 5.45 \text{ W/m}^2\text{K}$$

The correlation estimate is lower than the experimental result, and the difference between the two estimates is about 29%.

♦ The correct answer is **B**.

P.5 Solution

Part A: Consider the following schematic.



For Part A, the power required for the regimented heater follows from the relationship

$$P_e = \bar{h}_{x_1-x_2}\left(X_2 - X_1\right)w\left(T_s - T_\infty\right)$$

where $\bar{h}_{x_1-x_2}$ is the average heat transfer coefficient, *X* denotes distance from the left end, *w* is width, and *T* denotes temperature. The Nusselt number for turbulent flow over the plate is determined as

$$Nu_X = 0.0296\,\mathrm{Re}_{X_1}^{4/5}\,\mathrm{Pr}^{1/3} = 0.0296\left(\frac{u_\infty \times X_1}{\nu}\right)^{4/5}\mathrm{Pr}^{1/3}$$

$$\therefore Nu_X = 0.0296\times\left(\frac{10\times0.2}{1.67\times10^{-5}}\right)^{4/5}\times0.706^{1/3} = 304.5$$

Similarly, we have, for the position X_2 = 0.3 m,

$$Nu_2 = 0.0296\times\left(\frac{10\times0.3}{1.67\times10^{-5}}\right)^{4/5}\times0.706^{1/3} = 421.1$$

Then, the heat transfer coefficient in each case follows from the definition of the Nusselt number,

$$Nu_1 = \frac{h_1\times X_1}{k} \rightarrow h_1 = \frac{Nu_1\times k}{X_1}$$

$$\therefore h_1 = \frac{304.5\times0.0269}{0.2} = 40.96\ \mathrm{W/m^2K}$$

and

$$Nu_2 = \frac{h_2\times X_2}{k} \rightarrow h_2 = \frac{Nu_2\times k}{X_2}$$

$$\therefore h_2 = \frac{421.1\times0.0269}{0.3} = 37.76\ \mathrm{W/m^2K}$$

The average convection coefficient, $\bar{h}_{x_1-x_2}$, is the average of these two *h* values. Returning to the expression for the required power P_e, we find that

$$P_e = \left(\frac{h_1+h_2}{2}\right)\left(X_2 - X_1\right)w\left(T_s - T_\infty\right)$$

$$\therefore P_e = \left(\frac{40.96+37.76}{2}\right)\times\left(0.3-0.2\right)\times0.2\times\left(47-23\right) = \boxed{18.9\ \mathrm{W}}$$

The power required for the regimented heater is just short of 19 watts.

♦ The correct answer is **A**.

Part B: If the blower malfunctions, the equation to use when predicting the power of the device is

$$P_e = \left[h_{\text{fc}}\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_\infty^4\right)\right]\left(X_2 - X_1\right)w$$

In words, the forced convection heat transfer loses importance relative to heat transfer due to free convection and radiation. The heat transfer coefficient can be computed while supposing that we have free convection heat transfer over a horizontal plate. The pertaining formula for *Nu* is

$$Nu = 0.54Ra^{1/4}$$

where the Rayleigh number *Ra* is determined with the usual relation

$$Ra = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu\alpha}$$

Here, $\beta = 1/T_f = 1/(296 + 320)/2 = 0.00325$ K^{-1} and the characteristic length $L_c = (w\times L)/2(w + L)$. Substituting the pertaining variables, we obtain

$$Ra = \frac{9.81\times 0.00325\times\left(47-23\right)\times\left[\dfrac{0.2\times 0.5}{2\left(0.2+0.5\right)}\right]}{\left(1.67\times 10^{-5}\right)\times\left(2.37\times 10^{-6}\right)} = 7.05\times 10^5$$

Then, substituting in the expression for *Nu*, we find that

$$Nu = 0.54\times\left(7.05\times 10^5\right)^{1/4} = 15.6$$

Noting that $L_c = 0.0714$ m, the free convection heat transfer coefficient easily follows,

$$Nu = \frac{h_{\text{fc}}\times L_c}{k} \rightarrow h_{\text{fc}} = \frac{Nu\times k}{L_c}$$

$$\therefore h_{fg} = \frac{15.6\times 0.0269}{0.0714} = 5.88\ \text{W/m}^2\text{K}$$

Finally, we can return to the new expression for P_e, observing that $P_e = 18.9$ W as before,

$$P_e = \left[h_{\text{fc}}\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_\infty^4\right)\right]\left(X_2 - X_1\right)w$$

$$\therefore 18.9 = \left[5.88\times\left(T_s - 296\right) + 0.03\times\left(5.87\times 10^{-8}\right)\times\left(T_s^4 - 296^4\right)\right]\times\left(0.3-0.2\right)\times 0.2$$

$$\therefore 18.9 = \left[5.88\left(T_s - 296\right) + 1.76\times 10^{-9}\left(T_s^4 - 296^4\right)\right]\times 0.02$$

The relation above reads as a fourth-degree equation in the surface temperature T_s, which can be solved by means of the *Solve* command in Mathematica,

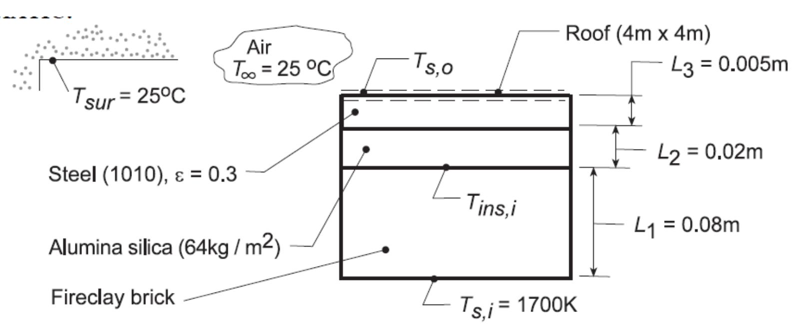
```
Solve[18.9 == (5.88(Ts − 296) + 1.76 * 10⁻⁹(Ts⁴ − 296⁴)) * 0.02, Ts]
```

This yields two imaginary solutions, along with $T_s = -1624$ K, which is impossible, and $T_s = 447$ K, which is the one feasible solution. We conclude that $T_s = 447$ K = 174°C.

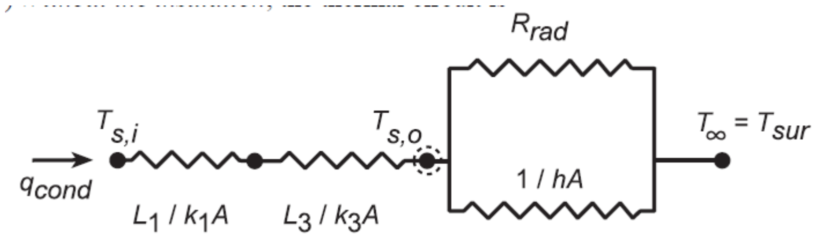
♦ The correct answer is **B**.

P.6 Solution

Part A: Below, we have a schematic of the present system.



Without the insulation, the thermal circuit is as shown.



The outer surface temperature $T_{s,o}$, which is our main unknown, is highlighted. Performing an energy balance at the outer surface, we write

$$q_{\text{cond}} = q_{\text{conv}} + q_{\text{rad}} \rightarrow \frac{T_{s,i} - T_{s,o}}{\frac{L_1}{k_1 A} + \frac{L_3}{k_3 A}} = hA\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma A\left(T_{s,o}^4 - T_{\text{sur}}^4\right) \text{ (I)}$$

The Rayleigh number is

$$Ra = \frac{g\beta\left(T_{s,o} - T_\infty\right)L_c^3}{\nu\alpha}$$

where the characteristic length associated with free convection is $L_c = A_s/P$ = $4^2/[2(4+4)]$ = 1 m. Substituting, we get

$$Ra = \frac{g\beta\left(T_{s,o} - T_\infty\right)L_c^3}{\nu\alpha} = \frac{9.81 \times 0.0025 \times \left(T_{s,o} - 298\right) \times 1^3}{\left(2.64 \times 10^{-5}\right) \times \left(3.83 \times 10^{-5}\right)} = 2.43 \times 10^7 \left(T_{s,o} - 298\right)$$

The Nusselt number for natural convection in the upper surface of a hot plate is, for the current range of Rayleigh numbers,

$$Nu = 0.15 Ra^{1/3}$$

$$\therefore Nu = 0.15 \times \left[2.43 \times 10^7 \times \left(T_{s,o} - 298\right)\right]^{1/3} = 43.45\left(T_{s,o} - 298\right)^{1/3}$$

From the definition of *Nu*, we have

$$Nu = \frac{h \times L_c}{k} \rightarrow h = \frac{k \times Nu}{L_c}$$

$$\therefore h = \frac{0.0338}{1} \times 43.45\left(T_{s,o} - 298\right)^{1/3} = 1.47\left(T_{s,o} - 298\right)^{1/3}$$

The energy balance can now be written as

$$\frac{T_{s,i} - T_{s,o}}{\frac{L_1}{k_1} + \frac{L_3}{k_3}} = h\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma\left(T_{s,o}^4 - T_{\text{sur}}^4\right)$$

$$\therefore \frac{\left(1700 - T_{s,o}\right)}{\frac{0.08}{1.8} + \frac{0.005}{48.8}} = \left[1.47\left(T_{s,o} - 298\right)^{1/3}\right] \times \left(T_{s,o} - 298\right) + 0.3 \times \left(5.67 \times 10^{-8}\right) \times \left(T_{s,o}^4 - 298^4\right)$$

$$\therefore \frac{\left(1700 - T_{s,o}\right)}{\frac{0.08}{1.8} + \frac{0.005}{48.8}} = 1.47\left(T_{s,o} - 298\right)^{4/3} + 0.3 \times \left(5.67 \times 10^{-8}\right) \times \left(T_{s,o}^4 - 298^4\right)$$

$$\therefore \frac{\left(1700 - T_{s,o}\right)}{\frac{0.08}{1.8} + \frac{0.005}{48.8}} = 1.47\left(T_{s,o} - 298\right)^{4/3} + 1.70 \times 10^{-8} \times \left(T_{s,o}^4 - 298^4\right)$$

$$\therefore 22.45\left(1700 - T_{s,o}\right) = 1.47\left(T_{s,o} - 298\right)^{4/3} + 1.70 \times 10^{-8}\left(T_{s,o}^4 - 298^4\right) \text{ (II)}$$

This equation can be solved iteratively. In Mathematica, the *Solve* command would do the trick. The one meaningful solution is $T_{s,o}$ = 894 K. Backsubstituting in Equation (I), we find that

$$\dot{q} = 16\left[1.47\left(894-298\right)^{4/3} + 0.3\times\left(5.67\times10^{-8}\right)\times\left(894^4-298^4\right)\right] = \boxed{289.7\text{ kW}}$$

The heat loss through the roof without insulation is about 290 kilowatts.

♦ The correct answer is **B**.

Part B: With the insulation, an additional conduction resistance is provided and the energy balance at the outer surface becomes

$$\frac{T_{s,i}-T_{s,o}}{\dfrac{L_1}{k_1 A}+\dfrac{L_2}{k_2 A}+\dfrac{L_3}{k_3 A}} = hA\left(T_{s,o}-T_\infty\right)+\varepsilon\sigma A\left(T_{s,o}^4-T_{\text{surr}}^4\right)$$

$$\therefore \frac{1700-T_{s,o}}{\dfrac{0.08}{1.8}+\dfrac{0.02}{0.125}+\dfrac{0.005}{48.8}} = 1.47\left(T_{s,o}-298\right)^{4/3}+1.70\times10^{-8}\times\left(T_{s,o}^4-298^4\right)$$

$$\therefore 4.89\left(1700-T_{s,o}\right) = 1.47\left(T_{s,o}-298\right)^{4/3}+1.70\times10^{-8}\left(T_{s,o}^4-298^4\right)$$

Applying an iterative procedure as before, we obtain $T_{s,o}$ = 610 K. Substituting in Equation (I), the result is

$$\dot{q} = 16\left[1.47\left(610-298\right)^{4/3} + 0.3\times\left(5.67\times10^{-8}\right)\times\left(610^4-298^4\right)\right] = \boxed{85.3\text{ kW}}$$

The heat loss through the roof with insulation is about 85 kilowatts, which corresponds to a reduction of about 70% relative to the initial value. The inner surface temperature of the insulation is given by

$$\dot{q} = \frac{T_{s,i}-T_{\text{ins},i}}{\dfrac{L_1}{k_1 A}} \rightarrow T_{\text{ins},i} = -\dot{q}\frac{L_1}{k_1 A}+T_{s,i}$$

$$\therefore T_{\text{ins},i} = -85{,}300\times\frac{0.08}{1.8\times16}+1700 = \boxed{1463\text{ K}}$$

♦ The correct answer is **A**.

P.7 Solution

Part A: The film temperature is T_f = (98+25)/2 = 61.5°C, and the thermal expansion coefficient is, accordingly, β = 1/(61.5+273) = 0.003 K^{-1}. The surface area of the pan is $A_s = \pi DL = \pi \times 0.25\times0.12 = 0.0942$ m^2. The characteristic length is the depth of the cylinder $L_c = L$ = 0.12 m. We proceed to compute the Rayleigh number,

$$Ra = \frac{g\beta\left(T_s-T_\infty\right)L_c^3}{\nu^2}\Pr = \frac{9.81\times0.003\times\left(98-25\right)\times0.12^3}{\left(1.91\times10^{-5}\right)^2}\times0.720 = 7.33\times10^6$$

The Grashof number, *Gr*, is the ratio of *Ra* to the Prandtl number, *Pr*; that is,

$$Gr = \frac{Ra}{\Pr} = \frac{\left(7.33\times10^6\right)}{0.720} = 1.02\times10^7$$

Consider the following inequality,

$$\frac{35L_c}{Gr^{1/4}} \le 25$$

If this inequality holds, the vertical cylinder can be treated as a vertical plate. Substituting the values available for L_c and *Gr* gives

$$\frac{35\times 0.12}{\left(1.02\times 10^{7}\right)^{1/4}} = 0.0743 < 25$$

Since the inequality is obeyed, the pan cylinder can be treated as a vertical plate. The Nusselt number is therefore given by

$$Nu = \left\{ 0.825 + \frac{0.387 Ra^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}} \right\}^{2}$$

$$\therefore Nu = \left\{ 0.825 + \frac{0.387\times\left(7.33\times 10^{6}\right)^{1/6}}{\left[1+\left(\frac{0.492}{0.720}\right)^{9/16}\right]^{8/27}} \right\}^{2} = 28.6$$

The heat transfer coefficient easily follows,

$$Nu = \frac{h\times L_c}{k} \to h = \frac{Nu\times k}{L_c}$$

$$\therefore h = \frac{28.6\times 0.0282}{0.12} = 6.72 \ \mathrm{W/m^2K}$$

The heat transfer rate is obtained with Newton's law of cooling,

$$\dot{q}_{\mathrm{conv}} = hA_s\left(T_s - T_\infty\right) = 6.72\times 0.0942\times\left(98-25\right) = \boxed{46.2 \ \mathrm{W}}$$

The rate of heat loss from the pan due to natural convection is close to 46 watts. The rate of heat loss due to radiation can be determined with the Stefan-Boltzmann law,

$$\dot{q}_{\mathrm{rad}} = \varepsilon A_s\sigma\left(T_s^4 - T_\infty^4\right) = 0.8\times 0.0942\times\left(5.67\times 10^{-8}\right)\times\left[\left(98+273\right)^4 - \left(25+273\right)^4\right]$$

$$\therefore \dot{q}_{\mathrm{rad}} = \boxed{47.3 \ \mathrm{W}}$$

The rate of heat loss from the pan due to radiation is quite close to that due to convection.

♦ The correct answer is **D**.

Part B: Water is boiling in the pan at a rate of 1.5 kg/hour, which corresponds to a mass flow $\dot{m}$ = 1.5/3600 = 4.17×10^{-4} kg/s. Given the enthalpy of vaporization, h_{fg} = 2257 kJ/kg, the rate of heat loss due to evaporation of water, $\dot{q}_{\mathrm{evap}}$, is

$$\dot{q}_{\mathrm{evap}} = \dot{m}h_{fg} = \left(4.17\times 10^{-4}\right)\times\left(2257\times 10^{3}\right) = 941.2 \ \mathrm{W}$$

The ratio R of heat lost from the side of the pan to that by the evaporation of water is then

$$R = \frac{\dot{q}_{\mathrm{conv}} + \dot{q}_{\mathrm{rad}}}{\dot{q}_{\mathrm{evap}}} = \frac{46.2+47.3}{941.2} = \boxed{9.9\%}$$

That is to say, the rate of heat loss on the side of the pan, combining convection and radiation, is about 10 percent of the heat lost by evaporation.

♦ The correct answer is **A**.

P.8 Solution

Part A: The film temperature is T_f = (25 + 0)/2 = 12.5°C, and the corresponding thermal expansion coefficient is β = 1/(12.5 + 273) = 0.00350 K^{-1}. We proceed to compute the Rayleigh number,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}$$

$$\therefore Ra = \frac{9.81\times 0.0035\times(25-0)\times 0.3^3}{(1.45\times 10^{-5})^2}\times 0.733 = 8.08\times 10^7$$

Then, the Nusselt number can be determined with the following correlation, which applies for a cylinder in the horizontal position,

$$Nu = \left\{0.6 + \frac{0.387(Ra)^{1/6}}{\left[1+\left(\frac{0.559}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\therefore Nu = \left\{0.6 + \frac{0.387(8.08\times 10^7)^{1/6}}{\left[1+\left(\frac{0.559}{0.733}\right)^{9/16}\right]^{8/27}}\right\}^2 = 53.2$$

The heat transfer coefficient is then

$$Nu = \frac{h\times D}{k} \rightarrow h = \frac{k\times Nu}{D}$$

$$\therefore h = \frac{0.0246\times 53.2}{0.3} = 4.36\ \mathrm{W/m^2K}$$

The total heat loss due to convection and radiation follows as

$$\dot{q} = \dot{q}_{\mathrm{conv}} + \dot{q}_{\mathrm{rad}}$$

$$\therefore \dot{q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^4 - T_{\mathrm{sky}}^4)$$

The surface area of the conduit is $A_s = \pi DL = \pi \times 0.3 \times 100 = 94.25$ m^2. Substituting this and other pertaining variables, it follows that

$$\dot{q} = 4.36\times 94.25\times(25-0) + 0.8\times 94.25\times(5.67\times 10^{-8})\times\left[(25+273)^4 - (-30+273)^4\right] = \boxed{28.9\ \mathrm{kW}}$$

The power rating of the electric resistance heater needed is 28.9 kilowatts.

♦ The correct answer is **C**.

Part B: The total heat loss during 10 hours, Q, is given by the product of the power rating, $\dot{q}$, and the operation time in hours, T,

$$Q = \dot{q}\times T = 28.9\ \mathrm{kW}\ \times 10\ \mathrm{h} = 289\ \mathrm{kWh}$$

The price of electricity is \$0.09 per kWh. The cost C of the electricity employed to heat the pipe in the specified time period is, accordingly,

$$C = 289\ \cancel{\mathrm{kWh}}\times\frac{\$0.09}{\cancel{\mathrm{kWh}}} = \boxed{\$26,00}$$

The cost of electricity used to heat the pipe over a 10-hour period is 26 dollars.

♦ The correct answer is **C**.

P.9 Solution

Part A: The recommended correlation for a horizontal rotating shaft is

$$Nu = 0.133\,\mathrm{Re}^{2/3}\,\mathrm{Pr}^{1/3} \quad ; \quad (\mathrm{Re}_D < 4.3\times10^5,\ 0.7 < \mathrm{Pr} < 670)$$

where the Reynolds number is

$$\mathrm{Re} = \frac{\Omega D^2}{\nu}$$

We have Ω = 5000 rpm = 523.6 rad/s, so that

$$\mathrm{Re} = \frac{\omega D^2}{\nu} = \frac{523.6\times0.02^2}{\left(1.89\times10^{-5}\right)} = 11{,}082$$

Using this value and Pr = 0.703, we can compute the Nusselt number,

$$Nu = 0.133\times11{,}082^{2/3}\times0.703^{1/3} = 58.8$$

and thence the convection coefficient,

$$Nu = \frac{h_{\mathrm{rot}}\times D}{k} \rightarrow h_{\mathrm{rot}} = \frac{Nu\times k}{D}$$

$$\therefore h_{\mathrm{rot}} = \frac{58.8\times0.0285}{0.02} = 83.79\ \mathrm{W/m^2K}$$

Finally, we can determine the heat transfer rate per unit length,

$$q'_{\mathrm{rot}} = h_{\mathrm{rot}}\times\pi D\times\left(T_s - T_\infty\right) = 83.79\times\pi\times0.02\times\left(87-27\right) = \boxed{315.9\ \mathrm{W/m}}$$

♦ The correct answer is **D**.

Part B: For the stationary shaft condition, the free convection coefficient can be estimated from the Churchill-Chu correlation. This, in turn, requires the Rayleigh number,

$$Ra = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha}$$

Given the film temperature $T_f = (T_s - T_\infty)/2$ = 57°C, we have $\beta = 1/T_f$ = 1/(57+273) = 0.00303 K^{-1}. Substituting these and other pertaining variables, we obtain

$$Ra = \frac{9.81\times0.00303\times\left(87-27\right)\times0.02^3}{\left(1.89\times10^{-5}\right)\times\left(2.69\times10^{-5}\right)} = 28{,}063$$

Using this quantity and Pr = 0.703, we can apply the correlation

$$Nu = \left\{0.60 + \frac{0.387\times Ra^{1/6}}{\left[1+\left(0.559/\mathrm{Pr}\right)^{1/6}\right]^{8/27}}\right\}$$

$$\therefore Nu = \left\{0.60 + \frac{0.387\times28{,}063^{1/6}}{\left[1+\left(0.559/0.703\right)^{1/6}\right]^{8/27}}\right\}^2 = 5.5$$

The heat transfer coefficient easily follows,

$$Nu = \frac{h_{\mathrm{fc}}\times D}{k} \rightarrow h_{\mathrm{fc}} = \frac{Nu\times k}{D}$$

$$\therefore h_{\mathrm{fc}} = \frac{5.5\times0.0285}{0.02} = 7.84\ \mathrm{W/m^2K}$$

Finally, we establish the following heat transfer rate per unit length,

$$q'_{\text{fc}} = 7.84\times(\pi\times 0.02)\times(87-27) = 29.6\ \text{W/m}$$

Mixed free and forced convection effects may be significant if the following inequality holds,

$$\text{Re} < 4.7\left(\frac{\text{Gr}^3}{\text{Pr}}\right)^{0.137}$$

where *Gr* = *Ra*/*Pr*. In the left-hand side, we have *Re* = 11,082; in the RHS, we have *Gr* = 28,063/0.703 = 39,919 and *Pr* = 0.702 as prescribed. Substituting in each side, we obtain

$$11{,}082 ? < ? 4.7\left(\frac{39{,}919^3}{0.702}\right)^{0.137} = 384$$

The inequality does not hold, and we conclude that free convection effects are not significant for rotational speeds above 5000 rpm.

P.10 Solution

Part A: The film temperature is T_f = (55 + 20)/2 = 37.5°C, and the corresponding thermal expansion coefficient is β = 1/(37.5 + 273) = 0.00322 K^{-1}. We proceed to calculate the Rayleigh number, which is given by

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{9.81\times 0.00322\times(55-20)\times 0.5^3}{\left(1.68\times 10^{-5}\right)^2}\times 0.726 = 3.55\times 10^8$$

Note that we have used the height of the bath as the characteristic dimension. The Nusselt number is determined with the correlation below, which applies for natural convection on a vertical plate,

$$Nu = \left\{0.825 + \frac{0.387Ra^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\therefore Nu = \left\{0.825 + \frac{0.387\left(3.55\times 10^8\right)^{1/6}}{\left[1+\left(\frac{0.492}{0.726}\right)^{9/16}\right]^{8/27}}\right\}^2 = 89.7$$

Next, the heat transfer coefficient can be obtained with the definition of *Nu*,

$$Nu = \frac{h\times L}{k} \to h = \frac{Nu\times k}{L}$$

$$\therefore h = \frac{89.7\times 0.0264}{0.5} = 4.74\ \text{W/m}^2\text{K}$$

where we have once again used the height of the bath as the characteristic length. The surface area A_s is

$$A_s = 2(0.5\times 1 + 0.5\times 3.5) = 4.5\ \text{m}^2$$

Then, the heat transfer rate due to natural convection follows from Newton's law of cooling,

$$\dot{q}_{\text{conv}} = hA_s(T_s - T_\infty) = 4.74\times 4.5\times(55-20) = 746.6\ \text{W}$$

The heat transfer rate due to radiation, in turn, is calculated with the Stefan-Boltzmann law,

$$\dot{q}_{\text{rad}} = \varepsilon A_s \sigma \left(T_s^4 - T_\infty^4\right) = 0.7 \times 4.5 \times \left(5.67 \times 10^{-8}\right) \times \left[(55+273)^4 - (20+273)^4\right] = 750.9 \text{ W}$$

Finally, the total rate of heat loss becomes

$$\dot{q} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}} = 746.6 + 750.9 = 1497.5 \text{ W}$$

The amount of heat lost during one year is

$$Q = \dot{q}\Delta t = 1.498 \text{ kW} \times \underbrace{8760 \text{ h}}_{=1 \text{ year}} = 13{,}123 \text{ kWh}$$

and the corresponding cost *C* is

$$C = 13{,}123 \cancel{\text{ kWh}} \times \frac{\$0.085}{\cancel{\text{kWh}}} = \boxed{\$1116/\text{year}}$$

That is to say, the annual cost of the heat losses from the container is over a thousand dollars.

♦ The correct answer is **C**.

Part B: We start the solution process by "guessing" the outer surface temperature to be 26°C. We will check the accuracy of this guess later and, if necessary, repeat the calculations with a better guess based on the obtained results. The film temperature is T_f = (26+20)/2 = 23°C, and the thermal expansion coefficient is β = 1/(23+273) = 0.00338 K^{-1}. We proceed to compute the Rayleigh number,

$$Ra = \frac{g\beta\left(T_s - T_a\right)L^3}{\nu^2}\text{Pr}$$

in which T_s = 26°C is the updated surface temperature. Substituting these and other pertaining variables, we obtain

$$Ra = \frac{9.81 \times 0.00338 \times (26-20) \times 0.5^3}{\left(1.54 \times 10^{-5}\right)^2} \times 0.730 = 7.65 \times 10^7$$

The Nusselt number can be obtained with the formula for flow over a vertical plate,

$$Nu = \left\{0.825 + \frac{0.387 Ra^{1/6}}{\left[1 + \left(\dfrac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\therefore Nu = \left\{0.825 + \frac{0.387\left(7.65 \times 10^7\right)^{1/6}}{\left[1 + \left(\dfrac{0.492}{0.730}\right)^{9/16}\right]^{8/27}}\right\}^2 = 56.6$$

The heat transfer coefficient can be obtained with the usual relation

$$Nu = \frac{h \times L_c}{k} \rightarrow h = \frac{Nu \times k}{L_c}$$

$$\therefore h = \frac{56.6 \times 0.0254}{0.5} = 2.88 \text{ W/m}^2\text{K}$$

The surface area of the plate (or, rather, the cans) is now

$$A_s = 2\left(0.5\times1.10+0.5\times3.60\right) = 4.7\ \text{m}^2$$

The total rate of heat loss from the outer surface of the insulated tank by convection and radiation becomes

$$\dot{q} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}}$$

$$\therefore \dot{q} = hA_s\left(T_s - T_a\right) + \varepsilon A_s\sigma\left(T_s^4 - T_{\text{surr}}^4\right)$$

where, in the second term on the right-hand side, we have the updated emissivity ε = 0.1 of the aluminum foil, and the updated surface temperature T_s = 26°C. Substituting the pertaining variables, the total heat loss from the outer surface of the insulated tank, considering both convection and radiation, is found as

$$\dot{q} = 2.88\times4.7\times\left(26-20\right)+0.1\times4.7\times\left(5.67\times10^{-8}\right)\times\left[\left(26+273\right)^4-\left(20+273\right)^4\right] = 97.8\ \text{W}$$

In steady operation, the heat lost by the side surfaces of the tank must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which in turn must be equal to the heat conducted through the insulation. The second condition requires the surface temperature to be

$$\dot{q} = \dot{q}_{\text{insul}} = k_{\text{ins}}A_s\frac{\left(T_{\text{tank}} - T_s\right)}{t} = 97.8\ \text{W}$$

$$\therefore 0.035\times4.7\times\frac{\left(55-T_s\right)}{0.05} = 97.8$$

$$\therefore 3.29\left(55-T_s\right) = 97.8$$

$$\therefore 55 - T_s = \frac{97.8}{3.29} = 29.7$$

$$\therefore T_s = 55 - 29.7 = 25.3$$

$$\therefore T_s = 25.3^\circ\text{C}$$

This result is close to the assumed surface temperature of 26°C. Accordingly, there is no need to repeat the calculations. The total amount Q of heat loss in one year is

$$Q = \dot{q}\Delta t = 97.8\ \text{W}\times\underbrace{8760\ \text{h}}_{=1\,\text{year}} = 856{,}728 = 856.7\ \text{kWh}$$

and the corresponding cost C is

$$C = 856.7\ \cancel{\text{kWh}}\times\frac{\$0.085}{\cancel{\text{kWh}}} = \$72.8$$

The money saved during a one-year period due to insulation becomes

$$\text{Money saved } = \text{Cost}_{\substack{\text{without}\\\text{insulation}}} - \text{Cost}_{\substack{\text{with}\\\text{insulation}}} = \$1116 - \$72.8 = \$1043.2$$

where \$1116 was obtained in Part A. The insulation will pay for itself in a time such that

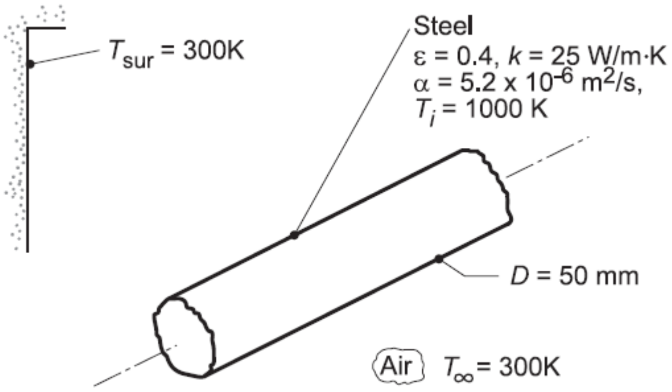
$$\text{Payback period} = \frac{C}{\text{Money saved}} = \frac{\$350}{\left(\dfrac{\$1043.2}{\text{year}}\right)} = 0.336\ \text{year}$$

$$\therefore \text{Payback period} = 123\ \text{days}$$

We would undoubtedly recommend use of insulation in this case.

P.11 Solution

Part A: A schematic of the problem is shown below.



The film temperature is $T_f = (T_i + T_\infty)/2 = (1000+300)/2 = 650$ K, and consequently $\beta = 1/T_f = 1/650 = 0.00154$ K^{-1}. The Rayleigh number is

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\alpha\nu} = \frac{9.81\times 0.00154\times(1000-300)\times 0.05^3}{(8.73\times 10^{-5})\times(6.02\times 10^{-5})} = 2.51\times 10^5$$

For free convection on a horizontal cylinder, the Nusselt number can be obtained with the correlation

$$Nu = \left\{0.60 + \frac{0.387 Ra^{1/6}}{\left[1+\left(\frac{0.559}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\therefore Nu = \left\{0.60 + \frac{0.387\times(2.51\times 10^5)^{1/6}}{\left[1+\left(\frac{0.559}{0.690}\right)^{9/16}\right]^{8/27}}\right\}^2 = 9.9$$

The convection coefficient easily follows,

$$Nu = \frac{h\times D}{k} \to h = \frac{Nu\times k}{D}$$

$$\therefore h = \frac{9.9\times 0.0497}{0.05} = \boxed{9.84\ \mathrm{W/m^2K}}$$

♦ The correct answer is **D**.

Part B: The radiation heat transfer coefficient can be approximated as

$$h_{rad} = \varepsilon\sigma(T_s + T_{surr})(T_s^2 + T_{surr}^2)$$

$$\therefore h_{rad} = 0.40\times(5.67\times 10^{-8})\times(1000+300)(1000^2+300^2) = 32.14\ \mathrm{W/m^2K}$$

We proceed to compute the Biot number,

$$Bi = \frac{(h+h_{rad})\frac{D}{4}}{k} = \frac{(9.84+32.14)\times\frac{0.05}{4}}{25} = 0.021$$

Hence, the lumped capacitance method can be applied. The equation to use is

$$\frac{T-T_\infty}{T_i - T_\infty} = \exp(-0.021\times \mathrm{Fo})$$

where T = 900 K is the temperature that the centerline of the rod must exceed, T_i = 1000 K, and T_∞ = 300 K. Substituting, we obtain

$$\frac{T-T_\infty}{T_i-T_\infty}=\frac{900-300}{1000-300}=0.857=\exp(-0.021\times\mathrm{Fo})$$

$$\therefore \mathrm{Fo}=7.34$$

To determine the conveyor time T, we resort to the equation for the Fourier number *Fo*,

$$Fo=\frac{\alpha\times T}{\left(\frac{r_o}{2}\right)^2}=7.34$$

in which α = 5.2×10^{-6} m^2/s for the steel and r_o = *D/2* = 0.025 m, so that

$$\frac{\left(5.2\times10^{-6}\right)\times T}{\left(\frac{0.025}{2}\right)^2}=7.34$$

$$\therefore 0.0333T=7.34$$

$$\therefore T=\frac{7.34}{0.0333}=\boxed{220\text{ s}}$$

The maximum allowable conveyor transit time is 3 minutes and 40 seconds.

♦ The correct answer is **C**.

P.12 Solution

Part A: The convection heat flux across the gap follows from Newton's law of cooling,

$$q''_{\mathrm{conv}}=h\left(T_h-T_c\right)$$

where the convection coefficient, h, can be established by treating the gap as a rectangular cavity. In this case, the Nusselt number can be determined with the Globe-Dropkin correlation,

$$Nu=0.069Ra^{1/3}\,\mathrm{Pr}^{0.074}$$

in which *Ra* is the Rayleigh number,

$$Ra=\frac{g\beta\left(T_h-T_c\right)L^3}{\nu\alpha}$$

$$\therefore Ra=\frac{9.81\times0.00251\times(200-50)\times0.05^3}{\left(2.62\times10^{-5}\right)\times\left(3.80\times10^{-5}\right)}=4.64\times10^5$$

Backsubstituting this quantity and *Pr* = 0.690 in the foregoing correlation, we obtain

$$Nu=0.069\times\left(4.64\times10^5\right)^{1/3}\times0.690^{0.074}=5.2$$

The heat transfer coefficient follows as

$$Nu=\frac{h\times L}{k}\rightarrow h=\frac{Nu\times k}{L}$$

$$\therefore h=\frac{5.2\times0.0337}{0.05}=3.50\ \mathrm{W/m^2K}$$

We can then determine the heat flux,

$$q''_{\text{conv}} = 3.50 \times (200 - 50) = \boxed{525\ \text{W/m}^2}$$

The convective heat flux across the gap without insulation is about half a kilowatt per square meter.

♦ The correct answer is **B.**

Part B: For free convection to become negligible, we must have Ra < 1708. The number of gaps is $N_g = N + 1$. The gap width is $L_g = L/(N + 1)$ and, as a first estimate, the temperature difference across each gap is $\Delta T_g = (T_h - T_g)/(N + 1)$. We can therefore pose the inequality

$$Ra < 1708 \rightarrow \frac{g\beta\left[(T_h - T_c)/(N+1)\right]\left[\dfrac{L}{(N+1)}\right]^3}{\nu\alpha} < 1708$$

$$\therefore \frac{9.81 \times 0.00251 \times \left[(200-50)/(N+1)\right] \times \left[\dfrac{0.05}{(N+1)}\right]^3}{\left(2.62 \times 10^{-5}\right) \times \left(3.80 \times 10^{-5}\right)} < 1708$$

$$\therefore \frac{463{,}251}{(N+1)^4} < 1708$$

$$\therefore N > 3.06$$

Therefore, we specify N = 4. The minimum number of foil sheets to be inserted in the gap to eliminate free convection is 4.

♦ The correct answer is **C**.

Answer Summary

Problem 1		**D**
Problem 2	2A	Open-ended pb.
	2B	**B**
Problem 3		**C**
Problem 4	4A	Open-ended pb.
	4B	**B**
	4C	**B**
Problem 5	5A	**A**
	5B	**B**
Problem 6	6A	**B**
	6B	**A**
Problem 7	7A	**D**
	7B	**A**
Problem 8	8A	**C**
	8B	**C**
Problem 9	9A	**D**
	9B	Open-ended pb.
Problem 10	10A	**C**
	10B	Open-ended pb.
Problem 11	11A	**D**
	11B	**C**
Problem 12	12A	**B**
	12B	**C**

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- ÇENGEL, Y. and GHAJAR, A. (2015). Heat and *Mass Transfer: Fundamentals and Applications.* 5th edition. New York: McGraw-Hill.

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